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COMPETITION BETWEEN OFFLINE AND ONLINE RETAILERS WITH HETEROGENEOUS CUSTOMERS

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Competition between offline and online retailers with heterogeneous customers♠

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Abstract

We consider the spatial competition between two traditional physical (or offline) retailers and an Internet (or online) retailer where the efficiency of the latter differs from that of the former. We assume consumers are heterogeneous across two dimensions: (i) the costs of traveling to either of the offline retailers and (ii) the costs of purchasing from the online retailer. Both dimensions depend on the spatial location of consumers and are independent of each other. We show that the online retailer maximizes its profit at an intermediate level of the consumer disutility of online purchase when its efficiency is low.

JEL codes: L13, D43

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1. Introduction

While online retailers have strongly developed their presence in retail markets, offline retailers continue to play an important role (Choi et al., 2012) for the following three reasons. First, some customers lack the ability to use the Internet, such as not knowing how to find the most suitable website for their needs. Second, purchasing from online retailers incurs several costs, including those relating to the waiting time for delivery or the limited information available on products. Lastly, some customers may prefer offline retailers because they prefer the feeling of shopping in physical places or they wish to share time shopping with family and friends.

The quality of service is a crucial element in the marketing mix of online retailers. Therefore, some e-retailers will attempt to improve the quality of their services, such as the quality of their website and the options available for consumers when browsing the website.[1] It seems intuitive that online retailers will want to improve the quality of their online services as much as possible, thus minimizing the disutility of consumers when making online purchases.

However, we also observe that at least in some cases, online retailers will not minimize the disutility costs of consumers purchasing from them, even if doing so would be without apparent excessive costs. For example, Dell's online retailers use highly technical language when describing product characteristics, easily understood by expert users, but not computer novices. Similarly, ArredoDesign Online, an online retailer specializing in home furnishings and interior design, offers a choice of different product colors. However, it depicts only one of the colors available for each product on its website.

A natural question is then why do some online retailers not appear to attempt to minimize consumer disutility costs, even when doing so does not apparently entail significant costs, as shown by the previous examples. To answer this question, we

[1] For example, Amazon.com allows customers to read parts of books; Yoox.com, an online clothing store, creates customer-specific e-shops based on personal purchase histories; and Poppin.com, an online office supplies and furniture store, has a section on its website that allows customers to personalize desks fully by assembling different items.

examine how an improvement in the usefulness of online retailers affects retailer profitability. We consider an oligopoly competition model in which two offline retailers compete with an online retailer that has some cost/quality (dis)advantage. In our model, consumers are heterogeneous across two dimensions: (i) the costs of traveling to either of the offline retailers as in standard Hotelling models and (ii) the costs of purchasing from the online retailer. The latter contrasts with the typical assumption that consumers are homogeneous in their disutility costs of using the Internet (see Balasubramanian, 1998), and reflects that consumers are heterogeneous in their Internet skills, in finding information on the web, etc.

We show that depending on the competitive position of the online retailer over the offline retailers, the consumer-specific disutility of online purchases provides quite different effects on the online and offline retailers. In particular, the online retailer's profit changes nonmonotonically with an increase in the degree of consumer disutility for online purchases if the competitive position of the online retailer is sufficiently weak. This implies that an increase in the consumer disutility of online purchase can benefit the online retailer if its competitive position is sufficiently weak, but always harms the online retailer if its competitive position is strong. In addition, this nonmonotonic relationship emerges only if the offline retailers compete directly with each other, that is, the online retailer serves only consumers with a high degree of familiarity with the Internet.

Several existing studies consider the competition between online and offline retailers, of which Balasubramanian (1998) was the first to introduce competition between online retailers and offline retailers in a circular setup (see also Bouckaert, 2000). Subsequently, Nakayama (2009) analyzes the situation where some consumers are constrained to purchase only from offline retailers in a linear city model,[2] while Guo and Lai (2014, 2017) consider the spatial positioning of offline retailers when facing competition with an online retailer. All of these studies, however, assume that consumers are homogeneous with respect to the disutility of online purchases.

[2] E-commerce also appears in the context of supply chain competition (see, for example, Chiang et al., 2003, Cattani et al., 2006, and Chen et al., 2013).

The remainder of the paper proceeds as follows. Section 2 introduces the model. Section 3 derives the equilibrium prices and profits. Section 4 discusses the effect of changes in the model parameters on equilibrium profits. Section 5 concludes.

2. Model

Three firms compete in price, two of which (Firms 1 and 2) are offline retailers and the third firm (Firm E) is an online retailer.[3] All firms produce at constant marginal costs, which we normalize to zero without loss of generality, with zero fixed costs.

Consumers, who we assume purchase one unit at most, are heterogeneous across two dimensions: (i) the costs of traveling to either of the offline retailers and (ii) the costs of purchasing from the online retailer. Therefore, a vector (x, y) on a two-dimensional square with x- and y-axes characterizes each consumer in our analysis.[4]

The first heterogeneity of consumers relates to the disutility from traveling to the offline retailers, represented by a horizontal line segment (x-axis). We indicate the locations of consumers in a spatial sense along the horizontal dimension, as well as in the location of the offline retailers. We assume that Firm 1 is located at point 0, whereas Firm 2 is located at point 1. The location of a consumer along the horizontal axis is indicated by $x \in [0,1]$.

Irrespective of y, which we explain later, the utility of a consumer located at x when purchasing from offline retailers 1 and 2 is respectively:

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[3] As is common in the online vs offline competition literature (see, for example, Balasubramanian, 1998, Bouckaert, 2000, and Nakayama, 2009), we consider only a single online retailer. In the case of many online retailers, their location irrelevancy would then determine price undercutting and zero profits unless we introduced an additional source of differentiation.

[4] Several other analyses also consider two-dimensional Hotelling models (Economides, 1986, Tabuchi, 1994, Veendorp and Majeed, 1995, Ansari et al., 1998, Irmen and Thisse, 1998). However, to our knowledge, none of these assumes the existence of a third (different) retailer and consumers holding heterogeneous preferences with respect to both of the two identical retailers (the first dimension of heterogeneity) or the different retailer (the second dimension of heterogeneity).

$$u_1 = v - p_1 - tx, \tag{1}$$

$$u_2 = v - p_2 - t(1-x), \tag{2}$$

where p_i is the price set by Firm $i = 1, 2$, $t(>0)$ represents the unit travel cost of moving toward an offline retailer, and v is the reservation price, which we assume is sufficiently high to ensure that each consumer purchases a positive quantity in equilibrium so that the market is covered.

The second heterogeneity of consumers relates to the disutility of purchasing from the online retailer (Firm E), as depicted by a vertical line segment (y-axis). For notational reasons, along the vertical dimension, we identify the "locations" of consumers along with the "location" of the online retailer in a spatial sense, even though this does not matter in e-commerce. We indicate the location of a consumer along the vertical axis with $y \in [0,1]$. Because Firm E is "located" at $y = 0$, it is available for each point on the horizontal axis.

In addition to the two-dimensional heterogeneity of consumers (x, y), we incorporate the disadvantages of the online retailer over the offline retailers as in Balasubramanian (1998). We can formulate this disadvantage in two ways: (i) the inherent quality disadvantage of the online retailer over the offline retailers and (ii) the marginal cost disadvantage of the online retailer. In this study, we employ the first formulation and parameterize the disadvantage using the exogenous parameter s.

We describe the utility of a consumer located at y and purchasing from Firm E with the following equation:

$$u_E = v - s - p_E - \tau y, \tag{3}$$

where p_E is the price set by Firm E, and $\tau(>0)$ is the unit "travel" cost of moving toward Firm E. τ can be modified by the online retailer, thus influencing the consumer-specific disutility cost.[5] For example, consider a consumer whose location in

[5] Discussion of the endogenous determination of disutility cost is available in the context of spatial competition (e.g., Hendel and Neiva de Figueiredo, 1997; Matsumura and Matsushima, 2007).

the vertical dimension is one (that is, $y = 1$). This means that the consumer may have low Internet skills so that purchasing online has a relevant innate cost. However, by reducing τ (for example, by providing user-friendly tutorials), the online retailer may significantly reduce the consumer-specific disutility costs of that consumer.

Note that we do not exclude the possibility that s is negative, that is, the online retailer has an advantage over the offline retailers.[6] Note also that in formulation (ii), we remove s from equation (3), but the marginal cost of the online retailer differs from that of each offline retailers by an amount equal to s. Because of the mathematical property of the spatial competition model, we can show that the equilibrium properties under the two formulations are qualitatively identical (see the Technical Appendix).

We assume the uniform distribution of consumers on the two-dimensional square with a density of one.[7] The objectives of the retailers are given by $\pi_i = p_i q_i(p_1, p_2, p_E)$ with $i = 1, 2, E$, where $q_i(p_1, p_2, p_E)$ is the demand function for firm i, which is determined by equations (1), (2), and (3). Section 3 provides further details of the function. Given the market structure and the exogenous parameters, the firms simultaneously set their prices. We discuss the optimal level of τ for the online retailer using comparative statics in Section 4.

3. Equilibrium outcome

[6] For example, the offline retailers and Firm E may provide goods with different qualities: when the good of each offline store is better (worse) than that of the online store, s is positive (negative) because *all* consumers obtain higher (lower) utility from purchasing from the offline retailers. Jiang and Balasubramanian (2014) empirically show that for some experiential products, consumers are likely to rely on a physical examination, which is possible only in offline retailers. In this case, s takes a positive value. Of course, the opposite can also hold. Consider a consumer who purchases a car with a certain paint job and other options. In an online store, consumers can visualize all possible color combinations, wheel types, and door appearances on their computer screens, which is impossible to do in offline retailers. In such a case, s is negative.

[7] Note that we can modify the consumer distribution assumption by changing the range of x or y from $[0,1]$ to $[0, k]$ without modifying the results qualitatively.

Before we provide the equilibrium outcome of this game, we derive the demand quantities for the three firms. First, by equating u_1 and u_2, and then solving with respect to x, we identify a consumer who is indifferent between the two offline retailers; that is:

$$\hat{x}(p_1, p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2t}. \tag{4}$$

Here, consumers located at $x \in [0, \hat{x}(.)]$ prefer Firm 1 to Firm 2, whereas consumers located at $x \in [\hat{x}(.), 1]$ prefer Firm 2 to Firm 1.

Second, equating u_i, with $i = 1, 2$, and u_E, and then solving with respect to y yields a consumer who is indifferent between Firm i and Firm E, that is:

$$\hat{y}_1(p_1, p_E, x) = \frac{p_1 - p_E - s + tx}{\tau}, \tag{5}$$

$$\hat{y}_2(p_2, p_E, x) = \frac{p_2 - p_E - s + t(1-x)}{\tau}. \tag{6}$$

Here, consumers located at $y \in [0, \hat{y}_i(.)]$ prefer Firm E to Firm i, with $i = 1, 2$, whereas consumers located at $y \in [\hat{y}_i(.), 1]$ prefer Firm i to Firm E.

Given the two-dimensional nature of our model, a pair (x, y) defines each consumer and identifies the level of demand for each firm by intersecting the indifferent consumers (4), (5), and (6). The full derivation of the levels of demand is in the Technical Appendix, where we provide the four relevant cases.[8] Figure 1 illustrates the demand levels in each case.

Remark 1. *Using the demand system, we explicitly derive the equilibrium prices for the four cases. Lemmas 1–4 in the Technical Appendix summarize the outcomes.*

[8] In an actual case, by focusing only on situations where all firms sell a positive quantity, eight cases are in principle possible. However, as the physical retailers are symmetric, we can focus only on the symmetric situations, thus reducing the relevant cases to four. See the derivation of the demand functions in the Technical Appendix for details.

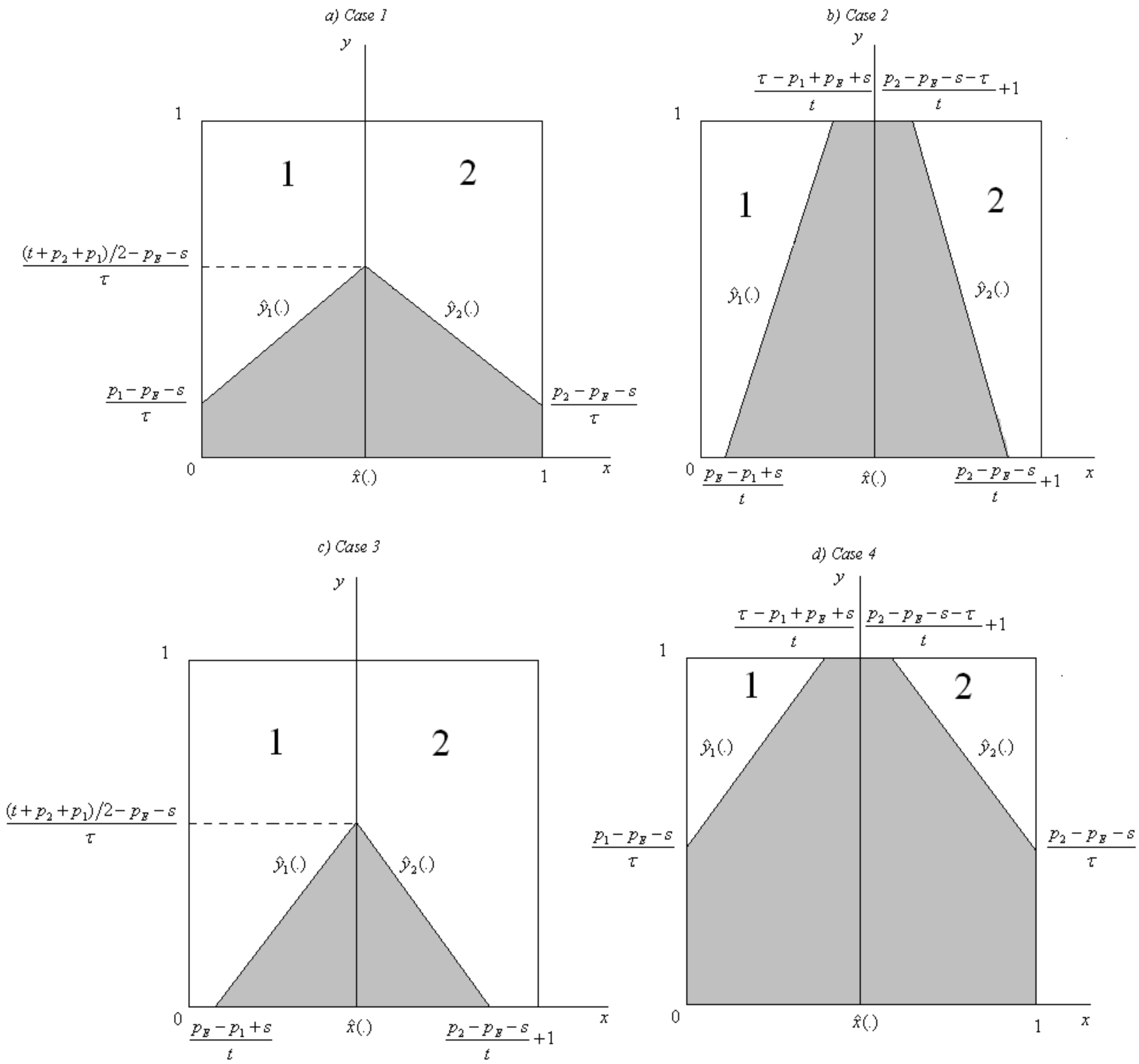


Figure 1: Demand for the three firms

In what follows, we explain the conditions under which each case emerges. In each image in Figure 1, the amount of demand for Firm 1 is represented by Area 1, that for Firm 2 by Area 2, and that for Firm *E* by the gray-shaded area. In Cases 1 and 3, each offline retailer competes with both the other offline retailer and the online retailer, whereas in cases 2 and 4, each offline retailer competes only with the online retailer.

