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CREDIT BOOMS, DEBT OVERHANG AND SECULAR STAGNATION

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Abstract

Why do advanced economies fall into prolonged periods of economic stagnation, particularly in the aftermath of credit booms? We present a model of persistent aggregate demand shortage based on strong liquidity preferences of households, in which we incorporate financial imperfections to study the interactions between debt, liquidity and asset prices. We show that financially more deregulated economies are more likely to experience persistent stagnation. In the short run, credit booms can mask this structural aggregate demand deficiency. However, the resulting debt overhang permanently depresses spending in the long run since deleveraging becomes self-defeating because of debt deflation. These findings are in line with the macroeconomic developments in Japan during its lost decades and other advanced economies before and during the Great Recession.

Keywords: Secular Stagnation, Aggregate Demand Deficiency, Liquidity Preferences, Financial Frictions, Leverage

JEL Classification: E13, E21, E32, E41, E51

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1 Introduction

Many advanced economies suffer from insufficient aggregate demand in the aftermath of the global financial crisis despite unconventional monetary policy actions of unprecedented scales. In addition, the experience of Japan shows that economic stagnation and deflationary tendencies can prevail for decades without any natural recovery. Hence, worries that economies might permanently fail to operate at full employment are widespread and expressed in the "secular stagnation" hypothesis.[1] Proponents of this view emphasize the importance of asset prices, credit availability and private sector debt. This is for at least two reasons.

On the one hand, credit booms or asset price booms are seen as a means to temporarily stimulate a stagnating economy. In particular, Summers (2014a,b) argues that the credit boom in the United States in the early 2000s was masking the underlying lack of aggregate demand by initiating unsustainable consumption spending of households. Similar effects were at play during the stock market boom of the 1990s. Therefore, Summers (2014a) concludes that "the difficulty that has arisen in recent years in achieving adequate growth has been present for a long time but has been masked by unsustainable finances" and "it has been close to 20 years since the American economy grew at a healthy pace supported by sustainable finance".[2]

On the other hand, the resulting indebtedness of the private sector is considered a major impediment to economic recovery. For instance, Eggertsson and Krugman (2012) emphasize the reduction in private demand due to debt overhang during balance sheet recessions.[3] In addition, housing wealth and leverage are main determinants of economic activity in the United States (cf. Mian and Sufi, 2011; Mian et al., 2013) and Japan (cf. Ogawa, 2003; Ogawa and Wan, 2007). Credit growth is also a strong indicator for financial crises (cf. Borio and Lowe, 2002; Shin, 2013). These crises are associated with substantially higher output losses than normal recessions despite forceful monetary policy actions because of the decoupling of monetary aggregates and the volume of credit (cf. Schularick and Taylor, 2012; Jordà et al., 2015).

It follows that interactions of asset prices, credit availability and private sector debt are important factors for the emergence and the severity of economic stagnation. But how does an economy fall into stagnation and what is the role of these factors?

[1] The term "secular stagnation" itself goes back to Hansen (1938) and was taken up by Larry Summers (2013). Yet, Keynes (1936) in Chapter 17 of the General Theory already argues that permanent demand shortage can exist as a steady state phenomenon in a monetary economy.

[2] A related argument is made by Krugman (2013): "In other words, you can argue that our economy has been trying to get into the liquidity trap for a number of years, and that it only avoided the trap for a while thanks to successive bubbles."

[3] Other theoretical treatments of deleveraging shocks at the zero lower bound include Eggertsson and Mehrotra (2015) among others.

In this paper, we develop a stylized dynamic macroeconomic model to analyze the interactions between asset prices, leverage and economic stagnation. The model features three types of assets and two types of households: Borrowers obtain funds from savers, but their borrowing ability is limited by the value of collateral that is endogenously determined in the housing market following Iacoviello (2005). Households gain utility from consumption, housing and money. The last follows Sidrauski (1967) and reflects, among other things, the demand for liquidity.

We follow the research line initiated by Ono (1994, 2001) and assume insatiable liquidity preferences: The marginal utility of money stays strictly positive even for very large money holdings, which prevents consumption of the saver from increasing as potential output rises. This in turn creates stagnation if consumption of the borrower is sufficiently restricted as is the case when the economy suffers from debt overhang. Hence, economies with a higher leverage are more prone to suffering from insufficient aggregate demand.

Our setting implies that asset price or credit booms can temporarily stimulate an economy that would otherwise suffer from demand deficiency. A credit or asset price boom, which is triggered by financial liberalization, enables borrowers to temporarily increase their consumption spending, stimulating aggregate demand and inflation. In addition, housing demand is stimulated and the real house price increases, thereby reinforcing the initial credit boom as the value of collateral increases. Yet, in the new steady state, borrowers' consumption is depressed by interest payments to savers. Savers however do not increase their consumption accordingly as they prefer to hoard money because of strong liquidity preferences. As a consequence, aggregate demand falls permanently short of potential output and the economy experiences persistent deflation.

In this steady state, borrowers do actually delever but these efforts are self-defeating due to deflation. Their real debt burden remains constant and permanently depresses spending so that the economy does not recover. Monetary policy becomes ineffective since injections of liquidity are held as cash by savers and do not stimulate spending.[4] Hence, there is a temptation for policymakers to stimulate sluggish growth by initiating lending booms that come at the cost of greater damage in the long run.

These findings are in line with the macroeconomic developments in Japan during its lost decades and other advanced economies during the Great Recession.

We proceed as follows. In Section 2, we present some stylized facts on private sector debt and asset prices and discuss our main assumption of insatiable liquidity preferences. Section 3 introduces a model of economic stagnation, which is analyzed in Section 4. We discuss the role of leverage for economic stagnation in section 5 and some extensions of the model as well as policy recommendations in section 6. The final section concludes.

[4]This is consistent with the apparent ineffectiveness of the Bank of Japan's unconventional monetary policy actions in the late 1990s and early 2000s in stimulating inflation (see Ugai, 2007; Ueda, 2012b).

2 Money, Credit and Stagnation

Which empirical patterns should a model of secular stagnation be able to replicate? A look at the transition of Japan from a high growth to a stagnating economy allows us to derive stylized facts on asset prices, credit, money and economic stagnation. We also show that recent developments in other advanced economies are reminiscent of the Japanese situation. In addition, we argue that our assumption of insatiable liquidity preferences is able to capture these patterns and relate our approach to the existing literature.

Stylized Facts Consider the macroeconomic development of Japan over the last three decades as illustrated in Figure 1. The transformation from a high growth to a stagnating economy is apparent in panels (a) and (b) which show real GDP growth and inflation: From 1980 to 1991, the Japanese economy grew at an average rate of 4.4% in real terms with an annual inflation rate of 1.9%. In contrast, real GDP grew at only 0.9% on average in the period since 1992 with inflation falling into negative territory.[5]

A distinguishing feature of the high growth and the stagnation period is the behavior of asset prices and credit. As shown in panels (c) and (d), Japan experienced a credit and asset price boom during its high growth period. Within ten years, the outstanding amount of private credit to the non-financial sector as a fraction of GDP increased from 1.4 in 1980 to 2.2 in 1992 primarily driven by bank lending to small and medium-sized corporations and declining lending standards (see Posen, 2003). Credit to the private sector grew on average by 7.9% in real terms during the period from 1980 to 1991 while residential property prices (as a proxy for collateralizeable assets) increased by 5.1% in real terms.[6] Credit expansion and asset price inflation in terms of stock, land and housing prices were at the core of Japan's bubble economy.[7]

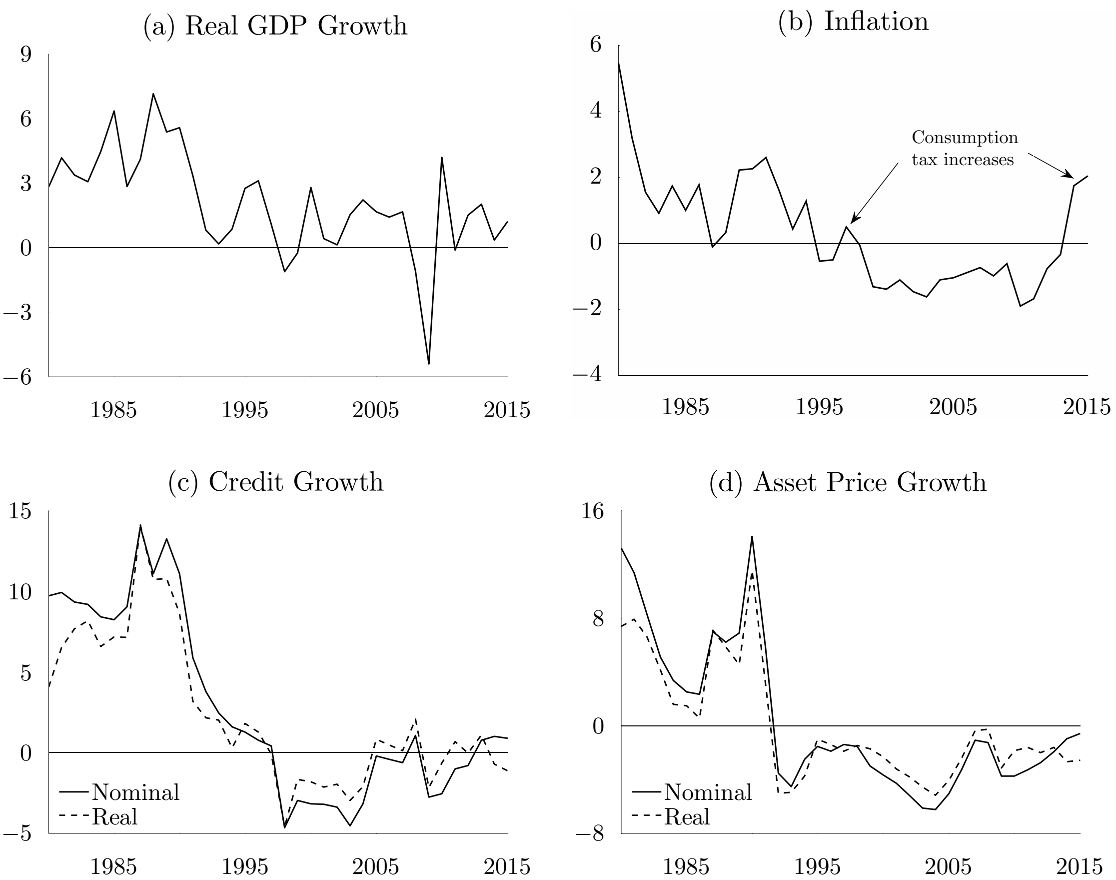
In contrast, asset prices declined and the private sector disencumbered in the stagnation period following the asset price crash of the early 1990s: Credit to the private sector declined by 0.8% on average each year after 1991 while the real amount of credit stagnated. Credit as a share of GDP declined by almost 20% from 2.2 in 1992 to a level of

[5]We measure inflation by the GDP Deflator. The patterns is the same for CPI inflation at 2.6% (1980-1991) and 0.25% (1992-2015). The tendency is the same when we exclude the financial crisis episode since 2008. Then real growth is slightly higher at 1.2% (1992-2007) but still substantially below the pre-1992 average. Similar developments hold for other measures of economic activity, like real consumption expenditure growth which declines from 4.0% to 1.0%. Note that the recent increase in inflation in panel (b), as well as the spike in 1997, can be explained by an increase in the consumption tax in April 2014 (and 1997). Apart from the tax effect, there is no indication of a persistent increase in inflation.

[6]Property price increases were higher for commercial property (6.0%) and in the six major cities (12.1%). Similarly, the subsequent decline was stronger for commercial property (-5.6%) and in cities (-4.8%). All housing price data comes from the Bank for International Settlement's (BIS) "Long series on nominal residential property prices" database, see Bank for International Settlements (2015).

[7]For further discussion of the Japanese experience and the similarities with the recent developments in the United States, see Tsuruta (1999), Shimizu and Watanabe (2010), Ueda (2012a), among others.

Figure 1: Macroeconomic Development in Japan, 1980-2015

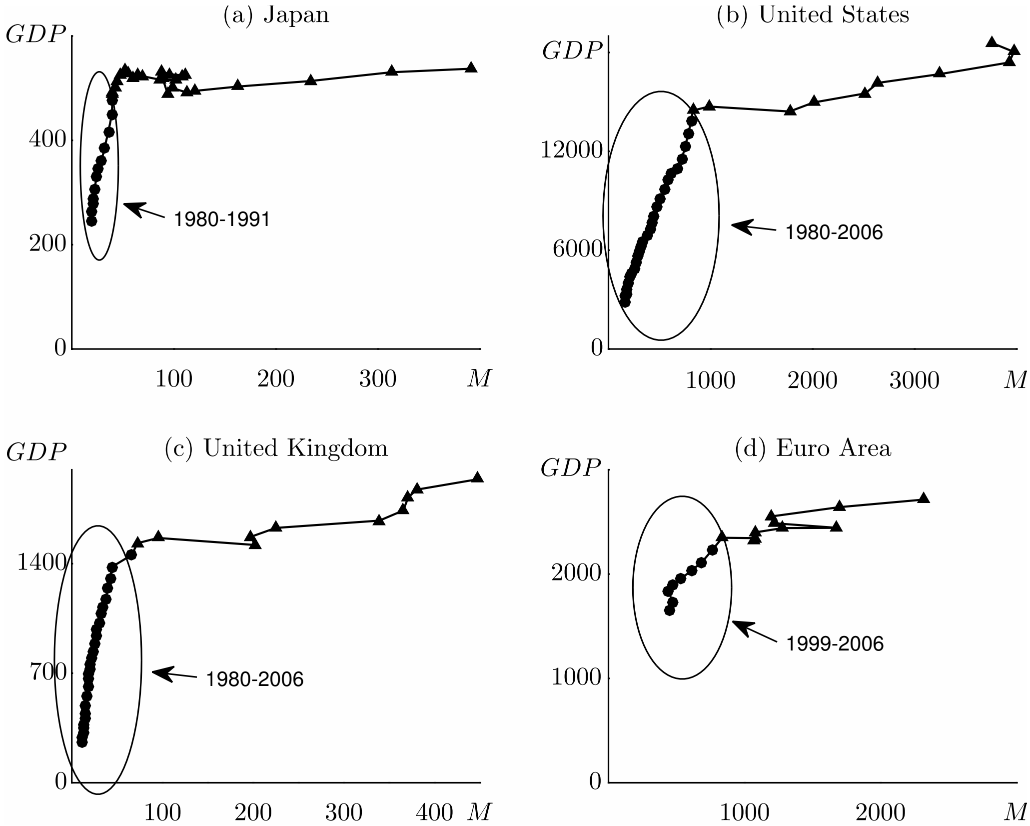


Notes: Real GDP and inflation from the World Bank. Inflation is measured by the GDP Deflator, in percent. Credit measures credit to the private non-financial sector and asset prices refer to the residential property price index. Both series are from the Bank for International Settlements (BIS). Real variables are deflated with the GDP Deflator.

