ON THE CASH-FLOW
AND CONTROL RIGHTS
OF CONTINGENT CAPITAL

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Abstract

This paper develops a model of banking to study the risk-taking consequences of contingent capital (CC). It begins with the observation that partial conversion of CC provides its owners with a portfolio of equity and debt. Since the former (latter) asset typically induces a preference for risk taking (safety), the net preference of CC-holders upon conversion should depend on their relative holdings of each asset, which in turn, depends on the amount of CC converted. In addition to acquiring cash-flow rights, these conversions provide CC-holders with equity control rights, which afford them greater influence over management’s portfolio selection. The paper demonstrates that rational shareholders - that anticipate these endogenous preferences and equity control rights - may be inclined to either: (1) dilute their own equity stakes through “excessive” risk taking in order to create risk-loving and influential CC-holders; or (2) rule-out conversion altogether through “excessive” safety, thereby preempting the creation of influential and safety-loving CC-holders. The results also suggest that higher CC-to-equity ratios can reduce the likelihood of reaching an “excessive” risk-taking equilibria.

Keywords: Contingent Capital, Corporate Governance, Bank Regulation, Blockholders, Shareholder Activism, Portfolio Choice.

JEL classification: G21, G28, G11, G34.
1 Introduction

For better or worse, capital-adequacy ratios have become the centerpiece of banking regulation. Proponents of these argue that bank capital reduces ex-ante risk taking in addition to bolstering loss-absorbing equity buffers. However, capital ratios also constrain lending during times of financial stress when equity is both depleted and expensive. A security with the potential to mitigate this problem can be found in Flannery (2005), which proposes the issuance of subordinated debentures that automatically convert into common equity when recapitalization is needed. These securities are generally referred to as contingent capital/convertibles (CC), and have been issued by a number of European banks. This paper analyzes the effects of CC cash-flow and control rights on bank risk taking. Specifically, it begins with the observation that partial conversion of CC provides its owners with a portfolio of equity and debt. Since the former asset is likely to induce a preference for risk taking, while the latter asset is likely to induce one for safety, the net preference of CC-holders upon conversion should depend on their relative holdings of each asset, which in turn, depends on the amount of CC converted. In addition to acquiring cash-flow rights, CC investors also receive equity control rights, which afford them greater influence over management’s portfolio selection. Anticipating these endogenous preferences and control rights, initial shareholders may find it optimal to dilute their own equity stakes through “excessive” risk taking in order to engender risk-loving and influential CC-holders. Conversely they may find it optimal to rule-out conversion altogether through “excessive” safety, thereby eliminating the influence of safety-loving CC-holders. This paper develops a model of banking to study the implications of these endogenous preferences and CC-holder control rights.

Prior to the 1980’s, US regulators refrained from imposing strict minimum capital requirements on banks, preferring instead to use judgment-based regulation tailored to specific institutions. However, in response to rapidly-diminishing capital levels at US banks during the 1970’s and early 1980’s, the federal banking agencies introduced explicit minimum capital requirements in 1981. And by 1985, the Federal Reserve, the Office of the Comptroller of the Currency, and the Federal Deposit Insurance Corporation agreed to harmonize these requirements at 5.5% of adjusted total bank assets.

However, US regulators were soon concerned that these 1985 requirements failed to adequately account for asset risk and off-balance sheet exposures, and desired to have uniform international regulation (to level the playing field). This led central bank governors of the G-10 countries to adopt the first Basel Accord (Basel I - fully implemented in 1993), which introduced an 8% capital requirement. In spite of appreciably higher capital levels among US banks around the implementation of Basel I (see Flannery and Rangan (2008) - which, incidentally, casts doubt on the effectiveness of Basel I for this trend) commentators cited a number of its shortcomings, and work began on Basel II, which added a capital charge for market and operational risk, and further disaggregated asset risk-weights. However, the 2007-2009 financial crisis hit before Basel II was fully implemented, and the subsequent analysis of this crisis exposed a number of Basel II’s limitations. As a result, work began on Basel III, which increased the minimum capital requirement to 10.5%, added

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1See Berg and Kaserer (2015) for a list of issuing banks, and Fiordelisi et al. (2018) for a list by country.
2For brief histories of US capital regulation see Alfriend (1988) and Burhouse et al. (2003).
3France, the UK and Germany, had risk-based capital standards in place by 1979, 1980 and 1985, respectively.
a counter-cyclical capital buffer (at the discretion of national regulators, up to 2.5%) and includes a simple leverage ratio (Tier 1 capital greater than 3% of non-risk-weighted assets).

The principle rationale for capital regulation has two components. First, equity and subordinated debt provide insulation against asset losses to senior creditors and deposit insurers. Second, and more controversially, higher capital levels are purported to incentivise lower ex-ante risk taking (for a recent dissenting view see Kashyap et al. 2008). This is typically rationalized by appealing to greater skin-in-the-game, as in Furlong and Keeley (1989), Keeley and Furlong (1990), Hellmann et al. (2000), Cooper and Ross (2002) and Repullo (2004), the ability of capital risk-weights to discourage banks from selecting inefficient and high-risk portfolios, as in Kim and Santomero (1988), discouraging high-risk firms from seeking bank charters by limiting their scalability, as in Morrison and White (2004), and reducing the probability that banks become undercapitalized and motivated to gamble for resurrection, as in Calem and Rob (1999).

Notwithstanding capital regulation’s potential effectiveness as a microprudential tool, it has mixed effects on lending efficiency. On the one hand, higher capital levels facilitate efficient lending through a “balance sheet” channel, whereby well-capitalized banks face lower wholesale-funding costs (Bernanke and Gertler 1995). While on the other hand, negative capital shocks that result in binding capital constraints often encourage banks to reduce lending, as opposed to raising additional equity, which is especially costly during times of financial stress due to intensified debt overhang (Myers 1977) and heightened informational asymmetries regarding bank value (Myers and Majluf 1984). Bernanke and Lown (1991) appears to be the first study to empirically test for this “capital crunch” effect, which is found using early 1990s US data. Peek and Rosengren (1997), and Watanabe (2007), corroborate this result using Japanese bank data, while also addressing the endogeneity of bank capital w.r.t. loan demand. Brinkmann and Horvitz (1995) address the endogeneity problem using US data, and also find evidence for a capital crunch, as does Gambacorta and Mistrulli (2004) using Italian data.

Given the primary importance of bank lending for economic activity (see Gorton and Winton 2002, among many others, for a discussion), policy tools that reduce lending disruptions during times of depleted bank capital are certainly needed. One such tool is found in Flannery (2005), which proposes the issuance of “reverse convertible debentures” that automatically convert into common equity when a bank’s capital ratio falls below a pre-specified level. This prompt recapitalization not only staves-off costly bankruptcy, but may also reduce the frequency and intensity of capital crunches. Support for this general idea is forthcoming among a number of academics, policymakers and market participants. However, disagreement remains concerning the security’s primary design features: the conversion trigger, the rate at which CC converts into equity, and the reference value of equity used for conversion.

In his original paper, Flannery recommended a market-value trigger, and a one-to-one conversion ratio of CC into common equity (both at current market prices). Market-value triggers are also recommended by McDonald (2013), Calomiris and Herring (2013) and Flannery (2016), and benefit from lower susceptibility to accounting manipulation, and are...
forward-looking measures of capital value. The primary alternative is using equity’s book value, as recommended by the Squam Lake Group (2009) and Pitt et al. (2011). This trigger benefits from its insensitivity to market manipulation, its applicability to private banks, and the avoidance of multiple/no equilibria, as pointed out by Sundaresan and Wang (2015). The model developed herein considers a book-value trigger, although it can also accommodate market-value triggers. With respect to the rate at which CC converts into equity, McDonald (2013) recommends a conversion ratio that benefit shareholders; this is meant to reduce market manipulation by short-sellers (also discussed in Flannery (2016) and Pennacchi (2010)). Berg and Kaserer (2015) report that most European issuances have this feature. Taking the opposite view is Calomiris and Herring (2013) and D’Souza et al. (2009), who argue that conversion terms should benefit CC holders, as this will incentivise preemptive equity injections when capital levels become depleted. In the middle lies Sundaresan and Wang (2015), which recommends no wealth-transfer upon conversion. The current model accommodates all three possibilities. Finally, with respect to the reference value of equity used for conversion, Calomiris and Herring (2013) and Flannery (2016) recommend using equity’s contemporaneous market value, which ensures that the value of converted-equity is proportional to the value of written-off CC, while McDonald (2013) recommends using fixed reference values (specifically, conversion into a fixed number of sharers), as does the Squam Lake Group (2009). Fixed reference values can reduce the chance of market manipulation by bondholders, who would otherwise have an incentive to depress stock prices in an attempt to acquire larger percentages of bank equity upon conversion. The current model accommodates both fixed- and market-based reference values, and can be described as follows.

Banks in the model are financed with equity, deposits and CC. Each bank holds an asset portfolio, whose risk can be adjusted in each period by way of a mean-preserving spread/contraction. In addition to changing the bank’s payoff structure, these mean-preserving spreads/contractions affect its regulatory-capital requirement by adjusting its asset-risk weights: larger spreads (contractions) make portfolios riskier (safer), and therefore, increase (decrease) the bank’s capital requirement. As considered by Glassermann and Nouri (2012) and Flannery (2005, 2016), when banks in the model trigger a capital-ratio violation, just enough CC is written-down to regain compliance. In this way, risk shifting also affects the proportion of bank equity and debt held by CC-investors upon conversion.

A number of papers, including Koziol and Lawrenz (2012) and Pennacchi et al. (2014), have argued that shareholders may be wary off CC due to their potential loss of control upon conversion. One implications of this, as discussed in Berg and Kaserer (2015), is

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5 McDonald (2013) also recommends a “dual price” trigger: conversion occurs if and only if a bank’s stock price and a pre-specified financial index fall below certain levels. This permits bankruptcy during “regular” times, while it provides support for troubled banks during times of systematic stress.

6 Many have principle write-down clauses. This is necessary for private banks, as noted in Flannery (2014).

7 Rothschild and Stiglitz (1970) demonstrates that many notions of “riskiness” are encompassed by mean-preserving spreads.

8 Control issues may be particularly acute for CC owing to: (1) high ownership-dispersion among banking institutions (Holderness (2003) provides references for the relevant evidence), which may be attributed to bank regulation substituting for large-shareholder monitoring, while simultaneously reducing the private benefits of control; and/or (2) the class of investor that is likely to purchase CC en masse, which may consist of activist investors such as hedge funds, due to the potential ownership-restrictions placed on traditional
that risk-averse CC-holders may shut down high-risk business lines upon gaining control of a bank, thus reducing the market value of equity. The current paper agrees with this conjecture, but only for certain cases. As noted above, partial conversion of CC leaves its owners with a portfolio of bank equity and debt, and due to equity’s (debt’s) convex (concave) payoff structure, its value is an increasing (decreasing) function of asset-risk. What separates this paper from the previous literature is that CC-holder risk-preferences are determined endogenously, as part-and-parcel to their rational objective of wealth maximization, which involves maximizing the total value of their portfolio of bank debt and equity. In this way, CC-holders remain risk-averse when CC write-downs and equity transfers are relatively small, but become risk-loving when write-downs and equity transfers are large. This dichotomy is important, because as noted above, CC-holder preferences can manifest themselves in further mean-preserving spreads/contractions via activism, with the associated implications for bank value and financial stability. Furthermore, given that initial shareholders will anticipate these endogenous preferences, and can partially affect them via risk shifting pre-conversion, the bank’s level of pre-conversion risk taking is also affected. Taken together, the model has two distinct equilibria. In the first, shareholders engage in “excessive” risk taking, which increases the value of equity and creates an influential and risk-seeking voting block that supports further risk-taking initiatives. In the second, shareholders engage in “excessive” safety, to avoid conversion altogether in an effort to prevent the creation of influential and risk-averse voting blocks.

