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WELFARE ANALYSIS AND POLICY IMPLICATIONS IN MELITZ-TYPE MODEL WHERE MARKUP DIFFERS ACROSS INDUSTRIES

Kazuyoshi Ohki

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The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

Welfare analysis and policy implications in Melitz-type model where markup differs across industries *

Kazuyoshi Ohki†

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Abstract

We construct a monopolistic competition model considering different markups across industries and firm-level heterogeneity of productivity. An excess entry occurs in low-markup (competitive) industry, and vice versa in high-markup (non-competitive) industry. To achieve the optimum allocation, a social planner should implement an appropriate mix of policies, whose requirement is tighter than the homogeneous-firm model under some situations. The total amount of optimum subsidy (tax) is dependent on the property of distribution when the elasticity of substitution between industries is above unity.

keyword: welfare, optimum policy, firm-level heterogeneity of productivity, non-synchronization of markups, Melitz-type model

JEL classification:D61, H21, L11

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†Graduate School of Economics, Osaka University, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan. emai:kazuyoshi.ohki@gmail.com

1 Introduction

Empirical studies substantiate noteworthy differences in both industry-level and firm-level. Aghion et al (2005) and Epifani and Gancia (2011) demonstrate that the monopolistic power (price-cost margins), hence the degree of market competition, varies widely across industries. Foster, Haltiwanger and Krizan (1998) shows that firm-level productivity largely differs even in the same industry. In the theoretical context, Melitz and Redding (2015) emphasizes afresh the effect of endogenous firm selection, which comes from firm-level heterogeneity, for welfare analysis.

Thus in this paper, we construct a monopolistic competition model considering different markups across industries and firm-level heterogeneity of productivity intra-industry. Then we analyze welfare and policy implications. Our model shows that an excess entry occurs in low-markup (competitive) industry, and vice versa in high-markup (non-competitive) industry, which is consistent with the result of the homogeneous-firm model as Dixit and Stiglitz (1977) and Epifani and Gancia (2011). To achieve the optimum allocation, a social planner should implement an appropriate mix of policies, whose requirement is tighter than the homogeneous-firm model under some conditions. This comes from an endogenous selection of firms, which is the key property of Melitz-type model. The intuition is as follows: if there are excess firms in low-markup industry, a social planner imposes some tax in this industry to reduce the number of firms. However this tax relaxes the competition, which can lower the productivity cutoff when it is determined endogenously. In this case, low-productivity firms, which should not participate in production from the point of social optimum, begin to produce. To make them exit, a social planner should impose another instrument.

Our paper is related to two standard literatures. The one is literatures studying welfare and policy implications using a model where the markup differs across industries. Dixit and Stiglitz (1977) constructs a model with a monopolistic competitive industry and a perfect competitive industry, and shows that excess resources are allocated in the perfect competitive industry. Bilbiie, Ghironi and Melitz (2006) considers a model with endogenous labor supply, and shows that a misallocation is caused by the non-synchronization of markups on consumption and leisure. Epifani and Gancia (2011) analyzes a monopolistic competition model considering different markups across industries. However these models do not consider firm-level heterogeneity. We consider not only industry-level heterogeneity but also firm-level heterogeneity.

The other is literatures studying welfare and policy implications using Melitz-type model, which is firstly presented in Melitz (2003). Although Melitz and Redding (2015) shows that a social optimal allocation coincides with a market equilibrium and that a policy intervention is not needed in basic Melitz model, deviating from the assumption of basic Melitz model causes a resource misallocation. We divide the causes of misallocation into three types. The first type of the cause is markup heterogeneity across goods. If markups of some goods are lower than others, excess resources concentrate on low-markup goods. Demidova, Rodriguez-Clare (2009) and Jung (2012) consider Melitz-type model in a small open economy. They show that the resource misallocation is caused by the non-synchronization of markups on domestic goods and imported goods, and that governments of small country have incentive to implement some policies to improve their welfare. The second type of the cause is static externalities. Even in basic Melitz model, there are some static externalities. However in this case, "consumer surplus effect" and "profit destruction effect" are identical because of the property of CES utility form and monopolistic competition. Then deviating from these assumptions causes a misallocation. Jung (2015) shows the misallocation and the necessity of policy intervention by studying Melitz-type model with CES-Benassy ustility. Dhingra and Morrow (2014) examines Melitz-type model with VES (variable elasticity of substitution) utility where an elasticity of utility and an elasticity of marginal utility are not constant, and shows that the market equilibrium is not optimum. The third type of the cause is dynamic externalities. If, for example, there is some positive (negative) spillover in R&D activity, excess resources are allocated to production (R&D) sector in the BGP equilibrium. Baldwin and Robert-Nicoud (2008) and Unel (2010) construct dynamic Melitz-type model, and studies welfare and policy implications. Besides, a policy maker has an incentive to implement some instrument for the purpose of strategy. Felbermayr, Jung, and Larch (2013) and Pfluger and Suedekum (2013) construct Melitz-type model in an open economy model consisting two large countries, and show that non-cooperative governments of large country have an incentive to encourage domestic firms.

Our purpose to analyze in this paper is categorized just in the first type. We construct a model with two monopolistic competitive industries which differ in monopolistic power (the degree of competition). To concentrate our attention on the "heterogeneous markup distortion," we assume CES utility form and monopolistic competition, and we focus our analysis on statics. In the Appendix C, we consider Melitz-type model with endogenous labor supply, and show that this setting is the special case in our model.

Related literatures discussed above use various instruments to improve welfare.[1] It seems that these instruments have different effects on endogenous variables in their model. Thus, we consider various instruments in our model. Some instruments, consumption subsidy (tax) and sales tax, affect only the number of entry. Other instruments, production subsidy (tax) and operating profit tax, affect two endogenous variables: the number of entry and the quantity of production. The other instruments, entry fixed cost subsidy (tax), operating fixed cost subsidy (tax) and ordinary profit tax, affect three endogenous variables: the number of entry, the quantity of production and the productivity cutoff. And, we find some ways to achieve the first best equilibrium by mixing these instruments appropriately. These optimum policies depend on whether lump-sum transfer (tax) is available, and whether the productivity cutoff determines endogenously.

The rest of this paper is structured as follows. Section 2 describes the model. Section 3 derives the market equilibrium, and Section 4 describes the social optimum allocation. We analyze policy implication in Section 5, and discuss the differences from homogeneous-firm model in Section 6. Section 7 provides concluding comments.

2 Model

In this paper, we focus on one-shot and closed economy. We normalize the wage rate as the numeraire. The number of representative households is exogenously given normalize as unity. They supply one unit of labor inelastically. The utility of them is given by:

$$U = \left[\gamma_1 \left[\int_0^{n_1} \left(x_1\left(j\right)\right)^{\frac{\sigma_1 - 1}{\sigma_1}} dj \right]^{\frac{\sigma_1}{\sigma_1 - 1}\frac{\rho - 1}{\rho}} + \gamma_2 \left[\int_0^{n_2} \left(x_2\left(j\right)\right)^{\frac{\sigma_2 - 1}{\sigma_2}} dj \right]^{\frac{\sigma_2}{\sigma_2 - 1}\frac{\rho - 1}{\rho}} \right]^{\frac{\rho}{\rho - 1}}, \quad (1)$$

where $x_i\left(j\right)$ is the consumption of differentiated goods of firm j in industry i, n_i is the number of producing firms in industry i, $\rho \geq 1$ is the elasticity of substitution between industries, and γ_i is a

[1] Demidova and Rodriguez-Clare (2009) considers consumption subsidy, export subsidy, and import tariff. Jung (2012) considers entry fixed cost subsidy and operating fixed cost subsidy. Felbermayr, Jung and Larch (2013) consider imported tariff. Pfluger and Suedekum (2013) consider entry fixed cost subsidy and operating profit tax. Jung (2015) consider operating fixed cost subsidy (tax).

parameter.[2,3] And $\sigma_i \geq \rho$ is the elasticity of substitution in industry i, which determines markup.[4]

