STICKY-WAGE MODELS
AND KNOWLEDGE CAPITAL

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Abstract

We present a sticky-wage model with two types of labors: while worker’s labor contributes to current production, researcher’s work helps develop new ideas to add to firm’s knowledge capital that enhances its productivity for many periods. The long-lived effect of knowledge capital on productivity is analogous to the long-lasting effect of consumer durables on utility in the sticky-price model of Barsky, House and Kimball (2007). Our sticky-wage model generates the near monetary neutrality result similar to the result in their sticky-price model, if returns to researchers’ labor are low in developing knowledge capital. We show, however, that the relative role of the pricing of the two production inputs analogous to consumption durables and nondurables in their sticky-price model is completely reversed in our sticky-wage model.

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1 Introduction

The importance of durable goods for economic fluctuations has long been studied in the business cycle models (e.g., Baxter 1996; Weder 1998). In a more recent paper, Barsky, House and Kimball (2007, hereafter BHK) emphasize the dominant role of the pricing of durable goods in generating monetary non-neutrality in a New Keynesian sticky-price model with durable and nondurable goods. They also provide an intriguing example of the near neutrality of money when prices of durables are flexible and those of nondurables are sticky.

This paper provides another example of perverse predictions of the monetary neutrality in a New Keynesian sticky-wage models with two types of labors. As emphasized by Galí (2011), the wage-setting block in the New Keynesian model plays a central role in determining the response of the economy to monetary shocks. As such, the sticky-wage approach has been considered as a workhorse in the New Keynesian literature along with the sticky-price approach.¹ In sticky-wage models, wages are set by households in a way symmetric to how prices are set by firms in sticky-price models (e.g., Erceg, Henderson and Levin 2000). Likewise, we turn BHK’s New Keynesian model on its head, replacing their two types of goods with our two types of labors.

We examine the roles of the pricing of the two types of labors analogous to consumption durables and nondurables in BHK’s sticky-price model. In BHK’s sticky-price model, firms set prices for two types of goods: once produced, one is immediately consumed, while the other adds to its stock that yields utility over time and wears out only gradually. In our sticky-wage model, households set wages for two types of labors: while worker’s labor immediately contributes to current production, researcher’s labor develops new ideas to add to firm’s knowledge capital that enhances its productivity for many periods and becomes obsolete only gradually.² The long-lived effect of knowledge capital on productivity in our sticky-wage model is therefore analogous to the long-lasting effect of consumer durables on utility in BHK’s sticky-price model.

Our findings are summarized as follows. First, we study the configuration of nominal rigidities when

¹Huang and Liu (2002), Huang, Liu and Phaneuf (2004), Christiano, Eichenbaum and Evans (2005) and many others find that wage rigidity is at least as important as price rigidity for explaining the empirical regularities in the U.S. economy.
²This structure differs from Carlstrom and Fuerst (2010) who simply added sticky wages in the BHK model with a single type of workers’ labor.
wages of durable input (i.e., researcher’s labor) are flexible and wages of nondurable input (workers’ labor) are sticky. We find that our sticky-wage model under this configuration does not predict the near neutrality of money. In fact, while this configuration of nominal rigidities corresponds to the case of the near neutrality of money in BHK’s sticky-price model, the non-neutrality in our sticky-wage model is as significant as when both researchers’ and workers’ wages are sticky. Second, we consider the other configuration of nominal rigidities under which researchers’ wages are sticky and workers’ wages are flexible. We find that the sticky-wage model under this configuration does predict the near neutrality of money as in the BHK’s sticky-price model, if returns to researchers’ labor in developing new ideas are reasonably low. This result means that the relative role of the pricing of the two production inputs analogous to the two consumption goods in BHK’s sticky-price model is completely reversed in our sticky-wage model: to generate a prediction similar to that of the standard New Keynesian model, the model needs stickiness in wages of workers rather than of researchers. In the model, the pricing of workers’ labor plays a dominant role in determining the response of aggregate output to monetary shocks.

2 A sticky-wage model with knowledge capital

The model features a continuum of households indexed by \( i \in [0, 1] \), each consisting of a worker and a researcher, and a continuum of firms indexed by \( j \in [0, 1] \), each producing a differentiated good. The labor services of the workers are differentiated and imperfectly substitutable, and so are the labor services of the researchers. There is a government conducting monetary policy.