1.6 in 2015. At the same time, nominal property prices decreased substantially by 3.1% per year on average, while price decreases were somewhat smaller in real terms due to deflation. The decline in asset prices continued throughout the stagnation period without any indication of a sustained recovery.

In addition, monetary policy became ineffective at stimulating output after 1991. Panel (a) in Figure 2 illustrates the relationship between base money and nominal GDP since 1980 for Japan. The neoclassical quantity equation seems to fit the data quite well during the 1980s as increases in the monetary base are associated with an increase in nominal spending. Yet, there clearly is a structural break in the early 1990s associated with the transition to economic stagnation. Subsequent substantial increases in the money supply - particularly during quantitative easing in the early 2000s and in the context of “Abenomics” - did not translate into higher nominal spending but simply resulted in a decline in the circulation velocity of money.

Figure 2: Breakdown of the Quantity Equation

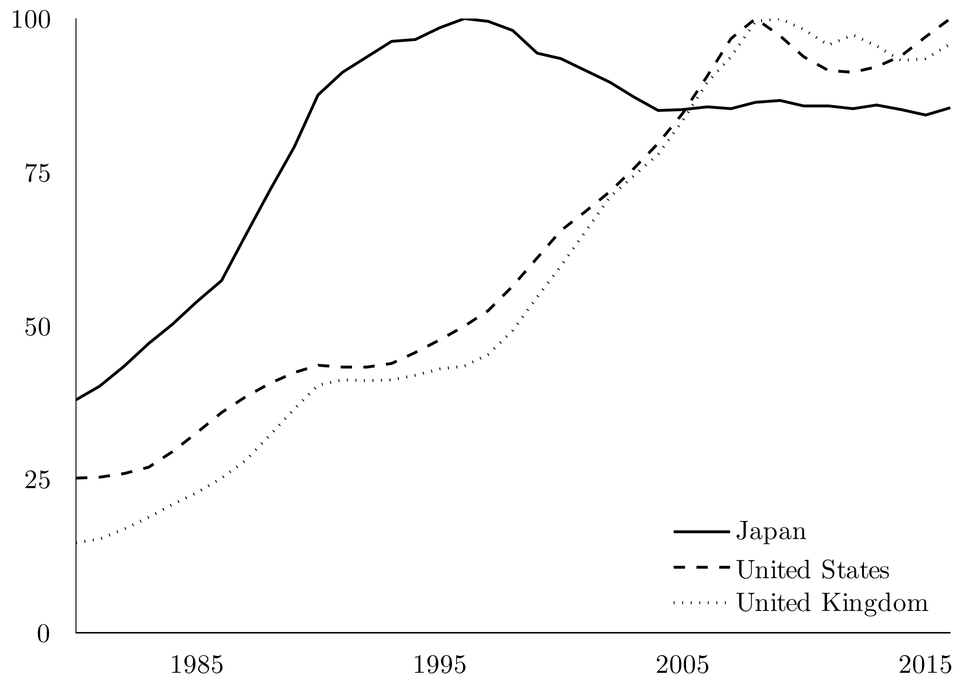


Notes: This figure shows the monetary base and nominal GDP for each country. Units are trillion of Yen (Japan), billion of US-Dollar (United States), billion of Pound Sterling (United Kingdom) and billion of Euro (ECB). Data from Federal Reserve Bank of St. Louis (FRED) Database (US and UK), Bank of Japan and World Bank (Japan), Eurostat and ECB. Data on the Euro Area includes other liquidity providing operations by national central banks.

Many advanced countries experience similar developments during the Great Recession. Panels (b) to (d) in Figure 2 show that the formerly stable relationship between base money and nominal spending substantially changed in the United States, the United Kingdom and the Euro Area as expansions in the money supply ceased to stimulate nominal spending. This structural change is associated with a prolonged period of depressed spending because of persistent debt overhang in the aftermath of a credit boom. The extent of the credit expansion and the subsequent debt overhang are reminiscent of Japan's experience as illustrated in Figure 3.

To sum up, we observe persistent economic stagnation despite unprecedented expansions of the monetary base. The emergence of stagnation is associated with the end of a credit boom that results in substantial debt overhang. In this paper we will present a dynamic macroeconomic model of aggregate demand shortage that theoretically explains these phenomena.

Figure 3: Real Credit to the Private Non-Financial Sector



Notes: This figure shows real credit to the private non-financial sector. Data from Bank for International Settlements (BIS). Nominal credit deflated by GDP Deflator and normalized to 100 in year of peak: 1996 (Japan), 2008 (US) and 2009 (UK).

Related Literature We contribute to the literature on persistent aggregate demand shortage based on the insatiability of liquidity or wealth preferences. This literature was initiated by Ono (1994, 2001) and substantially extended and microfounded by Ono and Ishida (2014). The main modification in these models is the idea that the marginal utility from holding real money balances has a strictly positive lower bound:

$$\lim_{m \to \infty} v'(m) = \beta > 0$$

As a consequence, increases in money holdings or wealth at some point cease to stimulate consumption spending as agents prefer to hoard money or wealth instead.

The idea of a causal relationship between aggregate demand shortage and the insatiability of liquidity preferences goes back as far as Chapter 17 in Keynes (1936) as described and analyzed in detail by Ono (2001). Moreover, Murota and Ono (2011) provide an explanation of this feature based on behavioral economics. Specifically, they show that this property can be linked to relative status preferences with respect to money. From an empirical point of view, Ono et al. (2004) offer support for the insatiability of liquidity preferences based on quarterly data in Japan using parametric and non-parametric methods.

Ono (1994) introduces heterogeneous households into this framework and redistributive policies are analyzed by Matsuzaki (2003) for consumption taxes and Hashimoto (2004) for intergenerational transfers.[8] In these models, financial markets are assumed to be perfect and agents are heterogeneous only with respect to their initial wealth.

We extend this framework to feature financial market imperfections and introduce heterogeneity in time preference rates to motivate borrowing. This allows us to analyze interactions of collateral, asset prices and aggregate demand. We implement these via a borrowing constraint in the spirit of Kiyotaki and Moore (1997) and Iacoviello (2005) such that the value of collateral is endogenously determined in the housing market.[9]

Research interest in models of secular stagnation has increased substantially in the aftermath of the financial crisis. Michaillat and Saez (2014) and Michau (2017) develop similar models of stagnation that build upon a constant marginal utility of wealth. Eggertsson and Mehrotra (2015) analyze stagnation and the effects of deleveraging in an OLG framework. Eggertsson et al. (2016) extend this setup to the open economy. In addition, some recent contributions analyze the effects of (the burst of) asset price bubbles on economic growth (cf. Boullot, 2017; Hanson and Phan, 2017; Biswas et al., 2017).

Our work is linked to the idea of balance sheet recessions. In our setup, households do continuously pay off debt. However, deflation makes these efforts self-defeating: Because contracts are in nominal terms, deflation increases the real value of debt. In equilibrium, the real debt burden remains unaffected and continues to impede the recovery. Balance sheets are not restored, even in the long run. Deleveraging shocks are also discussed in a similar borrower-saver framework by Eggertsson and Krugman (2012). However, the authors do not model persistent but only temporary stagnation.

Our approach differs from the liquidity trap literature.[10] This view explains aggregate demand shortages as the consequence of negative shocks in combination with a lower bound on the nominal interest rate. Stagnation is a temporary phenomenon. This is in stark contrast to the experience of Japan where deflationary forces already prevail for more than two decades. It is difficult to make the case for the prevalence of price rigidities over such a long period. In our model, stagnation occurs in steady state despite the possibility of continuous price adjustment.

[8]Further extensions include Ono (2006, 2014) and Hashimoto and Johdo (2009) who model persistent stagnation in a two-country framework to analyze the role of FDI and international spillovers of various policies as well as Rodríguez-Arana (2007) who analyzes fiscal deficits under stagnation. Moreover, Murota and Ono (2012) explain zero nominal interest rates and excess reserve holdings by commercial banks in a setup with preferences for deposit holdings. In addition, Ono (2015) applies this framework to the situation of Japan to explain the transition from high growth to secular stagnation based solely on the insatiability of wealth preferences.

[9]We do not explicitly model the asymmetric information problems that give rise to the financial friction (see Townsend, 1979; Stiglitz and Weiss, 1981, among others for a microfoundation).

[10]The modern treatment of this problem started with the seminal paper of Krugman (1998), which initiated an extensive literature, particularly since the global financial crisis.

3 The Model Economy

We use a continuous time model with money-in-the-utility that features competitive firms, two types of households and a central bank but abstracts from taxation or government expenditures. Agents have perfect foresight and there is no uncertainty in the model. We build on Ono (1994, 2001) for the idea of permanent demand shortage based on insatiable liquidity preferences and Iacoviello (2005) for modeling endogenous borrowing constraints with durable assets as collateral to introduce private sector debt.

3.1 Firms

The supply side is modeled as a Lucas tree. Firms are price takers and produce the amount $\bar{y}$ of the consumption good without any inputs or costs. This constitutes the economy's production capacity or a measure of potential output. Yet, actual sales are determined by aggregate demand C_t so that actual income y_t falls short of potential output in case of aggregate demand shortage. Firm sales are hence given by

$$y_t = \min\{C_t, \bar{y}\} \ . \tag{1}$$

Nominal firm profits are simply given by $P_t y_t$ as production is costless. These are distributed equally across households and show up as exogenous income in the budget constraints. When falling short of potential output $\bar{y}$, aggregate demand determines firm profits and household income. As a consequence, there are feedback loops between spending and income.

In addition, we abstract from the labor market and the wage-setting process and instead introduce a reduced-form Phillips curve for the inflation rate π_t. Specifically, the price level dynamics under full employment differ from those in the presence of aggregate demand shortage as follows:

$$\pi_t = \frac{\dot{P}_t}{P_t} = \begin{cases} \mu \ \text{ if } \ C_t = \bar{y} \ , \\ \alpha\left(\dfrac{C_t}{\bar{y}} - 1\right) \ \text{ if } \ C_t < \bar{y} \ . \end{cases} \tag{2}$$

Under full employment, the dynamics of the price level are similar to the standard MIU framework. The price level adjusts to clear the money market and the inflation rate is determined by the growth rate of the money supply μ, such that the quantity equation holds and money is neutral. In contrast, the output gap determines inflation in case of aggregate demand shortage, where the parameter $\alpha > 0$ governs the speed of price adjustment. A negative output gap will result in deflation. If the output gap persists in steady state, deflation will persist.

Similar relations are derived in standard macroeconomic models with a labor market based on downward nominal wage rigidity.[11] Specifically, Ono and Ishida (2014) and Ono (2015) provide the following microfoundation for equation (2) based on fairness concerns in the wage setting process:[12] In their model, the productivity of workers depends on their perception of being treated in a fair way. In particular, workers withhold effort when they are not remunerated at least with a "fair wage". Under full employment, competition among firms for workers determines the wage offer. Therefore, the dynamics of the price level determine the wage dynamics. The former are in turn dependent on the money supply growth. In contrast, firms have bargaining power when there is unemployment. However, the fair wage provides a lower bound on wage offers to prevent shirking. As a consequence, it is the dynamics of the fair wage that determine the wage and hence the price dynamics. These are in turn related to the level of unemployment or the output gap. Taken together, inflation is governed by an expression similar to equation (2), where α can be interpreted as the (exogenous) job separation rate faced by workers.

In addition, the Phillips curve in equation (2) is formally equivalent to wage setting frictions as in Eggertsson and Mehrotra (2015), Michau (2017) or Schmitt-Grohé and Uribe (2016, 2017). All of these contributions introduce some form of downward nominal wage rigidity that becomes binding in case of unemployment. Eggertsson and Mehrotra (2015) assume that wages cannot fall below a "wage norm", which is a linear combination of past wages and the marginal product of labor. In Michau (2017), wage demands of workers are guided by a reference rate of inflation, which creates an asymmetry in the wage dynamics similar to the one discussed above. Finally, Schmitt-Grohé and Uribe (2016, 2017) introduce an exogenous lower bound on the growth rate of the nominal wage that becomes binding in case of unemployment. The same mechanism is used by Hanson and Phan (2017) and Biswas et al. (2017).

It is worth pointing out that our conclusions on the role of asset prices and private sector debt for economic stagnation continue to hold in the presence of a richer modeling of the labor market. Specifically, the introduction of a production function and wage setting frictions in the spirit of Ono and Ishida (2014) does not alter our results qualitatively but comes at the cost of computational complexity. It is for this reason that we decided to rely on the reduced-form expression for inflation introduced above.

[11] As argued by Schmitt-Grohé and Uribe (2016): "There is abundant empirical evidence on downward nominal wage rigidity stemming mostly from developed countries." An overview of the empirical evidence is presented in section 8 of their paper.

[12] In these models, the representative household has a fixed labor endowment. In equilibrium, competitive firms make zero profits, which is why the real wage equals labor productivity, which is constant due to the linear production function. $P_t y_t$ then is not a lump-sum transfer of profits but labor wages and the deflation gap is related to the labor market instead of the commodity market. Yet, the implications for the emergence of stagnation are only modestly affected.

3.2 Households

There is a mass one of infinitely-lived households. Each household is one of two types based on his time preference rate ρ_i: A fraction n of households are savers ($i = 1$) whereas the remaining fraction $1 - n$ are borrowers ($i = 2$) in the sense that $\rho_1 < \rho_2$.[13] This setting will endogenously result in differences in wealth levels and we will hence model an economy in which the "rich" (savers) lend to the "poor" (borrowers).[14]

Households have three means of savings: money $M_{i,t}$, credit contracts $B_{i,t}$ and real assets in the form of housing $h_{i,t}$. Money yields an interest rate of $R_M = 0$ whereas loans are contracted at the non-negative nominal interest rate R_t. Let $B_{i,t} > 0$ denote savings in the form of loans issued and $B_{i,t} < 0$ debt in the form of credit. Let Q_t denote the nominal house price in period t. The return on housing depends on the resale value of the house in the following period.

Then total nominal wealth $A_{i,t}$ is given by the sum of the household's money holdings, bond holdings and the value of its housing stock: $A_{i,t} = B_{i,t} + M_{i,t} + Q_t h_{i,t}$. In real terms, total wealth is given by

$$a_{i,t} = b_{i,t} + m_{i,t} + q_t h_{i,t} \ , \tag{3}$$

where lowercase letters denote the respective variables in real terms such that q_t denotes the real house price defined as $Q_t = P_t q_t$. Households are the owners of firms and receive firm profits $P_t y_t$, where y_t is defined in equation (1). These profits are distributed equally across both types and considered exogenous by the households. In addition, households receive all income from seignorage in a lump-sum transfer $Z_{i,t}$. For the moment, this transfer is not important. Later, we will assume that $\mu = 0$ and hence $Z_{i,t} = 0$. Yet, it becomes relevant for the discussion of $\mu > 0$ in section 6. In real terms, the flow of funds constraint is given by[15]

$$\dot{a}_{i,t} = r_t a_{i,t} - R_t m_{i,t} - (r_t q_t - \dot{q}_t) h_{i,t} - c_{i,t} + y_t + z_{i,t} \ . \tag{4}$$

[13] The borrower-saver separation based on differences in time preference rates is a standard method to introduce borrowing incentives in macroeconomic models, see Sufi (2012). Since these differences are permanent, the roles of lenders and borrowers are static. Alternative ways of modeling include idiosyncratic income shocks or an uneven life-cycle income distribution.

