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MULTIPRODUCT FIRMS, CONSUMER SEARCH, AND DEMAND HETEROGENEITY

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Multiproduct Firms, Consumer Search, and Demand Heterogeneity

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This study constructs a consumer search model in which some consumers search for multiple products, whereas others search for a single product. A price difference arises because of a difference in the price elasticity for each group. We show that a positive demand shock to one of the products decreases the price of another product, whereas it increases its own price, and a negative correlation between the demands for each product strengthens these tendencies. Both prices decrease, however, following a positive demand shock when the demands for each product are positively correlated. We also show that multiproduct firms set a relatively high price for a more demanded product, as such a product's price tends to be more elastic with respect to search costs. A price difference between products increases as the demand gap between products increases or as economies of scale in search increase.

Keywords: consumer search, multiproduct pricing, demand heterogeneity
JEL classification: D11, D43, D83, L13

I. Introduction

Considering consumers' purchase behavior, multiproduct firms that sell in multiproduct markets must often consider a variety of consumers with different demands. Some consumers will want to purchase a large basket of products, whereas others will want to purchase only one of the products, *i.e.,* each consumer is heterogeneous in terms of demand. There exists a large literature investigating multiproduct firms' pricing in the context of differentiated products and demand-side heterogeneity.[1] From another perspective, consumers'

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[1]For example, Champsaur and Rochet (1989) and Johnson and Myatt (2003) analyze vertically differentiated products with multiple demand segments; Doraszelski and Draganska (2006) investigates the segmentation problem in horizontally differentiated products markets. All of these studies assume perfect information.

purchase behavior is inherently incurring various costs associated with information gathering, namely *search costs*. There is a growing literature focusing on the relationship between consumers' search behavior and firms' pricing in a single-product environment that shows that such search behavior significantly affects firms' strategy;[2] although, only a few studies adopt a multiproduct context (McAfee 1995, Zhou 2014, Rhodes 2014). Furthermore, somewhat surprisingly, no previous studies focus on the demand-side heterogeneity mentioned above in a multiproduct environment. The purpose of this study is to reveal how heterogeneity in demand affects multiproduct firms' pricing, and provide useful implications in a multiproduct environment.

In our model, we consider a market with two multiproduct firms and two products that are supplied by both firms. There are potentially three groups of consumers: consumers who wish to purchase just one product or the other; and consumers wishing to purchase both products. Each consumer is initially uninformed about actual prices and her valuation of the product she wants; hence, she must search to gather product information which incurs a search cost per firm. Search costs can be different between single-product and multiproduct searchers, and multiproduct searchers do not need to purchase all products in one place. The firms cannot discriminate prices among consumers. To highlight the demand heterogeneity effect, we mainly assume the two products are symmetric in the sense of the distribution of valuation.

First, we describe the optimal search rules of the three groups and the demand function for each group, and then derive equilibrium prices for given consumers. A key factor is that the price elasticity of each group will be different because there exists scale merit of multiproduct search behavior, namely the *joint search effect* so termed by Zhou (2014) and *economies of scale in search*. Hence, the ratio of consumers searching for multiple products to consumers searching for just one of the products matters the price that firms charge for each product. These effects provide firms with an extra incentive to lower prices to attract multiproduct searchers. Consequently, a positive demand shock to one of the products (an increase in the mass of multiproduct searchers) induces firms to lower the price of the other product. Not surprisingly, such an increase in demand for one product affects not only the price of the other product but also the own price because a price reduction for one product induces multiple purchases via the scale merit of multiproduct search. As a result, an increase in demand for one product also causes an increase in own price resulting in a price difference between products. We believe that these results bridge a gap between evidence and theory.[3]

[2]The basic theory of search was developed by Simon (1955), Stigler (1961), Kohn and Shavell (1974), and Weitzman (1979). The pioneering works of Diamond (1971), Varian (1980), and Stahl (1989) investigate models of price searching. Wolinsky (1986) and Anderson and Renault (1999) construct a model of search for both actual price and match utility. Recently, in laboratory experiments, Caplin, Dean and Martin (2011) develops a search-theoretic experimental technique and shows that subjects indeed search sequentially.

[3]For example, we often observe shifts in prices that are associated with a change in demand; that is, *countercyclical pricing* such as retail prices decreasing during high demand seasons. Indeed, there is a vast body of evidence that shows such demand-changing shifts in prices, as Chevalier, Kashyap and Rossi (2003) points out. Nevertheless, in standard price theory and consumer search theory, positive demand shocks only have either a positive effect or no effect on price. So there was a remaining puzzle.

In the latter part of this paper, we consider several extensions that are useful for both theoretical and empirical analyses: correlation among consumers' demand, comparison prices and price sensitivity of a popular and a less popular product. We show that if there is a negative correlation between demands, it strengthens the impact of a change in demand for one product on the two prices; that is, the price of one product increases more, while the price of the other product decreases more. However, a positive correlation negates the effect of an increasing price associated with an increase in own demand, and such an increase in demand leads to a decrease in both prices. Specifically, more multiple searchers result in lower prices for both products. We also show that firms charge higher prices for the popular product, whereas they lower the price for the other product (such as *cross-subsidization pricing*). Moreover, the price for a popular product is more sensitive to changes in search costs in many circumstances, which is intuitive and consistent with standard models of price determination.

The rest of the article proceeds as follows. In the remaining part of this section, we describe both the related theoretical and empirical literature. Section II presents the basic model of single- and multiproduct searches and gives the optimal pricing rule. Section III analyzes how a change in the proportion of consumers searching for one of the products (or simply a demand shock to one of the products) affects the prices; introduce correlation between demands; compare prices and price sensitivity of different popularity products. Section IV concludes and discusses some important extensions. All proofs and calculations are available in the appendix.

Related literature

Our model builds on a single-product search model and a multiproduct search model established in the seminal works of Anderson and Renault (1999) and Zhou (2014), respectively. These papers (and other existing papers on consumer search theory) assume that all consumers search for all products in the market. We combine these models to allow for consumers with different demands for product varieties. More precisely, our idea is similar to that of Zhou (2011), who produced the working paper version of the work above. A key difference is that we allow the demand for each product to be different, and investigate how this demand heterogeneity affects firms' pricing.

There is a growing (but still small) literature that investigates multiproduct search environments despite most firms selling many products. The pioneering work of Burdett and Malueg (1981) develops a multiproduct search model in which each consumer wants to purchase several products and must discover the vector of prices. They characterize the set of consumers' acceptable price vectors as the reservation price frontier. The idea of a reservation frontier in our multiproduct analysis (and that of Zhou (2014)) is similar to their work. McAfee (1995) (and Burdett and Judd (1983)) also studies a multiproduct environment

We show that if the demand for each product are positively correlated, then prices can indeed decrease in response to demand changes (see Section III-ii (Proposition 2)).

in which consumers can discover the vector of prices with a given probability. In those paper, firms adopt mixed strategies and there are multiple equilibria.

In recent works on multiproduct search, Rhodes (2014) and Rhodes and Zhou (2019) investigate consumers searching both prices and valuations for multiple products. Rhodes (2014) studies the relationship between the size of the product range and prices. He shows that larger stores charge lower prices to attract consumers having lower product valuations. The methodology of his work is different from that of Zhou (2014), and ours, but his results are similar. Rhodes and Zhou (2019) examine why small retailers that provide a single product can coexist with larger multiproduct firms.

Our paper is related to the literature on *cross-subsidization* strategy; that is, the strategy of charging a high price for one of the products and charging a low price for others to compensate for it. Chen and Rey (2012) consider consumers with homogeneous valuations and both large and small retailers that offer different varieties in a competitive market. They show that a cross-subsidization strategy will be adopted by the large retailer as an exploitative device. Chen and Rey (2019) also investigate cross-subsidization with consumers who are different in terms of search cost. In another related study, Lal and Matutes (1994) examine a multiproduct price-search environment in other ways; that is, using the Hotelling city model with advertising. They show that all consumers purchase all products at the same store in equilibrium (one-stop shopping occurs), which is different from our results. They also investigate *loss-leader pricing* (a strategy in which a very low price is charged (often below marginal cost) to attract multiple purchases), which is related to our results. We believe that our model may be complementary to their paper and enhances the explanatory power of multiproduct firms' strategies.

Our paper is also related to the literature on countercyclical pricing as mentioned above (Warner and Barsky 1995, Chevalier, Kashyap and Rossi 2003, Guler, Misra and Vilcassim 2014). Warner and Barsky (1995) find weekly price patterns, such that the mean price decreases on Friday, using retail store data in Ann Arbor. To examine the patterns, they adopt the idea of economies of scale in search, which is somewhat similar to our approach, to generate cyclical demand elasticities. They show that it is optimal for consumers to search during high purchase seasons because of the economies of scale in search, and as a result, lower prices exist in higher purchase seasons. Chevalier, Kashyap and Rossi (2003) also examine whether prices fall during high demand seasons using weekly store-level scanner data of Dominick's Finer Foods, the second largest supermarket in the Chicago area. Guler, Misra and Vilcassim (2014) finds that the difference of changes in consumer valuations result in countercyclical pricing. As Zhou (2011) points out, those paper consider only own-price elasticity. We show that the cross-price elasticity also does matter, and such countercyclical pricing strategies can be explained well by changes in the number of mass multiproduct searchers (relative to the mass of single-product searchers) in the market. Therefore, we should take both price elasticities into account in considering multiproduct firms' pricing.

II. The Model

We base our setting on studies by Anderson and Renault (1999) and Zhou (2014). There are two firms, I and II, supplying two products, products 1 and 2. Without loss of generality, we assume that their marginal costs are normalized to zero. We also exclude the possibility of product bundling (both *pure* and *mixed*), so consumers can choose any combination of firms' products.[4] The market size is normalized to one. Let λ_i ($0 \leq \lambda_i \leq 1$) denote the proportion of consumers who search for product i, for $i = 1, 2$.[5] Suppose that the consumer demand for the products is independently distributed; then, $\lambda_1\lambda_2$ of consumers search for both products.[6] Firms have perfect information about the mass of consumers. We assume initially that both λs are independent. We analyses the effects of the correlation between λs later.

