

**A SIMPLE AGGREGATE DEMAND ANALYSIS
WITH DYNAMIC OPTIMIZATION
IN
A SMALL OPEN ECONOMY**

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July 2019

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A simple aggregate demand analysis with dynamic optimization in a small open economy^{*}

by

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Abstract

We develop an aggregate demand analysis of a small open economy based on all agents' dynamic optimization. Murota and Ono (2015) present a simple Keynesian cross analysis with dynamic optimization. This paper extends it to a small-country setting with two factors and two commodities, of which the structure is as simple as the conventional Keynesian cross analysis. We apply the model to examine the effects of changes in various parameters, such as the terms of trade, foreign asset holdings and government purchases, on aggregate demand. They are quite different from those under full employment and those of the Mundell-Fleming model.

Keywords: aggregate demand shortage, unemployment, small open economy

JEL Classification Number: E12, E13, E21, E24, F32, F41

^{*} This research was financially supported by the program of the Joint Usage/Research Center for “Behavioral Economics” at ISER, Osaka University, and Grants-in-Aid for Scientific Research (A), (C) and (S) from the JSPS: grant numbers 15H05728, 16H02016, 16K03624, and 19K01638.

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1. Introduction

This paper presents a simple aggregate demand analysis with dynamic optimization of a two-factor two-commodity small open economy that suffers from secular stagnation of aggregate demand, and examines the effects of changes in fiscal and monetary policies, the terms of trade, foreign asset holdings, and preference and technology parameters, on consumption and aggregate demand.

A typical classical analysis on aggregate demand fluctuations in a small open economy is the “Mundell-Fleming” model (Mankiw, 2010; Ch.12). It is an extension of the conventional Keynesian model to an open-economy setting. Although it has widely been used in policy making thanks to its simplicity, probably, it neither considers the optimal firm and household behavior nor gives the dynamics of economic variables. Instead, it begins with assuming such ad-hoc functions as the consumption, investment, net export, and liquidity demand functions, and ignores the current account adjustment. Thus, it is essentially a short-run analysis. Dornbusch (1980) introduced price and exchange-rate dynamics into Mundell-Fleming model, but still ignored optimal firm and household behavior.

Since the Lucas critique (1976), some micro-foundations have always been required for macroeconomic analyses and those behavioral functions must endogenously be derived from optimizing behavior of agents. Unfortunately, however, most of the recent researches on macroeconomic dynamics with micro-foundations, such as RBC and DSGE models (e.g., Kydland and Prescott, 1982; Christiano, Eichenbaum and Evans, 1999; Hayashi and Prescott, 2002; Walsh, 2017), do not treat aggregate demand shortages, which many countries are now facing, but analyze dynamic adjustment processes without aggregate demand shortages.

A dynamic optimization model of secular stagnation due to aggregate demand shortages was first presented by Ono (1994, 2001) in a closed-economy setting. He showed that if the marginal utility of money (or wealth) holding is insatiable, aggregate demand deficiency and

deflation can persistently emerge. Although real money balances expand under deflation, the marginal rate of substitution between money and consumption does not change because of the insatiability, and thus the wealth effect of real money expansions on consumption disappears. While lower prices as a result of deflation will not stimulate consumption, deflation itself makes it more advantageous for people to save more and consume less, leading to a steady state with secular deflation and stagnation.¹ Using the model Murota and Ono (2015) obtain a consumption function with similar mathematical properties to the conventional Keynesian consumption function and examine the multiplier effect of macroeconomic policies on aggregate demand.

In this paper we extend the new Keynesian cross analysis by Murota and Ono (2015) to a small open economy setting with two factors and two commodities and propose a simple analytical framework like the Mundell-Fleming model. Using it we obtain the effects of changes in various policy, preference and technological parameters on consumption and aggregate demand, and show that they are quite different from those under full employment and those of the Mundell-Fleming model. There are open-economy extensions of the dynamic stagnation model, e.g. Ono (2006, 2007, 2014, 2018), Johdo and Hashimoto (2009) and Hashimoto (2011, 2015), but they use a two-country setting and have much more complicated frameworks. The present analysis treats a small country case and has a simple structure comparable to the Mundell-Fleming model.

¹ This model has widely been used in various analyses under persistent stagnation in a closed-economy setting. For example, Matsuzaki (2003) studies the effect of a consumption tax on effective demand in the presence of poor and rich people. Hashimoto (2004) examines the intergenerational redistribution effects in an overlapping-generations framework with the present type of stagnation. Johdo (2006) considers the relationship between R&D subsidies and unemployment. Rodriguez-Arana (2007) examines the dynamic path with public deficit in the present stagnation case and compares it with that of the neoclassical case. Johdo (2009) introduce habit formation preference on consumption with non-satiated liquidity preference, and Murota and Ono (2011) introduce status preference for asset holdings. Hashimoto and Ono (2011) examine the effects of various pro-population policies under this type of stagnation. Murota and Ono (2012) find the properties of zero interest rate in the stagnant economy. Illing, Ono and Schlegl (2018) analyze financial market imperfections in a stagnant economy. See Ono and Hashimoto (2012) for various extensions of this stagnation model. Recently, this stagnation mechanism has been discussed also by Michailat and Saez (2014) and Michau (2018).

The remainder of this paper proceeds as follows. Section 2 outlines the model structure. After discussing the case of full employment as a benchmark in section 3, we consider the case where secular unemployment and stagnation occur and examine effects of changes in the terms of trade and various policy, preference and technological parameters in section 4. It is found that those effects are opposite to those under full employment and significantly differ from those in the Mundell-Fleming model. The final section summarizes our findings and concludes the paper.