Representative households maximizes (1) under following budget constraint $\int_0^{n_1} s_1^c p_1(j)\, x_1(j)\, dj + \int_0^{n_2} s_2^c p_2(j)\, x_2(j)\, dj = E$, where $p_i(j)$ is the price of firm j in industry i, s_i^c is the consumption subsidy (tax) in industry i when $s < 1$ $(s > 1)$. E is the aggregate expenditure, which satisfies $E = 1 + T$, where T is the lump-sum transfer (tax) when $T > 0$ $(T < 0)$. Then the aggregate demand function is derived as

$$x_i(j) = \frac{1}{s_i^c p_i(j)} \left(\frac{P_i}{s_i^c p_i(j)} \right)^{\sigma_i - 1} \left(\frac{P}{P_i} \right)^{(\rho-1)} (\gamma_i)^\rho E, \tag{2}$$

where $P_i \equiv \left[M_i^e \int_0^{n_i} (s_i^c p_i(j))^{-(\sigma_i - 1)} dj \right]^{\frac{-1}{\sigma_i - 1}}$ is the price index in industry i, and $P \equiv \left[(\gamma_1)^\rho (P_1)^{-(\rho-1)} + (\gamma_2)^\rho (P_2)^{-(\rho-1)} \right]$ is the price index.[5]

We consider three policy variables related to cost: entry s_i^e, operating s_i^d, and production s_i^p. When $s < 1$ $(s > 1)$, the policy is subsidy (tax). Each differentiated goods are produced by producing firms, which has heterogeneous producing technology. Initially, they must hire f_i^e unit of labor (they pay $s_i^e f_i^e$), and draw a productivity from a distribution $G_i(\phi)$. Producing firms in industry i having productivity ϕ must hire $\frac{1}{\phi} x_i(\phi) + f_i^d$ unit of labor (they pay $\frac{1}{\phi} s_i^p x_i(\phi) + s_i^d f_i^d$) to produce $x_i(\phi)$ unit of goods, where f_i^d is the operating (overhead production) fixed cost in the domestic market.

Then the ordinary profit is expressed as $\pi_i(\phi) = \frac{1}{s_i^{op}} \left(\frac{1}{s_i^s} p_i(\phi) - \frac{s_i^p}{\phi} \right) x_i(\phi) - s_i^d f_i^d$, where $s_i^s \geq 1$ is the sales tax and $s_i^{op} \geq 1$ is the operating profit tax. They choose monopolistic price to maximize their ordinary profit as follows:[6]

$$p_i(\phi) = s_i^p s_i^s \frac{\sigma_i}{\sigma_i - 1} \frac{1}{\phi}, \tag{3}$$

where $s_i^p s_i^s \frac{\sigma_i}{\sigma_i - 1}$ is the markup from the firm's point of view in industry i, which rises with s_i^p and s_i^s. In our setting, the markup is not same across industries, which causes a misallocation.

[2]When $\rho = 1$, a relationship between industries is Cobb-Douglas form, and γ_i expresses the market share of industry i. When $\rho > 1$, a relationship between industries is CES form, and the market share of industry i is expressed endogenously as $\left(\frac{P}{P_i} \right)^{(\rho-1)} (\gamma_i)^\rho$.

[3]There are some papers using such CES-CES utility form: Devereux and Leem (2001), Doi and Mino (2005), Kucheryavyy, (2012), Lewis and Winkler (2015).

[4]We examine the situation where the markup differs across industries, the productivity differs across firms, the relationship between industries is CES form and the utility of each industries is also CES form.

[5]Note that P_i is the minimum expenditure to satisfy $\left[\int_0^{n_i} (x_i(j))^{\frac{\sigma_i - 1}{\sigma_i}} dj \right]^{\frac{\sigma_i}{\sigma_i - 1}} = 1$, and P is the minimum expenditure to satisfy $U = 1$.

[6]We assume that firms do not consider the influence on the price index (we assume a monopolistic competition).

Because of the existence of the operating fixed cost, producing firms in industry i having low productivity such as $\phi < \phi_i^d$ choose not to operate, where productivity cutoff, ϕ_i^d, is derived from following zero-cutoff condition:

$$\frac{1}{s_i^{or}} \pi_i \left(\phi_i^d\right) = 0, \tag{4}$$

where $s_i^{or} \geq 1$ is the ordinary profit tax.

We assume free entry, then the expected after-tax ordinary profit equals the entry fixed cost as follows:

$$\int_{\phi_i^d}^{\phi_i^{\max}} \frac{1}{s_i^{or}} \pi_i (\phi) \, dG_i (\phi) = s_i^e f_i^e. \tag{5}$$

Labor is demanded by entry, operating and production. Labor supply is exogenously given as unity. The labor market clearing condition is:

$$\sum_{i=1,2} \left\{ M_i^e f_i^e + \left[1 - G_i \left(\phi_i^d\right)\right] M_i^e f_i^d + M_i^e \int_{\phi_i^d}^{\phi_i^{\max}} \frac{1}{\phi} x_i (\phi) \, dG_i (\phi) \right\} = 1, \tag{6}$$

where M_i^e is the number of entry in industry i, and $n_i = \left[1 - G_i \left(\phi_i^d\right)\right] M_i^e$ is satisfied.

3 Market equilibrium

In this section, we derive the market equilibrium (ME). From (2), (3) and (4), the ordinary profit of firms in industry i having productivity ϕ is expressed as $\pi_i (\phi) = s_i^d f_i^d \left[\frac{(\phi)^{\sigma_i - 1}}{\left(\phi_i^d\right)^{\sigma_i - 1}} - 1 \right]$. Substituting this into (5) yields

$$\left[1 - G_i \left(\phi_i^{d,ME}\right)\right] \left\{ \frac{\int_{\phi_i^{d,ME}}^{\phi_i^{\max}} \left(\frac{\phi}{\phi_i^{d,ME}} \right)^{\sigma_i - 1} dG_i (\phi)}{\left[1 - G_i \left(\phi_i^{d,ME}\right)\right]} - 1 \right\} = \frac{s_i^e s_i^{or} f_i^e}{s_i^d f_i^d}. \tag{7}$$

This equation determines the market equilibrium value of productivity cutoff, ϕ_i^d.[7] The productivity cutoff rises with s_i^d, whereas falls with s_i^e and s_i^{or}. The same degree of change in s_i^d and s_i^e (s_i^d and s_i^{or}) does not affect ϕ_i^d.

[7] In Appendix A, we derive the explicit solution by imposing specific assumption.

Substituting (3), (2) is expressed as $x_i(\phi) = \frac{(P_i)^{-(\rho-1)} P^{(\rho-1)} (\gamma_i)^\rho E}{\sigma_i M_i^e} \frac{1}{s_i^c s_i^p s_i^s} \frac{(\phi)^{\sigma_i}}{\int_{\phi_i^d}^{\phi_i^{\max}} (\phi)^{\sigma_i - 1} dG_i(\phi)} (\sigma_i - 1)$, and the ordinary profit is expressed as $\pi_i(\phi) = \frac{(P_i)^{-(\rho-1)} P^{\rho-1} (\gamma_i)^\rho E}{\sigma_i M_i^e} \frac{1}{s_i^c s_i^{op} s_i^s} \frac{(\phi)^{\sigma_i - 1}}{\int_{\phi_i^d}^{\phi_i^{\max}} (\phi)^{\sigma_i - 1} dG_i(\phi)} - s_i^d f_i^d$. Substituting this expression of ordinary profit into (5) yields

$$\frac{(P_i)^{-(\rho-1)} P^{\rho-1} (\gamma_i)^\rho E}{\sigma_i M_i^e} = s_i^c s_i^{op} s_i^s \left\{ s_i^d f_i^d \left[1 - G_i\left(\phi_i^d\right)\right] + s_i^e s_i^{or} f_i^e \right\}. \tag{8}$$

Substituting (8) into the demand function, the production of firms in industry i having productivity ϕ is expressed as follows:

$$x_i^{ME}(\phi) = \left\{ s_i^d f_i^d \left[1 - G_i\left(\phi_i^d\right)\right] + s_i^e s_i^{or} f_i^e \right\} \frac{s_i^{op}}{s_i^p} \frac{(\phi)^{\sigma_i}}{\int_{\phi_i^{d,ME}}^{\phi_i^{\max}} (\phi)^{\sigma_i - 1} dG_i(\phi)} (\sigma_i - 1). \tag{9}$$

The production of firms in industry i increases with s_i^d, s_i^e, s_i^{or}, s_i^{op}, whereas decreases with s_i^p, and is not affected by s_i^c and s_i^s.[8]