Initially, each consumer has imperfect information about the actual price and the utilities of all products, and they sequentially gather information while incurring search cost s. According to Perloff and Salop (1985), and other standard consumer-search models, we treat both products as differentiated between firms I and II, and each consumer has idiosyncratic tastes for each product. We represent this by assuming that each consumer's valuation for each product is independently and randomly drawn from a common cumulative distribution function F, which is twice differentiable so it has a common probability density function f and is defined on the interval $[\underline{u}, \bar{u}]$. The valuation for product i is denoted by u_i for $i = 1, 2$ and we call the valuation the *match utility*. To obtain clear-cut results, we assume that the two products are neither complements nor substitutes in the sense that each consumer simply obtains the sum of the match utilities when she searches for both products. We also assume that each consumer has an independent valuation for the two products in the same firm. Additionally, we assume that each consumer's valuation is independent across consumers. Therefore, for the consumers who search for both products, once they go to a firm, they draw a pair of match values from $F(u_1, u_2) = F(u_1)F(u_2)$. The distribution is commonly known. We also impose the regularity condition (the increasing hazard rate condition) such that $\frac{f_i(u_i)}{1-F_i(u_i)}$ increases in u_i for $i = 1, 2$.

We allow search costs to be heterogeneous among groups. Specifically, we allow that the search cost *per product* for a multiproduct searcher can be less than that for a single-product searcher, which is based on the idea that there will be *economies of scale in search* for multiple search behaviors (*e.g.,* saving time, money, etc.). Let s^S be the search cost *per store* for a single-product searcher and

[4]For more details, see the discussion in Section IV.

[5]We assume that λ is exogenously given to simplify the model and make the analysis more transparent. This case corresponds to the case such that a consumer who searches only for product i (represented by $\lambda_i(1-\lambda_j)$) will obtain a large basic utility V_i in addition to the realization match utility u_i, whereas she obtains only a small basic utility V_j such that $V_j + \bar{u_j} - p_j < 0$; hence, she does not search for information about product j *ab initio.* This setting is consistent with many other studies and the assumption of a full covered market.

[6]Of course, another market expression can be considered. We adopt this setting in following the previous research, and simplify the notation. In our model, the single-product search model (*e.g.,* Anderson and Renault (1999)) corresponds to $\lambda_i = 1$ and $\lambda_j = 0$, and the multiproduct search model (*e.g.,* Zhou (2014)) corresponds to $\lambda_1 = \lambda_2 = 1$. See the discussion for more details.

$s\ (= s^M)$ for a multiproduct searcher. Define δ such that $s^S = \delta s\ (0.5 \leq \delta \leq 1)$.[7] An interpretation of δ is as follows. If $\delta = 1$, then the search costs *per store* are exactly the same among consumers, *i.e.,* a multiproduct searcher can obtain information for half the cost per product, and therefore, there exists economies of scale in search for multiproduct searchers. However, for $\delta = 0.5$, the search costs *per product* are the same among consumers. In that sense, δ represents the (relative) measure of economies of scale in search (*per product*). This is based on the following ideas. As the main component of a search cost is the *trip cost*, the search cost per product for multiproduct searchers may be less than it is for single-product searchers via saving money, time, and so on. In that sense, there exists economies of scale in search. In this case, δ approaches from 0.5 to 1 as the trip cost increases relative to the in-store search cost. However, when the main search cost is the cost for searching inside the store, the search cost per product for all consumers close to similar. In this case, δ approaches 0.5.

We also include other assumptions, such as *no replacement, full covered market, and free-recall.*[8] Consumers can memorize information and freely return to firms they've visited to purchase products. In equilibrium, all consumers purchase the products they want. These three assumptions are standard in the single-product search literature. In the multiproduct search models, however, the free-recall assumption implies some new features. Once a consumer who is searching for two products visits a firm, her options are (i) buying both products immediately and ending the search activity, or (ii) buying no products and going to another firm. In this scenario, the other possible option (iii) buying one of the products and then continuing to search, does not exist because of the free-recall assumption.

We also assume that each consumer's utility function is linear in that the net utility from purchasing product i at the price p_i is given by $(i = 1, 2)$:

$$u_i - p_i.$$

Because this is independent of any search cost, when a consumer purchases both products, her utility is the sum of the individual net utilities.

Each consumer's optimal search rule follows Kohn and Shavell (1974): in each period of search, a consumer compares the incremental expected benefit of one more search with the additional search cost. If the former is greater, she goes to another firm and then compares the observed utilities; otherwise, she immediately stops and purchases the product (products) at the current firm. We assume that a first observation is inevitable so the search cost for the first observation is sunk, and each consumer visits firms in a random order.

[7]This restriction on the parameter range is merely expedient. In general, multiproduct searchers should search for each product separately if $\delta < 0.5$. But theoretically, δ can take a value less than 0.5 and greater than 1, but such an extension does not affect our conclusions.

[8]Of course, in most markets, consumers incur not only a search cost but also a return cost. In addition, Janssen and Parakhonyak (2014) show that costly recall significantly affects firms' pricing strategies and consumers' behavior. However, considering such a costly recall makes our model more complicated, and moreover, as we will discuss in Section IV, our main insights are unchanged when we reconsider this assumption for smaller large costs. Hence, following most of the consumer search literature (including Zhou (2014)), we adopt this assumption for simplicity and tractability. It is also useful for comparison of our results with the models based on Wolinsky (1986) and Anderson and Renault (1999).

The timing of the game is as follows. In the first stage, firms set prices simultaneously. In the second stage, consumers search according to the optimal search rule. In this study, we focus on the symmetric equilibrium from the related literature. We adopt the perfect Bayesian equilibrium as a solution concept.

In equilibrium, each firm maximizes its own profit and each consumer maximizes her own consumer surplus. We assume that each consumer's belief is consistent in the equilibrium. We also assume the passive beliefs of consumers: each consumer does not change her belief, even if she observes a price that differs from her expectation. To make our analysis interesting, we focus on the case of a relatively small search cost, which ensures search activity.

Single-product search

First, we characterize the single-product search. Consider consumers who search for only a single product i $(i = 1, 2)$. Recall that $\lambda_i(1 - \lambda_j)$ $(i \neq j)$ represents the proportion of such consumers in the market population. The demand of these consumers is specified as in Anderson and Renault (1999). First, we define the *reservation match utility.* Let x represent the first observed value. Then, the incremental expected benefit from an additional search is given by:

$$\int_x^{\bar{u_i}} (u_i - x) dF(u_i) = \int_x^{\bar{u_i}} [1 - F(u_i)] du_i.$$

For simplicity, following McAfee (1995), we define the following expression.

$$\zeta_i(x) \equiv \int_x^{\bar{u_i}} [1 - F(u_i)] du_i, \text{ for } i = 1, 2.$$

We have the critical value with which an incremental expected benefit and an additional search cost s^S are indifferent for the consumer:

$$\zeta_i(\hat{u}_i) = s^S (= \delta s).$$

Hereafter, we refer to $\hat{u}_i$ as the reservation match utility for product i for the single-product search. Notice that $\hat{u}_i$ is a decreasing function of both s and δ. In the following analysis, we focus on the case in which each consumer has a relatively small search cost:

$$s^S \leq \bar{s}^S (= \delta \bar{s}) \equiv \zeta_i(\underline{u_i}), \quad \text{for} \quad i = 1, 2, \tag{1}$$

i.e., the incremental expected benefit of one more search is at least greater than the search cost for an additional search even if the consumer drew the lowest utility. We will state a formal condition in the next subsection. For the moment, we assume that the inequality above holds.

Now, consider a firm's incentive to deviate from the equilibrium pricing strategy. Suppose firms adopt a symmetric equilibrium pricing strategy, but firm I reduces its price by $\epsilon_i > 0$. Then, the demand for firm I from $\lambda_i(1-\lambda_j)$-type consumers is given by:

$$D_i^S(p_i) = \frac{1}{2}[1 - F(\hat{u}_i - \epsilon_i)][1 + F(\hat{u}_i)] + \int_{\underline{u_i}}^{\hat{u}_i - \epsilon_i} F(u_i + \epsilon_i) f(u_i) du_i. \quad (2)$$

The first term of the RHS represents a stopping probability at firm I. As a consumer searches in a random order and there are two firms, half of the consumers visit firm I first, and the rest of the consumers who had visited firm II will go to firm I with probability $F(\hat{u}_i)$. If firm I reduces the price by a small value ϵ_i, such a deviation induces a purchase with a relatively lower match utility; that is, a purchase by a consumer who observes a lower match utility than her reservation match utility $\hat{u}_i$. The second term of the RHS of equation (2) denotes the expected demand of the remaining (fully informed) consumers who finally decide to buy from firm I.

Multiproduct search

Now, consider $\lambda_1\lambda_2$ consumers who want to purchase both products. This part follows straightforwardly from Zhou (2014). For consistency, we adopt the same notation as in his model. First, we define the reservation match *utilities*, which we denote by the pair of u_1 and u_2 that satisfy the following equation:

$$\zeta_1(u_1) + \zeta_2(u_2) = s \ (= s^M).$$

To characterize the pair of reservation match utilities, we introduce function $\phi(u_1) = u_2$, which satisfies:

$$\zeta_1(u_1) + \zeta_2(\phi(u_1)) = s, \quad (3)$$

and call these pairs of utilities the *reservation frontier*.

Once the search cost is assumed to be small enough, all consumers searching for a single-product will be active. However, for a multiproduct search, such a discussion is complicated because there exists s' such that a consumer searching for multiproducts will do another search for both products, but will not do so for a single product. To avoid this complexity, we focus on a relatively small search cost that satisfies the following inequality:

$$s < \bar{s} \equiv \zeta_i(\underline{u_i}), \quad \text{for} \quad i = 1, 2, \quad (4)$$

i.e., each consumer searches for only one product. We refer to this inequality (4) as a *relatively small search cost condition.* Notice that when it holds, it also ensures search activities for a single-product search because $s^S = \delta s (= \delta s^M) < \bar{s}$.

By (4), we can define the minimum reservation match utility a_i that satisfies the following equality.

$$\zeta(a_i) = s, \quad \text{for} \quad i = 1, 2,$$

and by using the definition of a_i, we also obtain two expressions: $\phi(a_i) = \bar{u_j}$, and $\phi(\bar{u_i}) = a_j$. Note that (4) means $a_i > \underline{u_i}$. Figure 1 shows the above expressions

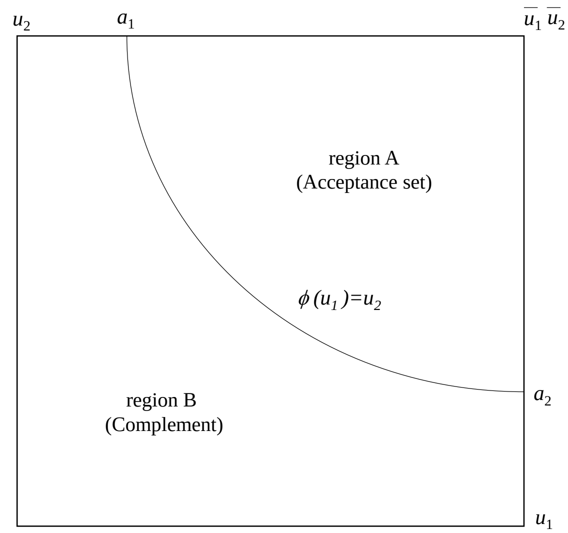

FIGURE 1. THE OPTIMAL STOPPING RULE FOR THE MULTIPRODUCT SEARCH

and definitions (this figure is the same as Zhou (2014)).