2. The Model

We consider a small open economy with two factors, labor and capital, and two commodities, 1 and 2, in a continuous infinite-time setting. The home country specializes in commodity 1 while the rest of the world produces both commodities, which are tradable.² The nominal home prices of the two commodities are P_1 and $P_2 (= \varepsilon P_2^*)$, where ε is the nominal exchange rate and P_2^* is the international price of the foreign commodity in terms of the foreign currency. Due to the small-country assumption, the international relative price $\bar{\omega}$ of commodity 2 in terms of commodity 1 is exogenously given to the home country and thus the nominal exchange rate ε changes so that it always satisfies

$$\bar{\omega} \equiv \frac{\varepsilon P_2^*}{P_1} = \frac{P_2}{P_1}.$$

Capital and assets freely move into, and out of, the country and thus the real interest rate equals the exogenously given world interest rate \bar{r} , which satisfies

$$r = R - \pi = \bar{r}, \tag{1}$$

where $\pi \equiv \dot{P}/P$ represents the inflation (or deflation if negative) rate of the home consumer

² Under free capital movement across countries, which we assume soon, a small country must specialize in one of the two commodities. Using a dynamic 2x2x2 model with capital accumulation Ono and Shibata (2006) prove that under free asset trade instead of free capital movement a country with a much smaller population specializes in one of the two commodities while the other country produces both commodities.

price index P and R is the nominal interest rate in terms of the home currency. The no-arbitrage condition requires

$$R = \dot{\varepsilon}/\varepsilon + \bar{R}, \quad (2)$$

where \bar{R} is the nominal interest rate in terms of the foreign currency, which is given to the home country.

2.1. Households

The population of home households is normalized to unity and their labor endowment is one unit, which is inelastically supplied. The lifetime utility of the representative household is

$$U = \int_0^\infty [\hat{u}(c_1, c_2) + v(m)] \exp(-\rho t) dt,$$

where $\hat{u}(c_1, c_2)$ represents homothetic utility of consumption, c_j ($j = 1, 2$) is consumption of commodity j , ρ is the subjective discount rate and $v(m)$ is utility of real money holdings m . It is maximized subject to the flow budget equation and asset constraint:

$$\begin{aligned} \dot{a} &= \bar{r}a + wx - p_1c_1 - p_2c_2 - Rm - \tau, \\ a &= m + b, \end{aligned} \quad (3)$$

where a is total asset consisting of real balances m and international asset-capital b , τ is the real lump-sum tax, $w \equiv W/P$ is the real wage, p_j ($j = 1, 2$) is the real price of commodity j , and x is the employment rate. The employment rate x is positive and may be lower than 1:

$$1 \geq x \geq 0,$$

because involuntary unemployment may appear. Because $\hat{u}(c_1, c_2)$ is homothetic, real prices p_j ($j = 1, 2$) depend on only the relative price \bar{w} and satisfy

$$p_j \equiv \frac{p_j}{P} = p_j(\bar{w}); \quad p'_1(\bar{w}) < 0, \quad p'_2(\bar{w}) > 0.$$

Given the current value Hamiltonian function H of the household optimizing behavior:

$$H = \hat{u}(c_1, c_2) + v(m) + \lambda[\bar{r}a + wx - p_1(\bar{w})c_1 - p_2(\bar{w})c_2 - Rm - \tau],$$

where λ is the costate variable for a , the first-order optimal conditions are

$$\hat{u}_1(c_1, c_2) = \lambda p_1, \quad \hat{u}_2(c_1, c_2) = \lambda p_2, \quad v'(m) = \lambda R, \quad \dot{\lambda} = (\rho - \bar{r})\lambda. \quad (4)$$

Because $\hat{u}(c_1, c_2)$ is homothetic, from (4) one has

$$\frac{p_2}{p_1} (= \bar{\omega}) = \frac{\hat{u}_2(c_1, c_2)}{\hat{u}_1(c_1, c_2)} = \frac{\hat{u}_2(1, c_2/c_1)}{\hat{u}_1(1, c_2/c_1)},$$

implying that c_2/c_1 is a function of only $\bar{\omega}$. Therefore, the ratio of consumption expenditure allocated to each commodity depends only on the relative price:

$$p_1(\bar{\omega})c_1 = \delta(\bar{\omega})c, \quad p_2(\bar{\omega})c_2 = [1 - \delta(\bar{\omega})]c, \\ c = p_1c_1 + p_2c_2, \quad (5)$$

where c is real total consumption and $\delta(\cdot)$ is the ratio of consumption expenditure allocated to commodity 1.

Because of the homothetic utility, $\hat{u}(c_1, c_2)$ in which c_1 and c_2 take the optimal levels given by (5) must be independent of the relative price $\bar{\omega}$ so that one can exhibit $\hat{u}(c_1, c_2)$ as $u(c)$:

$$u(c) \equiv \hat{u}\left(\frac{\delta}{p_1}c, \frac{1-\delta}{p_2}c\right).$$

From (5) and this equation we obtain

$$u'(c) = \delta \frac{\hat{u}_1}{p_1} + (1 - \delta) \frac{\hat{u}_2}{p_2} = \lambda.$$

From (1), (4) and the above property, we obtain the Euler equation and the money demand function:

$$\eta \frac{\dot{c}}{c} = R - \pi - \rho = \bar{r} - \rho, \quad (6)$$

$$R = \frac{v'(m)}{u'(c)}, \quad (7)$$

where $\eta \equiv -u''(c)c/u'(c)$ is the elasticity of marginal utility of consumption.

As is standard in small open economy models, we assume

$$\rho = r = \bar{r},$$

since otherwise this country will eventually be a large country (if $\rho < \bar{r}$) or disappears (if $\rho > \bar{r}$). By applying this property to (7) we find

$$\frac{\dot{c}}{c} = 0, \quad (8)$$

implying that consumption c is constant over time and that access to the international asset-capital market makes consumption completely smoothed. Finally, the transversality condition is

$$\lim_{t \rightarrow \infty} \lambda a \exp(-\bar{r}t) = 0.$$

2.2. Firms

The home country specializes in commodity 1 and the technology is of constant returns to scale with respect to labor x and capital k as follows:

$$\theta F(x, k) = \theta f(n)k, \quad n \equiv \frac{x}{k},$$

where θ is the total productivity. Because capital freely moves into the country, the first-order optimal conditions to maximize profits $p_1(\bar{\omega})\theta f(n)k - wx - \bar{r}k$ are

$$p_1(\bar{\omega})\theta f'(n) = w, \quad p_1(\bar{\omega})\theta[f(n) - nf'(n)] = \bar{r}. \quad (9)$$

Because the international relative price $\bar{\omega}$ and the real interest rate $\bar{r}(= \rho)$ are given to the home country, w is fully determined by exogenous parameters θ and $\bar{\omega}$ and satisfies

$$w = w(\theta, \bar{\omega}); \quad w_\theta = \frac{p_1(\bar{\omega})f(n)}{n} > 0, \quad w_{\bar{\omega}} = \frac{p'_1(\bar{\omega})\theta f(n)}{n} < 0, \quad (10)$$

where a subscript of w represents a partial derivative with respect to it. An increase in the labor productivity obviously increases the real wage. An increase in the relative price of the foreign commodity works as if the productivity of the home commodity decreases and thus the real wage declines.