Depending on whether (i) τ/t is high or low and (ii) *s* is high or low, one of the four cases in Figure 1 emerges. First, if τ/t is high (i.e., consumers are highly heterogeneous in the vertical dimension), Firm *E* is less attractive for consumers with a higher *y*, and then these consumers prefer either Firm 1 or 2 to Firm *E*, leading to the direct competition between Firms 1 and 2 for these consumers. That is, if τ / t is high, Cases 1 and 3 are more likely to emerge.

Second, if τ / t is low (i.e., consumers are highly heterogeneous in the horizontal dimension), Firm E is more attractive, even for consumers with a higher y, and then consumers far from the offline retailers prefer Firm E to either Firm 1 or 2, leading to no direct competition between Firms 1 and 2. That is, if τ / t is low, cases 2 and 4 are more likely to emerge.

In the previous comparisons between the two cases on τ / t, s also influences which of the two cases is more likely to emerge: Cases 1 or 3 when τ / t is high, and Cases 2 or 4 when τ / t is low. If s is high, Firm E is less competitive, and then the demand for Firm E is small. Thus, if s is high, Case 3 is more likely to emerge when τ / t is high, and Case 2 is more likely to emerge when τ / t is low. In contrast, if s is low, the demand for Firm E is large. Thus, if s is low, Case 1 is more likely to emerge when τ / t is high, and Case 4 is more likely to emerge when τ / t is low. Table 1 provides a summary.

Table 1: Relation between exogenous parameters and the four cases

	τ / t is low	τ / t is high
s is high	Case 2	Case 3
s is low or negative	Case 4	Case 1

Figure 2 illustrates the conditions under which each case emerges in equilibrium,[9] and Remark 2 summarizes:

[9] "Not available" indicates the case in which Firm E is inactive.

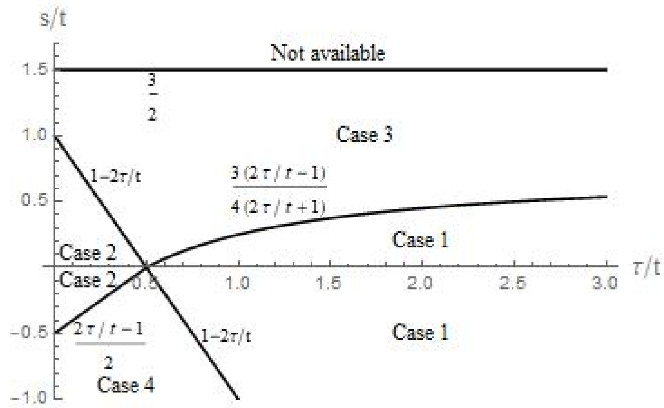


Figure 2: Parameter ranges for the four cases

Remark 2. *Case 1 is more likely to emerge in equilibrium when s is low and τ / t is high; Case 2 is more likely to emerge in equilibrium when s is high and τ / t is low; Case 3 is more likely to emerge in equilibrium when s is high and τ / t is high; and Case 4 is more likely to emerge in equilibrium when s is low and τ / t is low. Case 3 emerges only if s is positive, whereas Case 4 emerges only if s is negative.*

4. Comparative statics

In this section, we first perform comparative statics to evaluate the effect of the model parameters on the equilibrium profits of the offline retailers and the online retailer. Table 2 illustrates the effect of τ, s, and t on the equilibrium profits. In addition, we also depict the relationship between the consumer and social surpluses and the exogenous parameters.

4.1 Firm profits

To start, we consider the equilibrium profits of offline retailers. The effect of τ and s is obvious. A higher τ and a higher s make it more expensive for any consumer to purchase from Firm *E*: the online retailer is less competitive and the offline retailers earn higher profits.

Table 2: Effect of τ , s, and t on equilibrium profits

a) Case 1

Case 1	$\pi_1^1 * = \pi_2^1 *$	$\pi_E^1 *$
$\uparrow \tau$	positive	inverse U-shape
$\uparrow s$	positive	negative
$\uparrow t$	inverse U-shape	positive

b) Case 2

Case 2	$\pi_1^2 * = \pi_2^2 *$	$\pi_E^2 *$
$\uparrow \tau$	positive	negative
$\uparrow s$	positive	negative
$\uparrow t$	U-shape	positive

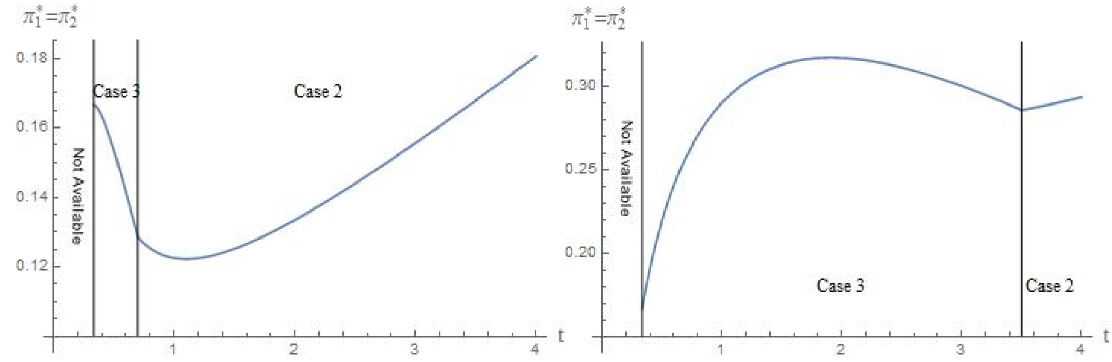
c) Case 3

Case 3	$\pi_1^3 * = \pi_2^3 *$	$\pi_E^3 *$
$\uparrow \tau$	positive	inverse U-shape
$\uparrow s$	positive	negative
$\uparrow t$	inverse U-shape	positive

d) Case 4

Case 4	$\pi_1^4 * = \pi_2^4 *$	$\pi_E^4 *$
$\uparrow \tau$	positive	negative
$\uparrow s$	positive	negative
$\uparrow t$	positive	positive

The effect of t is less obvious. When τ is sufficiently low, there is no direct competition between the offline retailers, as in Balasubramanian (1998). Indeed, the effect of t on offline retailer profits is similar to that in Balasubramanian (1998) (see the left-hand side image in Figure 3), a special version of our model for the case of $\tau = 0$.



$(\tau = 1/10, s = 1/2)$ $(\tau = 3/2, s = 1/2)$

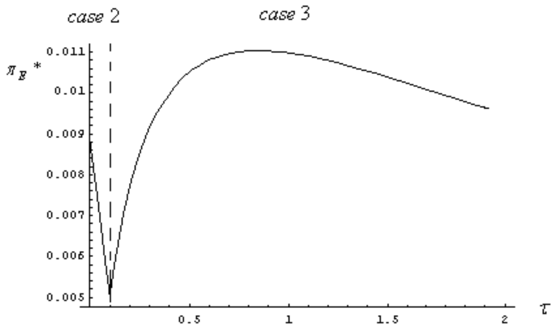
Figure 3: Profits of offline retailers

By contrast, when τ is large, there is direct competition between the offline retailers. The offline retailer profits change nonmonotonically with t and exhibit inverse U-shape relations (see the right-hand side image in Figure 3). An increase in t has primarily two contrasting effects: it mitigates competition between the offline retailers and diminishes their competitiveness over Firm *E*.

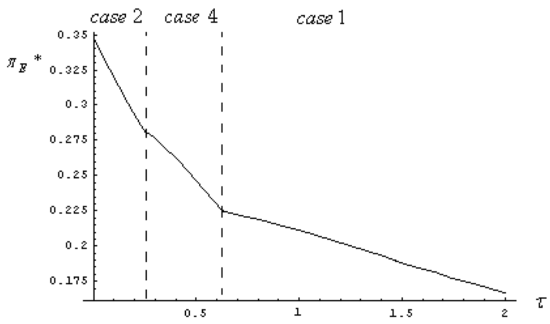
If t is small, the offline retailers dominate the market owing to their attractiveness, implying that direct competition between the offline retailers is more important than that with Firm *E*. Therefore, competition mitigation dominates demand loss through an increase in t. The converse also holds: if t is large, the dominance of the offline retailers is weak, and the demand loss dominates competition mitigation through an increase in t. Consequently, the profit of each offline retailer increases with t for a small t but decreases with t for a large t.

In contrast, if t is very high, direct competition between the offline retailers disappears, leading to Case 2. In this case, the profits of each offline retailer increase with t. This is because a higher t allows Firm *E* to set a higher price, which in turn allows the offline retailers to set higher prices because of the strategic complementarity of prices. This effect is similar to that in Balasubramanian (1998) given the similarity between Case 2 and the case discussed in Balasubramanian (1998). Indeed, in Case 2 we observe that for any y, Firm E directly competes with Firms 1 and 2, as in Balasubramanian (1998). This correspondence leads to similar equilibrium properties.[10]

We now discuss the equilibrium profits of Firm E. Not surprisingly, an increase in t benefits Firm E, whereas an increase in s harms Firm E. Here, we mainly consider the effect of τ on the profits of Firm E. In Figure 4, the curves plot the direct effect of τ on the profits of Firm E. To simplify the discussion, we mainly discuss two cases for the value of s: (*i*) where s is sufficiently large and (*ii*) where s is sufficiently negative.



$(s = 4/5, t = 1)$



$(s = -1/4, t = 1)$

Figure 4: Profit of the online retailer

Suppose s is sufficiently large, as represented by the left-hand side image in Figure 4. If τ is not too low, direct competition between the offline retailers emerges in equilibrium. In this case, an increase in τ involves a trade-off with the profits of Firm E: the mitigation of price competition through customer segmentation between Firm E and each offline retailer (on the positive side) and a shrinkage in demand for Firm E through the direct cost increment (on the negative side).[11] When the competitive power of Firm E is weak (i.e., s is large), the demand shrinkage is not effective because its

[10] The basic properties of the profits of the offline retailers do not change even when s is negative.

[11] There is another positive effect on the profits of Firm E: a price change for Firm E becomes less influential on the competition between Firm E and each offline store because direct competition between the offline retailers becomes more important.

price–cost margin is low. Therefore, up to some threshold, the profits of the online retailer increase and then decrease with τ.[12]

We can better appreciate the competition intensity and market effects using Figure 5, which illustrates the impact of changes in τ on firm demand. When τ is low, we have Case 2 (on the left-hand side). If τ increases, the straight lines rotate inward and are equal to 1 at $\hat{x}$ (in the center). When τ is high, we have Case 3 (on the right-hand side). In this case, if τ increases, the straight lines rotate outward. This implies that when τ increases, Firm *E* ceases serving consumers with a higher y whose locations are near the center of the horizontal segment but begins serving consumers with a lower y whose locations are near the edges of the horizontal segment. That is, Firm *E* leaves the less profitable consumers to the offline retailers and so effectively exits this consumer segment (the market effect). In this way, the competition between Firm *E* and the offline retailers is mitigated (the competition intensity effect), and Firm *E* earns higher profits by capturing those consumers predisposed to Firm *E*.

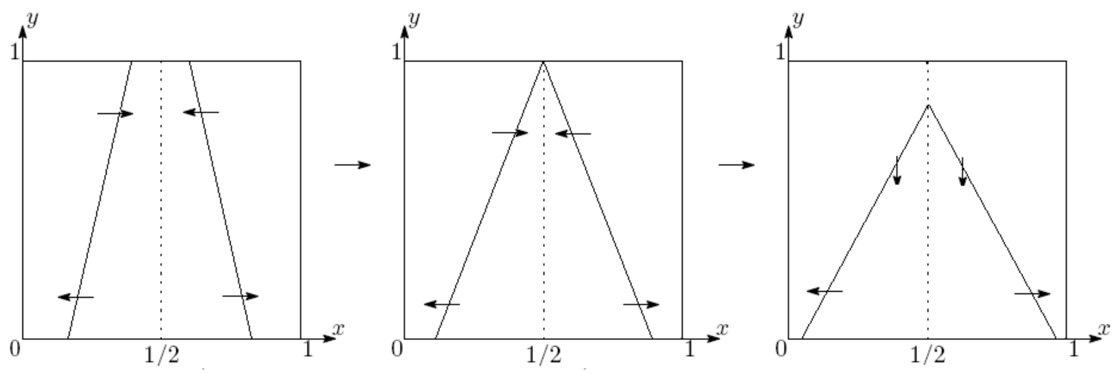


Figure 5: The effect of an increase in τ on demand

We should note that direct competition between offline retailers plays a crucial role in determining this result. Indeed, this condition is omitted from the basic model of Balasubramanian (1998), in which there is no direct competition between offline

[12] In other words, a greater value of τ could serve, to some extent, as a device to soften the competition between Firm *E* and Firms 1 and 2, even if a larger τ implies worse service and thereby lower demand for the online retailer.

retailers and only competition between each offline retailer and a direct marketer, as in Cases 2 and 4 of our model.[13]

Suppose now that s is sufficiently negative, as represented by the right-hand side image in Figure 4. Unless τ is very high, direct competition between the offline retailers now does not emerge. That is, Firm *E* retains a large market share and continues to be the direct competitor for each offline retailer. Therefore, demand size shrinkage through an increase in τ has a large negative effect on Firm *E* because the firm's strong competitive power leads to a higher price–cost margin. It follows that the profits of the online retailer decrease with τ.[14,15]

The following propositions summarize the above discussion. Proposition 1 states that if the competitive position of Firm *E* is sufficiently weak, the profits of the online retailer are maximized at an intermediate level of τ, whereas Proposition 2 is that if the competitive position of Firm *E* is sufficiently strong, the profits of the online retailer strictly decrease with τ.[16]

Proposition 1. *Firm E's profits are globally maximized at* $\tau^* = \dfrac{5s + \sqrt{-11s^2 + 36st - 9t^2}}{9}$ *if and only if* $s \geq 0.754949t$.

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[13] Considering the optimal amount of advertising, Balasubramanian (1998) also shows that in some cases fully informed advertising by the direct marketer is not optimal, which implies that some consumers do not know of the existence of the direct marketer. For this consumer segment, side-by-side physical retailers directly compete. However, in Balasubramanian (1998) the emergence of direct competition between side-by-side physical retailers depends only on the first-stage choice by the direct marketer. Therefore, direct competition emerges if, and only if, the direct marketer does not fully inform its product to all consumers. In addition, partial advertising seems difficult for online retailers, whereas it is possible for a direct marketer using catalog marketing.

[14] We note that if s is very low and τ is very high, direct competition emerges, but because τ is high, a further increase of τ is detrimental for profits because the shrinkage of demand effect dominates. Therefore, the profits of Firm *E* are strictly decreasing with τ.

[15] Relating to the vertical dimension, if there are only two types of consumers with (i) zero disutility and (ii) disutility level $\tau(>0)$, an increase in τ diminishes the profits of Firm *E* (see the Technical Appendix).

