ISSN (Print) 0473-453X
ISSN (Online) 2435-0982

Discussion Paper No. 1062

A MONETARY SEARCH MODEL WITH NON-UNITARY DISCOUNTING

Daiki Maeda

September 2019

The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

A Monetary Search Model with Non-unitary Discounting*

Daiki Maeda†

September 18, 2019

Abstract

Based on findings in the behavioral economics literature, we incorporate non-unitary discounting into a monetary search model to study optimal monetary policy. We apply non-unitary discounting, that is, discount rates that are different across goods. With this extension to the model, we find that there are cases where optimal monetary policy deviates from the Friedman rule.

Keywords: Non-unitary discounting; Search; Friedman rule

JEL *classification*: E52, E70

1 Introduction

Many researchers use a monetary search model to study optimal monetary policy. Lagos and Wright (2005)(hereafter, LW) is one of the most well-known studies in this literature. On the contrary, more recent behavioral economics researchers report that subjective discount rates are different between goods (hereafter "non-unitary discounting") and show that non-unitary discounting has significant impacts on agents' behavior and optimal monetary policy. Based on an experiment conducted in Uganda, Ubfal (2016) reports the existence of non-unitary discounting. Hori and Futagami (2019) show that, theoretically, optimal tax policy is different from the standard model because the agent's behavior reflects the time-inconsistency caused by non-unitary

*I would like to thank Koichi Futagami, Makoto Hanazono, Hirokazu Ishise, Noritaka Kudoh, Tarishi Matsuoka, Toshiki Miyashita, Kohei Okada, Yoshiyasu Ono, Akihisa Shibata, Takashi Shimizu, Katsuya Takii, Wataru Tamura, Makoto Watanabe, the Search Theory Workshop participants at Nagoya University, the conference participants at Osaka University, and 2019 Japanese Economic Association Spring Meeting participants in Musashi University for their helpful comments and suggestions on a prior version of the paper. I would especially like to thank Tatsuro Iwaisako and Takeshi Ojima for constructive remarks and advice on an earlier version of this paper. All remaining errors are mine.

†Institute of Social and Economic Research, Osaka University, 6-1 Mihogaoka, Ibaraki, Osaka, 567-0047, Japan. E-mail address: u035866j@ecs.osaka-u.ac.jp

discounting. Since non-unitary discounting exists and affects the optimal policy, it is important that we also study optimal monetary policy using a monetary search model with non-unitary discounting.

For this study, we incorporate non-unitary discounting into LW's framework. It has the following property. Each period is divided into two subperiods, day and night. In the day subperiod, the decentralized market (hereafter DM) is open and agents have to search for their partner to trade. They then use money to succeed in the trade because there exists the possibility of a single coincidence of wants. In the night subperiod, the market (hereafter we call the market a centralized market, CM), which is perfectly competitive, is opened. In this market, the agent can choose both the consumption and the production volume freely. In our study, we assume, for simplicity, that the DM is also competitive and that money is used to buy the goods as in Berentsen et al. (2007)[1]. Moreover, we add the assumption that the discount rates for the utility of consumption and disutility of production in each subperiod are different in this framework.

Using this extension to the model, we show that the Friedman (1969) rule is not optimal in the following two cases. The first case is when the discount rate for the utility of the consumption of goods traded in the DM (hereafter DM goods) is sufficiently low. The money demand is higher than in the unitary discounting case because the agents want to consume more during the next day subperiod because of the low discount rate of the DM goods. In this case, the consumption of the DM goods is higher than the optimal one. Since the high inflation rate decreases the money demand and the consumption of the DM goods, it improves welfare and the Friedman rule is not optimal. The second case is when the discount rate of the disutility from producing the DM goods is low. In this case, the agents produce the DM goods more so that they can work less in the future. However, if they do so, they excessively supply the DM goods. In this case, weakening the incentive for production by raising the inflation rate can improve welfare.

Hiraguchi (2018) is similar to our study; however, he incorporates temptation preference whereby the agents have a desire to spend all their money and experience disutility from suppressing that desire. This factor differentiates his model from ours. Hori and Futagami (2018) also study the monetary economy with non-unitary discounting. However, they assume that households face a cash-in-advance constraint. Our model has a finer micro-foundation for money demand than their model.

The remainder of this paper is organized as follows: Section 2 provides our model. Section

[1]However, in Appendix E, we show that we obtain the same result from the original settings given by LW.

3 addresses the individual's optimization and obtains the equilibrium. Section 4 presents an analysis of monetary policy.