[14] Alternatively, we could assume that agents differ in their initial wealth $a_{i,0}$ such that $a_{1,0} >> a_{2,0}$.

[15] Equation (4) is based on the following expressions for the evolution of nominal and real wealth where we use the composition of household assets to substitute for B_t:

$$\dot{A}_t = R_t B_t + \dot{Q}_t h_t - P_t c_t + P_t y_t = R_t A_t - R_t M_t - R_t Q_t h_t + \dot{Q}_t h_t - P_t c_t + P_t y_t + Z_{i,t}$$

$$\dot{Q}_t = P_t \dot{q}_t + q_t \dot{P}_t$$

$$\dot{a}_t = \left(\frac{\dot{A}_t}{P_t}\right) = \frac{\dot{A}_t}{P_t} - \frac{A_t}{P_t}\frac{\dot{P}_t}{P_t} = (R_t - \pi_t) a_t - R_t m_t - (R_t q_t - \pi_t q_t - \dot{q}_t) h_t - c_t + y_t + z_{i,t}$$

The household incurs opportunity costs when holding money because of the foregone interest income that would be associated with lending. Similar costs arise when investing in housing. Yet, housing investment involves the possibility of capital gains (or losses) associated with changes in the real house price captured by $\dot{q}_t$.

Impatient households have a strong motive to borrow. However, lenders require sufficient collateral in the form of housing because of problems of asymmetric information in the credit market. As a consequence, savers will only lend up to a fraction θ of the value of the borrower's collateralizeable assets.[16] In real terms, the associated borrowing constraint takes the form:

$$b_{2,t} \geq -\theta q_t h_{2,t} \ . \tag{5}$$

In our model, housing is the only durable asset that serves as collateral. In contrast, money is not collateralizable because it is too fungible to be effectively seized by lenders in case of missed repayment.[17]

Apart from differences in time preference, households have identical preferences. They choose consumption, real money holdings and housing to maximize their lifetime utility function:

$$U_i = \int_0^\infty \left[u(c_{i,t}) + v(m_{i,t}) + w(h_{i,t})\right] e^{-\rho_i t} dt \ , \tag{6}$$

where ρ_i denotes the subjective discount rate of the household of type i. Utility from consumption and housing satisfies the Inada conditions. For simplicity, we make the following functional form assumptions on these instantaneous utility functions:

$$u(c_{i,t}) = ln(c_{i,t}) \ ; \quad w(h_{i,t}) = \gamma ln(h_{i,t}) \ ,$$

where $\gamma > 0$ is an exogenous and positive constant. In contrast, the Inada conditions do not hold for the utility from real money balances. As discussed in the previous section and following Ono (1994, 2001), we deviate from the neoclassical assumptions and introduce insatiable liquidity preferences. Formally, the marginal utility of real money holdings does not converge to zero but approaches a strictly positive constant value:

$$\lim_{m \to \infty} v'(m) = \beta > 0 \ .$$

We will explain the consequences of this assumption in the following sections.

[16] We refer to the parameter θ as the loan-to-value ratio. Throughout this paper, we choose parameters to ensure that the borrowing constraint is always binding.

[17] It is easy to introduce a collateral value for money in this setting. The borrowing constraint then becomes $b_{2,t} \geq -\theta_1 q_t h_{2,t} - \theta_2 m_{2,t}$, where θ_2 determines the collateralizability of money. For the special case of $\theta_1 = \theta_2 = \theta$, this formulation, together with (3), implies that (5) becomes a pure wealth constraint where the composition of wealth is irrelevant, i.e. $b_{2,t} \geq -\theta(1-\theta)^{-1} a_{2,t}$. Our main results are unchanged (and even stronger) when using this formulation.

Rich Households (Savers): Savers maximize lifetime utility (6) subject to the wealth composition (3) and the flow budget constraint (4). From the Hamiltonian function:

$$H_1 = u(c_{1,t}) + v(m_{1,t}) + w(h_{1,t}) + \lambda_{1,t}(r_t a_{1,t} - c_{1,t} - R_t m_{1,t} - (r_t q_t - \dot{q}_t)h_{1,t} + y_t) \ ,$$

we obtain the following optimality conditions:

$$\frac{1}{c_{1,t}} = \lambda_{1,t} \ , \tag{7}$$

$$\lambda_{1,t} R_t = v'(m_{1,t}) \ , \tag{8}$$

$$\frac{\gamma}{h_{1,t}} = \lambda_{1,t}(r_t q_t - \dot{q}_t) \ , \tag{9}$$

$$\dot{\lambda}_{1,t} = (\rho_1 - r_t)\lambda_{1,t} \ , \tag{10}$$

$$\lim_{t\to\infty} \lambda_{1,t} a_{1,t} e^{-\rho_1 t} = 0 \ . \tag{11}$$

Equations (7) to (11) describe the optimal consumption, money holdings, housing investment and borrowing of the rich agent as well as the transversality condition for real wealth. For the saver, the nominal interest rate governs both the intertemporal allocation of consumption via (7) and (10) as well as the intra-temporal trade-off between money and consumption according to (7) and (8). This yields the following expression:

$$\frac{\dot{c}_{1,t}}{c_{1,t}} + \rho_1 + \pi_t = R_t = v'(m_{1,t})c_{1,t} \ . \tag{12}$$

In optimum, the rich household equates the marginal rate of substitution between present and future consumption to the marginal rate of substitution between present consumption and money holdings, i.e. the liquidity premium, which also equals the nominal interest rate that constitutes the opportunity cost of holding money. Under neoclassical assumptions, the liquidity premium is declining in $m_{1,t}$, all else equal, thereby stimulating consumption or decreasing the nominal interest rate.

In contrast, with insatiable liquidity preferences, the marginal utility of real money holdings will reach the positive lower bound if the wealth of the patient households is sufficiently high, i.e. $v'(m_1) = \beta$. Then the liquidity premium no longer declines with additional money holdings and $R_t = R_t(c_{1,t})$. As a consequence, consumption of the rich household is unaffected by changes in his money holdings for a given nominal interest rate. For that reason monetary policy becomes ineffective in single agent models such as Ono (2001): Additional money is stored as cash and does no longer stimulate consumption. The economy is trapped in a deflationary steady state despite an infinite expansion of the real money stock.

Poor Households (Borrowers): Borrowers maximize lifetime utility (6) subject to the wealth composition (3), the flow budget constraint (4) and the borrowing constraint (5). Therefore, their Hamiltonian function is given by:

$$H_2 = u(c_{2,t}) + v(m_{2,t}) + w(h_{2,t}) + \lambda_{2,t}(r_t a_{2,t} - c_{2,t} - R_t m_{2,t} - (r_t q_t - \dot{q}_t)h_{2,t} + y_t)$$

$$+\varphi_t(a_{2,t} - m_{2,t} - (1-\theta)q_t h_{2,t}) \ ,$$

from which the following optimality conditions are obtained:

$$\frac{1}{c_{2,t}} = \lambda_{2,t} \ , \tag{13}$$

$$\lambda_{2,t} R_t + \varphi_t = v'(m_{2,t}) \ , \tag{14}$$

$$\frac{\gamma}{h_{2,t}} = \lambda_{2,t}(r_t q_t - \dot{q}_t) + \varphi_t(1-\theta)q_t \ , \tag{15}$$

$$\dot{\lambda}_{2,t} = (\rho_2 - r_t)\lambda_{2,t} - \varphi_t \ , \tag{16}$$

$$\lim_{t\to\infty} \lambda_{2,t} a_{2,t} e^{-\rho_1 t} = 0 \ . \tag{17}$$

Equations (13) to (17) describe optimal consumption demand, money demand, housing investment and borrowing of the poor agent as well as the transversality condition. The borrower also equates the marginal rate of substitution between present and future consumption to the liquidity premium. This results from (13), (14) and (16) and gives

$$\frac{\dot{c}_{2,t}}{c_{2,t}} + \rho_2 + \pi_t = R_t + \varphi_t c_{2,t} = v'(m_{2,t})c_{2,t} \ . \tag{18}$$

The borrowing friction affects optimal money demand and the evolution of consumption. Impatience creates a strong motive to borrow funds for current consumption so that current funds have a higher value to the borrowers than to the savers. When these funds are used to increase liquidity instead of consumption, the household incurs an implicit cost of φ_t due to the borrowing constraint facing in fact a higher implicit interest rate than the saver. As a consequence, optimal money demand is reduced relative to the case without borrowing frictions.

Under neoclassical assumptions, the liquidity premium decreases with money holdings for the borrower, i.e. $v''(m_2) < 0$. In contrast, with insatiable liquidity preferences, $v'(m_2) = \beta > 0$ if the borrower becomes sufficiently wealthy. As a consequence, our model features different regions depending on the behavior of $v'(m_1)$ and $v'(m_2)$, which we will discuss in the following sections.

3.3 Asset Prices, Borrowing and Leverage

What determines the dynamics of the real house price? Households incur opportunity costs when investing in housing because of the opportunity loss of real interest income that is associated with the alternative of bond savings. Yet, agents gain utility from housing which is captured by the user cost, i.e. the marginal rate of substitution between consumption and housing. For the saver, this follows from (7) and (9). For the borrower, housing investment comes at a higher cost, his implicit interest rate being higher than for the saver. Yet, since housing serves as collateral, the associated borrowing costs are lower than those for money at $(1-\theta)\varphi_t$ which can be seen in (13) and (15).

Moreover, changes in the real house price affect the costs and benefits of housing due to valuation effects. In optimum, the real house price adjusts such that agents are indifferent between investing in an additional unit of housing and alternative uses. Hence, the real house price has to appreciate if the opportunity costs from housing exceed the user costs to compensate housing investors for the higher costs with capital gains. Similarly, the real house price has to depreciate if the benefits of housing exceed the opportunity costs resulting in capital losses for house owners. From equations (7), (9), (13) and (15), the dynamics of the real house price can be expressed as the difference between opportunity costs and housing benefits for both agents as

$$\dot{q}_t = r_t q_t - \frac{\gamma c_{1,t}}{h_{1,t}} = r_t q_t - \frac{\gamma c_{2,t}}{h_{2,t}} + \varphi_t (1-\theta) q_t c_{2,t} \ . \tag{19}$$

Throughout the analysis, we consider the case of a strictly binding borrowing constraint, i.e. $\varphi_t > 0$. Then, the borrower always takes on loans up to the maximum given by (5). It follows from (3) and (5) that total real assets of the borrower consist of his money holdings and housing investment, a fraction θ of which serves as collateral:

$$a_{2,t} = m_{2,t} + (1-\theta) q_t h_{2,t} \ . \tag{20}$$

Similarly, total real assets of the saver include loans to the borrower. From (18) and (19), the consumption value of borrowing $\varphi_t c_{2,t}$ - or equivalently, the consumption cost of debt-financed money holdings or housing investment - equals the difference in the liquidity premia and is proportional to the difference in the user cost of housing:

$$v'(m_{2,t})c_{2,t} - v'(m_{1,t})c_{1,t} = \varphi_t c_{2,t} = \frac{1}{1-\theta} \left(\frac{\gamma c_{2,t}}{q_t h_{2,t}} - \frac{\gamma c_{1,t}}{q_t h_{1,t}} \right) \ . \tag{21}$$

Hence, a binding borrowing constraint implies that it is more costly (in terms of consumption) for the borrower to hold money or invest in housing than for the saver.

3.4 Market Equilibrium Conditions

Aggregate demand C_t consists of the consumption demand of savers and borrowers:

$$C_t = nc_{1,t} + (1-n)c_{2,t} \ . \tag{22}$$

Aggregate demand relative to potential output determines the output gap which in turn is related to inflation via (2). In addition, aggregate demand determines firm profits and household income y_t in the flow budget constraints (4).

The central bank perfectly controls the nominal money supply M_t which grows at an exogenous rate μ. Hence, the real money supply m_t evolves as

$$\frac{\dot{m}_t}{m_t} = \mu - \pi_t \ . \tag{23}$$

In contrast, the nominal interest rate R_t is determined endogenously in the money market. It is related to inflation and the real interest rate via the Fisher Equation

$$R_t = r_t + \pi_t \ . \tag{24}$$

Total money demand is the weighted average of the individual money demands. Money market clearing requires that real money demand equals the real money supply m_t:

$$m_t = \frac{M}{P_t} = nm_{1,t} + (1-n)m_{2,t} \ . \tag{25}$$

Loans are financial claims among households. Hence, they are in zero net supply with

$$nb_{1,t} + (1-n)b_{2,t} = 0 \ . \tag{26}$$

In contrast, housing is a real asset. Following Iacoviello (2005), we assume a fixed supply of houses H and abstract from depreciation and construction both of which could easily be implemented in this setting.[18] Market clearing in the housing market then requires

$$nh_{1,t} + (1-n)h_{2,t} = H \ . \tag{27}$$

Equations (1) to (27) fully describe the model economy. The dynamics are summarized by a system of differential equations given by (4), (12), (18), (19) and (23) where (4) applies to both types. All other variables are derived from the solution to this system.

[18]This assumption seems reasonable for an economy like Japan that is characterized by land scarcity and a low price elasticity of the housing supply. A study by Shimizu and Watanabe (2010) concludes that the housing supply was very price inelastic during the Japanese housing boom of the late 1980s, partly due to the incentives given by the tax system as well as regulation on land utilization.

4 Analysis of the Model Economy

In the following analysis, we focus on the special case of a constant nominal money supply,

$$\mu = 0,$$

for simplicity. This implies a zero trend inflation rate under full employment as is evident from (2).[19] Yet, the qualitative conclusions of our analysis can be generalized and hold for any given level of μ as we will discuss in section 6 in greater detail.

Our model framework features three regions depending on the behavior of the marginal utilities $v'(m_1)$ and $v'(m_2)$. This is in turn related to the production capacity $\bar{y}$:

1. For low levels of potential output $\bar{y}$, the economy behaves as in the standard *neo-classical case*. The marginal utility of money is decreasing in money holdings for both households, i.e. $v''(m_i) < 0$, and aggregate demand equals potential output. The price level is constant and changes proportionally with the money supply.

2. For higher levels of $\bar{y}$, there is an *asymmetric* steady state under stagnation. In this region, the patient household's marginal utility of money is constant while the impatient household's liquidity premium still declines with additional money holdings, i.e. $v''(m_1) = 0$ and $v''(m_2) < 0$. Aggregate demand falls short of potential output and deflation occurs.

3. For very high levels of potential output, the *symmetric* steady state under stagnation might occur. In this region, the marginal utility of money has reached its lower bound for savers and borrowers, i.e. $v''(m_i) = 0$.