The model’s two equilibria have diametrically-opposed implications for risk taking, and therefore, studying which factors promote each - both bank-specific and regulatory - is warranted. Toward this end, the model is solved analytically, and a number of comparative statics are run. This exercise recommends that CC-to-equity ratios ought to be relatively high. For reasons explained below, the “excessive” risk-taking equilibria only materialize when CC-holders are expected to become risk-loving post-conversion. When outstanding CC is large (small) this requires a relatively large (small) write-down and equity transfer, which dilutes initial shareholders greatly (minimally) when CC-to-equity ratios are high (low); as a result, initial shareholders are more likely to select the “excessive” safety (risk-taking) equilibria. The comparative statics also suggest that high debt-to-equity ratios increase the likelihood of excessive risk taking, via more lucrative asset substitution (as in Jensen and Meckling 1973). Finally, the rate at which CC converts into equity, and the bank’s risk-adjusted capital requirement, are shown to have mixed effects on bank risk taking.

In addition to providing an analytic solution, the paper also presents a set of numerical results based on reasonable parameter values. These provide an indication of each equilibrium’s feasibility, while also helping to illustrate how the model’s variables interact with one another.

This study is related to a growing literature on the risk-taking incentives created by CC; for recent surveys of the CC literature see Calomiris and Herring (2013) and Flannery (2014). Glasserman and Nouri (2012), for instance, demonstrates that non-monotonic relationships may arise between asset volatilities and CC yields (initially decreasing - due to equity’s greater upside potential - and then increasing), implying that banks can lower their funding holders of conventional bank debt (both regulatory and self-imposed, see Pitt et al. 2011).
costs by adjusting asset risk - either down or up. The results of Koziol and Lawrenz (2012) go further, suggesting that a substitution of CC for straight debt will always incentivize greater risk taking on the part of banks, by lowering the expected cost of bankruptcy, while maintaining the interest-tax shield of subordinated debt. Hilscher and Raviv (2014), Berg and Kaserer (2015) and Himmelberg and Tsypaklov (2015) consider the conversion terms of CC, and how these will affect risk taking: banks in all three models attempt to force conversion when terms are favorable to shareholders by increasing asset volatilities or “burning” money, in the first two papers, and the last paper, respectively. Pennacchi et al. (2014) also suggests that favorable conversion terms will result in higher asset volatilities, and recommends a solution: issue CC with unfavorable conversion terms and provide shareholders with the option to repurchase converted equity at CC’s par value. The current paper differs from these studies in a number of important ways. First, it considers multi-period risk shifting by analyzing a bank’s optimal portfolio selection both before and after conversion. Second, it explicitly models the risk-preferences of CC-holders, and thereby challenges the tendency to assume that these preferences run counter to those of initial shareholders. Third, it considers the effects of corporate-control changes on a bank’s portfolio selection, and illustrates that initial shareholders may prefer to relinquish control as opposed to selecting overly-conservative investment strategies.

This paper is also related to the literature on blockholder preferences and corporate control; for a recent survey of the blockholder literature see Edmans and Holderness (2017). Two particularly-relevant studies are Dewatripont and Tirole (1994) and Dhillon and Rossetto (2014). The first demonstrates that optimal managerial contracts can be implemented when corporate control is given to investors with the “correct” proportion of a firm’s debt and equity, and therefore, have the “correct” risk preferences. In Dhillon and Rossetto (2014), a blockholder’s tolerance for risk is increasing in her level of portfolio diversification. As such, placing marginal control with well-diversified, and therefore risk-seeking investors (risk is efficient in this model), supports high firm value. Empirical support for the positive relationship between blockholder diversification and risk-seeking can be found in Facio et al. (2001) for European firms.

The remainder of this paper is organized as follows. Section 2 describes the modeling environment. Section 3 presents the model’s general solution. Section 4 derives an analytic solution to the model, and presents a number of comparative statics. Section 5 presents the paper’s numerical results, and Section 6 concludes.

2 The Model

All investors in the model are assumed to be risk-neutral expected-wealth maximizes, and interest rates are normalized to zero. Banks operate for two periods: 1 and 2. They enter period 1 with a riskless asset portfolio that pays $X_g$ at the end of period 2, and with deposit liabilities and CC liabilities that promise to pay $D$ and $C$ at the end of period 2, respectively.

9 The future tax status of interest payments on CC is still uncertain.

10 Coffee (2011) argues that CC liabilities ought to be converted into preferred shares with substantial voting rights, in order to create risk-averse blockholders. This argument likely presupposes that conversion of CC into common equity may not necessarily produce these risk-averse blockholders. However, Coffee (2011) does not study the preferences of “conventional” CC-holders, nor does it comment on how these preferences are likely to affect risk taking both before and after conversion.
Banks can adjust asset portfolios in each period by engaging in mean-preserving spreads or mean-preserving contractions. To make matters simple, there are three possible payoffs at the end of period 2: \( X_g \), \( X_b = X_g - \Delta \), and \( X_e = X_g + \Delta \), for some \( \Delta \in (0, X_g] \). Given this set of potential outcomes, mean-preserving spreads are equivalent to removing probability mass from the central outcome \( (X_g) \) and splitting it equally among the other two, whereas mean-preserving contractions are equivalent to removing equal amounts of probability mass from the extreme outcomes and adding it to the central one. It is assumed that \( X_b < D + C < X_g \), which implies that risky asset-portfolios entail risky debt. Banks liquidate their assets at the end of period 2, pay outstanding debt obligations (to the extent possible, with depositors receiving higher priority), and distribute anything that remains to shareholders.

In the absence of adjustment costs or constraints on risk shifting, shareholders would prefer maximum risk taking (see Rochet 1992) and seek to allocate half of the asset-portfolio’s probability mass to the outcome \( X_b \), and the other half to \( X_e \), whereas creditors would prefer maximum safety, and seek to allocate all of the portfolio’s probability mass to \( X_g \) (given that \( X_g > D + C \)). To address these extreme outcomes, it is assumed that controlling shareholders (whether they be initial shareholders or CC-holders, as discussed below) incur private adjustment costs when engaging in mean-preserving spreads/contractions. These may be thought of as time spent searching for new investments, or the mental cost of restructuring the workforce (e.g., firing staff that specialize in outgoing asset-classes). Denote these adjustment costs by the function \( \lambda_j A(\theta_i) \), where \( \theta_i \) is the mass of probability reallocated from the central outcome to the extreme outcomes (which may be negative) in period \( i \), and \( \lambda_j \geq 0 \) is a constant that may depend on the type of controlling shareholder (\( j = E \) for initial shareholders and \( j = C \) for CC-holders). It is assumed that \( A(0) = 0 \), \( A(\theta) = A(-\theta) \forall \theta \) (i.e., adjustment costs are symmetric for mean-preserving spreads and contractions), \( A'(|\cdot|) \geq 0 \), \( A''(|\cdot|) \geq 0 \) (where \( |\cdot| \) is the absolute-value operator, i.e., costs are convex in the magnitude of risk shifting), and that costs are additive across periods.

Banks are subject to capital regulation at the beginning of each period, which creates a role for CC (beyond loss-absorbing subordinated debt). A chief purpose of CC is to quickly “recapitalize” banks that fail to meet their capital-adequacy ratios (CARs). This is accomplished by reducing the face value of CC liabilities, thereby increasing the bank’s CAR. The amount by which CC is written down depends on the bank’s capital position pre-write-down and its capital requirement. Often, these requirements are increasing functions of asset risk (e.g., Basel III). To accommodate this, it is assumed that capital regulation is characterized by the function \( R(\Theta_i) = \hat{R} + \eta(\Theta_i) \), where \( \hat{R} \geq 0 \) is the risk-free capital requirement, \( \Theta_i \) is the aggregate mass of probability attributed to the extreme outcomes (i.e., not attributed to the outcome \( X_g \)) at the beginning of each period \( i \), and \( \eta(\cdot) \) is the additional capital requirement for \( \Theta_i \), where \( \eta(0) = 0 \) and \( \eta'(\cdot) \geq 0 \). It is assumed that banks satisfy their risk-free capital requirements at the beginning of period 1, which allows us to focus on period 2 write-downs. Given the characterization of \( R(\Theta_i) \), and the fact that banks enter period 1 with a risk-free portfolio (i.e., \( \Theta_{i-1} = 0 \)), the percentage of CC

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11 These may be thought of as exchanging assets with identical market \( \beta \)'s and different idiosyncratic risks.

12 Charter values typically moderate this type of excessive risk taking (see for example Keeley (1990) and Hellmann et al. (2000)).

13 Portfolio risk is fully characterized by the variable \( \Theta_i \), and thus, no other information is pertinent.
written-down in period 2, denoted by \( \omega(\cdot) \), must satisfy the following equality:

\[
\frac{X_g - D - C(1 - \omega(\theta_1))}{X_g} = \bar{R} + \eta(\theta_1),
\]

where the left-hand-side is the ratio of equity’s book value to the value of total assets (recall that portfolios are mean-preserving spreads/contractions of one another, and therefore, have valuations independent of \( \theta_1 \) - always equal to \( X_g \)). Isolating \( \omega(\theta_1) \) produces the CC write-down function:

\[
\omega(\theta_1) = \frac{1}{C} \left[ D + C - X_g(1 - \bar{R} - \eta(\theta_1)) \right].
\]

Finally, to account for all mean-preserving spreads that fail to trigger CAR violations, and all mean-preserving spreads that require outside equity to satisfy the bank’s CAR, the following condition is imposed: \( \omega(\theta_1) \in [0, 1] \).

Next we turn to the design features of CC. Unlike for standard debt contracts, the owners of CC are compensated for write-down concessions in the form of equity grants. As discussed in the introduction, the rate at which CC converts into equity, and the reference value of equity used for conversion, are topics of academic debate, and vary in practice. For the current analysis, it is assumed that CC converts into equity at the constant rate \( r \geq 1 \). This precludes the transfer wealth from CC-holders to initial shareholder via strategic conversion, and thereby streamlines the exposition.\(^{14}\) With regard to equity’s reference value, two quantities are considered: 1) equity’s book value at the start of period 1; and 2) the expected value of equity at the end of period 2 - after \( \theta_1 \) is selected and the expected value of \( \theta_2 \) is also considered. The first of these is synonymous with using fixed reference values, while the second is synonymous with using equity’s market value. Furthermore, since the first value permits an analytic solution, it is used for that purpose in Section 4, while both reference-value assumptions are used for the numerical results of Section 5.\(^{15}\) Once the reference value of equity is selected, the percentage of shares transferred to CC-holders upon conversion, denoted by \( \pi(\cdot) \), must satisfy the following equation:

\[
\pi(\theta_1) = \frac{rC \omega(\theta_1)}{E(\theta_1, \theta_2)} = \frac{r[D + C - X_g(1 - \bar{R} - \eta(\theta_1))]}{E(\theta_1, \theta_2)},
\]

where the reference value of equity \( (E(\theta_1, \theta_2)) \) may or may not depend on \( \theta_1 \) and \( \theta_2 \), as discussed above.

The last substantive modeling issue concerns the bank’s corporate governance and control. For simplicity, it is assumed that upon conversion, CC-holders use their newly-acquired block of shares (and any existing influence they have as long-term creditors - see Nini et al. (2012) for a discussion of creditor-influence over management) to fully-influence management’s portfolio selection in period 2. Although extreme, this assumption allows us to

\(^{14}\) This assumption is not necessary for the paper’s main results, it merely streamlines the exposition. For a treatment of strategic conversion that transfers wealth from CC-holders to shareholders, refer to the introduction’s references.

\(^{15}\) As discussed in Section 5, equity’s reference value is of second-order importance for most of the numerical results.
avoid the complicated and ad-hoc task of modeling relative influence among initial shareholders and new shareholders; a burgeoning research field in its own right, but one that has not yet produced a clear recommendation on how to model this interplay (see Holderness and Edmons 2017 for a discussion). The current paper’s concluding section offers a straightforward refinement to this assumption, and discusses some of the complexities and uncertainties surrounding blockholder dynamics.

With the modeling environment now fully-characterized, we turn to its general solution next.

