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MULTIPRODUCT FIRMS, CONSUMER SEARCH, AND DEMAND HETEROGENEITY

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Multiproduct Firms, Consumer Search, and Demand Heterogeneity*

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Abstract

This study constructs a sequential consumer search model with differentiated products in which some consumers search for a single product while the others search for multiple products. When the mass of consumers who demand one of the products decreases, the price for one product decreases while another price increases due to the joint-search effect, even if the products are neither complements nor substitutes. In addition, under some conditions, this decrease in demand causes *an increase* in each firm's profit.

Keywords: consumer search, multiproduct pricing, demand heterogeneity
JEL classification: D11, D43, D83, L13

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1. Introduction

Product information significantly influences consumers' purchase behavior. Consumers generally have incomplete information about the products they desire, for example, they must learn about the price, quality, necessity, existence, and so on. Thus, it is natural to study the process of consumers' search activities to gather information about products. For instance, consider products such as convenience products, which consumers purchase repeatedly. For such products, consumers may know the qualities, necessity, and existence, but they may not know the price at that moment due to frequent price changes by the firms. Therefore, they must search for prices (Stigler 1961, Diamond 1971, Varian 1980, Burdett and Judd 1983). As another example, consider other products such as shoes, clothes, furniture, and so on. Consumers may not be familiar with such products and must thus search not only for prices but also for their utilities, such as *match utilities* (Wolinsky 1986, Anderson and Renault 1999).

Firms consider other important factors, such as the heterogeneity of consumers' search costs (Stahl 1989, Moraga-González, Sándor and Wildenbeest 2017), consumers' risk preferences (Karle and Peitz 2014, Rosato 2016), and the purchase of multiple products (Burdett and Malueg 1981, McAfee 1995, Zhou 2014). In addition, it is natural to consider consumers'' potential heterogeneous demand for product varieties in that some consumers purchase multiple products (say, products A and B), although others demand one of the products (A or B); that is the demand for each product is heterogeneous. However, no prior studies focus on this type of heterogeneity.

To capture this feature, we develop a model that combines a single product search model and multiproduct search model established in the seminal works by Wolinsky (1986) and Zhou (2014), respectively. Unlike the above literature, we allow for heterogeneous demand for product varieties in our meaning. In the existing literature on consumer search, when the mass of consumers who demand one of the products changes, it does not affect the price at all. We show that this change in demand causes the price for one product to increase and the price for another to decrease due to the joint-search effect, even if the products are neither complements nor substitutes. In addition, we also show that such a change in demand would induce the phenomenon of profit-increasing consumer exit under some conditions.

For example, consider hypermarket stores such as Walmart, Target, and the like. These hypermarket stores are located in suburban or out-of-town areas and supply many products in their physical stores. Most consumers have heterogeneous preferences and search for more than one products with paying search costs such as gasoline, time, and so forth. Most of this search cost is a tripping cost rather than the search cost inside the stores. In that sense, the existing multiproduct search literature well captures these features. In addition, as we mentioned above, consumers may be heterogeneous in the type of product varieties they desire. Using our framework, we can see how heterogeneous preferences affect prices and profits.

For example, as in the above example, consider hypermarket stores and an indirect competitor such as Amazon. Amazon started in 1995, with strictly increasing revenue from 1995 to 2017. We assume that Amazon gained not a few of consumers from such physical hypermarket stores. Although one might think that the emergence of such indirect competitors hurts the revenues of physical stores, Walmart's revenue increased from 1995 to 2014 (it slightly decreased in 2015, but rebounded in 2017).[*1]

We can use our model to interpret this phenomenon as follows. When consumers search for more than one product, firms have an incentive to charge a lower price for one product to attract consumers to maximize their own joint profits. Now, a fraction of consumers purchases a portion of their necessities on Amazon.com. However, they must still purchase their remaining necessities such as food, clothes, and so forth at the physical stores because uncertainty about quality induces consumers to go to the physical stores. Thus, physical stores can charge a higher price for such products, and then each physical store's joint profit increases, even if the demand for one product decreases. In other words, in some cases, a decrease in this type of demand would soften price competition among stores.

The rest of the paper proceeds as follows. Section 2 presents the basic model of single and multiproduct search and gives the optimal pricing rule. Section 3 analyses the effects of changing the mass of consumers who demand for one of the products. Section 4 then concludes and discusses considerable extensions. All proofs and welfare analyses are available in the appendix.

2. The Model

We base our setting on Wolinsky (1986) and Zhou (2014). There are two firms, A and B, supplying two products, products 1 and 2. Without loss of generality, we assume that their marginal costs are normalized to zero.

The market size is normalized to one. Let λ_i ($0 \leq \lambda_i \leq 1$) denotes the *mass* of consumers who demand product i, for $i = 1, 2$. Suppose that the consumer demand for the products are

[*1] We use financial data from the Amazon.com and Walmart.com. See the references to find the data.

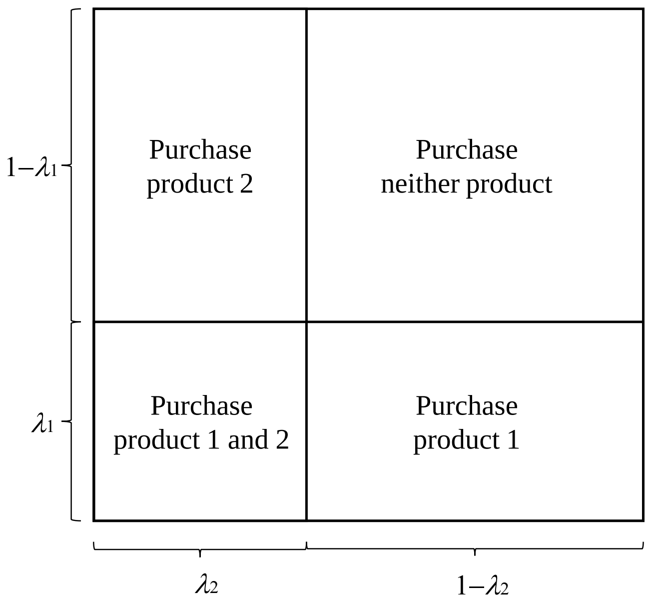

Fig. 1. Market demand structure.

independently distributed; then, $\lambda_1\lambda_2$ of consumers search for both products, but $(1-\lambda_1)(1-\lambda_2)$ of consumers demand nothing. Figure 1 shows the market demand structure.

According to Wolinsky (1986), we treat both products as differentiated between firms A and B, and each consumer has idiosyncratic tastes for each product. We represent this by assuming that each consumer's valuation for each product is independently and randomly drawn from a common cumulative distribution function F, which is twice differentiable so it has a common probability density function f and defined on the interval $[\underline{u}, \bar{u}]$. The valuation for product i is denoted by u_i for $i = 1, 2$ and we call the valuation the *match utility.* We assume that the two products are neither complements nor substitutes in the sense that each consumer simply obtains the sum of the match utilities when she searches both products. We also assume that each consumer has an independent valuation for the two products in the same firm. Additionally, assume that each consumer's valuation is independent across consumers. Therefore, for the consumers who search for both products, once they go to a firm, they draw a pair of match values from $F(u_1, u_2) = F(u_1)F(u_2)$. The distribution is commonly known.