Among the three mentioned above, we focus on the asymmetric steady state under stagnation for several reasons. First, this steady state features economic stagnation and deflation unlike the neoclassical case. Secondly, indebtedness and asset prices play an important role in affecting the severity of stagnation.[20] Thirdly, it is more in conformity with what has occurred in the Japanese economy as discussed in the introduction. The asymmetric steady state under stagnation is defined as follows:

> *Asymmetric Steady State*: The real and nominal interest rates are constant, the price level is declining at a constant rate, the real consumption level of each household is constant as is the real house price, and the borrower's asset level is constant while the saver's wealth expands infinitely:

$$\dot{r} = 0 \ , \ \dot{R} = 0 \ , \ \pi < 0 \ , \ \dot{c}_1 = 0 \ , \ \dot{c}_2 = 0 \ , \ \dot{q} = 0 \ , \ \dot{a}_1 > 0 \ , \ \dot{a}_2 = 0 \ . \tag{28}$$

[19] This parameterization also allows us to derive some expressions in closed-form that do not depend on the shape of the function $v(m)$, which helps to provide a more intuitive understanding of our results.

[20] In contrast, changes in leverage cease to affect aggregate demand in the symmetric steady state, but simply affect asset prices and the distribution of the housing stock.

4.1 The Occurrence of Persistent Stagnation

Intuitively, aggregate demand shortage occurs if potential output is so high that households are no longer willing to consume the available amount of $\bar{y}$ due to the insatiability of liquidity preferences of the saver.[21] For lower levels of potential output, the economy attains full employment at zero inflation (in the present case as $\mu = 0$) and the price level adjusts to clear the money market for a given level of the nominal money supply in equation (25). We define this full employment steady state as follows:

> *Neoclassical Equilibrium*: The real and nominal interest rates are constant, the price level is constant, the real house price is constant and the consumption and wealth of all households are constant:
>
> $$\dot{r} = 0 \ , \ \dot{R} = 0 \ , \ \pi = 0 \ , \ \dot{c}_1 = 0 \ , \ \dot{c}_2 = 0 \ , \ \dot{q} = 0 \ , \ \dot{a}_1 = 0 \ , \ \dot{a}_2 = 0 \ . \tag{29}$$

Consider first the case of homogeneous agents: Suppose there are only patient households. From (12) with $\dot{c}_1 = 0$, the economy attains full employment, i.e. $c_1 = \bar{y}$, and zero inflation as long as the marginal utility of money can adjust such that $v'(m_{1,t})\bar{y} = \rho_1$. With insatiable liquidity preferences, there is a lower bound β of the marginal utility of real money holdings. Once the production capacity $\bar{y}$ exceeds the level of $\rho_1\beta^{-1}$, there is no longer a solution to (12) that is compatible with $\pi = 0$ and $c_1 = \bar{y}$. This is because households are no longer willing to consume the available output but prefer to accumulate money instead. As a consequence, stagnation and deflation occur in equilibrium.

Similarly, suppose there were only impatient households.[22] From (18) with $\dot{c}_2 = 0$, the economy attains full employment as long as higher spending can be accommodated at zero inflation such that $v'(m_2)\bar{y} = \rho_2$. There is no solution to (18) consistent with full employment once $\bar{y}$ is above $\rho_2\beta^{-1}$. Taken together, the relevant condition for homogeneous agent models is given by $\bar{y} > \rho_i\beta^{-1}$ where ρ_i refers to the representative household. This condition is illustrated by the lower line in Figure 4.

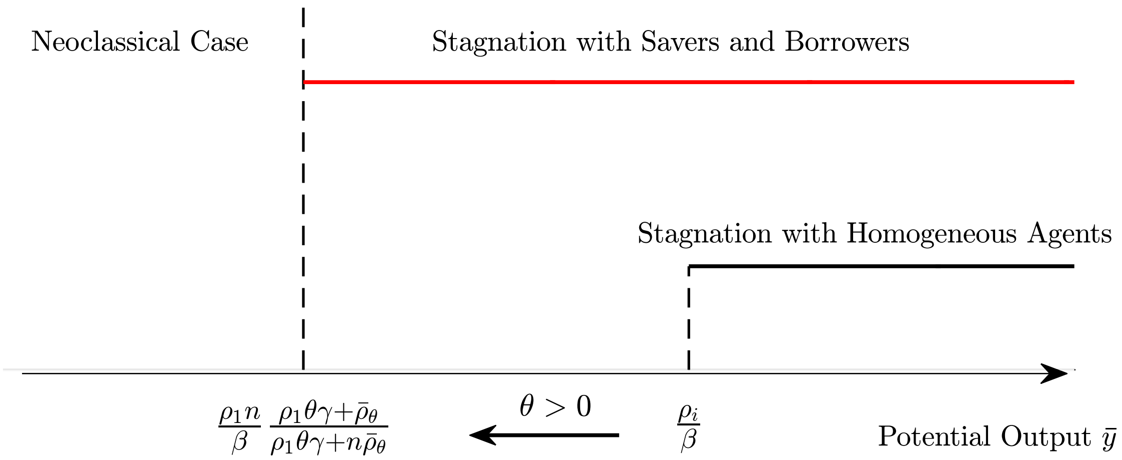
In an economy with n savers and $1 - n$ borrowers, the distribution of consumption spending under full employment determines the occurrence of stagnation. Since $\pi = 0$, we have from (12) and (24) that $R = r = \rho_1$. Then, consumption levels are derived from the flow budget constraints (4). The borrower consumes his income net of interest payments on debt. Income in turn depends on aggregate demand which equals potential output. His consumption in the neoclassical steady state is then given by

$$c_2^{NC} = \frac{\bar{\rho}_\theta}{\bar{\rho}_\theta + \theta\rho_1\gamma}\bar{y} \ , \quad \text{where} \ \ \bar{\rho}_\theta \equiv \theta\rho_1 + (1-\theta)\rho_2 \ . \tag{30}$$

[21]In addition, the liquidity premium of the borrower must still decline with additional money holdings for asymmetric (instead of symmetric) stagnation to occur. This requirement is not important at the moment and will be discuss in greater detail in section 6.

[22]Note that in this case, the borrowing constraint would cease to be binding, i.e. $\varphi_t = 0$.

Figure 4: Occurrence of Persistent Stagnation



Notes: This figure shows the equilibrium of the model for different values of $\bar{y}$ compared to homogeneous agent models. In particular, note that in the heterogeneous agent framework economic stagnation occurs for smaller levels of potential output.

Note that $\bar{\rho}_\theta$ can be interpreted as the debt-weighted average discount rate. This follows from (4), (20) and the requirements $\pi = 0$, $\dot{q} = 0$ and $\dot{a}_2 = 0$. The rich household behaves similarly, but receives interest income on its lending. Hence, steady state consumption of the saver exceeds consumption of the borrower in the neoclassical steady state due to the redistribution associated with ownership of financial assets:

$$c_1^{NC} = \frac{n\bar{\rho}_\theta + \theta\rho_1\gamma}{n\bar{\rho}_\theta + n\theta\rho_1\gamma}\bar{y} = \frac{n\bar{\rho}_\theta + \theta\rho_1\gamma}{n\bar{\rho}_\theta}c_2^{NC} > c_2^{NC} \ . \tag{31}$$

It is easy to see from these expressions that aggregate demand equals potential output. Yet, it follows from (12) and (18) with $\dot{c}_i = 0$ that the consumption levels of both agents in (30) and (31) are consistent with zero inflation only if the marginal utility of money falls sufficiently. In particular, for the neoclassical case to exist it has to hold that

$$v'(m_{1,t}) = \frac{\rho_1}{c_1^{NC}} \quad \text{and} \quad v'(m_{2,t}) = \frac{\rho_2}{c_2^{NC}} \ . \tag{32}$$

With insatiable liquidity preferences, there exists a lower bound on the marginal utility of money such that $v'(m_{i,t}) \geq \beta$. Hence, the neoclassical case is not feasible once $\beta c_i^{NC} > \rho_i$. So with rising levels of potential output $\bar{y}$, at some threshold the stagnation steady state will occur. Since $\rho_1 < \rho_2$ and $c_1^{NC} > c_2^{NC}$, it is the saver's marginal utility of money that will reach its lower bound first for rising levels of $\bar{y}$. Then, aggregate demand falls short of the production capacity and the economy enters the stagnation steady state. We derive the following proposition from combining (31) and (32):

Proposition 1 *The neoclassical equilibrium with full employment and zero inflation cannot be attained once potential output exceeds the following threshold:*

$$\widetilde{y} \equiv \frac{\rho_1 n}{\beta} \frac{\theta\rho_1\gamma + \bar{\rho}_\theta}{\theta\rho_1\gamma + n\bar{\rho}_\theta} < \frac{\rho_1}{\beta} \ . \tag{33}$$

The threshold $\widetilde{y}$ *is affected by the model parameters as follows (see Appendix A):*

$$\frac{\partial\widetilde{y}}{\partial\beta} < 0 \ , \quad \frac{\partial\widetilde{y}}{\partial\rho_1} > 0 \ , \quad \frac{\partial\widetilde{y}}{\partial\rho_2} > 0 \ , \quad \frac{\partial\widetilde{y}}{\partial\theta} < 0 \ , \quad \frac{\partial\widetilde{y}}{\partial n} > 0 \ , \quad \frac{\partial\widetilde{y}}{\partial\gamma} < 0 \ .$$

Once potential output exceeds $\widetilde{y}$, the economy is in the asymmetric steady state under stagnation defined in (28) and suffers from insufficient demand and deflation. Additional income does no longer stimulate consumption of the saver who chooses to accumulate wealth instead. This is represented by the upper line in Figure 4.

The lower the insatiability parameter β, the higher potential output needs to be for the economy to enter stagnation. Similarly, increases in the time preference rate of the saver ρ_1 or in their fraction of the population n also increase the income threshold. The same holds for a higher time preference rate ρ_2 of the borrower.

What we add is the insight that financially more developed countries, i.e. countries with higher leverage, drift into stagnation already at a lower level of potential output. This is because the higher debt is associated with lower steady state consumption demand from the borrower. To see this, note from (30) that if financial markets are closed and no borrowing is possible, i.e. if $\theta = 0$ or $\gamma = 0$, the consumption levels of both households are equal and given by $c_i = \bar{y}$ under full employment. Once we allow for borrowing, housing investment is associated with an increase in indebtedness of the borrower. This in turn results in a higher real interest burden on poor households and reduces their affordable consumption. This gives rise to a more unequal income distribution but does not affect aggregate demand as long as the rich households expand their consumption accordingly. If they invest in liquidity holdings instead, aggregate demand falls short of the economy's production capacity and stagnation occurs.[23]

Let us contrast this condition with the existence condition in single-agent models without lending and borrowing as in Ono (2001), which was discussed above. Condition (33) is reduced to this expression if we abstract from housing ($\gamma = 0$), if we do not allow for borrowing ($\theta = 0$) or if there are only rich households ($n = 1$). In all other cases, $\widetilde{y}$ is below the threshold of the single-agent model. Hence, the economy enters stagnation in an earlier stage, which is illustrated in Figure 4. The reason is that consumption of the saver is higher due to additional income associated with interest payments on loans.

[23]The same effect arises for a higher value of γ. Economies that invest more heavily in assets in fixed supply are hence more prone to stagnation. Also note that the effects of θ and γ are mutually reinforcing.

4.2 The Asymmetric Steady State under Persistent Stagnation

Under asymmetric stagnation, higher consumption spending of the borrower stimulates consumption spending of the saver. This follows from (12) with $\dot{c}_1 = 0$ and (2) to substitute for π. The reasoning is as follows: An increase in consumption of the borrower expands aggregate demand and mitigates deflation. Less deflation in turn increases the nominal interest rate via (24) since the real rate equals the saver's time preference rate ρ_1 as obtained from (10) with $\dot{\lambda}_1 = 0$. The nominal rate has to equal the marginal rate of substitution between consumption and money holdings for the saver and since the marginal utility of money is constant due to insatiability, consumption c_1^* has to increase. This is the same relation as in Ono (1994) and Matsuzaki (2003) and results from the insatiability of liquidity preferences in combination with sluggish price adjustment as manifested in the Phillips curve relation in (2):[24]

$$c_1^* = \frac{(\rho_1 - \alpha)\bar{y}}{\beta\bar{y} - \alpha n} + \frac{\alpha(1-n)}{\beta\bar{y} - \alpha n} c_2^* \ . \tag{34}$$

Spillovers from aggregate demand are stronger the higher the share of spending constrained households $(1-n)$ and the higher the speed of price adjustment α. In particular, steady state consumption c_1^* is not directly affected by the borrowing decision or asset composition of the impatient household. Yet, there are indirect effects via c_2^*.

In contrast, consumption spending of the borrower is affected by his money holdings in equilibrium. This follows from optimality condition (18) with $\dot{c}_2 = 0$ and (2) and (34) to substitute for π^* and c_1^* and implies

$$c_2^* = \frac{\chi}{v'(m_2^*)(\beta\bar{y} - \alpha n) - \beta\alpha(1-n)} \ , \tag{35}$$

$$\text{where } \chi \equiv \rho_2(\beta\bar{y} - \alpha n) - \alpha(\beta\bar{y} - \rho_1 n) \ .$$

More money induces more consumption of the borrower. This is necessary to equalize the liquidity premium to the nominal interest rate which is itself a function of c_2^* via (12) and (34). Note that this channel does not exist for the saver who accumulates money without expanding consumption.

Importantly, we require parameter restrictions to guarantee positive consumption levels c_1^* and c_2^* in steady state. From (34) and the denominator of (35), since $v'(m_2^*) > \beta$, we make the following assumptions throughout this paper:

$$\text{(i) } \rho_1 > \alpha \quad \text{and} \quad \text{(ii) } \beta\bar{y} > \alpha \ . \tag{36}$$

[24] Steady state values of the respective variables will be characterized by "*" for notational convenience.

In addition, the shadow value of borrowing is determined by the difference in discount rates. From (12) and (18) with $\dot{c}_{1,t} = \dot{c}_{2,t} = 0$, the Lagrange parameter on the borrowing constraint is given by

$$\varphi^* = \frac{\rho_2 - \rho_1}{c_2^*} > 0 \ . \tag{37}$$

Hence, the borrowing constraint is binding in equilibrium. Higher money holdings of the borrower reduce the value of additional funds since c_2^* increases in m_2^* as discussed above.

Total wealth of the borrower consists of money holdings and housing investment net of loans via (20). The steady state value of housing investment of each agent in turn is a constant fraction of its consumption level given by

$$q^* h_1^* = \frac{\gamma}{\rho_1} c_1^* \quad \text{and} \quad q^* h_2^* = \frac{\gamma}{\bar{\rho}_\theta} c_2^* \quad , \quad \text{where} \quad \bar{\rho}_\theta \equiv \theta \rho_1 + (1-\theta)\rho_2 \ . \tag{38}$$

This follows from (16) with $\dot{\lambda}_2 = 0$ and from (19). Increases in θ provide higher incentives for borrowers to invest in housing because of its role as collateral. Note that (5) and (38) imply that the real level of debt is constant. Hence, borrowers do delever in nominal terms, which however is self-defeating due to deflation. Substituting (35) solved for m_2^* and (38) into (20) implies the following expression for the real wealth of the borrower in steady state:

$$a_2^* = \frac{(1-\theta)\gamma}{\bar{\rho}_\theta} c_2^* + m_2^* \ , \tag{39}$$

where m_2^* is a function of c_2^* given by (35). It follows that real wealth and consumption demand are positively related for the borrower. An increase in consumption induces both higher money holdings and higher housing investment, and hence higher real wealth.