[16] We can also extend the results in Propositions 1 and 2 to the case of Stackelberg competition. Details available upon request.

Proof. See the Technical Appendix. ■

Proposition 2. *Firm E's profits decrease monotonically with* τ *if* $s < -0.1t$.

Proof. See the Technical Appendix. ■

We compare the equilibrium prices of offline retailers with the equilibrium prices of the online retailer in the various cases. We find that in Cases 1 and 3 (Case 4), the equilibrium price of the offline retailers is higher (lower) than the equilibrium price of the online retailer. We also find that in Case 2 the equilibrium price of the offline retailer is higher (lower) if τ is sufficiently high (low).

The ambiguous relation between prices could capture the conventional observation that online prices may be higher or lower than offline prices (Smith et al., 2000). Using the comparison in Cases 1 and 3, we connect the realized competition mode (Cases 1 and 3) to the price difference between the online and offline retailers. Finally, we note that variation in *s* involves unambiguous effects on the market shares of firms: in all cases, when *s* increases, the demand for the offline retailers (online retailer) increases (decreases).

4.2 Consumer surplus and welfare

In what follows, we briefly discuss the effect of τ on consumer surplus. Given symmetry, the equilibrium consumer surplus is given by the following functions (the superscript in CS^i indicates Case *i* = *1, 2, 3,* and *4* in Figure 1):[17]

$$CS^1{*} = 2[\int_0^{x\wedge *}\int_{\hat{y}_1 *}^1 (v - p_1^1 * - tx)dydx + \int_0^{x\wedge *}\int_0^{\hat{y}_1 *} (v - p_E^1 * - s)dydx] \tag{7}$$

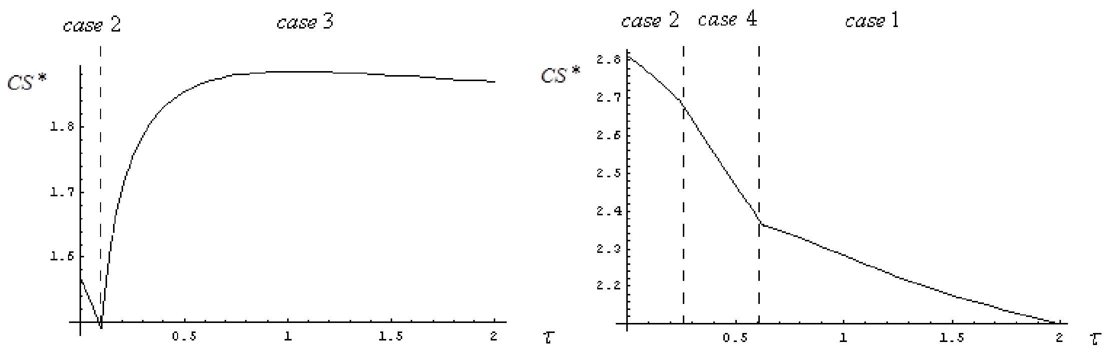
$$\begin{aligned} CS^2{*} = \quad & 2[\int_0^{\frac{p_E^2 * - p_1^2 * + s}{t}}\int_0^1 (v - p_1^1 * - tx)dydx + \int_{\frac{p_E^2 * - p_1^2 * + s}{t}}^{\frac{\tau + p_E^2 * - p_1^2 * + s}{t}}\int_{\hat{y}_1 *}^1 (v - p_1^1 * - tx)dydx + \\ & + \int_{\frac{p_E^2 * - p_1^2 * + s}{t}}^{\frac{\tau + p_E^2 * - p_1^2 * + s}{t}}\int_0^{\hat{y}_1 *} (v - p_E^2 * - s)dydx + \int_{\frac{\tau + p_E^2 * - p_1^2 * + s}{t}}^{\hat{x}*}\int_0^1 (v - p_E^2 * - s)dydx] \end{aligned} \tag{8}$$

[17] We omit the explicit expressions.

$$CS^3* = 2\Big[\int_0^{\frac{p_E^3*-p_1^3*+s}{t}} \int_0^1 (v - p_1^3 * - tx)dydx + \\ + \int_{\frac{p_E^3*-p_1^3*+s}{t}}^{\hat{x}*} \int_{\hat{y}_1*}^1 (v - p_1^3 * - tx)dydx \quad + \int_{\frac{p_E^3*-p_1^3*+s}{t}}^{\hat{x}*} \int_0^{\hat{y}_1*} (v - p_E^3 * - s)dydx\Big] \tag{9}$$

$$CS^4* = 2\Big[\int_0^{\frac{\tau+p_E^4*-p_1^4*+s}{t}} \int_{\hat{y}_1*}^1 (v - p_1^4 * - tx)dydx + \\ + \int_0^{\frac{\tau+p_E^4*-p_1^4*+s}{t}} \int_0^{\hat{y}_1*} (v - p_E^4 * - s)dydx \quad + \int_{\frac{\tau+p_E^4*-p_1^4*+s}{t}}^{\hat{x}*} \int_0^1 (v - p_E^4 * - s)dydx\Big] \tag{10}$$

We consider, by means of numerical simulation, the effect of τ in the two cases discussed in Figure 4, i.e., (*i*) when s is sufficiently large and (*ii*) when s is sufficiently negative.[18] We obtain Figure 6:



$(s = 4/5, t = 1)$ $\qquad$ $(s = -1/4, t = 1)$

Figure 6: Consumer surplus

As depicted, the effect of τ on consumer surplus is similar to the effect of τ on the equilibrium profits of the online retailer. Interestingly, when s is sufficiently high, consumer surplus is maximized for a positive value of τ.

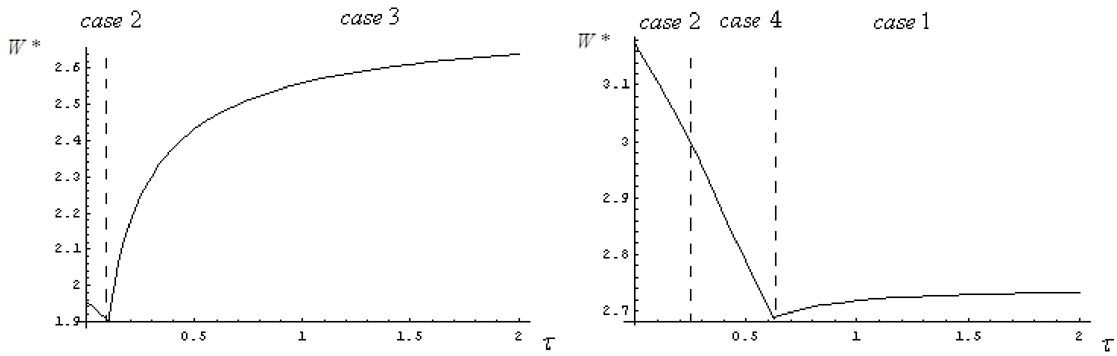
For intuition, we focus on Case 3. An increase in τ involves a trade-off in terms of consumer surplus: an increase in the disutility costs of purchasing from the online retailer, and a shift of purchases from the online retailer to one of the offline retailers. The former (negative) effect is large when consumers with a high y purchase from the

[18] See Larralde et al. (2009) for the use of numerical simulations when dealing with multidimensional models.

online retailer. This condition also holds for the latter (positive) effect. In addition, the latter effect still works if the locations of the marginal consumers purchasing from the online retailer are close to one of the offline retailers because their costs to travel to the retailer where they purchase are small. As a result, the latter effect dominates the former, and therefore the overall surplus of consumers purchasing from the online retailer increases with τ.[19]

In addition to this trade-off, higher consumer disutility from purchasing from the online retailer increases prices because of the weaker competition between retailers.[20] Therefore, the overall surplus of consumers purchasing from offline retailers decreases with τ. With Case 3, when τ is low, the positive effect on the surplus of the consumers purchasing from the online retailer prevails, whereas when τ is high, this is for the negative effect on the surplus of the consumers purchasing from the offline firms, thus explaining the inverse U-shape relation between CS^3* and τ.

Finally, we consider the impact of τ on welfare, as given by the sum of firm profits and consumer surplus and illustrated in Figure 7. As shown, when s is positive or negative, welfare increases with τ when τ is sufficiently high. The main driver here is the positive effect of τ on the equilibrium profits of the offline retailers (see Table 2).



$(s = 4/5, t = 1)$ $(s = -1/4, t = 1)$

Figure 7: Welfare

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[19] That is, the term $\int_{\frac{p_E^3*-p_1^3*+s}{t}}^{\hat{x}*}\int_{\hat{y}_1*}^{1}(v-p_1^3*-tx)dydx+\int_{\frac{p_E^3*-p_1^3*+s}{t}}^{\hat{x}*}\int_0^{\hat{y}_1*}(v-p_E^3*-s)dydx$ in (9) increases with τ.

[20] That is, the term $\int_0^{\frac{p_E^3*-p_1^3*+s}{t}}\int_0^{1}(v-p_1^3*-tx)dydx$ in (9) decreases with τ.

5. Conclusion

In this paper, we consider the interplay between online and offline retailers. We extend the existing literature by assuming that consumers are heterogeneous in their disutility when purchasing from online retailers. In particular, we analyze how the quality of service provided by the online retailer affects both its own profits and those of the offline retailers.

Our main finding is that even in the absence of quality-related costs, it can be optimal for the online retailer not to minimize the consumer disutility arising from the use of the Internet if the competitive power of the online retailer is weak. This is because a reduction in Internet-related consumer disutility induces fierce competition between the retailers, which outweighs the expansion in demand for the online retailer. However, this result does not hold if the online retailer's competitive power is strong because the demand expansion generates a significant benefit for a strong online retailer with a high price–cost margin.

Consider the following example. Amazon recently launched a new service, called Amazon Prime Now. In some countries, Amazon Prime Now requires a mobile device (like smartphones), thereby excluding personal computer users. Note that it would be relatively easy to improve the quality of Amazon Prime Now by allowing use by personal computer users. Why then is the quality of Amazon Prime Now not improved? Our model offers a possible explanation. By allowing only mobile device users to use Amazon Prime Now, Amazon is targeting its service at more Internet-oriented consumers and thereby leaving other consumers to offline retailers.

There are several possibilities for extending our model in future research. First, we could use the model to analyze the case of downstream competition when one or more manufacturers must decide whether to distribute their products through offline or online retailers (Chiang et al., 2003; Cattani et al., 2006; Yoo and Lee, 2011). Second, we could also employ the present setup to allow offline retailers to first choose their spatial location before competing on price, as in Guo and Lai (2014, 2017).

Our paper also involves several potential limitations deserving of note. To start, we assumed full market coverage (i.e., v is sufficiently large), a common assumption in the literature (e.g. Balasubramanian, 1998) for reason of tractability. However, when v is sufficiently low, some consumers with a higher y and an intermediate x will not be served in equilibrium. In particular, if we consider Cases 1 and 3, we observe that when we introduce an uncovered market, we remove any direct competition between offline retailers.[21] As direct competition between offline retailers is a necessary condition for the nonmonotonic relationship between τ and the online retailer's profits to emerge, we presume it will not arise when the market is not covered.

In addition, we did not explicitly consider the possibility that one of the offline retailers owns an online retailer, that is, it is a multichannel retailer (e.g., Bernstein et al., 2008; Huang and Swaminathan, 2009; Chen and Chen, 2017). However, we can provide an example where we preserve our main result even in this case. Consider the following situation. Suppose a multichannel retailer owns both Firm *E* and an offline retailer (say, Firm 3) such that it is located at 1/2. The price of Firm *E* may then differ from the price of Firm 3.

Accordingly, the utility function of a consumer purchasing from Firm 3 is given by $u_3 = v - p_3 - t(1/2 - x)$ if $x \le 1/2$ and $u_3 = v - p_3 - t(x - 1/2)$ if $x \ge 1/2$. Comparing the case of $\tau = 0$ with that of $\tau > 0$, we find a strictly positive value of τ where the profits of the multichannel retailer are higher than that under $\tau = 0$.[22] This implies that the multichannel retailer cannot maximize profits for a zero level of consumer-specific disutility from the Internet.

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[22] See the Technical Appendix for the proof.

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Technical Appendix (Not for Publication)

Equivalence of the two formulations for the online retailer's disadvantage (Section 2)

To understand intuitively the qualitative equivalence of the two formulations, we consider a standard Hotelling model in which only two offline retailers are respectively located at the endpoints. First, following the first formulation, we suppose that the reservation price for one retailer is v, whereas the reservation price for the other retailer is $v - c$. By routine calculation, it is immediately shown that the equilibrium prices are $t + c/3$ and $t - c/3$, yielding profits equal to $(3t+c)^2/18t$ and $(3t-c)^2/18t$, respectively. Second, following the second formulation, we suppose that the reservation prices of each consumer for both retailers are the same, but the marginal costs of the retailers are 0 and c, respectively. Standard calculations show that the equilibrium prices are now $t + c/3$ and $t + 2c/3$, but the equilibrium profits are still $(3t+c)^2/18t$ and $(3t-c)^2/18t$, respectively. Below, we show that we can apply this well-known result of the one-dimensional Hotelling model to the two-dimensional setup in this analysis.

Derivation of the demand functions (Section 3)

We state that a consumer (x, y) purchases from Firm 1 if the consumer belongs to the following set, $D_1 = \{(x, y) : x \in [0, \hat{x}(.)] \cap y \in [\hat{y}_1(.), 1]\}$. Similarly, a consumer purchases from Firm 2 if the consumer belongs to the set $D_2 = \{(x, y) : x \in [\hat{x}(.), 1] \cap y \in [\hat{y}_2(.), 1]\}$. Finally, a consumer purchases from Firm E if the consumer belongs to the set $D_E = \{(x, y) : (x, y) \notin D_1 \cup D_2\}$. We assume that in equilibrium, each firm has a positive demand. In addition, we assume that the model parameters are such that at least one consumer is served by one of the firms in equilibrium.[23] Instead of providing ex ante the necessary condition for this to be satisfied, we derive the equilibrium prices

[23] The consumer that is most keen to purchase from Firm 1 is located at $(x, y) = (0, 1)$. By substituting into u_1 and u_E and then comparing, we obtain that this consumer is served by Firm 1 in equilibrium when $p_1 - p_E < s + \tau$. The same condition applies for Firm 2. Conversely, the consumer that is most keen to purchase from Firm E is located at $(x, y) = (1/2, 0)$. By substituting this into u_1, u_2, and u_E and comparing, we find that this consumer is served by Firm E in equilibrium when $p_1 - p_E > s - t/2$ and $p_2 - p_E > s - t/2$.

under the hypothesis that each firm has positive demand, and then derive the conditions on the parameters such that the initial hypothesis is satisfied.