2.3. Government

Suppose that the government expands the money supply M^S at a constant rate $\mu \equiv \dot{M}^S/M^S$ and purchases g_1 and g_2 of the two commodities, respectively. Then, the government's budget constraint is

$$\tau + \frac{\mu M^S}{P} = g(\equiv p_1(\bar{w})g_1 + p_2(\bar{w})g_2), \quad (11)$$

where g represents aggregate government purchases.³

2.4. Market adjustments

Because the real money holdings m always satisfies

$$m = \frac{M^S}{P}, \quad (12)$$

the dynamics of m is

$$\frac{\dot{m}}{m} = \mu - \pi,$$

where μ is the money growth rate. Substituting this equation and the government budget constraint (11) into the flow budget equation in (3) yields the dynamics of international asset-capital holdings:

$$\dot{b} = \underbrace{\bar{r}b + w(\theta, \bar{w})x}_{\text{national income}} - (c + g), \quad (13)$$

where international asset-capital b is the sum of domestic capital k and foreign asset-debt b^f and hence

$$b = k + b^f, \quad \dot{b} = \dot{k} + \dot{b}^f.$$

Thus, (13) is equivalent to the current account equation:

$$\dot{b}^f = \bar{r}b^f + \underbrace{\bar{r}k + w(\theta, \bar{w})x}_{\text{GDP}} - \underbrace{(c + i + g)}_{\text{aggregate demand}}.$$

where i is real domestic investment \dot{k} .

³ The balanced budget is assumed merely for simplicity. Because we take into account the flow budget equation, the Ricardian equivalence holds. Thus, even if the government issues public bonds and adopts a deficit budget, the following analysis is valid.

The nominal wage adjustment in the labor market is perfect upward but sluggish downward and follows

$$\frac{\dot{W}}{W} = \alpha(x - 1) \quad \text{if } x < 1,$$

where α represents the adjustment speed of the nominal wage.⁴ It is because workers resist a decline in W but welcome a rise in W no matter how fast it is. Because real wage w is constant over time from (10), this wage adjustment yields the inflation rate of the commodity price as follows:

$$\pi \equiv \frac{\dot{P}}{P} = \begin{cases} \frac{\dot{W}}{W} = \alpha(x - 1) & \text{if } x < 1, \\ \mu & \text{if } x = 1. \end{cases} \quad (14)$$

This implies that in the presence of aggregate demand shortages P follows the movement of W while under full employment W follows the movement of P . The asymmetry in the inflation process is a fundamental element of stagnation models including among others the contributions of Eggertsson, Mehrotra and Robbins (2017), Schmitt-Grohé and Uribe (2016, 2017), Michau (2018) and Illing, Ono and Schlegl (2018). Stagnation typically results from some form of downward nominal wage rigidity that arises in case of unemployment.

3. Full employment

Using the model presented in the previous section, we will propose a simple analytical framework of aggregate demand fluctuations that can replace the conventional Mundell-Fleming model, and apply it to examine the effects of changes in various policy, preference and technological parameters and the terms of trade. Before doing so, this section treats the case of full employment as a benchmark and shows policy implications, which will

⁴ This assumption is imposed so that the possibility of unemployment is not intrinsically avoided. Obviously, this assumption does not eliminate the possibility of full employment steady state. Ono and Ishida (2014) give a micro-foundation of wage adjustment under which the adjustment converges to this form if stagnation occurs in steady state.

be later compared with those under secular stagnation.

From (13) in which $x = 1$, we find

$$\dot{b} = \bar{r}b + w(\theta, \bar{\omega}) - (c + g),$$

which is unstable with respect to b . Furthermore, from (8), c is constant over time. Hence, c must initially jump so that $\dot{b} = 0$ and then,

$$c = c_F \equiv \bar{r}b_0 + w(\theta, \bar{\omega}) - g, \quad (15)$$

where b_0 is the initial holding of international asset-capital. Because $\pi = \mu$ under full employment from (14), we replace c and π in (6) and (7) by c_F in (15) and μ respectively and obtain

$$\rho + \mu = R = \frac{v'(m)}{u'(c_F)}, \quad (16)$$

which gives the steady state level of m . The full-employment consumer price index P moves in parallel with M^S so that m satisfies (12).

Noting that $w(\theta, \bar{\omega})$ satisfies (10), from (15) we find the effect of changes in the parameters on the full-employment consumption c_F :

$$\theta \uparrow, \bar{\omega} \downarrow, b_0 \uparrow \Rightarrow c_F \uparrow; \quad g \uparrow \Rightarrow c_F \downarrow,$$

$$M^S \text{ or } \mu \text{ has no effect on } c_F. \quad (17)$$

An increase in productivity θ , an improvement in the terms of trade ($\bar{\omega} \downarrow$), and a larger b_0 naturally raise national income and hence increases consumption while an increase in government purchases g crowds out consumption. As for monetary policy, the super neutrality of money holds: Neither an instantaneous jump of money supply M^S nor an increase in the monetary expansion rate μ affects consumption.

From (2) and (16), we obtain

$$\frac{\dot{\varepsilon}}{\varepsilon} = \rho + \mu - \bar{R}. \quad (18)$$

Thus, an expansion in μ raises the depreciation speed of the home currency while changes in

$(\bar{\omega}, b_0, \theta, g)$ have no effect on it.

4. Secular demand stagnation

Let us now explore the possibility of persistent unemployment and secular stagnation. We will obtain the condition under which the steady state with full employment cannot be reached in a small open economy, and show that a liquidity trap plays a crucial role in leading the economy to this situation. Secular stagnation that arises under a liquidity trap in a dynamic optimization setting was first analyzed by Ono (1994, 2001) in a closed-economy setting.⁵ We apply the model to the present setting.