3 The General Solution

This section characterizes the model’s general solution using a subgame-perfect Nash equilibrium. For simplicity, and without loss of generality, \( \Delta = X_g \) (i.e., \( X_b = 0 \) and \( X_e = 2X_g \)). Furthermore, the term “shareholders” is used to denote all shareholders on record at the beginning of period 1. Given the model’s structure, it is solved backwards, starting with period-2 risk shifting.

3.1 Period 2

In period 2, the bank’s portfolio selection is either controlled by shareholders (when capital requirements are met) or by CC-holders (when CC is converted into equity). This brings about two sets of nodes/cases at the beginning of period 2.

Case 1: When capital requirements are met, management selects the period-2 mean-preserving spread/contraction that maximizes expected shareholder wealth, net of adjustment costs, for a given \( \theta_1 \). Thus, its problem is to:

\[
\max_{\theta_2} : \quad X_g - \left( 1 - \frac{(\theta_1 + \theta_2)}{2} \right) (D + C) - \lambda E A(\theta_2),
\]

S.t. \( \theta_2 \in [-\theta_1, \frac{1}{2} - \theta_1] \),

where the lower-bound constraint on \( \theta_2 \) reflects the complete reversal of period-1 risk taking, while its upper-bound constraint is the highest level of risk taking available for a given \( \theta_1 \). The first order condition is:

\[-A'(\theta_2) + \frac{D + C}{2\lambda E} = 0,
\]

which is independent of \( \theta_1 \), and implies:

\[\theta_{2,E}^* = A'^{-1} \left( \frac{D + C}{2\lambda E} \right),\]  

\[\text{(4)}\]

The intuition for this maximization-operand is as follows: maximizing the value of equity is equivalent to maximizing the value of total assets minus the value of total liabilities. The former is always equal to \( X_g \) (as discussed above), while the latter is equal to the face value of outstanding debt, multiplied by the probability that debt is repaid in full (given that \( X_b = 0 \)). This probability equals \( 1 - (\theta_1 + \theta_2)/2 \), given that \( \theta_1 + \theta_2 \) of probability mass is subtracted from the central outcome, and of this, one-half is allocated to the bad outcome \( X_b \).
where the $E$ subscript on $\theta_{2,E}^*$ indicates shareholder control in period 2. The value $\theta_{2,E}^*$ is necessarily positive (and strictly so when $A(\cdot)$ is continuous), and equates the marginal transfer of wealth from creditors to shareholders with the marginal adjustment cost. It is assumed throughout that $\theta_{2,E}^*$ is sufficiently large to violate the bank’s capital requirement if selected in period 1. If this were not the case, management’s intertemporal optimization problem becomes trivial ($\theta_1^* = \theta_2^* = \theta_{2,E}^*$), and CC never converts in equilibrium.

**Case 2:** In the absence of equity ownership, CC-holders would rationally seek to maximize the value of bank debt, net of personal adjustment costs, by selecting a weakly-negative $\theta_2$. However, arriving at case 2 implies that CC-holders have acquired equity states, and as such, their true objective is to maximize the total value of their portfolio of bank securities. Thus, management’s problem is to:

$$\max_{\theta_2} \pi(\theta_1) \left\{ Xg - \left(1 - \frac{(\theta_1 + \theta_2)}{2}\right)(D + C(1 - \omega(\theta_1))) \right\}$$

$$+ \left(1 - \frac{(\theta_1 + \theta_2)}{2}\right)C(1 - \omega(\theta_1)) - \lambda_C A(\theta_2), \quad (5)$$

where the first component of this maximization-operand is the value of equity held by CC-holders, the second is the value of outstanding CC, while the last is the adjustment cost. The same constraints on $\theta_2$’s upper and lower bounds continue to apply. The first-order condition is:

$$\frac{\pi(\theta_1)}{2}D - \frac{(1 - \pi(\theta_1))}{2}C(1 - \omega(\theta_1)) - \lambda_C A'(\theta_2) = 0. \quad (6)$$

The first component of Equation 6 is the marginal wealth extracted from (transferred to) depositors by increasing (decreasing) asset risk, multiplied by the percentage of equity held by CC-holders, the second component is the marginal reduction (increase) in the value of CC from increasing (decreasing) asset risk, net of the CC-holders’ share of the resulting increase (decrease) in equity value from reducing (increasing) the value of CC, whereas the third component is the marginal adjustment cost.

The determining factor for whether CC-holders select a mean-preserving spread or contraction in period 2 is the sign of:

$$NV(\theta_1) = \frac{\pi(\theta_1)}{2}D - \frac{(1 - \pi(\theta_1))}{2}C(1 - \omega(\theta_1)),$$  

i.e., the marginal net value of increasing asset risk from the perspective of CC-holders, conditional on $\theta_1$. $NV(\theta_1)$ is increasing in both $\omega(\theta_1)$ (the percentage of CC written down) and $\pi(\theta_1)$ (the percentage of equity transferred to CC-holders), which are, in turn, increasing functions of $\theta_1$. When $\theta_1$ is sufficiently small, the reduction in CC-value from increasing asset risk is greater in magnitude than the corresponding increase in converted-equity value,
and thus, \( NV(\theta_1) < 0 \Rightarrow \theta_2^*(\theta_1) < 0 \). Whereas when \( \theta_1 \) is sufficiently large, the opposite is true, and \( NV(\theta_1) > 0 \Rightarrow \theta_2^*(\theta_1) > 0 \). Therefore, solving for \( \theta_2^* \):

\[
\theta_{2,C}^* = 1_{NV(\theta_1)} A^{-1} \left( \frac{\pi(\theta_1)}{2\lambda_C} D - \frac{(1 - \pi(\theta_1))}{2\lambda_C} C(1 - \omega(\theta_1)) \right), \tag{8}
\]

where the \( C \) subscript on \( \theta_{2,C}^* \) indicates CC-holder control in period 2, and \( 1_{NV(\theta_1)} \) is an indicator function characterized as follows:

\[
1_{NV(\theta_1)} = \begin{cases} 
-1 & \text{if } NV(\theta_1) < 0 \\
1 & \text{if } NV(\theta_1) \geq 0.
\end{cases}
\]

Unlike for the previous case, the level (and sign) of \( \theta_{2,C}^* \) depends on the level of period-1 risk shifting through \( \omega(\theta_1) \) and \( \pi(\theta_1) \).

### 3.1.1 Combining Cases 1 and 2 of Period 2

Taken together, the choice of \( \theta_1 \) produces three qualitatively-distinct sets of outcomes for period-2 risk shifting, as summarized in Figure 1. When \( \theta_1 \) is sufficiently small, shareholders retain control of the bank’s portfolio selection in period 2, and management chooses the value of \( \theta_2^* \) that maximizes expected shareholder wealth, net of adjustment costs (\( \theta_{2,E}^* \) from Equation 4). Second, when \( \theta_1 \) is relatively small, but large enough to trigger conversion, \( \theta_{2,C}^* \) becomes negative, and reverses some, if not all, of the period-1 risk taking. Finally, when \( \theta_1 \) is relatively large, \( \theta_{2,C}^* \) becomes positive; further-increase the bank’s equity value. With these results established, we move on to the bank’s period-1 problem next.

#### Figure 1. Control over Period-2 Portfolio Selection and the Sign of \( \theta_{2,C}^* \)

<table>
<thead>
<tr>
<th>Shareholder Control</th>
<th>Creditor Control</th>
<th>Creditor Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_{2,E}^* \geq 0 )</td>
<td>( \theta_{2,C}^* &lt; 0 )</td>
<td>( \theta_{2,C}^* &gt; 0 )</td>
</tr>
<tr>
<td>0</td>
<td>low ( \theta_1 )</td>
<td>moderate ( \theta_1 )</td>
</tr>
<tr>
<td></td>
<td>high ( \theta_1 )</td>
<td>( \theta_1 )</td>
</tr>
</tbody>
</table>

This figure depicts the relationship between \( \theta_1 \) and: (1) which class of investor controls the bank’s portfolio selection in period 2, and (2) the sign of \( \theta_{2,C}^* \). When \( \theta_1 \) is relatively small, banks adhere to their capital requirements and management selects \( \theta_{2,E}^* \) from Equation 4. When banks fail to meet their capital requirements, and the value of \( \theta_1 \) is relatively low, CC-holders seek to increase the value of outstanding debt, and thereby, management selects a negative \( \theta_{2,C}^* \). Conversely, when \( \theta_1 \) is relatively large, CC-holders have relatively little bank debt, and a relatively large amount of bank equity; this induces CC-holders to increase the value of equity, and thereby, management selects a positive \( \theta_{2,C}^* \).

### 3.2 Period 1

Management’s problem in period 1 is to select the value of \( \theta_1 \) that maximizes expected shareholder wealth, conditional on correctly anticipating the best-response function of CC-holders and shareholders in period 2. This involves comparing two values: the maximum
expected wealth of shareholders when they retain control (case 1), and their maximum expected wealth when CC-holders gain control (case 2). As shown below, whenever the first value exceeds the second, management optimally selects the largest period-1 mean-preserving spread that fails to trigger conversion, whereas whenever the second value is larger, management always selects a mean-preserving spread in period 1 that is sufficiently large to induce additional risk taking on the part of CC-holders in period 2.

**Case 1:** Conditional on shareholders retaining control in period 2, management selects:

\[ \theta_{1,E}^* = \eta^{-1} \left( \frac{X_g - (D + C)}{X_g} - \hat{R} \right), \]

where the \( E \) subscript on \( \theta_{1,E}^* \) indicates anticipated shareholder control in period 2. \( \theta_{1,E}^* \) is the maximum value of \( \theta_1 \) that fails to trigger conversion (see Equation 1). Its optimality follows from the positive relationship between shareholder wealth and \( \theta_1 \) \( \forall \theta_1 < \theta_{2,E}^* \) (see Equation 3), and the previous assumption that \( \theta_1 = \theta_{2,E}^* \) triggers conversion if selected in period 1.

**Case 2:** Conditional on CC-holders gaining control of the bank’s portfolio selection in period 2, management selects:

\[ \theta_{1,C}^* = \arg \max_{\theta_1} \left( 1 - \pi(\theta_1) \right) \left( X_g - \left( 1 - \frac{\theta_1 + \theta_{2,C}(\theta_1)}{2} \right) \left( D + C(1 - \omega(\theta_1)) \right) \right) - \lambda_E A(\theta_1), \]

where the \( C \) subscript on \( \theta_{1,C}^* \) indicates anticipated CC-holder control in period 2. \( \theta_{1,C}^* \) maximizes the expected wealth of shareholders given the functions \( \omega(\theta_1) \), \( \pi(\theta_1) \) and \( \theta_{2,S}(\theta_1) \), derived above. As shown in Appendix A, \( \theta_{2,C}(\theta_{1,C}^*) > 0 \) is a necessary condition for management to select \( \theta_{1,C}^* \) in equilibrium. That is, whenever management selects a mean-preserving spread that triggers conversion, the resulting \( \omega(\theta_1) \) and \( \pi(\theta_1) \) must be sufficiently large that CC-holders become risk-loving, and prefer additional risk taking in period 2. This corresponds to the right-most region of Figure 1.

Given that \( \theta_{2,C}(\theta_{1,C}^*) > 0 \) is a necessary condition for conversion, all outcomes involving \( \theta_1 > \theta_{1,E}^* \) (conversion is triggered) and \( \theta_{2,C}(\theta_1) < 0 \) can be ruled-out (the middle region of Figure 1). This allows us to focus on the left- and right-most regions of this figure when searching for an equilibrium.

### 3.3 Equilibrium

Finally, to determine the equilibrium values of \( \theta_1 \) and \( \theta_2 \), management compares the expected wealth of shareholders under action set \( \{\theta_{1,E}^*, \theta_{2,E}^*\} \) (Case 1) to that under action set \( \{\theta_{1,C}^*, \theta_{2,C}^*\} \) (Case 2), and selects whichever one produces the highest value\(^{17}\).