At any date \( t \), the objective of household \( i \in [0, 1] \) is to maximize

\[
\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ U(C_s(i)) - V_W(N_{W,s}(i)) - V_R(N_{R,s}(i)) \right],
\]

where \( \mathbb{E}_t \) is the conditional expectations operator, \( \beta \in (0, 1) \) is a subjective discount factor, \( C_s(i) = \int_0^1 C_s(i, j)^{(\epsilon_P-1) / \epsilon_P} \, dj \) is the household \( i \)’s total consumption with \( \epsilon_P > 1 \), and \( N_{W,s}(i) \) and \( N_{R,s}(i) \) are worker’s and researcher’s labors, respectively. The functions \( U, V_W, \) and \( V_R \) are strictly increasing and twice continuously differentiable, with concave \( U \) and convex \( V_W \) and \( V_R \). The household’s budget
constraint in period $t$ is

$$
\int_0^1 P_t(j) C_t(i,j) dj \leq W_{W,t}(i) N_{W,t}(i) + W_{R,t}(i) N_{R,t}(i) - E_t[D_{t,t+1} B_{t+1}(i)] + B_t(i) + \Pi_t(i),
$$

(2)

where $P_t(j)$ is good $j$’s price, $W_{W,t}(i)$ and $W_{R,t}(i)$ are nominal wages of worker $i$ and researcher $i$, respectively, $D_{t,t+1}$ is the stochastic discount factor from date $t+1$ to $t$, $B_{t+1}(i)$ is a random quantity representing household $i$’s holdings of one-period state-contingent nominal bonds in period $t$, and $\Pi_t(i)$ is household $i$’s claim to firms’ profits.

Utility maximization gives rise to household $i$’s demand, $C_t(i,j) = \left[ P_t(j)/P_t \right]^{\varepsilon_P} C_t(i)$, where $P_t = \left[ \int_0^1 P_t(j)^{1-\varepsilon_P} dj \right]^{1/(1-\varepsilon_P)}$. The total demand for firm $j$’s good is given by $C_t(j) = \int_0^1 C_t(i,j) di = \left[ P_t(j)/P_t \right]^{\varepsilon_P} C_t$, where $C_t = \int_0^1 C_t(i) di$.

Firm $j$ produces its output, using workers’ labor inputs and its knowledge capital according to

$$
C_t(j) = F(N_{W,t}(j), K_t(j)),
$$

(3)

where $N_{W,t}(j) = \left[ \int_0^1 N_{W,t}(i,j)^{(\varepsilon_W-1)/\varepsilon_W} di \right]^{\varepsilon_W/(\varepsilon_W-1)}$ with $\varepsilon_W > 1$, and $K_t(j)$ is firm $j$’s knowledge capital, respectively. The function $F$ is homogenous of degree one, strictly increasing, concave, and continuously differentiable. Knowledge capital satisfies the following law of motion:

$$
K_t(j) = (1-\delta) K_{t-1}(j) + X_t(j),
$$

(4)

where $X_t(j)$ denotes firm $j$’s new ideas produced by R&D investment. Knowledge capital becomes obsolete at a rate $\delta \in (0,1)$. We consider a small positive value of $\delta$ to ensure the stationarity of $K_t(j)$.

New ideas are developed using researchers’ labor $N_{R,t}(j)$, together with a stock of aggregate knowledge capital $K_{t-1}$, according to

$$
X_t(j) = G(N_{R,t}(j), K_{t-1}),
$$

(5)

where $N_{R,t}(j) = \left[ \int_0^1 N_{R,t}(i,j)^{(\varepsilon_R-1)/\varepsilon_R} di \right]^{\varepsilon_R/(\varepsilon_R-1)}$ with $\varepsilon_R > 1$. The function $G$ is homogenous of degree one, strictly increasing, concave, and continuously differentiable. Knowledge capital is accumulated by

\footnote{See, for example, Comin and Gertler (2006) for an introduction of the possibility that knowledge becomes obsolete.}
individual firms and the economy-wide knowledge stock in turn helps each firm develop new ideas to add on its own knowledge capital (i.e., knowledge spillover). Therefore, $K_t(j)$ can also be thought of as a measure of intangible assets.\footnote{Examples include patents, copyrights, trademarks and trade names, blueprints or building designs, engineering drawings, and organizational expenses, as defined in the Compustat database. While not a pure public good, knowledge capital modeled here is only partially excludable or non-rival and represents a cost independent from the level of output (see Romer 1986, 1990; Arrow 2000). In a similar vein, it may also be reinterpreted as a form of organizational capital in the spirit of Beaudry and Devereux (1995), in the sense that its accumulation is an alternative rather than a complement to production.}

The specification here is in the spirit of the seminal works by Romer (1990) and Grossman and Helpman (1991), among others.

Cost minimization gives rise to firm $j$’s demand for worker $i$ and researcher $i$,

$$N_{W,t}(i,j) = \left[ \frac{W_{W,t}(i)}{W_{W,t}} \right]^{-\varepsilon_w} N_{W,t}(j) \quad \text{and} \quad N_{R,t}(i,j) = \left[ \frac{W_{R,t}(i)}{W_{R,t}} \right]^{-\varepsilon_R} N_{R,t}(j),$$

where $W_{W,t} = [\int_0^1 W_{W,t}(i)^{1-\varepsilon_w} \, di]^{1/(1-\varepsilon_w)}$ and $W_{R,t} = [\int_0^1 W_{R,t}(i)^{1-\varepsilon_R} \, di]^{1/(1-\varepsilon_R)}$. Because households are indifferent about working at different firms, wages $W_{W,t}(i)$ and $W_{R,t}(i)$ are independent of $j$.

