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THE LOCK-IN EFFECT
AND
THE CORPORATE PAYOUT PUZZLE

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Abstract

Taxes on capital gains are deferred until realization, whereas dividends are taxed upon accrual. This often makes dividends tax-disadvantaged relative to share repurchases, which leads to the payout puzzle: why do firms pay dividends? This paper demonstrates that tax deferment can also provide a solution to the payout puzzle: if shareholders demand repurchase premiums when selling equity back to a firm - as compensation for accelerated realizations - then dividends can become tax-efficient. This mechanism is appealing because it explains dividend payments without appealing to asymmetric information, incomplete contracts, repurchase constraints, or shareholder irrationality.

Keywords: Payout Policy, Capital Taxation.

JEL classification: G35, H24.

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1 Introduction

Shareholders derive income from dividends and capital gains. Both sources of income have identical value in perfect capital markets without taxation, but their values can diverge when capital income is taxed. In the United States, for instance, dividends and long-term capital gains have a top federal tax rate of 20%, and when you factor-in the ability to postpone taxes on accrued capital gains until realization, the latter's effective tax rate is lower (likely in the range of 11% - 16% for many shareholders, as discussed below). This tax differential makes dividend payments somewhat of a puzzle, since firms can repurchase equity and generate tax-favored capital gains for their shareholders; a concept enshrined in Fisher Black's 1976 paper "The Dividend Puzzle." The current paper develops a model of corporate payout policy to demonstrate that dividends may be paid (by some firms) for the very same reason that dividends are currently tax disadvantaged in the US: shareholders often postpone the realization of capital gains for tax purposes. This incentive to delay equity sales - known as the lock-in effect - may require some firms to "over pay" when repurchasing equity, which can make tax-disadvantaged dividends optimal. The argument below is built on evidence that both retail and institutional investors often behave in accordance with the lock-in effect, and that equity prices typically appreciate during repurchase programs.

Payout policy - the choice between dividends and share repurchases - is shown by Miller and Modigliani (1961) to be independent of firm value when capital markets are perfect, investment policy is fixed, investors are rational, and taxes are nil. This follows from the equilibrium condition that wealth-maximizing shareholders are indifferent between receiving $1 in cash (a dividend payment) and a stream of cash flows with present value $1 (via higher ownership concentration following a share repurchase). This irrelevance result fails to hold if one form of payout is tax disadvantaged, however, which is often thought to be the case with US dividends owing to: higher statutory tax rates prior to 2003; and the postponement of capital-gains taxes until realization, which lowers their effective tax rate. The second factor produces the lock-in effect mentioned above, which is manifest in the trading behavior of both retail and institutional investors.

Feldstein et al. (1980) appears to be the first study to provide empirical evidence that retail investors time their capital-gains realizations to lower tax burdens. This effect is also identified in Auten and Clotfelter (1982), which shows that realizations are particularly responsive to the transitory components of capital-gains taxation; a result confirmed by Burman and Randolph (1994) and Auerbach and Siegel (2000). This evidence is supplemented by Brown and Ryngaert (1992) and Landsman and Shackelford (1995), which show that shareholders typically demand higher prices when selling equity with larger accrued capital gains. Finally, the evidence in Barber and Odean (2003) and Ivkovic et al. (2005), that investors harvest capital losses in December, is also consistent with tax-motivated realizations. These empirical regularities are consistent with Chey et al.'s (2006) estimate that 90% of all directly-held stock is placed within taxable accounts.[1]

Retail investors have gradually substituted away from direct equity ownership and towards institutional ownership. This places greater emphasis on the latter's trading behavior when considering lock-in effects. Rather than being a homogeneous group, institutional investors differ greatly regarding their tax exposure: ranging from tax-exempt pension funds and non-profit organizations; to partially-taxed insurance companies (with tax relief for policyholder income); to fully taxable corporations, hedge funds, and mutual funds. Chetty and Saez (2005) estimates that only 15% of institutional investors are fully nontaxable on a dollar-weighted basis, and since mutual-fund inflows are positively related with a fund's tax efficiency (Bergstresser and Poterba, 2002 and Dimmock et al., 2018), and fund-manager compensation is often tied to asset size, these managers have an incentive to be tax efficient. This seems to be the case; Huddart and Narayanan (2002), Jin (2006), Sialm and Starks (2012), and Dimmock et al. (2018) all provide evidence that investment funds with tax-sensitive clients behave in accordance with the lock-in effect, and Sialm and Starks (2012) and Sialm and Zhang (2020) provide evidence that pre-tax returns are typically not sacrificed by tax-efficient funds.[2]

[1]However, some investors consistently make investment mistakes by selling appreciated assets too soon while failing to realize capital losses in a timely manner; Shefrin and Statman (1985) dub this phenomenon the disposition effect, and both Barber and Odean (2003) and Ivkovic et al. (2005) document its occurrence. The empirical evidence suggests that wealthier households are less disposed to making investment errors (see Ivkovic et al., 2005 and Campbell, 2006).