4.1. Steady state with secular stagnation

A liquidity trap in the present setting arises if the desire for money holding is insatiable:

$$\lim_{m \rightarrow \infty} v'(m) = \beta > 0,$$

where β is a positive constant.⁵ Then, the shape of the money demand curve represented by (7) is as illustrated in Figure 1. In this case it is clear that the solution of m in (16) does not exist if c_F is so large as to satisfy

$$\rho + \mu < \frac{\beta}{u'(c_F)} (< \frac{v'(m)}{u'(c_F)} \text{ for any } m). \quad (19)$$

From (15) c_F equals home national income minus government purchases, and thus (19) implies that a richer country tends to fall in secular stagnation.

Now we obtain the steady state with secular stagnation and involuntary unemployment. In the presence of unemployment ($x < 1$) nominal wages and prices continue to decline in the

⁵ Ono (1994: 4–8) gives an extensive survey on the insatiable utility of money in the history of economic thought (e.g., Veblen, Marx, Simmel, Keynes). Ono (1994: 34–8) uses the GMM (generalized method of moments) to show the validity of this property while Ono, Ogawa and Yoshida (2004) apply parametric and nonparametric methods to support this property.

way represented by (14), which keeps expanding real balances and lead to $v'(m) = \beta$.⁶ Thus, from (6), (7) and (14), we obtain

$$\frac{\beta}{u'(c)} = \rho + \alpha(x - 1). \quad (20)$$

Because consumption c is constant over time, as proven in (8), c initially jumps to the steady state value making $\dot{b} = 0$ and stays there. Otherwise, b keeps either expanding if initially $\dot{b} > 0$, violating the transversality condition, or decreasing if initially $\dot{b} < 0$, which is infeasible.

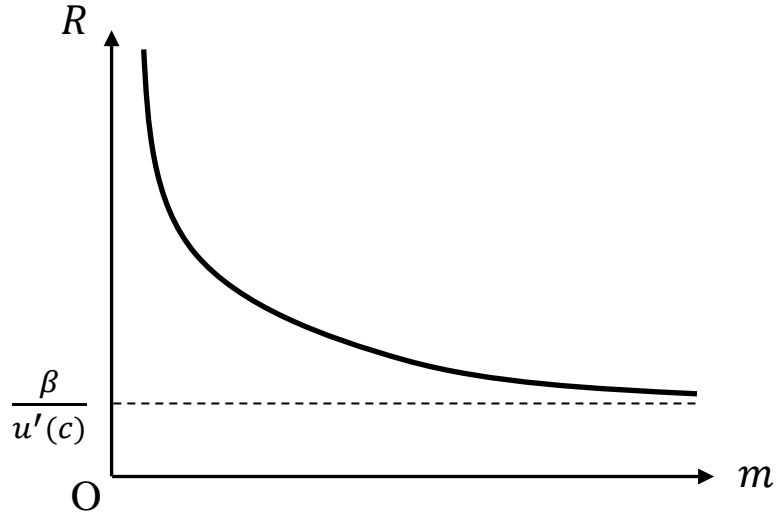


Figure 1: Money demand with a liquidity trap

As there is no dynamics of b , the international asset-capital remains at the initial level b_0 and hence from (13) we always have

$$\dot{b} = \bar{r}b_0 + w(\theta, \bar{\omega})x - c - g = 0,$$

which gives

⁶ This deflation path satisfies the transversality condition although real balances keep expanding because the nominal interest rate $R = \beta/u'(c)$ is positive and hence $\dot{m}/m = \mu - \pi = \mu - R + \bar{r} < \bar{r}$ as long as μ is smaller than R .

$$0 < x = \frac{c+g-\bar{r}b_0}{w(\theta, \bar{\omega})} < 1. \quad (21)$$

Substituting this x to (20) yields

$$\Phi(c) \equiv \frac{\beta}{u'(c)} - \left(\rho + \alpha \left(\frac{c+g-\bar{r}b_0}{w(\theta, \bar{\omega})} - 1 \right) \right) = 0, \quad (22)$$

which gives the equilibrium level of c .

From (15), (19) and (22) we find

$$\Phi(c_F) = \frac{\beta}{u'(c_F)} - \rho > 0,$$

in the present case. Thus, in order for the equilibrium c given by (22) to exist, the following inequality must be satisfied,

$$\Phi(c_L) = \frac{\beta}{u'(c_L)} - \left(\rho + \alpha \left(\frac{c_L+g-\bar{r}b_0}{w(\theta, \bar{\omega})} - 1 \right) \right) < 0, \quad c_L \equiv \max\{0, \bar{r}b_0 - g\},$$

where c_L is the lowest possible value of consumption, which is either zero or the disposable income in the case where employment is zero. Furthermore, if the equilibrium is unique, $\Phi(c)$ must be positively inclined in the neighborhood of the equilibrium c —i.e.,

$$\Phi'(c) = \frac{-\beta u''(c)}{(u'(c))^2} - \frac{\alpha}{w(\theta, \bar{\omega})} > 0. \quad (23)$$

4.2. Revised Keynesian cross in a small open economy

We now reformulate the present model to a small-country version of the new Keynesian cross analysis of Murota and Ono (2015). Let us define y to be aggregate demand $c + g$:

$$y = c + g,$$

where real investment i does not appear because the economy is always in steady state, as mentioned above (21). Replacing $c + g$ in (22) by y yields

$$\frac{\beta}{u'(c)} = \rho + \alpha \left(\frac{y-\bar{r}b_0}{w(\theta, \bar{\omega})} - 1 \right) \Rightarrow c = c(y; w(\theta, \bar{\omega}), \alpha, \beta, b_0). \quad (24)$$

This is the revised consumption function with dynamic optimization, showing the relationship

between aggregate demand y and consumption c . If $b_0 = 0$, this function reduces to

$$\frac{\beta}{u'(c)} = \rho + \alpha \left(\frac{y}{w} - 1 \right),$$

which is the same as the consumption function in the closed-economy model of Murota and Ono (2015). Using the conditions in (23) we obtain the following properties:

$$1 > c_y = \frac{\alpha/w(\theta, \bar{w})}{-\beta u''/(u')^2} > 0,$$

$$c(y_L; w(\theta, \bar{w}), \alpha, \beta, b_0) = u'^{-1} \left(\frac{\beta}{\rho + \alpha \left(\frac{y_L - \bar{r}b_0}{w(\theta, \bar{w})} - 1 \right)} \right) > 0, \quad y_L \equiv \max\{0, \bar{r}b_0\}, \quad (25)$$

which are mathematically the same as those of the conventional Keynesian consumption function: (i) the derivative with respect to y is positive and smaller than 1, and (ii) the consumption level is positive when $y = y_L$.