At any date $t$, firm $j$ chooses $\{N_{W,s}(j), N_{R,s}(j), K_t(j)\}_{s \geq t}$ to maximize

$$E_t \sum_{s=t}^{\infty} D_{t,s} \left[ P_s(j) C_s(j) - W_{W,s} N_{W,s}(j) - W_{R,s} N_{R,s}(j) \right],$$

subject to (3)–(5). Here $D_{t,s} = \prod_{\tau=1}^{s-t} D_{t+\tau-1,t+\tau}$ denotes the $s$-period stochastic discount factor from $s$ to $t$, for all $s > t$, with $D_{t,t} = 1$. This profit maximization problem takes into account the solution to the embodied cost minimization problem.

The first-order conditions for $N_{W,t}(j)$, $N_{R,t}(j)$, and $K_t(j)$ are given by

$$\frac{W_{W,t}}{P_t} = F_{N,t}^*, \quad Q_t = \frac{W_{R,t}}{G_{N,t}},$$

$$Q_t = P_t F_{K,t}^* + (1 - \delta) E_t D_{t,t+1} Q_{t+1},$$

where $F_{N,t}^*$ and $F_{K,t}^*$ are the marginal product of inputs adjusted by price markups to simplify the notations. In particular, $F_{N,t}^* \equiv \mu_p^{-1} [\partial F(N_{W,t}, K_t)/\partial N_{W,t}]$, $F_{K,t}^* \equiv \mu_p^{-1} [\partial F(N_{W,t}, K_t)/\partial K_t]$ where $\mu_p = \varepsilon_p / (\varepsilon_p - 1)$. In (8), $G_{N,t} \equiv \partial G(N_{R,t}, K_{t-1})/\partial N_{R,t}$. Here, due to symmetry across firms, we drop the index $j$ from the firm $j$’s variables. The Lagrange multiplier for (4) is $Q_t$, which is interpreted as the nom-

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inal marginal benefit of increasing $K_t$. In equilibrium, the nominal marginal benefit $Q_t$ is equalized to the nominal marginal cost of producing new ideas $X_t$, which is given by $W_{R,t}/G_{N,t}$.

Households are monopolistic competitors in the labor markets for workers and researchers. They set their wages $W_{h,t}(i)$ where $h$ represents either $W$ or $R$. The total demand for household $i$’s labor is given by

$$N_{h,t}(i) = \int_0^1 N_{h,t}(i,j) dj = \left[ \frac{W_{h,t}(i)}{W_{h,t}} \right]^{-\varepsilon_h} N_{h,t},$$

where $N_{h,t} = \int_0^1 N_{h,t}(j) dj$ for $h = W, R$. Taking the labor demand functions as given, households set $W_{h,t}(i)$ in a staggered fashion with a probability of wage adjustment $1 - \theta_h$, respectively. If worker (researcher) $i$ gets the chance to reset its wage in period $t$, then it will choose the wage to satisfy

$$W_{h,t}(i) = \frac{\varepsilon_h}{\varepsilon_h - 1} \frac{E_t \sum_{s=t}^\infty (\beta\theta_h)^{s-t} V_h([W_{h,t}(i)/W_{h,s}]^{-\varepsilon_h} N_{h,s}) W_{h,s}^{\varepsilon_h} N_{h,s}}{E_t \sum_{s=t}^\infty (\beta\theta_h)^{s-t} U'(C_s(i)) W_{h,s}^{\varepsilon_h} N_{h,s}/P_s}, \quad h = W, R. \quad (10)$$

Analogous to BHK’s sticky-price model in which households must use cash to purchase goods, our sticky-wage model assumes that firms, instead of households, must hold money since they face a cash-in-advance (CIA) constraint for the payment of labors. Therefore, money demand is introduced here via $M_t = W_{W,t}N_{W,t} + W_{R,t}N_{R,t}$. The money supply $M_t^s$ grows at a rate $e^{\xi_t}$, $M_t^s = e^{\xi_t} M_{t-1}^s$, where $\xi_t$ is a white-noise process.

Finally, define real GDP as $Y_t$:

$$Y_t = PC_t + QX_t, \quad (11)$$

where $P$ and $Q$ are the steady-state values of $P_t$ and $Q_t$, respectively. The latter is interpreted as the imputed value of new ideas in the steady state, since $Q_t$ is equal to the nominal marginal cost of producing $X_t$. Note that $Y_t$ includes R&D investment as it reflects recent changes in the definition of GDP. The nominal GDP is $P_tC_t + Q_tX_t$ and the GDP deflator is the nominal GDP divided by real GDP.

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5To conserve space, we did not explicitly introduce the CIA constraint into the firm problem. In an appendix, which is available upon request from the authors, we assume that firms must make wage payment in advance by money. That model with firms’ money demand yields identical first-order conditions described in the main text in this paper.