2 Model

Our model's setting is close to Berentsen et al.'s (2007) without banks. Time is discrete and runs from $t = 0$ to ∞. There exists a continuum of infinitely lived agents with a unit measure. Each period is divided into two subperiods, day and night. In the day subperiod, the agent becomes a "seller" who produces the goods with probability $n \in (0, 1)$ and a "buyer" who consumes it with probability $1 - n$. The instant utility is given by:

$$\mathcal{U}(q_t^b, q_t^s, x_t, h_t) = u(q_t^b) - c(q_t^s) + U(x_t) - h_t, \tag{1}$$

where $u(q)$ is the utility from consuming q units of the goods and $c(q)$ is the cost of producing q units of the goods in the day subperiod, and $U(x)$ is the utility from consuming x units of the goods and h is the disutility of suppling h units of labor in the night subperiod. The functions u, c, and U are twice continuously differentiable and satisfy $u(0) = c(0) = 0$, $u' > 0$, $c' > 0$, $U' > 0$, $u'' < 0$ $c'' \geq 0$, $U'' < 0$, $u'(0) = U'(0) = \infty$ and $u'(\infty) = U'(\infty) = 0$. h is positive and has upper bound $\bar{h}$. The labor productivity in the night subperiod is 1. Since buyers and sellers are anonymous, money is used to facilitate trade in the DM. Money is divisible and storable but intrinsically useless. M_t denotes the amount of money issued before period t, and the growth rate of money is $\gamma - 1$. Therefore, $M_{t+1}/M_t = \gamma$. Money is issued in the night subperiod, and all money is transferred to individuals equally. When we define T as the transfer of money to individuals, we obtain $T_t = (\gamma - 1)M_t$.

In the remainder of this section, we explain the features of our model that differ from Berentsen et al. (2007). We assume that the discount rates for the (dis)utility of consumption and production are different. Therefore, lifetime utility is given by:

$$Z = \sum_{t=0}^{\infty} \left[\beta_{db}^t u(q_t^b) - \beta_{ds}^t c(q_t^s) + \beta_{cb}^t U(x_t) - \beta_{cs}^t h_t \right], \tag{2}$$

where β_{bd} denotes the discount factor of $u(q^b)$, β_{ds} denotes the discount factor of $c(q^s)$, β_{cb} denotes the discount factor of $U(x)$, and β_{cs} denotes the discount factor of h.

3 Individual's optimization and the equilibrium

As mentioned in the previous section, there are two states associated with an agent: a seller and a buyer. In this section, we seek the optimal behavior in each state. Before we discuss optimal behavior, we mention notations. We omit the subscript t, which represents added variables, except for the case where we need the subscript. Moreover, we add the variable in the next period to +1, for example, z_{+1}.

First, we seek the seller's optimal behavior. The individual who becomes a seller in period t can obtain money and experiences disutility, $c(q^s)$ in the day subperiod. That agent also experiences net utility, $U(x) - h$, in the night subperiod. Therefore, the seller's value function, which she maximized, is given by:

$$V_0^s(m) = \max_{q^s, x, h, m_{+1}} [-c(q^s) + U(x) - h + V(m_{+1})], \tag{3}$$

$$\textit{where}$$

$$V(m_{+1}) \equiv \beta_{db} V_{db}(m_{+1}) - \beta_{ds} V_{ds}(m_{+1}) + \beta_{cb} V_{cb}(m_{+1}) - \beta_{cs} V_{cs}(m_{+1}). \tag{4}$$

$V(m)$ is the value that is obtained from future consumption and production. Each element, $V_{db}(m)$, $V_{ds}(m)$, $V_{cb}(m)$ and $V_{cs}(m)$, denotes the sum of the expected discounted present value of consuming and producing goods in the future day subperiod, and consuming and supplyng labor in the future night subperiod, respectively. These are given by:

$$V_{db}(m_t) = E \sum_{j=0}^{\infty} \beta_{db}^j u(q_{t+j}^b), \tag{5}$$

$$V_{ds}(m_t) = E \sum_{j=0}^{\infty} \beta_{ds}^j c(q_{t+j}^s), \tag{6}$$

$$V_{cb}(m_t) = E \sum_{i=0}^{\infty} \beta_{cb}^j U(x_{t+j}), \tag{7}$$

$$V_{cs}(m_t) = E \sum_{i=0}^{\infty} \beta_{cs}^j h_{t+j}, \tag{8}$$

where E is the expectation operator. The seller's budget constraint in the night subperiod is given by:

$$h = x + \phi m_{+1} - \phi p q^s - \phi(m + T) \tag{9}$$

where ϕ is the real price of money and p is the nominal price of the goods in the day subperiod. Substituting (9) into (3), we obtain:

$$V_0^s(m) = \max_{q^s}[\phi p q^s - c(q^s)] + \max_x[U(x) - x] + \phi(m+T) + \max_{m_{+1}}[-\phi m_{+1} + V(m_{+1})]. \quad (10)$$