[2]The down-side of tax efficiency is greater capital-gains overhang - higher levels of accrued capital gains within a fund, and thus, a higher potential for capital-gains taxes in the future - which is shown by Bergstresser and Poterba (2002) to reduce future inflows. However, since future realizations are timed by the fund itself, the actual impact on shareholder taxes is relatively small according to Sialm and Zhang

With this backdrop of shareholder lock-in effects, the model's primary mechanism is straightforward to describe. Firms in the model use profits to pay dividends, repurchase shares, and make capital investments. Managers act in the interest of shareholders by choosing investment/payout policy to maximize firm value. Once investment is determined, management decides what fraction of aggregate payout to distribute as dividends and what fraction to use repurchasing shares. Both forms of payout are subject to capital-income taxation, and dividends are taxed more heavily. Shareholders are heterogeneous regarding their desired holding periods and level of accrued capital gains on firm equity, and shareholder-wealth maximization is shown to create a wedge between equity's intrinsic value and the ask price of shareholders with an accrued capital gain and non-zero investment horizon - compensation for the lock-in effect. This differential - referred to as the "lock-in premium" herein - is weakly positive and increasing in both capital gains and investment horizon. Firms may therefore be required to pay lock-in premiums when repurchasing equity, which is acceptable provided these premiums remain small, since the alternative is paying a tax-disadvantaged dividend. However, since firms typically repurchase large quantities of equity, especially over multiple years,[3] the marginal shareholder's lock-in premium can become sufficiently large that paying a tax-disadvantaged dividend becomes optimal. At this point, firms pay dividends. This mechanism is consistent with Brav et al.'s (2005) finding that 86% of surveyed executives cite their company's stock price - relative to its true/intrinsic value - as an important, or very important, determinant of repurchase activity.

This mechanism is appealing because it explains dividend payments without appealing to asymmetric information, repurchase constraints, incomplete contracts and/or irrationality:[4]

(2020).

[3]The announcement size of a typical repurchase program is reported to be 6.6% in Ikenberry et al. (1995), 6.8% in Gaspar et al. (2012), 7% in Stephens and Weisbach (1998), 7.5% in Chan et al. (2010), and 8% in Jagannathan et al. (2000). In addition, Stephens and Weisback (1998) reports that many firms repurchase more equity than they announce, and Skinner (2008) reports that repurchasing firms do so every 2 out of 3 years on average (since 1990). Taken together, share repurchases can add significant demand-side price pressure.

[4]As discussed in Section 2, these factors are highlighted in the following studies. 1) Repurchase constraints: King (1977), Auerbach (1979), Bradford (1981), and Grullon and Michaely (2002). 2) Asymmetric information: Bhattacharya (1979), John and Williams (1985), Ofer and Thakor (1987), Brennan and Thakor (1990), Bernheim (1991), and Allen et al. (2000). 3) Incomplete contracting: Easterbrook (1984), Jensen (1986), Allen et al. (2000), and Morck and Yeung (2005). And 4) Irrationality: Shefrin and Statman (1984),

the only assumption from Miller and Modigliani (1961) to be relaxed is zero taxes.[5] This is not meant to question the validity of these restrictions, only to point out that skeptics of their applicability - for a particular firm at a particular time - may find the current model's paucity of qualifying assumptions appealing. In any case, manages hold heterogeneous views on which factors drive payout policy - as documented in Brav et al. (2005) - so any model of payout policy is unlikely to explain the behavior of all firms at all times, and many of the explanations offered (including this one) are not mutually exclusive.

The remainder of this paper is organized as follows. Section 2 discusses the previous literature and how this paper fits within it. Section 3 provides an empirical context for the model by illustrating that dividends are tax disadvantaged in the United States. Section 4 contains the main payout model. Section 5 extends this model by endogenizing the shareholder distribution. Section 6 concludes.

2 Previous Literature

The theoretical literature on why firms pay tax-disadvantaged dividends can be segmented by the four restrictions mentioned above, i.e., constraints on share repurchases, constraints on information sets, the incompleteness of contracts, and irrationality.[6] Starting with the first, King (1977), Auerbach (1979), and Bradford (1981) assume that firms avoid equity buybacks, in part, due to Section 302 of the Internal Revenue Code (implicit in King, 1977 and Bradford, 1981), whereby share repurchases that resemble dividend payments (proportional repurchases) are taxed as such.[7] In addition, Grullon and Michaely (2002) argues that some firms were concerned that repurchasing equity prior to the adoption of rule 10b-18 in

Bagwell and Shoven (1989), and Baker and Wurgler (2004).

[5]Some papers (e.g., Bagwell and Shoven 1989) note that firms may tailor payout policy to certain tax clienteles: firms that cater to shareholders with a dividend-tax advantage (e.g., certain non-pass-through corporations) will pay dividends, while those catering to shareholders with a dividend-tax disadvantage (e.g., hedge funds and mutual funds) will repurchase shares. However, since most of the market's capitalization is concentrated in firms that pay dividends *and* repurchase stock (Skinner 2008), shareholders that gravitate towards either dividend-only firms or repurchase-only firms will be grossly under diversified.

[6]Dividends may also be paid due to the transactions costs of buying and selling equity, where the latter affects shareholders that consume out of realized capital gains. For a survey of the payout literature, see Allen and Michaely (2003).

[7]See Brennan and Thakor (1990) for a description of Section 302's guidelines.

1982 would have triggered antimanipulative provisions of the Securities and Exchange Act.[8] The current paper places no restrictions on payout policy beyond the standard non-negative dividend constraint.

Regarding the second restriction, Bhattacharya (1979) and Ofer and Thakor (1987) argue that dividend payments signal firm value by exposing firms to costly external finance: dividend taxes are necessary for Bhattacharya's (1979) separating equilibria, while they affect the payout mix in Ofer and Thakor (1987). John and Williams (1985) and Bernheim (1991) also require dividend taxes to generate separating equilibira: dividend payments reduce equity dilution in John and Williams (1985), while they fine-tune payout taxes in Bernheim (1991). In addition to addressing information problems between managers and shareholders, Brennan and Thakor (1990) and Allen et al. (2000) show that dividends can address information problems *among* shareholders: dividend payments forestall the transfer of wealth from uninformed to informed shareholders during repurchase programs in Brennan and Thakor (1990), while they attract informed shareholders in Allen et al. (2000). The current paper assumes that all market participants have the same information.