Higher consumption demand implies higher housing demand, as is clear from (38), because they are substitutes. This is true for both types of households. Market clearing in the housing market then requires a higher equilibrium house price in response to an increase in aggregate demand. This follows from the market clearing condition (27) in combination with steady state housing demands. The real house price obtained from (27) and (38) is given by

$$q^* = \frac{\gamma}{H} \left[\frac{n}{\rho_1} c_1^* + \frac{1-n}{\bar{\rho}_\theta} c_2^* \right] \ . \tag{40}$$

From (34), $c_1^* = c_1^*(c_2^*)$ and hence $q^* = q^*(c_2^*)$. An increase in consumption of the borrower increases the real house price in steady state. Also note that the house price increases with the housing preference γ and decreases with a higher supply H, all else equal.

The borrower's real assets are constant in the asymmetric steady state. From the budget constraint (4) with $\dot{a}_2 = 0$ and (20), (22) and (38), we get

$$n c_1^* + (1-n) c_2^* + (\rho_1 - \beta c_1^*) m_2^* = \left(\frac{\theta \rho_1 \gamma}{\bar{\rho}_\theta} + 1 \right) c_2^* \ , \tag{41}$$

where $c_1^* = c_1^*(c_2^*)$ from (34) and $m_2^* = m_2^*(c_2^*)$ from (35). The borrower obtains real income from two sources: First, the household receives firm profits which are determined by aggregate demand. This is reflected in the term $nc_1^* + (1-n)c_2^*$ of (41). Secondly, inflation affects the real return on money. Since money does not pay interest, the real return is given by the rate of *deflation*, i.e. $-\pi^* = \rho_1 - \beta c_1^*$. This income is used to finance consumption expenditures c_2^* and to pay interest on debt. These interest payments depend on the household's borrowing capacity which is determined by the value of its housing collateral via (5). The collateral value in turn is related to consumption as is clear from (38). In equilibrium, real interest payments are a fraction $\theta\rho_1\gamma\bar{\rho}_\theta^{-1}$ of consumption and increase with θ, γ and ρ_1 but decrease with ρ_2, because $\bar{\rho}_\theta = \theta\rho_1 + (1-\theta)\rho_2$.

Finally, it follows from (25) that the real money stock becomes infinitely high because of the effects of deflation. The increase in the real money supply exclusively benefits the saver in the asymmetric steady state. Hence, his real wealth expands at the rate of deflation. However, this does not violate the transversality condition (13) since it holds that $\rho_1 > \alpha > -\pi^*$. In contrast, increases in the real money stock accrue to both agents in the symmetric steady state, again without violating the transversality conditions.

Equations (34) to (41) define the asymmetric steady state under persistent stagnation. Once m_2^*, c_1^* and c_2^* are jointly determined from the combination of (34), (35) and (41), all other variables are derived from these values.

Model Dynamics under Asymmetric Stagnation The model dynamics are represented by a system of six differential equations for consumption and real assets of savers and borrowers, the real house price and the real money supply. All other variables can be derived from this system: From (2) and (22), it follows that $\pi_t = \pi(c_{1,t}, c_{2,t})$. From (12) with $v'(m_{1,t}) = \beta$, we have $R_t = R(c_{1,t})$ and hence $r_t = r(c_{1,t}, c_{2,t})$ from (24). Given $c_{1,t}, c_{2,t}, q_t$ and $a_{2,t}$, equations (20), (21) with $v'(m_{1,t}) = \beta$ and (27) determine money holdings $m_{2,t}$ and housing investments $h_{1,t}$ and $h_{2,t}$. From (21), it follows that the implicit cost due to the borrowing constraint, φ_t, is a function of the same four variables.

The evolution of the saver's consumption is determined by (12) with $v'(m_{1,t}) = \beta$:

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \beta c_{1,t} - \pi_t - \rho_1 \ . \tag{42}$$

Since $\pi_t = \pi(c_{1,t}, c_{2,t})$, it follows that $\dot{c}_{1,t} = \dot{c}_1(c_{1,t}, c_{2,t})$. Similarly, the evolution of the borrower's consumption is determined by (18) with $v'(m_{2,t}) > \beta$:

$$\frac{\dot{c}_{2,t}}{c_{2,t}} = v'(m_{2,t})c_{2,t} - \pi_t - \rho_2 \ . \tag{43}$$

Since $\pi_t = \pi(c_{1,t}, c_{2,t})$ and $m_{2,t} = m_2(c_{1,t}, c_{2,t}, q_t, a_{2,t})$, we have $\dot{c}_{2,t} = \dot{c}_2(c_{1,t}, c_{2,t}, q_t, a_{2,t})$.

The evolution of the real house price is determined by (19):

$$\frac{\dot{q}_t}{q_t} = r_t - \frac{\gamma c_{1,t}}{q_t h_{1,t}} \ . \tag{44}$$

With $r_t = r(c_{1,t}, c_{2,t})$ and $h_{1,t} = h_1(c_{1,t}, c_{2,t}, q_t, a_{2,t})$, we have $\dot{q}_t = \dot{q}(c_{1,t}, c_{2,t}, q_t, a_{2,t})$. The evolution of the real wealth of both agents is determined by (4) where we use (19) to substitute for $\dot{q}_t - r_t q_t$, (18) for φ_t in (19) and (22) for y_t:

$$\dot{a}_{1,t} = -\pi_t a_{1,t} + \beta c_{1,t} \left[\theta n^{-1} q_t H + (1-\theta) q_t h_{1,t}\right] - (1-n+\gamma) c_{1,t} + (1-n) c_{2,t} \ , \tag{45}$$

$$\dot{a}_{2,t} = -\pi_t a_{2,t} + v'(m_{2,t}) c_{2,t} (1-\theta) q_t h_{2,t} - (n+\gamma) c_{2,t} + n c_{1,t} \ . \tag{46}$$

It is easy to see that $\dot{a}_{1,t} = \dot{a}_1(c_{1,t}, c_{2,t}, q_t, a_{1,t}, a_{2,t})$ and $\dot{a}_{2,t} = \dot{a}_2(c_{1,t}, c_{2,t}, q_t, a_{2,t})$. Finally, the real money supply decreases with the inflation rate as is clear from (25):

$$\frac{\dot{m}_t}{m_1} = -\pi_t \ . \tag{47}$$

Therefore, it holds that $\dot{m}_t = \dot{m}(c_{1,t}, c_{2,t}, m_t)$. Equations (42) to (47) fully describe the economy together with the initial asset levels $a_{1,0}$ and $a_{2,0}$. Given paths for these variables, we can derive the associated paths of all other variables. This system satisfies saddle-point stability around the asymmetric steady state as shown in Appendix B. In the following section, we analyze the dynamic and static properties of this steady state.

5 Asset Prices and Leverage Under Stagnation

As shown above, private sector debt affects the occurrence of persistent stagnation. In addition, leverage and asset prices also affect aggregate demand under stagnation. Specifically, we show that credit booms can temporarily mask aggregate demand insufficiency. However, this comes at the cost of more severe stagnation in the new steady state.

5.1 Credit and Asset Price Booms under Stagnation

We have argued that an economy can enter an equilibrium of persistent stagnation as a consequence of the debt burden of some households. However, an expansion of debt via financial liberalization can in the short run mask aggregate demand shortage by creating a temporary credit and asset price boom. Specifically, consider an economy that is suffering from insufficient aggregate demand. Suppose that lending standards loosen such that borrowers can take on more loans per unit of housing net worth. This setup is in line with the claims of Larry Summers about the U.S. economy during the early

2000s and also mirrors several features of the situation of Japan in the late 1980s that we described in the introduction.[25]

Figure 5 shows the associated model dynamics as deviations from the initial steady state for two values of the housing preference parameter. The increase in the loan-to-value ratio triggers a substantial credit boom. Borrowers can acquire new funds for a given collateral value some of which they consume and some of which they hold as money or invest in new housing. These funds are provided by savers and financed by their money holdings and the sale of houses. What follows is a temporary boom in both the real economy and the housing market.

The credit boom stimulates aggregate demand as borrowers increase their consumption. This creates inflation which lowers the real interest rate and stimulates consumption of the savers as well. As a consequence, the nominal interest rate increases. If the credit boom is sufficiently strong, the economy can temporarily return to full employment with aggregate spending being constrained by potential output.

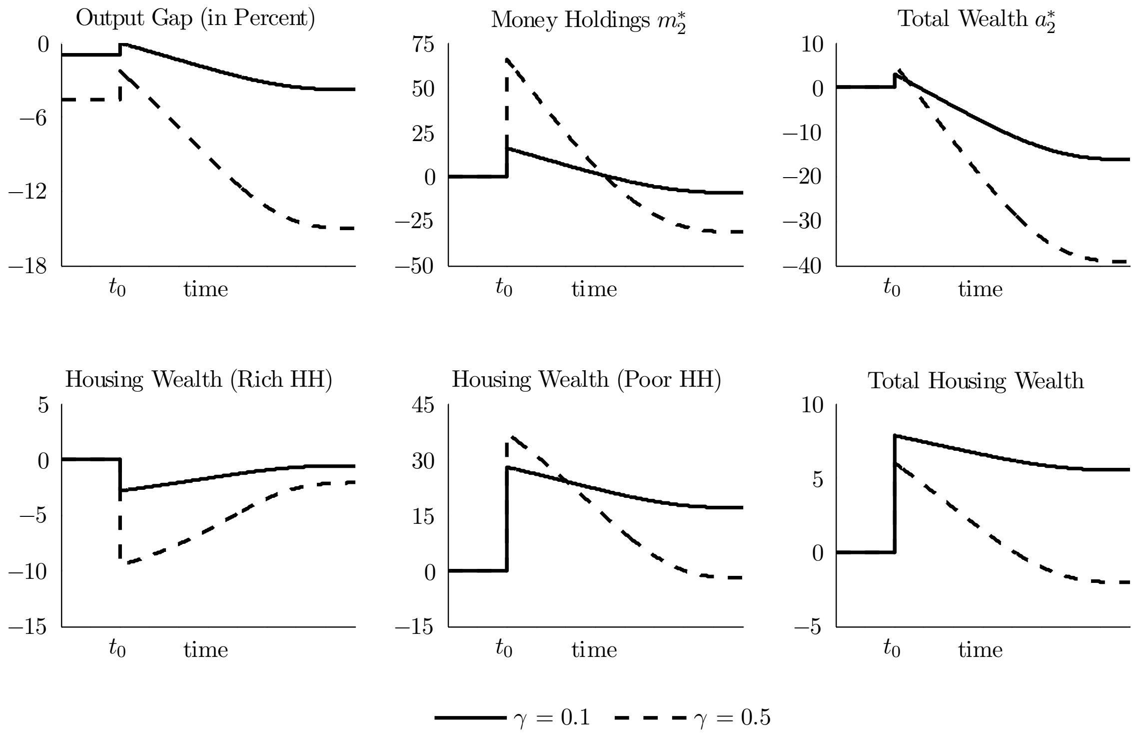
In addition, an asset price boom ensues since the real house price surges as housing demand of impatient households increases. The initial jump in the house price has a positive valuation effect on the housing holdings of both agents, which increases the real value of their assets. A feedback loop sets in with higher house prices increasing the collateral value of borrowers which in turn enhances their borrowing ability thereby reinforcing the initial credit boom. The housing allocation shifts in favor of the impatient households, which further strengthens the value of their collateral.[26].

The allocation of new funds among consumption, money and housing investment is guided by the parameters in the utility function of the borrower. Higher impatience implies a stronger increase in consumption and hence aggregate demand and inflation. In contrast, higher preferences for housing imply that more of the newly available funds are spent on purchasing fixed supply assets. As a consequence, higher preferences for housing imply a stronger amplification of the dynamics because of more pronounced collateral effects. In fact, aggregate consumption might actually fall during the credit boom for very high values of γ. Figure 5 illustrates the dependency of the dynamic responses on γ.

[25]Note that we proxy the credit boom by variations in θ but do not make explicit claims about the origin of this variation. The sources of the Japanese credit boom are still up to debate. Yet, Posen (2003) argues that both partial deregulation in corporate finance and a relaxation of lending standards in the mortgage market with mortgage limits rising from 65% of the home value on average to 100% played a major role for the Japanese credit boom. According to Posen (2003), "there is a consensus view among economists on how partial financial deregulation in Japan in the 1980s led to a lending boom". The effects of deregulation and financial liberalization are also well-documented in Tsuruta (1999).

[26]This is the same propagation mechanism as described in Kiyotaki and Moore (1997) and Iacoviello (2005) among others, which creates amplification and persistence of shocks. Note that we only consider the case of a binding borrowing constraint. For a treatment of occasionally binding constraints, see Guerrieri and Iacoviello (2015).

Figure 5: Dynamic Effects of Financial Liberalization



Notes: This figure shows the dynamics associated with a permanent increase in the loan-to-value ratio from $\theta = 0.15$ to $\theta = 0.5$. The output gap is given in percentage points. All other variables are depicted as deviations from the *initial* steady state in percent. We assume the following utility from money for the borrower: $v(m_{2,t}) = \beta m_{2,t} + \delta ln(m_{2,t})$. The figure is based on the following calibration: $\beta = 0.0005, y = 100, \rho_1 = 0.05, \rho_2 = 0.1, \alpha = 0.01, n = 0.5, H = 1$ and $\delta = 0.1$. Simulations are based on a modification of the relaxation algorithm of Trimborn et al. (2008).

Over time, some of the newly acquired assets are sold by the borrower to smooth its consumption and for interest payments. Therefore, the allocation of the housing stock reverts in favor of the saver and aggregate demand remains above its new equilibrium for a prolonged period, thereby masking the underlying demand deficiency. Yet, eventually the resulting debt overhang pushes the economy into persistent stagnation, which is worse than before the credit boom, as we will show in the next subsection.

Also note that a house price boom, which is typically modeled by an increase in γ (cf. Iacoviello, 2005), can temporarily stimulate the stagnating economy though at the cost of more severe stagnation in the long run. The argument is similar: Higher housing demand creates an immediate increase in the real house price resulting in valuation gains for both households. In addition, the value of collateral that borrowers can pledge for funds increases, which initiates a credit boom. These funds are used to increase consumption, money holdings and housing investment, the last of which feeds back into the value of borrowers' collateral. The dynamics are similar to those in Figure 5.