To understand the tradeoffs that shareholders face in period 1, it is instructive to separate the difference in expected shareholder wealth from implementing the first and second action sets (wealth under \( \{\theta_{1,E}^*, \theta_{2,E}^*\} \) minus wealth under \( \{\theta_{1,C}^*, \theta_{2,C}^*\} \)) into the following 5 components (with descriptions below):

\(^{17}\)For brevity, \( \theta_{1,C}^* \)’s argument is omitted here, and below, when it is evaluated at \( \theta_{1,C}^* \).
1. The value of equity transferred from shareholders to CC-holders. This component is necessarily positive: no equity is transferred when capital requirements are satisfied.

2. The reduced face value of CC. This component is necessarily negative: no CC is written-off when capital requirements are satisfied.

3. The period-1 adjustment-cost differential. This component is necessarily positive: greater period-1 risk shifting entails a higher adjustment cost, and $\theta^*_1, C > \theta^*_1, E$ from above.

4. The period-2 adjustment-cost differential. This component is necessarily negative: adjustment costs are only incurred by controlling shareholders, and original shareholders only retain control of the bank’s portfolio selection under the first action set.

5. The differential in total wealth extracted from creditors. This component may be positive or negative, depending on the relative magnitude of aggregate risk shifting arising from each action set, i.e., $(\theta^*_1, E + \theta^*_2, E)$ vs. $(\theta^*_1, C + \theta^*_2, C)$.

Shareholders will unambiguously recommend action set $\{\theta^*_1, E, \theta^*_2, E\}$ (shareholder control) when the bank’s total risk taking is lower under action set $\{\theta^*_1, C, \theta^*_2, C\}$ (i.e., a negative 5th component). In this case, both the total value of equity, and the percentage of equity held by shareholders, is lower. Conversely, if aggregate risk taking under CC-holder control is higher (i.e., a positive 5th component), shareholders face a legitimate tradeoff between owning 100% of a bank with relatively low equity value, and owning less than 100% of a bank with relatively high equity value. As discussed below, Components 1 (equity dilution) and 5 (wealth extracted from creditors) are the most important factors for determining $\theta^*_1$.

4 Analytic Solution and Comparative Statics

This section derives an analytic solution to the bank’s problem in order to evaluate the effect that each parameter has on the bank’s propensity to adjust portfolio risk in periods 1 and 2. To simplify the exposition, the following 7 assumptions are made. However, they are either made without loss of generality, or are justifiable based on empirical grounds.
1. Unit expected profit: \( X_g = 1 \).

2. Affine capital requirement: \( R(\theta_1) = \hat{R} + \hat{\eta}\theta_1 \) (\( \hat{\eta} \) relatively small).

3. No excess capital in period 1: \( \hat{R} = E_1 = 1 - D - C \) (where \( E_1 \) is period 1 book equity).\(^{18}\)

4. Banks are primarily deposit-funded: \( D >> C + E_1 \).

5. There are two magnitudes of risk shifting available in each period \( i \) (in addition to the null option of zero): \( \theta_i \in \{0, \pm \hat{\theta}, \pm 2\hat{\theta}\}, i = 1, 2 \) \(^{19}\)

6. The lower level of risk shifting is costless: \( A(\pm \hat{\theta}) = 0 \).

7. The higher level of risk shifting is costly: \( A(\pm 2\hat{\theta}) = \hat{\theta}(D + C) \) \( + \hat{A} \), \( \hat{A} > 0 \), \( \lambda_E = \lambda_C = 1 \).

4.1 Period 2

Following the process outlined in Section 2, the model is solved using backward induction, starting with period-2 risk shifting for both cases.

**Case 1:** When shareholders retain control of the bank’s portfolio selection in period 2 (i.e., \( \theta_1 = 0 \) from assumption 3) \( \theta_{2,E}^* = \hat{\theta} \) from assumptions 5, 6 and 7. In this case, expected shareholder wealth is:

\[
E_1 + \frac{\hat{\theta}(D + C)}{2}.
\]

I.e., the initial value of equity plus the amount of wealth extracted from creditors in period 2.

**Case 2:** When CC-holders assume control of the bank’s portfolio selection in period 2 (i.e., \( \theta_1 \) equals \( \hat{\theta} \) or \( 2\hat{\theta} \), from assumption 3) the value of \( \theta_{2}^*(\theta_1) \) depends on the functions \( \omega(\theta_1) \) and \( \pi(\theta_1) \) from Equations 7 and 8. From assumptions 1, 2 and 3, and Equations 2 and 3 these simplify to:

\[
\omega(\theta_1) = \frac{\hat{\eta}}{C}\theta_1, \tag{2.1}
\]

and

\[
\pi(\theta_1) = \frac{r\hat{\eta}}{E_1}\theta_1. \tag{3.1}
\]

From these two equations, and Equation 7 the threshold value of \( \theta_1 \) for which CC-holders are indifferent between increasing/decreasing portfolio risk solves:

\[
\frac{r\hat{\eta}}{2E_1}\theta_1(D + C - \hat{\eta}\theta_1) = \frac{C - \hat{\eta}\theta_1}{2},
\]

\(^{18}\)This assumption is natural given that bank equity is typically more expensive than bank debt. This cost-of-capital differential is often attributed to debt’s explicit/implicit government guarantee, its use as a medium-of-exchange, and its potential to improve corporate governance (see Kashyap et al. (2008) and Admati et al. (2013) for both sides of this corporate-governance argument).

\(^{19}\)Subject to: \( \hat{\theta} \in (0, 1/2], \theta_1 \geq 0, \theta_2 \geq -\theta_1 \) and that unit probability mass is maintained.
where the left-hand-side is the marginal benefit of increasing portfolio risk via a higher equity value, and the right-hand-side is the marginal cost of increasing portfolio risk via a lower CC value. Simplifying this expressing, and ignoring the higher-order term (i.e., \( \hat{\eta}^2 \theta^2 \approx 0 \)), produces the following condition:

\[
\theta^*_2, C(\theta_1) \geq 0 \text{ iff } C - \frac{\hat{\eta}}{E_1} \theta_1 (r(D + C) + E_1) \leq 0,
\]

and the following threshold value of \( \theta_1 \) that produces indifference:

\[
\theta^T_1 = \frac{CE_1}{\hat{\eta}[r(D + C) + E_1]}.
\]  

(10)

When \( \theta_1 \) exceeds this value, CC-holders optimally increase risk-taking in period 2, whereas whenever \( \theta_1 < \theta^T_1 \), they decrease if. Taking the first derivative of Equation 10 with respect to each parameter provides our first proposition and corollary:

**Proposition 1** Under assumptions 1-7, and for a given level of period-1 risk taking, management is more likely to select a mean-preserving spread (i.e., increase portfolio risk) when the initial value of equity and the original face value of CC are low, and when outstanding deposits, the conversion ratio and the capital requirement are high. Whereas management is more likely to select a mean-preserving contraction for the opposite.

**Corollary 1** As the bank substitutes more of its conventional debt with CC, management is more likely to select a mean-preserving contraction in period 2 (i.e., reduce portfolio risk) since \( \theta^T_1 \) increases. (Again, under assumptions 1-7, and for a given level of period-1 risk taking).

The intuition for these results is as follows. More stringent capital regulation (i.e., a higher \( \hat{\eta} \)) induces larger CC write-downs and equity transfers for any level of period-1 risk taking, which simultaneously increases the marginal benefit of further risk taking (via higher equity ownership by CC-holders) and reduces the marginal cost (by lowering outstanding CC). Taken together, higher values of \( \hat{\eta} \) increase the likelihood of period-2 risk taking. Furthermore, the initial book value of equity (\( E_1 \)), the amount of outstanding deposits (\( D \)) and the conversion rate (\( r \)), all affect the likelihood of period-2 risk taking via the marginal benefit of increasing asset risk: specifically, a higher value of \( r \), and a lower value of \( E_1 \), both increase the percentage of equity transferred to CC-holders for any CC write-down, while a higher value of \( D \) increases the rate at which wealth is extracted from depositors via mean-preserving spreads. Conversely, the original face value of CC (\( C \)) operates through the marginal cost of increasing risk: higher values of \( C \) occasion larger values of outstanding CC for any amount written off, and therefore, increase the marginal cost of risk taking. This also explains Corollary 1.

Equation 10 is also useful for identifying the set of \( \hat{\theta} \) values that should be focused on going forward. Notice that when \( \hat{\theta} < \theta^T_1 / 2 \), CC-holders always select a mean-preserving contraction upon gaining control of the bank, whereas when \( \hat{\theta} > \theta^T_1 \) they always select a mean-preserving spread. However, the interesting case is when \( \hat{\theta} \in (\theta^T_1 / 2, \theta^T_1) \). In this situation, CC-holders optimally select a mean-preserving contraction when period-1 risk taking
is low (due to relatively small values of $\omega(\theta_1)$ and $\pi(\theta_1)$) and select a mean-preserving spread when period-1 risk taking is high (due to relatively large values of $\omega(\theta_1)$ and $\pi(\theta_1)$). Therefore, in what follows, it is assumed that $\hat{\theta} \in (\theta_T^1 / 2, \theta_T^1)$.

The final step of this subsection is to determine the level of risk shifting selected by CC-holders upon gaining control in period 2. From assumptions 6 and 7, we can infer that CC-holders optimally select $\theta_{2,C}^* = -\hat{\theta}$ when $\theta_1 = \hat{\theta}$, and $\theta_{2,C}^* = \hat{\theta}$ when $\theta_1 = 2\hat{\theta}$. That is, CC-holders always avoid the two extreme risk-shifting options (i.e., $\pm 2\hat{\theta}$), due to the high adjustment-cost of selecting them (see Appendix B for a proof of this).

The above results are summarized in Table 1 and together characterize the set of optimal strategies at each period-2 decision-node/case. Notice that $\theta_2^*(\theta_1)$’s overall structure is similar to that presented in Figure 1.

Table 1. Optimal Strategies at Each Period-2 Decision Node: $\theta_2^*(\theta_1)$

<table>
<thead>
<tr>
<th>$\theta_1$</th>
<th>$\theta_2^*(\theta_1)$</th>
<th>Period-2 Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\hat{\theta}$</td>
<td>Shareholders</td>
</tr>
<tr>
<td>$\hat{\theta}$</td>
<td>$-\hat{\theta}$</td>
<td>CC-holders</td>
</tr>
<tr>
<td>$2\hat{\theta}$</td>
<td>$\hat{\theta}$</td>
<td>CC-holders</td>
</tr>
</tbody>
</table>

This table reports the best-response of management at each period-2 decision node/case. The first column specifies each possible decision node/case, the second reports the best-response at each of these nodes/cases, while the third reports the investor-type in control of the bank’s portfolio selection.

4.2 Period 1

With the set of period-2 strategies now derived, we can solve for the bank’s optimal level of period-1 risk shifting. Following the procedure outlined above, this involves comparing the expected wealth of shareholders when they retain control of the bank’s portfolio selection, to their maximum expected wealth when CC-holders gain control.

**Case 1:** From Section 3 we know that management always selects the highest $\theta_1$ that fails to trigger conversion whenever shareholder-control is optimal in period 2. Given Assumptions 3 and 5, this corresponds to $\theta_1 = 0$, which produces an expected shareholder wealth of:

$$\text{Wealth}(0) = E_1 + \frac{\hat{\theta}}{2}(D + C),$$

from above.

**Case 2:** We also know from Section 3 that management always selects a mean-preserving spread that induces additional risk taking on the part of CC-holders whenever transferring control in period 2 is optimal. This rules-out $\theta_1 = \hat{\theta}$, since $\theta_{2,C}^*(\hat{\theta}) = -\hat{\theta}$ from Table 1. This leaves $\theta_1 = 2\hat{\theta}$ as the only candidate.

$\text{To see this result more clearly, note that expected shareholder wealth when } \theta_1 = \hat{\theta} \text{ is:}$
When $\theta_1 = 2\hat{\theta}$, $\omega(2\hat{\theta})C = 2\hat{\theta}\eta$ of CC is written-down from Equation 2.1, and the percentage of equity transferred to CC-holders is $\pi(2\hat{\theta}) = 2\hat{\theta}\eta/E_1$ from Equation 3.1. Taken together, this results in an expected shareholder wealth of:

$$Wealth(2\hat{\theta}) = \left(1 - \frac{2r\hat{\theta}}{E_1}\right)\left(E_1 + \frac{3}{2}\hat{\theta}(D + C - 2\hat{\theta})\right) - \left(\frac{\hat{\theta}(D + C)}{2} + \hat{A}\right),$$

where the first bracket is the percentage of equity retained by shareholders, $3\hat{\theta}/2$ is the probability of reaching the excellent state, $D + C - 2\hat{\theta}$ is the remaining face value of aggregate debt in period 2, while the right-most bracket contains the adjustment cost of selecting $\theta_1 = 2\hat{\theta}$ in period 1.