Regarding the third restriction, Jensen (1986) seems to be the quintessential argument in favor of using dividend payments to extract free cash flow, although it does champion interest payments for this purpose. Morck and Yeung (2005) argues that bankruptcy concerns make dividend payments the superior option. Allen et al. (2000) and Morck and Yeung (2005) argue that dividends attract large institutional investors that reduce collective-action problems (Morck and Yeung 2005) and increase monitoring effort. Easterbrook (1984) also highlights monitoring, but focuses on the external monitoring brought about because high dividend payments lead to external finance more often (as in Bhattacharya, 1979 and Ofer and Thakor, 1987). Easterbrook (1984) also notes that dividends can offset managerial risk-aversion by increasing leverage, and therefore, equity value. The current paper assumes that managers act in the interest of shareholders and only invest in positive net-present-value projects.

Finally, there are two sets of explanations pertaining to irrationality: irrational managers

[8] Rule 10b-18 provides guidelines for repurchasing equity; see Jagannathan et al. (2000) for details.

and irrational shareholders. Regarding the first, Bagwell and Shoven (1989) propose that managers have gradually learned to appreciate the benefits of share repurchases over time, and that Nixon's price and wage ceilings of the 1970s - which led to "voluntary" dividend controls according to Ofer and Thakor (1987) - catalyzed their learning-by-doing process. When this argument is coupled with Lintner's (1956) dividend-stickiness model, the payment of current dividends is understandable.[9] With regard to shareholder irrationality, Shefrin and Statman (1984) argues that firms pay dividends because shareholders suffer from regret aversion and/or self-control issues; and since preferences are guided by prospect theory (rather than expected-utility theory), mental accounting creates a value for dividends. In addition, Baker and Wurgler (2004) argues that shareholders have time-varying preferences for dividend-paying firms (or conversely, growth-oriented firms), and that firms cater to these inexplicable preferences by initiating, continuing, or omitting dividend payments. The current paper assumes that both managers and shareholders are perfectly rational, and that shareholders are expected-wealth maximizes.

The current study is also related to theoretical work on share repurchases and equity-supply curves. Two relevant papers are Stulz (1988) and Bagwell (1991), which study upward-sloping equity-supply curves in the context of corporate takeovers, and note that lock-in effects may contribute to their positive gradients. Stulz (1988) argues that share repurchases are useful for increasing takeover-bid premiums but reduce success probability, whereas Bagwell (1991) highlights their usefulness as a takeover defense, since repurchasing equity increases acquisition cost. Evidence for upward-sloping equity-supply curves, in the context of Dutch repurchase auctions, is provided by Bagwell (1992). The current paper differs from Stulz (1988) and Bagwell (1991) in a number of important ways. First, it deals with dividend policy (a regularly-occurring consideration for most firms) and not takeover strategy (a comparatively less-frequent consideration). Second, payout policy in the current model involves a trade off between taxes on dividends and capital gains, whereas in Stulz (1988) and Bagwell (1991) it involves wealth transfers between target-firm shareholders and acquiring-firm shareholders. Third, a greater emphasis is placed on characterizing the relationship between shareholder characteristics and lock-in premiums in the current study,

[9] Brav et al. (2005) also documents that some managers pay dividends because of inertia; wishing they had never started.

which is important for understanding its static and dynamic properties. And furthermore, the current model provides a more holistic analysis of the firm's investment/payout policy and how dividends and share repurchases evolve over time.

In addition to Stulz (1988) and Bagwell (1991), Barclay and Smith (1988), Chowdhry and Nanda (1994), and Huang and Thakor (2013) all highlight the positive relationship between share repurchases and stock prices. This relationship is attributable to higher bid-ask spreads in Barclay and Smith (1988), the information content of share repurchases in Chowdhry and Nanda (1994), and the reduction in shareholder-manager disagreement in Huang and Thakor (2013). The current paper differs from these studies because it considers a complete-information framework with perfectly-rational agents.

3 Empirical Payout

This section provides an empirical context for the payout model by illustrating the US payout puzzle over the period 1972-2017. This is done by establishing both the prevalence of dividend payments over this period and their tax disadvantage relative to share repurchases.

Since capital gains are taxed upon realization, an investor's *effective* tax rate on *accrued* capital gains is weakly lower than their *statutory* tax rate on *realized* capital gains (denoted by τ_g). To see this, suppose that an asset appreciates by \$1 today and is liquidated in T periods. The tax liability thus created - due in T periods - can be satisfied by investing $\tau_g/(1+r_f)^T$ in a risk-free security with after-tax interest rate r_f. As such, one measure of the effective tax rate is $\tau_g/(1+r_f)^T \leq \tau_g$, which is decreasing in r_f and T.[10] This raises the question: what is the "marginal investor's" effective tax rate - denoted by τ_e - i.e., the investor who sets equity prices? This question is partially addressed by Protopapadakis (1983) and Chay et al. (2006), which suggest that US equities have effective tax rates of approximately 55%-80% of the statutory rate on average.[11]

[10]This explanation is similar to that in Constantinides (1983) and is well articulated by King (1977, p. 59): "Deferral is equivalent to an interest-free loan from the revenue authorities to the taxpayer of an amount equal to the tax liability on the accrued gain, and hence is also equivalent to a reduction in the effective rate of tax." Section 5 below provides a detailed discussion of effective tax rates on accrued capital gains.

