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PRODUCTIVITY GROWTH, INDUSTRY LOCATION PATTERNS AND LABOR MARKET FRICTIONS

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Abstract

This paper constructs a two-country model of international trade to study how labor market frictions affect industry location patterns, unemployment rates, and fully endogenous productivity growth. We show that when the larger country offers subsidies to labor search costs or reduces unemployment benefits, the domestic unemployment rate falls, causing greater industry concentration and faster productivity growth, but higher unemployment for the smaller country. When similar labor market policies are implemented in the smaller country, however, the resulting fall in domestic unemployment leads to lower industry concentration and slower productivity growth, while lowering unemployment in the larger country.

JEL Classifications: E24; J64; O41; R11
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1 Introduction

In an increasingly integrated world economy, firms are less constrained by national borders as they search for the best geographic locations for production. The shift toward more flexible location patterns often collides with economic policy on two fronts. Regional policy tends to emphasize employment and innovation-based growth, hand in hand, as demonstrated by the Europe 2020 Integrated Guidelines for the Economic and Employment Policies of Member States. At the same time, however, policy makers also tend to support the existence of innovation and production clusters across regions (Martin 2008) as having a positive effect on growth. Further research is needed to shed light on how industry location patterns affect unemployment and growth in order to clarify the links between regional policy objectives.

With this objective in mind, we develop an endogenous market structure and endogenous growth framework (Aghion and Howitt 1998; Laincz and Peretto 2006; Etro 2009) to consider the links between industry location patterns, national unemployment rates, and long-run productivity growth in a two-country model of international trade. In particular, we combine the process innovation framework of Smulders and van de Klundert (1995) and Peretto (1996) with the equilibrium unemployment framework of Mortensen and Pissarides (1999). First, in an intermediate sector, monopolistically competitive firms produce horizontally differentiated varieties for sale to final good firms and invest in process innovation with the aim of reducing future production costs. Then, in the final good sector, firms employ workers to manage the assembly of intermediate products into final goods for both consumption by households and employment in the production and innovation activities of intermediate firms.

Production and innovation are footloose between countries (Martin and Rogers 1995), but trade in intermediate goods is subject to trade costs. Thus, the country with the larger market size, as measured by the size of the employed labor force, attracts greater shares of both intermediate and final good production through a standard home market effect (Krugman 1980). In addition, technical knowledge spillovers from production into the innovation activities of intermediate firms diminish with distance. As a result, all intermediate firms locate their innovation activities in the country with the larger market, as the greater share of intermediate production leads to higher knowledge spillovers that lower the cost of process innovation.

The labor market of each country features a matching process in which final good firms incur search costs as they look for unemployed workers and then engage in Nash bargaining to determine the division of profit with their employees. Free entry and exit in the final good sector reduces firm value to the capitalized cost of hiring a worker, and national labor markets are subsequently linked through a common interest rate that is determined in an international financial market. Domestic labor market policies then influence unemployment rates in both countries through the response of capitalized hiring costs to adjustments in the interest rate.

We use the framework to consider the effects of economic integration. An improvement in international knowledge diffusion directly raises knowledge spillovers into innovation, while a reduction in trade costs raises knowledge spillovers indirectly by increasing the concentration of intermediate production in the larger country. In both cases, the rate of productivity growth accelerates. The interest rate also rises, however, increasing the capitalized cost of hiring workers and adversely influencing the unemployment rates of the two countries. Thus, improved economic integration leads to a higher rate of economic growth at a cost of lower employment levels.

The framework is also used to study two types of national labor market policies: subsidies to the search costs associated with hiring workers and unemployment benefits for workers without jobs. Search subsidies allay hiring costs, placing downward pressure on the unemployment rate, while unemployment benefits reduce the incentive to look for work placing upward pressure on the unemployment rate. Beyond these direct effects on employment levels, however, lies the indirect effect of changes in industry concentration on labor market outcomes through adjustments in the interest rate. In the larger country, either an increase in search subsidies or a decrease in unemployment benefits increases the number of employed workers and expands market size, raising the country's share of intermediate production. As a result, knowledge spillovers improve and productivity growth accelerates. The interest rate once again rises, however, placing upward pressure on the unemployment rates of both countries. Overall, the unemployment rate rises in the smaller country, but falls in the larger country as the direct effect dominates the indirect effect.

The effects of national labor market policies implemented in the smaller country operate through similar mechanisms with opposite results. An increase in search subsidies, or a decrease in unemployment benefits, expands the market size of the smaller country as employment rises, attracting intermediate production and reducing industry concentration in the larger country. Hence, lower knowledge spillovers lead to slower productivity growth. The interest rate also falls, causing the unemployment rate to decrease in each country.

An analytical study of national welfare effects proves to be intractable within our framework. But we are able to use simple numerical examples to present several welfare results. On the one hand, these numerical examples suggest that improved economic integration, through lower trade costs or a higher knowledge diffusion, is always welfare improving for both countries. On the other hand, increases in search subsidies tend to benefit the welfare of the implementing country, while hurting the welfare of the other country. Raising unemployment benefits tends to improve the welfare of both countries. In this manner, the numerical examples indicate the potential for national labor market policy to influence the welfare of neighboring countries through the effects of shifts in industry location on long-run growth.

This paper contributes to the theoretical literature examining the relationship between innovation-based endogenous growth and unemployment in open economy settings. Building on the closed economy models of Aghion and Howitt (1994) and Mortensen (2005), this literature generally adopts the quality ladders framework (Grossman and Helpman 1991) of creative destruction. Sener (2001) demonstrates that trade liberalization promotes economic growth, but raises unemployment for low-skilled workers as faster innovation leads to a higher job separation rate. Moreover, Stepanok (2016) shows that a reduction in trade costs may hurt or benefit employment depending on the size of the R&D sector relative to the economy. Similarly, introducing a North-South product cycles framework, Stepanok (2018) finds that stronger intellectual property rights lower unemployment in the advanced North, with the effects of trade liberalization on employment depending on the outside options of workers in negotiations with employers. While the above literature mainly focuses on the implications of trade liberalization for growth and unemployment, the endogenous market structure and endogenous growth framework developed in this paper enables a study of the interdependence of national labor markets that arises when industry location patterns shift in response to changes in domestic labor market policy, allowing us to provide new insights into the relationship between growth and unemployment in open economic settings.

The remainder of the paper is organized as follows. In Section 2 we introduce our theoretical model of industry location, labor market frictions, and endogenous productivity growth. Section 3 then characterizes the balance growth path that arises in long-run equilibrium. Section 4 applies the model to a study of the effects of improvements in economic integration and adjustments in labor market policy on unemployment rates, industry location, productivity growth, and national welfare levels. We conclude the paper in Section 5. All proofs are relegated to the Appendix.

2 The Model

We now introduce our theoretical framework in which two countries, home and foreign, each produce intermediate and final goods. Final goods are produced using labor and intermediate goods both for consumption by households and for employment in the innovation and production activities of intermediate firms. Labor is the sole factor of production. We present the framework with a focus on the home country, with analogous conditions for the foreign country. Foreign variables are indicated by an asterisk.

2.1 Households

In each country, there is a dynastic representative household with a large number of members L, of whom $(1-u)L$ are employed and uL are unemployed, with $u \in (0,1)$ denoting the unemployment rate. As such, the household receives an expected wage income of $w(1-u)L$ and unemployment benefits of buL, where w is the wage rate and b is the unemployment benefit. The household eliminates consumption and employment uncertainty by providing perfect consumption insurance, with equal consumption levels for all household members, regardless of whether they are employed or not (Merz 1995; Andolfatto 1996).