As in Berentsen et al. (2007), the seller treats p as given. Assuming an interior solution for h, the optimality condition of (10) is given by:

$$\phi p = c'(q^s) \quad (11)$$

$$U'(x) = 1, \quad (12)$$

$$\phi = V'(m_{+1}). \quad (13)$$

Let x^* denote the value of satisfying (12). Moreover, we assume that $U(x^*) > x^*$ is satisfied for x^* to have a positive value.

Next, we seek the buyer's optimal behavior. Since buyers and sellers are anonymous, buyers hold money before they consume. Therefore, buyers face the following constraint:

$$pq^b \leq m. \quad (14)$$

Using a similar calculation to the seller's problem, we obtain the value function, which the buyer maximizes, as follows:

$$V_0^b(m) = \max_{pq^b \leq m}[u(q^b) - \phi p q^b] + \max_x[U(x) - x] + \phi(m+T) + \max_{m_{+1}}[-\phi m_{+1} + V(m_{+1})]. \quad (15)$$

From (15), the optimal conditions of the buyer are the same as (12) and (13). We also find that the optimal consumption in the DM satisfies $u'(q^b) \geq \phi p$. Since (11) is satisfied in the equilibrium, $\frac{u'(q^b)}{c'(q^s)} \geq 1$.

Since we have acquired the seller's and buyer's solution, we can also obtain $V(m_{+1})$. From (11) to (13), q^s, x and m_{+1} do not depend on m. Therefore, (4) is rewritten as follows:

$$V(m_{+1}) = (1-n)v(m_{+1}) + \beta_{cs}\phi_{+1}m_{+1} + constantterm, \quad (16)$$

$$where$$

$$v(m_{+1}) \equiv \beta_{db}u(q^b(m_{+1})) - \beta_{cs}\phi_{+1}p_{+1}q^b(m_{+1}) \quad (17)$$

where *constant term* is the term that does not depend on m_{+1}[2]. Substituting this equation into the last term of (10) and (15), we obtain:

$$\max_{m_{+1}}[-\phi m_{+1} + (1-n)v(m_{+1}) + \beta_{cs}\phi_{+1}m_{+1}]. \tag{18}$$

We can show that in the equilibrium, (14) is binding, as in LW[3]. Therefore, we obtain the optimality condition of (18) as follows:

$$\frac{\phi}{\phi_{+1}} = (1-n)\beta_{db}\frac{u'(q^b_{+1})}{c'(q^s_{+1})} + n\beta_{cs}. \tag{19}$$

Since (11) is satisfied in the equilibrium, we have substituted (19) into (11).

We mention the Friedman rule. If (14) is not binding, $\beta_{cs}\phi_{+1} = \phi$ is satisfied in the equilibrium. This condition means that the cost of holding money is zero. Since $\frac{\phi}{\phi_{+1}} - 1$ denotes the inflation rate, $\frac{\phi}{\phi_{+1}} = \gamma$ in the steady state. Using these two conditions, we obtain the following Lemma.

Lemma 1. *The Friedman rule in this economy is that* $\gamma \to \beta_{cs}$.

We provide the market equilibrium conditions. The market equilibrium condition in the DM is given by $q^s = \frac{1-n}{n}q^b$. The money market equilibrium condition is given by $m = M$. Therefore, $m_{+1} = (1+\pi)M = M + T$. The aggregate labor supply in the CM is given by $nh^s + (1-n)h^b$, where h^s and h^b is the labor supply of the seller and buyer. Using other market equilibrium conditions, we get $nh^s + (1-n)h^b = x$. This implies that the CM goods market also clears.