The intersections of the indifferent consumers $\hat{x}(.)$, $\hat{y}_1(.)$, and $\hat{y}_2(.)$ have three properties. First, $\hat{y}_1(\hat{x}) = \hat{y}_2(\hat{x})$ (this equality is derived by substituting $x = \hat{x}$ in (4) into $\hat{y}_1$ and $\hat{y}_2$ in (5) and (6)). Although there are three cases concerning $\hat{y}_1(\hat{x}) = \hat{y}_2(\hat{x})$ in principle— $\hat{y}_1(\hat{x}) = \hat{y}_2(\hat{x}) \geq 1$, $\hat{y}_1(\hat{x}) = \hat{y}_2(\hat{x}) \in (0,1)$, and $\hat{y}_1(\hat{x}) = \hat{y}_2(\hat{x}) \leq 0$ —we can exclude the final case because it implies that Firm E has zero demand.[24] Second, at the left endpoint of $\hat{y}_1(.)$, i.e., $\hat{y}_1(0)$, three cases are possible in principle: $\hat{y}_1(0) \geq 1$, $\hat{y}_1(0) \in (0,1)$, and $\hat{y}_1(0) \leq 0$. However, we can exclude the first case because it implies that Firm 1 has zero demand. Finally, at the right endpoint of $\hat{y}_2(.)$, i.e., $\hat{y}_2(1)$, three cases are possible in principle: $\hat{y}_2(1) \geq 1$, $\hat{y}_2(1) \in (0,1)$, and $\hat{y}_2(1) \leq 0$. However, we can exclude the first case because it implies that Firm 2 sells nothing.

Because two possible situations exist for each of the three intersections, $\hat{y}_1(\hat{x}) = \hat{y}_2(\hat{x})$, $\hat{y}_1(0)$, and $\hat{y}_2(1)$, the combination of all possible cases yields eight cases in principle. However, because the offline retailers are symmetric, we focus only on symmetric equilibria. This excludes the cases for which the endpoints $\hat{y}_1(0)$ and $\hat{y}_2(1)$ are different.[25] Therefore, the relevant cases reduce to four. In particular, the possible cases under symmetry are in Table 1A. In addition, Figure 1 in the main text illustrates the four relevant cases.

Table 1A: The four possible cases under symmetry

Case	$\hat{y}_1(\hat{x}) = \hat{y}_2(\hat{x})$	$\hat{y}_1(0)$	$\hat{y}_2(1)$
1	$\hat{y}_1(\hat{x}) = \hat{y}_2(\hat{x}) \in [0,1]$	$\hat{y}_1(0) \in [0,1]$	$\hat{y}_2(1) \in [0,1]$
2	$\hat{y}_1(\hat{x}) = \hat{y}_2(\hat{x}) \geq 1$	$\hat{y}_1(0) \leq 0$	$\hat{y}_2(1) \leq 0$
3	$\hat{y}_1(\hat{x}) = \hat{y}_2(\hat{x}) \in [0,1]$	$\hat{y}_1(0) \leq 0$	$\hat{y}_2(1) \leq 0$
4	$\hat{y}_1(\hat{x}) = \hat{y}_2(\hat{x}) \geq 1$	$\hat{y}_1(0) \in [0,1]$	$\hat{y}_2(1) \in [0,1]$

[24] Because we are interested in the situation where all firms are active in the market, we consider only those cases where all firms sell a positive quantity in equilibrium.

[25] That is, under symmetry, it must be that $\hat{y}_1(0) \in [0,1] \Leftrightarrow \hat{y}_2(1) \in [0,1]$ and $\hat{y}_1(0) \leq 0 \Leftrightarrow \hat{y}_2(1) \leq 0$.

Derivation of the equilibrium prices and profits and the necessary conditions for each case (Remark 1 in Section 3)

This section provides the proofs of the mathematical solutions for the equilibrium prices. To find the equilibrium prices in each of the four cases, we check the following four conditions (in *italics*). First, we check the *first-order* and *second-order* conditions, yielding the "candidate local" equilibrium prices for Case *j*. Afterward, we confirm whether the outcome under these prices is consistent with Case *j* (the *consistency* condition). Finally, we need to prove that the candidate price maximizes profits not only locally (that is, within Case *j*) but also globally. That is, we prove that under the local equilibrium outcome in Case *j*, no firm has an incentive to set a price that changes the case from Case *j* to another case to increase its profits (the *no-deviation* condition).

(Case 1)

Figure 1(*a*) in the main text illustrates the first case. Therefore, we express demand as follows:

$$q_1^1 = \frac{(1 - \frac{p_1^1 - p_E^1 - s}{\tau} + 1 - \frac{t + p_1^1 + p_2^1}{2\tau} + \frac{p_E^1 + s}{\tau})(\frac{1}{2} + \frac{p_2^1 - p_1^1}{2t})}{2}$$

$$= \frac{(t + p_2^1 - p_1^1)(4s - p_2^1 - 3p_1^1 - t + 4p_E^1 + 4\tau)}{8t\tau}, \tag{A1}$$

$$q_2^1 = \frac{(1 - \frac{p_2^1 - p_E^1 - s}{\tau} + 1 - \frac{t + p_1^1 + p_2^1}{2\tau} + \frac{p_E^1 + s}{\tau})(1 - \frac{1}{2} - \frac{p_2^1 - p_1^1}{2t})}{2},$$

$$= \frac{(p_1^1 - p_2^1 + t)(4s - p_1^1 - 3p_2^1 - t + 4p_E^1 + 4\tau)}{8t\tau}, \tag{A2}$$

$$q_E^1 = 1 - q_1^1 - q_2^1 = \frac{-p_1^1(p_1^1 - 2t) - p_2^1(p_2^1 - 2t) + 2p_1^1 p_2^1 - 4ts + t^2 - 4tp_E^1}{4t\tau}. \tag{A3}$$

The profit functions of the three firms are, therefore: $\pi_1^1 = p_1^1 q_1^1$, $\pi_2^1 = p_2^1 q_2^1$, and $\pi_E^1 = p_E^1 q_E^1$. By solving $\frac{\partial \pi_z^1}{\partial p_z^1} = 0$ ($z = 1,2$), we have the following roots:

$$\dot{p}_z^1(p_{-z}^1, p_E^1) = \frac{1}{9}(2p_{-z}^1 + 4s + 2t + 4p_E^1 + 4\tau - \frac{1}{2}\sqrt{\begin{aligned}&36(p_{-z}^1 + t)(p_{-z}^1 - 4s + t - 4p_E^1 - 4\tau) + \\ &+16(p_{-z}^1 + 2s + t + 2p_E^1 + 2\tau)^2\end{aligned}}), \tag{A4}$$

$$\ddot{p}_z^1(p_{-z}^1, p_E^1) = \frac{1}{9}(2p_{-z}^1 + 4s + 2t + 4p_E^1 + 4\tau + \frac{1}{2}\sqrt{\begin{matrix} 36(p_{-z}^1 + t)(p_{-z}^1 - 4s + t - 4p_E^1 - 4\tau) + \\ +16(p_{-z}^1 + 2s + t + 2p_E^1 + 2\tau)^2 \end{matrix}}). \quad \text{(A5)}$$

The second-order derivative is: $\frac{\partial^2 \pi_z^1}{\partial {p_z^1}^2} = \frac{9p_z^1 - 2(p_{-z}^1 + 2s + t + 2p_E^1 + 2\tau)}{4t\tau}$. By substituting $\dot{p}_z^1$ and $\ddot{p}_z^1$ into the second-order derivative, it is easy to see that: $\frac{\partial^2 \pi_z^1}{\partial {p_z^1}^2}(\dot{p}_z^1) \le 0$ and $\frac{\partial^2 \pi_z^1}{\partial {p_z^1}^2}(\ddot{p}_z^1) \ge 0$. Therefore, we can consider only the first root.

By solving $\frac{\partial \pi_E^1}{\partial p_E^1} = 0$ we have: $\dot{p}_E(p_1^1, p_2^1) = \frac{2t(p_1^1 + p_2^1) - (p_1^1 - p_2^1)^2 + t(t - 4s)}{8t}$. The second-order condition is always satisfied because $\frac{\partial^2 \pi_E^1}{\partial {p_E^1}^2} = -\frac{2}{\tau} < 0$.

By solving the system composed of equations $\dot{p}_1^1(.)$, $\dot{p}_2^1(.)$, and $\dot{p}_E^1(.)$, and restricting the analysis to symmetric equilibria for the offline retailers and positive prices, we end up with two "candidate equilibria." The first is given by:

$$p_1^1{*} = p_2^1{*} = \frac{9t + 8\tau + 4s - \sqrt{16s^2 + 97t^2 + 16t\tau + 64\tau^2 + 8s(t + 8\tau)}}{8}, \quad \text{(A6)}$$

$$p_E^1{*} = \frac{11t + 8\tau - 4s - \sqrt{16s^2 + 97t^2 + 16t\tau + 64\tau^2 + 8s(t + 8\tau)}}{16}. \quad \text{(A7)}$$

The second candidate equilibrium is given by:

$$p_1^1{\wedge} = p_2^1{\wedge} = \frac{9t + 8\tau + 4s + \sqrt{16s^2 + 97t^2 + 16t\tau + 64\tau^2 + 8s(t + 8\tau)}}{8}, \quad \text{(A8)}$$

$$p_E^1{\wedge} = \frac{11t + 8\tau - 4s + \sqrt{16s^2 + 97t^2 + 16t\tau + 64\tau^2 + 8s(t + 8\tau)}}{16}. \quad \text{(A9)}$$

From Table 1A, the *consistency* conditions of Case 1 require us to verify the following *consistency* conditions for the two candidate equilibria: $\hat{y}_1(., \hat{x}(.)) \in [0,1]$, $\hat{y}_1(.,0) \in [0,1]$, and $\hat{y}_2(.,1) \in [0,1]$. Plugging $p_1^1{*}$, $p_2^1{*}$, and $p_E^1{*}$ into the *consistency* conditions, the above conditions are satisfied when $s \in [\hat{s}, \tilde{s}]$, where $\hat{s} \equiv t - 2\tau$ and $\tilde{s} \equiv \frac{3t(2\tau - t)}{4(2\tau + t)}$.

Alternatively, plugging $p_1^1\wedge$, $p_2^1\wedge$, and $p_E^1\wedge$ into the *consistency* conditions, we observe that they cannot be simultaneously satisfied. Therefore, the unique triplet of local equilibrium prices in Case 1 is represented by p_1^1*, p_2^1*, and p_E^1*.

We now verify that the prices at (A6) and (A7) also satisfy the *no-deviation* condition, and therefore they represent a global equilibrium. To start, suppose that one of the offline retailers, for example, Firm 2, deviates from the equilibrium price indicated at (A6). Clearly, the indifferent consumer $\hat{y}_1$ is unaffected by this variation. Instead, we observe a shift of $\hat{y}_2$ and $\hat{x}$ along $\hat{y}_1$. Based on this, we have three different possible shapes of Firm 2's demand when Firm 2 deviates from the Case 1 equilibrium prices.[26] Figure A1 illustrates this point.

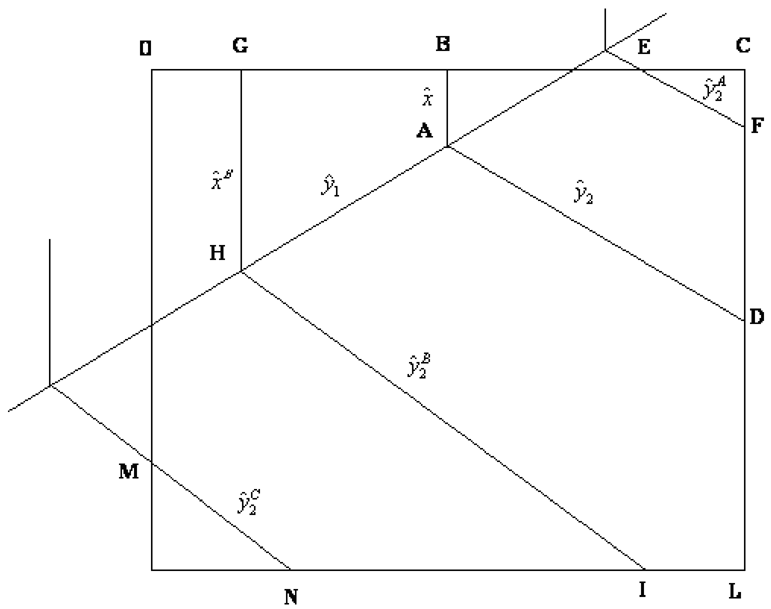

Figure A1: Deviations of Firm 2 from Case 1 equilibrium prices

Suppose that the initial situation is represented by lines $\hat{y}_1, \hat{y}_2$, and $\hat{x}$. Firm 2's demand is then given by area ADCB. Suppose Firm 2 increases its price. If the increase is high, the shape of Firm 2's demand changes: the new situation is represented by $\hat{y}_1$ and $\hat{y}_2^A$, and the demand of Firm 2 is given by area ECF. Suppose that Firm 2 decreases the price. If the decrease is moderate, the shape of Firm 2's demand changes: the new

[26] Note that the various cases caused by firm 2's deviation do not coincide with the four cases in the main text.

situation is represented by $\hat{y}_1$, $\hat{y}_2^B$, and $\hat{x}^B$, and the demand is given by area GHILC. If the decrease is high, the shape of Firm 2's demand changes again: the new situation is represented by $\hat{y}_2^C$ and the demand is given by area OMNLC.

To verify the *no-deviation* condition, we have to proceed as follows. Suppose that Firm 2's deviation induces $\hat{y}_2^A$. Then, given p_1^1* and p_E^1*, Firm 2 maximizes $\pi_2 = p_2 q_2$ with respect to p_2, where q_2 is given by area ECF. We obtain $p_2^d = \frac{24s + 22t + 48\tau - 2\Gamma - \sqrt{160s^2 + 272st + 218t^2 + 640s\tau + 544t\tau + 640\tau^2 - \Gamma(22t + 8\tau + 24s)}}{48}$, where $\Gamma \equiv \sqrt{16s^2 + 97t^2 + 16t\tau + 64\tau^2 + 8s(t + 8\tau)}$. However, the triplet p_1^1*, p_2^d, and p_E^1* cannot sustain the demand ECF for Firm 2 when $s \in [\hat{s}, \tilde{s}]$, which is the necessary condition for p_1^1* and p_E^1* to be equilibrium prices in Case 1. Therefore, there is no interior solution for deviation inducing demand ECF for Firm 2. Similarly, no interior deviation price for Firm 2 can sustain either demand GHILC or demand OMNLC for Firm 2.

Consider now a deviation of Firm *E* from (A7), while Firm 1 and Firm 2 set p_1^1* and p_2^1*, respectively. Note that in this case, the indifferent consumer $\hat{x}$ is unaffected by this deviation. Therefore, there is a symmetric and parallel shift of $\hat{y}_1$ and $\hat{y}_2$ along $\hat{x}$. Based on this, we have two different possible Firm *E*'s demand shapes when it deviates from Case 1 equilibrium prices. Figure A2 illustrates this point.

Suppose that the initial situation is represented by lines $\hat{y}_1$, $\hat{y}_2$, and $\hat{x}$. Firm *E*'s demand is given by area AFMNO. Suppose that Firm *E* increases its price. If such an increase is sufficiently high, the shape of Firm *E*'s demand changes: the new situation is represented by $\hat{y}_1^B$, $\hat{y}_2^B$, and $\hat{x}$, and the demand is given by area HIL. In contrast, suppose that Firm *E* decreases its price. If the decrease is sufficiently high, the shape of Firm *E*'s demand changes: the new situation is represented by $\hat{y}_1^A$ and $\hat{y}_2^A$, and the demand is given by area GDMNEC.