We also include other assumptions, including *economies of scale of search, free-recall, no replacement* and *fully-covered.* The first assumption means that a consumer's search cost s is constant, whether she searches for a single product or both products. This assumption is easily justified if we consider that most of the sources of search cost are a *tripping cost.* Next three assumptions are quite usual in the singleproduct search literature.*2 Whereas in the multiproduct search model, these assumptions imply some new features. By the free-recall assumption, once a consumer who searches two products visits a firm, her options are (i) Buying both products immediately and ending the search activity, or (ii) Buying no products and going to another firm. In this scenario, the other possible option (iii) Buying one of the products and then continuing to search, does not exist due to the free-recall assumption. Note that we implicitly exclude the possibility of product bundling, so consumers can choose any combination of firms' products. We also assume that each consumer's utility function is linear in that the net utility from purchasing product i at the price of p_i is given by ($i = 1, 2$):

$$u_i - p_i$$

Because it is independent of any search cost, when a consumer purchases both products, her utility is the sum of each net utility.

The consumer's optimal search rule is as follows. (i) A first observation is inevitable, so the search cost for the first observation is sunk. (ii) A consumer compares an incremental expected benefit with an additional search cost from one more search. If the former is greater, she goes to another firm and then compares the observed utilities; otherwise, she immediately stops and purchases the product (products). We assume that each consumer visits firms in a random order.

The timing of the game is as follows. In the first stage, firms simultaneously set prices. In the second stage, consumers search with the optimal search rule. In this study, we focus on the *symmetric equilibrium* from the related literature. We adopt the Perfect Bayesian Equilibrium as a solution concept.

In equilibrium, each firm maximizes its own profit and each consumer maximizes her own consumer surplus, and her belief is consistent. We also assume that a consumer does not change her mind (belief), even if she observes a price that differs from that in her mind. To make our analysis interesting, we focus on the case with a relatively small search cost, which ensures search activity.

*2 We can also justify the free-recall assumption when we consider the search cost as the trial cost or other costs with the following properties: there are some difficulties in obtaining the match utility of the product indirectly; hence, the consumer needs a direct observation. After observing the match utility, there is no need to pay an additional cost, even if she went to the other firm and returned. Consider, for example, trying on clothes.

2.1. Single product search

First, we characterize the single product search. Consider consumers who search for only a single product i ($i = 1, 2$). Recall that $\lambda_i(1-\lambda_j)$ ($i \neq j$) represents the proportion of such consumers in the market population. For these consumers, the demand is a part of Wolinsky's (1986) model. First, we define the *reservation match utility.* Suppose each consumer has a (correct) belief that firms adopt the same optimal pricing strategy in equilibrium. Let x represent the first observed value. Then, the incremental expected benefit from an additional search is given by:

$$\int_{x}^{\bar{u_i}} (u_i - x) dF(u_i) = \int_{x}^{\bar{u_i}} [1 - F(u_i)] du_i$$

For simplicity, according to McAfee (1995), define the following expression.

$$\zeta_i(x) \equiv \int_{x}^{\bar{u_i}} [1 - F(u_i)] du_i, \text{ for } i = 1, 2$$

We have the critical value with which an incremental expected benefit and an additional search cost s are indifferent for the consumer:

$$\zeta_i(\hat{u}_i) = s$$

We refer to $\hat{u}_i$ as the reservation match utility for product i for the single product search. In the following analysis, we suppose that the search cost s is relatively small, which ensures the validity of search activity for a single product. Let $\bar{s}$ denote the maximum search cost, which we define as:

$$\zeta_i(\underline{u_i}) = \bar{s} \quad \text{for} \quad i = 1, 2$$

Then, the search cost must satisfy the following inequality.

$$s \leq \bar{s} \tag{1}$$

We refer to this inequality as the *relatively small search cost condition.*

Now, consider a firm's incentive to deviate from the equilibrium pricing strategy. Suppose firms adopt a symmetric equilibrium pricing strategy, but firm A cuts reduces its price by $\epsilon_i > 0$. Then, the demand for firm A from $\lambda_i(1-\lambda_j)$ type consumers is given by:

$$D_i(p_i) = \frac{1}{2}[1 - F(\hat{u}_i - \epsilon_i)][1 + F(\hat{u}_i)] + \int_{\underline{u_i}}^{\hat{u}_i - \epsilon_i} F(u_i + \epsilon_i) f(u_i) du_i \tag{2}$$

The first term of the RHS represents a stopping probability. Since a consumer searches in a random order and there are two firms, half of the consumers visit firm A first. If firm A reduces the price by a small value ϵ_i, such a deviation induces purchasing with a relatively lower match utility; that is, a purchase by a consumer who observes a lower match utility than her reservation match utility $\hat{u_i}$. The second term of the RHS of equation (2) denotes the expected demand of fully-informed consumers who finally decide to buy from firm A.

2.2. multiproduct search

Now, consider $\lambda_1\lambda_2$ consumers who want to purchase both products. This part follows straightforwardly from Zhou (2014). For consistency, we adopt the same notation as in his model. First, we define the reservation match *utilities*, which we denote by the pair of u_1 and u_2 that satisfy the following equation:

$$\zeta_1(u_1) + \zeta_2(u_2) = s$$

To characterize the pair of reservation match utilities, we introduce function $\phi(u_1) = u_2$, which satisfies:

$$\zeta_1(u_1) + \zeta_2(\phi(u_1)) = s \tag{3}$$

and call these pairs of utilities the *reservation frontier*.

In the last subsection, we assumed a relatively small search cost. If such a condition holds, then it ensures the validity of search activity for $\lambda_1\lambda_2$ consumers due to the *economies of scale of search* assumption. For example, suppose a consumer observes the highest match utility $\bar{u_2}$, but u_1 is not enough to satisfy the reservation match utilities in the first search; then, she continues to search, even if she searches only for product 1 due to the search cost condition. We can define the minimum reservation match utilities a_1 and a_2 as:

$$\zeta_1(a_1) = s \quad \text{and} \quad \zeta_2(a_2) = s$$

Figure 2 shows the above expressions and definitions. We call region A the *Acceptance (Acceptable) set*, in line with Burdett and Malueg (1981). Region B is just a complement of A. Once a consumer observes a pair of utilities, which are an element of Acceptance set A, the consumer stops and purchases both products immediately because she knows that an incremental expected benefit from one more search never overcomes the additional search cost.

Suppose firm A deviates by ϵ_1 and ϵ_2. These deviations move the reservation frontier down to the lower-left. (In other words, more deviations lead to more immediate acceptances). There are trade-offs between the benefit due to increasing demand and the loss due to the lower prices. Note that unlike the single product model, the deviation of ϵ_1 boosts not only demand for product 1, but also that for product 2, because the reservation frontier moves

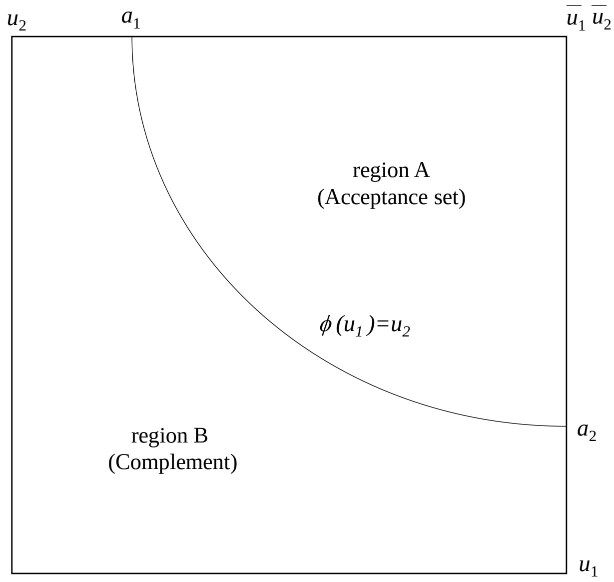

Fig. 2. The optimal stopping rule for the multiproduct search

down.