The lifetime utility of the representative household takes the following constant intertemporal elasticity of substitution formulation:

$$W = \int_0^\infty e^{-\rho t} \frac{C(t)^{1-1/\sigma} - 1}{1 - 1/\sigma} dt, \tag{1}$$

where C is home consumption of the final good, ρ is the subjective discount rate, and $\sigma \in (0, 1)$ is the intertemporal elasticity of substitution.[1] The household selects an expenditure-saving path with the aim of maximizing lifetime utility subject to the flow budget constraint

$$\dot{A}(t) = r(t)A(t) + (1 - u(t))w(t)L + u(t)b(t)L - C(t) - T(t), \tag{2}$$

where A is asset wealth, r is the interest rate, and a dot over a variable denotes time differentiation. Government policies are funded through a lump-sum tax T levied on the household.

The optimal expenditure-saving path follows the standard Euler condition for the evolution of household expenditure:

$$\frac{\dot{C(t)}}{C(t)} = \sigma(r(t) - \rho), \tag{3}$$

where with a perfectly integrated international financial market, interest rates and expenditure dynamics are common across countries; that is, $\dot{C}/C = \dot{C}^*/C^* = \sigma(r - \rho)$. In order to simplify the model exposition, henceforth we suppress time notation where possible.

[1] See Hall (1988), Guvenen (2006), and Chiappori and Paiella (2011) for empirical estimates of the intertemporal elasticity of substitution that are consistent with $\sigma \in (0, 1)$.

2.2 Final good Sector

The final good sector produces a homogeneous good for supply to an international market characterized by free trade. Selecting the final good as the model numeraire, we set the price to unity, under the assumption that aggregate demand is sufficiently large to ensure that both countries always have active final good sectors.

Each final good firm is managed by a single worker and employs intermediate goods in production. The technology of a representative firm in the home country is

$$y = \left(\int_0^n x^\varepsilon(i) di + \int_0^{n^*} x^{*\varepsilon}(i^*) di^* \right)^{\alpha/\varepsilon}, \tag{4}$$

where $\alpha \in (0, 1)$, $x(i)$ and $x(i^*)$ are the input quantities associated with the masses of intermediate varieties n and n^* produced in home and foreign, and $\varepsilon \in (0, 1)$ is the degree of product differentiation between any given pair of varieties.

The profit of a final good producer located in the home country is

$$\Pi_Y = y - \int_0^n p(i)x(i)di - \int_0^{n^*} \tau p(i^*)x(i^*)di^*, \tag{5}$$

with $p(i)$ and $p(i^*)$ denoting the prices of intermediate varieties produced in home and foreign. International trade in intermediate goods involves iceberg trade costs whereby $\tau > 1$ units must be shipped for every unit sold in an export market (Samuelson 1954).

Profit maximization generates the following firm-level demands in the home country for the intermediate varieties produced in home and foreign:

$$x(i) = \frac{p(i)^{-\frac{1}{1-\varepsilon}} \alpha y}{\int_0^n p(j)^{\frac{-\varepsilon}{1-\varepsilon}} dj + \varphi \int_0^{n^*} p(j^*)^{\frac{-\varepsilon}{1-\varepsilon}} dj^*}, \qquad i \in n \tag{6}$$

$$x(i^*) = \frac{(\tau p(i^*))^{-\frac{1}{1-\varepsilon}} \alpha y}{\int_0^n p(j)^{\frac{-\varepsilon}{1-\varepsilon}} dj + \varphi \int_0^{n^*} p(j^*)^{\frac{-\varepsilon}{1-\varepsilon}} dj^*}, \qquad i^* \in n^* \tag{7}$$

where the level of trade costs is indexed by $\varphi \equiv \tau^{-\varepsilon/(1-\varepsilon)} \in (0, 1)$, which as a decreasing function of trade costs captures the freeness of trade: $\varphi = 0$ describes prohibitively high trade costs and $\varphi = 1$ indicates free trade. Substituting the demand conditions into the profit function (5) yields the profit for a final good producer located in home as $\Pi_Y = (1-\alpha)y$. In addition, with a single worker employed at each firm, the number of producing firms matches the level of employment $(1-u)L$, and aggregate final good output in home is $Y \equiv y(1-u)L$.

2.3 Intermediate Sector

Firms in the intermediate sector employ final goods in both production (I_X) and in process innovation (I_R) as they supply unique differentiated varieties for sale to final good producers in a market characterized by monopolistic competition (Dixit and Stiglitz 1977), while investing in process innovation to improve their production technologies (Smulders and van de Klundert 1995; Peretto 1996). Although intermediate firms do not incur costs with market entry, at each moment in time they do face fixed costs (I_F) measured in units of final goods.

The technology of an intermediate firm i with production located in home is

$$X(i) = z(i)^{\gamma} I_X(i), \tag{8}$$

where X is output, z is firm-level productivity, I_X is the quantity of final goods employed in production, and $\gamma \in (0,1)$ is the output elasticity of productivity. Given the CES production technology employed in the final good sector, intermediate firms maximize profit by setting price equal to

$$p(i) = \frac{1}{\varepsilon z(i)^{\gamma}}, \tag{9}$$

where $1/\varepsilon > 1$ is the constant markup and $1/z^{\gamma}$ is the unit production cost. Matching supply with the combined demands of home and foreign final good producers, home-based intermediate production is $X = x(1-u)L + \tau x^*(1-u^*)L^*$. As a result, the optimal operating profit on sales ($\pi_X = pX - I_X = (1-\varepsilon)pX$) of an intermediate firm with production located in home is

$$\pi_X(i) = \frac{\alpha(1-\varepsilon)p(i)^{\frac{-\varepsilon}{1-\varepsilon}}Y}{\int_0^n p(j)^{\frac{-\varepsilon}{1-\varepsilon}}dj + \varphi\int_0^{n^*}p(j^*)^{\frac{-\varepsilon}{1-\varepsilon}}dj^*} + \frac{\alpha(1-\varepsilon)\varphi p(i)^{\frac{-\varepsilon}{1-\varepsilon}}Y^*}{\varphi\int_0^n p(j)^{\frac{-\varepsilon}{1-\varepsilon}}dj + \int_0^{n^*}p(j^*)^{\frac{-\varepsilon}{1-\varepsilon}}dj^*}. \tag{10}$$

As introduced above, each period intermediate firms employ I_R units of labor in process innovation with the aim of reducing production costs. The evolution of the productivity of firm i is then governed by

$$\dot{z}(i) = KI_R(i). \tag{11}$$

Importantly, the productivity of R&D investment depends on knowledge spillovers from production technologies into the innovation process. Adapting the specification of Baldwin and Forslid (2000), we model the knowledge spillovers received by an intermediate firm with innovation located in home as

$$K = \frac{1}{N}\left(\int_0^n z(j)dj + \lambda\int_0^{n*} z^*(j^*)dj^*\right), \tag{12}$$

where $N \equiv n + n^*$ represents the total mass of intermediate varieties. Knowledge spillovers are measured as a weighted average of the productivities of the production technologies observable by the firm, with the degree of international diffusion $\lambda \in (0, 1)$ leading to a greater weighting for knowledge spillovers from production technologies located in proximity to the innovation activity of the firm. This specification is supported by an empirical literature that documents the existence of international knowledge spillovers that diminish with distance (Bottazi and Peri 2003; Thompson 2006; Mancusi 2008).

In order to simplify the analysis, we assume that although the production processes associated with individual production lines are unique, initial productivity levels are symmetric across the intermediate firms operating in both countries. As such, we henceforth drop the firm index i with $z = z^*$, and define the level of knowledge spillovers into innovation activity located in home as $K = zk$, where

$$k = s + \lambda s^*, \tag{13}$$

with $s \equiv n/N$ and $s^* \equiv n^*/N$ denoting the shares of intermediate firms with production located in home and foreign.