We call region A of Figure 1 the *Acceptance (Acceptable) set*, following Burdett and Malueg (1981). Region B is the complement of A. Once a consumer observes a pair of utilities, which are an element of Acceptance set A, the consumer stops and purchases both products immediately because she knows that the incremental expected benefit from one more search never exceeds the additional search cost.

Suppose now that firm I deviates by ϵ_1 and ϵ_2. These deviations move the reservation frontier to the lower left of the figure (in other words, more deviations lead to more immediate acceptances). There are tradeoffs between the benefit from increasing demand and the loss from the lower prices. Note that unlike the single-product model, the deviation of ϵ_i increases not only demand for product i, but also that for product j, because the reservation frontier moves down.

We use the notations A and B, which denote the region of the acceptance set and its complement, and give the demand for each product i for firm I as:

$$
\begin{aligned}
D_i^M(p_1, p_2) = \frac{1}{2}\int_{A^I(\epsilon)} dF(\mathbf{u}) + \int_{B^I(\epsilon)} F(u_i + \epsilon_i^I) dF(\mathbf{u}) \\
+ \int_{B^{II}} (1 - F(v_i - \epsilon_i^I) dF(\mathbf{v}),
\end{aligned}
\tag{5}
$$

where u_1 and u_2 are the match utilities from firm I, v_1 and v_2 are from firm I I, $(\mathbf{u})$ and $(\mathbf{v})$ express (u_1, u_2), (v_1, v_2), and $A(\epsilon)$ and $B(\epsilon)$ are the regions after deviating (ϵ_1, ϵ_2), respectively. The first term of the RHS represents the expected demand for a consumer who purchases both products immediately. The second term represents the expected demand for a consumer who first visits firm I and decides not to purchase any products, and then goes to firm II, but finally becomes fully informed and returns to firm I, and purchases product i there. The third term is almost the same as the second term and represents a consumer who visits firm II the first time.

Equilibrium

We can now derive the first-order condition for firm I when it deviates. We combine equations (2) and (5) and obtain the profit function for firm I:

$$\pi^I(p_i, p_j) = \frac{1}{2} \sum_{i=1, i \neq j}^{2} (p_i - \epsilon_i^I) \left\{ \lambda_i (1 - \lambda_j) D_i^S + \lambda_i \lambda_j D_i^M \right\}. \tag{6}$$

In the symmetric equilibrium, no firm has an incentive to deviate. Thus, we have the following Lemma 1.

Lemma 1. *In the symmetric equilibrium in which consumers adopt an optimal search rule and under a relatively small search cost, as in condition (4), the equilibrium prices p_1 and p_2 are given by the solution to the following equations.*

$$\begin{aligned} & p_1 \left\{ (1 - \lambda_2) \left[f(\hat{u_1})(1 - F(\hat{u_1})) + 2 \int_{\underline{u_1}}^{\hat{u_1}} f(u_1)^2 du_1 \right] \right. \\ & \qquad \left. + \lambda_2 \left[2 \int_B f_1(u_1) dF(\mathbf{u}) + \int_{a_2}^{\bar{u_2}} (1 - F(\phi^{-1}(u_2))) f(\phi^{-1}(u_2)) f(u_2) du_2 \right] \right\} \\ & \qquad = 1 - \lambda_2 p_2 \int_{a_2}^{\bar{u_2}} (1 - F(u_2)) f(\phi^{-1}(u_2)) f(u_2) du_2, \end{aligned} \tag{7}$$

and,

$$\begin{aligned} & p_2 \left\{ (1 - \lambda_1) \left[f(\hat{u_2})(1 - F(\hat{u_2})) + 2 \int_{\underline{u_2}}^{\hat{u_2}} f(u_2)^2 du_2 \right] \right. \\ & \qquad \left. + \lambda_1 \left[2 \int_B f_2(u_2) dF(\mathbf{u}) + \int_{a_1}^{\bar{u_1}} (1 - F(\phi(u_1))) f(u_1) f(\phi(u_1)) du_1 \right] \right\} \\ & \qquad = 1 - \lambda_1 p_1 \int_{a_1}^{\bar{u_1}} (1 - F(u_1)) f(u_1) f(\phi(u_1)) du_1. \end{aligned} \tag{8}$$

where $\phi(u_1)$ and $\phi^{-1}(u_2)$ are defined in equation (3).

As Zhou (2014) mentions in his seminal work, it is not easy to show the existence of an equilibrium. Even if we assume the log-concavity of $f(u)$, we cannot check whether the profit function (6) is quasi-concave or not. Therefore, we employ a technique developed by Zhou (2014) to check the existence of a symmetric equilibrium. To save space, we show the uniqueness and existence of a symmetric equilibrium in the online appendix, where we show that when $\delta = 1$, and when the match utilities are distributed symmetrically and exponentially, there exists a unique symmetric pure-strategy equilibrium. By following the same procedure, we may also check the existence of other distributions *e.g.*, uniform, log-normal, Weibull, Gumbel, logistic, and so on.

III. Analysis

In the last section, we characterized the symmetric equilibrium. In this section, we derive the equilibrium prices and analyze how the presence of three groups of consumers who are heterogeneous in demand for products affects the equilibrium prices and profit. First, we begin by analyzing how a change in the proportion of consumers searching for one of the products (or simply a demand shock to one of the products), say a change in λ_i, affects the prices of both products. In that part, we will explain the phenomenon in which both prices fall as demand increases (*countercyclical pricing*). Then we introduce correlation (both negative and positive) between the λs and check how the correlation affects the pricing. Second, we compare the prices when there is a difference between the two λs (*i.e.,* differences in popularity), and also compare the price sensitivities after a change in search cost. Finally, we analyze how a change in λ affects each firm's profit, by using some examples with specific distributions of match utilities.

We first note that each product's price tends to be lower in the multiproduct search environment than in the single-product search environment because of the presence of the joint search effect (and implicitly, the presence of economies of scale in search) as mentioned by Zhou (2011). However, the mechanism of price adjustment and the characteristics of prices relating to demand in the multiproduct environment are still unclear. That is why we first investigate how a demand shock (a change in λ) affects prices. Hereafter, we refer to a demand shock to product i simply as a change in λ_i. There are many examples of positive or negative demand shocks. For example, suppose a new indirect competitor supplying one of the products appears outside of the market. A fraction of consumers may now purchase that product outside of the market for many reasons, so the demand for the product would fall in the market. Then, how does such a (negative) demand shock affect both prices? In our model, a decrease in λ_i reflects such a negative demand shock. As a second example, we often observe that consumers' shopping patterns change over time and seasons, such as weekends, holidays, and vacations. Consumers who usually purchase only a single product may purchase multiple products during weekends, holidays, and the Christmas and Easter seasons. Furthermore, in those periods, some consumers who usually purchase nothing may also purchase single or multiple products. Such behavior can be expressed as an increased λ_i or increase in both λs in our model. Then, how does such a change in λ affect prices? What about the magnitude of the impacts on each price?

i. Equilibrium prices

First, we derive the symmetric equilibrium prices. To simplify the notation, we first define the following.

$$\alpha_i \equiv f(\hat{u_i})(1-F(\hat{u_i})) + 2\int_{\underline{u_i}}^{\hat{u_i}} (f(u_i))^2 du_i \text{ (for } i=1.2),$$
$$\beta_i \equiv \int_{a_j}^{\bar{u_j}} (1-F(u_j))f(\phi^{-1}(u_j))f(u_j)du_j,\ \beta_j \equiv \int_{a_i}^{\bar{u_i}} (1-F(u_i))f(u_i)f(\phi(u_i))du_i,$$
$$\gamma_i \equiv 2\int_B f_i(u_i)dF(\mathbf{u}) + \int_{a_j}^{\bar{u_j}} (1-F(\phi^{-1}(u_j)))f(\phi^{-1}(u_j))f(u_j)du_j,$$
$$\gamma_j \equiv 2\int_B f_j(u_j)dF(\mathbf{u}) + \int_{a_i}^{\bar{u_i}} (1-F(\phi(u_i)))f(u_i)f(\phi(u_i))du_i.$$

Note that all terms above are nonnegative. We first explain what each notation represents. Notice that α_i is a partial derivative of the demand for product i by consumers searching for a single product with respect to p_i, *i.e.,* the marginal effect of a change in p_i on the mass of $\lambda_i(1-\lambda_j)$ consumers. Suppose α_i increases while the other parameters are constant, then it induces firms to reduce p_i. In that sense, α_i reflects the effect of a single-product searcher on price i. Hereafter, we call α the (standard) effect of a single-product search. Similarly, γ is a standard effect of a multiproduct search. We emphasize that α_i and γ_i can take different values, and the latter becomes larger in general. This is because multiproduct searchers enjoy *economies of scale in search* (there are diseconomies of scale in search when $\delta < 0.5$), and therefore the impacts of a product's price adjustment on each consumer group will be different.[9] In addition, there exists another important factor that arises in the multiproduct search environment, namely the *joint search effect*, β, which was introduced by Zhou (2014). It highlights a new feature of multiproduct search pricing in that an undercutting of price p_i induces multiproduct searchers to purchase not only product i but also product j. In that sense, undercutting one product's price boosts the demand for both products, and because of the presence of this effect, multiproduct firms have extra incentive to reduce prices. β represents this tendency.

In the following analysis, we assume that the two distributions of match utilities are symmetric.[10] Hence, slightly abusing the notation, we can express $\alpha = \alpha_i = \alpha_j$, and so forth. Then, from Lemma 1, and by using the expressions above, we obtain the equilibrium prices as follows:

$$p_i^* = \frac{\alpha(1-\lambda_i) + \gamma\lambda_i - \beta\lambda_j}{X} \quad \text{for } i = 1, 2,\ i \neq j, \tag{9}$$

[9]We note that δ affects only α. As we show in the appendix, α is decreasing in δ, as is $\hat{u}$.

[10]An asymmetric distribution in the match utilities will cause various interesting phenomena; however, this is inconsistent with our objective. Moreover, we have not proved the existence of such an asymmetric distribution, so we focus only on the symmetric distribution.

where $X \equiv \alpha^2(1-\lambda_i)(1-\lambda_j) + \alpha\gamma(\lambda_i + \lambda_j - 2\lambda_i\lambda_j) + \lambda_i\lambda_j\left(\gamma^2 - \beta^2\right)$.