For firm A, we use the notations A and B in which denote the region of the acceptance set and the complement, and give the demand function as:

$$\begin{aligned} D_{12}(p_1, p_2) =& \frac{1}{2}\left\{\int_{A(\epsilon)} dF(\mathbf{u}) + \int_{B(\epsilon)} [F(u_1+\epsilon_1) + F(u_2+\epsilon_2)] dF(\mathbf{u}) \right. \\ & \left. + \int_{B} [(1 - F(v_1-\epsilon_1)) + (1 - F(v_2-\epsilon_2))] dF(\mathbf{v}) \right\} \end{aligned} \tag{4}$$

Where u_1 and u_2 are the match utilities from firm A, v_1 and v_2 are from firm B, $(\mathbf{u})$ and $(\mathbf{v})$ express (u_1, u_2), (v_1, v_2), and $A(\epsilon)$ and $B(\epsilon)$ are the region after deviating (ϵ_1, ϵ_2), respectively. The first term of the RHS represents the expected demand for a consumer who purchases both products immediately. The second term represents the expected demand for a consumer who first visits firm A and decides not to purchase, and then goes to firm B, but finally becomes fully-informed and returns to firm A, and purchases at least one product. The third term is almost the same as the second term and represents a consumer who visits firm B the first time.

2.3. Equilibrium

We can now derive the first-order condition for firm A when it deviates. We combine equations (2) and (4) and obtain the profit function for firm A:

$$
\begin{aligned}
&\pi(p_i, p_j) \\
&= \sum_{i=1, i\neq j}^{2} \frac{1}{2}\lambda_i(1-\lambda_j)(p_i-\epsilon_i)\Big\{[1-F(\hat{u_i}-\epsilon_i)][1+F(\hat{u_i})] + \int_{\underline{u_i}}^{\hat{u_i}-\epsilon_i} F(u_i+\epsilon_i)f(u_i)du_i\Big\} \\
&+ \sum_{i=1, i\neq j}^{2} \frac{1}{2}\lambda_i\lambda_j(p_i-\epsilon_i)\left\{\int_{A(\epsilon)} dF(\mathbf{u}) + \int_{B(\epsilon)} F(u_i+\epsilon_i)dF(\mathbf{u}) + \int_{B}[1-F(v_i-\epsilon_i)]dF(\mathbf{v})\right\}
\end{aligned}
\tag{5}
$$

In the symmetric equilibrium, no firm has an incentive to deviate. Thus, we have the following Lemma.

Lemma 1. *In the symmetric equilibrium in which consumers adopt an optimal search rule and under a relatively small search cost, as in condition (1), the equilibrium prices p_1 and p_2 are given by the solution to the following equations.*

$$
\begin{aligned}
p_1 \Bigg\{ &(1-\lambda_2)\left[f(\hat{u_1})(1-F(\hat{u_1})) + 2\int_{\underline{u_1}}^{\hat{u_1}} f(u_1)^2 du_1\right] \\
&+\lambda_2\left[2\int_B f_1(u_1)dF(\mathbf{u}) + \int_{a_2}^{\bar{u_2}} (1-F(\phi^{-1}(u_2)))f(\phi^{-1}(u_2))f(u_2)du_2\right]\Bigg\} \\
&= 1-\lambda_2 p_2 \underbrace{\int_{a_2}^{\bar{u_2}} (1-F(u_2))f(\phi^{-1}(u_2))f(u_2)du_2}_{\text{joint-search effect}}
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
p_2 \Bigg\{ &(1-\lambda_1)\left[f(\hat{u_2})(1-F(\hat{u_2})) + 2\int_{\underline{u_2}}^{\hat{u_2}} f(u_2)^2 du_2\right] \\
&+\lambda_1\left[2\int_B f_2(u_2)dF(\mathbf{u}) + \int_{a_1}^{\bar{u_1}} (1-F(\phi(u_1)))f(u_1)f(\phi(u_1))du_1\right]\Bigg\} \\
&= 1-\lambda_1 p_1 \underbrace{\int_{a_1}^{\bar{u_1}} (1-F(u_1))f(u_1)f(\phi(u_1))du_1,}_{\text{joint-search effect}}
\end{aligned}
\tag{7}
$$

where $\phi(u_1)$ and $\phi^{-1}(u_2)$ are defined in equation (3).

If (1) does not hold, then there is a trivial equilibrium wherein firms charge monopoly prices and consumers never search beyond the first trip.[*3]

[*3] Namely, the Diamond paradox occurs.

Uniqueness and existence of a symmetric equilibrium:

As Zhou (2014) mentions in his seminal work, it is not easy to show the existence of an equilibrium. Even if we assume the log-concavity of $f(u)$, we cannot check whether the profit function (5) is quasi-concave or not. Therefore, we employ a technique developed by Zhou (2014) to check the existence of a symmetric equilibrium. To save space, we show the uniqueness and existence of a symmetric equilibrium in the online appendix, where we show that when the match utilities are distributed symmetrically and exponentially, there exists a unique symmetric pure-strategy equilibrium. [*4]

3. Effect of changing the mass of consumer who demand one of the products on prices and profit

In the last section, we characterized the symmetric equilibrium and derive the symmetric equilibrium pricing strategy. In this section, we analyze how changing the mass of consumers who demand one of the products affects the prices for both products and profit. For simplicity, in the following analysis, we refer to the changing of the mass of consumers who demand products i, as change in λ_i.

3.1. Effect of changing the mass of consumer who demand one of the products on prices

First, we investigate how the change in λ affects prices. For example, suppose that λ_1 decreases. This corresponds to the case in which a new indirect competitor supplies product 1 appears outside of this market, and a fraction of consumers now purchase product 1 in this outside market. Since such an entry occurred outside the market and each product is independent, the incumbents still maintain market power. Moreover, in the single product case, this sort of entry does not affect the price of product 1 since the consumers who left the market have no outside option, so they must still gather information. However, as we will see, the change in λ affects the prices of both products 1 and 2 through the joint-search effect in the multiproduct case, even if the two products are neither complements nor substitutes. We begin with the following claim.

[*4] We may also check the existence for other distributions by the same procedure, e.g., uniform, log-normal, Weibull, Gumbel, logistic and so on.

Claim: *In the symmetric equilibrium under the relatively small search cost condition (1), the change the mass of consumers who demand one product affects prices for both products and each firm's profit when there are consumers who search for both products and there is a joint-search effect.*

In the following context, we focus on product 1. From the system of equations (6) and (7), we obtain the following proposition.