Dividing (8) of industry 1 by that of indusry 2, we obtain following relationship of the number of entry:

$$\frac{\left(M_1^{e,ME}\right)^{\frac{\sigma_1 - \rho}{\sigma_1 - 1}}}{\left(M_2^{e,ME}\right)^{\frac{\sigma_2 - \rho}{\sigma_2 - 1}}} = \left(\frac{\sigma_1 - 1}{\sigma_1} \frac{\sigma_2}{\sigma_2 - 1} \frac{s_2^c s_2^p s_2^s}{s_1^c s_1^p s_1^s} \right)^\rho \frac{\left\{ \left[1 - G_2\left(\phi_2^{d,ME}\right)\right] s_2^d f_2^d + s_2^e s_2^{or} f_2^e \right\} \frac{s_2^{op}}{s_2^p}}{\left\{ \left[1 - G_1\left(\phi_1^{d,ME}\right)\right] s_1^d f_1^d + s_1^e s_1^{or} f_1^e \right\} \frac{s_1^{op}}{s_1^p}} \left(\frac{\sigma_2 - 1}{\sigma_1 - 1} \right) \left(\frac{\gamma_1}{\gamma_2} \right)^\rho \frac{\left[\int_{\phi_1^{d,ME}}^{\phi_1^{\max}} \phi^{(\sigma_1 - 1)} dG_1(\phi) \right]^{\frac{\rho - 1}{\sigma_1 - 1}}}{\left[\int_{\phi_2^{d,ME}}^{\phi_2^{\max}} \phi^{(\sigma_2 - 1)} dG_2(\phi) \right]^{\frac{\rho - 1}{\sigma_2 - 1}}}. \tag{10}$$

The relative numbers of entry in industry i decreases with s_i^c, s_i^d, s_i^e, s_i^s, s_i^{or}, s_i^{op} and s_i^p. (6) and (10) determine the market equilibrium value of the number of entry, M_i^e.[9]

[8]Various instruments affect $x_i(\phi)$ through two channels. The one is through the market share of firm j in industry i, $\left(\frac{P_i}{s_i^c p_i(\phi)}\right)^{\sigma_i - 1} \left(\frac{P}{P_i}\right)^{(\rho-1)} (\gamma_i)^\rho E = \left\{ s_i^d f_i^d \left[1 - G_i\left(\phi_i^d\right)\right] + s_i^e s_i^{or} f_i^e \right\} s_i^c s_i^s s_i^{op} \sigma_i \frac{(\phi)^{\sigma_i - 1}}{\int_{\phi_i^{d,ME}}^{\phi_i^{\max}} (\phi)^{\sigma_i - 1} dG(\phi)}$. Subsiding (taxing) s_i^c, s_i^d, s_i^e, s_i^{or}, s_i^{op} and s_i^s induces (impedes) competition, and decreases (increases) the market share of firm j in industry i, which decreases (increases) $x_i(\phi)$. The other is thorough the price from the household's point of view, $s_i^c p_i(\phi) = s_i^c s_i^p s_i^s \frac{\sigma_i}{\sigma_i - 1} \frac{1}{\phi}$. Subsiding (taxing) s_i^c, s_i^p and s_i^s drops (raises) the price from the household's point of view, which increases (decreases) $x_i(\phi)$. A change in s_i^c or s_i^s have above two opposite effects, and completely offset each other.

[9]When $\rho = 1$, the market share in each industry is exogenously given, and the ratio of the numbers of entry is expressed as $\frac{M_1^e}{M_2^e} = \frac{\frac{\sigma_2}{\gamma_2} \left\{ \left[1 - G_2\left(\phi_2^{d,ME}\right)\right] s_2^d f_2^d + s_2^e s_2^{or} f_2^e \right\} s_2^c s_2^{op} s_2^s}{\frac{\sigma_1}{\gamma_1} \left\{ \left[1 - G_1\left(\phi_1^{d,ME}\right)\right] s_1^d f_1^d + s_1^e s_1^{or} f_1^e \right\} s_1^c s_1^{op} s_1^s}$. Subsiding (taxing) s_i^c, s_i^{or}, s_i^{op} and s_i^s increases (decreases) the expected operating profit in industry i, which induces (impedes) entry, and increases (decreases) the relative numbers of entry in industry i. Subsiding (taxing) s_i^d and s_i^e decreases (increases) the expected fixed cost, which induces (impedes) entry, and increases (decreases) the relative numbers of entry in industry i. When $\rho > 1$, the market share in industry i is now endogenous. Subsiding (taxing) s_i^c, s_i^s and s_i^p drops (raises) the price index in industry i, which increases (decreases) the market share in industry i and increases (decreases) the expected operating profit in industry i. Then this increases (decreases) the relative numbers of entry in industry i.

4 Social optimum allocation

In this section, we derive the social optimal allocation (SO). A social planner allocates resources to maximize (1) such as:[10]

$$\begin{array}{ll}\max\limits_{x_i(\phi),\phi_i^d,M_i^e} & U=\left[\gamma_1\left[\int\limits_{\phi_1^d}^{\phi_1^{\max}}(x_1(\phi))^{\frac{\sigma_1-1}{\sigma_1}}dG_1(\phi)\right]^{\frac{\sigma_1}{\sigma_1-1}\frac{\rho-1}{\rho}}+\gamma_2\left[\int\limits_{\phi_2^d}^{\phi_2^{\max}}(x_2(\phi))^{\frac{\sigma_2-1}{\sigma_2}}dG_2(\phi)\right]^{\frac{\sigma_2}{\sigma_2-1}\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \\ s.t. & \sum\limits_{i=1,2}\left\{M_i^e f_i^e+\left[1-G_i\left(\phi_i^d\right)\right]M_i^e f_i^d+M_i^e\int\limits_{\phi_i^d}^{\infty}\frac{1}{\phi}x_i(\phi)\,dG_i(\phi)\right\}\leq 1\end{array}. \tag{11}$$

Solving this problem yields

$$\left[1-G_i\left(\phi_i^{d,SO}\right)\right]\left\{\frac{\int\limits_{\phi_i^{d,SO}}^{\phi_i^{\max}}\left(\frac{\phi}{\phi_i^{d,SO}}\right)^{\sigma_i-1}dG_i(\phi)}{\left[1-G_i\left(\phi_i^{d,SO}\right)\right]}-1\right\}=\frac{f_i^e}{f_i^d}, \tag{12}$$

$$x_i^{SO}(\phi)=\left\{\left[1-G\left(\phi_i^{d,SO}\right)\right]f_i^d+f_i^e\right\}\frac{(\phi)^{\sigma_i}}{\int\limits_{\phi_i^{d,SO}}^{\phi_i^{\max}}(\phi)^{\sigma_i-1}dG_i(\phi)}(\sigma_i-1), \tag{13}$$

$$\frac{\left(M_1^{e,SO}\right)^{\frac{\sigma_1-\rho}{\sigma_1-1}}}{\left(M_2^{e,SO}\right)^{\frac{\sigma_2-\rho}{\sigma_2-1}}}=\frac{\left[1-G\left(\phi_2^{d,SO}\right)\right]f_2^d+f_2^e}{\left[1-G\left(\phi_1^{d,SO}\right)\right]f_1^d+f_1^e}\left(\frac{\sigma_2-1}{\sigma_1-1}\right)\left(\frac{\gamma_1}{\gamma_2}\right)^{\rho}\frac{\left[\int\limits_{\phi_1^{d,SO}}^{\phi_i^{\max}}(\phi)^{\sigma_1-1}dG_1(\phi)\right]^{\frac{\rho-1}{\sigma_1-1}}}{\left[\int\limits_{\phi_2^{d,SO}}^{\phi_2^{\max}}(\phi)^{\sigma_2-1}dG_2(\phi)\right]^{\frac{\rho-1}{\sigma_2-1}}}. \tag{14}$$

5 Policy implication

5.1 Comparison between the market equilibrium and the social optimum allocation

To begin with, we make a comparison between the market equilibrium and the social optimum allocation under laissez-faire economy ($s_i^c=s_i^d=s_i^e=s_i^{op}=s_i^{or}=s_i^p=s_i^s=1$). From (7) and (12), $\phi_i^{d,ME}=$

[10]In the appendix B, we derive this maximization problem in detail.