Before moving to the analyses, we describe the following *search effect condition*:

$$\gamma + \beta > \alpha, \tag{10}$$

i.e., the sum of the effect of a multiproduct search on prices is greater than that of a single-product search. We emphasize that this *search effect condition* predominantly depends on δ, and more importantly, it easily holds in broader cases because there are two unique effects of multiproduct search: economies of scale in search and joint search effect.[11][12]

ii. Effect of a change in λ on prices

According to the discussion above, we now investigate how a demand shock (a change in λ) affects prices. We first show that the effect of a change in λ for one of the products on each price is not the same. From (9), we obtain the following lemma.

Lemma 2: *Suppose the search cost condition (4) holds, and two products are symmetric. Then, in the symmetric equilibrium, the partial derivatives with respect to equilibrium prices are given as follows.*

$$\frac{\partial p_i}{\partial \lambda_i} = \frac{\beta\lambda_j(\gamma+\beta-\alpha)A_j}{X^2}, \tag{11}$$

$$\frac{\partial p_j}{\partial \lambda_i} = -\frac{\{(\gamma+\beta-\alpha)(\alpha(1-\lambda_j)+\gamma\lambda_j)A_j\}}{X^2}, \tag{12}$$

where $A_j = \alpha(1-\lambda_j) + \lambda_j(\gamma - \beta) \geq 0$.[13]

From Lemma 2 we can see that the signs of each partial derivative depend on the term $(\gamma+\beta-\alpha)$: the search effect condition (10). If (10) holds, the sign of (11) is positive unless β and $\lambda_j \neq 0$, and the sign of (12) is negative. If not, the opposite result holds. This yields the following proposition.

Proposition 1: *Suppose the search cost condition (4) and the search effect condition (10) hold, and the two products are symmetric. Then, in the symmetric equilibrium,*

[11] For example, it holds for any δ and s in the exponential case such as $F(u) = 1 - e^{-u}$. As another example, it is violated for $\delta < 0.082$ for $\bar{s}^M$ in the uniform case such as $F(u) = u$, but such a δ is implausible. We can easily verify that (10) holds except for an extreme case in other usual distributions.

[12] This condition is not a *necessary* condition. Some of following results are opposite when (10) does not hold.

[13] See the proof of proposition 1 in the appendix.

(a) *the equilibrium price p_j always decreases in λ_i.*

(b) *p_i increases in λ_i unless both β and λ_j equal zero. If not, p_i does not change in λ_i.*

If (10) does not hold, the directions of change are opposite.

The interpretation of Proposition 1 is as follows. Suppose both β and λ_j do not equal zero. This means that there exist consumers searching for multiproducts, and undercutting one product price affects such consumers' behavior via the joint search effect. When each firm cannot price discriminate among consumers, each firm must take all three effects above (α, β, and γ) into account in setting prices. For the moment, suppose that the search effect condition (10) holds. Then, under a given λ_j, a positive demand shock to product i (an increase in λ_i) results in changes of three masses: an increase in the mass of consumers searching for both products ($\lambda_i\lambda_j$) and those for product i ($\lambda_i(1-\lambda_j)$), whereas the mass of consumers searching only for product j ($\lambda_j(1-\lambda_i)$) decreases. This means that the effect of a multiproduct search relative to the effect of a single-product search on price j becomes large as long as (10) holds. Thus, an increase in λ_i induces firms to reduce p_j. However, such a reduction in p_j boosts multiple purchases when the joint search effect exists. Hence, it enables firms to increase p_i slightly. Therefore, an increase in λ_i causes a decrease in p_j first, then an increase in p_i; however, the latter effect disappears when β or λ_j equals zero. We note that such a price increase effect is not larger than the price reduction for another product, which is implied by the result of Zhou (2011) which we mentioned above; that is, firms can make money back slightly by increasing the price, but they cannot compensate for all the price reductions (we will confirm this in Proposition 3). Furthermore, a decrease in δ (meaning a decline in the relative efficiency of a multiproduct search) implies that it mitigates the effect of a change in λ_i on p_j (and p_i) because it causes an increase in α. It should be noted that Proposition 1-(a) holds even if $\beta = 0$; that is, a decrease in λ_i always affects (and usually decreases) p_j. This is because of the presence of economies of scale in search which is represented by the difference between γ and δ, and it always intensifies price competition unless (10) holds. We also note that if (10) does not hold, then the effect of a multiproduct search relative to the effect of a single-product search on price j becomes *small*, and therefore the opposite results hold. In summary, the directions and sizes of price adjustments hinge on the relative impacts of single-product search and multiproduct search.

Proposition 1 provides answers to the earlier questions; a decrease in demand for one of the products enables firms to charge higher prices for the other products because there is a smaller joint search effect and economies of scale in search;[14]

[14]For one example, consider hypermarket stores such as Walmart, and an indirect competitor such as Amazon. Amazon started in 1995, with strictly increasing revenue from 1995 to 2017. Although one might think that the emergence of Amazon would hurt the revenues of physical stores, while specialized stores did suffer severely, Walmart's revenue increased constantly from 1990 to 2014 (it decreased slightly in 2015, but rebounded in 2017). Proposition 1 can provide (at least partly) a plausible answer to

a positive demand shock for the product strengthens the joint search effect and economies of scale in search, and therefore, it induces firms to reduce other product's price to induce multiple purchases. This implies that at least one of the equilibrium prices must be lower during high demand seasons, which is consistent with the literature (we consider more related cases in the next part such as how simultaneous demand shocks affect prices).

These results suggest that multiproduct search may play an important role in multiproduct firms' price adjustments, and therefore, we should take all of those effects into account in a multiproduct environment.

Now we introduce the correlation between the two λs, *i.e.,* a change in the demand for one of the products occurs along with a change in the demand for another product. Such a correlation can be either positive or negative. For example, when consumers face a constraint of some kind (*e.g.,* budget, time, baggage, etc.), the λs should be correlated negatively. As in the previous example, consumers tend to purchase more products in some seasons (*e.g.* weekends; holidays; vacations) than in other seasons. In this case, demand may be positively correlated.

When there is correlation between the λs, Lemma 2 must be modified as follows.

Lemma 2': *Suppose two products are symmetric and the two λs are correlated. Then, in the symmetric equilibrium under the search cost condition (4),*

$$\frac{\partial p_i^*}{\partial \lambda_i} = \frac{(\gamma+\beta-\alpha)\left(\beta\lambda_j A_j - \left(\frac{\partial \lambda_j}{\partial \lambda_i}\right)(\alpha(1-\lambda_i)+\gamma\lambda_i)A_i\right)}{X^2}, \tag{13}$$

$$\frac{\partial p_j^*}{\partial \lambda_i} = -\frac{(\gamma+\beta-\alpha)\left((\alpha(1-\lambda_j)+\gamma\lambda_j)A_j - \left(\frac{\partial \lambda_j}{\partial \lambda_i}\right)\beta\lambda_i A_i\right)}{X^2}. \tag{14}$$

Notice that all the expressions above except for the second terms in the large parentheses are the same as in Lemma 2. However, (13) no longer equals zero for $\beta = 0$, unlike (11). Note that the second term in the large parenthesis of (13) is almost the same as the first term of it in (14). This means that when there is correlation between the λs, a change in λ_i now affects p_i not only via the joint search effect but also via a change in λ_j and *vice versa.* We can confirm that

this puzzle. In a multiproduct environment, physical stores tend to charge lower prices via the joint search effect and economies of scale in search than in a single-product environment. Now, a fraction of consumers purchase a portion of their necessities on Amazon.com, and they purchase few products in physical stores (they still purchase some products such as food, clothes, and so forth at physical stores because of various factors, *e.g.*, uncertainty about quality, risk, ability to obtain products immediately). It reduces the joint search effect, and the equilibrium prices increase. Consequently, multiproduct firms can maintain constant revenue (while specialized stores suffer). Indeed, we confirmed whether or not the number of stores increased following the emergence of Amazon, which in turn increased Walmart's profits by using an AR model and store data from 1978 to 2019 (customer numbers data were not available). The results showed that the emergence of Amazon had no significant impact on store numbers or on store growth rates, which implicitly supports our hypothesis.

both $\beta\lambda A$ terms above are (generally) smaller than the $(\alpha(1-\lambda)+\gamma\lambda)A$ terms; that is, the price increase effect is smaller than the price reduction effect for each equation, as we mentioned in the interpretation of Proposition 1. From Lemma 2', we obtain the following.

Proposition 2: *Suppose the search cost condition (4) and the search effect condition (10) hold, and the two products are symmetric. Suppose also that the two λs are correlated. Then, in the symmetric equilibrium,*

(a) *a negative correlation strengthens the impacts of a change in λ_i on both prices, which was shown in Proposition 1.*

(b) *a positive correlation weakens the price-increasing effect of a change in λ_i on p_i. If the correlation is not weak, then both prices decrease in λ_i.*

If (10) does not hold, the opposite of (a) holds.

The interpretation of Proposition 2-(a) is as follows. Suppose the λs are negatively correlated and suppose the search effect condition (10) holds. Suppose also that λ_i increases. Then, there are two forces that induce firms to reduce p_j. According to Proposition 1-(a), such an increase in λ_i reduces p_j, whereas p_i increases. Moreover, a decrease in λ_j also affects p_j in an opposite way, *i.e.,* it reduces p_j, whereas it increases p_i. As a result, firms charge lower p_j than the case with no correlation. Moreover, a decrease in λ_j induces firms to increase p_i because it mitigates the effect of a multiproduct search (relative to the effect of a single search) on p_i. The latter effect remains when $\beta = 0$, and therefore a change in λ_i always affects both prices, unlike in Proposition 1. Consequently, a change in λ_i induces firms to increase p_i and decrease p_j, and such price adjustments are greater than in the case of no correlation. However, Proposition 2-(b) tells us that it is difficult to obtain clear-cut predictions about price changes when a positive correlation between the λs exists. This is because a change in λ_i now has an impact through not only an increase in the mass of consumers searching for multiproducts, but also through an increase in the mass of consumers searching for each product. If the correlation between the λs is sufficiently weak, then the signs of the price changes are the same as in Proposition 2-(a), but the price increase effect on prices becomes small, because there still exists room for price increases via the joint search effect. However, if the correlation is not sufficiently weak, firms no longer increase p_i as λ_i increases because the effect of a price reduction relating to multiproduct search dominates. As a result, both products' prices decrease.[15]

[15]For example, suppose $\partial\lambda_i/\partial\lambda_j = 0.1$ (weak positive correlation) and $s = \bar{s}$. Then we can confirm that both prices decrease in λ_i for small λ_j ($\lambda_j < 0.55$ for any λ_i) in the exponential case that we will specify later. As another example, both prices decrease in λ_i for $\lambda_j < 0.18$ in the uniform case.

iii. Popularity

Now consider how the difference in λ affects the prices. Such a difference in λ can be interpreted as a difference in popularity. Thus, we refer to the product with the higher λ as the more popular product in this subsection. Suppose that $\lambda_i > \lambda_j$ (*i.e.,* product i is more popular than product j). Also suppose that the two λs are not correlated.[16] Without loss of generality, we normalize λ_i to one, *i.e.,* all consumers want to purchase product i, whereas some of them do not want product j. In that sense, we can interpret λ_j as representing a measure of the unpopularity of product j relative to product i.