Proposition 1: *In the symmetric equilibrium under the relatively small search cost condition (1), the following relation holds.*

$$\frac{\partial p_1}{\partial \lambda_1} \lesseqgtr 0 \Leftrightarrow \beta_1\lambda_2\left\{(\gamma_2-\alpha_2)[\gamma_1\lambda_2+\alpha_1(1-\lambda_2)]+\beta_2(\alpha_2-\beta_1\lambda_2)\right\} \lesseqgtr 0, \tag{8}$$

where

$$\alpha_1 \equiv f(\hat{u_1})(1-F(\hat{u_1}))+2\int_{\underline{u_1}}^{\hat{u_1}}(f(u_1))^2du_1, \quad \beta_1 \equiv \int_{a_2}^{\bar{u_2}}(1-F(u_2))f(\phi^{-1}(u_2))f(u_2)du_2$$

$$\gamma_1 \equiv 2\int_B f_1(u_1)dF(\mathbf{u})+\int_{a_2}^{\bar{u_2}}(1-F(\phi^{-1}(u_2)))f(\phi^{-1}(u_2))f(u_2)du_2$$

$$\alpha_2 \equiv f(\hat{u_2})(1-F(\hat{u_2}))+2\int_{\underline{u_2}}^{\hat{u_2}}(f(u_2))^2du_2, \quad \beta_2 \equiv \int_{a_1}^{\bar{u_1}}(1-F(u_1))f(u_1)f(\phi(u_1))du_1$$

$$\gamma_2 \equiv 2\int_B f_2(u_2)dF(\mathbf{u})+\int_{a_1}^{\bar{u_1}}(1-F(\phi(u_1)))f(u_1)f(\phi(u_1))du_1$$

Note that β_1 and β_2 are the joint-search effect. Note also that when there is no joint-search effect or λ_2 equals zero, the RHS of (8) also equals zero, so the change in λ_1 does not affect the price of product 1, which corresponds to the case in which all consumers search for each product separately, even if they desire both products.

Although the mass of consumer who demand for product 1 increases, we cannot immediately conclude that the sign is non-negative since it depends on the distribution of the match utilities and the search cost. Actually, it would also be negative. To see this, consider the first and the last blanket of the RHS of (8). Suppose s is sufficiently small (implying that competition is considerably severe). When s is sufficiently small, the second term of γ_2 becomes small, since a small search cost implies a higher a_1. Conversely, the second term of α_2 becomes large. Therefore, when the reservation frontier is not too long (the first term of γ) and the probability that consumers draw around the reservation utility a_2 is high, the first blanket of the above expression would be small. Likewise, when the joint-search effect is sufficiently large, the last blanket would also be negative, and it dominates the remaining parts. Therefore, the above expression would be negative when consumers are not price-sensitive, s is small, and the joint-search effect is sufficiently large. However, as we will see in the next subsection, when the match utilities are distributed exponentially or uniformly, the RHS of (8) becomes positive. We omit the same expression for the product 2 because we can derive it in the same manner.

Now, consider the effect of the change λ_2 on the price of product 1.

Proposition 2 : *In the symmetric equilibrium under the relatively small search cost condition (1), the following relation holds.*

$$\frac{\partial p_1}{\partial \lambda_2} \lesseqgtr 0 \Leftrightarrow -[(\gamma_2 - \alpha_2)\lambda_1 + \alpha_2]\left\{(\gamma_1 - \alpha_1)[\gamma_2\lambda_1 + \alpha_2(1-\lambda_1)] + \beta_1(\alpha_1 - \beta_2\lambda_1)\right\} \lesseqgtr 0 \quad (9)$$

The notations are the same as Proposition 1. Note that when the joint-search effects or λ_1 is not too large, the above expression would be negative.[*5] The intuition is as follows. Suppose that both the joint-search effect affecting price 1 and the mass of consumers who demand product 1 are not too large. In this case, competition is not severe, so the firm could obtain enough benefit from selling product 2 because it charges a high price for product 2. On the other hand, the price reduction for product 1 is beneficial since the loss from this reduction is not too large, but it induces more immediate purchasing for both products. Therefore, the price for product 1 decreases as the mass of consumers who demand product 2 increases. When the above expression is negative, the effect of the change in λ_2 on the price of product 1 behaves as complementary products do, even if the two products are independent.

3.2. Effect of changing the mass of consumers who demand one of the products on profit

Second, we investigate how the change in λ affects each firm's profit. For simplicity, in the following analysis, we assume that the two products are symmetric.[*6] In the symmetric equilibrium, each firm obtains half of the demand. Therefore, each firm's expected profit is

[*5] In the appendix, we show that $(\gamma_1 - \alpha_1)$ and $(\gamma_2 - \alpha_2)$ are non-negative. Additionally, when the joint-search effect affecting price 1 or λ_1 is not too large, $\beta_1\beta_2\lambda_1$ closes to zero, so the curly blanket may always be positive, so the expression becomes negative.

[*6] An asymmetric distribution in the match utilities will cause various interesting phenomena, such as the price decreasing in the search cost, even if the match utilities are distributed uniformly or follow another distribution. However, it will deviate from the objective. Moreover, we have not proved an existence of such an asymmetric distribution, so we here focus only on the symmetric distribution.

given by:[*7]

$$\pi = p_1 \frac{1}{2} \lambda_1 + p_2 \frac{1}{2} \lambda_2, \tag{10}$$

where p_1 and p_2 are the solutions to equations (6) and (7). The following example demonstrates the effect of a change in λ on profit (and prices).

The uniform example:

Suppose that the match utilities are distributed uniformly as $f(u_i) = 1$ and $F(u_i) = u_i$ on the interval $[0, 1]$ for $i = 1, 2$. This case corresponds to a situation wherein various consumers with diverse preferences exist, so firms cannot focus on any specific group, or a situation wherein each consumer evaluates the match value(s) the same as they do the price(s). We can find plenty of such uniform examples in practice. For simplify, in the following analysis, we remove the subscript. The reservation match value $\hat{u} = a$ is given by:

$$\int_a^1 [1 - F(u)] du \Leftrightarrow a = 1 - \sqrt{2s}$$

To ensure the validity of search activity, s must be at most 0.5. Define $\int_A$ and $\int_B$ as the areas of the acceptance set A and a complement of A, respectively. In this case, area A is a quarter-circle with radius $\sqrt{2s}$. Using these expressions, we obtain the equilibrium prices p_1^* and p_2^*, as follows.

$$p_1^* = \frac{1 - \frac{\lambda_2 s}{(1-\lambda_1)(2-\sqrt{2s})+\lambda_1(2\int_B + \int_A)}}{(1-\lambda_2)[(2-\sqrt{2s})] + \lambda_2[2\int_B + \int_A] - \frac{\lambda_1 \lambda_2 s^2}{(1-\lambda_1)(2-\sqrt{2s})+\lambda_1(2\int_B + \int_A)}} \tag{11}$$

$$p_2^* = \frac{1 - \frac{\lambda_1 s}{(1-\lambda_2)\left[(2-\sqrt{2s})\right]+\lambda_2(2\int_B + \int_A)}}{(1-\lambda_1)[(2-\sqrt{2s})] + \lambda_1[2\int_B + \int_A] - \frac{\lambda_1 \lambda_2 s^2}{(1-\lambda_2)[(2-\sqrt{2s})]+\lambda_2(2\int_B + \int_A)}} \tag{12}$$

We can obtain the symmetric equilibrium profit by substituting the above equilibrium prices into (10). We can see that the price is increasing in s as Zhou (2014) shows. The other comparative statics are as follows.

$$\frac{\partial \pi}{\partial s} > 0 \ , \ \ \frac{\partial p_1^*}{\partial \lambda_1} > 0 \ , \ \ \frac{\partial p_1^*}{\partial \lambda_2} < 0$$

These expressions are consistent with Propositions 1 and 2. The intuition is quite straight-

[*7] We can also obtain the profit from equation (5) directly by specifying the distribution.