4 Policy analysis

In this section, we analyze monetary policy in the steady state. We consider a one-shot policy, which does not change forever. We define $V_0(m) \equiv (1-n)V^b_0(m) + nV^s_0(m)$ as a welfare function. This definition is natural because agents in each period face the problem (10) and (15), and maximizing this function is the same as maximizing the expected value of the lifetime utility, (2). Calculated as in Appendix C, we then obtain the welfare function in the steady state as

[2]The derivation of (16) is provided in Appendix A.
[3]See Appendix B.

follows:

$$v_0 = (1-n)\frac{u(\bar{q})}{1-\beta_{db}} - n\frac{c\left(\frac{1-n}{n}\bar{q}\right)}{1-\beta_{ds}} + \frac{U(x^*)}{1-\beta_{cb}} - \frac{x^*}{1-\beta_{cs}}, \tag{20}$$

where $\bar{q}$ is the amount of the buyer's consumption in the steady state. Rewriting (19), in the steady state, we get

$$\frac{\gamma - \beta_{cs}}{\beta_{cs}} = (1-n)\left(\frac{\beta_{db}}{\beta_{cs}}\frac{u'(\bar{q})}{c'\left(\frac{1-n}{n}\bar{q}\right)} - 1\right). \tag{21}$$

From (20) and (21), we obtain the following proposition.

Proposition 1. *The Friedman rule is not optimal if either case is satisfied:*

1. $\beta_{db} > \beta_{cs}$
2. $\frac{1-\sqrt{1-4\beta_{cs}(1-\beta_{ds})}}{2} < \beta_{db} < \frac{1+\sqrt{1-4\beta_{cs}(1-\beta_{ds})}}{2}$ *and* $\beta_{db} \leq \beta_{cs}$.

Proof. See Appendix D. □

The intuition for the first case of the proposition is the following. The higher β_{db} implies that the agents demand more money for more future consumption. Then, the consumption of the DM goods is higher than the optimal one. Since a high inflation rate decreases money demand and consumption of the DM goods, it improves welfare. However, in the second case, the Friedman rule is not optimal although money demand is low. This is why excessive production occurs in the DM. Shown in Appendix D, this case happens when β_{ds} is high. The value maximizing (20) of $\bar{q}$ is given by:

$$\frac{u'(\bar{q})}{c'\left(\frac{1-n}{n}\bar{q}\right)} = \frac{1-\beta_{db}}{1-\beta_{ds}}.$$

This equation implies that the value maximizing (20) of $\bar{q}$ is small if β_{ds} is high. In other words, if β_{ds} is high, excessive production tends to occur. In this case, weakening the incentive for production by raising the inflation rate can improve welfare.

Appendix

A The derivation of (16)

We add +2 to the variable after two periods, for example z_{+2}. If the agent in the current period becomes the buyer after two periods, then she gets the following discounted value:

$$\begin{aligned} V^b(m_{+1}) =& \beta_{db} \underbrace{(u(q^b_{+1}) + \beta_{db} V_{db}(m_{+2}))}_{V_{db}(m_{+1})} - \beta_{ds} \cdot \underbrace{\beta_{ds} V_{ds}(m_{+2})}_{V_{ds}(m_{+1})} \\ &+ \beta_{cb}(\underbrace{U(x_{+1}) + \beta_{cb} V_{cb}(m_{+2})}_{V_{cb}(m_{+1})}) - \beta_{cs} \underbrace{(h_{+1} + \beta_{cs} V_{cs}(m_{+2})}_{V_{cs}(m_{+1})}. \end{aligned} \tag{A.1}$$

The individual faces the same problem after two periods as the individual faces now, (10) and (15). Therefore, future behavior, x_{+1}, m_{+2} and q^s_{+1}, does not depend on m_{+1}. From this fact, we obtain:

$$\begin{aligned} V^b(0) =& \beta_{db} \cdot \underbrace{\beta_{db} V_{db}(m_{+2})}_{V_{db}(0)} + \beta_{ds} \cdot \underbrace{\beta_{ds} V_{ds}(m_{+2})}_{V_{ds}(0)} + \beta_{cb}(\underbrace{U(x_{+1}) + \beta_{cb} V_{cb}(m_{+2})}_{V_{cb}(0)}) \\ &- \beta_{cs} \underbrace{(x_{+1} + \phi_{+1} m_{+2} - \phi_{+1} T_{+1} + \beta_{cs} V_{cs}(m_{+2}))}_{V_{cs}(0)}, \end{aligned} \tag{A.2}$$

where we have already substituted the budget constraint in the night subperiod. Using (A.2), we express (A.1) as follows:

$$V^b(m_{+1}) = \beta_{db} u(q^b_{+1}) - \beta_{cs}(\phi_{+1} p_{+1} q^b_{+1} - \phi m_{+1}) + V^b(0). \tag{A.3}$$

In addition to the buyer, we obtain the value of the seller as follows:

$$V^s(m_{+1}) = \beta_{cs} \phi_{+1} m_{+1} + V^s(0). \tag{A.4}$$

Since $V(m_{+1}) = (1-n)V^b(m_{+1}) + nV^s(m_{+1})$, substituting (A.3) and (A.4) into this equation, we obtain (16).