### 4.3 Equilibrium

Finally, to determine the equilibrium values of $\theta_1$ and $\theta_2$, management compares the expected wealth of shareholders under action set $\{0, \hat{\theta}\}$ (Case 1) to that under action set $\{2\hat{\theta}, \hat{\theta}\}$ (Case 2). Subtracting $Wealth(0)$ from $Wealth(2\hat{\theta})$, and ignoring the higher-order terms (i.e., $3r\hat{\theta}^2/2$ and $3r\hat{\theta}\eta\hat{\theta}$) produces the following condition:

$$\theta_1^* = 2\hat{\theta} \text{ iff } \hat{\theta}(D + C) - \left[\frac{\hat{\theta}}{2}(D + C) + \hat{A}\right] - 2r\hat{\eta} - \frac{3r\eta\hat{\theta}^2}{E_1}(D + C) > 0.$$  

The first term of this condition’s right-hand-side captures the additional benefit of selecting $2\hat{\theta}$ in period 1: the probability of reaching the excellent state increases by $\hat{\theta}$ (i.e., $3\hat{\theta}/2$ for Case 1 vs. $\hat{\theta}/2$ for Case 2). The bracketed term captures the direct adjustment cost of selecting $2\hat{\theta}$ (from Assumption 7), while the last two terms capture the indirect costs: $2r\hat{\eta}\hat{\theta}$ of equity’s book value is transferred to CC-holders during the initial conversion, while $3r\eta\hat{\theta}^2(D + C)/E_1$ of equity’s capital gain accrues to CC-holders and not shareholders. Taking the first derivative of this condition with respect to each parameter provides our second proposition and corollary:

**Proposition 2** Under assumptions 1-7, and given $\hat{\theta} \in (\theta_1^T/2, \theta_1^T)$, management is more likely to select a high level of period-1 risk taking when outstanding deposits, the original face value of CC and the initial value of equity are high, and when the conversion ratio, the capital requirement and the adjustment cost parameter ($\hat{A}$) are low. Whereas management is more likely to forgo period-1 risk taking altogether for the opposite.

$$Wealth(\theta) = \left(1 - \frac{r\hat{\eta}}{E_1}\right)E_1 = E_1 - r\hat{\eta}\hat{\theta},$$

which is strictly less than $Wealth(0)$ from above. By selecting $\theta_1 = \hat{\theta}$, the bank fails to meet its period-1 capital requirement, which brings about CC conversion, and dilutes shareholders. Furthermore, the low level of period-1 risk shifting results in relatively small values of $\omega(\theta_1)$ and $\pi(\theta_1)$, which induces CC-holders to reduce asset risk in period 2 in an effort to increase the value of outstanding debt. This, in turn, fully-reverses all period-1 risk shifting, which results in an aggregate equity value of $E_1$. Thus, by selecting $\theta_1 = \hat{\theta}$ instead of $\theta_1 = 0$, shareholders simultaneously reduce the total value of equity, and dilute their own equity stakes. This is precisely the scenario that shareholders wish to avoid.

$^{21}$These terms are 1 and 2 orders of magnitude lower than the others, respectively, for reasonable parameter values.
**Corollary 2** As the bank substitutes more of its conventional debt with CC, management is more likely to forgo period-1 risk taking. (Again, under assumptions 1-7 and given \( \hat{\theta} \in (\theta_T^1/2, \theta_T^1) \))

The results of Proposition 2 can be explained as follows. As leverage increases from an increase in either \( D \) or \( C \), the rate at which wealth is extracted from creditors, via mean-preserving spreads, increases. This makes risk taking more desirable on the margin. Conversely, as \( r \) and \( \hat{\eta} \) increase, and as \( E_1 \) decreases, a larger percentage of the bank is transferred to CC-holders upon conversion for a given capital-ratio violation; the resulting equity dilution makes risk taking less desirable on the margin. Furthermore, risk taking is clearly less desirable when adjustment costs are higher.

Corollary 2 follows from the fungibility of deposits and outstanding CC with respect to creditor-wealth extraction via asset substitution,\textsuperscript{22} and that shareholders will only advocate for CC conversion when equity dilution is sufficiently low (Component 1 of Expression 9). This last requirement places an upper bound on \( \hat{\theta} \) (call this \( \hat{\theta}^U \)); see Appendix C for a proof of this. Furthermore, as the bank substitutes its conventional debt with CC, \( \theta_T^1 \) increases (from Corollary 1), which causes the set \( \{ \hat{\theta} | \hat{\theta} > \theta_T^1/2 \text{ and } \hat{\theta} < \theta_T^1 \} \) to shift upwards. This makes \( \hat{\theta} < \hat{\theta}^U \) less likely to occur.

### 4.4 Discussion

In the preceding analysis, management either selects action set \( \{0, \hat{\theta}\} \) or action set \( \{2\hat{\theta}, \hat{\theta}\} \), with corresponding aggregate risk shifting of \( \hat{\theta} \) and \( 3\hat{\theta} \), respectively. In the first case, management avoids period-2 conversion altogether in an effort to prevent the creation of influential and risk-averse blockholders; the type of blockholder championed by Coffee (2011). In the second case, management selects a relatively high level of period-1 risk taking, to both increase the value of equity, and to create an influential and risk-seeking voting block that supports management’s future risk-taking initiatives.

In order to reach the second equilibrium, two conditions must be satisfied. First, CC-holders must find it optimal to select a mean-preserving spread in period 2, and second, shareholders must find it optimal to permit conversion. Both of these requirements are less likely to be satisfied when banks substitute relatively large quantities of subordinated debt with CC - from Corollaries 1 and 2 - which recommends that CC-to-equity ratios ought to be relatively high.\textsuperscript{23}

With respect to the second equilibrium’s existence, Propositions 1 and 2 indicate that most of the model’s parameters have a contradictory effect on the likelihood of satisfying both conditions simultaneously, with the exception of outstanding deposits \( D \) (having a uniformly-positive effect) and the adjustment-cost parameter \( \hat{A} \) (having a negative effect). For the other parameters, the two conditions are only satisfied when their values lie within a jointly-determined subset of the parameter space. Specifically, the first condition is only

\textsuperscript{22}That is, once \( \theta_1 \) is selected (and thus \( \omega(\theta_1) \) and \( \pi(\theta_1) \) are determined), the market value of equity increases at the same rate regardless of whether wealth is extracted from depositors or remaining CC-holders via asset substitution. Thus, a higher amount of \( C \) - given a fixed total debt/equity ratio - leaves the quantity \( \hat{\theta}(D + C) \) unchanged.

\textsuperscript{23}To see this result numerically, compare the results of Section 5 to those of Appendix D.
satisfied when CC investors have a sufficiently large equity stake and/or hold a sufficiently small amount of debt upon conversion. These requirements place upper bounds on \( E_1 \) and \( C \), and lower bounds on \( r \) and \( \hat{\eta} \). Conversely, the second condition requires that equity dilution be relatively modest and/or that gains from risk taking be relatively high. These requirements place lower bounds on \( E_1 \) and \( C \), and upper bounds on \( r \) and \( \hat{\eta} \).

Given these restrictions, a natural question that arises is whether the “excessive” risk taking equilibria actually exist for reasonable parameter values. To provide an indication of this, we turn to the numerical results next.

5 Numerical Results

This section presents a set of numerical results that illustrate the paper’s main theory: CC-holders may petition the bank’s management to increase or decrease asset-risk depending on their portfolio of bank debt and equity. Shareholders will respond to these endogenous preferences by petitioning management to either select high or low (possibly zero) mean-preserving spreads in period 1. In accordance with the analytic solution derived above, the first set of numerical results is based on discrete risk shifting, i.e., \( \theta_i \in \{0, \pm \hat{\theta}, \pm 2\hat{\theta}\} \), \( i = 1, 2 \), while the second set of results is based on continuous risk shifting.

5.1 Discrete Risk Shifting

As before, \( X_g \) is normalized to 1. It is assumed that deposits constitute the vast majority of bank funding, and as such, \( D = .85 \). Of the remaining .15 in asset value, one-third is allocated to contingent capital \( (C = .05) \), while two-thirds are allocated to equity’s book value \( (E_1 = .1) \). Banks have no excess capital in period 1, and thus, \( \hat{R} = .1 \). Furthermore, \( \hat{\eta} \) is set equal to .1. Banks may select one of five risk-shifting levels in each period: 0; \( \pm .05 \); and \( \pm .1 \), subject to maintaining a unit measure of probability. There is no personal adjustment cost for \( \theta_i = \pm .05 \), while it is .025 for \( \theta_i = \pm .1 \). These parameter values are reported in Table 2.

The first set of numerical results are reported in Table 3. These were derived using both reference-value assumptions: the results contained in column (1) correspond to the use of equity’s book value, while those in column (2) correspond to the use of its market value.

Capital requirements are adhered to in both cases when \( \theta_1 = 0 \), and thus, no CC is written down. Conversely, when \( \theta_1 = .05 \) \( (\theta_1 = .1) \) a capital shortfall of .005 (.01) arises, which triggers a 10% (20%) CC write-down. These results are reported in rows (1) - (3) of Table 3. The reference value of equity under the first assumption (book value) is .1 regardless

---

24To the extent possible using numerical methods.
25As pointed out by Sundaresan and Wang (2015), it is difficult to calibrate the parameters of CC properly using available bank data, as CC is not yet widely used by banks. However, assigning 15% of a bank’s asset-value to its total capital (CC plus equity) is consistent with the estimates of Pennacchi et al. (2014) for: Bank of America, Citygroup and JPMorgan Chase (12.4% of total capital on average: common stock, preferred stock and subordinated debt). It is also consistent with Berg and Kaserer (2015), which uses 85% for deposits, 5% for CC and 10% for equity, and similar to Glasserman and Nouri (2012) (equity of 10% and CC between 5%-15%), Flannery (2005, 2016), Sundaresan and Wang (2015) and Himmelberg and Tsyplakov (2015) (all assuming equity of 8% and CC of 5%).
26Note: \( \frac{\hat{\theta}(D+C)}{2} = .0225 \). Adding \( \hat{A} = .0025 \) to this value equals .025. See Assumption 7 from Section 4.
Table 2. Parameter Values for the Discrete Risk-Shifting Numerical Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_g$</td>
<td>1</td>
<td>$\hat{\theta}$</td>
<td>.05</td>
</tr>
<tr>
<td>$D$</td>
<td>.85</td>
<td>$A(\pm\theta)$</td>
<td>0</td>
</tr>
<tr>
<td>$C$</td>
<td>.05</td>
<td>$A(\pm2\hat{\theta})$</td>
<td>.025</td>
</tr>
<tr>
<td>$E_1/\tilde{R}$</td>
<td>.1</td>
<td>$\lambda_E$</td>
<td>1</td>
</tr>
<tr>
<td>$\tilde{\eta}$</td>
<td>.1</td>
<td>$\lambda_C$</td>
<td>1</td>
</tr>
</tbody>
</table>

This table reports the base-case parameter values used for the discrete risk-shifting numerical results.

of $\theta_1$. Therefore, $\pi(.05) = 5\%$ and $\pi(.1) = 10\%$ in this case, as reported in rows (4) - (6) of Table 3. Under the second assumption (market value), both the reference value of equity, and the percentage of equity transferred to CC-holders upon conversion, are solutions to a fixed-point problem. This problem is solved by noting that $\theta^*_{2,C}(.05) = -.05$ while $\theta^*_{2,C}(.1) = .05$ for both reference value assumptions (see rows (7) - (9) of Table S), due to the relatively high (low) value of outstanding CC, and the relatively low (high) percentage of equity transferred to CC-holders, when $\theta_1 = .05$ ($\theta_1 = .1$). As a result, there is no aggregate risk shifting when $\theta_1 = .05$, and equity’s only bump in market value stems from the reduction in outstanding CC. In this case, the market value of equity is .105 (i.e., $E_1$ plus 10\% of $C$). With a market value of .105, $\pi(.05) = 4.8\%$ under the second reference-value assumption, which is marginally lower than under the first. Conversely, there is a relatively large amount of risk shifting when $\theta_1 = .1$ since $\theta_1 + \theta^*_{2,C}(\theta_1) = .15$. This, in addition to the relatively large reduction in outstanding debt (20\% of $C$), results in a high market value of equity, which equals .1768 in this case. Therefore, $\pi(.1) = 5.7\%$ under the second reference-value assumption, which is 4.3\% lower than under the first (see rows (4) - (6) of Table 3 for the percentage of equity transferred upon conversion, and rows (10) - (12) for the market value of equity).