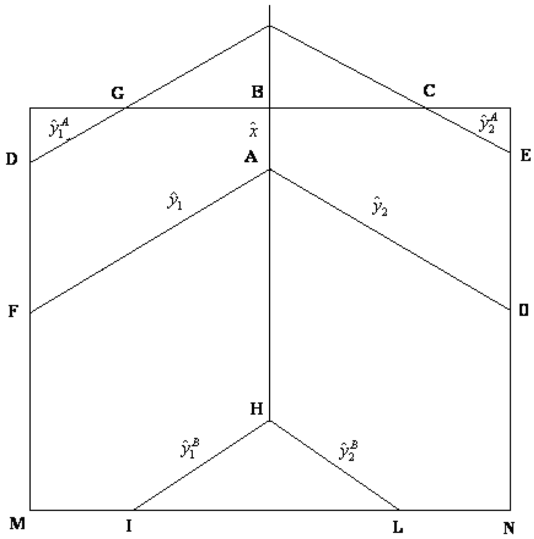

Figure A2: Deviations of Firm *E* from Case 1 equilibrium prices

Suppose that Firm E's deviation induces $\hat{y}_1^B$ and $\hat{y}_2^B$. Then, given p_1^1* and p_2^1*, Firm *E* maximizes $\pi_E = p_E q_E$ with respect to p_E, where q_E is given by HIL. We get $p_E^d = \dfrac{18t - 8s - 2\Gamma + \sqrt{2}\sqrt{16s^2 + 89t^2 + 104t\tau + 32\tau^2 - 9t\Gamma + 4s(8\tau - 8t + \Gamma)}}{24}$. However, the triplet p_1^1*, p_2^1*, and p_E^d cannot sustain the demand HIL for Firm *E* when $s \in [\hat{s}, \tilde{s}]$, which is the necessary condition for p_1^1* and p_2^1* to be equilibrium prices in Case 1. Therefore, there is no interior solution for a deviation inducing demand HIL for Firm *E*. Similarly, we can show that any interior deviation price for Firm *E* cannot sustain the demand GDMNEC for Firm *E*.

We summarize this in the following lemma:

Lemma 1. *In Case 1, a (unique) equilibrium triplet of prices exists,* $\mathbf{P^1}* \equiv \{p_1^1*, p_2^1*, p_E^1*\}$, *if and only if* $s \in [\hat{s}, \tilde{s}]$, *where* $\hat{s} \equiv t - 2\tau$ *and* $\tilde{s} \equiv \dfrac{3t(2\tau - t)}{4(2\tau + t)}$, *and*

$$p_1^1* = p_2^1* = \frac{9t + 8\tau + 4s - \sqrt{16s^2 + 97t^2 + 16t\tau + 64\tau^2 + 8s(t + 8\tau)}}{8},$$

$$p_E^1* = \frac{11t + 8\tau - 4s - \sqrt{16s^2 + 97t^2 + 16t\tau + 64\tau^2 + 8s(t + 8\tau)}}{16}.$$

The resulting equilibrium profits in Case 1 are:

$$\pi_1^1{*} = \pi_2^1{*} = \frac{t[-4s-49t-8\tau+5\sqrt{8s(2s+t+8\tau)+97t^2+16\tau(t+4\tau)}\,]}{64\tau}, \tag{A10}$$

$$\pi_E^1{*} = \frac{[4s-11t-8\tau+\sqrt{8s(2s+t+8\tau)+97t^2+16\tau(t+4\tau)}\,]^2}{256\tau}. \tag{A11}$$

Checking the sign of the derivative of $\pi_1^1{*}$, $\pi_2^1{*}$, and $\pi_E^1{*}$ with respect to each parameter within the parameter set yields the outcome in Table 2 of the main text.

Next, we show that we can interpret parameter s as the marginal cost disadvantage of Firm E. First, we set $s=0$ and see that the profit function of Firm E is now $\pi_E^1 = (p_E^1 - c)q_E^1$, where c is the marginal cost of Firm E.[27] The equilibrium prices are:

$$p_1^1{*} = p_2^1{*} = \frac{9t+8\tau+4c-\sqrt{8c(2c+t+8\tau)+97t^2+16\tau(t+4\tau)}}{8},$$

$$p_E^1{*}-c = \frac{11t+8\tau+12c-\sqrt{16c^2+97t^2+16t\tau+64\tau^2+8c(t+8\tau)}}{16}-c.$$

It is immediately seen that when $c=s$ the equilibrium profits are identical under both interpretations.

(Case 2)

Figure 1(b) in the main text represents the second case. Using the figure, we immediately derive the demand function of each firm:

$$q_1^2 = \frac{\left(\frac{\tau-p_1^2+p_E^2+s}{t}+\frac{p_E^2-p_1^2+s}{t}\right)}{2} = \frac{\tau+2s+2(p_E^2-p_1^2)}{2t}, \tag{A12}$$

$$q_2^2 = \frac{\left(1-1-\frac{p_2^2-p_E^2-s-\tau}{t}+1-1-\frac{p_2^2-p_E^2-s}{t}\right)}{2} = \frac{\tau+2s+2(p_E^2-p_2^2)}{2t}, \tag{A13}$$

$$q_E^2 = 1-q_1^2-q_2^2 = \frac{p_1^2+p_2^2-2p_E^2-2s+t-\tau}{t}. \tag{A14}$$

[27] Clearly, the situation where $s<0$ can be described by assuming that the offline retailers have positive marginal cost, that is: $\pi_1^1 = (p_1^1-c)q_1^1$ and $\pi_2^1 = (p_2^1-c)q_2^1$.

Therefore, the profit functions of the firms are given by $\pi_1^2 = p_1^2 q_1^2$, $\pi_2^2 = p_2^2 q_2^2$, and $\pi_E^2 = p_E^2 q_E^2$, respectively. By solving $\frac{\partial \pi_z^2}{\partial p_z^2} = 0$ (*z* = *1, 2*), we have the following root: $\dot{p}_z^2(p_E^2) = \frac{2p_E^2 + \tau + 2s}{4}$. The second-order derivative is always satisfied because $\frac{\partial^2 \pi_z^2}{\partial p_z^{2\,2}} = -\frac{2}{t} < 0$. By solving $\frac{\partial \pi_E^2}{\partial p_E^2} = 0$ we have: $\dot{p}_E^2(p_1^2, p_2^2) = \frac{p_1^2 + p_2^2 + t - \tau - 2s}{4}$. The second-order condition is always satisfied because $\frac{\partial^2 \pi_E^2}{\partial p_E^{2\,2}} = -\frac{4}{t} < 0$. By solving the system composed of equations $\dot{p}_1^2(.)$, $\dot{p}_2^2(.)$, and $\dot{p}_E^2(.)$, we end up with the following candidate equilibrium:

$$p_1^2{*} = p_2^2{*} = \frac{t + \tau + 2s}{6}, \tag{A15}$$

$$p_E^2{*} = \frac{2t - \tau - 2s}{6}. \tag{A16}$$

Given Figure 1(*b*), we have to verify the following *consistency* conditions for the candidate equilibrium: $\hat{y}_1(.,\hat{x}(.)) \geq 1$, $\hat{y}_1(.,0) \leq 0$, and $\hat{y}_2(.,1) \leq 0$. Plugging $p_1^2{*}$, $p_2^2{*}$, and $p_E^2{*}$ into the *consistency* conditions, the conditions above are satisfied when $s \in [\breve{s}, \hat{s}]$, where $\hat{s} \equiv (2\tau - t)/2$. Thus, $p_1^2{*}$, $p_2^2{*}$, and $p_E^2{*}$ represent a local equilibrium.

We now have to verify that the prices at (A15) and (A16) also satisfy the *no-deviation* condition, and therefore they represent a global equilibrium. To start, suppose that one of the offline retailers, for example, Firm 2, deviates from the equilibrium price indicated at (A15). Therefore, suppose a deviation of p_2. The indifferent consumer $\hat{y}_1$ is unaffected by this deviation. Instead, we observe a parallel shift of $\hat{y}_2$ and $\hat{x}$ along $\hat{y}_1$. Based on this, we have three different possible shapes of Firm 2's demand when Firm 2 deviates from the Case 2 equilibrium prices. Figure A3 illustrates this point.

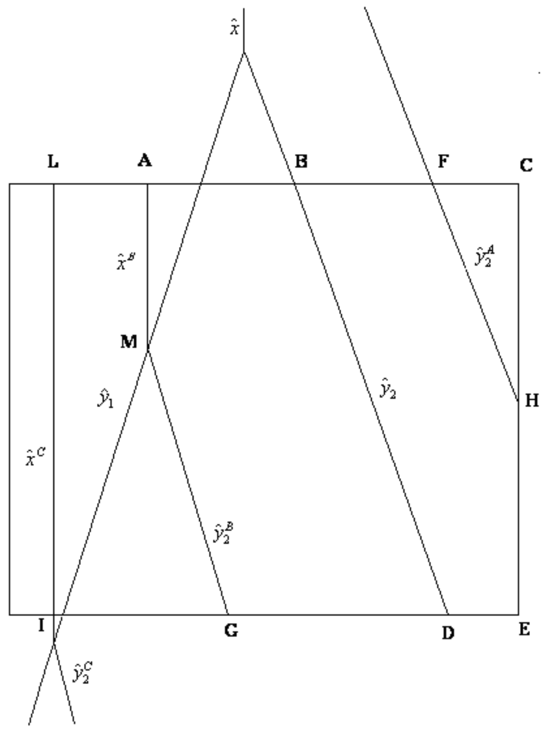

Figure A3: Deviations of Firm 2 from Case 2 equilibrium prices

Suppose that the initial situation is represented by lines $\hat{y}_1$ and $\hat{y}_2$. Firm 2's demand is then given by area BDEC. Suppose Firm 2 increases its price. If the increase is high, the shape of Firm 2's demand changes: the new situation is represented by $\hat{y}_2^A$, and the demand of Firm 2 is given by area FCH. Suppose that Firm 2 decreases the price. If the decrease is moderate, the shape of Firm 2's demand changes: the new situation is represented by $\hat{y}_1$, $\hat{y}_2^B$, and $\hat{x}^B$, and the demand is given by area ACEGM. If the decrease is high, the shape of Firm 2's demand changes again: the new situation is represented by $\hat{x}^C$ and the demand is given by area LIEC.

To prove the *no-deviation* condition, we proceed as follows. Suppose that Firm 2's deviation induces $\hat{y}_2^A$. Then, given p_1^2* and p_E^2*, Firm 2 maximizes $\pi_2 = p_2 q_2$ with respect to p_2, where q_2 is given by area FCH. Maximizing this π_2, we obtain $p_2^d = \dfrac{2t + 5\tau + 4s}{18}$. However, the triplet p_1^2*, p_2^d, and p_E^2* cannot sustain demand FCH for Firm 2 when $s \in [\hat{s}, \hat{s}]$, which is the necessary condition for p_1^2* and p_E^2* to be the equilibrium prices in Case 2, and there is no interior solution for any deviation that would induce the demand FCH for Firm 2. In the same way, we can show that no interior deviation price for Firm 2 can sustain demand ACEGM or demand LIEC for Firm 2.

Consider now the deviation of Firm *E* from (A16), while Firm 1 and Firm 2 set p_1^2* and p_2^2*, respectively. In this case, the indifferent consumer $\hat{x}$ is unaffected by Firm *E*'s deviation. Therefore, there is a symmetric and parallel shift of $\hat{y}_1$ and $\hat{y}_2$ along $\hat{x}$. Based on this, we have two different possible shapes of Firm *E*'s demand when it deviates from Case 2 equilibrium prices. Figure A4 illustrates this point.

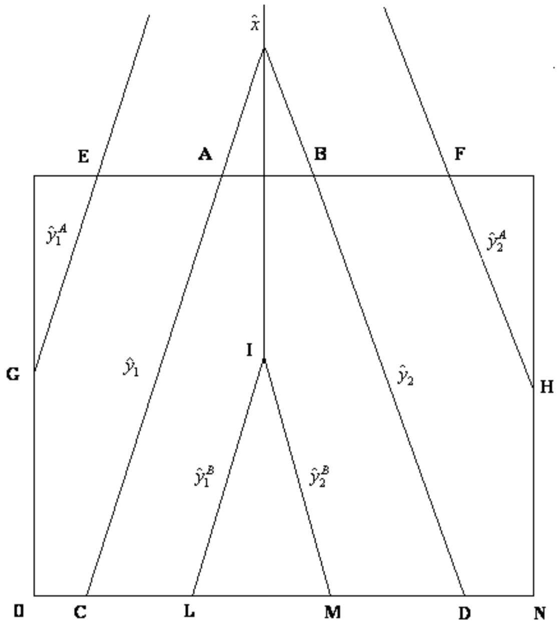

Figure A4: Deviations of Firm *E* from Case 2 equilibrium prices

Suppose that the initial situation is represented by lines $\hat{y}_1$ and $\hat{y}_2$. Firm *E*'s demand is given by area ACDB. Suppose that Firm *E* increases its price. If such an increase is sufficiently high, the shape of Firm *E*'s demand changes: the new situation is represented by $\hat{y}_1^B$, $\hat{y}_2^B$, and $\hat{x}$, and the demand is given by area ILM. In contrast, suppose that Firm *E* decreases its price. If the decrease is sufficiently high, the shape of Firm *E*'s demand changes: the new situation is represented by $\hat{y}_1^A$ and $\hat{y}_2^A$, and the demand is given by area EGONHF.

Suppose that Firm E's deviation induces $\hat{y}_1^B$ and $\hat{y}_2^B$. Then, given p_1^2* and p_2^2*, Firm *E* maximizes $\pi_E = p_E q_E$, where q_E is given by area ILM. The maximization problem leads to $p_E^d = \dfrac{2t - 10\tau - 8s + \sqrt{16s^2 - 8st + t^2 + 40s\tau + 98t\tau + 25\tau^2}}{18}$. However, the triplet p_1^2*, p_2^2*, and p_E^d cannot sustain the demand ILM for Firm *E* when $s \in [\hat{s}, \hat{s}]$, which is the necessary condition for p_1^2* and p_2^2* to be equilibrium prices in Case 2. Therefore, there is no interior solution for a deviation inducing demand ILM for Firm *E*. Similarly, it can be shown that the demand EGONHF for Firm *E* cannot be sustained by any interior deviation price for Firm *E*.

We summarize in the following lemma:

Lemma 2. *In Case 2, a (unique) equilibrium triplet of prices exists,* $\mathbf{P}^{2}* \equiv \{p_1^2*, p_2^2*, p_E^2*\}$, *if and only if* $s \in [\hat{s}, \hat{s}]$, *where* $\hat{s} \equiv (2\tau - t)/2$, *and*

$$p_1^2* = p_2^2* = \frac{t + \tau + 2s}{6},$$

$$p_E^2* = \frac{2t - \tau - 2s}{6}.$$

The resulting equilibrium profits in Case 2 are:

$$\pi_1^2* = \pi_2^2* = \frac{(2s + t + \tau)^2}{36t}, \tag{A17}$$

$$\pi_E^2* = \frac{(2s - 2t + \tau)^2}{18t}. \tag{A18}$$

Checking the sign of the derivative of π_1^2*, π_2^2*, and π_E^2* with respect to each parameter within the parameter set yields the outcome in Table 2 in the main text.