5.2 Debt Overhang and Stagnation

While increases in θ and γ can temporarily stimulate aggregate demand by initiating a credit boom, they also affect the properties of the stagnation steady state. The former represents financial liberalization - or the degree of sustainable finance - whereas the latter is a proxy for the level of asset prices. Higher leverage θ and a higher house price reduce aggregate demand in the asymmetric steady state and hence worsen economic stagnation. This is summarized in the following proposition (see Appendix C for the proof):

Proposition 2 *In the asymmetric steady state under stagnation, an increase in the loan-to-value ratio reduces aggregate demand and worsens deflation. It holds that*

$$\frac{dC^*}{d\theta} < 0 \ , \quad \frac{dc_1^*}{d\theta} < 0 \ , \quad \frac{dc_2^*}{d\theta} < 0 \ , \quad \frac{dm_2^*}{d\theta} < 0 \ , \quad \frac{da_2^*}{d\theta} < 0 \ , \quad \frac{d\pi^*}{d\theta} < 0 \ .$$

The same effects arise from an increase in the housing preference γ. It holds that

$$\frac{dC^*}{d\gamma} < 0 \ , \quad \frac{dc_1^*}{d\gamma} < 0 \ , \quad \frac{dc_2^*}{d\gamma} < 0 \ , \quad \frac{dm_2^*}{d\gamma} < 0 \ , \quad \frac{da_2^*}{d\gamma} < 0 \ , \quad \frac{d\pi^*}{d\gamma} < 0 \ .$$

Consider intuitively the effects of an increase in the loan-to-value ratio θ. Initially, the borrowing constraint (5) is relaxed allowing the borrower to acquire new funds, as described above. However, the new steady state is associated with higher debt and hence higher real interest payments as the steady state real interest rate is not affected. These payments are a fraction $\theta\rho_1\gamma\bar{\rho}_\theta^{-1}$ of the borrower's consumption spending where $\bar{\rho}_\theta$ is decreasing in θ. This can be seen from (41). Therefore, higher leverage is associated with higher interest costs per unit of consumption which implies that the borrower's income is not sufficient to cover expenditures for a given c_2^* once θ increases. As this would violate the lifetime budget constraint of the borrower, his consumption spending has to decline.

This implies that the expenditures of the borrower are reduced ("spending effect"), raising disposable income. However, the lower spending negatively affects the borrower's income since aggregate demand declines ("demand effect"). This partially offsets the first effect. In addition, the real return on money holdings is affected ("capital gains effect"): Higher deflation increases the return on money. Yet, lower consumption discourages money holdings. The first effect is stronger, the higher money holdings, but the net effect is always negative in the asymmetric stagnation case (see Appendix B). This implies that a decrease in consumption reduces expenditures. Hence, consumption must decline in response to an increase in θ. These effects can be seen from the total differential of (41):

$$\underbrace{\left(n\frac{dc_1^*}{dc_2^*} + 1 - n\right)\frac{dc_2^*}{d\theta}}_{\text{Demand Effect}} + \underbrace{\left(-\pi^*\frac{dm_2^*}{dc_2^*} - m_2^*\frac{d\pi^*}{dc_2^*}\right)\frac{dc_2^*}{d\theta}}_{\text{Capital Gains Effect}} - \underbrace{\left(1 + \frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\right)\frac{dc_2^*}{d\theta}}_{\text{Spending Effect}} = \underbrace{\frac{\rho_1\rho_2\gamma}{\bar{\rho}_\theta^2}c_2^*}_{\text{Interest Cost}} \ . \quad (48)$$

The decrease in consumption of the borrower feeds back into the other variables of the model. Aggregate demand decreases, which aggravates deflation via (2). Deflation in turn reduces the nominal interest rate via (24) since the real rate is determined by ρ_1. This reduces consumption of the saver, which can be seen in (34). In addition, money demand of the borrower declines, as is clear from (35), as does the borrower's real wealth, which can be seen in (39).

The effects of a higher θ on the real house price and the distribution of the housing stock are ambiguous because of two opposing effects on q^*: On the one hand, investment in housing becomes more attractive for a given level of consumption c_2^* since housing becomes more collateralizeable. Higher housing demand bids up the house price. On the other hand, there is an indirect effect on the house price because lower consumption spending decreases housing demand of both agents which in turn lowers the real house price. This can be seen from (40).[27]

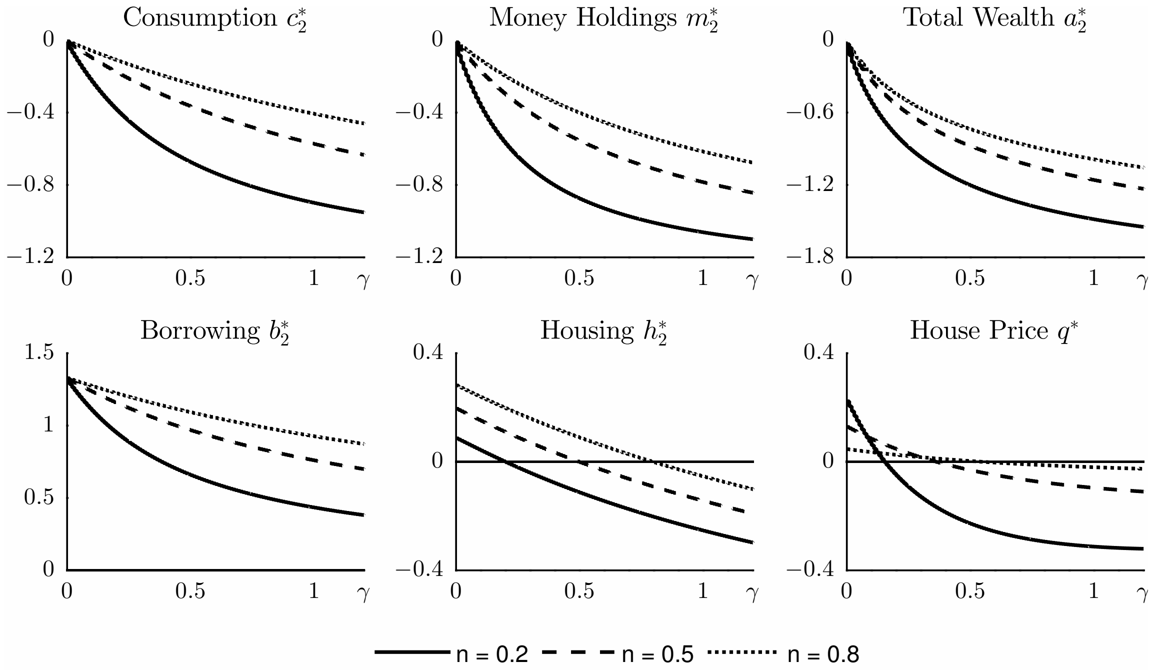
The preference for housing γ determines the relative strength of these effects. The higher γ, the weaker the effects associated with the higher collateral value relative to the negative effect on consumption. If γ is sufficiently high, the indebtedness of the borrower might actually *decline* in response to financial liberalization since housing is reallocated to the saver. The reason is that higher levels of γ are associated with a higher collateral value and hence higher household debt. A given change in θ needs to be balanced by a stronger decline in consumption and hence a stronger reduction in housing demand.

Figure 6 illustrates the effects of a rise in the loan-to-value ratio θ on the steady state. Each subplot shows the elasticity of the respective variable to a rise in θ as a function of the housing preference parameter γ for three different values of n. We set parameters such that the economy is at full employment for $\theta = 0$. In particular, note the negative effect on the borrower's housing investment and the real house price for large values of γ. This in turn implies that financial liberalization is associated with a substantial decrease in the real wealth of the poor household. Also, the responses of consumption and asset prices are stronger the higher the share of poor households.

In the literature, γ is typically calibrated to match empirical observations on the housing market. In a similar framework, Iacoviello (2005) chooses a value of $\gamma = 0.1$ to match the value of residential housing to output in the United States. Guerrieri and Iacoviello (2015) follow a similar approach in a model with endogenous housing supply and select a value of $\gamma = 0.04$. When we apply the same criterion, the implied value of γ ranges between 0.08 and 0.1 and is substantially below unity. This implies the dominance of the collateral channel and hence financial liberalization raises asset prices and credit-financed housing investment.

[27]What is clear from (38), is that the equilibrium value of the saver's housing investment declines with θ while the effect on the value of the borrower's housing investment is unclear.

Figure 6: Elasticities with Respect to θ



Notes: This figure shows the elasticities of the model variables with respect to the loan-to-value ratio θ as a function of the housing preference γ and for three different values of n. The y-axis shows the %$-$change in each variable in response to a 1% increase in θ in steady state. We assume the following utility from money for the borrower: $v(m_{2,t}) = \beta m_{2,t} + \delta ln(m_{2,t})$. The calibration is as follows: $\beta = 0.0005, \bar{y} = 100, \rho_1 = 0.05, \rho_2 = 0.1, \alpha = 0.01, \delta = 0.1, \theta = 0.5$ and $H = 1$.

These effects are in stark contrast to the standard neoclassical case with $v''(m_i) < 0$ for both types of households. From (30), it is clear that an increase in indebtedness reduces the consumption demand of the borrower in the neoclassical steady state since this agent faces higher real interest payments. Yet, aggregate demand is unaffected by variations in θ or γ because the saver increases his consumption level accordingly as long as his liquidity premium is decreasing in real money holdings, which can be seen from (31). As a consequence, changes in these parameters do result in a redistribution of available income and hence of consumption spending and housing investment. However, they do not trigger deviations from full employment because aggregate demand is not affected by these changes. In addition, the price level will adjust to clear the money market, which can be inferred from (25).

Similarly, aggregate demand is no longer affected by variations in these parameters once the model economy is in the symmetric steady state under stagnation. Then, variations in θ or γ cease to affect the consumption spending of both agents and simply lead to a redistribution of the housing stock and changes in the real house price. This case as well as other extensions of the model will be discussed in the next section.

6 Model Extensions and Discussion

In this section, we analyze two extensions of the model that have been turned off so far in order to focus on the core mechanism. In addition, we discuss policy recommendations.

6.1 Asymmetric and Symmetric Stagnation

From Proposition 1, we know that stagnation does not occur for sufficiently low levels of potential output. In addition, it is clear from (32) and the discussion in the previous section that the borrower will also eventually choose to accumulate money holdings if his consumption level is sufficiently high. More specifically, it follows from (35) that symmetric stagnation will occur once the borrower's consumption level has reached the critical threshold of $\chi/\beta(\beta\bar{y}-\alpha)$. We first derive a sufficient condition for asymmetric stagnation to prevail and then give an intuition for the occurrence of the symmetric case.

Under symmetric stagnation, $v'(m_{1,t}) = v'(m_{2,t}) = \beta$ and both households accumulate wealth infinitely. Consumption of neither type is stimulated by additional money. Formally, the symmetric steady state is defined as follows:

> *Symmetric Steady State*: The real and nominal interest rates are constant, the price level is declining at a constant rate, the real house price is constant, the real consumption levels are constant but the wealth of each household expands infinitely:

$$\dot{r} = 0\ ,\ \dot{R} = 0\ ,\ \pi < 0\ ,\ \dot{c}_1 = 0\ ,\ \dot{c}_2 = 0\ ,\ \dot{q} = 0\ ,\ \dot{a}_1 > 0\ ,\ \dot{a}_2 > 0\ . \tag{49}$$

The economy enters stagnation once potential output exceeds $\widetilde{y}$ defined in (33). Then, $v'(m_{1,t}) = \beta$ and there is deflation and demand shortage, i.e. $\pi < 0$ and $C < \bar{y}$ from (2) and (22). Consider the population-weighed average of (12) and (18) with $\dot{c}_1 = \dot{c}_2 = 0$:

$$n\rho_1 + (1-n)\rho_2 = \beta n c_1 + v'(m_2)(1-n)c_2 - \pi\ . \tag{50}$$

Symmetric stagnation cannot occur if $\beta\bar{y} < n\rho_1 + (1-n)\rho_2$. To see this, suppose we have $v'(m_{2,t}) = \beta$ and $\beta\bar{y} < n\rho_1 + (1-n)\rho_2$. Then from (2), (22) and (50), we get

$$\beta\bar{y} - \alpha < n\rho_1 + (1-n)\rho_2 - \alpha = \beta[nc_1 + (1-n)c_2] - \pi - \alpha = (\beta\bar{y}-\alpha)\frac{C}{\bar{y}}\ .$$

This only holds for $C > \bar{y}$ which is not the case. Hence, we always have $v'(m_{2,t}) > \beta$ for $\beta\bar{y} < n\rho_1 + (1-n)\rho_2$. Together with Proposition 1 and Condition (36), this yields the following proposition:

Proposition 3 *Given the parameter restrictions $\rho_1 > \alpha$ and $\beta\bar{y} > \alpha$, the following condition is sufficient for the asymmetric steady state under stagnation to occur:*

$$n\rho_1 \left(\frac{\theta\rho_1\gamma + \bar{\rho}_\theta}{\theta\rho_1\gamma + n\bar{\rho}_\theta} \right) < \beta\bar{y} < n\rho_1 + (1-n)\rho_2 \ . \tag{51}$$

The first inequality in (51) follows from (33) and ensures that aggregate demand falls short of potential output and the second inequality ensures asymmetry. Intuitively, the second condition requires that the time preference rate ρ_2 is sufficiently high so that borrowers still strive for higher consumption. Yet, note that an increase in ρ_2 also tightens the first inequality, which is clear from Proposition 1.

Importantly, (51) is a sufficient condition for the existence of the asymmetric steady state but not a necessary condition. Under certain conditions, the asymmetric stagnation case will prevail for higher values of potential output. This is the case when further increases in potential output do not stimulate the borrower's consumption to exceed the threshold discussed above. We discuss the necessary existence condition in detail in Appendix B and only provide some intuition here.

Intuitively, the borrower's consumption depends on two factors as can be seen from (41): Income from firm profits which are determined by aggregate demand and capital gains on money holdings which depend on the rate of deflation. Under stagnation, an increase in the economy's production capacity worsens deflation which has two effects on the borrower's income. On the one hand, deflation reduces the consumption incentives of the saver. This reduces the income of the borrower since aggregate demand declines ("aggregate demand effect"). On the other hand, the purchasing power of money holdings rises which stimulates the borrower's consumption ("capital gains effect"). The second effect is stronger the higher his money holdings. If the capital gains effect dominates, the borrower's consumption increases with a higher production capacity as do his money holdings. Then, the marginal utility of money eventually reaches the lower bound and symmetric stagnation occurs.[28] But the asymmetric case may persist even for high levels of potential output $\bar{y}$ as long as the capital gains effect is weak or negative.

To summarize, our model features three regions depending on $\bar{y}$: If potential output is below the threshold $\widetilde{y}$ given by (33), the neoclassical case applies and there is no demand shortage. In contrast, stagnation occurs for $\bar{y} > \widetilde{y}$ because of the insatiability of liquidity preferences. The asymmetric case always occurs if condition (51) holds and might prevail for even higher values of potential output. Finally, the symmetric case occurs if consumption of the borrower under stagnation becomes sufficiently high.