If management selects $\theta_1 = 0$, expected shareholder wealth becomes .1225 under both reference-value assumptions, as all equity is retained, and .0225 of wealth is extracted from creditors in period 2 - since $\theta^*_{2,E} = .05$ (i.e., $\theta^*_{2,E}(D + C)/2 = .0225$). If management selects $\theta_1 = .05$, expected shareholder wealth becomes .0998 (.1) under the first (second) reference-value assumption, since shareholders retain 95\% (95.2\%) of the bank’s equity in period 2, which is only worth .105 in aggregate (i.e., $E_1$ plus the 10\% of $C$ that was written down). Finally, if management selects $\theta_1 = .1$, expected shareholder wealth becomes .1341 (.1418) under the first (second) reference-value assumption, since shareholders own 90\% (94.3\%) of

27It should be noted that control changes do not imply that CC-holders have acquired a majority of bank equity - see Berle and Means (1932) for an early analysis of the “working control” exerted by minority interests of diffusely-held corporations - only that CC-holders have gained a sufficient level of influence through their block of shares (the same point is made in Berg and Kaserer (2015), for instance). It is beyond the scope of the current analysis to estimate the percentage of shares needed to gain this influence - which depends on the particular bank in question, its corporate charter, its legal environment, the composition of its original shareholders, etc. - and therefore, no attempt was made to calibrate the numerical results to specific values of $\pi(\theta_1)$. As such, all results pertaining to CC-holder influence and $\pi(\theta_1)$ should be viewed as qualitative in nature, and not quantitative.
Table 3. Numerical Results: Discrete Risk Shifting

<table>
<thead>
<tr>
<th></th>
<th>(1) Book Value</th>
<th>(2) Market Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega(\theta_1)$ (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Equity Controlled: $\theta_1 = 0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(2) CC Controlled: $\theta_1 = 0.05$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>(3) CC Controlled: $\theta_1 = 0.1$</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$\pi(\theta_1)$ (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) Equity Controlled: $\theta_1 = 0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(5) CC Controlled: $\theta_1 = 0.05$</td>
<td>5</td>
<td>4.8</td>
</tr>
<tr>
<td>(6) CC Controlled: $\theta_1 = 0.1$</td>
<td>10</td>
<td>5.7</td>
</tr>
<tr>
<td>$\theta_2^*(\theta_1)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7) Equity Controlled: $\theta_1 = 0$</td>
<td>.05</td>
<td>.05</td>
</tr>
<tr>
<td>(8) CC Controlled: $\theta_1 = 0.05$</td>
<td>-.05</td>
<td>-.05</td>
</tr>
<tr>
<td>(9) CC Controlled: $\theta_1 = 0.1$</td>
<td>.05</td>
<td>.05</td>
</tr>
<tr>
<td>Market Value of Equity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10) Equity Controlled: $\theta_1 = 0$</td>
<td>.1225</td>
<td>.1225</td>
</tr>
<tr>
<td>(11) CC Controlled: $\theta_1 = 0.05$</td>
<td>.1050</td>
<td>.1050</td>
</tr>
<tr>
<td>(12) CC Controlled: $\theta_1 = 0.1$</td>
<td>.1768</td>
<td>.1768</td>
</tr>
<tr>
<td>Expected Shareholder Wealth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(13) Equity Controlled: $\theta_1 = 0$</td>
<td>.1225</td>
<td>.1225</td>
</tr>
<tr>
<td>(14) CC Controlled: $\theta_1 = 0.05$</td>
<td>.0998</td>
<td>.1000</td>
</tr>
<tr>
<td>(15) CC Controlled: $\theta_1 = 0.1$</td>
<td>.1341</td>
<td>.1418</td>
</tr>
</tbody>
</table>

This table reports the numerical results for the discrete risk-shifting case. Column (1) reports the set of results derived using a constant reference value (book value), while Column (2) reports the set of results derived using equity’s market value. $\omega(\theta_1)$ is the percentage of CC written down, $\pi(\theta_1)$ is the percentage of equity transferred to CC-holders upon conversion, while $\theta_2^*(\theta_1)$ is the value of period-2 risk shifting.

Given that expected shareholder wealth under action set $\{\theta_{1,E}, \theta_{2,E}\} = \{0, .05\}$ is .1225 for both reference-value assumptions, while it is .1341 (.1418) under action set $\{\theta_{1,C}, \theta_{2,C}\} = \{.1, .05\}$ for the first (second) reference-value assumption, the “excessive” risk-taking equilibrium is reached in both cases, and aggregate risk taking is .15.

Appendix D contains an alternative, but plausible, set of parameter values (.05 of $C$ is substituted for $D$) that supports an “excessive” safety equilibrium.
5.2 Continuous Risk Shifting

The next set of numerical results are analogous with continuous risk shifting. These are meant to provide a clearer picture of how the model’s variables interact with one another. For this exercise, all parameter values remain unchanged from Section 5.1, except that personal adjustment costs are remodeled to accommodate continuous risk shifting. Toward this end, it is assumed that \( A(\theta_i) = \theta_i^2 \) for \( i = 1, 2 \), \( \lambda_E = 2.9 \), and \( \lambda_C = \pi(\theta^*_1,C) \Lambda_E \). The first assumption produces a simple, convex and symmetric cost function, the second generates a set of results that are quantitatively similar to the ones presented above, while the last assumption accommodates a proportional adjustment cost (proportional to ownership share).

The first set of continuous risk-shifting numerical results are derived using equity’s market value as its reference value, and are reported in Figure 2. Panel 1 of this figure plots the bank’s capital requirement as a function of period 1 risk shifting (left axis). Banks satisfy these requirements with initial book equity when \( \theta_1 = 0 \). In this case, shareholders retain control of the bank’s portfolio selection in period 2, and management selects the value of \( \theta_2 \) that maximizes expected shareholder wealth. This is plotted in Panel 2 as a function of \( \theta_2 \). In this case, expected shareholder wealth is maximized at \( \theta^*_2,E = .078 \) (see row (7) of Table 4).

When \( \theta_1 > 1 \), banks fail to satisfy their capital requirements at the beginning of period 2, and some CC is written down; the amount by which this happens is plotted in Panel 1 (right axis). As before, the percentage of equity transferred to CC-holders upon conversion, and the period-2 mean-preserving spread/contraction, are solutions to a fixed-point problem involving equity’s market value. After solving this problem for each grid-point of \( \theta_1 > 0 \), we obtain the graphs of \( \pi(\theta_1) \) and \( \theta^*_2,C(\theta_1) \). These are plotted in Panels 3 and 4, respectively.\(^{28}\) For all values of \( \theta_1 < .067 \), CC-holders select a mean-preserving contraction in period 2 - as indicated by negative values of \( \theta^*_2,C(\theta_1) \) - due to the relatively low values of \( \omega(\theta_1) \) and \( \pi(\theta_1) \). Conversely, CC-holders select a mean-preserving spread when \( \theta_1 \geq .067 \).

Now that we have the functions \( \omega(\theta_1) \), \( \pi(\theta_1) \), \( \theta^*_2,C(\theta_1) \) and \( \lambda_E A(\theta_1) \), we can calculate the expected wealth of shareholders for each grid-point of \( \theta_1 > 0 \); the graph of this mapping is plotted in Panel 5. Expected shareholder wealth, conditional on \( \theta_1 > 0 \), is maximized at \( \theta^*_1,C = .104 \) (row (6) of Table 4), and in this case, CC-holders select \( \theta^*_2,C = .022 \) in period 2 (row (8) of Table 4), bringing the total mean-preserving spread to \( \theta^*_1,C + \theta^*_2,C = .126 \).

To determine which action set is chosen by management (i.e., \{\( \theta_{1,E}, \theta_{2,E} \)\} = \{0, .078\} or \{\( \theta_{1,C}, \theta_{2,C} \)\} = \{.104, .022\}) we can overlay both shareholder-wealth functions (from Panels 2 and 5) and compare their peak values; this is done in Panel 6. Since action set \{0, .078\} produces an expected shareholder wealth of .1175, while action set \{.104, .022\} produces an expected shareholder wealth of .1247 (see rows (11) and (12) of Table 4 respectively), the “excessive” risk-taking equilibrium is arrived at, and aggregate risk shifting is .126.

Finally, for the sake of comparison, Figure 3 overlays the contents of Panel 6 with the shareholder-wealth function derived using equity’s book value as its reference value. Both

\(^{28}\) Note that \( \pi(\theta_1) \) and \( \theta^*_2,C(\theta_1) \)’s kink-points at \( \theta_1(\%) = 3.1\% \) are the result of the restriction \( \theta_2 \geq \theta_1 \), which preserves a unit measure of probability.
This figure plots the continuous risk-shifting numerical results. Panel 1: the bank’s capital-adequacy ratio (left axis), and the percentage of CC written down (right axis), as a function of $\theta_1$ (in percentage terms). Panel 2: the relationship between shareholder wealth and $\theta_2$ (in percentage terms) when $\theta_1 = 0$. Panel 3: The percentage of equity transferred to CC-holders upon conversion, as a function of $\theta_1$ (in percentage terms), when the bank uses equity’s market value as the reference value. Panel 4: the relationship between period-2 risk shifting and $\theta_1 > 0$ (both in percentage terms). Panel 5: the relationship between shareholder wealth and $\theta_1 > 0$ (in percentage terms). Panel 6: Overlay of Panels 2 and 5.
Table 4. Numerical Results: Continuous Risk Shifting

<table>
<thead>
<tr>
<th></th>
<th>(1) Book Value</th>
<th>(2) Market Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega(\theta_1)$ (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Equity Controlled: $\theta_1 = 0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(2) CC Controlled: $\theta_1 &gt; 0$</td>
<td>18.0</td>
<td>20.1</td>
</tr>
<tr>
<td>$\pi(\theta_1)$ (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) Equity Controlled: $\theta_1 = 0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(4) CC Controlled: $\theta_1 &gt; 0$</td>
<td>9.0</td>
<td>6.3</td>
</tr>
<tr>
<td>$\theta_1^*$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5) Equity Controlled: $\theta_1 = 0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(6) CC Controlled: $\theta_1 &gt; 0$</td>
<td>.090</td>
<td>.104</td>
</tr>
<tr>
<td>$\theta_2^*(\theta_1)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7) Equity Controlled: $\theta_1 = 0$</td>
<td>.078</td>
<td>.078</td>
</tr>
<tr>
<td>(8) CC Controlled: $\theta_1 &gt; 0$</td>
<td>.038</td>
<td>.022</td>
</tr>
<tr>
<td><strong>Final Equity Value</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(9) Equity Controlled: $\theta_1 = 0$</td>
<td>.1351</td>
<td>.1351</td>
</tr>
<tr>
<td>(10) CC Controlled: $\theta_1 &gt; 0$</td>
<td>.1658</td>
<td>.1665</td>
</tr>
<tr>
<td><strong>Expected Shareholder Wealth</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(11) Equity Controlled: $\theta_1 = 0$</td>
<td>.1175</td>
<td>.1175</td>
</tr>
<tr>
<td>(12) CC Controlled: $\theta_1 &gt; 0$</td>
<td>.1274</td>
<td>.1247</td>
</tr>
</tbody>
</table>

This table reports the numerical results for the continuous risk-shifting case. Column (1) reports the set of results derived using a constant reference value (book value), while Column (2) reports the set of results derived using equity’s market value. $\omega(\theta_1)(\%)$ is the percentage of CC written down, $\pi(\theta_1)(\%)$ is the percentage of equity transferred to CC-holders upon conversion, while $\theta_2^*(\theta_1)$ is the value of period-2 risk shifting.

shareholder-wealth functions for $\theta_1 > 0$ are similar, and produce identical qualitative results - management selects the action set $\{\theta_{1,C}, \theta_{2,C}\}$ in equilibrium. In the case of a fixed reference value, the equilibrium action set is $\{.09, .038\}$, and aggregate risk shifting is .128. See column (1) of Table 4 for comparable results using the fixed-reference-value assumption.