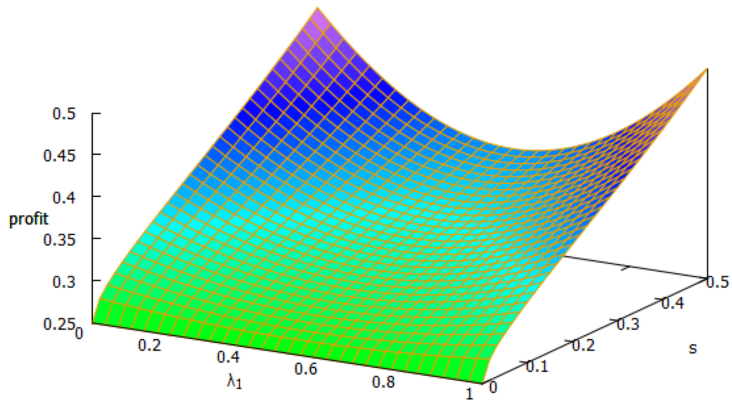

Fig. 3. Profit is convex in λ_1 and increasing in s.

forward. Since the realization of match values is randomly drawn and the utility function is linear, firms have sufficiently large market power in terms of pricing. Hence, firms will not lose all consumers, even if they raise prices and firms can thus charge high prices as the search cost increase. Likewise, prices increase in own demand. On the other hand, when the mass of consumers who demand one product increases, the joint-search effect induces a price reduction of the other product. This is a peculiar effect for the multiproduct search model.

To see how the joint-search effect affects profit, suppose $\lambda_1 + \lambda_2 = 1$. The following figure shows the effect of changing the proportion of λ. Profit is convex in λ and increases in s, implying that profit reaches its minimum when the joint-search effect reaches its maximum.

The exponential example:

Next, we consider exponentially distributed match values. This case corresponds to the situation wherein consumers are sensitive to product suitability or there are many low-quality products. In this case, consumers think that it is unlikely to find a better product from an additional search in terms of *match value*; that is, price has an important role. Suppose that the match values are distributed as $f(u) = be^{-bu}$ and $F(u) = 1 - e^{-bu}$ on the interval $[0, \infty)$. A larger b implies that consumers are more sensitive. The reservation match value $\hat{u} = a$ is defined as:

$$\int_a^\infty [1 - F(u)]du = e^{-ba} = s$$

Note that to ensure search activity, the search cost must satisfy $e^0 = \frac{1}{b} \geq s$. We obtain the equilibrium prices p_1^* and p_2^* as follows.

$$p_1^* = \frac{\frac{1}{b}(1 - \frac{1}{6}\lambda_2 s^3 b^4)}{1 - \frac{1}{36}\lambda_1 \lambda_2 s^6 b^6} \tag{13}$$

$$p_2^* = \frac{\frac{1}{b}(1 - \frac{1}{6}\lambda_1 s^3 b^4)}{1 - \frac{1}{36}\lambda_1 \lambda_2 s^6 b^6} \tag{14}$$

Thus, the equilibrium profit is given by:

$$\pi(\lambda_1, \lambda_2, s, b) = \frac{6\left\{(b^4 s^3 \lambda_1 \lambda_2) - 3\lambda_2 - 3\lambda_1\right\}}{(b^6 s^6 \lambda_1 \lambda_2 - 36)\, b} \tag{15}$$

The comparative statics are as follows.

$$\frac{\partial \pi}{\partial s} < 0\ , \quad \frac{\partial p}{\partial s} < 0\ , \quad \frac{\partial p_1^*}{\partial \lambda_1} > 0\ , \quad \frac{\partial p_1^*}{\partial \lambda_2} < 0 \tag{16}$$

As we mentioned above, price plays a more important role in the exponential case than the uniform case. As in the uniform case, the equilibrium prices simply increase in own λ. However, since profit is the sum of the profit for each product and a price reduction for one product boosts the demand for both products, this price reduction attracts more consumers than in the uniform case. A firm can improve profit by reducing the price for either product. Thus, when the mass of consumers who demand product 1 increases, firms have an incentive to charge a lower price for product 2 to restrain consumers from leaving. Moreover, when the search cost s is relatively high, firms must charge less, for the same aim. This is why equilibrium price p_i decrease in λ_j $(i \neq j)$ and the search cost s.

Now, suppose $\lambda_1 + \lambda_2 = 1$. The following figure shows how changing the proportion of λ affects profit. From the above figure, in the exponential case, the price decreases in s, as does profit.

In this case, we can find an interesting phenomenon: *profit-increasing consumer exit* may occur. To illustrate this, we fix the mass of consumers who demand product 2 and change λ_1. Then, we obtain the following proposition.

Proposition 3. *Suppose the match utilities are distributed symmetrically and exponentially. In the symmetric equilibrium, the following relation holds.*

$$\frac{\partial \pi}{\partial \lambda_1} < 0 \Leftrightarrow (b^6 s^6 \lambda_2^2 - 12 b^4 s^3 \lambda_2 + 36) < 0 \tag{17}$$

When the above relation holds, profit increases as the λ_1 decreases; that is, profit-increasing

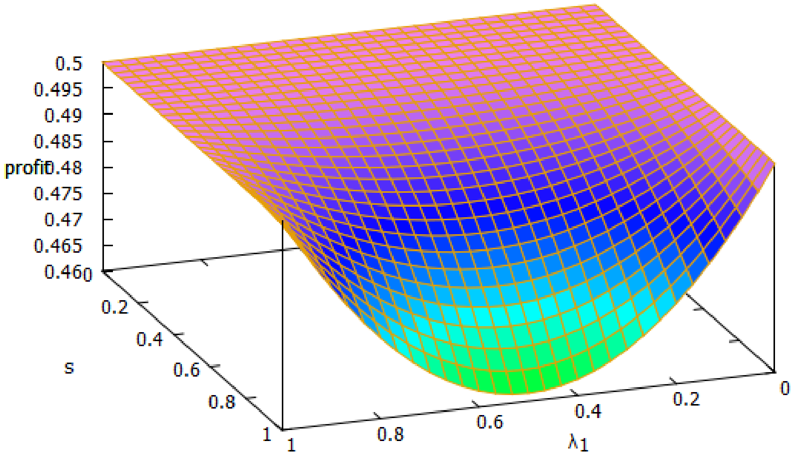

Fig. 4. Profit is convex in λ_1 and decreasing in s.

consumer exit occurs.[*8] Note that the threshold of b decreases in λ_2 and s. In other words, the threshold decreases as the joint-search effect increases. To understand this phenomenon, suppose that the joint-search effect and the search cost are large. For the exponential distribution, price has an important role.[*9] When the joint-search effect is sufficiently large, firms must charge a lower price to keep a consumer from leaving, or the consumer would find better products and purchase both of them from the rival. Additionally, the large b implies that consumers are more sensitive, so firms fall into severe price competition. Therefore, the exit of consumers who search for both products from one side of the market is desirable in terms of the firm's profit. Figure 5 shows how the effect of changing λ on profit moves from positive to negative when s is sufficiently high.