[28] Thus, there will be an implicit threshold $\hat{y}$ such that there is symmetric stagnation for $\bar{y} > \hat{y}$. This threshold depends on the model parameters, particularly on those affecting equilibrium money holdings of the borrower. These in turn depend on the shape of the utility function $v(m)$. Therefore, we cannot give a closed-form expression for this threshold.

6.2 Stagnation with Positive Money Growth

So far, we have focused on the case of zero trend inflation as a result of a constant money supply, i.e. $\mu = 0$, under full employment. Two considerations need to be taken into account when considering the case of $\mu > 0$ that affect the occurrence of stagnation as well as the existence of the stagnation steady state. For the general conditions and proofs, we refer to Ono and Ishida (2014) for the case of homogeneous households. Here, we will provide an intuitive discussion of the effects of $\mu > 0$ for the case of heterogeneous agents in the borrower-saver framework.

First, as argued above, stagnation occurs once one of the households is no longer willing to consume the amount consistent with full employment because of his insatiable desire for holding liquidity, i.e. once the following threshold is reached for any household:

$$\tilde{c}_i^{NC} > \frac{\rho_i + \mu}{\beta} \ . \tag{52}$$

This is a generalization of condition (32) for the case of positive money growth. Two effects emerge relative to the case of $\mu = 0$ that has been discussed so far.

Positive nominal money growth raises the nominal interest rate under full employment, due to the Fisher equation (24). This increases the opportunity cost of holding money for both agents, which stimulates their consumption, thereby increasing the liquidity premium. As a consequence, full employment can be sustained for higher levels of potential output and stagnation occurs at a later stage. In fact, for every level of potential output $\bar{y}$ there exists a nominal money growth rate μ such that full employment prevails. However, this comes at the cost of higher inflation.

In addition, there is a more subtle effect as positive money growth might affect both households' consumption levels $\tilde{c}_i^{NC}$ under full employment. This crucially depends on the assumption about the distribution of seignorage profits $z_t = \mu m_t$. If these are distributed in proportion to each agents money holdings, there is no effect on the full employment levels of consumption, given by (30) and (31).[29] However, if seignorage income is distributed equally across households, the household with lower money holdings benefits at the expense of the household with higher money holdings. For reasonable parameter specifications, it will be the saver whose consumption will be lowered by this effect, while the borrower benefits. This further increases the income threshold for stagnation.[30]

[29] The intuition is simple: Each household incurs implicit costs of money holdings due to inflation. In turn, the household benefits from inflation via the seignorage profits. If profits are distributed in proportion to money holdings, these effects exactly offset each other.

[30] It could even be the case that the saver's consumption is actually *lower* under full employment than the borrower's consumption level because of the redistributive effect of inflation. Yet, note that this effect only occurs for a very restrictive parameterization. Specifically, both the difference in discount rates and the money growth rate need to be sufficiently high. In addition, the loan-to-value ratio or the housing preference parameter need to be sufficiently low.

Secondly, the existence condition of the asymmetric steady state is affected. Because of persistent deflation, the money supply expands indefinitely and so does the wealth level of the saver. With $\mu = 0$, the rate of expansion is given by the rate of deflation as is clear from (23). Since the deflation rate is below the real interest rate, as we assume $\rho_1 > \alpha$, the transversality condition (11) holds despite this expansion. With positive nominal money growth, however, the expansion of the real money supply increases to $\mu - \pi$ as does the growth rate of household wealth. For the transversality condition to hold, we need to require that this rate of expansion is below the time preference rate of the saver that determines the real interest rate. Specifically, for a steady state to exist, it has to hold that

$$0 > \frac{\dot{m}_t}{m_t} - \rho_1 = \mu - \beta \tilde{c}_1^* \,, \tag{53}$$

where $\tilde{c}_1^*$ denotes the saver's consumption in the asymmetric steady state with $\mu > 0$.

On top of that, the occurrence condition of the symmetric stagnation steady state is affected by introducing positive money growth. The effects depend again on the assumption on the distribution of seignorage income. If this income is distributed in proportion to each household's money holdings, then there are no effects as the borrower's consumption under asymmetric stagnation is not affected. In contrast, if this income is distributed equally across households, the borrower's consumption will be stimulated under asymmetric stagnation. As the money supply expands, so does his exogenous income, which allows for higher consumption. Then, the symmetric stagnation case will eventually occur if condition (53) holds.

In conclusion, the equilibrium of the economy is conditional on the money growth rate. A sufficiently high rate of money growth may help to restore full employment. Since this comes at the cost of high inflation, policymakers are likely to be inclined to prefer a scenario of persistent stagnation and take measures to improve aggregate demand within that equilibrium. However, for any rate of money growth μ, there exists a level of potential output above which stagnation occurs. Sustainable full employment will hence require an ever-increasing expansion of the money growth rate. Even worse, the interplay of conditions (52) and (53) also implies that multiple equilibria can emerge with both stagnation and full employment as steady state equilibria for the same parameterization. It might also be the case that no equilibrium exists at all. So once the economy has reached stagnation, it will be very hard and costly in terms of high inflation to move towards the full employment steady state.

For that reason, our analysis has focused primarily on the stagnation case with $\mu = 0$. Note, however, that the conclusions also hold for a low inflation scenario which requires sufficiently low levels of monetary growth.[31]

[31] This is similar to the assumptions of Michaillat and Saez (2014) and Michau (2017) that the central bank follows a sufficiently low inflation target.

6.3 Policy Recommendations and the Nature of the Friction

Two features in our model prevent the economy from reaching full employment - insatiable liquidity preferences and debt overhang. Insatiable liquidity preferences imply that stagnation always occurs for sufficiently high levels of potential output, even in the absence of financial frictions. The reason is that agents prefer to hold excessive money instead of consumption. This implies that expansionary monetary policy is ineffective in the stagnation steady state of our model.[32] In fact, the deflationary steady state is characterized by an infinite expansion of the real money stock.

In contrast, the case for fiscal policy as a potential cure to stagnation is straightforward. The government is not constrained by the same liquidity motives as the private sector and can expand its spending.[33] Redistributive policies work by transferring resources from rich agents to poor ones. The latter expand their consumption while spending of the former is not directly affected (unless at the margin). Therefore, targeted redistributive interventions can help to stimulate the economy. In reality, targeted transfers might not be feasible though. Yet, Matsuzaki (2003) shows in a similar setting that lump-sum transfers financed by a consumption tax can increase aggregate demand if the fraction of poor households is sufficiently small.[34]

Private debt overhang is another factor that depresses aggregate demand since indebted households reduce their consumption spending. In fact, borrowers do delever in nominal terms in the deflationary steady state. But this is self-defeating because of the effects of debt deflation on the real value of their outstanding obligations. Hence, policies that limit household indebtedness and help to repair balance sheets of spending-constrained households are another option to expand aggregate demand. Yet, they include a potentially costly adjustment process in the short run.

Similar conclusions hold when we impose the borrowing constraint on the supply side. Although this analysis is beyond the scope of this paper, the following thought experiment clarifies this point: Suppose the collateralizeable asset is a factor of production and producers are constrained in their borrowing ability. As above, financial liberalization is associated with higher equilibrium collateral holdings by the borrower under certain parameter constellations. These in turn imply a higher equilibrium production capacity. Therefore, financial liberalization may improve equilibrium output under neoclassical as-

[32] Yet, a sufficiently large expansion of the money supply might restore the full employment case though at the cost of inflation as discussed before.

[33] Note that the expansionary effect of government spending has nothing to do with deficit-budget financing or balanced-budget financing. It works through a direct creation of demand. We refer to Ono (1994, 2001) for an explicit modeling of government spending. We abstract from public debt in our framework since we focus primarily on private debt.

[34] Note that this discussion uses aggregate demand as the relevant policy criterion rather than a welfare function based on individual utilities. The latter is complicated by household heterogeneity and the infinite expansion of the saver's money holdings in steady state.

sumptions. However, the economy is demand-constrained in our model because of the insatiability of liquidity preferences so that the implied improvements in the supply side actually worsen the output gap and deflation. An increase in indebtedness hence deteriorates equilibrium income for reasonable parameter ranges, irrespective of the modeling of the borrowing friction on the demand side or supply side.

Finally, our results continue to hold with insatiable wealth preferences instead of liquidity preferences. Unlike the latter, wealth preferences affect the equilibrium real interest rate by encouraging household savings (cf. Ono, 2015; Kumhof et al., 2015). As a consequence, the natural real rate of interest can turn negative in steady state (cf. Michau, 2017). In our setting, this would imply a redistribution from savers to borrowers as the real cost of debt becomes negative. However, the very existence of housing as a durable asset without depreciation prevents the real rate from turning negative in our setup. This can be easily seen from (19), which is unaffected by the introduction of wealth preferences. Housing yields a positive "dividend" stream in the form of the user cost of housing while the cost of housing investment are given by the real opportunity cost, since there is no depreciation. The real house price adjusts to make agents indifferent between housing investment and other uses of funds. Hence, from (19) a negative real rate of interest would require a decline of the real house price in steady state:

$$r^* < 0 \quad \Leftrightarrow \quad \frac{\dot{q}}{q} < -\frac{\gamma c_1^*}{h_1^*} < 0 \; .$$

This is not consistent with a stationary steady state. Moreover, it would imply that the real house price eventually converges to zero and hence that the current asset price itself is not well-defined. We can therefore exclude the possibility of a negative real rate in our model under wealth preferences. Hence, there cannot be a redistribution from savers to borrowers via negative interest cost of debt in steady state.

7 Conclusion

Many developed countries, e.g. Japan, EU and the USA, have been suffering from persistent stagnation of aggregate demand under which some households do not increase consumption and keep wealth while others do not increase consumption because they are severely indebted. It typically occurred after a credit and stock price boom. To analyze this phenomenon, we have introduced private indebtedness into a model with two types of agents that have different time patience and insatiable preferences for money holding.

The less patient households borrow funds from the more patient ones but face a borrowing constraint that depends on the value of their housing. Therefore their consumption is restricted by this constraint. The more patient households earn interests from the lending and hence can expand consumption, but in fact do not because of high preference for money holding. Thus, aggregate demand shortages arise and deflation occurs. The deflation makes it more advantageous for the lenders to reduce consumption and hold money. It in turn expands the real value of debt of the borrowers and decreases their consumption because they have to pay high interests to the lenders.

If the borrowers could consume more, deflation would mitigate and stimulate the lenders consumption as well, leading to an expansion of total income. Thus, a government that faces this situation may be tempted to ease the borrowing constraint. It will indeed enable the borrowers to consume more and mitigate deflation, which also stimulates the lenders consumption by lowering the advantage of holding money. Moreover, easing the borrowing constraint makes the borrowers think housing investment to be more valuable because an increase in the value of housing enables them to borrow more for consumption. Thus, it triggers a housing price boom.

However, those positive effects occur only in the short run. In the long run the borrowers are more indebted so that they have to reduce consumption, which worsens deflation and makes the lenders to decrease consumption and save more because money holding is more profitable. The decrease in total consumption stops the housing price boom. The economy eventually falls into secular stagnation of aggregate demand. Thus, direct transfers from the richer to the poorer, which does not create debt overhang, will be more promising.

Appendices

Appendix A: Proof of Proposition 1

The model parameters affect the stagnation threshold $\widetilde{y}$ in (33) as follows:

$$\frac{\partial \widetilde{y}}{\partial \beta} = -\frac{n\rho_1}{\beta^2}\frac{\gamma\theta\rho_1 + \bar{\rho}_\theta}{\gamma\theta\rho_1 + n\bar{\rho}_\theta} < 0 \ ,$$

$$\frac{\partial \widetilde{y}}{\partial \rho_1} = \frac{n[(1+\gamma)(\gamma+n)\theta^2\rho_1^2 + 2(1+\gamma)n\theta(1-\theta)\rho_1\rho_2 + (1-\theta)^2 n\rho_2^2]}{\beta[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} > 0 \ ,$$

$$\frac{\partial \widetilde{y}}{\partial \rho_2} = \frac{n\rho_1}{\beta}\frac{(1-n)\gamma\theta(1-\theta)\rho_1}{[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} > 0 \ ,$$

$$\frac{\partial \widetilde{y}}{\partial \theta} = -\frac{n\rho_1}{\beta}\frac{\rho_1\rho_2\gamma(1-n)}{[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} < 0 \ ,$$

$$\frac{\partial \widetilde{y}}{\partial n} = \frac{\gamma\theta\rho_1^2[\gamma\theta\rho_1 + \bar{\rho}_\theta]}{\beta[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} > 0 \ ,$$

$$\frac{\partial \widetilde{y}}{\partial \gamma} = -\frac{n\rho_1}{\beta}\frac{(1-n)\rho_1\theta\bar{\rho}_\theta}{[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} < 0 \ .$$

Appendix B: Existence and Stability of the Asymmetric Steady State

In this appendix, we derive the necessary existence condition for the asymmetric steady state under stagnation and show that it satisfies saddle-point stability.