6See also Schmitt-Grohé and Uribe (2006, 2007) who assume the CIA constraint on the wage bills of firms.
3 Results

In this section, we describe parametrization and present results. We then provide the intuition for our results.

3.1 Parametrization

In simulating our model, we closely follow the baseline parametrization in BHK unless otherwise noted. The subjective discount factor $\beta$ is set to ensure that the annual risk-free rate equals 2 percent. The utility function is parametrized as $U(C_t(i)) = C_t(i)^{1-\sigma} / (1 - \sigma)$, $V_h(N_{h,t}(i)) = N_{h,t}(i)^{1+\psi_h} / (1 + \psi_h)$, where $\sigma = \psi_h = 1$, for $h = W, R$. The parameter $\varepsilon_P$ is set to 11 as in BHK, while $\varepsilon_W$ and $\varepsilon_R$ are set to 5, consistent with the previous studies on sticky-wage models (e.g., Erceg, Henderson and Levin 2000; Huang and Liu 2002). When we assume wage stickiness in simulations, the average duration between wage changes is set to four quarters.

The production function of consumption goods is specified as $F(N_{W,t}(j), K_t(j)) = [N_{W,t}(j)]^{1-\alpha} [K_t(j)]^{\alpha}$. Here, $\alpha$ is set to 0.5, due to the lack of a broad consensus on the knowledge input share, but our results are robust to the choices of this parameter value. We choose the depreciation rate of knowledge capital $\delta$ to ensure that the steady-state GDP share of R&D investment equals 0.03, consistent with the U.S. data.\footnote{In our model, the steady-state GDP share of R&D investment is $QX / (PC + QX)$ written as a function of $\delta$. In particular, it is given by $a\delta [1 - \beta (1 - \delta)]^{-1} / (\mu_P + a\delta [1 - \beta (1 - \delta)]^{-1})$.}

The production function of new ideas is specified as $G(N_{R,t}(j), K_{t-1}) = [N_{R,t}(j)]^\phi K_{t-1}^{1-\lambda}$, where $\phi$ and $\lambda$ measure the degree of returns to researchers’ labor and the degree of knowledge spillovers, respectively. For the latter parameter $\lambda$, we set $\lambda = 0.5$, again due to a lack of broad consensus on knowledge spillovers in the production of new idea. In parameterizing the former parameter $\phi$, however, we allow for $\phi = 1$ and 0.5 and contrast the impulse responses of output under both cases. In the next section, we will examine the model under high returns ($\phi = 1$) and low returns ($\phi = 0.5$) to researchers’ labor in knowledge production and discuss the roles of nominal wage rigidities on real effects of monetary shocks.
3.2 Main findings

We first investigate the real effect of a monetary shock in the sticky-wage model with high returns to researchers’ labor \((\phi = 1)\) in producing new idea. Figure 1 plots impulse responses of output to a one percent increase in the money supply.\(^8\) In the left panel, we show the impulse responses under \(\phi = 1\) for three configurations of nominal wage rigidities. The dotted line represents output responses when both workers’ and researchers’ wages are sticky (All wages sticky). The dashed line corresponds to output responses when workers’ wages are sticky but researchers’ wages are flexible (Sticky workers’ wages). In both configurations, the monetary non-neutrality is significant and output rises above the steady state by 0.47 and 0.48 percent on impact of the monetary shock, respectively. The impacts of the monetary shock on output take more than a year for them to go back to the steady state. The solid line points to output responses when researchers’ wages are sticky but workers’ wages are flexible (Sticky researchers’ wages). In this configuration of nominal wage rigidities, the impact converges to zero somewhat more quickly than those in the other two configurations. Nevertheless, the output response on impact is very close to those in the other configurations.

The results suggest that the sticky-wage model with two types of labors does not generate the near neutrality of money even when prices of durable inputs are flexible. This is a sharp contrast to BHK’s sticky-price model. The model also predicts the substantial non-neutrality when prices of nondurable inputs are flexible. In fact, when \(\phi = 1\), monetary non-neutrality is substantial in any configurations of nominal rigidities.

However, when returns to researchers’ labor are reasonably low (i.e., \(\phi = 0.5\)), the real effect of money can be dramatically weak. In the right panel of Figure 1, we again show the impulse responses of output but in the economy with \(\phi = 0.5\). If researchers’ wages are sticky and workers’ wages are flexible, output increases by a small amount (e.g., only 0.03 percent on impact), much smaller than the other two configurations of nominal rigidities.

It should be emphasized that the near neutrality of money in our sticky-wage model is generated under

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\(^8\)In all figures of this paper, simulations are based on a period of 100th of a year, as in BHK. For convenience, quarters are marked on the horizontal axes.
the exactly opposite configuration of nominal rigidities to that in BHK’s sticky-price model, regardless of the similarity between our sticky-wage and BHK’s sticky-price models. Monetary neutrality in BHK’s model occurs with flexible durable prices even if nondurable prices are sticky, whereas the neutrality in our model occurs with flexible nondurable input prices even if durable input prices are sticky.