However, the implication completely differs between the two consumption functions. The conventional consumption function regards y as income and assumes that a household spends only a part of income on consumption in the same period. Thus, a tax straightforwardly reduces consumption while a subsidy expands it. However, this assumption is severely criticized because of its ad-hocness. The present consumption function, in contrast, is derived from household and firm optimizing behavior, market equilibrium conditions and wage and price adjustments. In this function y is not income but aggregate demand. It creates employment, raises inflation (or mitigates deflation) and thereby stimulates consumption.

Because aggregate demand y equals $c + g$, using the revised consumption function given by (24) we find

$$y = c(y; w(\theta, \bar{w}), \alpha, \beta, b_0) + g, \quad (26)$$

which determines the equilibrium levels of y and c . From (25) and (26) we find the multiplier effect of government purchases like the conventional one and the effects of the other parameters:

$$\frac{dy}{dg} = \frac{1}{1-c_y} > 1, \quad \frac{dc}{dg} = \frac{c_y}{1-c_y} > 0;$$

$$\frac{dy}{dM^S} = 0, \quad \frac{dy}{d\mu} = 0;$$

$$\frac{dy}{dz} = \frac{dc}{dz} = \frac{c_z}{1-c_y} \quad \text{for } z = w(\theta, \bar{w}), \alpha, \beta, b_0,$$

Noting that $x < 1$ under stagnation, from (24) we have

$$\begin{aligned} c_w &= -\frac{\alpha x}{-w\beta u''/(u')^2} < 0, \quad c_\alpha = \frac{x-1}{-\beta u''/(u')^2} < 0, \\ c_\beta &= -\frac{u'}{-\beta u''} < 0, \quad c_{b_0} = -\frac{\rho\alpha/w}{-\beta u''/(u')^2} < 0. \end{aligned} \quad (27)$$

Because w increases as θ is larger and decreases as \bar{w} rises, as shown in (10), the effects of the parameter changes are summarized as follows:

$$\begin{aligned} g \uparrow &\Rightarrow y \uparrow, c \uparrow; \\ M^S \text{ or } \mu &\text{ has no effect on } y, c; \\ \theta \uparrow, \bar{w} \downarrow &\Rightarrow w \uparrow, y \downarrow, c \downarrow; \\ \alpha \uparrow, \beta \uparrow, b_0 \uparrow &\Rightarrow y \downarrow, c \downarrow. \end{aligned} \quad (28)$$

As shown in (17), when full employment prevails, government purchases crowd out consumption c_F while an increase in productivity θ , an improvement in the terms of trade $\bar{w} \downarrow$ and an increase in international asset-capital holdings b_0 expand c_F . Thus, the results in (28) are opposite to the results under full employment. They are also quite different from those of the Mundell-Fleming model. In the Mundell-Fleming model a fiscal expansion has no effect on consumption or national income while a monetary expansion expands both of them (see Mankiw, 2010, Ch.12).

Using (26) we propose a Keynesian-cross-like analysis that diagrammatically gives the results in (28). Figure 2 illustrates the 45 degree line showing the left-hand side of (26) and the curve of $c + g$ given by the right-hand side of (26). From (26) and (27), we find that increases in g and \bar{w} shifts the right-hand side of (26) upward while increases in θ , α , β and b_0 shifts it

downward, and hence the results on y given in (28) obtain in the figure.

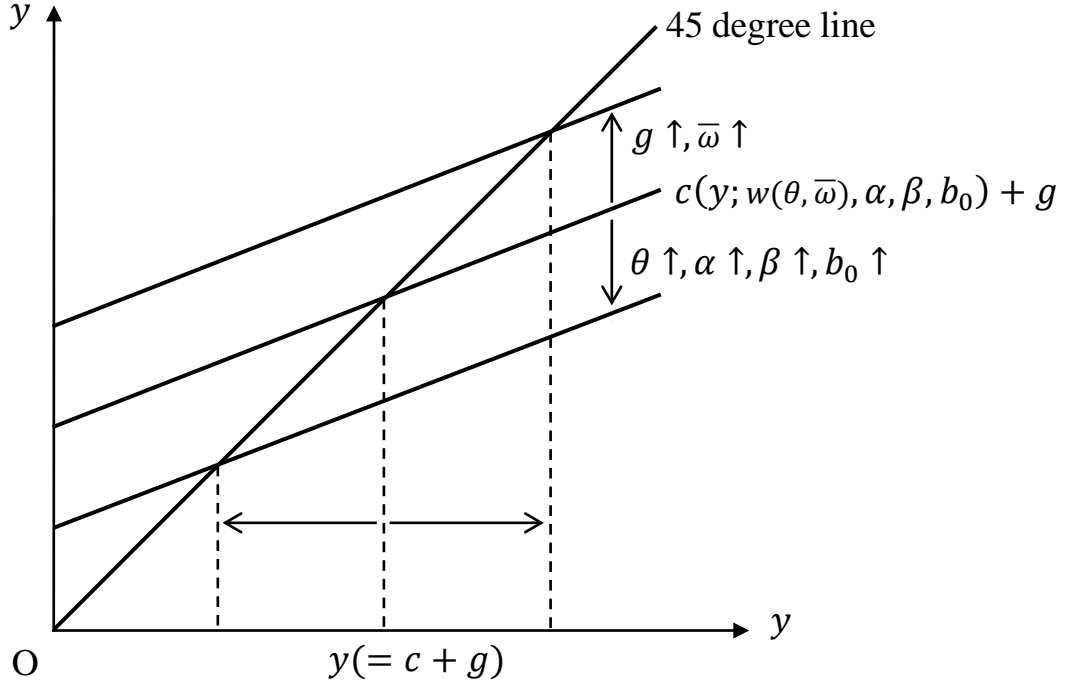


Figure 2: Keynesian cross in a small open economy

Let us intuitively discuss the implications of the above results. An increase in government purchases worsens the current account given by (13), which creates a depreciation pressure on the home exchange rate. However, because the country is small, even an infinitesimal currency depreciation increases demand for the home commodity and expand employment, which mitigates deflation and thereby stimulates consumption. Note that the present multiplier effect of government purchases on consumption emerges although the government budget is balanced while in the conventional Keynesian model it does only under a deficit budget.⁷ An increase in the money stock M^S or the monetary expansion rate μ has no impact on any

⁷ In the present model the multiplier effect emerges regardless of whether the government adopts a balanced budget or a deficit budget because the Ricardian equivalence holds.

variable.