By substituting $\lambda_i = 1$ into (9) for i and j, we obtain the following proposition.

Proposition 3: *Suppose the search cost condition (4) and the search effect condition (10) hold, and two products are symmetric. Then, in the symmetric equilibrium,*

(a) *the equilibrium price for the more popular product is relatively high compared with that for the less popular product:*

(15) $$p_i^* > p_j^* \text{ for } \lambda_i > \lambda_j \ (i \neq j),$$

(b) *the (relative) price difference becomes large as λ_j decreases,*

(c) *the (relative) price difference becomes large as δ increases.*

If (10) does not hold, the opposite result holds.

The interpretation is intuitive. Proposition 3-(a) means that firms tend to charge a relatively high (low) price for a more (less) popular product in equilibrium. As we have seen above, each firm must place more focus on consumers searching for multiple products when (10) holds. As each firm can adjust two prices to maximize its own joint profit, it charges a lower price for less popular products to attract consumers searching for multiple products, whereas it charges higher prices for more popular products and makes money back from these products. Proposition 3-(b) shows that firms tend to lower the price of less popular products as the popularity of such products decreases (*i.e.,* the mass of consumers searching for multiple products becomes small). This is because the impact of such a price reduction (say, reducing p_j) is not large; hence, each firm tends to drive p_j down more to induce multiple purchases, whereas it drives p_i up to make money back. The mechanism of such a price adjustment follows from Proposition 1-(a). The interpretation of Proposition 3-(c) is simple, and the mechanism is almost the same as for Proposition 3-(b). As δ increases, the effect of a multiproduct search becomes large relative to that of a single-product search. Hence, firms have to

[16] Of course, the same arguments of Proposition 2 hold when we introduce the correlation between the λs.

drive p_j down to attract multiproduct searchers.

There are various implications for multiproduct firms' strategies via Proposition 3. For example, we often observe *cross-subsidization* pricing such that multiproduct firms charge a high price for one product, whereas they charge low prices for other products. This strategy is often adopted by multiproduct firms *e.g.,* (fast food) restaurants, supermarkets, and the like. We can partly explain such a pricing strategy using Proposition 3 and a consumer search-theoretic approach. Firms can charge marked-up prices because of consumers' search costs, and in a multiproduct environment, they tend to charge a relatively high price for popular products, whereas they charge less for less popular products to stimulate profitable purchases and make money back. The intensity of such pricing depends on both the size of the mass of multiproduct searchers and the size of the economies of scale in search. Firms charge a very high price for more popular products and charge a low price for other products when the mass of multiproduct searchers is small and when the multiproduct search behavior is sufficiently beneficial to such consumers.[17]

Now let us consider the sensitivity (the price elasticity) of the two prices to a change in search costs. Search costs may vary for many reasons: psychological factors, technological innovation, physical factors, and so on. As in the case of prices we described, the effect of such a change in search costs on prices will be different among products.

Before we move on to derive the elasticity, we first discuss the preliminary results.

$$\frac{\partial p_i^*}{\partial s} \lesseqgtr 0 \quad \text{for } i = 1.2. \tag{16}$$

This is almost the same as Zhou (2014). Recall that our model is basically the convex combination of his model and Anderson and Renault (1999)'s model, and hence, the same argument about the search costs may hold. We note that $\frac{\partial \alpha}{\partial s} \leq 0$ and $\frac{\partial \gamma}{\partial s} \leq 0$ under a regularity condition.[18] These imply that the standard effect of a single-product search and a multiproduct search on prices will be mitigated as s becomes large, and hence, prices tend to increase as s increases. The intuition is simple. As each consumer must pay a search cost for gathering information about products, firms have sufficiently large market power in terms of pricing. When search costs increase, each consumer becomes more reluctant to search further. Hence, firms will not lose all consumers, even if they raise prices and firms can

[17] In practice, firms sometimes charge lower prices for *more* popular products in contrast with the result above. To apply our model to such a counterexample, we should take into account other factors. For example, introducing product asymmetry is plausible and feasible. As another example, if we consider a (positive) correlation between the λs as explained in Proposition 2-(b), then the desired results will be obtained. We can also consider *ex ante* information heterogeneity, (*e.g.,* consumers know the match utility of one of the products because they purchase it frequently), correlation between match utilities (consider complements), and so on. Even with these additional factors, the main insights (and mechanism of price adjustment) of our model remain true.

[18] See the proof of Proposition 4 in the appendix.

thus charge higher prices as the search cost increases. However, $\frac{\partial\beta}{\partial s}$ can change in either direction.[19] This implies that the equilibrium price decreases in s only if $\frac{\partial\beta}{\partial s} > 0$, and such an effect is sufficiently large relative to the other two.[20] In that case, firms have a greater incentive to reduce prices substantially to attract multiple purchases.

Let us consider price elasticity further. Let e_i be a price elasticity of product i:

$$e_i = \frac{\partial p_i^*}{\partial s} \cdot \frac{s}{p_i^*}, \quad \text{for } i = 1, 2. \tag{17}$$

Then, we obtain the following proposition.

Proposition 4: *Suppose the search cost condition (4) and the search effect condition (10) hold, and two products are symmetric. Then, in the symmetric equilibrium, the price of the more popular product is more elastic with respect to the search cost than the price of the less popular product if and only if,*

$$\left\{-\frac{\partial\alpha}{\partial s}\right\}(\gamma - \beta\lambda_j) + \left\{\frac{\partial\beta}{\partial s}\right\}(\gamma + \lambda_j(\gamma - \alpha)) > \left\{-\frac{\partial\gamma}{\partial s}\right\}(\alpha - \beta(1 + \lambda_j)). \tag{18}$$

This shows that the price sensitivity to a change in search costs for each product will be different even if two products are symmetric in the sense of match utility. Recall that the first and the last sets of curly brackets are nonnegative under a regularity condition, and the first term is nonnegative as proved in Lemma 1. However, the remaining terms can change in either direction, depending on the distribution of match utilities and δ. We first present two examples in which (18) holds.

Uniform example: Suppose the match utilities are uniformly distributed as $F(u) = u$ on the interval $[0,\ 1]$. Also suppose $\delta = 1$. This case corresponds to a situation wherein various consumers with diverse preferences exist, so firms cannot focus on any specific group, or a situation wherein each consumer evaluates the match value(s) the same as they do the price(s). We can find many such uniform examples in practice. Let Δ_e denote the difference between e_i and e_j. In this case, Δ_e takes a complicated form. Instead, we present a figure in which $\Delta_e \geq 0$. The price for a popular product is more elastic with respect to s than the price for a less popular product when the match utilities are distributed uniformly.

Exponential example: Suppose the match utilities are exponentially distributed as $F(u) = 1 - e^{-u}$ on the interval $[0,\ \infty)$. This case corresponds to the situation wherein consumers are sensitive to product suitability or there are many low-quality products. In this case, consumers think that it is unlikely they will find a better product from an additional search in terms of match value; that is, price

[19] $\frac{\partial\beta}{\partial s} > 0$ holds when the density function is decreasing because $\frac{\partial\beta}{\partial s} = f(a)f(\bar{u}) - \int_a^{\bar{u}} f'(u)f(\phi(u))du$.
[20] Choi, Dai and Kim (2018) present a similar result with advertised prices.

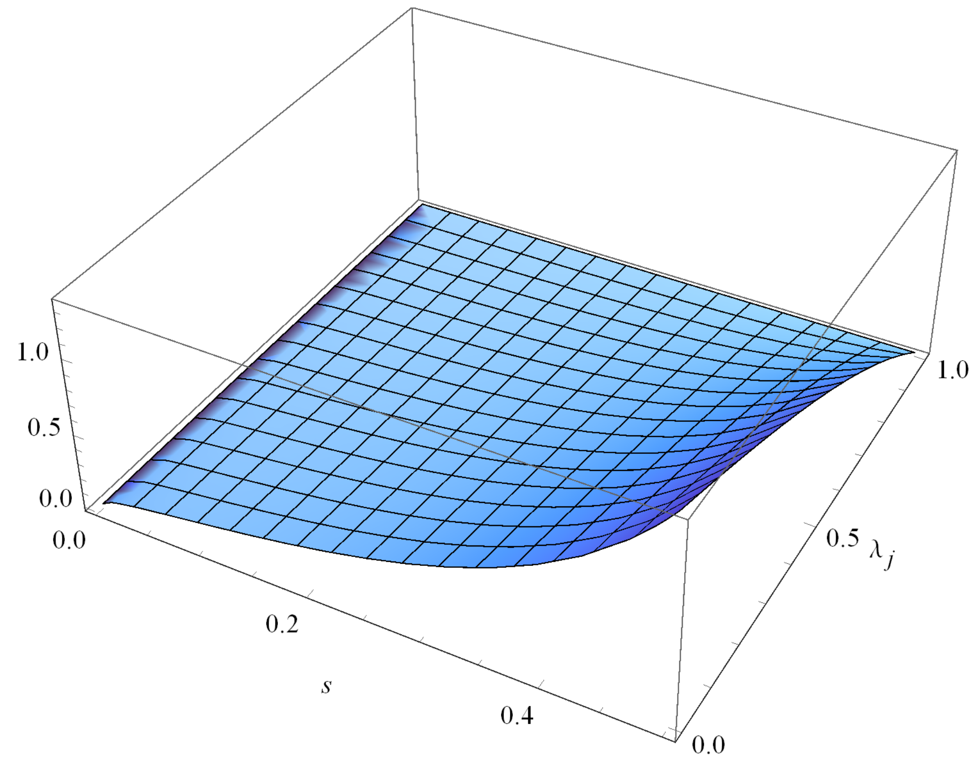

Figure 2. Uniform example Δ_e is positive for $s > 0$ when $\delta = 1$.

plays an important role. Then, Δ_e is given by:

$$\Delta_e = \frac{18(1-\lambda_j)s^3}{(6-s^3)(6-\lambda_j s^3)} > 0, \quad \text{for } s > 0, \ \forall \delta.$$

There are many other types of distributions for which Proposition 4 holds, as in the above examples. Although (18) does not give us clear-cut predictions, we argue that it may hold in many general cases. From (18), one can see that the opposite of (18) holds if: the density function is an increasing function in u ($\frac{\partial \beta}{\partial s} < 0$); the standard effect of a multiproduct search is sufficiently small relative to that of a single-product search; and δ is sufficiently small (α is large enough relative to γ). However, as we have mentioned in connection with the search effect condition, the second and third conditions above are not plausible. Hence, the opposite result might not be sustained in broader cases; that is, the price of a popular product tends to be more sensitive to a change in s than it is for a less popular product.

iv. Profit analysis

In the remaining part, we show in brief how a change in λ affects each firm's profit. For simplicity, in the following analysis, we assume that there is no correlation between the λs. Note first that each firm obtains half of the demand in the symmetric equilibrium. Therefore, each firm's expected (joint) profit is given by:

$$\pi = \frac{1}{2}\sum_{i=1}^{2} p_i \lambda_i, \tag{19}$$

where p_i is given by (9).