See Appendix D for an alternative, but plausible, set of parameter values (where .05 of $C$ is once again substituted for $D$) that supports an “excessive” safety equilibrium for the case of continuous risk shifting.
Figure 3. Shareholder Wealth When: $\theta_1 = 0$; and $\theta_1 > 0$, for both Reference-Value Assumptions

This figure plots the relationship between expected shareholder wealth and $\theta_2$ when shareholders retain control (i.e., $\theta_1 = 0$), and the relationship between expected shareholder wealth and $\theta_1$ when CC-holders gain control (i.e., $\theta_1 > 0$), for both reference-value assumptions.

6 Conclusion

Contingent-capital securities (CC) have the potential to mitigate lending disruptions caused by negative shocks to bank capital. However, they also have the potential to incentivize risk taking on the part of banks. This paper develops a model of banking to study this issue. Shareholders in the model can influence bank risk taking in each period by way of their equity-control rights. This is relevant for studying CC, because CC-investors become shareholders upon conversion, and can likewise influence risk taking via their newly-acquired equity-control rights. A second building block of the model is that partial conversion of CC - just enough to regain capital-ratio compliance - leaves its holders with a portfolio of bank debt and equity. In this way, CC-holder preferences for risk taking will rationally depend on their relative holdings of bank debt and equity post-conversion: a higher (lower) proportion of equity will incentivize more (less) risk taking. These endogenous preferences will, in turn, manifest themselves in post-conversion risk taking via endogenous CC-holder control rights.

In addition to affecting the level of risk taking post-conversion, these endogenous preferences and equity-control rights will affect the level of pre-conversion risk taking as well. This operates through the wealth-maximizing behavior of rational shareholders, who will anticipate these preferences, and can partially affect them via pre-conversion risk taking; since higher (lower) pre-conversion risk taking leads to higher (lower) capital requirements via asset risk-weights, and therefore, higher (lower) levels of CC written off. Taken together, the model has two distinct equilibria. In the first, shareholders engage in “excessive” risk taking, which increases the value of equity and creates a voting-block of risk-seeking CC-
holders that support further risk-taking initiatives. In the second, shareholders engage in “excessive” safety to avoid conversion altogether, thereby preventing the creation of an influential and risk-averse voting block.

The model is solved analytically, and comparative statics are run. These demonstrate that relatively high CC-to-equity ratios can reduce the likelihood of reaching an “excessive” risk-taking equilibrium, which provides a rational for substituting material quantities of conventional bank debt with CC. On the other hand, the comparative statics provide mixed results on the overall merits of higher capital requirements, and the optimal rate at which CC is converted into equity.

In addition to providing an analytic solution, the paper also presents a set of numerical results based on reasonable parameter values; these provide an indication of each equilibria’s feasibility.

Going forward, an interesting avenue of further study is the interaction among pre-existing blockholders, and those blockholders created via CC conversion. In this paper’s first endeavor at studying endogenous CC-holder preferences and control rights, it used a fairly narrow assumption regarding the dynamics of corporate-governance. As such, further refinements/extensions of this assumption may generate valuable new insights. The challenge, however, is selecting an appropriate refinement. As pointed out in the blockholder literature, this is a non-trivial matter given the co-dependencies among blockholders with regard to activism (see Edmans and Holderness 2017 for a discussion), the non-uniform propensity of investor-types to engage in activism (see Yermack (2010) and Edmans and Holderness (2017)) and the general opacity of blockholder activism (McCahery et al. (2016) makes an important contribution in this respect by surveying institutional investors on their role in corporate governance). Finally, in addition to leveraging insights from the blockholder literature, that literature may also do well by studying CC, given the unique endogeneity of blockholder-creation it entails.

References


29 One such refinement may be the incorporation of partial CC-holder influence, which depends, in a positive fashion, on the percentage of equity held post-conversion. This would likely incentivise higher risk taking in the “excessive” safety equilibria - since very small capital-ratio violations would no longer trigger control changes - but would probably leave the other qualitative results unaffected.

30 For instance, the observation that certain banks are inclined to prevent (permit) conversion via “excessive” safety (risk taking), may convey information about the level of activism/passivity inherent in pre-existing blockholders.


27


A Proof of Additional CC-Holder Risk Taking

This is a sketch of the proof. Shareholders always have the option to prevent conversion in period 1, and select $\theta_{2,E}^*$ in period 2 (from Equation 4) - call this Case 1. If management violates the bank’s CAR - call this Case 2 - aggregate risk shifting must be greater than $\theta_{2,E}^*$: only then will the bank’s aggregate equity value in Case 2 exceed that in Case 1 (net of the appreciation in equity value stemming from written-off CC - which accrues entirely to CC-holders given the assumption $r \geq 1$), which is necessary given shareholders dilute their own equity holdings in Case 2. If $\theta_{2,C}^* < 0$ (i.e., CC-holders select a mean-preserving contraction upon acquiring control) it must be that $\theta_1 > \theta_{2,E}^*$ (i.e., management selects a mean-preserving spread that exceeds $\theta_{2,E}^*$ in period 1). However, this can never happen in equilibrium since $\theta_1 > \theta_{2,E}^*$ is sub-optimal from Equation 4 when $\theta_{2,C}^* < 0$. ■

B Proof of Optimal CC-Holder Risk Shifting Level

The aggregate value of equity and debt held by CC-holders, conditional on $\theta_1$ and $\theta_2$, is:

$$\pi(\theta_1) \left\{ 1 - \left( 1 - \frac{(\theta_1 + \theta_2)}{2} \right) (D + C(1 - \omega(\theta_1))) \right\} + \left( 1 - \frac{(\theta_1 + \theta_2)}{2} \right) C(1 - \omega(\theta_1)),$$

from Expression 5. Rearranging this expression gives us:

$$\pi \left[ \frac{\theta_1 + \theta_2}{2} \right] D - (1 - \pi) \left[ \frac{\theta_1 + \theta_2}{2} \right] C(1 - \omega) + C(1 - \omega) + \pi(1 - D - C(1 - \omega)),$$

where the arguments of $\pi(\cdot)$ and $\omega(\cdot)$ are suppressed for brevity. Only the first two components are affected by $\theta_2$; the first is increasing in $\theta_2$ - wealth extracted from depositors - while the second is decreasing - reduction in CC value, net of the corresponding appreciation in equity value. An increase in the CC-holder’s portfolio value, from an increase in $\theta_2$, is highest when $\pi = 1$. If CC-holders select $\theta_2 = 2\theta$ instead of $\theta_2 = \theta$ when $\pi = 1$ (i.e., increase portfolio risk when it is the most advantageous to do so) their wealth - net of adjustment costs - changes by:

$$-\hat{\theta}C - \hat{A} < 0,$$

from Assumption 7. Therefore, CC-holders will never select $2\hat{\theta}$. Conversely, an increase in the CC-holder’s portfolio value, from a decrease in $\theta_2$, is highest in the limit as $\pi \to 0$. If CC-holders select $\theta_2 = -2\hat{\theta}$ instead of $\theta_2 = -\hat{\theta}$ as $\pi \to 0$ (i.e., decrease portfolio risk when it is the most advantageous to do so) their wealth - net of adjustment costs - changes by:

$$-\hat{\theta}D - \hat{A} < 0,$$

from Assumption 7 and the fact that $\pi \to 0 \iff \omega \to 0$. Therefore, CC-holders will never select $-2\hat{\theta}$. ■
C Proof Supporting Corollary 2

The right-hand-side of condition:

\[ \theta_1^* = 2\hat{\theta} \iff \hat{\theta}(D + C) - \left[ \frac{\hat{\theta}}{2}(D + C) + \hat{A} \right] - 2r\hat{\eta}\hat{\theta} - \frac{3r\hat{\eta}\hat{\theta}^2}{E_1}(D + C) > 0 \]

can be rewritten as:

\[ -\hat{A} + \hat{\theta} \left[ \frac{D + C}{2} - 2r\hat{\eta} \right] - \hat{\theta}^2 \left[ \frac{3r\hat{\eta}}{E_1}(D + C) \right] > 0. \]

Both bracketed terms are positive under Assumptions 1 - 7, and therefore, this quadratic is concave down, and necessarily negative for large enough $\hat{\theta}$. As such, there exists a $\hat{\theta}^U$ such that $\hat{\theta} < \hat{\theta}^U$ is necessary for $\theta_1^* = 2\hat{\theta}$.

D Alternative Numerical Results

The following set of numerical results illustrate that substituting CC for conventional bank debt can preclude the “excessive” risk-taking equilibria (leading to the “excessive” safety equilibria instead). The only change from Section 5’s base-case parameterization is that $0.05$ of $C$ is substituted for $D$ (i.e., CC is doubled). As before, the numerical results are derived using both discrete and continuous risk shifting.

D.1 Discrete Risk Shifting

Table 5 reports the set of parameter values (changes in boldface) used for the discrete risk-shifting case, and Table 6 reports the associated numerical results. These were derived using both reference-value assumptions: the results contained in column (1) correspond to the use of equity’s book value, while those contained in column (2) correspond to the use of equity’s market value.

Table 5. Alternative Parameter Values for the Discrete Risk-Shifting Numerical Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_g$</td>
<td>1</td>
<td>$\hat{\theta}$</td>
<td>.05</td>
</tr>
<tr>
<td>$D$</td>
<td>.8</td>
<td>$A(\pm \hat{\theta})$</td>
<td>0</td>
</tr>
<tr>
<td>$C$</td>
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<td>$A(\pm 2\hat{\theta})$</td>
<td>.025</td>
</tr>
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<td>$E_1/\hat{R}$</td>
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<td>$\lambda_E$</td>
<td>1</td>
</tr>
<tr>
<td>$\hat{\eta}$</td>
<td>.1</td>
<td>$\lambda_C$</td>
<td>1</td>
</tr>
</tbody>
</table>

This table reports the alternative parameter values used to derive the discrete risk-shifting numerical results. The only change from the base-case parameterization is that $0.05$ of $C$ is substituted for $D$.