A deterioration in the terms of trade ($\bar{\omega} \uparrow$) increases world demand for the home commodity and expands employment, which mitigates deflation and stimulates consumption. An increase in the wage adjust speed (α) in the presence of involuntary unemployment worsens deflation and lowers consumption. An increase in the liquidity preference (β) directly shifts the consumption function downward and thus worsens deflation, which further reduces consumption. An improvement in the productivity (θ) widens the deflationary gap and worsens deflation, urging households to reduce consumption. An increase in international asset-capital holdings b_0 raises the interest earnings and improves the current account. It yields an appreciation pressure on the home currency and worsens home employment. Consequently, deflation worsens and consumption declines.

Finally, we obtain the effect on the nominal exchange rate ε . From the no-arbitrage condition of the international asset-capital in (2), the depreciation speed of the home currency is

$$\frac{\dot{\varepsilon}}{\varepsilon} = \frac{\beta}{u'(c)} - \bar{R},$$

i.e., the depreciation speed changes in the same direction as c :

$$d\left(\frac{\dot{\varepsilon}}{\varepsilon}\right) = \frac{-\beta u''(c)}{(u'(c))^2} dc.$$

Therefore, a parameter change stimulating consumption has a positive impact on the depreciation speed of the home currency. This result is also quite different from that under full employment. Under full employment the depreciation speed of the home currency depends only on the monetary expansion rate μ and a larger μ increases it, as presented in (18). Under secular stagnation, however, μ has no effect on the depreciation speed because it affects neither c nor the nominal interest rate $R(= \beta/u'(c))$.

4.3. Numerical example

To get the intuition of the magnitudes of the multiplier effects let us present quantitative examples of the effects of changes in various parameters, such as government purchases g , international asset-capital holdings b_0 , total productivity θ and the relative price (or the terms of trade) \bar{w} . To do so, we specify the production and utility functions as follows:

$$\begin{aligned}\theta F(x, k) &= \theta x^\gamma k^{1-\gamma}, \quad \theta f(n) = \theta n^\gamma, \\ \hat{u}(c_1, c_2) &= \ln(\kappa_1 c_1^\sigma + \kappa_2 c_2^\sigma)^{1/\sigma}.\end{aligned}\tag{29}$$

By applying the production function in (29) to the optimizing condition of firms in (9), we find the real wage rate given in (10) to be

$$w = w(\theta, \bar{w}) = [\theta p_1(\bar{w}) \gamma^\gamma (1 - \gamma)^{1-\gamma} \bar{r}^{-(1-\gamma)}]^\frac{1}{\gamma}.\tag{30}$$

The consumption utility function in (29) gives the consumer price index P , the real price $p_1(\bar{w})$, and the expenditure share $\delta(\bar{w})$ of commodity 1 as follows:

$$\begin{aligned}P &= \left[\kappa_1^{1/(1-\sigma)} P_1^{-\sigma/(1-\sigma)} + \kappa_2^{1/(1-\sigma)} (\varepsilon P_2^*)^{-\sigma/(1-\sigma)} \right]^{-(1-\sigma)/\sigma}, \\ p_1(\bar{w}) &= \left[\kappa_1^{1/(1-\sigma)} + \kappa_2^{1/(1-\sigma)} \bar{w}^{-\sigma/(1-\sigma)} \right]^{(1-\sigma)/\sigma}, \\ \delta(\bar{w}) &= \frac{\kappa_1^{1/(1-\sigma)}}{\kappa_1^{1/(1-\sigma)} + \kappa_2^{1/(1-\sigma)} \bar{w}^{-\sigma/(1-\sigma)}}.\end{aligned}\tag{31}$$

Substituting c_1 and c_2 given in (5) into the consumption utility (29) and applying $p_1(\bar{w})$ and $\delta(\bar{w})$ in (31) to the result yields

$$\hat{u}(c_1, c_2) = \ln(\kappa_1 c_1^\sigma + \kappa_2 c_2^\sigma)^{1/\sigma} = \ln c \quad (\equiv u(c)).$$

Thus, the consumption function given in (24) turns to be

$$c = \hat{c}(y; b_0, w(\theta, \bar{w})) \equiv \frac{1}{\beta} \left[\rho - \alpha + \alpha \frac{y - \bar{r} b_0}{w(\theta, \bar{w})} \right].\tag{32}$$

From the Keynesian cross equation:

$$y = \hat{c}(y; b_0, w(\theta, \bar{w})) + g,$$

where \hat{c} is given by (32) and $w(\theta, \bar{w})$ by (30), we obtain the effects on aggregate demand of

changes in government purchases g , international asset-capital holdings b_0 , total productivity θ , and the terms of trade \bar{w} :

$$\begin{aligned} \frac{dy}{dg} &= \frac{1}{1 - \frac{\alpha}{\beta w}}, & \left(\frac{b_0}{y}\right) \frac{dy}{db_0} &= \frac{-\alpha \frac{\bar{r} b_0}{w}}{\rho - \alpha - \alpha \frac{\bar{r} b_0}{w} + \beta g}, \\ \left(\frac{\theta}{y}\right) \frac{dy}{d\theta} &= -\frac{\alpha x}{\rho - \alpha - \alpha \frac{\bar{r} b_0}{w} + \beta g} \frac{1}{\gamma}, & \left(\frac{\bar{w}}{y}\right) \frac{dy}{d\bar{w}} &= \frac{\alpha x}{\rho - \alpha - \alpha \frac{\bar{r} b_0}{w} + \beta g} \frac{1}{\gamma} (1 - \delta). \end{aligned} \quad (33)$$

where $1 - \delta = -p_1'(\bar{w})\bar{w}/p_1(\bar{w})$ from (31).