$\phi_i^{d,SO}, i = 1,2$ is satisfied. Moreover, from (9) and (13), we obtain $x_i^{ME}(\phi) = x_i^{SO}(\phi), \forall\phi, i = 1,2$. However, from (10) and (14), $\frac{M_1^{e,ME}}{M_2^{e,ME}} > \frac{M_1^{e,SO}}{M_2^{e,SO}}$ is satisfied when $\sigma_1 > \sigma_2$. From $n_i = \left[1 - G_i\left(\phi_i^d\right)\right] M_i^e$, $\frac{n_1^{ME}}{n_2^{ME}} > \frac{n_1^{SO}}{n_2^{SO}}$ is also satisfied when $\sigma_1 > \sigma_2$. Then we get the folowing proposition:

Proposition 5.1. *When the markup differs across industries and the productivity differs between firms, the quantity of firms' production and the productivity cutoff are optimum. However, the number of entry and the product variety are excess in low-markup (competitive) industry, and vice versa in high-markup (non-competitive) industry.*

This proposition suggests that intra-industry allocation is optimum, whereas inter-industry allocation is suboptimum. The former result is reflected our assumption; CES utility form, and monopolistic competition.[11] The latter result is caused by markup heterogeneity, which is consistent with "the Lerner-Samuelson intuition" saying that synchronization of markup is needed for efficiency allocation. [12]

5.2 Optimum policy with lump-sum transfer (tax)

Next, we want to find the optimum policy to achieve the first best equilibrium. First of all, we consider the situation where the lump-sum transfer (tax) is available. From (7) and (12),

$$\frac{s_i^e s_i^{or}}{s_i^d} = 1 \tag{15}$$

must be satisfied to achieve $\phi_i^{d,ME} = \phi_i^{d,SO}$. Since the productivity cutoff is optimum in the market equilibrium, a social planner should choose instruments so as not to affect ϕ_i^d. Then the same degree of change in s_i^d and s_i^e or (s_i^d and s_i^{or}) are required if (s)he adopts the policy related to operating fixed cost, entry fixed cost and ordinary profit. The intuition is as follows: If, for example, a social planner subsidizes only entry fixed cost, the number of entry increases. This tightens competition, which raises productivity cutoff. Then not high-productivity firms, however which should participate in production from the point of social optimum, decide not to produce goods. Therefore, in order to make them operate, a social planner should subsidize operating fixed cost too.

[11]Deviating from the assumption of CES utility form or monopolistic competition distorts intra-industry allocation. Jung (2015) considers CES-Benassy utility, and Dhingra and Morrow (2014) considers VES utility with Melitz-type model. In their model, an intra-industry resource allocation is suboptimum because "consumer surplus effect" and "profit destruction effect" is not equal. Considering other settings analyzed in homogeneous-firm model such as translog utility (as in Feenstra (2003)) and oligopolistic competition in prices or quantity (as in Lewis and Winkler (2015)), one should find that these factors make intra-industry allocation suboptimum.

[12]See Bilbiie, Ghironi and Melitz (2006, p15) for details.

From (9) and (13),

$$\left\{\left[1-G_i\left(\phi_i^{d,ME}\right)\right]s_i^d f_i^d + s_i^e s_i^{or} f_i^e\right\}\frac{s_i^{op}}{s_i^p} = \left[1-G_i\left(\phi_i^{d,SO}\right)\right]f_i^d + f_i^e \tag{16}$$

must be satisfied to achieve $x_i^{ME}(\phi) = x_i^{SO}(\phi)$. Since the quantity of firms' production is also optimum in the market equilibrium, a social planner should choose instruments so as not to affect $x_i(\phi)$. Then the same degree of change in s_i^{op} and s_i^p, (s_i^d, s_i^e and s_i^p) or (s_i^d, s_i^{or} and s_i^p) are required if (s)he adopts the policy related to operating fixed cost, entry fixed cost, production cost, operating profit and ordinary profit. [13]

From (10), (14), (15) and (16),

$$\frac{\sigma_1-1}{\sigma_1}\frac{\sigma_2}{\sigma_2-1}\frac{s_2^c s_2^p s_2^s}{s_1^c s_1^p s_1^s} = 1 \tag{17}$$

must be satisfied to achieve $M_i^{e,ME} = M_i^{e,SO}$. This condition is equivalent to $s_1^c p_1(\phi) = s_2^c p_2(\phi)\ \forall\phi$, which says that synchronization of the price from the household's point of view is needed for efficiency allocation. [14]

(15), (16) and (17) are a necessary condition to achieve the optimum allocation. Since we consider various instruments, there are some options to achieve the first best equilibrium. Hereafter, we assume that industry 1 is more competitive than industry 2, $\sigma_1 > \sigma_2$, for convenience' sake.

The first instrument is using a consumption subsidy (tax). Since consumption subsidy (tax) does not affect the productivity cutoff and the quantity of firms' production, a social planner can achieve the first best equilibrium controlling only consumption subsidy (tax) to synchronize the price from the household's point of view by imposing the consumption subsidy (tax) in noncompetitive (competitive) industry as

$$\frac{s_1^c}{s_2^c} = \frac{\sigma_1-1}{\sigma_1}\frac{\sigma_2}{\sigma_2-1} > 1 \qquad \begin{array}{l} s_1^d = s_1^e = s_1^{op} = s_1^{or} = s_1^p = s_1^s = 1 \\ s_2^d = s_2^e = s_2^{op} = s_2^{or} = s_2^p = s_2^s = 1. \end{array} \tag{18}$$

As far as (18) is satisfied, we can also achieve the first best equilibrium to intervene only one industry (i.e. the first best equilibrium can be achieved by setting $s_1^c = \frac{\sigma_1-1}{\sigma_1}\frac{\sigma_2}{\sigma_2-1}$ and $s_2^c = 1$).[15]

[13] See footnote 8.

[14] It is worth emphasizing that the first best equilibrium can be achieved even when the markup from the firm's point of view, $s_i^p s_i^s \frac{\sigma_i}{\sigma_i-1}$, is not synchronized.

[15] One can easily generalize this point that a social planner must intervene in $K-1$ industry to achieve the first best

The second instrument is using a sales tax. Since the sales tax does not affect the productivity cutoff and the quantity of firms' production, we can achieve the first best equilibrium controlling only sales tax to synchronize the price from the household's point of view by imposing the sales tax in competitive industry as

$$s_1^s = \frac{\sigma_1 - 1}{\sigma_1}\frac{\sigma_2}{\sigma_2 - 1} > 1 \qquad \begin{array}{c} s_1^c = s_1^d = s_1^e = s_1^{op} = s_1^{or} = s_1^p = 1 \\ s_2^c = s_2^d = s_2^e = s_2^{op} = s_2^{or} = s_2^p = s_2^s = 1. \end{array} \tag{19}$$

The third instrument is using a production cost subsidy (tax). Since the production cost subsidy (tax) increases (decreases) the quantity of firms' production, a social planner must control not only the production cost subsidy (tax) to synchronize the price from the household's point of view, but also other instruments not to make the quantity of firms' production suboptimum. One way to control firms' production is using an operating profit tax, which increases the quantity of firms' production. To synchronize the price from the household's point of view, a social planner imposes production cost tax in competitive industry. However, this raises the price, which decreases the production. Then in order to keep the firms' production optimum, (s)he also imposes the operating profit tax as

$$s_1^p = s_1^{op} = \frac{\sigma_1 - 1}{\sigma_1}\frac{\sigma_2}{\sigma_2 - 1} > 1 \qquad \begin{array}{c} s_1^c = s_1^d = s_1^e = s_1^{or} = s_1^s = 1 \\ s_2^c = s_2^d = s_2^e = s_2^{op} = s_2^{or} = s_2^p = s_2^s = 1. \end{array} \tag{20}$$

Since the same degree of change in s_i^d and s_i^e (s_i^d and s_i^{or}) has equivalent effect to the change in s_i^{op}, we can achieve the first best equilibrium by controlling the operating fixed cost subsidy (tax), the entry fixed cost subsidy (tax) and the ordinary profit tax instead of the operating profit tax as

$$\frac{s_1^p}{s_2^p} = \frac{\sigma_1 - 1}{\sigma_1}\frac{\sigma_2}{\sigma_2 - 1} > 1 \qquad \begin{array}{cc} s_1^d = s_1^e = s_1^p & s_1^c = s_1^{op} = s_1^{or} = s_1^s = 1 \\ s_2^d = s_2^e = s_2^p & s_2^c = s_2^{op} = s_2^{or} = s_2^s = 1, \end{array} \tag{21}$$

$$s_1^p = s_1^d = s_1^{or} = \frac{\sigma_1 - 1}{\sigma_1}\frac{\sigma_2}{\sigma_2 - 1} > 1 \qquad \begin{array}{c} s_1^c = s_1^e = s_1^{op} = s_1^s = 1 \\ s_2^c = s_2^d = s_2^e = s_2^{op} = s_2^{or} = s_2^p = s_2^s = 1. \end{array} \tag{22}$$

equilibrium when the number of industry is K.