(i) Existence: Using (34) to substitute for c_1^* and (35) for c_2^*, we rewrite (41) as

$$F(m_2) \equiv \alpha[(\beta\bar{y} - \rho_1 n)v'(m_2) - (1-n)\beta\rho_2]m_2 + n(\rho_1 - \alpha)\bar{y}[v'(m_2) - \beta] = A \ , \quad \text{(B.1)}$$

$$\text{where} \ \ A \equiv \frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) > 0 \ ,$$
$$\text{and} \ \chi \equiv \rho_2(\beta\bar{y} - \alpha n) - \alpha(\beta\bar{y} - \rho_1 n) > 0 \ .$$

The asymmetric steady state under stagnation exists for $\bar{y} > \widetilde{y}$ if there exists a finite and strictly positive value of m_2 as a solution to this equation. Note that the RHS of this expression is a positive constant that is independent of m_2. In contrast, the LHS of this expression is a function of m_2. It holds that:

$$\lim_{m_2 \to 0} F(m_2) = \infty \ ;$$
$$\lim_{m_2 \to \infty} F(m_2) \ = \ \begin{cases} -\infty \ \text{ if } \ \beta\bar{y} < n\rho_1 + (1-n)\rho_2 \ , \\ \lim\limits_{m_2 \to \infty} \alpha(\beta\bar{y} - \rho_1 n)(v'(m_2) - \beta)m_2 \ \text{ if } \ \beta\bar{y} = n\rho_1 + (1-n)\rho_2 \ , \\ +\infty \ \text{ if } \ \beta\bar{y} > n\rho_1 + (1-n)\rho_2 \ . \end{cases}$$

Under otherwise standard assumptions on $v(m)$, it follows that there is a unique solution of (B.1) if $\beta\bar{y} < n\rho_1 + (1-n)\rho_2$. This is the sufficient condition in Proposition 3. For higher values of $\bar{y}$, there may be two solutions, exactly one solution or no solution to (B.1). Existence of the asymmetric steady state then requires that the minimum (or limit if $\beta\bar{y} = n\rho_1 + (1-n)\rho_2$) of $F(m_2)$ is smaller than or equal to the RHS:

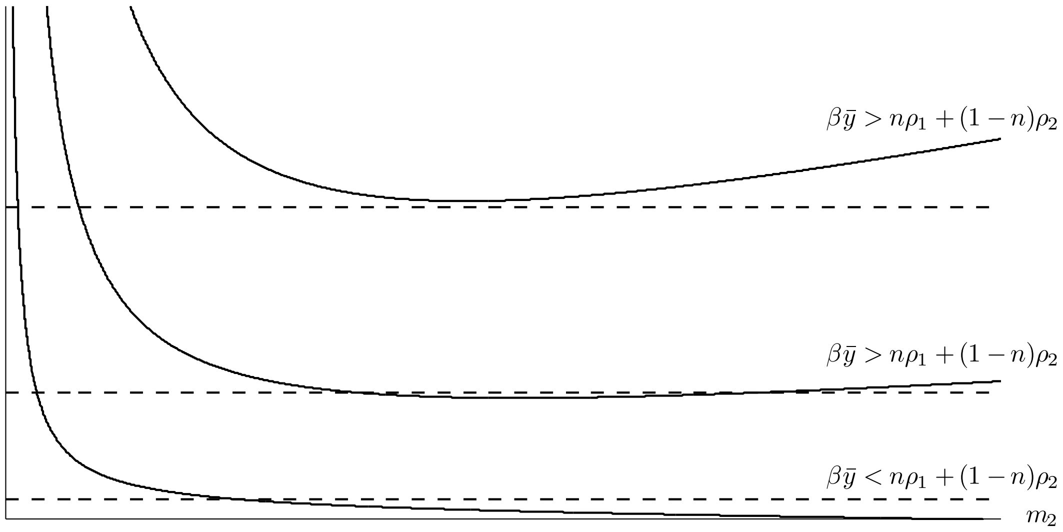
$$\min_{m_2} F(m_2) < \frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) \ . \quad \text{(B.2)}$$

This condition guarantees the existence of at least one solution to (B.1). In case of multiple solutions, we choose the solution that satisfies $F'(m_2^*) < 0$. This is for two reasons: First, it is consistent with continuous variations in $\bar{y}$. Second, this solution satisfies saddle-point stability, whereas the other solution is unstable. Therefore, a necessary condition for the asymmetric steady state under stagnation to occur is given by

$$\left.\frac{\partial F(m_2)}{\partial m_2}\right|_{m_2^*} < 0 \ . \quad \text{(B.3)}$$

Finally, if there is no finite value of m_2 that solves (B.1), we must have $\dot{a}_{2,t} > 0$ which implies that the economy is in the symmetric stagnation steady state.

Figure 7: Existence of the Asymmetric Steady State under Stagnation



Note: This figure illustrates the LHS (solid line) and RHS (dotted line) of (B.1) for different values of potential output $\bar{y}$ and for the specific utility function $v(m_2) = \beta m_2 + \delta ln(m_2)$.

To summarize: *The asymmetric steady state under stagnation exists for $\beta\bar{y} > n\rho_1 + (1-n)\rho_2$ if there exists a finite, positive value of m_2 that solves (B.1). Moreover, (B.2) is a sufficient condition for the existence of the asymmetric steady state under stagnation given $\beta\bar{y} - n\rho_1 - (1-n)\rho_2 \geq 0$. In addition, (B.3) is a necessary condition for the asymmetric steady state under stagnation to occur.*

For illustration, consider the specific utility function $v(m_2) = \beta m_2 + \delta ln(m_2)$. Figure 7 shows the behavior of the two sides of (B.1) as a function of m_2, which is given by

$$\alpha\beta(\beta\bar{y} - \rho_1 n - (1-n)\rho_2)m_2 + \delta\alpha(\beta\bar{y} - \rho_1 n) + \frac{\delta n(\rho_1 - \alpha)\bar{y}}{m_2} = \frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) \ .$$

For $\beta\bar{y} - n\rho_1 - (1-n)\rho_2 = 0$, the existence of the asymmetric steady state requires a sufficiently low value of δ:

$$\delta < \bar{\delta} \equiv \frac{1}{\alpha(\beta\bar{y} - \rho_1 n)} \left[\frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1)\right] \ . \tag{B.4}$$

For $\beta\bar{y} - n\rho_1 - (1-n)\rho_2 > 0$, we require in addition that (B.2) holds which implies

$$\left[\frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) - \alpha\delta(\beta\bar{y} - \rho_1 n)\right]^2 > 4\alpha\beta(\beta\bar{y} - n\rho_1 - (1-n)\rho_2)n(\rho_1 - \alpha)\delta\bar{y} \ . \tag{B.5}$$

(ii) Stability: The dynamic system is characterized by six differential equations for c_1, c_2, q, a_1, a_2 and m given by (42), (43), (44), (46), (45) and (47) and by the static equations (20), (21) and (27) for m_2, h_1 and h_2. The asymmetric steady state under stagnation is characterized by a diverging real money supply and real assets of the saver. Define $z_{1,t} \equiv a_{1,t}^{-1}$ and $z_{2,t} \equiv m_t^{-1}$. Then the steady state of $\{c_{1,t}, c_{2,t}, q_t, a_{2,t}, z_{1,t}, z_{2,t}\}$ is given by $\{c_1^*, c_2^*, q^*, a_2^*, 0, 0\}$. We linearize the system around this steady state using a first-order Taylor approximation:

$$\begin{pmatrix} \dot{c}_{1,t} \\ \dot{c}_{2,t} \\ \dot{q}_t \\ \dot{a}_{2,t} \\ \dot{z}_{1,t} \\ \dot{z}_{2,t} \end{pmatrix} = \begin{pmatrix} v_{11} & v_{12} & 0 & 0 & 0 & 0 \\ v_{21} & v_{22} & v_{23} & v_{24} & 0 & 0 \\ v_{31} & v_{32} & v_{33} & v_{34} & 0 & 0 \\ v_{41} & v_{42} & v_{43} & v_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & v_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & v_{66} \end{pmatrix} \begin{pmatrix} c_{1,t} - c_1^* \\ c_{2,t} - c_2^* \\ q_t - q^* \\ a_{2,t} - a_2^* \\ z_{1,t} - z_1^* \\ z_{2,t} - z_2^* \end{pmatrix} ,$$

where the entries v_{ij} in the transition matrix V refer to the respective terms in the linearized system. The eigenvalues ξ_i of V determine the stability of this system and are given by the solution to

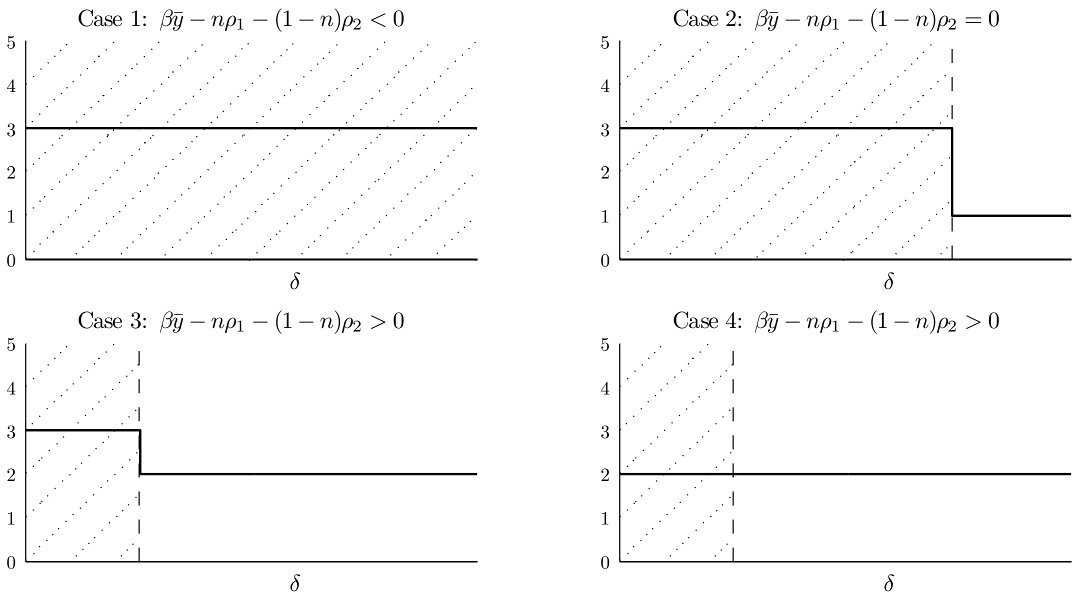
$$\left((v_{11} - \xi) \left\| \begin{matrix} v_{22} - \xi & v_{23} & v_{24} \\ v_{32} & v_{33} - \xi & v_{34} \\ v_{42} & v_{43} & v_{44} - \xi \end{matrix} \right\| - v_{12} \left\| \begin{matrix} v_{21} & v_{23} & v_{24} \\ v_{31} & v_{33} - \xi & v_{34} \\ v_{41} & v_{43} & v_{44} - \xi \end{matrix} \right\| \right) (v_{55} - \xi)(v_{66} - \xi) = 0 ,$$

where $\left\|Q\right\|$ is the determinant of Q. Since only $c_{1,t}$, $c_{2,t}$ and q_t are jumpable, there must be three positive and three negative eigenvalues for the system to exhibit saddlepoint stability. $\xi_i = \beta c_1^* - \rho_1 = \pi^*$ is a solution and under stagnation $\pi^* < 0$. Thus, these two eigenvalues are negative. We use a numerical analysis for the other solutions.

Based on the functional form $v(m_2) = \beta m_2 + \delta ln(m_2)$, we simulate V for three cases determined by $\beta \bar{y} - n\rho_1 - (1-n)\rho_2$. For each case, we vary δ (and implicitly m_2^*), which determines the strength of the capital gains channel in (41). We then determine the number of negative eigenvalues. The results are summarized in Figure 8 which also highlights the threshold parameter $\bar{\delta}$ in (B.4) or (B.5).

For $\beta \bar{y} - n\rho_1 - (1-n)\rho_2 < 0$ (case 1), the system is saddlepoint-stable for all $\delta > 0$. This corresponds to condition (51). For $\beta \bar{y} - n\rho_1 - (1-n)\rho_2 = 0$ (case 2), the system is saddlepoint-stable for $0 < \delta < \bar{\delta}$. Hence, under existence condition (B.4) the steady state exhibits saddle-point stability. For $\beta \bar{y} - n\rho_1 - (1-n)\rho_2 > 0$, there are two solutions to (B.1) shown in cases 3 and 4. Both solutions require condition (B.5) to hold. Yet, only one of these solutions shows saddle-point stability. This is the solution that fulfills condition (B.3). We therefore conclude that the model is saddlepoint-stable around the asymmetric stagnation steady state under conditions (B.1), (B.2) and (B.3).

Figure 8: Stability of the Saddle Path



Notes: This figure shows the number of negative eigenvalues in V for the function $v(m_2) = \beta m_2 + \delta ln(m_2)$. Case 1 refers to existence condition (51) and cases 2 to 4 refer to conditions (B.4) and (B.5) which are represented by vertical lines. Variations in δ are shown on the x-axis. The calibration is as follows: $\beta = 0.0005; \rho_1 = 0.05; \rho_2 = 0.1; \alpha = 0.01; n = 0.5; H = 1$, $\theta = 0.5$ and $\bar{y} = 120$ (case 1), $\bar{y} = 150$ (case 2) and $\bar{y} = 200$ (cases 3 and 4). In the dashed areas, the existence conditions for the asymmetric steady state are fulfilled.

Appendix C: Proof of Proposition 2

The effects of variations in the model parameters on the asymmetric steady state are derived from the total differential of (B.1). Define $\Omega(m_2, x) \equiv 0$ where x is any parameter in the model as

$$\Omega(m_2, x) = \alpha[(\beta\bar{y} - \rho_1 n)v'(m_2^*) - (1-n)\beta\rho_2]m_2^* + n(\rho_1 - \alpha)\bar{y}[v'(m_2^*) - \beta] - A \ ,$$

$$\text{where} \ \ A \equiv \frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) > 0 \ ,$$

$$\text{and} \ \chi \equiv \rho_2(\beta\bar{y} - \alpha n) - \alpha(\beta\bar{y} - \rho_1 n) > 0 \ .$$

From this expression, we can recover the effect on money demand of the borrower as follows:

$$\frac{\partial\Omega(m_2, x)}{\partial\theta}dx + \frac{\partial\Omega(m_2, x)}{\partial m_2}dm_2 = 0 \ \Leftrightarrow \ \frac{dm_2}{dx} = -\frac{\frac{\partial\Omega(m_2,x)}{\partial x}}{\frac{\partial\Omega(m_2,x)}{\partial m_2}} \ . \tag{C.1}$$

From (B.3) and the discussion in Appendix B, it follows that

$$\frac{\partial\Omega(m_2, x)}{\partial m_2} = F'(m_2) < 0 \ . \tag{C.2}$$

Consider the effects of variations in the loan-to-value ratio θ and the housing preference parameter γ on the asymmetric steady state under stagnation:

$$\frac{\partial \Omega(m_2, \theta)}{\partial \theta} = -\frac{\rho_1 \rho_2 \gamma}{\bar{\rho}_\theta^2} \chi < 0 \ , \tag{C.3}$$

$$\frac{\partial \Omega(m_2, \gamma)}{\partial \gamma} = -\frac{\theta \rho_1}{\bar{\rho}_\theta} \chi < 0 \ . \tag{C.4}$$

It hence follows from (C.1), (C.2), (C.3) and (C.4) that

$$\frac{dm_2^*}{d\theta} < 0 \ , \quad \frac{dm_2^*}{d\gamma} < 0 \ .$$

These results imply together with (35) that

$$\frac{dc_2^*}{d\theta} < 0 \ , \quad \frac{dc_2^*}{d\gamma} < 0 \ .$$

The effects on the steady state value of the other variables can be derived from their relation with c_2^* and m_2^*. We get from (34) that

$$\frac{dc_1^*}{d\theta} < 0 \ , \quad \frac{dc_1^*}{d\gamma} < 0 \ ,$$

and from (39) that

$$\frac{da_2^*}{d\theta} < 0 \ , \quad \frac{da_2^*}{d\gamma} < 0 \ .$$

Finally, these results imply in combination with (2) and (22) that increases in θ or γ reduce aggregate demand and worsen deflation in the asymmetric steady state:

$$\frac{dC^*}{d\theta} < 0 \ , \quad \frac{dC^*}{d\gamma} < 0 \ ,$$

$$\frac{d\pi^*}{d\theta} < 0 \ , \quad \frac{d\pi^*}{d\gamma} < 0 \ .$$

Also note that the cross-derivative is strictly negative which implies mutually reinforcing effects of γ and θ as illustrated in Figure 6:

$$\frac{\partial^2 \Omega(m_2, \theta)}{\partial \theta \ \partial \gamma} = -\frac{\rho_1 \rho_2}{\bar{\rho}_\theta^2} \chi < 0 \ .$$

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