[11]Poterba (1987) suggests 25%, which is partially supported by Ivkovic et al. (2005).

Since share repurchases generate accrued capital gains for most investors, it is standard to apply the marginal investor's effective tax rate τ_e when valuing this form of income.[12] Alternatively, since dividends are taxed upon accrual, no adjustment is needed for the dividend tax rate (denoted by τ_d). If we characterize the "dividend tax preference parameter" from Poterba (2004) as:

$$\theta_t = \frac{(1 - \tau_{d,t})}{(1 - \tau_{e,t})}, \tag{1}$$

where $\tau_{d,t}$ is the top federal tax rate on dividends in year t, and $\tau_{e,t}$ is 80% of the top federal tax rate on long-term capital gains in year t (top rates are used because capital income mostly accrues to high-income individuals), then Table 1 illustrates that dividends were likely to be tax-disadvantaged in every year between 1972-2017, since the after-tax income from a $1 gross dividend payment was approximately $\theta_t < 1$ of the after-tax income from a $1 share repurchase in year t.[13]

From Table 1 it would appear that US corporations should have avoided dividend payments altogether between 1972-2017. This, of course, was not the case. Despite being tax disadvantaged, dividends were a significant component of corporate payout in every year. This is illustrated by Figures 1 and 2, which plot aggregate dividend payments and share repurchases (in 2017 dollars), and the percentage of firms paying each, respectively, over the period 1972-2017 by Compustat-listed firms headquartered in the US.[14] As illustrated by Figure 1, dividends were the largest component of corporate payout until 1997, and although share repurchases assumed this position afterwards - except in 2009 - aggregate dividends remained sizable and grew in most years (average growth rate of 4.8% per year from 1997 to 2017). Furthermore, as illustrated by Figure 2, more firms paid dividends than repurchased

[12] Share repurchases also generate realized capital gains for selling shareholders; an important feature of the analysis below.

[13] Appendix A describes each tax-rate series and discusses θ_t. Also, see Jacob and Jacob (2013) for evidence that dividends were tax-disadvantaged relative to accrued capital gains in the following G-10 countries over the period 1990-2008: France, Germany, Japan, the Netherlands, Sweden (with the possible exception of 1994), Switzerland, and the United States.

[14] In keeping with the previous literature, both utilities and financial institutions - which typically pay high dividends - are excluded from the sample due to their unique regulatory environments (Standard Industrial Classification codes 4900-4949 and 6000-6999, respectively). See Appendix A for a description of each payout variable.

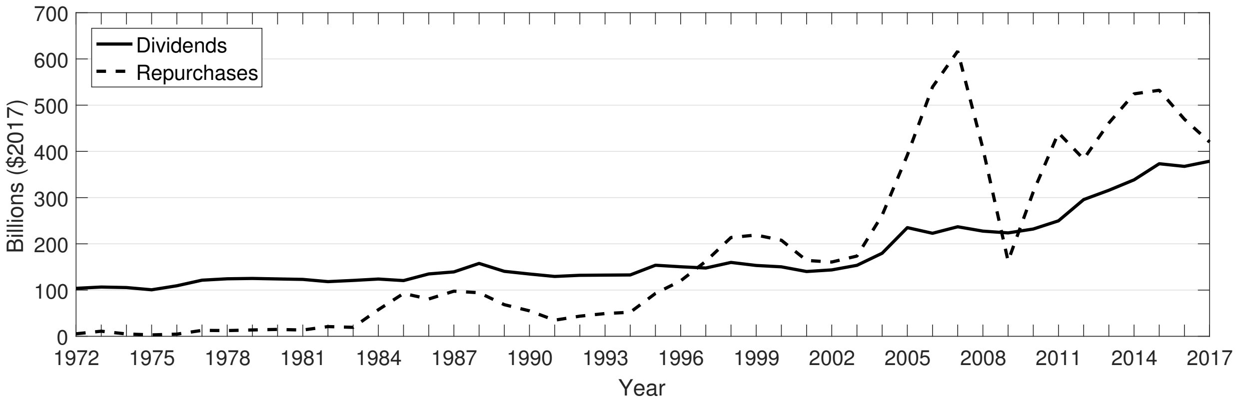
Table 1. Top Federal Tax Rate on Dividends and Long-Term Capital Gains (US: 1972-2017)