4. Concluding Remarks

In this paper, we construct a sequential consumer search model with differentiated products, where some consumers search for a single product, while others search for multiple products.

[*8] The phenomenon is somewhat similar to the well-known phenomenon in the standard oligopolistic competition model; see Ishibashi and Matsushima (2009), and Pazgal, Soberman and Thomadsen (2013), among others. However, the mechanism of the phenomenon is entirely different. In the existing literature, *ex ante* heterogeneity among consumers is important, e.g., preference, location, and so on. In our model, the important factor is the joint-search effect and it is unique to the multiproduct search model.

[*9] In that sense, the phenomenon also likely to occur in the case of the log-normal distribution, Weibull distribution, Gumbel distribution, logistic distribution, and so on. In any distribution, the joint-search effect is an essential matter.

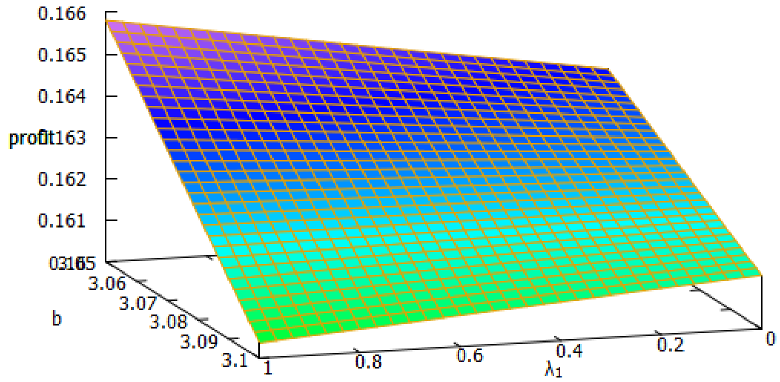

Fig. 5. The effect of changing λ_1 becomes negative from positive as b increases.

We show that the joint-search effect remains with such demand heterogeneity, and it induces a decrease in prices and profit. We also show that when the mass of consumers who demand one product changes, it affects the prices for both products due to the joint-search effect, even if the products are neither complements nor substitutes. We find that when consumers are price sensitive and the joint-search effect is sufficiently large, firms must charge lower prices, so a profit-increasing consumer exit may occur. Finally, we discuss two considerable extensions.

More than two products (or more than two firms)— We can extend the model for more than two products or more than two firms. Likewise, in Zhou (2014), an increasing number of products may strengthen the joint search effect. When the amount of consumers who search for more than two product increases, the joint search effect may also increase. Therefore, firms will fall into severe price competition as the joint-search effect increases, and hence profit may decrease. This implies that the profit-increasing consumer exit may become more likely in a case with more than two products. On the other hand, the effect of increasing the number of firms is hazy.

Costly recall— Most literature assumes that a consumer can go back without an additional search cost. It is justified if we consider calling in an order and online shopping, where a consumer can reserve previous web pages or take notes about prices and contact information. We can also justify the assumption when we consider some specific sort of products, as we mentioned in footnote 2. However, it is only one aspect of the real market. For more applicability, we can consider *costly recall.* Janssen and Parakhonyak (2014) analyze costly recall in a single product search model and discover a small but important effect that depresses prices. However, there are some complicated analytic problems caused by multiproduct search. We predict that we can also find this sort of effect in the multiproduct search model, and the effect should be larger than in the single product model.

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Appendix

This section consists of two parts. In former part A, we provide the proofs which are omitted in the main context. In latter part B, we demonstrate how changing the mass of consumers who demand one of the products (or both products) effects on the social welfare.

A. Omitted Proofs

Proof of Lemma 1:

The following proof is almost same as Zhou (2014). Let us start from the former part of Lemma 1. Suppose the maximum search cost condition holds. To simplify, we derive the F.O.C. of equation (5) separately between the single-product part and the multiproduct part. Of course, we can obtain Lemma 1 by differentiating equation (5) directly. Equation (7) is obtained by following the same procedure.

single-product part: We start from the former part of equation (6). To obtain the F.O.C, we differentiate (5) with respect to ϵ_1 and evaluate $\epsilon_1 = 0$. Then we obtain:

$$\frac{\partial \pi}{\partial \epsilon_1} = -\lambda_1(1-\lambda_2)\left\{\frac{1}{2}[1-F(\hat{u_1})][1+F(\hat{u_1})] + \int_{\underline{u_1}}^{\hat{u_1}} F(u_1)f(u_1)du_1\right\}$$
$$+ p_1\lambda_1(1-\lambda_2)\left\{\frac{1}{2}f(\hat{u_1})[1+F(\hat{u_1})] + \int_{\underline{u_1}}^{\hat{u_1}} f(u_1)^2du_1 - F(\hat{u_1})f(\hat{u_1})\right\} = 0$$
$$\Leftrightarrow p_1\lambda_1(1-\lambda_2)\left\{\frac{1}{2}f(\hat{u_1})[1-F(\hat{u_1})] + \int_{\underline{u_1}}^{\hat{u_1}} f(u_1)^2du_1\right\} = \lambda_1(1-\lambda_2)\frac{1}{2} \quad (18)$$

multiproduct part: Likewise, we can obtain the latter part of (6). Recall that $A(\epsilon)$ and $B(\epsilon)$ are the region when firm A debiates (ϵ_1, ϵ_2). By using Leibnitz's rule, the F.O.C. is **a half** of the following equation times $\lambda_1\lambda_2$.

$$-\int_{A(\epsilon)} dF(\mathbf{u}) + (p_1+p_2)\int_C f(\mathbf{u})du_2 - \int_{B(\epsilon)} F(u_1+\epsilon_1)dF(\mathbf{u})$$
$$+ p_1\int_{B(\epsilon)} f(u_1)dF(\mathbf{u}) - \int_C [p_1F(u_1+\epsilon_1) + p_2F(u_2+\epsilon_2)]\, f(\mathbf{u})du_2$$
$$-\int_B [1-F(v_1-\epsilon_1)]dF(\mathbf{v}) + \int_B p_1f(v_1)dF(\mathbf{v}) = 0,$$

where $\int_C$ denotes the line-integral. We applied Green's theorem such that $\int_A \frac{\partial g}{\partial x}dydx = \int_C gdy$. Evaluating $\epsilon_1 = 0$, we obtain:

$$\frac{1}{2}\lambda_1\lambda_2 = p_1\lambda_1\lambda_2\int_B f(u_1)dF(\mathbf{u}) + \frac{1}{2}\lambda_1\lambda_2\int_C \{p_1[1-F(u_1)] + p_2F[1-F(u_2)]\}\, f(\mathbf{u})du_2 \quad (19)$$

Combining equations (A1) and (A2), we obtain equation (6). Note that we can restate the line-integral part by using the definition of reservation frontier as follows:

$$\int_C [1-F(u_2)]f(\mathbf{u})du_1 = \int_{a_1}^{\bar{u_1}} [1-F(\phi(u_1))]f(u_1)f(\phi(u_2))du_1$$

The latter part of the Lemma 1. is quite obvious. Suppose the maximum search cost condition (1) doesn't hold. That is:

$$\zeta_i(\underline{u_i}) < s$$

It means that the incremental expected benefit never exceeds the search cost. Therefore, as long as each firm charges the equilibrium price, no consumer decides to continue searching. Let $\hat{p_i}$ denotes the monopoly price for product i (i=1,2). For any equilibrium price $p_i' \neq \hat{p_i}$,

$$\pi(\hat{p_1}, \hat{p_2}) \geq \pi(p_1', p_2')$$

holds. Therefore the firms have no incentive to charge prices except for the monopoly prices.