To obtain the quantitative results, the parameter values applied in the analysis are derived from per-capita data in Japan from 2004 to 2013 (the derivation is set out in the appendix).⁸ They are

$$\begin{aligned} \rho = \bar{r} &= 0.05, \quad \gamma = 0.6, \quad \alpha = 1.18 \times 10^{-2}, \quad \beta = 1.82 \times 10^{-4}, \\ x &= 0.956, \quad \delta = 0.937, \quad g = 32.3, \quad \bar{r} b_0 = 128.0, \quad w = 183.8. \end{aligned}$$

Applying these values to (33) yields

$$\frac{dy}{dg} = 1.54, \quad \left(\frac{b_0}{y}\right) \frac{dy}{db_0} = -0.230, \quad \left(\frac{\theta}{y}\right) \frac{dy}{d\theta} = -0.526, \quad \left(\frac{\bar{w}}{y}\right) \frac{dy}{d\bar{w}} = 0.033.$$

The multiplier of government purchases g on y ($= c + g$) is 1.54 and hence the multiplier on consumption is 0.54. Increases in b_0 and θ by 1% respectively reduce aggregate demand by 0.230% and 0.526% while a 1% increase in \bar{w} (viz. a deterioration in terms of trade) expands aggregate demand by 0.033%.

5. Conclusion

Using a dynamic optimization framework in a closed economy, Murota and Ono (2015) establishes a new Keynesian cross analysis, which is as simple as the conventional Keynesian cross analysis. This paper extends it to a small open economy setting and proposes a new Keynesian cross analysis of a small open economy with dynamic optimization.

⁸ Because the consumption tax was increased in 2014, we use 10-year data of 2004-2013 to avoid its influence.

In the new analysis consumption is shown to be a function of aggregate demand, as in the conventional consumption function, but the economic implication is quite different. While the conventional consumption represents the relationship between disposable income and consumption, the present consumption function is derived from household and firm optimizing behavior, market equilibrium conditions and wage and price adjustments. It shows the relationship between consumption and the inflation (or deflation if negative) rate determined by the employment rate, which in turn is a function of aggregate demand. The employment rate is determined so that the current account is balanced. Applying this consumption function to the Keynesian cross equation ($y = c + g$) yields a new Keynesian cross analysis with dynamic optimization in a small-country setting.

This analytical framework gives quite different economic and policy implications from those under full employment and those of the Mundell-Fleming model. An improvement in the terms of trade decreases world demand for the home commodity and decreases employment, which worsens deflation and reduces consumption and aggregate demand. An increase in international asset-capital holdings (due to international transfer, for example) improves the current account. It yields an appreciation pressure on the home currency and lowers home employment. Consequently, deflation worsens and consumption and aggregate demand decline. Government purchases worsen the current account, create a depreciation pressure on the home currency and hence increase employment. Thus, deflation mitigates and consumption is stimulated.

Those results are opposite to those under full employment. Under full employment an improvement in terms of trade and an increase in international asset-capital holdings increase consumption. Government purchase crowds out private consumption. They are also very different from those of the Mundell-Fleming model. In the Mundell-Fleming model an increase in government purchases reduces net export so that consumption is not affected. Furthermore,

it cannot deal with the effects of an improvement in the terms of trade and an increase in international capital-asset holdings because it does not consider the current account adjustment.

Appendix: Parameter values for the numerical example

The time preference rate ($\rho = 0.05$) is the value adopted by Benhabib and Farmer (1996) and Mankiw and Weinzierl (2006). The labor share in output is $\gamma = 0.6$, which is presented by Cooley and Prescott (1995). Following Katagiri and Takahashi (2017), we assume the share of imported consumption goods to be $1 - \delta = 0.063$. The macroeconomic data used in the following calculations are the averages of 2004-2013 in Japan.

The price adjustment speed α equals 1.18×10^{-2} . It is derived by substituting the annual inflation rate of the consumer price index for all items, $\pi = -5.20 \times 10^{-4}$ (Ministry of Internal Affairs and Communications, Statistics Bureau of Japan), and the average of unemployment rate, $1 - x = 0.044$ (Labor Force Survey, Statistics Bureau of Japan), to $\pi = \alpha(x - 1)$ given by (14).

Under the utility specification ($u'(c) = 1/c$) in (29), (20) reduces to $\beta c = \rho + \pi$. By substituting π and ρ given above and the average of actual final consumption expenditure per capita $c = 271.5$ from National Account for 2017 (Cabinet Office, Government of Japan) into this equation, we obtain the liquidity preference parameter $\beta = 1.82 \times 10^{-4}$.

Under the Cobb-Douglas production function given in (29) we have

$$\frac{wx}{\bar{r}k} = \frac{\gamma}{1-\gamma} = 1.5, \quad (\text{A1})$$

where $\gamma = 0.6$ (see Cooley and Prescott, 1995). Because $b = b^f + k$, (21) and (A1) yield

$$c + g - \bar{r}b^f = wx + \bar{r}k = 2.5 \times \bar{r}k. \quad (\text{A2})$$

The average of government actual final consumption per capita g (Cabinet Office,

Government of Japan) and the average of income account per capita $\bar{r}b^f$ (Ministry of Finance, Japan) are

$$g = 32.3, \quad \bar{r}b^f = 10.9.$$

Because $c = 271.5$ and $x(= 1 - 0.044) = 0.956$, as stated above, substituting them to (A2) gives

$$\bar{r}k = 117.1, \quad w = 183.8, \quad \bar{r}b_0(= \bar{r}b^f + \bar{r}k) = 128.0.$$