The total per-period profit of a firm with production located in home is composed of operating profit on sales (10), less the cost of final goods employed in process innovation (I_R) and the per-period fixed cost (I_F); that is,

$$\Pi_X = \pi_X - I_R - I_F. \tag{14}$$

Firm value (Q) is therefore equal to the present discounted value of future profits:

$$Q = \int_0^\infty e^{-\int_0^t r(t')dt'} \Pi_X(t)dt. \tag{15}$$

The firm selects its level of investment in innovation with the aim of maximizing firm value at each moment in time. This optimization problem is resolved using a current-value Hamiltonian function: $H = \Pi_X + qKI_R$, with q capturing the internal value of the firm's stock of technical knowledge in home. The optimal level of investment in process innovation then satisfies the following static and dynamic efficiency conditions:

$$q = \frac{1}{kz}, \qquad r = \frac{\varepsilon\gamma\pi_X k}{1-\varepsilon} - \frac{\dot{k}}{k} - \frac{\dot{z}}{z}, \tag{16}$$

where the small market shares that arise under monopolistic competition lead firms to ignore the effects of productivity improvements on the intermediate good price indexes and the level of knowledge spillovers (12) when determining their optimal investment levels.

Free market entry and exit drives firm value to zero in the intermediate sector. In particular, the time derivative of firm value (15) yields a no-arbitrage condition ($rQ = \Pi_X + \dot{Q}$) which equates the rate of return on market entry with the risk free rate of return. Firms enter the market when firm value is positive ($Q > 0$), decreasing per-period profit (14) and lowering firm value, but exit the market when firm value is negative ($Q < 0$) increasing per-period profits and raising firm value (Novshek and Sonnenchen 1987). The adjustment is immediate ($\dot{Q} = 0$), with the number of firms jumping to a level that satisfies $Q = 0$. Setting per-period profit equal to zero ($\Pi_X = 0$) then yields a free market entry condition

$$\pi_X = I_R + I_F, \tag{17}$$

which determines the equilibrium scales for production and process innovation.

2.4 Labor Market

The labor market features two types of workers: those looking for jobs and those employed with final good firms. Similarly, there are two types of final good producers: those with job vacancies and those employing workers. Defining the vacancy rate $v \in (0, 1)$ as the number of firms with vacancies relative to the overall labor force, the number of vacant final good firms searching for workers is vL.

Vacant final good firms incur time dependent search costs h while looking for an employee. As a result of this search effort, at each moment in time vacant firms are matched with unemployed workers according to the Poisson arrival rate

$$m \equiv \frac{M(vL, uL)}{vL},$$

where the matching function $M(vL, uL)$ is homothetic and increasing in both of its arguments (Pissarides 2000). The matching rate for vacant firms is therefore written as $m(\theta) \equiv M(1, 1/\theta)$, with $\theta \equiv v/u$ describing labor market tightness. We assume that $m(\theta)$ is continuously differentiable in $(0, \infty)$ with $m'(\theta) < 0$.

Denoting the value of a producing final good firm by J, and a vacant firm by V, the no-arbitrage conditions associated with each firm type are

$$rJ = \Pi_Y - w - \delta(J - V) + \dot{J}, \tag{18}$$

$$rV = -(1 - \triangle)h + m(\theta)(J - V) + \dot{V}, \tag{19}$$

where $\delta \in (0, 1)$ is the exogenous probability of an adverse shock that leads to job separation. The government supports the job matching process through the provision of a partial subsidy $\triangle \in (0, 1)$ to the search costs of vacant firms.

Similarly, denoting the present values of the income streams associated with employment and unemployment by E and U, the no-arbitrage conditions for each worker type are

$$rE = w - \delta(E - U) + \dot{E}, \tag{20}$$

$$rU = b + \theta m(\theta)(E - U) + \dot{U}, \tag{21}$$

where $\theta m(\theta)$ captures the matching rate of unemployed workers looking for jobs, and is increasing in market tightness: $\partial[\theta m(\theta)]/\partial\theta > 0$, as $M(vL, uL)/(uL) = M(\theta, 1) = \theta m(\theta)$.

Given the operating profit of final good firms and labor market tightness, wages are continuously renegotiated between the firm and its employee through Nash bargaining (Pissarides 2000). More specifically, wages are set to maximize the Nash product $(E - U)^{\beta}(J - V)^{1-\beta}$, where $\beta \in (0, 1)$ is the employee's bargaining power, and $J(w)$ and $E(w)$ are determined through the no-arbitrage conditions (18) and (20). The first order condition yields $(1 - \beta)(E - U) = \beta(J - V)$, which regulates the balance between the net values of employment and operating a final good firm.

Vacant firms incur the net search cost $(1 - \triangle)h$ at each moment in time, with an average time frame of $1/m(\theta)$ required to find a worker. But free market entry and exit in the final good sector drives the value of vacant firms to zero ($V = 0$). As a result, from the no-arbitrage conditions (18) and (19), we have

$$J = \frac{(1-\triangle)h}{m(\theta)}, \tag{22}$$

$$(r+\delta)J = \Pi_Y - w + \dot{J}. \tag{23}$$

Thus, the value of a producing firm is reduced to the expected cost of hiring a worker.

Following the equilibrium unemployment literature (Mortensen and Pissarides 1999; Pissarides and Vallanti 2007; Miyamoto and Takahashi 2011; Hashimoto and Im 2019), we assume that search costs are determined proportionately with the output of final good producers: $h = \bar{h}y$, with $\bar{h} \in (0,1)$. In addition, we set unemployment benefits as a percentage of the wage rate: $b = \bar{b}w$ with $\bar{b} \in [0,1)$. Then, using $\Pi_Y = (1-\alpha)y$ with $(1-\beta)(E-U) = \beta(J-V)$, in Appendix A, we derive the equilibrium wage rate for the home country as

$$w = \frac{\beta\left(1-\alpha+(1-\triangle)\bar{h}\theta\right)y}{1-(1-\beta)\bar{b}}. \tag{24}$$

Thus, the wage rate depends on labor market tightness and the output of final good firms (y).

Given the matching and separation processes discussed above, the evolution of the unemployment rate follows: $\dot{u} = \delta(1-u) - \theta m(\theta)u$, with the first and second terms respectively describing the flows into and out of unemployment. Setting this differential equation equal to zero yields the steady-state employment rate as

$$1-u = \frac{\theta m(\theta)}{\delta+\theta m(\theta)}. \tag{25}$$

Long-run employment is therefore directly linked with labor market tightness through the Beveridge curve (Pissarides 2000). Importantly, an increase in θ raises the matching rate $\theta m(\theta)$ for workers looking for employment, causing the employment rate to improve.

2.5 Production and Innovation Location Patterns

We model location patterns following the footloose capital literature that builds on Martin and Rogers (1995). In particular, the freedom of intermediate firms to shift production and innovation independently between countries intrinsically links national shares of final good production, intermediate production, and innovation activity.

Looking first at the pattern of final good production, we take the ratio of the production functions (4) for firms in home and foreign, and substitute in the demands for intermediate goods (6) and (7) to obtain

$$\frac{y}{y^*} = \left(\frac{s + \varphi s^*}{\varphi s + s^*}\right)^{\frac{\alpha(1-\varepsilon)}{\varepsilon(1-\alpha)}}. \tag{26}$$

Next, the free movement of intermediate production leads to the equalization of operating profit across home and foreign; that is, $\pi_X = \pi_X^*$, where we assume that both countries produce intermediate goods. Using the pricing rule (9) with operating profit (10) and the final good production ratio (26), the home share of intermediate production is found as

$$s = \frac{Y/Y^* - \varphi}{(1-\varphi)(1+Y/Y^*)}, \qquad Y/Y^* = \left(\frac{(1-u)L}{(1-u^*)L^*}\right)^{\frac{\varepsilon(1-\alpha)}{\varepsilon-\alpha}}, \tag{27}$$

where we will find that $\varepsilon > \alpha$ is required for a positive rate of productivity growth under stable market entry (see Section 3.2). Thus, the country with the larger employed labor force attracts the greater share of intermediate production through a standard home market effect (Krugman 1980); a result that is magnified by a fall in trade costs (Baldwin et al. 2003). Substituting the production shares back into operating profit (10) yields $\pi_X = \alpha(1-\varepsilon)(Y + Y^*)/N$ for all firms.