Moreover, there are other ways to obtain the first best equilibrium by mixing above policies appropriately. For example, a social planner can achieve the optimum allocation by imposing the sales tax in competitive industry, and imposing the consumption subsidy in noncompetitive industry as

$$\frac{s_1^s}{s_2^c} = \frac{\sigma_1 - 1}{\sigma_1}\frac{\sigma_2}{\sigma_2 - 1} > 0 \qquad \begin{array}{l} s_1^c = s_1^d = s_1^e = s_1^{op} = s_1^{or} = s_1^p = 1 \\ s_2^d = s_2^e = s_2^{op} = s_2^{or} = s_2^p = s_2^s = 1. \end{array} \tag{23}$$

Proposition 5.2. *When the markup differs across industries, the productivity differs between firms and the lump-sum transfer (tax) is available, the first best equilibrium can be achieved by implementing following instruments:*

- *Imposing the consumption subsidy (tax), s_i^c, in noncompetitive (competitive) industry.*
- *Imposing the sales tax, s_i^s, in competitive industry.*
- *Imposing the production cost tax, s_i^p, and the operating profit tax, s_i^{op}, in competitive industry.*
- *Imposing the production cost subsidy (tax), s_i^p, the operating fixed cost subsidy (tax), s_i^d, and the entry fixed cost subsidy (tax), s_i^e, in noncompetitive (competitive) industry.*
- *Imposing the production cost tax, s_i^p, the operating fixed cost tax, s_i^d, and the ordinary profit tax, s_i^{or}, in competitive industry.*
- *Mixing policies appropriately.*

5.3 Optimum policy without lump-sum transfer (tax)

In the previous subsection, we allow the lump-sum transfer (tax). However in practice, this assumption seems not to be feasible. Thus in this subsection, we find the optimum policy to achieve the first best equilibrium under the situation where the lump-sum transfer (tax) is not available. In this subsection, we focus on the case of $\rho = 1$ to obtain explicit results.

As we consider various instruments, the lump-sum transfer (tax) is expressed as

$$T=\sum_{i=1,2} M_i^e \left\{ \begin{array}{l} \left(s_i^c-1\right) \int_{\phi_i^d}^{\phi_i^{\max}} p_i\left(\phi\right) x_i\left(\phi\right) dG_i\left(\phi\right)+\left(s_i^d-1\right)\left[1-G_i\left(\phi_i^d\right)\right] f_i^d+\left(s_i^e-1\right) f_i^e \\ +\left(1-\frac{1}{s_i^{op}}\right) \int_{\phi_i^d}^{\phi_i^{\max}}\left(\pi_i\left(\phi\right)+s_i^d f_i^d\right) dG_i\left(\phi\right)+\left(1-\frac{1}{s_i^{or}}\right) \int_{\phi_i^d}^{\phi_i^{\max}} \pi_i\left(\phi\right) dG_i\left(\phi\right) \\ +\left(s_i^p-1\right) \int_{\phi_i^d}^{\phi_i^{\max}} \frac{1}{\phi} x_i\left(\phi\right) dG_i\left(\phi\right)+\left(1-\frac{1}{s_i^s}\right) \int_{\phi_i^d}^{\phi_i^{\max}} p_i\left(\phi\right) x_i\left(\phi\right) dG_i\left(\phi\right) \end{array} \right\}. \tag{24}$$

If the lump-sum transfer (tax) is not available,

$$T=0 \tag{25}$$

must be satisfied. Then four necessary conditions, (15), (16), (17) and (25), should be satisfied in order to achieve the optimum allocation.

Even when the lump-sum transfer (tax) is not available, there are some options to achieve the first best equilibrium.[16] The first instrument is imposing the consumption tax in competitive industry, and imposing the consumption subsidy in noncompetitive industry satisfying

$$\begin{array}{ll} s_1^c=\frac{\sigma_1-1}{\sigma_1}\left(\gamma_1 \frac{\sigma_1}{\sigma_1-1}+\gamma_2 \frac{\sigma_2}{\sigma_2-1}\right)>1 & s_1^d=s_1^e=s_1^{op}=s_1^{or}=s_1^p=s_1^s=1 \\ s_2^c=\frac{\sigma_2-1}{\sigma_2}\left(\gamma_1 \frac{\sigma_1}{\sigma_1-1}+\gamma_2 \frac{\sigma_2}{\sigma_2-1}\right)<1 & s_2^d=s_2^e=s_2^{op}=s_2^{or}=s_2^p=s_2^s=1. \end{array} \tag{26}$$

The second instrument is imposing the production cost tax, the operating fixed cost tax and the entry fixed cost tax in competitive industry, and imposing the production cost subsidy, the operating fixed cost subsidy and the entry fixed cost subsidy in noncompetitive industry satisfying

$$\begin{array}{ll} s_1^p=s_1^d=s_1^e=\frac{\sigma_1-1}{\sigma_1}\left(\gamma_1 \frac{\sigma_1}{\sigma_1-1}+\gamma_2 \frac{\sigma_2}{\sigma_2-1}\right)>1 & s_1^c=s_1^{op}=s_1^{or}=s_1^s=1 \\ s_2^p=s_2^d=s_2^e=\frac{\sigma_2-1}{\sigma_2}\left(\gamma_1 \frac{\sigma_1}{\sigma_1-1}+\gamma_2 \frac{\sigma_2}{\sigma_2-1}\right)<1 & s_2^c=s_2^{op}=s_2^{or}=s_2^s=1. \end{array} \tag{27}$$

If the lump-sum transfer (tax) in not available, the requirements for optimum allocation become tight.[17] And if we use instruments which cannot be the form of subsidy, we cannot achieve the first best equilibrium in the same way like as (19), (20) and (21). However, mixing polices appropriately enable us

[16] We implicitly assume that the sales subsidy, the operating profit subsidy and the ordinary profit subsidy are not available. However, if these instruments are available, there are more instruments to obtain first best allocation where the lump-sum transfer (tax) is not available.

[17] Compare (18) and (26) ((22) and (27)).

to achieve the optimum allocation. For example, we can achieve the first best equilibrium by imposing the sales tax in competitive industry, and imposing the production cost subsidy, the operating fixed cost subsidy and the entry fixed cost subsidy in noncompetitive industry as

$$\begin{array}{rr} s_1^s = \frac{\sigma_1-1}{\sigma_1}\left(\gamma_1\frac{\sigma_1}{\sigma_1-1}+\gamma_2\frac{\sigma_2}{\sigma_2-1}\right) > 1 & s_1^c = s_1^d = s_1^e = s_1^{op} = s_1^{or} = s_1^p = 1 \\ s_2^p = s_2^d = s_2^e = \frac{\sigma_2-1}{\sigma_2}\left(\gamma_1\frac{\sigma_1}{\sigma_1-1}+\gamma_2\frac{\sigma_2}{\sigma_2-1}\right) < 1 & s_2^c = s_2^{op} = s_2^{or} = s_2^s = 1. \end{array} \tag{28}$$

Proposition 5.3. *When the lump-sum transfer (tax) is not available, the requirement for optimum allocation is tighter than the case where the lump-sum transfer (tax) is available. Even so, the first best equilibrium can be achieved by imposing instruments appropriately.*

6 Comparison with homogeneous-firm model

Finally, we discuss the differences from the homogeneous-firm model. In the homogeneous-firm model, the productivity in industry i is identical and exogenous, $\bar{\phi}_i$, and the proportion of the number of variety to the entry is also exogenously given, $\left[1-G\left(\bar{\phi}_i\right)\right]$. Then (9) and (13) change as

$$\begin{aligned} x_i^{ME}(\phi) &= \left\{\left[1-G_i\left(\bar{\phi}_i\right)\right]s_i^d f_i^d + s_i^e s_i^{or} f_i^e\right\}\frac{s_i^{op}}{s_i^p}\bar{\phi}_i(\sigma_i-1) \\ x_i^{SO}(\phi) &= \left\{\left[1-G_i\left(\bar{\phi}_i\right)\right]f_i^d + f_i^e\right\}\bar{\phi}_i(\sigma_i-1). \end{aligned} \tag{29}$$