We can examine whether the result in our sticky-wage model is numerically close to the monetary neutrality result in BHK’s sticky-price model. Figure 2 indicates that, in comparisons, the neutrality result is more striking in our sticky-wage model than in BHK’s sticky-price model. For the sake of compatibility, we set the wage rigidity such that impulse responses of output from the standard sticky-wage model match those from the standard sticky-price model. Furthermore, physical capital is abstracted from BHK’s sticky-price model. The dashed lines display the impulse responses from the standard sticky-price and sticky-wage models (which by construction are identical across the two standard models). The solid line in the left panel reproduces BHK’s simulation results under the assumption that prices of durables are flexible and prices of nondurables are sticky. The solid line in the right panel presents the impulse response of output in our sticky-wage model, but with sticky researchers’ wages and flexible workers’ wages. The figure indicates that output increases by about 0.03 percent in our model, but it rises by about 0.06 percent in BHK’s model.

The contrast between our sticky-wage model and BHK’s sticky-price model goes beyond the above two instructive cases of nominal rigidities. To generate monetary non-neutrality, BHK’s sticky-price model depends on stickiness in prices of durables rather than of nondurables. However, our sticky-wage model depends on stickiness in wages of workers rather than of researchers. As the dashed and dotted lines in the right panel of Figure 1 illustrate, once workers’ wages become sticky, the monetary non-neutrality is as significant as when all wages are sticky. The similarity of output responses even suggests the irrelevance of pricing of durable input in our sticky-wage model. As we will show below, the irrelevance of pricing of durable input also holds broadly, unless returns to researchers’ labor are high.

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9 The standard models are the sticky-price model with only nondurables and the sticky-wage model with only workers’ labor. We match the slopes of the New Keynesian Phillips curves in the two standard models to facilitate the comparison.

10 The irrelevance of pricing of durable input is fairly robust to changes in the model, including incorporation of sticky prices into our model. The additional checks other than reported below are available upon request from the authors.
3.3 Analytical discussion

To understand the mechanism behind the monetary neutrality, we focus on dynamics of knowledge capital $K_t$, workers’ labor $N_{W,t}$, and researchers’ labor $N_{R,t}$. This is because GDP is given by $Y_t = PC_t + QX_t = PF(N_{W,t}, K_t) + QG(N_{R,t}, K_{t-1})$. To help us understand the mechanism, Figure 3 plots the impulse responses of a variety of variables to a monetary injection under the two configurations of nominal rigidities.

First, let us look at knowledge capital $K_t$. In a similar spirit as discussed in BHK, if the depreciation rate $\delta$ is low, a flow-stock ratio is so low that even large changes in the production of ideas have small effects on the total stock of knowledge capital. Therefore, as shown in the first row of Figure 3, a monetary injection does not produce a sizable increment in the stock of knowledge capital, independent of configurations of nominal rigidities. Consequently, it is helpful to treat knowledge capital as roughly constant: $K_t \simeq K$.

We then look at workers’ labor $N_{W,t}$. In equilibrium, workers’ wage markups $\mu_{W,t}$ are equal to the gap between the marginal product of workers’ labor $F^*_N(N_{W,t})$ in (7) and its marginal rate of substitutions for consumption $V''_W(N_{W,t})/U'(C_t)$. Therefore, we have

$$\mu_{W,t} \frac{V''(N_{W,t})}{U'(F(N_{W,t}, K_t))} = F^*_N(N_{W,t}, K_t), \quad (12)$$

where we used the market clearing condition for the composite good, $C_t = F(N_{W,t}, K_t)$.

Equation (12) suggests that $N_{W,t}$ effectively has a one-to-one relationship to wage markups for workers if $K_t$ is treated as constant. Thus, $N_{W,t}$ does not respond to a monetary injection if workers’ wages are flexible (i.e., $\mu_{W,t}$ is constant for all $t$). Similarly, consumption exhibits extremely small movement due to $C_t = F(N_{W,t}, K_t)$ along with the muted responses of $K_t$ and $N_{W,t}$ to the monetary shock.\footnote{The validity of our analysis and basic conclusion does not depend on how we parametrize preferences and technology. In fact, it is sufficient to assume $U'' > 0, U''' \leq 0, V''_W > 0, V'''_W \geq 0, F^*_N > 0, F^*_K \geq 0, F^*_N K \geq 0, \text{ and } F^*_N N \leq 0.$} The responses of $N_{W,t}$ and $C_t$ are reconfirmed by the solid lines in the second and third rows of Figure 3.

Finally, look at researchers’ labor $N_{R,t}$. Again, researchers’ wage markups $\mu_{R,t}$ are equalized to the gap between marginal product of researchers’ labor $(Q_t/P_t)G_{N,t}$ (in terms of consumption goods) and its
marginal rate of substitutions for consumption $V_t'(N_{R,t})/U_t'(C_t)$. This relationship can be expressed as

$$\mu_{R,t} V_t'(N_{R,t}) = \gamma_t G_N(N_{R,t}, K),$$  \hspace{1cm} (13)

where $\gamma_t \equiv Q_t U_t'/P_t$. In the equation, we also set $K_t \simeq K$.