Turning now to the innovation location decision, we show that all intermediate firms locate process innovation in the country with the larger market. With constant national employment levels, intermediate production shares (27) and knowledge spillovers (13) are constant in the steady state ($\dot{k} = 0$). Accordingly, the no-arbitrage condition for investment in process innovation (16) indicates that with symmetric initial productivity levels ($z = z^*$), the location with a higher level of knowledge spillovers (k) attracts all innovation activity, and all firms experience the same rate of productivity growth ($\dot{z}/z = \dot{z}^*/z^*$). Thus, innovation concentrates fully in the country with the larger employed labor force.

2.6 Government

National governments support unemployment benefits (b) and subsidies ($\triangle$) to search costs (h) through a lump-sum tax levied on household income. The balanced government budget in the home country is

$$T = buL + \triangle hvL, \tag{28}$$

where the first term on the righthand side is the total payment of benefits to unemployed workers, and the second term is the total subsidies provided to vacant final good firms.

2.7 Market Clearing Conditions

Given the behavior of households, firms, and governments discussed above, the asset market adjusts perfectly to ensure that at all moments in time we have

$$A + A^* = J(1-u)L + J^*(1-u^*)L^*. \tag{29}$$

The righthand side represents the total supply of asset value, with J denoting the value of an operating final good firm and $(1-u)L$ measuring the number of operating firms. Combining the government budget constraint (28) and the household budget constraint (2) with (22), (25), and (29) yields a market clearing condition for final goods:

$$Y + Y^* = C + C^* + vhL + v^*h^*L^* + N(I_X + I_R + I_F), \tag{30}$$

where we have used the fact that all intermediate firms have the same scales of production and innovation (see Appendix B for the derivation).

3 Long-run Equilibrium

This section establishes a steady-state equilibrium with a balanced growth path, along which output (Y) and consumption (C) grow at the same rate, and the interest rate (r), the unemployment rate (u), and labor market tightness (θ) are constant over time. In order to simplify the analysis, we assume that relative labor endowments and labor market frictions ensure that home always has a larger market ($(1-u)L > (1-u^*)L^*$), with greater shares of intermediate and final good production ($s > 1/2$), and that home therefore hosts all process innovation.

First, to derive the balanced rate of output growth, we combine the intermediate good demands (6) and (7) with final good production (4) and $x(i) = \varphi^{1/\varepsilon}x(i^*)$ to obtain

$$y = (\alpha\varepsilon)^{\frac{\alpha}{1-\alpha}}(s+\varphi s^*)^{\frac{\alpha(1-\varepsilon)}{\varepsilon(1-\alpha)}}N^{\frac{\alpha(1-\varepsilon)}{\varepsilon(1-\alpha)}}z^{\frac{\alpha\gamma}{1-\alpha}}. \tag{31}$$

We then show that all macroeconomic variables (Y, Y^*, C, C^*, N) grow at the same rate:

$$\frac{\dot{Y}}{Y} = \frac{\dot{Y}^*}{Y^*} = \frac{\dot{C}}{C} = \frac{\dot{C}^*}{C^*} = \frac{\dot{y}}{y} = \frac{\dot{y}^*}{y^*} = \frac{\dot{N}}{N} = \frac{\alpha\varepsilon\gamma}{\varepsilon-\alpha}g, \tag{32}$$

where $g \equiv \dot{z}/z$ denotes the productivity growth rate, and we have used $\dot{s} = 0$ (see Appendix C for the derivation).

3.1 National Labor Market Equilibrium

The long-run equilibrium associated with the labor market of each country is described by two job creation conditions which implicitly determine labor market tightness for home and foreign. For example, combining $\Pi_Y = (1-\alpha)y$, $h = \bar{h}y$, and $\dot{J}/J = \dot{y}/y = \sigma(r-\rho)$ with

(22), (23), and (24) yields a job creation condition for the home country:

$$\frac{(1-\triangle)\bar{h}\left((1-\sigma)r+\sigma\rho+\delta\right)}{m(\theta)}=\frac{(1-\alpha)(1-\beta)(1-\bar{b})-\beta(1-\triangle)\bar{h}\theta}{1-(1-\beta)\bar{b}}, \tag{33}$$

with an analogous expression for job creation in the foreign country. The lefthand side of (33) captures the expected capitalized cost of hiring a worker, while the righthand side describes the net profit derived from employing the worker (Pissarides 2000).

For a given level of the interest rate, domestic labor market policy determines labor market tightness. From (33), it is then immediate that:

Lemma 1 *A decrease in search costs ($\bar{h}$), unemployment benefits ($\bar{b}$), labor market power (β), or the separation rate (δ), or an increase in the search subsidy ($\triangle$), causes a rise in labor market tightness (θ).*

Within each country, the job creation condition (33) regulates the influence of domestic labor market policy on labor market tightness. First, returning to (19), an increase in the search subsidy ($\triangle$), or alternatively a decrease in the search cost ($\bar{h}$), lowers the cost of finding a new worker, temporarily increasing the value of a vacant firm (V). As a result, firms enter the final good sector, causing labor market tightness to rise until vacant firm value has returned to zero through a fall in the value of an operating firm (J), as shown in (22). Second, a decrease in employee negotiating power through either a fall in unemployment benefits ($\bar{b}$), or a direct decrease in labor market power (β), leads to lower wages that reduce the expected cost of hiring a worker. As such, labor market tightness rises.[2] Finally, a rise in the job separation rate lowers the values of both operating firms and employed workers, causing the number of vacancies and employed workers to decrease and labor market tightness to fall.

[2] See Peretto (2012) and Chu et al. (2016) for theoretical studies of the relationship between union power and economic growth.

3.2 Long-run Productivity Growth

The domestic labor markets of home and foreign are linked through the interest rate, which is determined in the international financial market and is influenced by industry concentration through the level of knowledge spillovers. Following Smulders and van de Klundert (1995), we describe the international financial market equilibrium using supply and demand conditions for investment funds. First, equating (3) and (32), the household supply of investment funds is described by a *preference* condition

$$r = \frac{\alpha\varepsilon\gamma}{\sigma(\varepsilon - \alpha)} g + \rho, \tag{34}$$

within which a positive relationship arises between the interest rate and the rate of productivity growth, as households require a higher return on savings to compensate them for delaying consumption. Second, the demand for investment funds from intermediate firms is captured with a *technology* condition that is obtained by reorganizing the second expression of (16) with (11) and (17):

$$r = \frac{\varepsilon\gamma I_F k}{1-\varepsilon} - \frac{(1-\varepsilon(1+\gamma))g}{1-\varepsilon}. \tag{35}$$

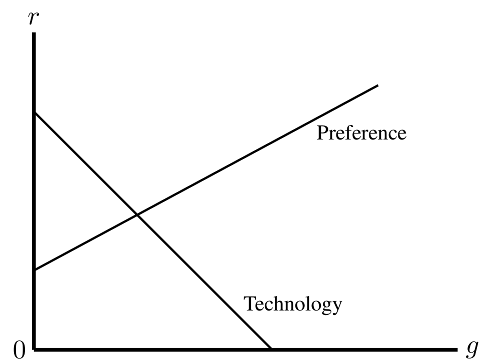
An increase in the interest rate puts downward pressure on the productivity growth rate as optimal investment in process innovation falls for a given level of knowledge spillovers (k).