	Div.	Cap. Gains				Div.	Cap. Gains		
	$\tau_{d,t}\%$	$\tau_{g,t}\%$	$\tau_{e,t}\%$	θ_{t}		$\tau_{d,t}\%$	$\tau_{g,t}\%$	$\tau_{e,t}\%$	θ_{t}
Year	(1)	(2)	(3)	(4)	Year	(5)	(6)	(7)	(8)
1972	70.0	35.0	28.0	**0.42**	1995	39.6	28.0	22.4	**0.78**
1973	70.0	35.0	28.0	**0.42**	1996	39.6	28.0	22.4	**0.78**
1974	70.0	35.0	28.0	**0.42**	1997	39.6	28.0	22.4	**0.78**
1975	70.0	35.0	28.0	**0.42**	1998	39.6	20.0	16.0	**0.72**
1976	70.0	35.0	28.0	**0.42**	1999	39.6	20.0	16.0	**0.72**
1977	70.0	35.0	28.0	**0.42**	2000	39.6	20.0	16.0	**0.72**
1978	70.0	35.0	28.0	**0.42**	2001	39.1	20.0	16.0	**0.73**
1979	70.0	28.0	22.4	**0.39**	2002	38.6	20.0	16.0	**0.73**
1980	70.0	28.0	22.4	**0.39**	2003	15.0	15.0	12.0	**0.97**
1981	70.0	28.0	22.4	**0.39**	2004	15.0	15.0	12.0	**0.97**
1982	50.0	20.0	16.0	**0.60**	2005	15.0	15.0	12.0	**0.97**
1983	50.0	20.0	16.0	**0.60**	2006	15.0	15.0	12.0	**0.97**
1984	50.0	20.0	16.0	**0.60**	2007	15.0	15.0	12.0	**0.97**
1985	50.0	20.0	16.0	**0.60**	2008	15.0	15.0	12.0	**0.97**
1986	50.0	20.0	16.0	**0.60**	2009	15.0	15.0	12.0	**0.97**
1987	38.5	28.0	22.4	**0.79**	2010	15.0	15.0	12.0	**0.97**
1988	28.0	28.0	22.4	**0.93**	2011	15.0	15.0	12.0	**0.97**
1989	28.0	28.0	22.4	**0.93**	2012	15.0	15.0	12.0	**0.97**
1990	28.0	28.0	22.4	**0.93**	2013	20.0	20.0	16.0	**0.95**
1991	31.0	28.0	22.4	**0.89**	2014	20.0	20.0	16.0	**0.95**
1992	31.0	28.0	22.4	**0.89**	2015	20.0	20.0	16.0	**0.95**
1993	39.6	28.0	22.4	**0.78**	2016	20.0	20.0	16.0	**0.95**
1994	39.6	28.0	22.4	**0.78**	2017	20.0	20.0	16.0	**0.95**

Columns 1 and 5 (2 and 6) report the top federal tax rate on dividends (long-term capital gains) in the United States from 1972 to 2017 (in years when the maximum tax rate changed mid-year, the highest rate is used), while columns 3 and 7 report estimates of the effective tax rate on accrued capital gains using an effective-to-statutory ratio of 0.8. Finally, columns 4 and 8 report the dividend tax preference parameter (Equation 1) for each year; it is below unity in all 46 years, which implies that dividends were tax-disadvantaged relative to share repurchases over the entire period.

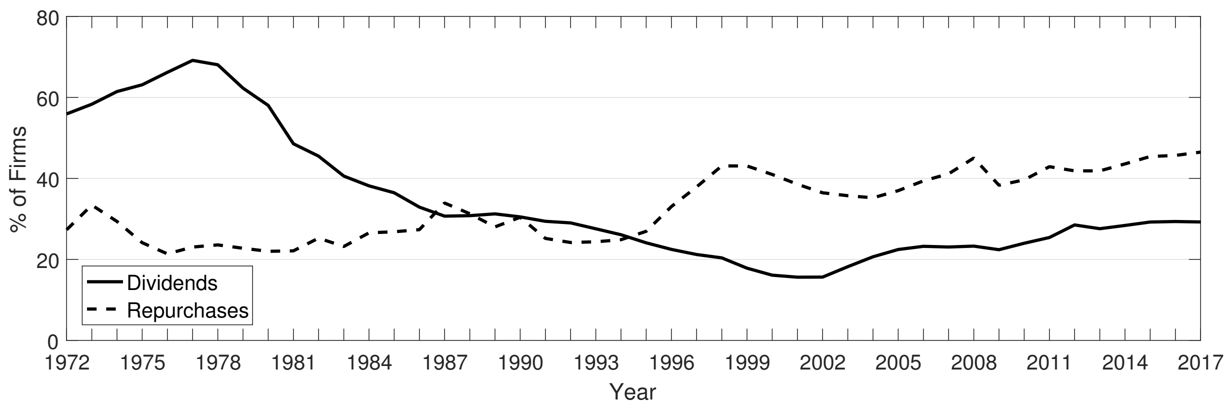
shares during most of the period 1972-1994. However, the percentage of dividend-paying firms has declined substantially over time: from a maximum of 69.1% in 1977 to a minimum of 15.6% in 2001 (Fama and French, 2001 attributes much of this decline to firm-composition effects). The percentage of firms paying dividends has picked up recently, however, reaching 29% by 2017.

Figure 1. Aggregate Dividends and Share Repurchases (US: 1972-2017)



This figure plots the aggregate level of real dividends (solid line) and share repurchases (dashed line) by Compustat-listed firms headquartered in the United States (excluding utilities and financial institutions) over the period 1972-2017 (in 2017 dollars).

Figure 2. Fraction of Firms Paying Dividends and Repurchasing Equity (US: 1972-2017)



This figure plots the fraction of Compustat-listed firms headquartered in the United States (excluding utilities and financial institutions) that paid dividends (solid line) and repurchased equity (dashed line) in each year between 1972-2017.

Although share repurchases have become more prominent in recent years, dividends have not disappeared and remain a significant component of corporate payout. The model developed below provides an explanation for this.

4 Model

This section develops the payout model. It begins by characterizing firms and shareholders, and then it derives the model's steady-state equilibria.