And, (10) and (14) also change as

$$\begin{aligned} \frac{\left(M_1^{e,ME}\right)^{\frac{\sigma_1-\rho}{\sigma_1-1}}}{\left(M_2^{e,ME}\right)^{\frac{\sigma_2-\rho}{\sigma_2-1}}} &= \left(\frac{\sigma_1-1}{\sigma_1}\frac{\sigma_2}{\sigma_2-1}\frac{s_2^c s_2^p s_2^s}{s_1^c s_1^p s_1^s}\right)^\rho \frac{\left\{\left[1-G_2\left(\bar{\phi}_2\right)\right]s_2^d f_2^d + s_2^e s_2^{or} f_2^e\right\}\frac{s_2^{op}}{s_2^p}}{\left\{\left[1-G_1\left(\bar{\phi}_1\right)\right]s_1^d f_1^d + s_1^e s_1^{or} f_1^e\right\}\frac{s_1^{op}}{s_1^p}}\left(\frac{\sigma_2-1}{\sigma_1-1}\right)\left(\frac{\gamma_1}{\gamma_2}\right)^\rho\left(\frac{\bar{\phi}_1}{\bar{\phi}_2}\right)^{\rho-1} \\ \frac{\left(M_1^{e,SO}\right)^{\frac{\sigma_1-\rho}{\sigma_1-1}}}{\left(M_2^{e,SO}\right)^{\frac{\sigma_2-\rho}{\sigma_2-1}}} &= \frac{\left\{\left[1-G_2\left(\bar{\phi}_2\right)\right]s_2^d f_2^d + s_2^e s_2^{or} f_2^e\right\}\frac{s_2^{op}}{s_2^p}}{\left\{\left[1-G_1\left(\bar{\phi}_1\right)\right]s_1^d f_1^d + s_1^e s_1^{or} f_1^e\right\}\frac{s_1^{op}}{s_1^p}}\left(\frac{\sigma_2-1}{\sigma_1-1}\right)\left(\frac{\gamma_1}{\gamma_2}\right)^\rho\left(\frac{\bar{\phi}_1}{\bar{\phi}_2}\right)^{\rho-1}. \end{aligned} \tag{30}$$

In the following, we consider two questions by comparing the heterogeneous-firm model and the homogeneous-firm model. The one is whether the requirement to achieve the first best equilibrium differs. The other is whether the total amount of subsidy (tax) to achieve the optimum allocation differs.

We will begin by considering the former question. In the homogeneous-firm model, from (29),

$$\left\{\left[1-G_i\left(\bar{\phi}_i\right)\right]s_i^d f_i^d + s_i^e s_i^{or} f_i^e\right\}\frac{s_i^{op}}{s_i^p} = \left[1-G_i\left(\bar{\phi}_i\right)\right]f_i^d + f_i^e \tag{31}$$

must be satisfied. From (30), (17) must be satisfied. Then two necessary conditions, (17) and (31), should be satisfied in order to achieve the optimum allocation. The most important difference between the heterogeneous-firm model and the homogeneous-firm model is the existence of endogenous firm selection (the existence of equation (7) and (12), hence the condition (15)). In the heterogeneous-firm model, the productivity cutoff and the average productivity are determined endogenously. Then a government intervention, even when this policy seems to be desirable, can create another distortion through endogenous firm selection. To control this distortion, tighter restriction is required than the homogeneous-firm model. In particular, it is noteworthy when we use instruments affecting productivity cutoff: s_i^d, s_i^e, and s_i^{or}. In the homogeneous-firm model, (21) and (22) change as

$$\frac{s_1^p}{s_2^p} = \frac{\sigma_1 - 1}{\sigma_1}\frac{\sigma_2}{\sigma_2 - 1} > 1 \quad \begin{array}{l} s_1^d = \frac{s_1^p\left\{\left[1-G_1\left(\bar{\phi}_1\right)\right]f_1^d + f_1^e\right\} - s_1^e f_1^e}{\left[1-G_1\left(\bar{\phi}_1\right)\right]f_1^d} \\ s_2^d = \frac{s_2^p\left\{\left[1-G_2\left(\bar{\phi}_2\right)\right]f_2^d + f_2^e\right\} - s_2^e f_2^e}{\left[1-G_2\left(\bar{\phi}_2\right)\right]f_2^d} \end{array} \quad \begin{array}{l} s_1^c = s_1^{op} = s_1^{or} = s_1^s = 1 \\ s_2^c = s_2^{op} = s_2^{or} = s_2^s = 1, \end{array} \tag{32}$$

$$s_1^p = \frac{\sigma_1 - 1}{\sigma_1}\frac{\sigma_2}{\sigma_2 - 1} > 1 \quad s_1^d = \frac{s_1^p\left\{\left[1-G_1\left(\bar{\phi}_1\right)\right]f_1^d + f_1^e\right\} - s_1^{or} f_1^e}{\left[1-G_1\left(\bar{\phi}_1\right)\right]f_1^d} \quad \begin{array}{l} s_1^c = s_1^e = s_1^{op} = s_1^s = 1 \\ s_2^c = s_2^d = s_2^e = s_2^{op} = s_2^{or} = s_2^p = s_2^s = 1. \end{array} \tag{33}$$

Note that the optimum conditions of heterogeneous-firm model, (21) and (22), are included by those of homogeneous-firm model, (32) and (33). Thus, the necessary condition of heterogeneous-firm model becomes tight than the homogeneous-firm model. Then in the homogeneous-firm model, we can achieve the optimum allocation by using instruments by which we cannot achieve the optimum allocation in the heterogeneous-firm model. For example, we can achieve the first best equilibrium to control only s_i^p and s_i^d by setting $s_1^p = \frac{\sigma_1 - 1}{\sigma_1}\frac{\sigma_2}{\sigma_2 - 1}$ and $s_1^d = \frac{s_1^p\left\{\left[1-G_1\left(\bar{\phi}_1\right)\right]f_1^d + f_1^e\right\} - f_1^e}{\left[1-G_1\left(\bar{\phi}_1\right)\right]f_1^d}$.

Proposition 6.1. *When firms' productivity is heterogeneous, the requirement for optimum allocation is tighter than the case where firms' productivity is homogeneous.*

Next, we shall consider the latter question about the total amount of optimum subsidy (tax). From (24), the lump-sum transfer (tax) is a function of M_i^e, $\int_{\phi_i^d}^{\phi_i^{\max}} p_i(\phi)\, x_i(\phi)\, dG_i(\phi) = \frac{(P_i)^{-(\rho-1)} P^{(\rho-1)} (\gamma_i)^\rho E}{M_i^e}\frac{1}{s_i^c}$, $\int_{\phi_i^d}^{\phi_i^{\max}} \frac{1}{\phi} x_i(\phi)\, dG_i(\phi) = \frac{(P_i)^{-(\rho-1)} P^{(\rho-1)} (\gamma_i)^\rho E}{M_i^e}\frac{\sigma_i - 1}{\sigma_i}\frac{1}{s_i^c s_i^p s_i^s}$, and $\int_{\phi_i^d}^{\phi_i^{\max}} \pi_i(\phi)\, dG_i(\phi) = \frac{(P_i)^{-(\rho-1)} P^{\rho-1} (\gamma_i)^\rho E}{\sigma_i M_i^e}\frac{1}{s_i^c s_i^{op} s_i^s} - s_i^d f_i^d \left[1 - G_i\left(\phi_i^d\right)\right]$.

For the present, we shall confine our attention to the case where the elasticity of substitution between industries is unity: $\rho = 1$. From (10) and (30), as long as the proportion of the number of variety to the entry is identical: $\left[1 - G_i\left(\phi_i^{d,ME}\right)\right] = \left[1 - G_i\left(\bar{\phi}_i\right)\right]$, $\frac{M_1^e}{M_2^e}$ of the heterogeneous-firm model is also identical with that of the homogeneous-firm model. Moreover, $\int_{\phi_i^d}^{\phi_i^{\max}} p_i\left(\phi\right) x_i\left(\phi\right) dG_i\left(\phi\right) = \frac{\gamma_i E}{M_i^e}\frac{1}{s_i^c}$, $\int_{\phi_i^d}^{\phi_i^{\max}} \frac{1}{\phi} x_i\left(\phi\right) dG_i\left(\phi\right) = \frac{\gamma_i E}{M_i^e}\frac{1}{s_i^c s_i^p s_i^s}\frac{\sigma_i - 1}{\sigma_i}$, and $\int_{\phi_i^d}^{\phi_i^{\max}} \pi_i\left(\phi\right) dG_i\left(\phi\right) = \frac{\gamma_i E}{\sigma_i M_i^e}\frac{1}{s_i^c s_i^{op} s_i^s} - s_i^d f_i^d \left[1 - G_i\left(\phi_i^d\right)\right]$ are independent from the property of distribution as long as M_i^e is not affected by it. Therefore, the property of distribution does not affect the total amount of subsidy (tax) when $\rho = 1$.