The preference and technology conditions are illustrated in Figure 1. Matching the demand (34) and supply (35) for investment funds, we derive the equilibrium rate of productivity growth as follows:

$$g = \sigma(\varepsilon - \alpha)(\varepsilon\gamma I_F k - (1-\varepsilon)\rho)\psi^{-1}, \tag{36}$$

where $\psi = \alpha\varepsilon(1-\varepsilon)\gamma + \sigma(\varepsilon - \alpha)(1 - \varepsilon(1+\gamma))$. This expression yields the following lemma outlining the requirements for positive productivity growth under stable market entry.

Figure 1: Equilibrium Productivity Growth



Lemma 2 *Positive productivity growth requires* $k \in ((1-\varepsilon)\rho/(\varepsilon\gamma I_F), 1)$.

In Appendix D, we show that a stable level of market entry into the intermediate sector requires a $\psi > 0$ in (36) to ensure that firm value responds correctly to market entry and exit ($d\Pi_X/dN < 0$) in the spirit of Novshek and Sonnenchein (1987); a condition that is synonymous with a greater slope for the preference condition than the technology condition. Therefore, the expression $\varepsilon\gamma I_F k - (1-\varepsilon)\rho$ dictates the range of knowledge spillovers over which positive productivity growth arises in long-run equilibrium. With the conditions for stable market entry satisfied, we find that an increase in the level of knowledge spillovers raises the rate of productivity growth. For $k \leq (1-\varepsilon)\rho/(\varepsilon\gamma I_F)$, however, there is no productivity growth, as intermediate firms do not find it optimal to invest in process innovation.

In addition, we find that the rate of economic growth is not biased by a scale effect, since proportionate increases in the populations of each country are fully absorbed by an expansion of the level of market entry, and therefore do not influence the innovation rate. To see this, using (16), (17), (27), and (31), with $\pi_X = \alpha(1-\varepsilon)(Y+Y^*)/N$, we obtain the entry level associated with current productivity (z) as follows:

$$N(z) = \frac{(\alpha\varepsilon^{\alpha}\psi^{1-\alpha})^{\frac{\varepsilon}{\varepsilon-\alpha}}(1+\varphi)^{\frac{\alpha(1-\varepsilon)}{\varepsilon-\alpha}} z^{\frac{\alpha\varepsilon\gamma}{\varepsilon-\alpha}} \left(((1-u)L)^{\frac{\varepsilon(1-\alpha)}{\varepsilon-\alpha}} + ((1-u^*)L^*)^{\frac{\varepsilon(1-\alpha)}{\varepsilon-\alpha}}\right)}{((\sigma(\varepsilon-\alpha)+\alpha\varepsilon\gamma)I_F - \sigma(\varepsilon-\alpha)\rho/k)^{\frac{\varepsilon(1-\alpha)}{\varepsilon-\alpha}}}, \quad (37)$$

where the denominator is positive under the conditions outlined in Lemma 2. Thus the framework is consistent with the empirical literature that supports endogenous market structure and endogenous growth frameworks that are not biased by scale effects (Laincz and Peretto 2006; Ha and Howitt 2007; Madsen et al. 2010).

4 Policy Analysis

We now use the model to evaluate the effects of improvements in economic integration and adjustments in the domestic labor market policies of home and foreign on unemployment rates, industry location patterns, and long-run productivity growth.

4.1 Economic Integration

Beginning with an analysis of the effects of improvements in economic integration, we obtain the following results.

Proposition 1 *An improvement in economic integration, through either lower trade costs (τ) or a higher degree of knowledge diffusion (λ), accelerates the rate of productivity growth (g), but raises the unemployment rates of both countries (u and u^*).*
Proof: See Appendix E.

Improved economic integration affects the location pattern of intermediate production, and subsequently unemployment rates and productivity growth, through two channels: a *knowledge spillover* effect and a *job creation* effect. Addressing first the knowledge spillover effect, from (13), an increase in the degree of knowledge diffusion (λ) directly raises the level of knowledge spillovers (k). Similarly, from (27), an increase in the freeness of trade (φ) strengthens the home market effect, causing the home share of intermediate production (s) and the level of knowledge spillovers (k) to both rise. In either case, the technology condition (35) shifts to the right in Figure 1, with greater knowledge spillovers allowing for a faster rate of productivity growth (36).

Turning next to the job creation effect, the resulting increase in the interest rate (r) induces a fall in labor market tightness and raises the unemployment rates of both home and foreign, as the capitalized cost of hiring workers increases in (33). The job creation effect has an ambiguous influence on the concentration of industry in the home country (s), however, as the fall in employment could be greater for either country. Overall, the knowledge spillover effect dominates, and the home share of intermediate production increases, allowing us to conclude that policies designed to promote economic integration lead to faster economic growth, but at a cost of higher unemployment rates.

4.2 National Labor Market Policy

In this section, we consider how changes in national labor market policy affect unemployment rates and long-run productivity growth. Looking first at the labor market policy of the home country, we obtain the following proposition.

Proposition 2 *In the larger home country, increases in search subsidies ($\triangle$) and decreases in unemployment benefits ($\bar{b}$) both accelerate productivity growth (g), while lowering the unemployment rate of home (u) and raising the unemployment rate of foreign (u^*).*

Proof: See Appendix E.

Referring to the job creation condition (33), and the results of Lemma 1, the direct effect of either an increase in search subsidies, or a decrease in unemployment benefits, is greater labor market tightness and a rise in the number of employed workers in home. The increase in home employment raises the home share of intermediate production, shifting the technology condition to the right in Figure 1, and enabling a faster rate of productivity growth through the knowledge spillover effect. The interest rate also rises, however, indirectly putting downward pressure on employment in both countries through the job creation effect. The overall result is greater industry concentration in home and faster productivity growth, while the unemployment rate rises in foreign but falls in home as the direct effect dominates the indirect job creation effect.

We next derive the following results for the labor market policy of the foreign country.

Proposition 3 *In the smaller foreign country, increases in search subsidies ($\triangle^*$) and decreases in unemployment benefits ($\bar{b}^*$) both dampen productivity growth (g), while reducing the unemployment rates of home and foreign (u and u^*).*
Proof: See Appendix E.

Changes in foreign labor market policy have similar, but opposite, effects to those of the home country. Through the job creation condition (33), the direct effect of an increase in search subsidies, or a decrease in unemployment benefits, is an increase in foreign labor market tightness and a rise in foreign employment that reduces the concentration of intermediate production in the home country. Accordingly, the technology condition shifts leftward in Figure 1 through the knowledge spillover effect, depressing the rate of productivity growth. The interest rate also falls, however, indirectly putting upward pressure on the employment rates of both home and foreign through the job creation effect. Overall, the home share of intermediate production falls and productivity growth slows, with reductions in the unemployment rates of both countries.

4.3 National Welfare

We briefly discuss how economic integration and labor market policy influence national welfare levels. With consumption and employment uncertainty eliminated through the household provision of perfect consumption insurance (Merz 1995; Andolfatto 1996), all household members in a given country have the same level of welfare. For example, lifetime utility (1) is used to derive the steady-state welfare of home at time zero as follows:

$$W_0 = \frac{(\varepsilon-\alpha)C_0^{1-1/\sigma}}{(1-1/\sigma)((\varepsilon-\alpha)\rho-(1-1/\sigma)\alpha\varepsilon\gamma g)} - \frac{1}{(1-1/\sigma)\rho}, \tag{38}$$

with current household consumption determined from the budget flow constraint (2) as $C_0 = w(1-u)L - \triangle hvL + \kappa\Theta$, where $\kappa \in [0,1]$ is the exogenous home share of total asset wealth, and $\Theta \equiv (\Pi_Y - w)(1-u)L + (\Pi_Y^* - w^*)(1-u^*)L^* - (1-\triangle)hvL - (1-\triangle^*)h^*v^*L^*$ denotes investment income earned on total asset holdings (see Appendix F). We also derive an analogous set of expressions for the national welfare of foreign.