In what follows, we show that parameter *s* can be interpreted also as a marginal cost disadvantage of Firm *E*. We set $s = 0$ and $\pi_E^1 = (p_E^1 - c)q_E^1$. The equilibrium prices are:

$$p_1^2* = p_2^2* = \frac{t + \tau + 2c}{6},$$

$$p_E^2* - c = \frac{2t - \tau + 4c}{6} - c.$$

It can be easily seen that when $c = s$ the equilibrium profits are the same under both interpretations.

(Case 3)

Figure 1(*c*) in the main text represent the third case. Using this figure, the explicit expression of the demand function of each firm is given by:

$$q_1^3 = (1 - \frac{t + p_1^3 + p_2^3}{2\tau} + \frac{p_E^3 + s}{\tau})(\frac{1}{2} + \frac{p_2^3 - p_1^3}{2t}) + \frac{(\frac{1}{2} + \frac{p_2^3 - p_1^3}{2t} + \frac{p_E^3 - p_1^3 + s}{t})(\frac{t + p_1^3 + p_2^3}{2\tau} - \frac{p_E^3 + s}{\tau})}{2}$$
$$= \frac{4s(t-s) - p_1^3 - p_1^3 - t^2 - 4p_E^3(2s - t + p_E^3) + 4t\tau - 2p_1^3(p_2^3 - 2s + t - 2p_E^3 + 2\tau) + 2p_2^3(2s - t + 2p_E^3 + 2\tau)}{8t\tau}$$

(A19)

$$q_2^3 = (1 - \frac{t + p_1^3 + p_2^3}{2\tau} + \frac{p_E^3 + s}{\tau})(\frac{1}{2} - \frac{p_2^3 - p_1^3}{2t}) + \frac{(\frac{1}{2} - \frac{p_2^3 - p_1^3}{2t} + \frac{p_E^3 - p_2^3 + s}{t})(\frac{t + p_1^3 + p_2^3}{2\tau} - \frac{p_E^3 + s}{\tau})}{2}$$
$$= \frac{4s(t-s) - p_1^3 - p_1^3 - t^2 - 4p_E^3(2s - t + p_E^3) + 4t\tau - 2p_1^3(p_2^3 - 2s + t - 2p_E^3 - 2\tau) + 2p_2^3(2s - t + 2p_E^3 - 2\tau)}{8t\tau}$$

(A20)

$$q_E^3 = 1 - q_1^3 - q_2^3 = \frac{(p_1^3 + p_2^3 - 2p_E^3 - 2s + t)^2}{4t\tau}. \quad \text{(A21)}$$

Therefore, the profits functions of the three firms are: $\pi_1^3 = p_1^3 q_1^3$, $\pi_2^3 = p_2^3 q_2^3$, and $\pi_E^3 = p_E^3 q_E^3$. By solving $\frac{\partial \pi_z^3}{\partial p_z^3} = 0$ ($z = 1, 2$), we have the following roots:

$$\dot{p}_z^3(p_{-z}^3, p_E^3) = \frac{1}{3}(-2p_{-z}^3 + 4s - 2t + 4p_E^3 - 4\tau - \sqrt{\begin{matrix} {p_{-z}^3}^2 + 4s^2 + t^2 + 4p_E^3(p_E^3 - t) + 28t\tau - 32\tau p_E^3 + 16\tau^2 + \\ -4s(t - 2p_E^2 + 8\tau) + 2p_2^3(-2s + t - 2p_E^3 + 14\tau) \end{matrix}}) \quad \text{(A22)}$$

$$\dot{p}_z^3(p_{-z}^3, p_E^3) = \frac{1}{3}(-2p_{-z}^3 + 4s - 2t + 4p_E^3 - 4\tau + \sqrt{\begin{matrix} {p_{-z}^3}^2 + 4s^2 + t^2 + 4p_E^3(p_E^3 - t) + 28t\tau - 32\tau p_E^2 + 16\tau^2 + \\ -4s(t - 2p_E^3 + 8\tau) + 2p_2^3(-2s + t - 2p_E^3 + 14\tau) \end{matrix}}) \quad \text{(A23)}$$

The second-order derivative is: $\frac{\partial^2 \pi_z^3}{\partial {p_z^3}^2} = -\frac{3p_z^3 + 2(p_{-z}^3 - 2s + t - 2p_E^3 + 2\tau)}{4t\tau}$. By substituting $\dot{p}_Z^3$ and $\ddot{p}_Z^3$ into the second-order derivative, it is easy to see that: $\frac{\partial^2 \pi_z^3}{\partial {p_z^3}^2}(\dot{p}_z^3) \geq 0$ and $\frac{\partial^2 \pi_z^3}{\partial {p_z^3}^2}(\ddot{p}_z^3) \leq 0$. Therefore, we can consider only the second root.

By solving $\frac{\partial \pi_E^3}{\partial p_E^3}=0$ we have the following roots: $\dot{p}_E^3(p_1^3, p_2^3)=\frac{1}{6}(p_1^3+p_2^3-2s+t)$ and $\ddot{p}_E^3(p_1^3, p_2^3)=\frac{1}{2}(p_1^3+p_2^3-2s+t)$. The second-order condition is the following: $\frac{\partial^2 \pi_E^3}{\partial {p_E^3}^2}=-\frac{2(p_1^3+p_2^3+t-2s-3p_E^3)}{t\tau}$.

First, consider the system composed of equations $\ddot{p}_1^3(.)$, $\ddot{p}_2^3(.)$, and $\dot{p}_E^3(.)$. Solving the system by restricting the analysis to symmetric equilibria for the offline retailers and positive prices, we end up with two "candidate" equilibria. The first is given by:

$$p_1^3{*}=p_2^3{*}=\frac{14s-7t-9\tau+3\sqrt{(2s-t+3\tau)^2-20\tau(2s-3t)}}{20}, \tag{A24}$$

$$p_E^3{*}=\frac{t-2s-3\tau+\sqrt{(2s-t+3\tau)^2-20\tau(2s-3t)}}{20}. \tag{A25}$$

The second candidate equilibrium is given by:

$$p_1^3{\wedge}=p_2^3{\wedge}=\frac{14s-7t-9\tau-3\sqrt{(2s-t+3\tau)^2-20\tau(2s-3t)}}{20}, \tag{A26}$$

$$p_E^3{\wedge}=\frac{t-2s-3\tau-\sqrt{(2s-t+3\tau)^2-20\tau(2s-3t)}}{20}. \tag{A27}$$

Given Figure 1(*c*), we have to verify the following *consistency* conditions for the two candidate equilibria, as well as the second-order condition for Firm *E*. Therefore: $\hat{y}_1(.,\hat{x}(.))\in[0,1]$, $\hat{y}_1(.,0)\le 0$, $\hat{y}_2(.,1)\le 0$, and $\frac{\partial^2 \pi_E^3}{\partial {p_E^3}^2}(.)\le 0$. Plugging $p_1^3{*}$, $p_2^3{*}$, and $p_E^3{*}$, the conditions above are satisfied when $s\in[\max[\tilde{s},\hat{s}],\frac{3t}{2}]$. Alternatively, plugging $p_1^3{\wedge}$, $p_2^3{\wedge}$, and $p_E^3{\wedge}$ into the above conditions, we observe that also in this case they cannot be simultaneously satisfied.

Next, consider the system composed of equations $\ddot{p}_1^3(.)$, $\ddot{p}_2^3(.)$, and $\ddot{p}_E^3(.)$. Solving the system, we end up with the following candidate equilibrium:

$$p_1^3{\wedge\wedge}=p_2^3{\wedge\wedge}=t, \tag{A28}$$

$$p_E^3{\wedge\wedge}=\frac{3t}{2}-s. \tag{A29}$$

However, plugging p_1^3^^, p_2^3^^, and p_E^3^^ into the *consistency* conditions and the second-order condition for Firm E, we observe that they cannot be simultaneously satisfied. Therefore, only p_1^3*, p_2^3*, and p_E^3* represent a local equilibrium.

Now, we have to verify that the prices at (A24) and (A25) also satisfy the *no-deviation* condition, and therefore they represent a global equilibrium. To start, suppose that one of the offline retailers, for example, Firm 2, deviates from the equilibrium price indicated at (A24). Therefore, suppose a deviation of p_2. Once again, the indifferent consumer $\hat{y}_1$ is unaffected by this deviation. Instead, we observe a parallel shift of $\hat{y}_2$ and $\hat{x}$ along $\hat{y}_1$. Based on this, we have three different possible shapes of Firm 2's demand when Firm 2 deviates from Case 3 equilibrium prices. Figure A5 illustrates this point.

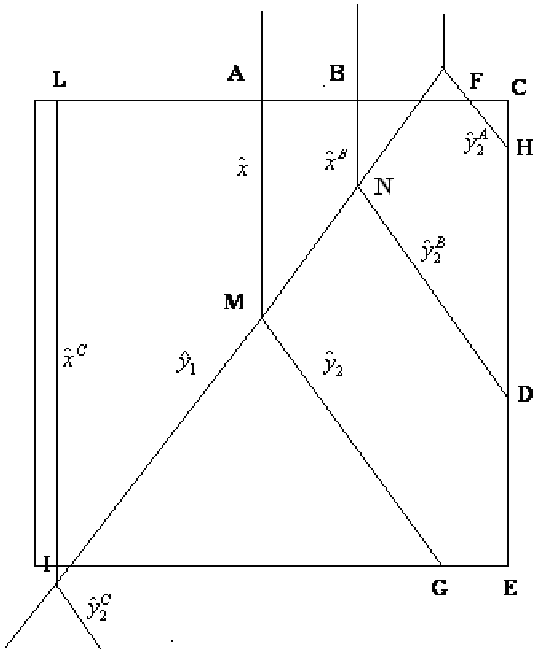

Figure A5: Deviations of Firm 2 from Case 3 equilibrium prices

Suppose that the initial situation is represented by lines $\hat{y}_1$ and $\hat{y}_2$. Firm 2's demand is then given by area AMGEC. Suppose Firm 2 increases its price. If the increase is intermediate, the shape of Firm 2's demand changes: the new situation is represented by $\hat{y}_2^B$, and the demand of Firm 2 is given by area BNDC. If the increase is high, the new situation is represented by $\hat{y}_2^A$ and the demand for Firm 2 becomes FHC. Suppose that Firm 2 decreases the price. If the decrease is high, the shape of Firm 2's demand changes: the new situation is represented by $\hat{y}_2^C$ and the demand is given by area LIEC.

We check the *no-deviation* condition. Suppose that Firm 2's deviation induces $\hat{y}_2^B$. Then, given p_1^3 * and p_E^3 *, Firm 2 maximizes $\pi_2 = p_2 q_2$, where q_2 is given by area BNDC. The maximization problem leads to

$$p_2^d = \frac{100s + 30t + 50\tau + 10\overline{\Gamma} - \sqrt{2984s^2 + 2026t^2 - 2796t\tau + 3934\tau^2 - 726t\overline{\Gamma} - 998\tau\overline{\Gamma} + 4s(14t + 738\tau + 77\overline{\Gamma})}}{180},$$

where $\overline{\Gamma} = \sqrt{4s^2 + t^2 + 54t\tau + 9\tau^2 - 4s(t + 7\tau)}$. However, the triplet p_1^3 *, p_2^d, and p_E^3 * cannot sustain the demand BNDC for Firm 2 neither when $s \in [\max[\tilde{s}, \hat{s}], \frac{3t}{2}]$, which is the necessary condition for p_1^3 * and p_E^3 * to be the equilibrium prices in Case 3, and there is no interior solution for a deviation that induces demand BNDC for Firm 2. In the same way, we can show that no interior deviation price for Firm 2 can sustain either demand FHC or demand LIEC for Firm 2.

Consider now a deviation of Firm *E* from (A25), while Firm 1 and Firm 2 set p_1^3 * and p_2^3 *, respectively. In this case, the indifferent consumer $\hat{x}$ is unaffected by the deviation of Firm *E*'s price. Therefore, there is a symmetric and parallel shift of $\hat{y}_1$ and $\hat{y}_2$ along $\hat{x}$. We now have two different possible shapes for Firm *E*'s demand when it deviates from Case 3 equilibrium prices. Figure A6 illustrates this point.

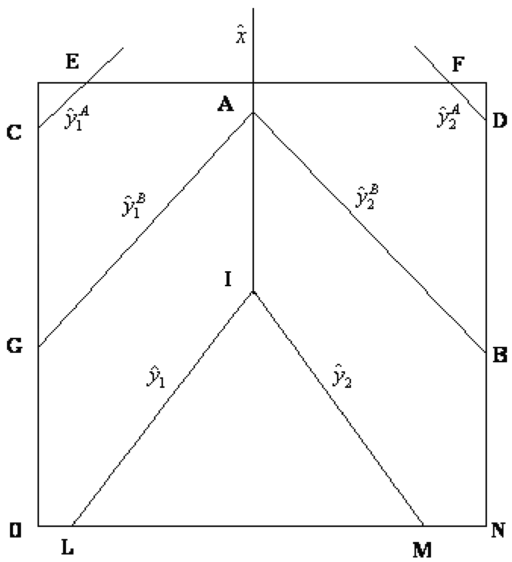

Figure A6: Deviations of Firm *E* from Case 3 equilibrium prices

Suppose that the initial situation is represented by lines $\hat{y}_1$ and $\hat{y}_2$. Firm E's demand is given by area ILM. Suppose that Firm E decreases its price. If this decrease is moderate, the shape of Firm E's demand changes: the new situation is represented by $\hat{y}_1^B$, $\hat{y}_2^B$, and $\hat{x}$, and the demand is given by area AGONB. In contrast, suppose that the decrease of Firm E's price is high. The new situation is represented by $\hat{y}_1^A$, $\hat{y}_2^A$, and $\hat{x}$, and the demand is given by area ECONDF.

Suppose that Firm E's deviation induces $\hat{y}_1^B$ and $\hat{y}_2^B$. Then, given p_1^3* and p_2^3*, Firm E maximizes $\pi_E = p_E q_E$, where q_E is given by area AGONB. The maximization problem leads to $p_E^d = \dfrac{36s + 2t + 34\tau + 2\Gamma - \sqrt{2}\sqrt{164s^2 + t^2 + 44t\tau + 149\tau^2 + t\Gamma + 17\tau\Gamma + 2s(8t + 146\tau + 9\Gamma)}}{60}$.

However, the triplet p_1^3*, p_2^3*, and p_E^d cannot sustain the demand AGONB for Firm E when $s \in [\max[\tilde{s}, \hat{s}], \frac{3t}{2}]$, which is the necessary condition for p_1^3* and p_2^3* to be equilibrium prices in Case 3. Therefore, there is no interior solution for a deviation inducing demand AGONB for Firm E. Similarly, we can show that no interior deviation price for Firm E can sustain the demand ECONDF for Firm E.

We summarize this in the following lemma:

Lemma 3. *In Case 3, a (unique) equilibrium triplet of prices exists,* $\mathbf{P}^3* \equiv \{p_1^3*, p_2^3*, p_E^3*\}$, *if and only if* $s \in [\max[\tilde{s}, \hat{s}], \frac{3t}{2}]$, *and*

$$p_1^3* = p_2^3* = \frac{14s - 7t - 9\tau + 3\sqrt{(2s - t + 3\tau)^2 - 20\tau(2s - 3t)}}{20},$$

$$p_E^3* = \frac{t - 2s - 3\tau + \sqrt{(2s - t + 3\tau)^2 - 20\tau(2s - 3t)}}{20}.$$

The resulting equilibrium profits in Case 3 are:

$$\pi_1^3* = \pi_2^3* = \frac{(9\tau - 14s + 7t - 3\tilde{\Gamma})[4s(s - t) + t^2 - \tau(8s + 26t - 9\tau) - \tilde{\Gamma}(2s - t + 3\tau)]}{2000t\tau}, \tag{A30}$$

$$\pi_E^3* = \frac{(t - 2s - 3\tau + \tilde{\Gamma})^3}{2000t\tau}, \tag{A31}$$

where $\widetilde{\Gamma} \equiv \sqrt{(2s-t+3\tau)^2 - 20\tau(2s-3t)}$.