To calculate the profits, the distributions must be specified. We consider the uniform case and the exponential case. In the following, we assume the search effect condition (10) holds.

Uniform example: Similar to the last subsection, suppose that the match utilities are distributed uniformly as $F(u_i) = u_i$ on the interval $[0, 1]$ for $i = 1, 2$. In the uniform case, we can confirm $\frac{\partial \beta}{\partial s} = 1$. By applying this result to the argument above, it immediately leads to $\frac{\partial p_i^*}{\partial s} > 0$. We also have:

$$\frac{\partial \pi}{\partial s} > 0, \quad \frac{\partial \pi}{\partial \delta} \geq 0 \ (= 0 \text{ for } \lambda_i = \lambda_j = 1), \quad \frac{\partial \pi}{\partial \lambda_i} > 0. \tag{20}$$

Details are in the appendix. These expressions are quite intuitive, and the intuition is also quite straightforward. As we have mentioned above, an increase in s makes consumers more reluctant to search further (it mitigates all three effects). Hence, the incentive for firms to charge higher prices as s increases is always stronger than the incentive to reduce prices because of the joint search effect. As a result, prices and each firm's profit increase monotonically in s. We note that $\alpha = 2 - \sqrt{2s\delta}$: that is, an increase in δ reduces the effect of a single-product search on price, and therefore such an increase enables firms to drive up prices as same as s unless all the consumers search both products ($\lambda_1 = \lambda_2 = 1$). The third part of (20) arises from a market expansion in λ_i.

Exponential example: Next, we consider general exponentially distributed match values. Suppose that the match values are distributed as $F(u) = 1 - e^{-u}$ on the interval $[0, \infty)$. We first note that $\beta = \frac{1}{6}s^3$; hence, $\frac{\partial \beta}{\partial s} < 0$. We also note that:

$$\frac{\partial p_i^*}{\partial s} = -\frac{18\lambda_j s^2 \left(36 + \lambda_i \lambda_j s^6 - 12\lambda_i s^3\right)}{\left(36 - \lambda_i \lambda_j s^6\right)^2} \leq 0.$$

Notice that the inequality above holds strictly unless $s \neq 0$ and $\lambda_j \neq 0$. Zhou (2014) shows that the equilibrium prices *decrease* in s when $\lambda_1 = \lambda_2 = 1$ (*i.e.*, the standard effect of the single-product search is absent), because the marginal joint search effect with respect to s on price is greater than the marginal standard effect (of the multiproduct search) on price. Unlike Zhou (2014), our model includes standard effects that cause firms to charge higher prices (because $\gamma + \beta > \alpha$ holds), although the same results hold.[21] This example implies that the presence of the joint search effect plays an important role in multiproduct firms' pricing. In particular, this applies for the case in which the density function is decreasing in u.

Now consider the equilibrium profit:[22]

$$\pi(\lambda_1, \lambda_2, s) = \frac{6\left(3(\lambda_1 + \lambda_2) - \lambda_1\lambda_2 s^3\right)}{36 - \lambda_1\lambda_2 s^6}. \tag{21}$$

The comparative statics are given as follows.

$$\frac{\partial \pi}{\partial s} < 0, \quad \frac{\partial \pi}{\partial \delta} = 0, \text{ and } \frac{\partial \pi}{\partial \lambda_i} > 0. \tag{22}$$

The first and second results follow from the argument above. As an increase in s does not make consumers reluctant to search further ($\frac{\partial \alpha}{\partial s} = 0$) in the exponential case, firms cannot increase prices as s increases. However, firms still have an incentive to reduce prices to restrain consumers from leaving and to induce multiple purchases (an increase in s strengthens this incentive). As a result, each firm's profit decreases in s, whereas it is constant in δ. The third one simply arises from a market expansion in λ_i. The following figure 3 shows how much the joint search effect affects profit, in the case that $\lambda_1 + \lambda_2 = 1$ (to fix the market size).[23]

IV. Concluding Remarks

In this study, we construct a sequential consumer-search model with differentiated products where there are three groups of consumers: some consumers search for multiple products, whereas others search for one of two products. We show that the difference in price elasticity between single-product searchers and multiproduct searchers affects firms' pricing, which arises from two unique effects of multiproduct search: joint search effect, and economies of scale in search. These effects induce firms to reduce the price more as the mass of multiproduct searchers increases, and therefore, firms adjust the prices such that they reduce the price for one product (in general, a less demanded product) to attract multiple purchases, while they slightly increase the price for another product. If demands are

[21] The marginal standard effect of both single-product and multiproduct searches with respect to s are zero in the exponential case.

[22] The reservation match value $\hat{u}$ can be derived by using,

$$\int_{\hat{u}}^{\infty} [1 - F(u)]du = e^{-\hat{u}} = s.$$

Note that to ensure search activity, the search cost must satisfy $e^0 = 1 \geq s$. We also note that α is independent of δ, and $\alpha = \gamma = 1$. By summarizing these results, we obtain (21).

[23] That is, there is negative correlation between the λs. Proposition 2 tells us that the effect of an increase in λ_i is strengthened.

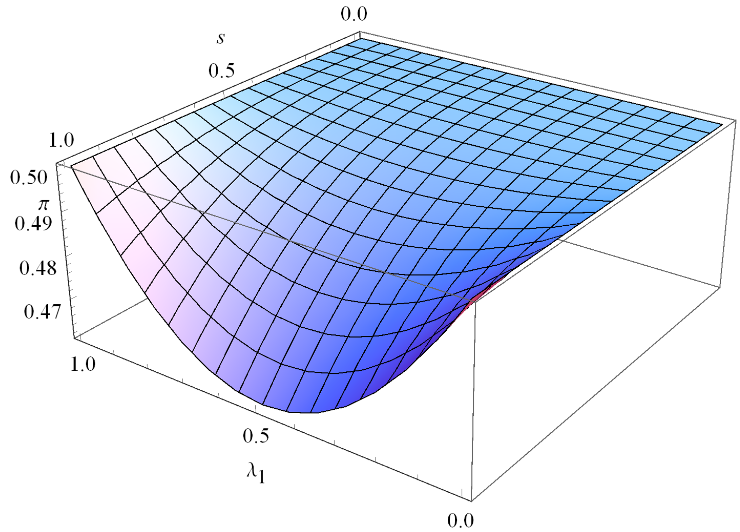

Figure 3. Profit is decreasing in s, and decreases as the joint search effect increases.

negatively correlated, these tendencies will be strengthened. However, if there is positive correlation between demands, then both prices will decrease in demand, such as in countercyclical pricing.

Finally, we discuss some important issues.

Another market expression— As we mentioned in the footnote 6, we represent a group of consumers by two λs to compare our results with those of previous studies and to simplify the model. In this setting, a change in one λ represents a change in the mass of each group of single and multiproduct searchers. It is of course possible to treat those searchers as separate groups, and to consider separately the effects on prices of changes in the mass of each group. For example, we can divide the groups of single and multiple searchers into three independent groups: λ_1, λ_2, and $\bar{\lambda}$. The expression in this paper can be expressed as a special case. We emphasize that as we have seen in the main text, the key factor is the relative magnitude of the effects of single-product search (α) and multiproduct search (γ and β), so our conclusion will be unchanged in the expression.

Costly recall— The free-recall assumption in our model means we did not have to consider consumers who purchase one of the products at the first store and then continue searching for the other product. There are various markets in which this assumption is justified, but in some cases, it may be necessary to consider such that a recall is costly. Janssen and Parakhonyak (2014) consider consumers' behavior and firms' pricing strategies without the free-recall assumption in the single-product environment and show that the equilibrium price will be different when each consumer must pay a cost for recalling (in addition, the equilibrium price is no longer stationary when there are more than two firms). They also show that when both costs are not large, an increase in the returning cost has a similar effect to that of increasing the search cost. By applying their results to our model, it can be anticipated that if both the search cost and returning cost are not large, and both λs are positive, there exists scope for multiproduct search. Therefore, our result survives (but the equilibrium prices now depend on both costs). We emphasize that even though we consider a costly recall, we can treat consumers who purchase one of the products at the first firm and continue searching for the other one as if they search only for a single product at the second firm. In that sense, our model is robust, although we adopt a free-recall assumption instead of costly recall. We predict that when we introduce a costly recall assumption, it will strengthen the effect of single-product search (α), whereas it weakens the standard effect of multiproduct search (γ). Hence, the impact of a change in demand on prices will be smaller. However, as we have seen above, our main insights still hold if the effect of the single-product search becomes sufficiently large relative to that of the multiproduct search (even if the search effect condition is violated).

Pure, mixed bundling and market structure— One plausible extension of our model is allowing a *bundling* strategy. There are two types of bundling strategy: *pure bundling* strategy such that multiproduct firms sell products only in a package; and *mixed bundling* strategy such that firms sell both the package and individual products. In our environment, mixed bundling is more plausible because a firm adopting a pure bundling strategy loses all the consumers searching for a single product, and more importantly, the search intensity of such consumers is less than that of multiproduct searchers and therefore the opportunity cost will be large. Despite the plausibility of mixed bundling, such an extension is challenging because it increases the strategy space and makes the problem extremely complicated even if we consider a duopoly case. Moreover, we argue that our model can be decomposed into two single-product search parts and a multiproduct search part when we allow firms to adopt mixed bundling, and therefore, Zhou (2011)'s argument is applicable.