Improvements in economic integration and adjustments in labor market policy affect welfare through a growth effect (g) and a level effect (C_0). Looking first at the growth effect, as we have seen from (36), the rate of productivity growth is positively linked with industry concentration (s) through knowledge spillovers (k). Accordingly, the growth effect causes the welfare of both countries to improve with either an increase in the labor market tightness of home (θ) or a decrease in the labor market tightness of foreign (θ^*).

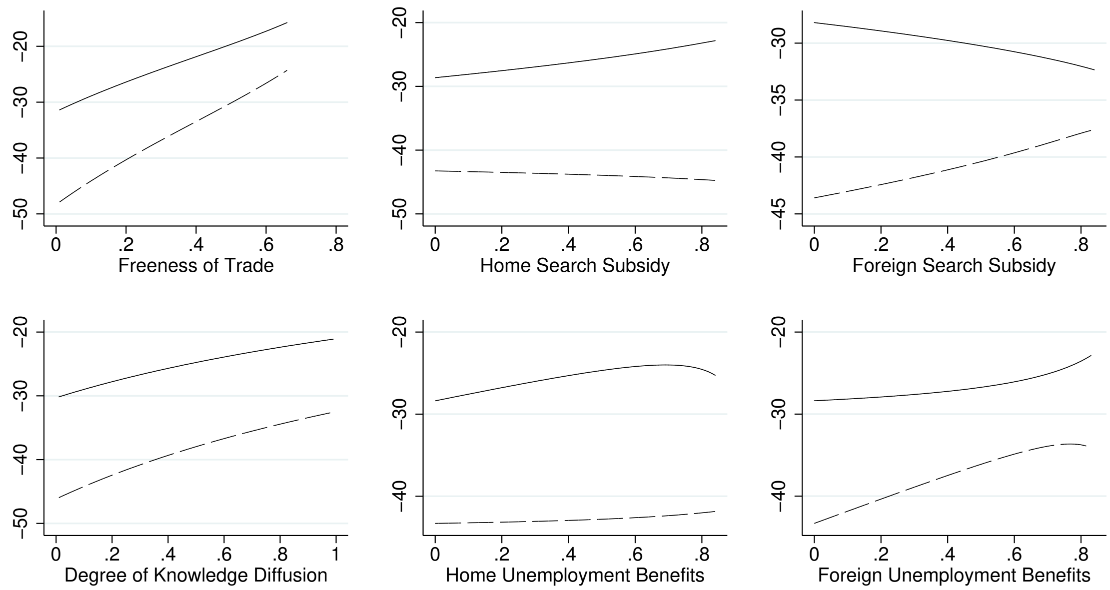
Turning to the level effect, we combine (22), (23), (24), (25), and (33) to rewrite current consumption as follows:

$$C_0 = \left(1 - \alpha - \frac{\delta \triangle \bar{h}}{m(\theta)}\right) Y + (\sigma\rho + (1-\sigma)r)\left(\frac{\kappa(1-\triangle^*)\bar{h}^* Y^*}{m(\theta^*)Y} - \frac{\kappa^*(1-\triangle)\bar{h}}{m(\theta)}\right) Y,$$

where we have normalized current productivity ($z = 1$). The first term captures the contribution of labor to final good output less the cost of subsidizing search efforts in the domestic labor market, while the second term describes the net investment income earned on asset wealth held in the final good firms of each country. From this expression, we find that labor market tightness influences current consumption through four channels: the cost and benefit of search subsidies, the interest rate, relative final good production, and aggregate final good production. The ambiguous and opposing nature of these channels, combined with the growth effect, renders a general analysis of national welfare intractable.

As an alternative, we present several numerical examples, specifying the matching rate as $m(\theta) = 0.15\theta^{-0.6}$ following evidence presented by Petrongolo and Pissarides (2001). For the demand parameters, we set $\rho = 0.05$, $\sigma = 0.5$, and $\kappa = 0.5$, where the intertemporal elasticity of substitution is consistent with estimates by Hall (1988), Guvenen (2006), and Chiappori and Paiella (2011). Technology parameters are set as $\alpha = 0.75$, $\varepsilon = 0.8$, $\gamma =$

Figure 2: National Welfare Effects



The solid line measures home welfare and the dashed line measures foreign welfare. Benchmark parameters are $\alpha = 0.75$, $\beta = 0.5$, $\gamma = 0.5$, $\delta = 0.01$, $\varepsilon = 0.8$, $\kappa = 0.5$, $\lambda = 0.15$, $\rho = 0.05$, $\sigma = 0.5$, $\varphi = 0.12$, $\bar{b} = \bar{b}^* = 0.001$, $\bar{h} = 0.05$, $\bar{h}^* = 0.08$, $\triangle = \triangle^* = 0.05$, $I_F = 0.4$, $L = 1.08$, and $L^* = 1$.

0.5, and $I_F = 0.4$, with the degree of product differentiation matching with values for the elasticity of substitution $1/(1-\varepsilon)$ reported by Bernard et al. (2003) and Broda and Weinstein (2006). For the labor market, we set benchmark parameter values to $\beta = 0.5$, $\delta = 0.01$, $\bar{h} = 0.05$, $\bar{h}^* = 0.08$, $\triangle = \triangle^* = 0.05$, $\bar{b} = \bar{b}^* = 0.001$, $L = 1.08$, and $L^* = 1$, ensuring that home always has a greater share of intermediate production. Following the mid-range of estimates for the degree of knowledge diffusion provided by Bloom et al. (2013), we assume $\lambda = 0.15$. The level of trade costs is set to $\tau = 1.7$ following Anderson and van Wincoop (2004) and Novy (2013), yielding $\varphi = 0.12$ for the freeness of trade. The benchmark parameter set generates unemployment rates of $u = 0.063$ and $u^* = 0.082$, an intermediate production share of $s = 0.62$, and a productivity growth rate of $g = 0.045$.

The numerical examples are presented in Figure 2, where the solid line measures home welfare and the dashed line measures foreign welfare. The ranges of policy parameters measured on the horizontal axes are limited to ensure that intermediate production shares and matching rates lie in the unit interval. First, improved economic integration raises the welfare of both countries, suggesting that the positive growth effects dominate the ambiguous level effects associated with decreases in trade costs and increases in knowledge diffusion. Second, adjustments in labor market policies have symmetric effects for home and foreign, with an increase in search subsidies improving the welfare of the implementing country, while lowering the welfare of the other country. Increases in unemployment benefits raise the welfare of both countries up to a point, after which they become detrimental to the implementing country. These numerical results suggest that the level effects of labor market policy adjustments tend to dominate the growth effects over the assumed parameter set. Importantly, we find that there is potential for national labor market policies to influence the welfare of neighboring countries through the effects of shifts in industry location on long-run growth.