Equation (13) implies that $N_{R,t}$ effectively has a one-to-one relationship to wage markups for researchers because $\gamma_t$ is near constant under flexible workers’ wages. To understand the near constancy of $\gamma_t$, use the fact that $D_{t,t+1} = \beta [U'(C_{t+1})/U'(C_t)](P_t/P_{t+1})$ and rewrite (9) as

$$\gamma_t = U'(C_t) F_{K}^{N}(N_{W,t}, K) + \beta (1 - \delta) E_t \gamma_{t+1},$$  \hspace{1cm} (14)

which indicates that $\gamma_t$ is stable over time since $N_{W,t}$ and $C_t$ are both near constant under the flexible workers’ wages. Therefore, $N_{R,t}$ has a one-to-one relationship with $\mu_{R,t}$ when workers’ wages are flexible. If researchers’ wages are sticky, a monetary injection moves researchers’ wage markups countercyclically and thus $N_{R,t}$ increases. Because $K_{t-1}$ is slowly moving to the monetary shock and $X_t = G(N_{R,t}, K)$, $X_t$ is also mainly driven by $\mu_{R,t}$.

Returns to researchers’ labor critically influences the sensitivity of $X_t$ and $Y_t$ to a monetary shock. In particular, when returns to researchers’ labor is high (i.e., $\phi = 1$), the incentive to employ researchers is extremely strong. In fact, an initial increase in $X_t$ is 15.26 percent to a one percent increase in the money supply. As a result, fluctuations in researchers’ labor matter significantly in business cycle frequencies and GDP increases by 0.47 percent, despite a small steady-state GDP share of R&D investment. When returns to researchers’ labor are reasonably low (i.e., $\phi = 0.5$), by contrast, the incentive to hire researchers is limited. An increase in $X_t$ on impact of the monetary injection is now 0.96 percent, which is transmitted to an increase in $Y_t$ by 0.03 percent. Consequently, together with essentially constant consumption, the output response is near zero in response to an unexpected increase in the money supply.

The relative wage response is helpful to understand the sensitivity of researchers’ labor. Based on (7), (8), and the definition $\gamma_t = Q_t U_t'/P_t$, the relative wage $W_{R,t}/W_{W,t}$ equals the ratio of the marginal product of researchers’ labor $(Q_t/P_t) G_{N,t} = \gamma_t G_{N,t}/U_t'$ to that of workers’ labor $F_{N,t}^{N}$. Then, since responses of $\gamma_t$,
$K_{t-1}, N_{W,t},$ and $C_t$ are all muted when workers’ wages are flexible, we set $\gamma_t \simeq \gamma, K_{t-1} \simeq K, N_{W,t} \simeq N_W,$ and $C_t \simeq C$ and obtain

$$\frac{W_{R,t}}{W_{W,t}} = \frac{1}{U'(C)} \frac{\gamma G_N(N_{R,t}, K)}{F_N(N_W, K)}.$$  

Therefore, $N_{R,t}$ moves together with the relative wages. When workers’ wages are flexible and researchers’ wages are sticky, the adjustment of researchers’ wages is slower than that of workers’ wages, so a monetary injection decreases the relative wage (as shown in the solid line in the bottom right corner of Figure 3).

Indeed, using our specification of the function $G(N_{R,t}, K_{t-1})$, we can approximate the above equation by

$$\hat{N}_{R,t} \simeq -\frac{1}{1-\phi} \left( \hat{W}_{R,t} - \hat{W}_{W,t} \right),$$

where a variable with a hat represents the log-deviation of that variable from its steady-state value. The researchers’ labor is less sensitive to the relative wage as $\phi$ decreases (i.e., as the marginal product of researchers’ labor becomes more diminishing). This sensitivity of researchers’ labor to the relative wage critically influences the significance of the monetary non-neutrality.

The mechanism of our monetary non-neutrality result under the opposite configuration, that is to say, with sticky workers’ wages and flexible researchers’ wages, is more straightforward to explain. Under sticky workers’ wages, the wage markups decrease in response to a monetary injection. Due to the one-to-one relationship between $N_{W,t}$ and $\mu_{W,t}$, $N_{W,t}$ increases. The increase in $N_{W,t}$ leads to an increase in $C_t (= F(N_{W,t}, K_t))$. Consumption is dominant in the increase in GDP $Y_t = PC_t + QX_t$. See the dashed lines for consumption and R&D in Figure 3.\footnote{The effect of $X_t$ on $Y_t$ is negligible as long as $\sigma$ is close to unity. In particular, under our assumption of the functional form of $U$ and $F$, the first term of the right-hand side of (14) becomes $\alpha C_t^{1-\sigma} / K \simeq \alpha / K$, implying the near constancy of $\gamma_t$. Together with the constancy of $\mu_{R,t}$ in (13) in this configuration, $N_{R,t}$ is near constant and thus $X_t$ is small in response.}

Given the dominance of $C_t$, the monetary non-neutrality in this opposite configuration hinges upon whether workers’ wages are flexible or sticky, but has little to do with rigidity in researchers’ wages and with the degree of returns to researchers’ labor.