Although pure bundling seems less plausible as we mentioned above, many interesting questions still remain open. If firms cannot discriminate price among consumer groups and cannot adopt mixed bundling, then we can pose two questions. (i) Do firms have an incentive to adopt pure bundling, and if so, in what case? (ii) Is there room for the coexistence of multiproduct firms (generalists) and single-product firms (specialists)? The latter question is also related to the issue of market structure in multiproduct environments.

Advertising and ordered search— There is a growing body of literature that considers advertising and ordered search in search environments, but it mainly focuses on a single-product environment. In practice, however, we often receive advertisements by multiproduct firms rather than single-product firms. For example, multiproduct firms often advertise such that some products are sold at very low prices, and such ads are generally nontargeted. We can modify our model to discuss many aspects of this issue. For example, we show that multiproduct firms' pricing hinges on the relative importance of single and multiproduct searchers. Then, which products the firms should advertise? How differences in the size of consumer groups affect advertising performance? Which firm should consumers visit first? How much product information should they provide? Is there any possibility of asymmetric equilibria? Can small firms coexist with multiproduct firms as in the example above? What about the relationship between the market population and coexistence? These questions are interesting from both theoretical and empirical perspectives, and analyses of them would help bridge the gap between the literature on economics and marketing by providing many insights into these fields.

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V. Appendix

Proof of Lemma 1:

The following proof follows Wolinsky (1986) (Anderson and Renault (1999)) and Zhou (2014). Let us begin with the former part.

Suppose the search cost condition holds. Let us decompose the first-order condition (FOC) of equation (6) into a single-product part and a multiproduct part.[24] Equation (8) is obtained by following the same procedure.

[24] We can obtain the lemma by differentiating equation (6) directly.

Single-product part: We begin with the former part of equation (7). Let us focus on product 1. By differentiating (6) with respect to ϵ_1 and evaluating $\epsilon_1 = 0$, we obtain:

$$\frac{\partial \pi}{\partial \epsilon_1} = -\lambda_1(1-\lambda_2)\left\{\frac{1}{2}[1-F(\hat{u_1})][1+F(\hat{u_1})] + \int_{\underline{u_1}}^{\hat{u_1}} F(u_1)f(u_1)du_1\right\}$$
$$+ p_1\lambda_1(1-\lambda_2)\left\{\frac{1}{2}f(\hat{u_1})[1+F(\hat{u_1})] + \int_{\underline{u_1}}^{\hat{u_1}} f(u_1)^2du_1 - F(\hat{u_1})f(\hat{u_1})\right\} = 0.$$

This gives:

$$\text{(23)} \qquad p_1\lambda_1(1-\lambda_2)\left\{\frac{1}{2}f(\hat{u_1})[1-F(\hat{u_1})] + \int_{\underline{u_1}}^{\hat{u_1}} f(u_1)^2du_1\right\} = \lambda_1(1-\lambda_2)\frac{1}{2}.$$

Multiproduct part: Likewise, we can obtain the latter part of (7) by following the same procedure. Recall that $A(\epsilon)$ and $B(\epsilon)$ are the regions where firm I deviates (ϵ_1, ϵ_2). By using Leibnitz's rule, the FOC is **a half** of the following equation times $\lambda_1\lambda_2$,

$$-\int_{A(\epsilon)} dF(\mathbf{u}) + (p_1+p_2)\int_C f(\mathbf{u})du_2 - \int_{B(\epsilon)} F(u_1+\epsilon_1)dF(\mathbf{u})$$
$$+ p_1\int_{B(\epsilon)} f(u_1)dF(\mathbf{u}) - \int_C \left[p_1F(u_1+\epsilon_1) + p_2F(u_2+\epsilon_2)\right] f(\mathbf{u})du_2$$
$$-\int_B [1-F(v_1-\epsilon_1)]dF(\mathbf{v}) + \int_B p_1 f(v_1)dF(\mathbf{v}) = 0,$$

where $\int_C$ denotes the line integral. We applied Green's theorem such that $\int_A \frac{\partial g}{\partial x}dydx = \int_C gdy$. By evaluating $\epsilon_1 = 0$, we obtain:

$$\text{(24)} \quad \frac{1}{2} = p_1\int_B f(u_1)dF(\mathbf{u}) + \frac{1}{2}\int_C \left\{p_1[1-F(u_1)] + p_2F[1-F(u_2)]\right\} f(\mathbf{u})du_2.$$

By multiplying $\lambda_1\lambda_2$ and combining equations (V) and (V), we obtain equation (7). We note that the line-integral part can be restated by using the definition of the reservation frontier as follows:

$$\int_C [1-F(u_2)]f(\mathbf{u})du_1 = \int_{a_1}^{\bar{u_1}} [1-F(\phi(u_1))]f(u_1)f(\phi(u_2))du_1.$$

□

Calculation of Lemma 2:
From Lemma 1 and the definitions, and by applying a symmetric assumption, we can restate the equilibrium price p_i^* $(i = 1, 2,\ i \neq j)$ as follows.

$$p_i^* = \frac{\alpha(1-\lambda_i) + \gamma\lambda_i - \beta\lambda_j}{X}, \tag{25}$$

where $X \equiv \alpha^2(1-\lambda_i)(1-\lambda_j) + \alpha\gamma(\lambda_i + \lambda_j - 2\lambda_i\lambda_j) + \lambda_i\lambda_j\left(\gamma^2 - \beta^2\right)$. Note that a change in λ_i (or λ_j) does not affect both the region of integrals and the distribution functions directly. By differentiating p_i with respect to λ_i we obtain:

$$\frac{\partial p_i^*}{\partial \lambda_i} = \frac{\beta\lambda_j(\gamma + \beta - \alpha)A_j}{X^2}, \tag{26}$$

where $A_j = \alpha(1-\lambda_j) + \lambda_j(\gamma - \beta)$. By following the same procedure, we obtain the following partial derivative about p_j^*:

$$\frac{\partial p_j^*}{\partial \lambda_i} = -\frac{(\gamma + \beta - \alpha)(\alpha(1-\lambda_j) + \gamma\lambda_j)A_j}{X^2}. \tag{27}$$

□

Proof of Proposition 1:
Recall that α, β, and γ are all nonnegative. Thus, the sign of $\partial p_i^*/\partial \lambda_i$ depends on the signs of both $(\gamma + \beta - \alpha)$ and A_j. The former one depends on the scale of δ (it moves in either direction), and hence, we just consider the sign of A_j.

We show that $\gamma > \beta$.[25] From the definitions and by applying the symmetry of distribution, we have:

$$\begin{aligned}\gamma - \beta &> \int_a^{\bar{u}} [1 - F(\phi(u))] f(\phi(u)) f(u) du - \int_a^{\bar{u}} [1 - F(u)] f(u) f(\phi(u)) du \\ &= \int_a^{\bar{u}} [F(u) - F(\phi(u))] f(\phi(u)) f(u) du.\end{aligned}$$

Let u' solve $u = \phi(u)$ (hence, it is defined on the interval $[a, \bar{u}]$). Then, the last expression above can be decomposed into:

$$\underbrace{\int_{u'}^{\bar{u}} [F(u) - F(\phi(u))] f(\phi(u)) f(u) du}_{(i)} + \underbrace{\int_a^{u'} [F(u) - F(\phi(u))] f(\phi(u)) f(u) du}_{(ii)}. \tag{28}$$

By changing the integral variable from u to $u = \phi^{-1}(u)$ $(= \phi(u))$, the second term

[25] The following procedure follows straightforwardly from the working paper version of Zhou (2014).

can be rewritten as:

$$\int_{u'}^{\bar{u}} (-\phi'(u))[F(\phi(u)) - F(u)]f(u)f(\phi(u))du.$$

From the definition, $\phi'(t) = -\frac{1-F(t)}{1-F(\phi(t))} < 0$. As $\phi(u)$ is a decreasing function in u, and because u' solves $u = \phi(u)$, $\phi(u) < u$ for $u \in (u', \bar{u})$. This leads to $F(\phi(u)) < F(u)$, and $-\phi'(t) > 0$. Hence, we have:

$$(i) + (ii) > (i) + \int_{u'}^{\bar{u}} [F(\phi(u)) - F(u)]f(u)f(\phi(u))du = 0.$$

This result follows from $\gamma > \beta$, and hence, $A_{i,j} > 0$ holds. Therefore, the sign of (26) is the same as the sign of $(\gamma + \beta - \alpha)$, and the sign of (27) is just the opposite.

□

Proof of Proposition 2:

When the λs are correlated, equation (26) and equation (27) no longer hold. From (25), we have two new expressions (Lemma 2'):

$$\frac{\partial p_i^*}{\partial \lambda_i} = \frac{(\gamma + \beta - \alpha)\left(\beta\lambda_j A_j - \left(\frac{\partial \lambda_j}{\partial \lambda_i}\right) A_i(\alpha(1 - \lambda_i) + \gamma\lambda_i)\right)}{X^2}. \tag{29}$$

$$\frac{\partial p_j^*}{\partial \lambda_i} = -\frac{(\gamma + \beta - \alpha)\left(A_j(\alpha(1 - \lambda_j) + \gamma\lambda_j) - \left(\frac{\partial \lambda_j}{\partial \lambda_i}\right) \beta\lambda_i A_i\right)}{X^2}. \tag{30}$$

By applying similar arguments that are proved above, the proof of Proposition 2 is completed.

□

Proof of Proposition 3:

Suppose $\lambda_i = 1$. Then, from (25), we have:

$$p_i^* = \frac{\gamma - \beta\lambda_j}{\alpha\gamma(1 - \lambda_j) + \lambda_j(\gamma^2 - \beta^2)}, \quad p_j^* = \frac{\alpha(1 - \lambda_j) - \beta + \lambda_2\gamma}{\alpha\gamma(1 - \lambda_j) + \lambda_j(\gamma^2 - \beta^2)}.$$

Let Δ_p denote the difference between p_i^* and p_j^*:

$$\Delta_p \equiv p_i^* - p_j^* = \frac{(1 - \lambda_j)(\gamma + \beta - \alpha)}{\alpha\gamma(1 - \lambda_j) + \lambda_j(\gamma^2 - \beta^2)}. \tag{31}$$

As $\gamma > \beta$, the sign depends on $(\gamma + \beta - \alpha)$. This completes Proposition 3-(a).

Let us consider Proposition 3-(b). By differentiating (31) with respect to λ_j, we have:

$$\frac{\partial \Delta_p}{\partial \lambda_j} = -\frac{(\gamma^2 - \beta^2)(\gamma + \beta - \alpha)}{\left(\alpha\gamma(1-\lambda_j) + \lambda_j\left(\gamma^2 - \beta^2\right)\right)^2} < 0,$$

which means that the (relative) price difference becomes small in λ_j.