5 Conclusion

We construct a two-country model of industry location and trade and investigate the relationships between national labor market frictions, unemployment rates and fully endogenous productivity growth. With perfect capital mobility, firms are free to shift their production and innovation activities between countries. Trade costs and imperfect knowledge spillovers result in the partial concentration of production and the full concentration of innovation in the country with the larger market size as measured by the employed labor force. Focusing on national labor market policies, we consider how unemployment benefits and subsidies to firm-level search costs affect productivity growth through adjustments in national unemployment rates, shares of industry, and knowledge spillovers from production into innovation. We show that when the larger country offers subsidies to search costs or reduces unemployment benefits in the domestic labor market, the resulting decrease in the unemployment rate causes an increase in the concentration of industry and accelerates productivity growth as the level of knowledge spillovers rises. If similar domestic labor market policies are implemented in the smaller country, productivity growth is dampened as the unemployment rate decreases leading to lower industry concentration and a fall in knowledge spillovers.

Appendix A: Derivation of the Wage Condition

First, taking the difference of (20) and (21), we have

$$(\dot{E} - \dot{U}) = \frac{\beta(r + \delta + \theta m(\theta))J}{1 - \beta} + b - w.$$

Into this expression, we substitute the time derivative of $(1 - \beta)(E - U) = \beta(J - V)$ with (22), (23), and $\dot{V} = 0$. Then, rearranging the result obtains

$$w = (1 - \beta)b + \beta(\Pi_Y + (1 - \triangle)h\theta).$$

Finally, using $b = \bar{b}w$, $h = \bar{h}y$, and $\Pi_Y = (1 - \alpha)y$ yields the wage condition (24).

Appendix B: Market Clearing Condition

Aggregating the household budget constraints (2) of home and foreign gives

$$\dot{A} + \dot{A^*} = r(A + A^*) + (1 - u)wL + (1 - u^*)w^*L^* - \triangle hvL - \triangle^* h^* v^* L^* - C - C^*.$$

Next, time differentiation of (29) yields

$$\dot{A} + \dot{A^*} = \dot{J}(1 - u)L - J\dot{u}L + \dot{J^*}(1 - u^*)L^* - J^*\dot{u^*}L^*.$$

Then, combining the above two expressions with $\dot{u} = \delta(1 - u) - \theta m(\theta)u$, (22), (23), and (29), we have

$$\Pi_Y(1 - u)L + \Pi_Y^*(1 - u^*)L^* = C + C^* + vhL + v^*h^*L^*.$$

Finally, using $\Pi_Y = (1-\alpha)y$, $\Pi_Y^* = (1-\alpha)y^*$, and $pXN = \alpha(Y+Y^*) = N(I_X + I_R + I_F)$, we obtain the market clearing condition for final goods (30).

Appendix C: Balanced Growth Path

First, using $h = \bar{h}y$, $v = u\theta$, $Y = y(1-u)L$, and $pXN = \alpha(Y + Y^*) = N(I_X + I_R + I_F)$ in (30), we have

$$Y\left((1-\alpha)\left(1 + \frac{Y^*}{Y}\right) - \left(\frac{u\bar{h}\theta}{1-u} + \frac{u^*\bar{h}^*\theta^*}{1-u^*}\frac{Y^*}{Y}\right)\right) = C\left(1 + \frac{C^*}{C}\right).$$

In the steady state, the interest rate (r), the unemployment rate (u), and labor market tightness (θ) are constant, ensuring that the relative production scale of final goods (Y/Y^*) is also constant. Also, as $\dot{C}/C = \dot{C}^*/C^* = \sigma(r - \rho)$ from (3), relative consumption (C/C^*) does not change over time. Thus, we find that output and consumption grow at the same rate $\dot{C}/C = \dot{C}^*/C^* = \dot{Y}/Y = \dot{Y}^*/Y^* = \dot{y}/y = \dot{y}^*/y^*$. Then, as intermediate firm production (I_X) and innovation (I_R) scales are constant, given the interest rate (r), $\alpha(1 + Y^*/Y)Y = N(I_X + I_R + I_F)$ yields $\dot{N}/N = \dot{Y}/Y$.

Appendix D: Stable Market Entry

Combining the second expression of (16) with $\dot{k} = 0$, (11), (14), and (34), we solve for the per-period profit of an intermediate firm as

$$\Pi_X = \frac{\psi\pi_X}{(1-\varepsilon)(\alpha\varepsilon\gamma + \sigma(\varepsilon - \alpha))} + \frac{\sigma(\varepsilon - \alpha)\rho}{(\alpha\varepsilon\gamma + \sigma(\varepsilon - \alpha))k} - I_F,$$

where $\pi_X = \alpha(1-\varepsilon)(Y + Y^*)/N$. Thus, given $(Y + Y^*)$, as $d\pi_X/dN = -\pi_X/N < 0$, $\psi = \alpha\varepsilon(1-\varepsilon)\gamma + \sigma(\varepsilon - \alpha)(1 - \varepsilon(1+\gamma)) > 0$ is required to ensure $d\Pi_X/dN < 0$. Then, the numerator of (39) yields the required range for k described in Lemma 1.

Appendix E: Comparative Statics

Combining (33), (34), and (36), we describe the steady-state system implicitly in terms of θ and θ^* using the following allocation curves for home and foreign: $\Gamma_K = \Gamma_J - (\rho+\delta)/(1-\sigma)$

and $\Gamma_K = \Gamma_J^* - (\rho+\delta)/(1-\sigma)$, where

$$\begin{aligned}
\Gamma_K &= \frac{\alpha\varepsilon\gamma\left(\varepsilon\gamma I_F k(\theta,\theta^*) - (1-\varepsilon)\rho\right)}{\alpha\varepsilon(1-\varepsilon)\gamma + \sigma(\varepsilon-\alpha)(1-\varepsilon(1+\gamma)}, \\
\Gamma_J &= \frac{\left((1-\alpha)(1-\beta)(1-\bar{b}) - \beta(1-\triangle)\bar{h}\theta\right)m(\theta)}{(1-\sigma)(1-(1-\beta)\bar{b})(1-\triangle)\bar{h}}, \\
\Gamma_J^* &= \frac{\left((1-\alpha)(1-\beta)(1-\bar{b}^*) - \beta(1-\triangle^*)\bar{h}^*\theta^*\right)m(\theta^*)}{(1-\sigma)(1-(1-\beta)\bar{b}^*)(1-\triangle^*)\bar{h}^*}.
\end{aligned}$$

We take the total derivatives of the allocation curves to obtain:

$$\begin{bmatrix} \frac{\partial\Gamma_K}{\partial\theta} - \frac{\partial\Gamma_J}{\partial\theta} & \frac{\partial\Gamma_K}{\partial\theta^*} \\ \frac{\partial\Gamma_K}{\partial\theta} & \frac{\partial\Gamma_K}{\partial\theta^*} - \frac{\partial\Gamma_J^*}{\partial\theta^*} \end{bmatrix} \begin{bmatrix} d\theta \\ d\theta^* \end{bmatrix} = \begin{bmatrix} -\frac{\partial\Gamma_K}{\partial\varphi} & -\frac{\partial\Gamma_K}{\partial\lambda} & \frac{\partial\Gamma_J}{\partial\triangle} & \frac{\partial\Gamma_J}{\partial\bar{b}} & 0 & 0 \\ -\frac{\partial\Gamma_K}{\partial\varphi} & -\frac{\partial\Gamma_K}{\partial\lambda} & 0 & 0 & \frac{\partial\Gamma_J^*}{\partial\triangle^*} & \frac{\partial\Gamma_J^*}{\partial\bar{b}^*} \end{bmatrix} \begin{bmatrix} d\varphi \\ d\lambda \\ d\triangle \\ d\bar{b} \\ d\triangle^* \\ d\bar{b}^* \end{bmatrix},$$