This condition is necessary but not sufficient. $\hat{\theta}$ must also be large enough to compensate shareholders for $\hat{A}$. In addition, any $\hat{\theta}^U > 1/2$ is not the least-upper-bound.
Table 6. Numerical Results: Discrete Risk Shifting (High CC-to-Equity Ratio)

<table>
<thead>
<tr>
<th></th>
<th>(1) Book Value</th>
<th>(2) Market Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ω(θ1)</strong> (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Equity Controlled: θ1 = 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(2) CC Controlled: θ1 = 0.05</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>(3) CC Controlled: θ1 = 0.1</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td><strong>π(θ1)</strong> (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) Equity Controlled: θ1 = 0</td>
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<td>0</td>
</tr>
<tr>
<td>(5) CC Controlled: θ1 = 0.05</td>
<td>5</td>
<td>4.8</td>
</tr>
<tr>
<td>(6) CC Controlled: θ1 = 0.1</td>
<td>10</td>
<td>7.6</td>
</tr>
<tr>
<td><strong>θ∗2(θ1)</strong></td>
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<td></td>
</tr>
<tr>
<td>(7) Equity Controlled: θ1 = 0</td>
<td>.05</td>
<td>.05</td>
</tr>
<tr>
<td>(8) CC Controlled: θ1 = 0.05</td>
<td>-.05</td>
<td>-.05</td>
</tr>
<tr>
<td>(9) CC Controlled: θ1 = 0.1</td>
<td>-.05</td>
<td>-.05</td>
</tr>
<tr>
<td><strong>Market Value of Equity</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10) Equity Controlled: θ1 = 0</td>
<td>.1225</td>
<td>.1225</td>
</tr>
<tr>
<td>(11) CC Controlled: θ1 = 0.05</td>
<td>.1050</td>
<td>.1050</td>
</tr>
<tr>
<td>(12) CC Controlled: θ1 = 0.1</td>
<td><strong>.1323</strong></td>
<td><strong>.1323</strong></td>
</tr>
<tr>
<td><strong>Expected Shareholder Wealth</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(13) Equity Controlled: θ1 = 0</td>
<td>.1225</td>
<td>.1225</td>
</tr>
<tr>
<td>(14) CC Controlled: θ1 = 0.05</td>
<td>.0998</td>
<td>.1000</td>
</tr>
<tr>
<td>(15) CC Controlled: θ1 = 0.1</td>
<td><strong>.0940</strong></td>
<td><strong>.0973</strong></td>
</tr>
</tbody>
</table>

This table reports the numerical results for the discrete risk-shifting case using the alternative parameter values contained in Table 5. Column (1) reports the set of results derived using a constant reference value (book value), while Column (2) reports the set of results derived using equity’s market value. ω(θ1)(%) is the percentage of CC written down, π(θ1)(%) is the percentage of equity transferred to CC-holders upon conversion, while θ∗2(θ1) is the value of period-2 risk shifting. All of the differences between the values in this table and those of Table 5 are reported in boldface.

When the original face value of CC is increased by 100%, from .05 to .1, the percentage of CC written down when θ1 = .05 (θ1 = .1) is reduced by 50%, from 10% (20%) to 5% (10%) (see rows (2) and (3) of Table 6). Since the book value of equity, and the absolute value of CC written-down, are both unchanged from Section 5, it follows that the percentage of equity transferred to CC-holders upon conversion remains unchanged as well when using the first reference-value assumption (book value), i.e., π(.05) = 5% and π(.1) = 10% (see rows (5) and (6) of Table 6). Under the second reference-value assumption (market value), both the reference value of equity, and the percentage of equity transferred to CC-holders upon conversion, are once again solutions to a fixed-point problem involving θ∗2,C and equity’s market value. In contrast to the base-case results, CC-holders in this situation optimally select a mean-preserving contraction in period 2 (i.e., θ∗2,C = -.05) for both θ1 = .05 and
$\theta_1 = .1$, and they do this for both reference-value assumptions (see rows (8) and (9) of Table 6). This follows from the relatively-large amount of outstanding CC post-conversion compared with the amount of transferred equity (i.e., a relatively low $\omega(\theta_1)$ in comparison with $\pi(\theta_1)$). We know from Section 3 that management never permits CC-conversion when $\theta_2^* < 0$; this implies that $\theta_1 = 0$ in period 1, and that the “excessive” safety equilibrium is achieved.

To see this result more clearly, note that equity’s market value when $\theta_1 = .1$ is .1323 (i.e., $E_1 (.1)$, plus the 10% of $C$ that was written down (.01), plus $(\theta_1 + \theta_2^*)(D + C(1 - \omega(\theta_1))/2 = (.1 - .05)(.8 + .1(1 - .1))/2 = .0223$ of wealth extracted from creditors via aggregate risk shifting - see row (12) of Table 6). Therefore, $\pi(.1) = 7.6$ under the second reference-value assumption (see row (6) of Table 6). Although more equity is transferred to CC-holders under the alternative parameterization - due to equity’s lower market value - the percentage of equity transferred is smaller relative to outstanding CC. This holds true for both reference-value assumptions. In both of these cases $NV(.1) < 0$ from Equation 7, which implies that $\theta_2^* < 0$ from Equation 8, and that $\theta_2^*(.05) = - .05$ from Appendix B. In this situation, expected shareholders wealth is .094 (.0973) under the first (second) reference-value assumption, as shareholders retain 90% (92.4%) of the bank’s equity in period 2, which is worth .1323, and they also need to pay an adjustment cost of .025 for selecting $\theta_1 = .1$ (see rows (6), (12) and (15) of Table 6). Therefore, management optimally selects $\theta_1 = 0$, which produces an expected shareholder wealth of .1225 (see rows (4), (10) and (14) of Table 6).

D.2 Continuous Risk Shifting

As in Section 5, the first set of continuous risk-shifting numerical results are derived using equity’s market value as it’s reference value. In addition, all parameter values and functional forms are carried-over from Section 5 except that $\omega$ of $C$ is substituted for $D$, as before.

Panel 1 of Figure 4 plots the bank’s capital requirement as a function of $\theta_1$ (left axis); this function is unchanged from Section 5. Similarly, when banks meet their capital requirements in period 2 (i.e., $\theta_1 = 0$), the level of expected shareholder wealth, as a function of $\theta_2$, remains unchanged also, since no CC is written down (see Panel 2). Therefore, expected shareholder wealth is similarly maximized at $\theta_2^* = .078$ (see row (7) of Table 7).

When banks fail to meet their capital requirements in period 2, the amount of CC written-down for each $\theta_1 > 0$ remains unchanged. However, and importantly, the percentage of CC written-off is reduced by 50% (right axis of Panel 1) since 100% more CC was issued. Since a larger amount of CC is outstanding for each $\theta_1 > 0$, CC-holders are now more inclined to select a mean-preserving contraction in period 2; thereby enhancing the value of their outstanding CC. This can be seen from a comparison of Panel 4 from Figures 2 and

32Note that equity’s market value, the percentage of equity transferred to CC-holders upon conversion, and the expected wealth of shareholders, all remain unchanged when $\theta_1 = .05$, since $\theta_2^*(.05) = - .05$ as before. Therefore, other than the percentage of CC written down, the numerical results are identical in that case.

33This follows from the fungibility of outstanding CC and deposits with respect to creditor-wealth extraction via asset substitution in period 2 (see footnote 22).
Figure 4. Numerical results: Continuous Risk Shifting (High CC-to-Equity Ratio)

Panel 1: Capital Requirement and Write-Downs

Panel 2: Wealth: Shareholder Controlled

Panel 3: Equity Transfer

Panel 4: Period-2 Risk Shifting

Panel 5: Wealth: CC-Holder Controlled

Panel 6: Overlay of Panels 2 & 5

This figure plots the continuous risk-shifting numerical results when .05 of C is substituted for D (compared with the base-case parameterization of Section 5). Panel 1: the bank’s capital-adequacy ratio (left axis), and the percentage of CC written down (right axis), as a function of $\theta_1$ (in percentage terms). Panel 2: the relationship between shareholder wealth and $\theta_2$ (in percentage terms) when $\theta_1 = 0$. Panel 3: The percentage of equity transferred to CC-holders upon conversion, as a function of $\theta_1$ (in percentage terms), when the bank uses equity’s market value as the reference value. Panel 4: The relationship between period-2 risk shifting and $\theta_1 > 0$ (both in percentage terms). Panel 5: the relationship between shareholder wealth and $\theta_1 > 0$ (in percentage terms). Panel 6: Overlay of Panels 2 and 5.
Table 7. Numerical Results: Continuous Risk Shifting (High CC-to-Equity Ratio)

<table>
<thead>
<tr>
<th>(1) Book Value</th>
<th>(2) Market Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ω(θ₁) (%)</strong></td>
<td></td>
</tr>
<tr>
<td>(1) Equity Controlled: θ₁ = 0</td>
<td>0</td>
</tr>
<tr>
<td>(2) CC Controlled: θ₁ &gt; 0</td>
<td>10.7</td>
</tr>
<tr>
<td><strong>π(θ₁) (%)</strong></td>
<td></td>
</tr>
<tr>
<td>(3) Equity Controlled: θ₁ = 0</td>
<td>0</td>
</tr>
<tr>
<td>(4) CC Controlled: θ₁ &gt; 0</td>
<td>10.7</td>
</tr>
<tr>
<td><strong>θ₁∗</strong></td>
<td></td>
</tr>
<tr>
<td>(5) Equity Controlled: θ₁ = 0</td>
<td>0</td>
</tr>
<tr>
<td>(6) CC Controlled: θ₁ &gt; 0</td>
<td>.107</td>
</tr>
<tr>
<td><strong>θ₂∗(θ₁)</strong></td>
<td></td>
</tr>
<tr>
<td>(7) Equity Controlled: θ₁ = 0</td>
<td>.078</td>
</tr>
<tr>
<td>(8) CC Controlled: θ₁ &gt; 0</td>
<td>.005</td>
</tr>
<tr>
<td><strong>Final Equity Value</strong></td>
<td></td>
</tr>
<tr>
<td>(9) Equity Controlled: θ₁ = 0</td>
<td>.1351</td>
</tr>
<tr>
<td>(10) CC Controlled: θ₁ &gt; 0</td>
<td>.1604</td>
</tr>
<tr>
<td><strong>Expected Shareholder Wealth</strong></td>
<td></td>
</tr>
<tr>
<td>(11) Equity Controlled: θ₁ = 0</td>
<td>.1175</td>
</tr>
<tr>
<td>(12) CC Controlled: θ₁ &gt; 0</td>
<td>.1100</td>
</tr>
</tbody>
</table>

This table reports the numerical results for the continuous risk-shifting case. Column (1) reports the set of results derived using a constant reference value (book value), while Column (2) reports the set of results derived using equity’s market value. ω(θ₁)(%) is the percentage of CC written down, π(θ₁)(%) is the percentage of equity transferred to CC-holders upon conversion, while θ₂∗(θ₁) is the value of period-2 risk shifting. All of the differences between the values in this table and those of Table 4 are reported in boldface.

This weakly-reduces the expected wealth of shareholders for every θ₁ > 0 by: (1) reducing equity’s market value, and (2) diluting shareholders to a greater extent. This is reflected in Panel 5, which plots expected shareholder wealth as function of θ₁ > 0, which is noticeably lower than its counterpart from Section 5 (Panel 5 of Figure 2). In this case, expected shareholder wealth is maximized at θ₁∗ = .116, which induces θ₂∗ = -.024 (see rows (6) and (8) of Table 7).
We know from Appendix A that \( \{\theta^*_1, C, \theta^*_2, C\} = \{.116, -.024\} \) cannot constitute an equilibrium action set since \( \theta^*_2, C < 0 \). However, another way of seeing this is to overlay both shareholder wealth functions (from Panels 2 and 5) and compare their peak values. This is done in Panel 6. As this figure illustrates, the shareholder-wealth function for \( \theta_1 = 0 \) achieves a higher maximum value than the shareholder-wealth function for \( \theta_1 > 0 \), and as such, management selects the action set \( \{\theta^*_{1,E}, \theta^*_{2,E}\} = \{0, .078\} \), thus achieving the “excessive” safety equilibrium.

Once again, for the sake of comparison, Figure 5 overlays the contents of Panel 6 from Figure 4 with the shareholder-wealth function derived using equity’s book value as it’s reference value. As before, both shareholder-wealth functions for \( \theta_1 > 0 \) produce the same equilibrium result - management selects the action set \( \{\theta^*_{1,E}, \theta^*_{2,E}\} \) in both cases. However, since more equity is transferred to CC-holders upon conversion in the second case (book value) it turns out that \( \theta^*_2, C > 0 \) (see row (8) of Table 7). That is, CC-holders select a mean-preserving spread in period 2.\(^{34}\) Even so, given that \( \theta^*_1, C \) is sufficiently low under the alternative parameterization (compare row (8) of Tables 4 and 7), it is sub-optimal for shareholders to dilute their own equity stakes in order to achieve a higher aggregate equity value (i.e., Component 5 of Expression 9 is sufficiently small in comparison to Component 1). See column (1) of Table 7 for comparable results using the fixed-reference-value assumption.

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\(^{34}\)Note, however, that \( \theta^*_2, C(\theta_1) \) shifts downward for the second reference-value assumption also (see Panel 3 of Figure 4).