Once we generalize the assumption of the elasticity of substitution between industries: $\rho > 1$, however, $\frac{M_1^e}{M_2^e}$ of the heterogeneous-firm model is now dependent on the property of distribution: $\int_{\phi_i^d}^{\phi_i^{\max}} \left(\phi\right)^{\sigma_i - 1} dG_i\left(\phi\right)$. Then $\frac{M_1^e}{M_2^e}$ of the heterogeneous-firm model does not coincide with that of the homogeneous-firm model unless $\left[\int_{\phi_i^d}^{\phi_i^{\max}} \left(\phi\right)^{\sigma_i - 1} dG_i\left(\phi\right)\right]^{\frac{1}{\sigma_i - 1}} = \bar{\phi}_i$ is satisfied by chance. Moreover, $\int_{\phi_i^d}^{\phi_i^{\max}} p_i\left(\phi\right) x_i\left(\phi\right) dG_i\left(\phi\right)$, $\int_{\phi_i^d}^{\phi_i^{\max}} \frac{1}{\phi} x_i\left(\phi\right) dG_i\left(\phi\right)$, and $\int_{\phi_i^d}^{\phi_i^{\max}} \pi_i\left(\phi\right) dG_i\left(\phi\right)$ are influenced by the property of distribution through the price index. Thus the total amount of subsidy (tax) is dependent on the property of distribution when $\rho > 1$.

Proposition 6.2. *Although the property of the distribution does not affect the total amount of subsidy (tax) to achieve the optimum allocation when the elasticity of substitution between industries is unity, it affects the total amount of subsidy (tax) when the elasticity of substitution between industries is more than unity.*

7 Conclusion

We analyzed the situation where the markup differs across industries with considering firm-level heterogeneity. As Melitz and Redding (2015) emphasized, we found the situation that the effect of endogenous firm selection plays the important role for welfare analysis when we consider the production cost subsidy (tax), the operating fixed cost subsidy (tax) and the entry fixed cost subsidy (tax). In practice, these instruments are often used as a major tool to control the degree of competition in particular industry. And many theoretical researches analyze the effect of such policies. Our results imply that a firm-level diversity matters in such situations, and that a policy mix can be useful tool to improve welfare.

In this paper, we focused on the difference of the markup across industries to analyze the "heterogeneous markup distortion." Of course, there are other factors to distort allocation. Especially, studying the static externalities by generalizing the assumption of CES utility form and the monopolistic competition is of special interest to us. Thus, analyzing the effect of endogenous firm selection on welfare and the optimum policy in such situations can be important future work.

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A Appendix

In this appendix, we derive the explicit market equilibrium solution by assuming a Pareto distribution with $G_i(\phi) = 1 - \phi^{-k_i}, \phi \in (0, +\infty)$ where $k_i > \sigma_i - 1$ is a parameter.

Then, (7) can be expressed explicitly as

$$\phi_i^{d,ME} = \left[\frac{\sigma_i - 1}{k_i - (\sigma_i - 1)} \frac{s_i^d f_i^d}{s_i^e s_i^{or} f_i^e} \right]^{\frac{1}{k_i}}.$$

(9) can be expressed as

$$x_i^{ME}(\phi) = (\phi)^{\sigma_i} \left[\frac{\sigma_i - 1}{k_i - (\sigma_i - 1)} \frac{s_i^d f_i^d}{s_i^e s_i^{or} f_i^e} \right]^{-\frac{\sigma_i - 1}{k_i}} \frac{s_i^d s_i^{op}}{s_i^p} f_i^d (\sigma_i - 1).$$

When $\rho = 1$, (10) can be expressed as

$$\frac{M_1^e}{M_2^e} = \left(\frac{\sigma_2}{\sigma_1} \frac{\gamma_1}{\gamma_2} \frac{s_2^c s_2^p s_2^s}{s_1^c s_1^p s_1^s} \right) \frac{\left\{ \left[1 - G_2 \left(\phi_2^{d,ME} \right) \right] s_2^d f_2^d + s_2^e s_2^{or} f_2^e \right\} \frac{s_2^{op}}{s_2^p}}{\left\{ \left[1 - G_1 \left(\phi_1^{d,ME} \right) \right] s_1^d f_1^d + s_1^e s_1^{or} f_1^e \right\} \frac{s_1^{op}}{s_1^p}}.$$

Combining this equation and (6) yields

$$M_i^{e,ME} = \frac{\gamma_i}{s_i^c s_i^{op} s_i^s \sigma_i \left\{ s_i^d f_i^d \left[1 - G_i \left(\phi_i^{d,ME} \right) \right] + s_i^e s_i^{or} f_i^e \right\}} \left\{ \sum_{h=1,2} \gamma_h \frac{f_h^e + \left[1 - G_h \left(\phi_h^{d,ME} \right) \right] f_h^d + \left[s_h^d f_h^d \left[1 - G_h \left(\phi_h^{d,ME} \right) \right] + s_h^e s_h^{or} f_h^e \right] \frac{s_h^{op}}{s_h^p} (\sigma_h - 1)}{s_h^c s_h^{op} s_h^s \sigma_h \left\{ s_h^d f_h^d \left[1 - G_h \left(\phi_h^{d,ME} \right) \right] + s_h^e s_h^{or} f_h^e \right\}} \right\}^{-1}.$$

Moreover, if we assume a Pareto distribution, we get

$$M_i^{e,ME} = \frac{\sigma_i - 1}{\sigma_i} \frac{\gamma_i}{s_i^c s_i^e s_i^{op} s_i^{or} s_i^s f_i^e k_i} \left\{ \sum_{h=1,2} \frac{\gamma_h}{\sigma_h} \frac{\left(s_h^d s_i^e s_h^{op} s_i^{or} k_i + s_h^d s_h^p - s_h^e s_h^{or} s_h^p \right) (\sigma_h - 1) + s_h^e s_h^{or} s_h^p k_h}{s_h^c s_h^d s_h^e s_h^{op} s_h^{or} s_h^p s_h^s k_h} \right\}^{-1}.$$

B Appendix

In this appendix, we solve the social optimum problem (11). We solve this optimization problem by dividing it into the following three steps.

We let χ_i denote the labor devoted into the production activities in industry i and η_i denote the proportion of labor devoted in industry i. First, given M_i^e, ϕ_i^d and χ_i, we solve the following problem of the production in industry i:

$$\max_{x_i(\phi)} \quad \left[M_i^e \int_{\phi_1^d}^{\phi_i^{\max}} (x_i(\phi))^{\frac{\sigma_i - 1}{\sigma_i}} dG(\phi) \right]^{\frac{\sigma_i}{\sigma_i - 1}}$$
$$s.t. \qquad M_i^e \int_{\phi_i^d}^{\phi_i^{\max}} \frac{1}{\phi} x_i(\phi) \, dG(\phi) = \chi_i.$$

Solving this probrem yields $x_i(\phi) = \frac{(\phi)^{\sigma_i}}{M_i^e \int_{\phi_i^d}^{\phi_i^{\max}} (\phi)^{\sigma_i - 1} dG(\phi)} \chi_i$. Then $X_i \equiv \left[M_i^e \int_{\phi_i^d}^{\phi_i^{\max}} (x_i(\phi))^{\frac{\sigma_i - 1}{\sigma_i}} dG(\phi) \right]^{\frac{\sigma_i}{\sigma_i - 1}}$ can be expressed as $X_i = \left[M_i^e \int_{\phi_i^d}^{\phi_i^{\max}} (\phi)^{\sigma_i - 1} dG(\phi) \right]^{\frac{1}{\sigma_i - 1}} \chi_i$.

Second, given η_i, we solve the following problem of the allocation in industry i:

$$\max_{\phi_i^d, M_i^e} \quad \left[M_i^e \int_{\phi_i^d}^{\phi_i^{\max}} (\phi)^{\sigma_i - 1} \, dG(\phi) \right]^{\frac{1}{\sigma_i - 1}} \chi_i$$

$$s.t. \quad \chi_i + M_i^e \left[1 - G\left(\phi_i^d\right)\right] f_i^d + M_i^e f_i^e = \eta_i.$$

Solving this problem, we get (12) and $M_i^e = \frac{\eta_i}{\sigma_i} \frac{1}{\left[1-G\left(\phi_i^d\right)\right] f_i^d + f_i^e}$. Substituting this expression of M_i^e into $x_i(\phi) = \frac{(\phi)^{\sigma_i}}{M_i^e \int_{\phi_i^d}^{\phi_i^{\max}} (\phi)^{\sigma_i - 1} dG(\phi)} \chi_i$ yields (13).

Then X_i can be expressed as $X_i = (\eta_i)^{\frac{\sigma_i}{\sigma_i - 1}} \left[\frac{1}{\sigma_i} \frac{1}{\left[1-G\left(\phi_i^d\right)\right] f_i^d + f_i^e} \int_{\phi_i^d}^{\phi_i^{\max}} (\phi)^{\sigma_i - 1} \, dG(\phi) \right]^{\frac{1}{\sigma_i - 1}} \frac{\sigma_i - 1}{\sigma_i}$.