Now we move on to Proposition 3-(c). Notice that (31) becomes large as α decreases. As a change in δ affects only α via $\hat{u}$, we have:

$$\frac{\partial \alpha}{\partial \delta} = \frac{d\hat{u}}{d\delta}\left(f'(\hat{u})[1 - F(\hat{u})] + f(\hat{u})^2\right).$$

From the definition of the reservation match utility, $\hat{u} = \zeta^{-1}(\delta s^s)$. By using the formula for the derivative of the inverse function, we obtain:

$$\frac{1}{\frac{d\hat{u}}{d\delta}} = \frac{d\delta}{d\hat{u}} = \frac{d}{d\hat{u}}\left(\frac{1}{s}\int_{\hat{u}}^{\bar{u}}[1 - F(u)]du\right) = -\frac{1}{s}[1 - F(\hat{u})].$$

Thus, we have:

$$\frac{\partial \alpha}{\partial \delta} = -\frac{s}{1 - F(\hat{u})}\left(f'(\hat{u})[1 - F(\hat{u})] + f(\hat{u})^2\right) < 0.$$

The second inequality is given by applying the regularity condition. Therefore, the (relative) price difference becomes large as δ increases.

□

Proof of Proposition 4:

We first define the difference between the two elasticities $\Delta_e \equiv e_i - e_j$. Then, from (17), we have:

$$\Delta_e = \frac{s(1-\lambda_j)\left\{\left(\frac{\partial \beta}{\partial s}\right)(\gamma + \lambda_j(\gamma - \alpha)) + \left(\frac{\partial \gamma}{\partial s}\right)(\alpha - \beta(1+\lambda_j)) - \left(\frac{\partial \alpha}{\partial s}\right)(\gamma - \beta\lambda_j)\right\}}{(\gamma - \beta\lambda_j)\left((\gamma - \alpha)\lambda_j + \alpha - \beta\right)}.$$

As the denominator is positive, Δ_e becomes positive if and only if the expression in the curly brackets above is positive:

$$\Delta_e > 0 \Leftrightarrow \left\{-\frac{\partial \alpha}{\partial s}\right\}(\gamma - \beta\lambda_j) + \left\{\frac{\partial \beta}{\partial s}\right\}(\gamma + \lambda_j(\gamma - \alpha)) + \left\{\frac{\partial \gamma}{\partial s}\right\}(\alpha - \beta(1+\lambda_j)) > 0.$$

We note:

$$\frac{\partial \alpha}{\partial s} = -\frac{\Lambda(\hat{u})}{1 - F(\hat{u})}, \quad \frac{\partial \gamma}{\partial s} = -\int_a^{\bar{u}} \frac{\Lambda(\phi(u))}{1 - F(\phi(u))} dF(u),$$

$$\frac{\partial \beta}{\partial s} = f(a)f(\bar{u}) - \int_a^{\bar{u}} f'(u) f(\phi(u)) du,$$

where $\Lambda(x) = f(x)^2 + f'(x)[1 - F(x)]$. Under a regularity condition, $\Lambda(\cdot) > 0$, and hence, both $\frac{\partial \alpha}{\partial s}$ and $\frac{\partial \gamma}{\partial s}$ are nonpositive.

□

Calculation of examples in Section III-iv:

Suppose that the match utilities are distributed uniformly as $F(u_i) = u_i$ on the interval $[0, 1]$ for $i = 1, 2$. The reservation match value $\hat{u}$ is given by:

$$\int_{\hat{u}}^{1} [1 - F(u)]du = \delta s^S \Leftrightarrow \hat{u} = 1 - \sqrt{2\delta s^S}.$$

To ensure the validity of the search activity, $s \leq 0.5$. Then, the profit of each firm is given by:

$$\pi^S = \frac{\alpha(\lambda_1 + \lambda_2) + \lambda_1^2(\gamma - \alpha) + \lambda_2^2(\gamma - \alpha) - 2\beta\lambda_2}{2\left(\alpha^2(1 - \lambda_1)(1 - \lambda_2) + \alpha\gamma(\lambda_1 + \lambda_2 - 2\lambda_1\lambda_2) + \lambda_1\lambda_2\left(\gamma^2 - \beta^2\right)\right)}. \tag{32}$$

From the definitions, $\alpha = 2 - \sqrt{2s\delta}$ and $\beta = s$. Now we consider the region of acceptance set A and the complement B, to derive γ. In this case, the acceptance area A is a quarter-circle with radius $\sqrt{2s}$. It follows that $\gamma = 2 - \frac{1}{2}\pi s$. By substituting these results into (32), we obtain each firm's (joint) profit function. Unfortunately, such an expression is too complicated to characterize using comparative statics. Instead of mathematical calculation, we just show the numerical results of the partial derivatives (32) with respect to s and δ for $\delta = 1$, $s = 0.5$, and λ_i in the following figures.

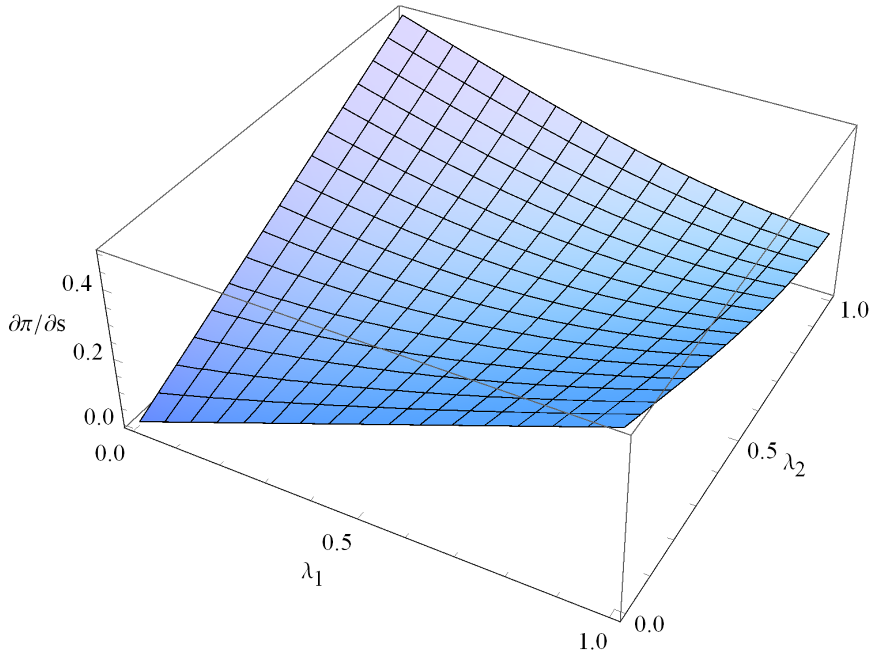

Figure 4. The partial derivative w.r.t. s is positive, and reaches its maximum when one $\lambda = 1$ and the other λ equals zero.

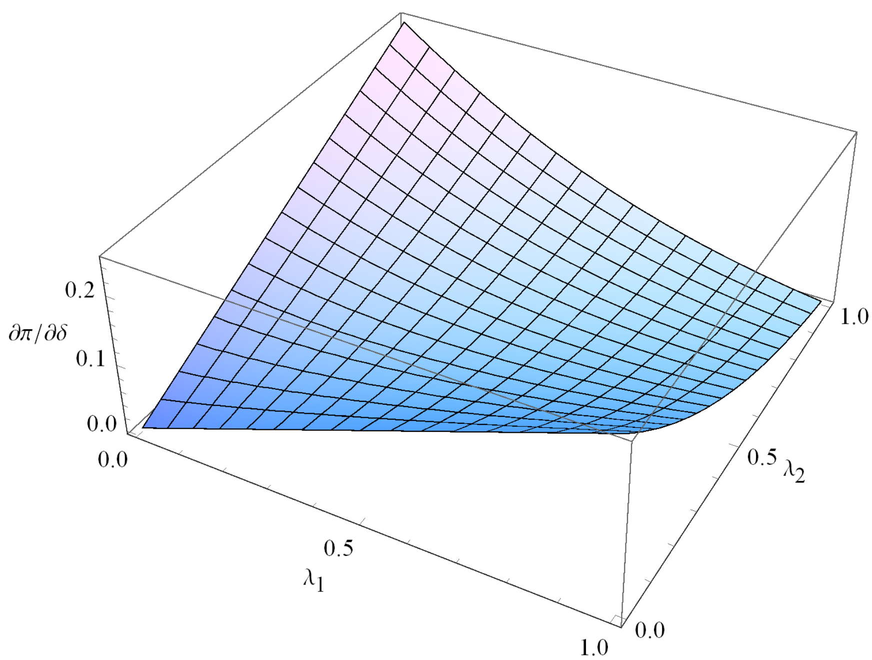

Figure 5. The partial derivative w.r.t. δ behaves almost the same as that of s (it reaches zero as both λs approach 1).

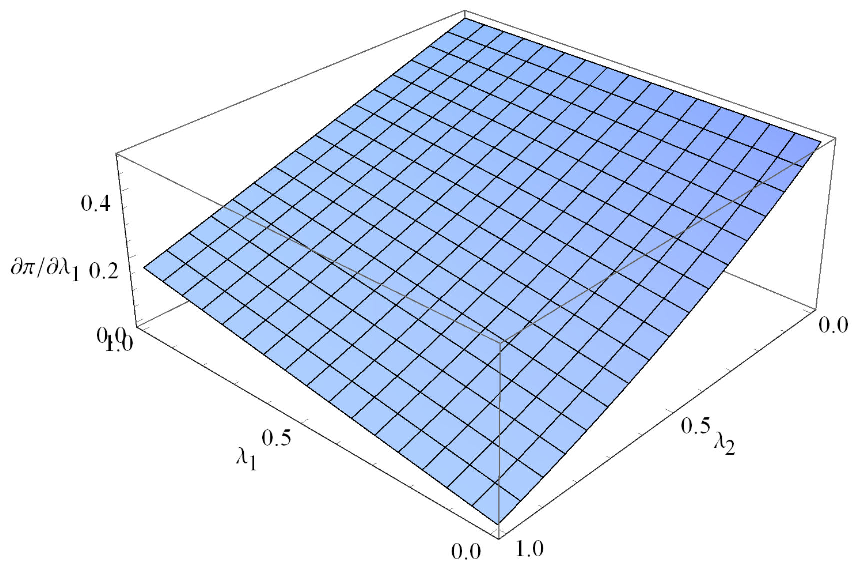

FIGURE 6. The partial derivative w.r.t. λ_1 is positive but decreases in λ_1 when λ_2 is large.