4.1 Firms

Firms in the model are all-equity financed; this allows us to abstract from issues related to capital structure and to focus on payout policy. Shareholders discount income generated by the firm, net of all applicable taxes, at the constant rate $1/(1+\rho)$ per-period, where $\rho > 0$. Consistent with investor rationality, shareholders value after-tax income from dividends and capital gains identically. There are two levels of taxation: firms pay corporate-profits tax at the rate τ_c, while shareholders pay income tax at the rate τ_d on dividends and τ_g on realized capital gains. These rates are constant across firms, shareholders, and time. Furthermore, there is no risk in the model, all information is symmetric, and firms live forever.

Firms begin each period t with a given level of non-depreciating capital K_t carried over from the previous period. This is used to generate end-of-period profits according to the function $\pi(K)$, where $\pi'(K) > 0$ and $\pi''(K) < 0$. When gross profits are generated, firms simultaneously pay corporate-profits tax, issue dividends, repurchase equity, and make capital investments, in that order. Firms can also issue equity (negative share repurchases) and divest capital (negative investment), but they cannot pay negative dividends. Managers act in the interest of shareholders by maximizing firm value (V). This is accomplished by selecting a feasible sequence of dividend payments, share repurchases, and capital investments subject to the firm's beginning-of-period capital stock.

Given these assumptions, an end-of-period dividend payment D_t is worth $(1-\tau_d)D_t/(1+\rho)$ at the beginning of period t, whereas an end-of-period capital gain ΔV_t is typically worth more than $(1-\tau_g)\Delta V_t/(1+\rho)$, since, as discussed in Section 3, the effective tax rate on ΔV_t is typically lower than τ_g.[15] If we continue to denote the marginal investor's effective tax rate by τ_e, then these gains are worth $(1-\tau_e)\Delta V_t/(1+\rho)$ at the beginning of period t.

[15]Note that V_t is the firm's *beginning*-of-period market value, whereas D_t and ΔV_t are the *end*-of-period dividend payment and capital gain (ex-dividend), respectively.

Taken together, the firm's intrinsic/market value at the beginning of period t is:[16]

$$V_t = \frac{(1-\tau_d)D_t}{(1+\rho)} + \frac{(1-\tau_e)\Delta V_t}{(1+\rho)} + \frac{V_t}{(1+\rho)}. \tag{2}$$

Capital gains can be generated in two ways within the model: higher continuation values, i.e., $V_{t+1} > V_t$; and higher ownership concentration following a share repurchase, i.e., $V_{t+1}/(1-\delta_t) > V_{t+1}$, where $\delta_t > 0$ is the fraction of equity repurchased in period t, which equals:

$$\delta_t = \frac{R_t}{R_t + V_{t+1}}, \tag{3}$$

where R_t is the intrinsic value of repurchased equity.[17] Therefore, capital gains at the end of period t are:

$$\Delta V_t = V_{t+1}/\left(1 - \frac{R_t}{V_{t+1} + R_t}\right) - V_t,$$

which simplifies to:

$$\Delta V_t = V_{t+1} + R_t - V_t. \tag{4}$$

Substituting Equation 4 into Equation 2 produces the following firm-value identity:

$$V_t = \frac{(1-\tau_d)D_t}{(1+\rho)} + \frac{(1-\tau_e)(V_{t+1} + R_t - V_t)}{(1+\rho)} + \frac{V_t}{(1+\rho)},$$

and solving for V_t produces:

$$V_t = \left[1 + \frac{\rho}{(1-\tau_e)}\right]^{-1} \left(\frac{(1-\tau_d)}{(1-\tau_e)}D_t + R_t + V_{t+1}\right). \tag{5}$$

This is a common firm-value equation but deserves a brief explanation. The end-of-period value of a gross dividend payment D_t is equal to the after-tax cash receipt $(1-\tau_d)D_t$ plus

[16]The terms "intrinsic value" and "market value" are used interchangeably. Section 5 refines the latter concept by introducing market transactions that involve outside investors.

[17]This equation can be derived as follows. Denote by N_t the number of perfectly-divisible outstanding shares at the beginning of period t, then $R_t = (N_t\delta_t)\,(V_{t+1}/N_t(1-\delta_t))$. I.e., the firm repurchases $N_t\delta_t$ of its shares during the repurchase program, and each of these is worth $V_{t+1}/N_t(1-\delta_t)$, where this valuation follows from the firm's continuation value of V_{t+1} being split equally among the remaining $N_t(1-\delta_t)$ shares. Canceling the N_t terms, and rearranging this expression, produces Equation 3.

the subsequent capital-loss offset that is created once the stock becomes ex-dividend. When both of these are fully capitalized into the stock's price, the value of a marginal dividend - just prior to its payment - is equal to $(1-\tau_d)/(1-\tau_e)$. Conversely, share repurchases - just prior to the repurchase program - have no tax implications for marginal investors, since the program's full value is already capitalized into the stock's price. This implies that both R_t and V_{t+1} have a unit coefficient at the end of period t. However, the story is different when capital gains are expected to materialize in the future. Here, marginal investors rightly anticipate the future tax liability on these gains, and the discount factor is adjusted (reduced) accordingly. This can be seen from the square-bracketed term of Equation 5, which has ρ inflated by the factor $1/(1-\tau_e)$.[18] This modified discount factor is applied to end-of-period dividends and share repurchases equally, as both constitute a capital gain when viewed from the beginning of a period (i.e., both increase equity's market value).