□

Proof of Proposition 1.

From the Lemma 1. and the definitions, we can rearrange the expressions of price 1 and price 2 as follows.

$$p_1 = \frac{1 - \frac{\lambda_2 \beta_1}{(1-\lambda_1)\alpha_2 + \lambda_1 \gamma_2}}{(1-\lambda_2)\alpha_1 + \lambda_2 \gamma_1 - \frac{\lambda_1 \lambda_2 \beta_2 \beta_1}{(1-\lambda_1)\alpha_2 + \lambda_1 \gamma_2}} \tag{20}$$

$$p_2 = \frac{1 - \frac{\lambda_1 \beta_2}{(1-\lambda_2)\alpha_1 + \lambda_2 \gamma_1}}{(1-\lambda_1)\,\alpha_2 + \lambda_1 \gamma_2 - \frac{\lambda_1 \lambda_2 \beta_2 \beta_1}{(1-\lambda_2)\alpha_1 + \lambda_2 \gamma_1}} \tag{21}$$

Note that the changing of λ_1 or λ_2 does not affects the region of integrals or the distribution functions directly. So we can differentiate p_i with respect to λ_i for $i = 1, 2$ as follows.

$$\frac{\partial p_1}{\partial \lambda_1} = \frac{\beta_1 \lambda_2 \{(\gamma_2 - \alpha_2)[\gamma_1 \lambda_2 + \alpha_1(1-\lambda_2)] + \beta_2(\alpha_2 - \beta_1 \lambda_2)\}}{\{[((\gamma_1 - \alpha_1)\gamma_2 - \beta_1 \beta_2 + (\alpha_1 - \gamma_1)\alpha_2)\lambda_1 + (\gamma_1 - \alpha_1)\alpha_2]\lambda_2 + (\alpha_1 \gamma_2 - \alpha_1 \alpha_2)\lambda_1 + \alpha_1 \alpha_2\}^2} \tag{22}$$

$$\frac{\partial p_2}{\partial \lambda_2} = \frac{\beta_2 \lambda_1 \{(\gamma_1 - \alpha_1)[\gamma_2 \lambda_1 + \alpha_2(1-\lambda_1)] + \beta_1(\alpha_1 - \beta_2 \lambda_1)\}}{\{[((\gamma_1 - \alpha_1)\gamma_2 - \beta_1 \beta_2 + (\alpha_1 - \gamma_1)\alpha_2)\lambda_1 + (\gamma_1 - \alpha_1)\alpha_2]\lambda_2 + (\alpha_1 \gamma_2 - \alpha_1 \alpha_2)\lambda_1 + \alpha_1 \alpha_2\}^2} \tag{23}$$

Since all notations above are non-negative, we have to check the sign of $(\gamma_1 - \alpha_1), (\gamma_2 - \alpha_2), (\alpha_2 - \beta_1 \lambda_2)$ and $(\alpha_1 - \beta_2 \lambda_1)$. First, we show that $\gamma_1 \geq \alpha_1$ (and $\gamma_2 \geq \alpha_2$) always hold. From the definition, we have,

$$\begin{aligned}
\gamma_1 - \alpha_1 \;&=\; 2\int_{\underline{u_2}}^{\bar{u_2}} \left\{ \int_{\underline{u_1}}^{\phi^{-1}(u_2)} f(u_1) f(\phi^{-1}(u_2)) du_1 + [1 - F(\phi^{-1}(u_2))] f(\phi^{-1}(u_2)) \right\} dF(u_2) \\
&\quad - f(a_1)[1 - F(a_2)] - 2\int_{\underline{u_1}}^{a_1} (f(u_1))^2 du_1 \\
&\geq 2\int_{\underline{u_2}}^{\bar{u_2}} \left\{ \int_{\underline{u_1}}^{a_1} f(u_1) f(\phi^{-1}(u_2)) du_1 - \int_{\underline{u_1}}^{a_1} (f(u_1))^2 du_1 \right\} dF(u_2) \geq 0
\end{aligned}$$

We use the fact that $\int_{\underline{u_2}}^{\bar{u_2}} f(\phi^{-1}(u_2)) f(u_2) du_2 = f(u_1)$ and $\phi^{-1}(u_2)$ defined in $[a_1, \bar{u_1}]$. $\gamma_2 \geq \alpha_2$ would be proved by following the same procedure. Thus, the sign of effect of changing λ_1 on price 1 (price 2) depends on the sign of,

$$\begin{aligned}
&\alpha_2 - \beta_1 \lambda_2 \quad \text{(for price 1)} \\
&\alpha_1 - \beta_2 \lambda_1 \quad \text{(for price 2)}
\end{aligned}$$

If the sign of the above expression is non-negative, then the effect of changing own λ on the price is positive, i.e., the price increases in the mass of consumers who demand for the product.

□

Proof of Proposition 2.

From the definition, We have,

$$\frac{\partial p_1}{\partial \lambda_2} = -\frac{((\gamma_2-\alpha_2)\lambda_1+\alpha_2)\left\{(\gamma_1-\alpha_1)[\gamma_2+\alpha_2(1-\lambda_1)]+\beta_1(\alpha_1-\beta_2\lambda_1)\right\}}{\left\{[((\gamma_1-\alpha_1)\gamma_2-\beta_1\beta_2+(\alpha_1-\gamma_1)\alpha_2)\lambda_1+(\gamma_1-\alpha_1)\alpha_2]\lambda_2+(\alpha_1\gamma_2-\alpha_1\alpha_2)\lambda_1+\alpha_1\alpha_2\right\}^2} \tag{24}$$

Since $\gamma_2-\alpha_2$ and $\gamma_1-\alpha_1$ are non-negative which is proved above, the sign of above expression depends on the sign of the $\beta_1(\alpha_1-\beta_2\lambda_1)$.

□

Proof of Proposition 3.