References

- Benhabib, J. and R. E. A. Farmer (1996) “Indeterminacy and sector-specific externalities”, *Journal of Monetary Economics*, 37 (3), pp. 421-443.
- Christiano, L., M. Eichenbaum, and C. L. Evans (1999) “Monetary policy shocks: What have we learned and to what end?” in J. B. Taylor and M. Woodford eds., *Handbook of Macroeconomics*, Vol. 1, Amsterdam: Elsevier, pp. 65-148.
- Cooley, T. F. and E. C. Prescott (1995) “Economic growth and business cycles,” in T. F. Cooley eds., *Frontiers of Business Cycle Research*, Princeton: Princeton University Press, pp. 1-38.
- Dornbusch, R. (1980) *Open Economy Macroeconomics*, New York: Basic Books.
- Eggertsson, G.B., N. R. Mehrotra, and J. A. Robbins (2017), “Model of secular stagnation: Theory and quantitative evaluation”, Working Paper, Brown University.
- Hashimoto, K. (2004) “Intergenerational transfer and effective demand”, *Economics Bulletin*, 5 (1), pp. 1-13.
- Hashimoto, K. (2011) “International outsourcing, the real exchange rate and effective demand”, *Metroeconomica*, 62 (2), pp. 305-327.
- Hashimoto, K. (2015) “Tariffs, offshoring and unemployment in a two country model”, *Japanese Economic Review*, 66 (3), pp. 371-392.

- Hashimoto, K. and Y. Ono (2011) “Does pro-population policy raise per capita consumption?”, *Japanese Economic Review*, 62 (2), pp. 151-169.
- Hayashi, F. and E. C. Prescott (2002) “The 1990s in Japan: A lost decade”, *Review of Economic Dynamics*, 5 (1), pp. 206-235.
- Katagiri, M. and K. Takahashi (2017) “Do term premiums matter? Transmission via exchange rate dynamics”, Working Paper Series No.17-E-7, Bank of Japan.
- Kydland, F. E. and E. C. Prescott (1982) “Time to build and aggregate fluctuations”, *Econometrica*, 50 (6), pp. 1345-1370.
- Illing, G., Y. Ono, and M. Schlegl (2018) “Credit booms, debt overhang and secular stagnation”, *European Economic Review*, 108, pp. 78-104.
- Johdo, W. (2006) “Geographical space and effective demand under stagnation”, *Australian Economic Papers*, 45 (4), pp. 286-298.
- Johdo, W. (2009) “Habit persistence and stagnation”, *Economic Modelling*, 26 (5), pp. 1110-1114.
- Johdo, W. and K. Hashimoto (2009) “International relocation, the real exchange rate and effective demand”, *Japan and the World Economy*, 21 (1), pp. 39-54.
- Lucas, R. (1976) “Econometric policy evaluation: A critique”, *Carnegie-Rochester Conference Series on Public Policy*, 1, pp. 19-46.
- Mankiw, G. (2010) *Macroeconomics*, 7th edition, New York: Worth Publishers.
- Mankiw, G. and M. Weinzierl (2006) “Dynamic scoring: A back-of-the-envelope guide”, *Journal of Public Economics*, 90 (8-9), pp. 1415-1433.
- Matsuzaki, D. (2003) “The effects of a consumption tax on effective demand under stagnation”, *Japanese Economic Review*, 54 (1), pp. 101-118.
- Michaillat, P. and E. Saez (2014) “An economical business-cycle model”, NBER Working Paper No. 19777.

- Michau, J. (2018) “Secular stagnation: Theory and remedies”, *Journal of Economic Theory*, 176, pp. 552-618.
- Murota, R. and Y. Ono (2011) “Growth, stagnation and status preference”, *Metroeconomica*, 62 (1), pp. 112-149.
- Murota, R. and Y. Ono (2012) “Zero nominal interest rates, unemployment, excess reserves and deflation in a liquidity trap”, *Metroeconomica*, 63 (2), pp. 335-357.
- Murota, R. and Y. Ono (2015) “Fiscal policy under deflationary gap and long-run stagnation: Reinterpretation of Keynesian multipliers”, *Economic Modelling*, 51, pp. 596-603.
- Ono, Y. (1994) *Money, Interest, and Stagnation – Dynamic Theory and Keynes’s Economics*, Oxford: Clarendon Press.
- Ono, Y. (2001) “A reinterpretation of Chapter 17 of Keynes’s General Theory: Effective demand shortage under dynamic optimization”, *International Economic Review*, 42 (1), pp. 207-236.
- Ono, Y. (2006) “International asymmetry in business activity and appreciation of a stagnant country’s currency”, *Japanese Economic Review*, 57 (1), pp. 101-120.
- Ono, Y. (2007) “International transfer under stagnation”, in L. Sajal ed., *Theory and Practice of Foreign Aid*, Amsterdam: Elsevier, pp. 155-171.
- Ono, Y. (2014) “International economic interdependence and exchange-rate adjustment under persistent stagnation”, *Japanese Economic Review*, 65 (1), pp. 70-92.
- Ono, Y. (2018) “Macroeconomic interdependence between a stagnant and a fully employed country”, *Japanese Economic Review*, 69 (4), pp. 450-477.
- Ono, Y. and J. Ishida (2014) “On persistent demand shortages: A behavioural approach”, *Japanese Economic Review*, 65 (1), pp. 42-69.
- Ono, Y., K. Ogawa, and A. Yoshida (2004) “The liquidity trap and persistent unemployment with dynamic optimizing agents: Empirical evidence”, *Japanese Economic Review*, 55

(4), pp. 355-371.

Ono, Y. and A. Shibata (2006) “Long-run specialization”, *Review of International Economics*, 14 (1), pp. 1-15.

Ono, Y. and K. Hashimoto (2012) *Fukyo no Keizai Riron (Economic Theory of Stagnation)*, Tokyo: Iwanami, in Japanese.

Rodriguez-Arana, A. (2007) “Inflation and the public deficit when the utility of money is insatiable”, *Japanese Economic Review*, 8 (2), pp. 238-254.

Schmitt-Grohé, S. and M. Uribe (2016) “Downward nominal wage rigidity, currency pegs, and involuntary unemployment”, *Journal of Political Economy* 124 (5), pp. 1466-1514.

Schmitt-Grohé, S. and M. Uribe (2017) “Liquidity traps and jobless recoveries”, *American Economic Journal: Macroeconomics* 9 (1), pp. 165-204.

Walsh, C. E. (2017) *Monetary Theory and Policy*, 4th ed., Cambridge MA.: MIT Press.