There is a final remark on our neutrality result. In BHK’s sticky-price model, the negative comovement between labor inputs for producing nondurable and durable goods is critical in producing their neutrality result. In our model, the neutrality result does not hinge upon any specific comovement patterns between
workers’ labor and knowledge capital or researchers’ labor. To see this more clearly, we invoke the functional forms for \( U, V_W, \) and \( F \) postulated in our numerical simulations. Under flexible workers’ wages in which \( \mu_{W,t} \) is constant for all \( t \), the log-linearized equation of (12) can be written as

\[
- [\alpha + \psi_W + \sigma (1 - \alpha)] \hat{N}_{W,t} = -\alpha (1 - \sigma) \hat{K}_t.
\]

(16)

Here, the coefficient on \( \hat{N}_{W,t} \) is negative whereas that on \( \hat{K}_t \) can be negative, zero, or positive. By virtue of (16), the correlation of workers’ labor with knowledge capital (thus with researchers’ labor as well) is negative, zero, or positive, if \( \sigma \) is greater than, equal to, or smaller than unity. However, as we will see in the next section on robustness, the sign of the correlation does not exert any quantitatively significant impact on the real effect of money.

4 Robustness checks

We here check the robustness of the results from our sticky-wage model with \( \phi = 0.5 \) to alternative values of parameters. The parameters we consider are the degrees of relative risk aversion, knowledge capital contribution in production, and researchers’ wage stickiness. Basically, comparisons in figures of this section are made for two configurations of nominal rigidities: sticky researchers’ wages and flexible workers’ wages (left panel) versus sticky workers’ wages and flexible researchers’ wages (right panel).

Degree of relative risk aversion  We first show that our results are robust to the degree of relative risk aversion \( \sigma \), which is set to unity in the baseline calibration. However, a lower value of \( \sigma \) (i.e., more elastic intertemporal substitution in consumption) can weaken households’ consumption smoothing motive. This may result in greater fluctuations in aggregate demand following monetary shocks and in stronger real effects of monetary shocks.

Figure 4 plots the impulse responses of output to a one percent increase in the money supply for a wide range of values of \( \sigma \) under the two configurations of nominal rigidities. As the right panel of the figure shows, when \( \sigma \) declines from 5 to 1, and then to 0.01, the monetary injection has a stronger real effect. However,  

13 We confirm that output responses when all wages are sticky are similar to those when only workers’ wages are sticky. Therefore, we will present the results for the two configurations of nominal rigidities.
the three impulse response functions in the left panel shows that money remains nearly neutral for all the
values of $\sigma$ when workers’ wages are flexible. We can also reconfirm from (16) that $\tilde{N}_{H,t}$ is almost invariant
to the monetary shock because knowledge capital is slowly moving ($\tilde{K}_t \approx 0$) and thus money remains nearly
neutral, given the relatively small response of $X_t$.

Degree of knowledge capital contribution in production We next show that our results are robust
to knowledge input share in production $\alpha$, which is set to 0.5 in the baseline calibration. The robustness
check is important because of a lack of consensus on the value of $\alpha$.

Figure 5 displays the impulse responses of output for a wide range of values of $\alpha$ under the two alternative
configurations of nominal rigidities. As the left panel of the figure shows, the impulse response of output
suggests that money remains nearly neutral with sticky researchers’ wage, even when the importance of
knowledge input in production increases from 0.05 to 0.70. On the other hand, when workers’ wages are
sticky, output responses substantially differ across different values of $\alpha$ (shown in the right panel). Therefore,
we conclude that, at least for the near neutrality of money when researchers’ wages are sticky, our model’s
predictions are robust to $\alpha$.\footnote{We also confirmed the robustness of the near neutrality of money to the degree of knowledge spillovers $\lambda \in [0,1)$, since our calibration of $\lambda = 0.5$ does not necessarily rely on solid empirical findings. Changes in $\lambda$ have only a tiny effect on both $K_{t-1}$ and $N_{R,t}$ so that output responses remain essentially unaltered under the two configurations of nominal rigidities.}

Degree of researchers’ wage stickiness Recall that in the baseline calibration we set the degree
of wage stickiness to ensure that the average duration between wage changes is four quarters. Figure 6
plots output responses with varying average durations of researchers’ wages, from one to eight quarters,
under flexible (left panel) and sticky (right panel) workers’ wages. When workers’ wages are flexible, output
responses remain uniformly small so money remaining nearly neutral. When workers’ wages are sticky (with
an average duration of four quarters), significant monetary non-neutrality is present. In both panels, increases
in researchers’ wage stickiness only marginally strengthen the real effects of monetary shocks, reaffirming the
importance of workers’ wage stickiness (and the unimportance of researchers’ wage stickiness) in determining
output responses.
5 Concluding remarks

In an influential paper, Barsky, House and Kimball (2007) show that the pricing of durable goods plays a dominant role, whereas the pricing of nondurable goods is immaterial, in determining the real effects of monetary shocks in a New Keynesian sticky-price model. Specifically, money can be neutral or have real effects depending on whether prices of durable goods are sticky or flexible, but independent of rigidity in nondurable goods’ prices. After concluding that “durables are the most important element in sticky-price models,” they urge that “researchers must devote more effort to empirical investigation of the pricing of these goods.”