Moving on, the firm's intrinsic value from Equation 5 can be transformed into the following infinite sequence of payout:

$$V_t = \sum_{s=t}^{\infty} \left[1 + \frac{\rho}{(1-\tau_e)}\right]^{-(s-t+1)} \left(\frac{(1-\tau_d)}{(1-\tau_e)} D_s + R_s\right), \tag{6}$$

by applying the transversality condition:

$$\lim_{T\to\infty} \left[1 + \frac{\rho}{(1-\tau_e)}\right]^{-T} V_T = 0.$$

Equation 6 is useful for describing the corporate payout puzzle. Recall that firms use after-tax profits to pay dividends, repurchase shares, and/or make capital investments. It is usually assumed that firms can repurchase all equity at its intrinsic value when capital markets are

[18] For the interested reader, the discount factor $[1+\rho/(1-\tau_e)]^{-1}$ can be explained as follows. An increase in either dividends or share repurchases translates into a higher end-of-period cum-dividend share price, i.e., a capital gain. The value of a marginal capital gain is $(1-\tau_e)$ at the end of a period, which translates into a beginning-of-period value of $(1-\tau_e)/(1+\rho)$. However, when this amount is capitalized into the firm's beginning-of-period market value, it not only increases the stock price, it also reduces the marginal investor's future tax liability, since the stock now appreciates by $(1-\tau_e)/(1+\rho)$ less. This creates a future tax savings of $\tau_e(1-\tau_e)/(1+\rho)$ with a present value of $\tau_e(1-\tau_e)/(1+\rho)^2$. This, in turn, increases the firm's stock price once again and further reduces the marginal investor's future tax liability. Continuing this process indefinitely produces an infinite sequence of progressively smaller tax savings with present value $[\tau_e(1-\tau_e)]/[(1+\rho)(1+\rho-\tau_e)]$. When this sum is added to the discounted value of the original capital gain (i.e., $(1-\tau_e)/(1+\rho)$), we arrive at the discount factor in Equation 5.

perfect (i.e., complete, competitive, frictionless, and free of informational asymmetries). This assumption is inconsistent with capital-gains taxation, however - with important implications below - but for the sake of describing the payout puzzle, assume that it holds for the moment, i.e., that R_s - the intrinsic value of repurchased equity in period s - equals the amount spent repurchasing shares in that period (denoted by A_s henceforth). In this case, a firm's period-s budget constraint is:

$$(1-\tau_c)\pi(K_s) = D_s + R_s + I_s. \tag{7}$$

It becomes immediately obvious from Equations 6 and 7 that setting $D_s = 0 \ \forall \ s$ is optimal whenever $\tau_d > \tau_e$, since a marginal substitution of dividends with share repurchases in period $s \geq t$ raises the firm's period-t value by $[1+\rho/(1-\tau_e)]^{-(s-t+1)}(\tau_d - \tau_e)/(1-\tau_e) > 0$. Given that τ_d is generally greater than τ_e (see Table 1 of Section 3) the payout puzzle follows immediately.[19]

Since $D_s = 0 \ \forall \, s$ is empirically violated (see Figures 1 and 2 of Section 3) the perfect-information literature has typically appealed to either: exogenous factors that place lower bounds on dividends (either explicit or implicit), such as the "intrinsic value of dividends" argument in Shefrin and Statman (1984); or exogenous factors that place upper bounds on share repurchases, such as appeals to Section 302 of the Internal Revenue Code. The former explanation is commonly referred to as the Traditional view or Conventional view, while the latter is commonly referred to as the New view or Tax Capitalization view.

In contrast to these explanations, the one pursued here does not require exogenous bounds on dividends or share repurchases. The crucial assumption is that capital gains are taxed upon *realization* rather than *accrual*. In this way, the model provides a self-contained rational for dividend payments. This is accomplished by deriving the firm's "repurchase function" (the mapping from A_t into R_t), which is based on shareholder-wealth maximization within a realization-based capital-gains tax system. This is done next.

[19] Conversely, if $\tau_d < \tau_e$, firm-value maximization entails unlimited equity issuance and unlimited dividend payments. Only in the special case when $\tau_d = \tau_e$ could there be an interior solution.

4.2 Shareholders and the Mapping from A_t into R_t

Shareholders are identical in all respects except for their investment horizons and level of accrued capital gains on firm equity. The mechanisms that produce each source of heterogeneity are not modeled in this section (they are modeled in Section 5 when the shareholder distribution is endogenized); in practice, the former depends on factors such as retirement planning, portfolio management, and the timing of consumption expenditures, while the latter depends on the timing of past share purchases. Consistent with the notion of an "investment horizon," shareholders in the model reinvest all dividend income in the same (or identical) stock and fully liquidate their equity positions upon reaching that horizon.[20] In this way, shareholders seek to maximize the amount of after-tax wealth they have upon reaching their investment horizon. Pursuing this objective within a realization-based capital-gains tax system produces the lock-in effect described above: i.e., the incentive to postpone the sale of equity with an accrued capital gain and non-zero investment horizon.