The firm's profit function in the symmetric equilibrium when the match values distribute exponentially as $f(u) = be^{-bu}$ is given by:

$$\pi(\lambda_1,\lambda_2,s,b) = \frac{6\left\{(b^4s^3\lambda_1\lambda_2)-3\lambda_2-3\lambda_1\right\}}{(b^6s^6\lambda_1\lambda_2-36)b} \tag{25}$$

Differentiating the above expression with respect to λ_1, we have,

$$\frac{\partial\pi}{\partial\lambda_1} = \frac{18(b^6s^6\lambda_2^2-12b^4s^3\lambda_2+36)}{b\left(b^6s^6\lambda_1\lambda_2-36\right)^2}$$

It implies that the firm's profit decreases as the λ increases if,

$$(b^6s^6\lambda_2^2-12b^4s^3\lambda_2+36)<0$$

□

B. Welfare Analysis

According to Zhou (2011), with our settings, we obtain consumer surplus as:

$$CS(s,p) = CS(0,p) - \int_0^s t(s)ds,$$

where $t(s)$ expresses the number of searching which is given by search cost s and p denotes the price vector. The first term of the RHS is consumer surplus in which s equals to zero and the second term denotes the inefficiency caused by imperfect information. Consumer surplus for the consumer who searches two products with a given s is given by:

$$CS(0,p) = \sum_{i=1}^{2}(E[max(u_i, v_i)] - p_i),$$

where u and v denote the match utilities from firm A and B respectively, and i denotes the index for a product $i = 1, 2$. Likewise, consumer surplus for the consumer who searches a single product, e.g., searching for product 1 is:

$$CS(0,p) = E[max(u_1, v_1)] - p_1$$

Let $t_i(s)$ denotes the number of searching for singleproduct i and $t_{12}(s)$ denotes the number of searching for multiproduct. Thus we have the aggregate consumer surplus as:

$$\begin{aligned} CS(s,p) = \lambda_1\lambda_2 &\left\{\sum_{i=1}^{2}(E[max(u_i, v_i)] - p_i) - \int_0^s t_{12}(s)ds\right\} \\ + \sum_{i=1, i\neq j}^{2} &\lambda_i(1-\lambda_j)\left\{(E[max(u_i, v_i)] - p_i - \int_0^s t_i(s)ds\right\} \end{aligned} \tag{26}$$

On the other hand, the aggregate producer surplus is derived as:

$$PS(s, p, \lambda_1, \lambda_2) = \lambda_1 p_1(s, \lambda_1, \lambda_2) + \lambda_2 p_2(s, \lambda_1, \lambda_2) \tag{27}$$

Therefore, the effect of changing λ_1 is given by differentiating the total surplus, which is combination of two surpluses. More precisely:

$$\begin{aligned} &\frac{\partial TS(s,p)}{\partial \lambda_1} = \frac{\partial CS(s,p)}{\partial \lambda_1} + \frac{\partial PS(s,p)}{\partial \lambda_1} \\ &= \lambda_2 \sum_{i=1}^{2}(E[max(u_i, v_i)] - p_i) + (1-\lambda_2)(E[max(u_1, v_1)] - p_1) - \lambda_2(E[max(u_2, v_2)] - p_2) \\ &- \lambda_2 \int_0^s t_{12}(s)ds - (1-\lambda_2)\int_0^s t_1(s)ds + \lambda_2 \int_0^s t_2(s)ds - \frac{\partial p_1}{\partial \lambda_1}\lambda_1\lambda_2 - \frac{\partial p_2}{\partial \lambda_1}\lambda_1\lambda_2 \\ &- \frac{\partial p_1}{\partial \lambda_1}\lambda_1(1-\lambda_2) - \frac{\partial p_2}{\partial \lambda_1}\lambda_2(1-\lambda_1) + p_1 + \lambda_1\frac{\partial p_1}{\partial \lambda_1} + \lambda_2\frac{\partial p_2}{\partial \lambda_1} \\ &\Leftrightarrow E[max(u_1, v_1)] + \int_0^s \{\lambda_2[t_2(s) - t_{12}(s)] - (1-\lambda_2)t_1(s)\}\, ds \end{aligned}$$

Recall that the distribution of match utilities are symmetry between firms. Denote the c.d.f. of match utility as $F_i(u_i)$ and define the c.d.f. of $max(u_i, v_i)$ as $G_i(x)$, we have:

$$E[max(u_i, v_i)] = \int_{\underline{u_i}}^{\bar{u_i}} dG_i(x) = 2\int_{\underline{u_i}}^{\bar{u_i}} xF_i(x)f_i(x)dx$$

Hence,

$$\frac{\partial TS(s)}{\partial \lambda_1} = 2\int_{\underline{u_1}}^{\bar{u_1}} xF_1(x)f_1(x)dx + \int_0^s \{\lambda_2[t_2(s) - t_{12}(s)] - (1-\lambda_2)t_1(s)\}\, ds \qquad (28)$$

We first derive the sign of the second term of the RHS. Let $A_i^S(s)$ denote the normalized length of the acceptance region for the model of singleproduct and $A_{12}^M(s)$ denote the normalized measure of the acceptance set. Then we have:

$$A_2^S(s) \equiv \int_{a_2}^1 du_2 \geq \int_{a_2}^1 [1 - \phi^{-1}(u_2)]du_2 \equiv A_{12}^M(s)$$

For the duopoly model, $t_2(s) = 2 - A_2(s)$ and $t_{12}(s) = 2 - A_{12}(s)$. Thus $t_2(s) \leq t_{12}(s)$. It implies that under the small search cost condition and free-recall assumption, the total number of searching for multiproduct is greater than or equal to the total number of searching for singleproduct. Therefore, the second term of the equation (A11) is non-positive.

Note that the cross partial derivative is given by:

$$\frac{\partial^2 TS}{\partial \lambda_1 \partial \lambda_2} = \int_0^s [t_1(s) + t_2(s) - t_{12}(s)]ds \geq 0$$

since $1 \leq t(s) \leq 2$. Hence, the effect of changing λ_i on social welfare increases in λ_j, implying that the multiproduct searching saves the search cost.

The uniform example

Suppose the match utilities are distributed uniformly and symmetrically over $[0,1]$ such as $F(u) = u$. Then $E[max(u_i, v_i)] = \frac{2}{3}$. $t_i(s)$ for $i = 1,2$ and $t_{12}(s)$ are given as:

$$t_1(s) = t_2(s) = 2 - \int_a^1 du = 2 - \sqrt{2s}$$
$$t_{12}(s) = 2 - \int_a^1 [1 - \phi(u)]du = 2 - \frac{1}{2}\pi s$$

Therefore,

$$\frac{\partial TS(s)}{\partial \lambda_1} = \frac{2}{3} + \lambda_2[2s + \frac{1}{4}\pi s^2 - \frac{4\sqrt{2}}{3}s^{\frac{3}{2}}] - 2s + \frac{2\sqrt{2}}{3}s^{\frac{3}{2}}$$

The following figure shows that how s and λ_2 affect the effect of changing λ_1.
It increases in λ_2 and decreases in s.

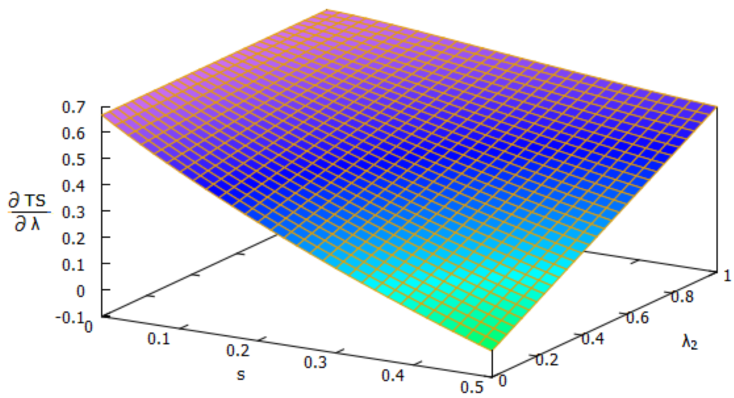

Fig. 6. The effect of changing λ_1 on welfare.