Checking the sign of the derivative of $\pi_1^3 *$, $\pi_2^3 *$, and $\pi_E^3 *$ with respect to each parameter within the parameter set yields the outcome in Table 2 in the main text.

Next, we show that we can also interpret parameter s as the marginal cost disadvantage of Firm *E*. First, we set $s=0$ and $\pi_E^1 = (p_E^1 - c)q_E^1$. The equilibrium prices are:

$$p_1^3 * = p_2^3 * = \frac{14c - 7t - 9\tau + 3\sqrt{(2c-t+3\tau)^2 - 20\tau(2c-3t)}}{20},$$

$$p_E^3 * - c = \frac{t + 18c - 3\tau + \sqrt{4c^2 + t^2 + 54t\tau + 9\tau^2 - 4c(t+7\tau)}}{20} - c.$$

It is immediately seen that when $c = s$ the equilibrium profits are identical under both interpretations.

(Case 4)

Let us consider Case 4, as represented in Figure 1(*d*) in the main text. The demand function of each firm is given by:

$$q_1^4 = (\frac{1}{2} - \frac{p_1^4 - p_E^4 - s}{2\tau})(\frac{k - p_1^4 + p_E^4 + s}{2t}) = \frac{(\tau + s + p_E^4 - p_1^4)^2}{2t\tau}, \tag{A32}$$

$$q_2^4 = \frac{1}{2}(1 - \frac{p_2^4 - p_E^4 - s}{\tau})(1 - 1 - \frac{p_2^4 - p_E^4 - s - k}{t}) = \frac{(\tau + s + p_E^4 - p_2^4)^2}{2t\tau}, \tag{A33}$$

$$q_E^4 = 1 - q_1^4 - q_2^4$$

$$= \frac{2(s + \tau + p_E^4)(p_1^4 + p_2^4) - p_1^{4^2} - p_2^{4^2} - 2[s^2 + p_E^4(p_E^4 + 2\tau) - \tau(t-\tau) + 2s(p_E^4 + \tau)]}{2t\tau}. \tag{A34}$$

Therefore, the profits functions are given by: $\pi_1^4 = p_1^4 q_1^4$, $\pi_2^4 = p_2^4 q_2^4$, and $\pi_E^4 = p_E^4 q_E^4$. By solving $\frac{\partial \pi_z^4}{\partial p_z^4} = 0$, ($z$ = *1, 2*), we have the following roots: $\dot{p}_z^4(p_E^4) = \frac{s + p_E^4 + \tau}{3}$ and $\ddot{p}_z^4(p_E^4) = p_E^4 + \tau + s$. The second-order derivative is: $\frac{\partial^2 \pi_z^4}{\partial p_z^{4^2}} = \frac{3p_Z^4 - 2(s + p_E^4 + \tau)}{t\tau}$. By substituting $\dot{p}_z^4$ and $\ddot{p}_z^4$ into the second-order derivative, it is easy to see that: $\frac{\partial^2 \pi_z^4}{\partial {p_z^4}^2}(\dot{p}_z^4) \le 0$ and $\frac{\partial^2 \pi_z^4}{\partial {p_z^4}^2}(\ddot{p}_z^4) \ge 0$. Therefore, we consider only the first root.

By solving $\frac{\partial \pi_E^4}{\partial p_E^4} = 0$ we have the following roots:

$$\dot{p}_E^4(p_1^4, p_2^4) = \frac{1}{6}[2(p_1^4 + p_2^4) - 4(s+\tau) + \sqrt{2}\sqrt{\begin{matrix} -({p_1^4}^2 + {p_2^4}^2) - 2(s+\tau)(p_1^4 + p_2^4) + \\ +4p_1^4 p_2^4 + 2[s(s+2\tau) + \tau(3t+\tau)] \end{matrix}}], \tag{A35}$$

$$\ddot{p}_E^4(p_1^4, p_2^4) = \frac{1}{6}[2(p_1^4 + p_2^4) - 4(s+\tau) - \sqrt{2}\sqrt{\begin{matrix} -({p_1^4}^2 + {p_2^4}^2) - 2(s+\tau)(p_1^4 + p_2^4) + \\ +4p_1^4 p_2^4 + 2[s(s+2\tau) + \tau(3t+\tau)] \end{matrix}}]. \tag{A36}$$

The second-order derivative is: $\frac{\partial^2 \pi_E^4}{\partial {p_E^4}^2} = \frac{2(p_1^4 + p_2^4 - 2s - 3p_E^4 - 2\tau)}{t\tau}$. By substituting $\dot{p}_E^4$ and $\ddot{p}_E^4$ into the second-order derivative, it is easy to see that: $\frac{\partial^2 \pi_E^4}{\partial {p_E^4}^2}(\dot{p}_E^4) \le 0$ and $\frac{\partial^2 \pi_E^4}{\partial {p_E^4}^2}(\ddot{p}_E^4) \ge 0$. Therefore, we consider only the first root.

By solving the system composed of equations $\dot{p}_1^4(.)$, $\dot{p}_2^4(.)$, and $\dot{p}_E^4(.)$, and restricting the analysis to symmetric equilibria for the offline retailers and positive prices, we end up with the following candidate equilibrium:

$$p_1^4{*} = p_2^4{*} = \frac{s + \tau + \sqrt{s^2 + 2s\tau + \tau(4t+\tau)}}{8}, \tag{A37}$$

$$p_E^4{*} = \frac{-5(s+\tau) + 3\sqrt{4t\tau + (s+\tau)^2}}{8}. \tag{A38}$$

Using Figure 1(*d*), we have to verify the *consistency* conditions for the two candidate equilibrium: $\hat{y}_1(.,\hat{x}(.)) \ge 1$, $\hat{y}_1(.,0) \in [0,1]$, and $\hat{y}_2(.,1) \in [0,1]$. Plugging $p_1^4{*}$, $p_2^4{*}$, and $p_E^4{*}$ into the *consistency* conditions, we observe that they are satisfied when $s \le \min[\widehat{s}, \hat{s}]$. Therefore, only $p_1^4{*}$, $p_2^4{*}$, and $p_E^4{*}$ represent a local equilibrium.

We now verify that the prices at (A37) and (A38) also satisfy the *no-deviation* condition. To start, suppose that one of the offline retailers, say Firm 2, deviates from the equilibrium price indicated at (A37). That is, suppose a deviation of p_2. The indifferent consumer $\hat{y}_1$ is unaffected by this variation. Instead, we observe a parallel shift of $\hat{y}_2$ and $\hat{x}$ along $\hat{y}_1$. Based on this, we have three different possible shapes of Firm 2's demand when Firm 2 deviates from Case 4 equilibrium prices. Figure A7 illustrates this point.

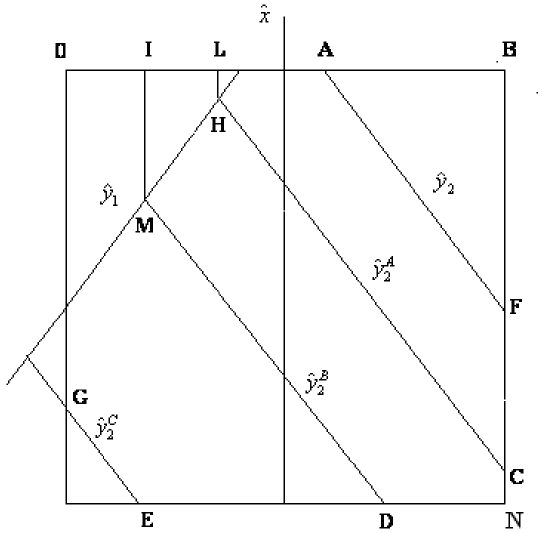

Figure A7: Deviations of Firm 2 from Case 4 equilibrium prices

Suppose that the initial situation is represented by lines $\hat{y}_1$ and $\hat{y}_2$. Firm 2's demand is then given by area ABF. Suppose Firm 2 decreases its price. If the increase is low, the shape of Firm 2's demand changes: the new situation is represented by $\hat{y}_2^A$, and the demand of Firm 2 is given by area BLHC. If the increase is intermediate, the new situation is represented by $\hat{y}_2^B$ and the demand for Firm 2 becomes IBNDM. If the increase is high, the new situation is represented by $\hat{y}_2^C$ and the demand for Firm 2 becomes OGENB.

We prove the *no-deviation* condition. Suppose that Firm 2's deviation induces $\hat{y}_2^A$. Then, given $p_1^4 *$ and $p_E^4 *$, Firm 2 maximizes $\pi_2 = p_2 q_2$, where q_2 is given by area BLHC. The maximization problem leads to $p_2^d = \dfrac{14s + 16t + 14\tau + 14\overline{\overline{\Gamma}} - \sqrt{2}\sqrt{97s^2 + 416t^2 + 58t\tau - 136t\overline{\overline{\Gamma}} + 97\tau^2 + 97\tau\overline{\overline{\Gamma}} - s(136t - 194\tau + 97\overline{\overline{\Gamma}})}}{72}$, where $\overline{\overline{\Gamma}} = \sqrt{s^2 + 2s\tau + 4t\tau + \tau^2}$. However, the triplet $p_1^4 *$, p_2^d, and $p_E^4 *$ cannot sustain the demand BLHC for Firm 2 when $s \leq \min[\overset{\frown}{s}, \hat{s}]$, which is the necessary condition for $p_1^4 *$ and $p_E^4 *$ to be the equilibrium prices in Case 4, and there is no interior solution for a deviation that induces demand BLHC for Firm 2. In the same way, we can show that no interior deviation price for Firm 2 can induce either demand IBNDM or demand OGENB for Firm 2.

Consider now a deviation of Firm *E* from (A38), while Firm 1 and Firm 2 set $p_1^4 *$ and $p_2^4 *$, respectively. In this case, the indifferent consumer $\hat{x}$ is unaffected by the variation of Firm *E*'s price. Therefore, there is a symmetric and parallel shift of $\hat{y}_1$ and $\hat{y}_2$ along $\hat{x}$. Based on this, there are two different possible Firm *E*'s demand shapes when it deviates from Case 4 equilibrium prices. Figure A8 illustrates this point.

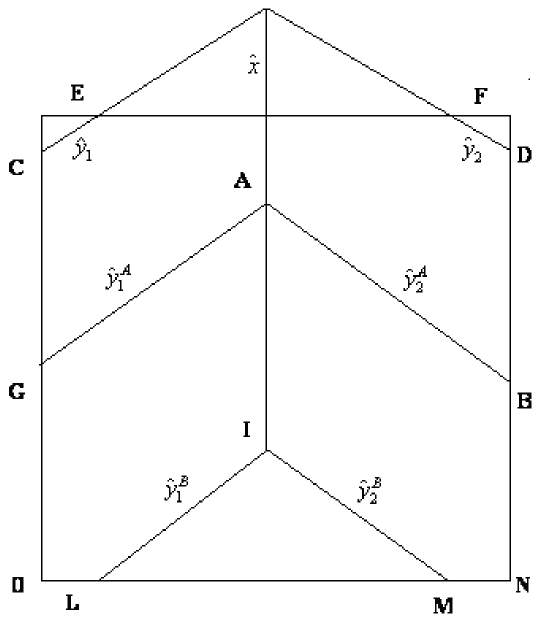

Figure A8: Deviations of Firm *E* from Case 4 equilibrium prices

Suppose that the initial situation is represented by lines $\hat{y}_1$ and $\hat{y}_2$. Firm *E*'s demand is given by area ECONDF. Suppose that Firm *E* increases its price. If such an increase is moderate, the shape of Firm *E*'s demand changes: the new situation is represented by $\hat{y}_1^A$, $\hat{y}_2^A$ and $\hat{x}$, and the demand is given by area AGONB. In contrast, suppose that the increase in Firm *E*'s price is high. The new situation is now represented by $\hat{y}_1^B$, $\hat{y}_2^B$, and $\hat{x}$, and the demand is given by area ILM.

Suppose that Firm E's deviation induces $\hat{y}_1^A$ and $\hat{y}_2^A$. Then, given p_1^4* and p_2^4*, Firm *E* maximizes $\pi_E = p_E q_E$, where q_E is given by area AGONB. The maximization problem leads to $p_E^d = \frac{-7s + 2t + \tau + \overline{\overline{\overline{\Gamma}}}}{16}$. However, the triplet p_1^4*, p_2^4*, and p_E^d cannot sustain the demand AGONB for Firm *E* when $s \le \min[\hat{s}, \hat{s}]$, which is the necessary condition for p_1^4* and p_2^4* to be equilibrium prices in Case 4. Therefore, there is no interior solution for deviation inducing demand AGONB for Firm *E*. Similarly, we can show that no interior deviation price for Firm *E* can sustain the demand ILM for Firm *E*.

We summarize this in the following lemma:

Lemma 4. *In Case 4, a (unique) equilibrium triplet of prices exists,* $\mathbf{P^4}* \equiv \{p_1^4*, p_2^4*, p_E^4*\}$, *if and only if* $s \le \min[\hat{s}, \hat{s}]$, *and*

$$p_1^4* = p_2^4* = \frac{s + \tau + \sqrt{s^2 + 2s\tau + 4t\tau + \tau^2}}{8},$$

$$p_E^4* = \frac{-5s - 5\tau + 3\sqrt{4t\tau + (s + \tau)^2}}{8}.$$

The resulting equilibrium profits in Case 4 are:

$$\pi_1^4* = \pi_2^4* = \frac{(s + \tau + \overline{\overline{\overline{\Gamma}}})^3}{256t\tau}, \tag{A39}$$

$$\pi_E^4* = \frac{(5s + 5\tau + 3\overline{\overline{\overline{\Gamma}}})(s^2 + 2s\tau - 6t\tau + \tau^2 + s\overline{\overline{\overline{\Gamma}}} + \tau\overline{\overline{\overline{\Gamma}}})}{64t\tau}. \tag{A40}$$

Checking the sign of the derivative of π_1^4*, π_2^4*, and π_E^4* with respect to each parameter within the parameter set yields the outcome in Table 2 in the main text.

Given Case 4 only emerges when s is negative, we now show that we can also interpret parameter s as the marginal cost disadvantage of Firms 1 and 2. First, we set $s = 0$ and $\pi_1^4 = (p_1^4 - c)q_1^4$ and $\pi_2^4 = (p_2^4 - c)q_2^4$, where $c > 0$ is the marginal cost of Firm 1 and Firm 2. The equilibrium prices are:

$$p_1^4* = p_2^4* = \frac{7c + \tau + \sqrt{c^2 - 2c\tau + 4t\tau + \tau^2}}{8},$$

$$p_E^4 * -c = \frac{5c - 5\tau + 3\sqrt{4t\tau + (-c+\tau)^2}}{8} - c.$$

It is immediately seen that when $c = -s$ the equilibrium profits are identical under both interpretations.