B Proof of binding (14) in the equilibrium

Suppose that $\beta_{cs}\phi_{+1} > \phi$. Then, from (18), we find that the optimal behavior is $m_{+1} = \infty$. Therefore, this case is not the equilibrium and $\beta_{cs}\phi_{+1} \leq \phi$ in the equilibrium. When $\beta_{cs}\phi_{+1} < \phi$, (14) is binding because the agents suffer losses if they hold more money than necessary. Therefore, we check if in the case where $\beta_{cs}\phi_{+1} = \phi$, there is not equilibrium or (14) binds. In the case that $v'(m_{+1}) = 0$ is satisfied. This occurs in the following two cases. The first case is that $\phi_{+1}p_{+1}q^* \leq m_{+1}$, where q^* satisfies $u'(q^*) = \phi p$. In this case, the future self does not increase the consumption of the DM goods if m_{+1} increases, that is $q'(m_{+1}) = 0$. Therefore, $v'(m_{+1}) = 0$. When we seek the left-hand limit of $v'(m_{+1})$ at $\phi_{+1}p_{+1}q^*$, we get $\lim_{m_{+1}\to\phi_{+1}p_{+1}q^*-0} v'(m_{+1}) = -\phi_{+1}p_{+1} < 0$. Hence, the case where $\phi_{+1}p_{+1}q^* = m_{+1}$ is not the equilibrium, and the case where $\beta_{cs}\phi_{+1} < \phi$ is the only equilibrium. The second case is that $m_{+1} = \phi p q^{**}$, where q^{**} denotes the maximizing value of $v(m_{+1})$. The agent does not have an incentive to increase the present holding of money because the increase will simply be used in the future. Therefore, (14) is binding in this case. By the above discussion, we showed that (14) is binding in the equilibrium.

C Derivation of (20)

We find that (5), which is the expected future value, can be expressed as follows:

$$\begin{aligned} V_{db}(m) &= (1-n)(u(\bar{q}) + V_{db}(m_{+1})) + n(0 + V_{db}(m_{+1})) \\ &= (1-n)u(\bar{q}) + V_{db}(m_{+1}) \end{aligned} \tag{C.1}$$

where the first term is the value of becoming a buyer, and the second term is the value of becoming a seller, in the next period in the DM. From (C.1), we obtain the future value of $V_{db}(m)$ in the steady state as follows:

$$v_{db} = (1-n)\frac{u(\bar{q})}{1-\beta_{db}}. \tag{C.2}$$

As well as $V_{db}(m)$, from (6) to (8) we obtain the steady state values of $V_{ds}(m)$, $V_{cb}(m)$ and V_{ds} as follows:

$$v_{ds} = n\frac{c(\bar{q})}{1-\beta_{ds}},\ v_{cb} = \frac{U(x^*)}{1-\beta_{cb}},\ v_{cs} = \frac{x^*}{1-\beta_{cs}}. \tag{C.3}$$

Using (C.2) and (C.3), we have:

$$\begin{aligned} v_0 &= (1-n)u(\bar{q}) - nc(\bar{q}) + U(x) - x^* + \beta_{db}v_{db} - \beta_{ds}v_{ds} + \beta_{cb}v_{cb} - \beta_{cs}v_{cs} \\ &= (1-n)\frac{u(\bar{q})}{1-\beta_{db}} - n\frac{c(\bar{q})}{1-\beta_{ds}} + \frac{U(x^*)}{1-\beta_{cb}} - \frac{x^*}{1-\beta_{cs}}. \end{aligned} \tag{C.4}$$

This is (20).

D Proof of Proposition 1

Since $\frac{u'(q^b)}{c'(q^s)} \geq 1$ in the equilibrium, the right hand side of (21) is positive if $\beta_{db} > \beta_{cs}$. Therefore, the Friedman rule is not optimal in the first case.