where

$$\begin{aligned}
\frac{\partial\Gamma_K}{\partial\theta} &= \frac{\alpha(\varepsilon\gamma)^2 I_F}{\psi}\frac{\partial k}{\partial s}\frac{\partial s}{\partial u}\frac{\partial u}{\partial\theta} > 0, \\
\frac{\partial\Gamma_K}{\partial\theta^*} &= \frac{\alpha(\varepsilon\gamma)^2 I_F}{\psi}\frac{\partial k}{\partial s}\frac{\partial s}{\partial u^*}\frac{\partial u^*}{\partial\theta^*} < 0, \\
\frac{\partial\Gamma_J}{\partial\theta} &= \frac{\left((1-\alpha)(1-\beta)(1-\bar{b}) - \beta(1-\triangle)\bar{h}\theta\right)\partial m(\theta)/\partial\theta - \beta(1-\triangle)\bar{h}m(\theta)}{(1-\sigma)(1-(1-\beta)\bar{b})(1-\triangle)\bar{h}} < 0, \\
\frac{\partial\Gamma_J^*}{\partial\theta^*} &= \frac{\left((1-\alpha)(1-\beta)(1-\bar{b}^*) - \beta(1-\triangle^*)\bar{h}^*\theta^*\right)\partial m(\theta^*)/\partial\theta^* - \beta(1-\triangle^*)\bar{h}^*m(\theta)^*}{(1-\sigma)(1-(1-\beta)\bar{b}^*)(1-\triangle^*)\bar{h}^*} < 0.
\end{aligned}$$

While the slope of the home allocation is strictly positive in (θ, θ^*) space, the foreign allocation curve is generally ambiguous. In order to ensure the existence of long-run equilibrium, we focus on cases for which the foreign allocation curve has a negative slope; that is, $\partial\Gamma_K/\partial\theta^* - \partial\Gamma_J/\partial\theta^* > 0$. Then the determinant of the Jacobian matrix given above is strictly positive: $|J| = -(\partial\Gamma_K/\partial\theta)(\partial\Gamma_J^*/\partial\theta^*) - (\partial\Gamma_J/\partial\theta)((\partial\Gamma_K/\partial\theta^*) - (\partial\Gamma_J^*/\partial\theta^*)) > 0$.

Cramer's rule then yields the following comparative static results for each proposition. For Proposition 1 we have

$$\frac{d\theta}{d\varphi} = \frac{\partial \Gamma_K}{\partial \varphi}\frac{\partial \Gamma_J^*}{\partial \theta^*}\frac{1}{|J|} < 0, \qquad \frac{d\theta^*}{d\varphi} = \frac{\partial \Gamma_K}{\partial \varphi}\frac{\partial \Gamma_J}{\partial \theta}\frac{1}{|J|} < 0,$$
$$\frac{d\theta}{d\lambda} = \frac{\partial \Gamma_K}{\partial \lambda}\frac{\partial \Gamma_J^*}{\partial \theta^*}\frac{1}{|J|} < 0, \qquad \frac{d\theta^*}{d\lambda} = \frac{\partial \Gamma_K}{\partial \lambda}\frac{\partial \Gamma_J}{\partial \theta}\frac{1}{|J|} < 0,$$

where

$$\frac{\partial \Gamma_K}{\partial \varphi} = \frac{\alpha(\varepsilon\gamma)^2 I_F}{\psi}\frac{\partial k}{\partial s}\frac{\partial s}{\partial \varphi} > 0, \qquad \frac{\partial \Gamma_K}{\partial \lambda} = \frac{\alpha(\varepsilon\gamma)^2 I_F}{\psi}\frac{\partial k}{\partial \lambda} > 0.$$

Next, the results for Proposition 2 are

$$\frac{d\theta}{d\triangle} = \frac{\partial \Gamma_J}{\partial \triangle}\left(\frac{\partial \Gamma_K}{\partial \theta^*} - \frac{\partial \Gamma_J^*}{\partial \theta^*}\right)\frac{1}{|J|} > 0, \qquad \frac{d\theta^*}{d\triangle} = -\frac{\partial \Gamma_J}{\partial \triangle}\frac{\partial \Gamma_K}{\partial \theta}\frac{1}{|J|} < 0,$$
$$\frac{d\theta}{d\bar{b}} = \frac{\partial \Gamma_J}{\partial \bar{b}}\left(\frac{\partial \Gamma_K}{\partial \theta^*} - \frac{\partial \Gamma_J^*}{\partial \theta^*}\right)\frac{1}{|J|} < 0, \qquad \frac{d\theta^*}{d\bar{b}} = -\frac{\partial \Gamma_J}{\partial \bar{b}}\frac{\partial \Gamma_K}{\partial \theta}\frac{1}{|J|} > 0,$$

where

$$\frac{\partial \Gamma_J}{\partial \triangle} = \frac{(1-\alpha)(1-\beta)(1-\bar{b})m(\theta)}{(1-\sigma)(1-(1-\beta)\bar{b})(1-\triangle)^2\bar{h}} > 0,$$
$$\frac{\partial \Gamma_J}{\partial \bar{b}} = -\frac{(1-\beta)\beta(1-\alpha+(1-\triangle)\bar{h}\theta)m(\theta)}{(1-\sigma)(1-(1-\beta)\bar{b})^2(1-\triangle)\bar{h}} < 0.$$

Finally, the results for Proposition 3 are

$$\frac{d\theta}{d\triangle^*} = -\frac{\partial \Gamma_J^*}{\partial \triangle^*}\frac{\partial \Gamma_K}{\partial \theta^*}\frac{1}{|J|} > 0, \qquad \frac{d\theta^*}{d\triangle^*} = \frac{\partial \Gamma_J^*}{\partial \triangle^*}\left(\frac{\partial \Gamma_K}{\partial \theta} - \frac{\partial \Gamma_J}{\partial \theta}\right)\frac{1}{|J|} > 0,$$
$$\frac{d\theta}{d\bar{b}^*} = -\frac{\partial \Gamma_J^*}{\partial \bar{b}^*}\frac{\partial \Gamma_K}{\partial \theta^*}\frac{1}{|J|} < 0, \qquad \frac{d\theta^*}{d\bar{b}^*} = \frac{\partial \Gamma_J^*}{\partial \bar{b}^*}\left(\frac{\partial \Gamma_K}{\partial \theta} - \frac{\partial \Gamma_J}{\partial \theta}\right)\frac{1}{|J|} < 0,$$

where

$$\frac{\partial \Gamma_J^*}{\partial \triangle^*} = \frac{(1-\alpha)(1-\beta)(1-\bar{b}^*)m(\theta^*)}{(1-\sigma)(1-(1-\beta)\bar{b}^*)(1-\triangle^*)^2\bar{h}^*} > 0,$$
$$\frac{\partial \Gamma_J^*}{\partial \bar{b}^*} = -\frac{(1-\beta)\beta(1-\alpha+(1-\triangle^*)\bar{h}^*\theta)m(\theta^*)}{(1-\sigma)(1-(1-\beta)\bar{b}^*)^2(1-\triangle^*)\bar{h}^*} < 0.$$

Appendix F: Initial Consumption

First note that the initial asset position of the home household is

$$A_0 = \kappa\left(J(1-u)L + J^*(1-u^*)L^*\right).$$

The time derivative of this expression gives

$$\dot{A} = \kappa\left(\dot{J}(1-u)L - J\dot{u}L + \dot{J}^*(1-u^*)L^* - J^*\dot{u^*}L^*\right).$$

Then, using $v = u\theta$ with the above two equations and (22), (23), and (25), we obtain

$$rA - \dot{A} = \kappa\left((\pi_Y - w)(1-u)L + (\pi_Y^* - w^*)(1-u^*)L^* - (1-\triangle)hvL - (1-\triangle^*)h^*v^*L^*\right).$$

Substituting this expression into (2) yields $C_0 = w(1-u)L - \triangle hvL + \kappa\Theta$.

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