The recent development in the New Keynesian literature has assigned a central role to the sticky-wage approach, along with the sticky-price approach. Against the background, we turned BHK’s sticky-price model on its head, replacing their two types of goods with our two types of labors. We have used a New Keynesian sticky-wage model to study how the pricing of the two types of labors may affect the real effects of monetary shocks.

We showed that the model’s prediction critically depends on stickiness in wages of workers (prices of nondurable input) rather than wages of researchers (prices of durable inputs) in determining the real effects of monetary shocks.

Under low returns to researchers’ labor in producing new ideas, the relative role of the pricing of the two production inputs analogous to consumption durables and nondurables in BHK’s sticky-price model is completely reversed in our sticky-wage model: whether money is neutral or not hinges upon rigidity or lack thereof in workers’ wages rather than in researchers’ wages. At the very general level, it is the pricing of workers’ not researchers’ labor that plays a dominant role in shaping aggregate dynamics following a monetary shock. We demonstrated that this conclusion holds quite generally regardless of other details of our model. Our results in this paper suggest that future research on the heterogeneity in wage stickiness across labors should be a priority at least as high as that on the heterogeneity in price stickiness across goods.
References


Figure 1: Responses of GDP to the Monetary Shock

Notes: The left panel shows responses of GDP to a one percent increase in the money supply when returns to researchers’ labor are high in knowledge production ($\phi = 1.0$). The right panel presents those when returns to researchers’ labor are low ($\phi = 0.5$). In each panel, dotted lines represent responses of GDP when both workers’ and researchers’ wages are sticky; dashed lines correspond those when workers’ wages are sticky, but researchers’ wages are flexible; solid lines are those when workers’ wages are flexible, but researchers’ wages are sticky. Vertical axis measures percentage deviations from the steady state. Time in quarters is on horizontal axis.
Figure 2: The Near Neutrality of Money in Sticky-price and Sticky-wage Models

Notes: The solid line on the left panel corresponds to the near money neutrality result in BHK’s model with sticky nondurable and flexible durable prices. The solid line on the right panel shows responses of GDP to a one percent increase in the money supply in our model with sticky researchers’ wages and flexible workers’ wages. The dashed lines represent the responses of GDP in the standard sticky-price model (left panel) and those in the standard sticky-wage model (right panel). Vertical axes measure percentage deviations from the steady state. Time in quarters is on horizontal axes.
Figure 3: Responses to a One Percent Increase in the Money Supply

Notes: Each panel shows responses of corresponding variables in our sticky-wage model to a one percent increase in the money supply under two alternative configurations on wage stickiness. Solid lines represent the case when workers’ wages are flexible and researchers’ wages are sticky. Dashed lines show responses with sticky workers’ wages and flexible researchers’ wages. Vertical axes measure percentage deviations from the steady state. Time in quarters is on horizontal axes.
Figure 4: Responses of GDP with Different Degrees of Relative Risk Aversion

Notes: The left panel shows responses of GDP to a one percent increase in the money supply when workers’ wages are flexible, but researchers’ wages are sticky. The right panel displays those when workers’ wages are sticky but researchers’ wages are flexible. Lines in each panel are for different values of \( \sigma \). Vertical axes measure percentage deviations from the steady state. Time in quarters is on horizontal axes.
Figure 5: Responses of GDP with Different Degrees of Knowledge Capital Contribution

Notes: The left panel shows responses of GDP to a one percent increase in the money supply when workers’ wages are flexible, but researchers’ wages are sticky. The right panel displays those when workers’ wages are sticky but researchers’ wages are flexible. Lines in each panel are for different values of $\alpha$. Vertical axes measure percentage deviations from the steady state. Time in quarters is on horizontal axes.
Figure 6: Responses of GDP with Different Degrees of Researchers’ Wage Stickiness

Notes: The left panel shows responses of GDP to a one percent increase in the money supply when workers’ wages are flexible, but the stickiness of researchers’ wages varies across lines. The right panel displays those when the average duration for workers’ wage changes is four quarters. Solid, dashed, and dotted lines in each panel represent the average duration between researchers’ wage changes being one quarter, four quarters, and eight quarters, respectively. Vertical axes measure percentage deviations from the steady state. Time in quarters is on horizontal axes.