Next, we consider the case in which $\beta_{db} \leq \beta_{cs}$. This implies that $\frac{\beta_{cs}}{\beta_{db}} = \frac{u'(q^{**})}{c\left(\frac{1-n}{n}q^{**}\right)} \geq \frac{u'(q^*)}{c\left(\frac{1-n}{n}q^*\right)} = 1$. Since $\frac{u'(q^b)}{c\left(\frac{1-n}{n}q^b\right)}$ is a decreasing function of q^b from the assumption, $q^{**} < q^*$. From Appendix B, since the amount of q^b in the equilibrium is smaller than q^{**}. Therefore, if q^{***}, which maximizes (20), is larger than q^{**}, the optimal q^b is equal to q^{**}. Substituting $\frac{u'(\bar{q})}{c\left(\frac{1-n}{n}\bar{q}\right)} = \frac{\beta_{cs}}{\beta_{db}}$ into (21), then the right hand side is zero. This implies that the Friedman rule is optimal. The condition that satisfies $q^{***} < q^{**}$ is given by $\frac{u'(q^{***})}{c'\left(\frac{1-n}{n}q^{***}\right)} = \frac{1-\beta_{db}}{1-\beta_{ds}} > \frac{\beta_{cs}}{\beta_{db}}$. This condition is equivalent to the condition of (21) becoming positive. Solving this, we obtain the second case of proposition 1. Moreover, since $\beta_{db} \leq \beta_{cs}$, $\frac{1-\beta_{db}}{1-\beta_{ds}} > \frac{\beta_{cs}}{\beta_{db}} > 1 \ \Leftrightarrow \beta_{ds} > \beta_{db}$.

E The case where the DM goods of the price and quantity are determined by bargaining

In this section, we assume, for simplicity, that the probability of the agents becoming the buyer and the seller is 1/2. Since the agent's problem in the night subperiod does not change, the optimality conditions in the night subperiod are same to (12) and (13), and x and m_{+1} do not depend on m. Therefore, the bargaining problem is

$$(u(q) - \phi z)^{\theta}(-c(q) + \phi z)^{1-\theta}, \tag{E.1}$$

$$s.t. \quad z \leq m, \tag{E.2}$$

where $z \equiv pq$ and θ is the buyer's bargaining weight. Since this is the same as LW, the solution is given by:

$$\phi z = g(q) \equiv \frac{\theta c(q)u'(q) + (1-\theta)u(q)c'(q)}{\theta u'(q) + (1-\theta)c'(q)}. \tag{E.3}$$

Notice that $g'(q) > 0$ because there is the assumption of instant utility.

(16) does not change except $n = 1/2$. Therefore, we can rewrite (18) as follows:

$$\max_{m_{+1}} \left[-\phi m_{+1} + \frac{1}{2}v(m_{+1}) + \beta_{cs}\phi_{+1}m_{+1}\right]. \tag{E.4}$$

From Appendix B, we find that $z_{+1} = m_{+1}$. Then, from (E.3), $q'(m_{+1}) = \frac{\phi}{g'(q_{+1})}$. Using this equation, we obtain (19) in the case:

$$\frac{\phi}{\phi_{+1}} = \frac{1}{2}\left[\beta_{db}\frac{u'(q_{+1})}{g'(q_{+1})} + \beta_{cs}\right]. \tag{E.5}$$

If $\theta \to 1$, $g(q) = c(q)$. Then, (E.5) is the same as (19). In other words, if the price and quantity are determined by bargaining, we can obtain the same result in our paper.

References

[1] Berentsen, A., Camera, G., Waller, C. (2007) Money, Credit and Banking. Journal of Economic Theory, 135, 171-195.

[2] Friedman, M. (1969) The Optimum Quantity of Money and Other Essays. Chicago: Aldine Publishing Company.

[3] Hiraguchi, R. (2018) Temptation and Self-Control in a Monetary Economy. Macroeconomic Dynamics, 22, 1076-1095.

[4] Hori, T., Futagami, K. (2018) Time-Inconsistent Discounting and the Friedman Rule: The Role of Non-Unitary Discounting. Discussion Papers in Economics and Business Graduate

School of Economics and Osaka School of International Public Policy (OSIPP), Osaka University, 18-04.

[5] Hori, T., Futagami, K. (2019) A Non-Unitary Discount Rate Model. Economica, 86(341), 139-165.

[6] Lagos, R., Wright, R. (2005) A Unified Framework for Monetary Theory and Policy Analysis. Journal of Political Economy 113, 463-484.

[7] Ubfal, D. (2016) How General Are Time Preferences? Eliciting Good-Specific Discount Rates. Journal of Development Economics, 118, 150-170.