Finally, we solve the problem of the allocation between industries:

$$\max_{\eta_1, \eta_2} \quad U = \left[\gamma_1 (X_1)^{\frac{\rho - 1}{\rho}} + \gamma_2 (X_2)^{\frac{\rho - 1}{\rho}} \right]^{\frac{\rho}{\rho - 1}}$$

$$s.t. \quad \eta_1 + \eta_2 = 1$$

Solving this problem, we get $\frac{(\eta_1)^{\frac{\sigma_1 - \rho}{\sigma_1 - 1}}}{(1-\eta_1)^{\frac{\sigma_2 - \rho}{\sigma_2 - 1}}} = \frac{\left[\frac{1}{\sigma_1} \frac{1}{\left[1-G_1\left(\phi_1^d\right)\right] f_1^d + f_1^f} \int_{\phi_1^d}^{\phi_i^{\max}} (\phi)^{\sigma_i - 1} dG_1(\phi) \right]^{\frac{\rho - 1}{\sigma_1 - 1}} (L)^{\frac{\sigma_1(\rho - 1)}{\sigma_1 - 1}}}{\left[\frac{1}{\sigma_2} \frac{1}{\left[1-G_2\left(\phi_2^d\right)\right] f_2^d + f_2^f} \int_{\phi_2^d}^{\phi_2^{\max}} (\phi)^{\sigma_2 - 1} dG_2(\phi) \right]^{\frac{\rho - 1}{\sigma_2 - 1}} (L)^{\frac{\sigma_2(\rho - 1)}{\sigma_2 - 1}}} \left(\frac{\sigma_2 - 1}{\sigma_2} \frac{\sigma_1}{\sigma_1 - 1} \right) \left(\frac{\gamma_1}{\gamma_2} \right)^{\rho}$.

From $M_i^e = \frac{\eta_i}{\sigma_i} \frac{1}{\left[1-G\left(\phi_i^d\right)\right] f_i^d + f_i^e}$, we obtain $\frac{(M_1^e)^{\frac{\sigma_1 - \rho}{\sigma_1 - 1}}}{\left(M_2^e\right)^{\frac{\sigma_2 - \rho}{\sigma_2 - 1}}} = \frac{(\eta_1)^{\frac{\sigma_1 - \rho}{\sigma_1 - 1}}}{(1-\eta_1)^{\frac{\sigma_2 - \rho}{\sigma_2 - 1}}} \frac{\left\{ \frac{1}{\sigma_1} \frac{L}{\left[1-G\left(\phi_1^d\right)\right] f_1^d + f_1^e} \right\}^{\frac{\sigma_1 - \rho}{\sigma_1 - 1}}}{\left\{ \frac{1}{\sigma_2} \frac{L}{\left[1-G\left(\phi_2^d\right)\right] f_2^d + f_2^e} \right\}^{\frac{\sigma_2 - \rho}{\sigma_2 - 1}}}$. Combining these two equations yields (14).[18]

C Appendix

In this appendix, we consider the situation where households supply labor elastically. And we show that this setting is a special case of our model in the sense that households consume high-markup differentiated goods and no-markup leisure good.

Each household has one unit of time, and distributes it into labor and leisure. Then, we rewrite the

[18] When $\rho = 1$, we obtain $\eta_i = \frac{\gamma_i \frac{\sigma_i}{\sigma_i - 1}}{\gamma_1 \frac{\sigma_1}{\sigma_1 - 1} + \gamma_2 \frac{\sigma_2}{\sigma_2 - 1}}$. Substituting this into $M_i^e = \frac{\eta_i}{\sigma_i} \frac{L}{\left[1-G\left(\phi_i^d\right)\right] f_i^d + f_i^e}$, we obtain $M_i^{e,SO} = \frac{\gamma_i L}{\sigma_i \left(\left[1-G\left(\phi_i^{d,SO}\right)\right] f_i^d + f_i^e \right)} \frac{\frac{\sigma_i}{\sigma_i - 1}}{\gamma_1 \frac{\sigma_1}{\sigma_1 - 1} + \gamma_2 \frac{\sigma_2}{\sigma_2 - 1}}$.

maximization problem of households as

$$
\begin{array}{ll}
\max\limits_{x_1(j),L} & U=\left[\gamma_1\left[\int\limits_0^{n_1}\left(x_1\left(j\right)\right)^{\frac{\sigma_1-1}{\sigma_1}}dj\right]^{\frac{\sigma_1}{\sigma_1-1}\frac{\hat{\rho}-1}{\hat{\rho}}}+\gamma_L\left(1-L\right)^{\frac{\hat{\rho}-1}{\hat{\rho}}}\right]^{\frac{\hat{\rho}}{\hat{\rho}-1}} \\
s.t. & \int\limits_0^{n_1}s_1^c x_1\left(j\right)p_1\left(j\right)dj=\frac{1}{s^L}L+T
\end{array}
$$

where L is the aggregate labor supply, $\hat{\rho}\leq\sigma_1$ is the elasticity of substitution between the differentiated goods and the leisure good, and s^L is the labor income subsidy (tax) when $s<1$ ($s>1$). Solving this maximization problem yields

$$
\begin{aligned}
x_1\left(j\right)&=\frac{1}{s_1^c p_1(j)}\left(\frac{P_1}{s_1^c p_1(j)}\right)^{\sigma_1-1}\left(\frac{P}{P_1}\right)^{\rho-1}\left(\gamma_1\right)^{\rho}\left(\frac{1}{s^L}+T\right) \\
1-L&=\frac{1}{P_L}\left(\frac{P}{P_L}\right)^{\rho-1}\left(\gamma_L\right)^{\rho}\left(\frac{1}{s^L}+T\right)
\end{aligned}
$$

where $P_L\equiv\left(\frac{1}{s^L}\right)$ is the price of leisure good. $\hat{P}\equiv\left[\left(\gamma_1\right)^{\hat{\rho}}\left(P_1\right)^{-(\hat{\rho}-1)}+\left(\gamma_L\right)^{\hat{\rho}}\left(P_L\right)^{-(\hat{\rho}-1)}\right]^{\frac{-1}{\hat{\rho}-1}}$ is the price index. Note that the price of the leisure good is equal to the after-subsidy (tax) wage, which is no-markup. Thus, we can interpret the leisure good as the homogenous good produced in perfect competitive industry.

The market equilibrium value of the productivity cutoff and the production of firms in industry 1 are expressed as (7) and (9). The first best equilibrium value of these are expressed as (11) and (12). The labor market clearing condition is

$$
M_1^e\int\limits_{\phi_1^d}^{\phi_1^{\max}}\frac{1}{\phi}x_1\left(\phi\right)dG_1\left(\phi\right)+M_1^e f_1^e+\left[1-G_1\left(\phi_1^d\right)\right]M_1^e f_1^d+\left(1-L\right)=1,
$$

(10) and (13) can be rewritten as

$$
\begin{aligned}
\frac{\left(M_1^{e,ME}\right)^{\frac{\sigma_1-\hat{\rho}}{\sigma_1-1}}}{\left(1-L^{ME}\right)}&=\left(\frac{\sigma_1-1}{\sigma_1}\frac{1}{s_1^c s^L s_1^p s_1^s}\right)^{\hat{\rho}}\frac{1}{\left\{s_1^d f_1^d\left[1-G_1\left(\phi_1^d\right)\right]+s_1^e s_1^{or}f_1^e\right\}\frac{s_1^{op}}{s_1^p}}\frac{1}{\sigma_1-1}\left(\frac{\gamma_1}{\gamma_L}\right)^{\hat{\rho}}\left[\int\limits_{\phi_1^d}^{\phi_1^{\max}}\left(\phi\right)^{\sigma_1-1}dG_1\left(\phi\right)\right]^{\frac{\hat{\rho}-1}{\sigma_1-1}} \\
\frac{\left(M_1^{e,SO}\right)^{\frac{\sigma_1-\hat{\rho}}{\sigma_1-1}}}{\left(1-L^{SO}\right)}&=\frac{1}{\left[1-G_1\left(\phi_1^d\right)\right]f_1^d+f_1^e}\frac{1}{\left(\sigma_1-1\right)}\left(\frac{\gamma_1}{\gamma_L}\right)^{\hat{\rho}}\left[\int\limits_{\phi_1^d}^{\phi_1^{\max}}\left(\phi\right)^{\sigma_i-1}dG_1\left(\phi\right)\right]^{\frac{\hat{\rho}-1}{\sigma_1-1}}.
\end{aligned}
$$

Obviously, these expressions are basically the same as discussed in section 3 and section 4, and we can achieve the optimum allocation in the same way discussed in section 5. Therefore, Melitz-type model with endogenous labor supply is a special case of our model, and "labor distortion" is included in "heterogeneous markup distortion."