<u>A different model with two types of consumers (Footnote 15 in Section 4.1)</u>

Instead of considering the continuous heterogeneity of consumers with respect to the disutility arising from the Internet (y distributed from 0 to 1), we consider a different model in which consumers have a disutility either equal to τ or to 0.[28] There is a quota equal to $\alpha\,(1-\alpha)$ of the first (second) type of consumers. Figure A9 illustrates one of the possible market structures.

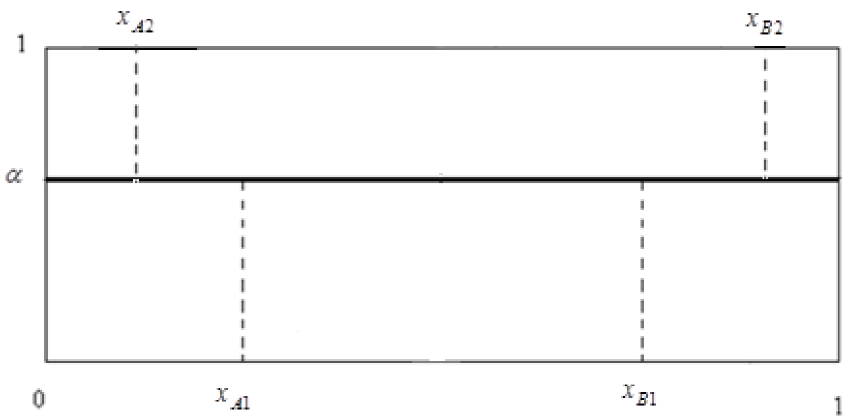

Figure A9: Discrete heterogeneity (I)

x_{A1} and x_{A2} in Figure A9 are obtained by solving $v - p_1 - tx = v - s - p_E - \tau$ and $v - p_1 - tx = v - s - p_E$, respectively. x_{B1} and x_{B2} are also obtained similarly. In Figure A9, there is no face-to-face competition between the offline retailers because they are separate. The profit functions are:

$$\pi_1 = p_1[\alpha x_{A1} + (1-\alpha)x_{A2}],\ \pi_2 = p_2[\alpha(1-x_{B1}) + (1-\alpha)(1-x_{B2})],$$

$$\pi_E = p_E[\alpha(x_{B1} - x_{A1}) + (1-\alpha)(x_{B2} - x_{A2})].$$

[28] The results would be the same for two positive disutility costs, that is τ_1 and τ_2.

Using the maximization problems, we obtain $p_1{*} = p_2{*} = \dfrac{2s+t+2\alpha\tau}{6}$ and $p_E{*} = \dfrac{t-s-\alpha\tau}{3}$. By using these prices into the profits and taking the derivative with respect to τ in the relevant parameter space (i.e., the parameter space that supports the market structure in Figure A9), it is immediately seen that the profits of Firm *E* decrease with τ.

There is another market structure in which Firms 1 and 2 operate in face-to-face competition (see Figure A10). It is immediately seen that τ plays no role in the analysis because it does not enter in the equations of the indifferent consumers. Therefore, the equilibrium profits of Firm *E* do not depend on τ.

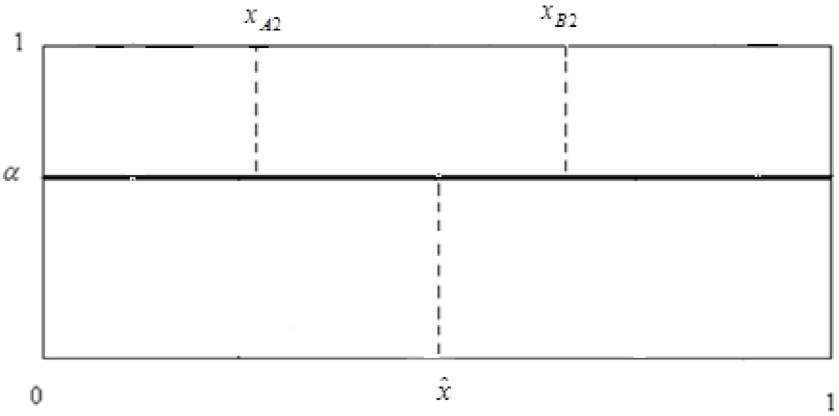

Figure A10: Discrete heterogeneity (II)

<u>Proof of Proposition 1 (Section 4.1)</u>

Using the profit of Firm *E* under Case 3, we calculate the first-order derivative of Firm *E*'s profit with respect to τ, leading to the candidate of the optimal τ, $\tau^* = \dfrac{5s+\sqrt{-11s^2+36st-9t^2}}{9}$. This τ satisfies the second-order condition. The profit of Firm *E* under τ^* is given by

$$\pi_E\big|_{\tau=\tau^*} = \frac{\{-11s+3t-\sqrt{-11s^2+36st-9t^2}+3\sqrt{2}\sqrt{(3t-s)(5s+\sqrt{-11s^2+36st-9t^2})}\}^3}{6000t(5s+\sqrt{-11s^2+36st-9t^2})}.$$

When $s \geq t$, Case 3 is always realized (see Figure 2 in the main text), and then τ^* is globally optimal. When $s < t$, however, we need to compare $\pi_E\big|_{\tau=\tau^*}$ with Firm *E*'s profit when $\tau = 0$, $\pi_E\big|_{\tau=0} = \dfrac{2(s-t)^2}{9t}$. Comparing the = profits, we find that

$\pi_E\big|_{\tau=\tau^*} \geq \pi_E\big|_{\tau=0}$ if and only if $s \geq 0.754949t$.

<u>Proof of Proposition 2 (Section 4.1)</u>

First, note that when $s < 0$, only Cases 1, 2, and 4 are possible (see Figure 2 in the main text). Moreover, the profits of Firm *E* in Case 2 strictly decrease with τ (see Table 2 in the main text). By calculating the first-order derivative of $\pi_E^4 *$ with respect to τ, we can see that it is always negative when $s < 0$. Finally, explicitly calculating the first-order derivative of $\pi_E^1 *$ with respect to τ, we confirm its sign and find that the first-order derivative is always negative if $s < -t/10$ (a sufficient condition).

<u>Uncovered market (Footnote 21 in Section 5)</u>

We consider the case in which not all consumers purchase the final product (known as an *uncovered market*). We focus on Cases 1 and 3 to show that in the uncovered market case, face-to-face competition between offline retailers is not possible. From (1) and (2), we observe that a consumer prefers not to buy instead of buying from Firm 1 if $x > \dfrac{v - p_1}{t}$, whereas a consumer prefers not to purchase instead of buying from Firm 2 if $x < 1 - \dfrac{v - p_2}{t}$. Provided v is sufficiently low, a nonempty subset of consumers located close to the center of the horizontal axis exists such that the consumers purchase from none of the offline retailers. At the same time, from (3), we observe that a consumer prefers not to purchase instead of buying from Firm *E* if $y > \dfrac{v - s - p_E}{\tau}$. If v is sufficiently low, some consumers (those with a high innate disutility from buying on the Internet), do not purchase from the online retailer. The black-shaded area in Figure A11 depicts the resulting subset of consumers that purchase from no firm.[29] We can

[29] Note that the existence of consumers that do not buy from Firm 1 and Firm 2 implies the existence of consumers that do not buy from Firm *E* under the structures in Case 1 and 3, that is, $y > (v - s - p_E)/\tau$ must lie below the intersection between $\hat{y}_1$ and $\hat{y}_2$.

then see that face-to-face competition between Firm 1 and Firm 2 is impossible in uncovered markets.

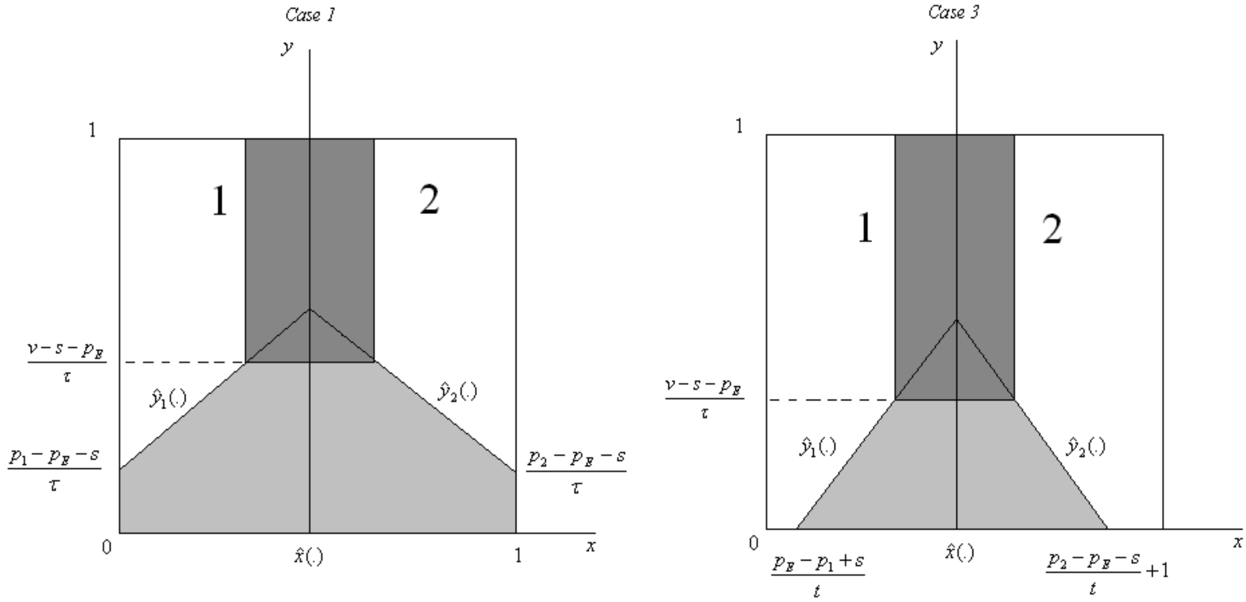

Figure A11: Case 1 and Case 3 with uncovered markets

<u>**Multichannel retailer (Footnote 22 in Section 5)**</u>

We are interested in showing that minimizing consumer-specific disutility may not be the optimal choice for the multichannel retailer. For this purpose, we perform a numerical comparison between a situation in which $\tau = 0$ and one in which $\tau > 0$. Assume that $t = 1/10$ and $s = 1/10000$. We begin by considering the case of $\tau = 0$, in which the demand functions are represented in Figure A12.

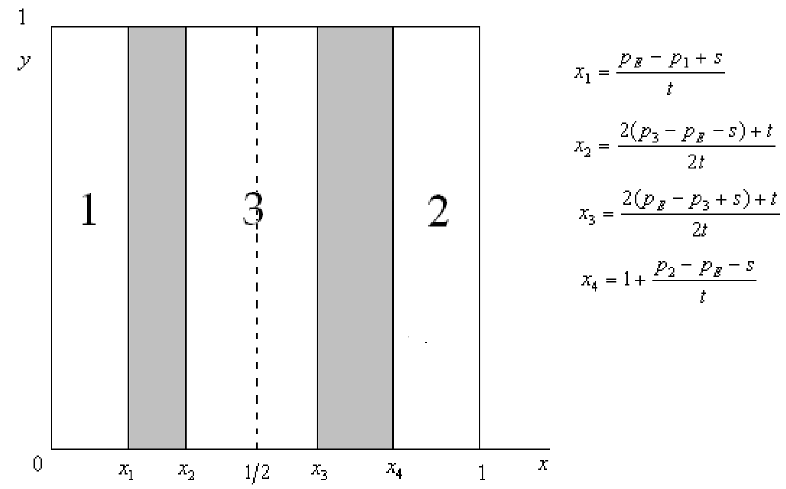

Figure A12: A multichannel retailer when τ = 0

In Figure A12, x_1 (resp. x_4) represents the indifferent consumer between offline retailer 1 (resp. 2) and the online retailer. In addition, x_2 and x_3 represent the consumers who are indifferent between the online retailer and offline retailer 3. Therefore, the multichannel retailer's demand is given by Area 3—the demand of the offline retailer—plus the gray-shaded area—the demand of the online retailer. Under the previous parameter specification, the equilibrium prices are $p_1{*} = p_2{*} = 0.0167$, $p_3{*} = 0.03335$, and $p_E{*} = 0.033$.[30] Consequently, the profits of the multichannel retailer are $\pi_{3+E}{*} = 0.0222$.

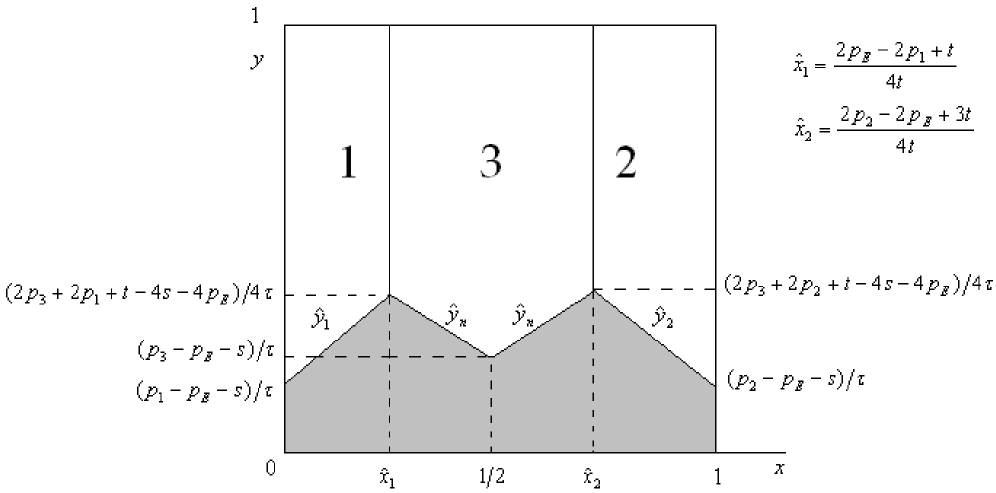

Figure A13: A multichannel retailer when τ>0

Consider now a possible situation in which $\tau > 0$. In particular, suppose that $\tau = 1/8$ and that the demand configuration is as represented in Figure A13. In Figure A13, $\hat{y}_1$ (resp. $\hat{y}_2$) represents the consumer indifferent between offline retailer 1 (resp. 2) and the online retailer, $\hat{x}_1$ (resp. $\hat{x}_2$) represents the consumer indifferent between the offline retailer 1 (resp. 2) and online retailer 3, and $\hat{y}_n$ represents the consumer indifferent between the offline retailer 3 and online retailer 3. Under the previous parameter specification, the equilibrium prices of each retailer are easily shown to be $p_1{*} = p_2{*} = 0.0402$, $p_3{*} = 0.0458$, and $p_E{*} = 0.0378$,[31] whereas the equilibrium profit of the mixed firm is $\pi_{3+E}{*} = 0.0225$. A comparison with the case in which $\tau = 0$

[30] Note that the equilibrium prices support the demand configuration represented in Figure A12.
[31] Note that these prices support the demand functions as represented in Figure A13.

immediately indicates that the profits of the multichannel retailer are higher when $\tau = 1/8$. Therefore, the multichannel retailer will not minimize profits when the consumer-specific disutility from the Internet is set to zero.