To illustrate this point, consider two scenarios regarding a period-t equity position with unit market value and proportional tax basis β (the position's acquisition cost dividend by its current market value): 1) the position is held for H periods (where H is the investment horizon); and 2) the same position is sold at the beginning of period t, and the after-tax proceeds are reinvested for H periods (in the same equity). Note that $\beta < 1$ indicates a capital gain, whereas $\beta > 1$ indicates a capital loss. If we denote the rate-of-return from dividends (capital gains) by r_d (r_g), then under Scenario 1, the position's after-tax value at the beginning of period $t + H$ will be:

$$(1-\tau_g)\left[(1+r_g+(1-\tau_d)r_d)^H\right]+\tau_g\left[\beta+(1-\tau_d)r_d\sum_{h=1}^{H}(1+r_g+(1-\tau_d)r_d)^{h-1}\right], \quad (8)$$

where the first part of this expression is the position's market value multiplied by $(1-\tau_g)$, while the second is the position's tax basis - the original tax basis (β) plus the reinvested dividend income - multiplied by τ_g; i.e., since we applied $(1-\tau_g)$ to the position's entire value (part one), we must add back its tax basis scaled by τ_g, since this part of the position

[20] The full-reinvestment assumption is not necessary for the main results below, however, it seems to be the most natural. Furthermore, reinvestment in debt securities is also permissable - as in Section 5.

escapes taxation. See Appendix B for a derivation of Expression 8. With regard to the second scenario, the period-t equity sale produces after-tax wealth of:

$$(1-\tau_g)+\tau_g\beta, \tag{9}$$

i.e., the position's period-t market value (1) multiplied by $(1-\tau_g)$ plus the period-t tax basis (β) scaled by τ_g. When this amount is reinvested in the same equity, the new position's after-tax value becomes:

$$(1-\tau_g)\left[((1-\tau_g)+\tau_g\beta)(1+r_g+(1-\tau_d)r_d)^H\right] + \tau_g\left[((1-\tau_g)+\tau_g\beta)\left(1+(1-\tau_d)r_d\sum_{h=1}^{H}(1+r_g+(1-\tau_d)r_d)^{h-1}\right)\right] \tag{10}$$

at the beginning of period $t+H$. This expression is similar to 8 above, except that the position's market value and tax basis are both $(1-\tau_g)+\tau_g\beta$ at the beginning of period t (instead of 1 and β, respectively) because of the period-t tax payment (refund) when $\beta<1$ ($\beta>1$). See Appendix B for a derivation of Expression 10.

The difference in after-tax wealth across these two scenarios (Expression 8 minus Expression 10) is characterized by the following function:

$$\Omega(H,\beta)=\begin{cases}\tau_g(1-\beta)[(1-\tau_g)r_g+(1-\tau_d)r_d]\sum_{h=1}^{H}(1+r_g+(1-\tau_d)r_d)^{h-1} & \text{if } H>0,\\ 0 & \text{if } H=0,\end{cases} \tag{11}$$

which has the following five relevant properties:

1. $\Omega(H,\beta)\geq 0$ if $\beta<1$: shareholders have a disincentive to sell equity with an accrued capital gain,
2. $\Omega(H,\beta)\leq 0$ if $\beta>1$: shareholders have an incentive to sell equity with an accrued capital loss,[21]

[21] The tax refund from selling equity with a capital loss requires offsetting income (e.g., a capital gain on other assets).

3. $\Omega(H, \beta) = 0$ if $\beta = 1$: shareholders are indifferent between selling/retaining equity in the absence of a capital gain or loss,

4. $\Omega'_\beta(H, \beta) \leq 0$: the wealth differential is decreasing in the investor's tax basis (increasing in the capital gain), where $\Omega'_\beta(\cdot)$ is the derivative of $\Omega(H, \beta)$ with respect to β,

5. $\Omega(H + 1, \beta) - \Omega(H, \beta) \geq 0$: the wealth differential is increasing in the investor's investment horizon.

See Appendix B for a derivation of Equation 11. The first property of $\Omega(H, \beta)$ is due to the foregone capital income that would have accrued on the shareholder's gross wealth used to pay the capital-gains tax in period t, whereas the second property is due to the additional income that is generated on the period-t tax refund.

Given the properties of $\Omega(H, \beta)$, shareholders with a capital loss are happy to liquidate their equity positions immediately, whereas those with a capital gain are not indifferent between: 1) liquidating their position at the firm's intrinsic value, and 2) retaining it for a desired number of periods. In order to create such indifference, a shareholder must be offered a "lock-in premium" in addition to equity's intrinsic value. This premium is shareholder-specific owing to differences in tax basis and investment horizon (properties 4 and 5, respectively) and is characterized by the following function (as a fraction of equity's intrinsic value):

$$L(H, \beta) = \begin{cases} (1 - \beta)\frac{\tau_g}{1-\tau_g}\left[\frac{\Omega(H,\beta)}{\Omega(H,\beta)+\tau_g(1-\beta)}\right] & \text{if } \beta < 1, \\ 0 & \text{if } \beta \geq 1, \end{cases} \tag{12}$$

where $\Omega(H, \beta)$ is from Equation 11. See Appendix C for a derivation of this result. As with $\Omega(H, \beta)$, the lock-in premium $L(H, \beta)$ is increasing in H and decreasing in β. When shareholders are offered this premium (in addition to equity's intrinsic value) they become indifferent between liquidating their equity position and holding it for a desired number of periods. This indifference makes $L(H, \beta)$ a pure cost from the perspective of firms and shareholders during a repurchase program. The only beneficiary is the tax authority via accelerated realizations.

It will be assumed throughout that each investor is paid their specific lock-in premium when selling equity back to a firm, which is the minimum compensation that a wealth-maximizing shareholder is willing to accept.[22] This minimizes the cost of repurchasing equity while maintaining shareholder rationality, which in turn, reduces the likelihood that dividends are paid in equilibrium. As such, the current pricing assumption is taken to be a "worst-case scenario" for dividend payments.

Recall that shareholders are heterogeneous with respect to H and β; this gives rise to a diverse set of shareholder types. Denote the density of this set by $f(H, \beta)$, which is assumed to be distributed discretely over H (in one-period intervals) and continuously over β. Furthermore, assume that $f(H, \beta)$ has full support over the domain $\{H, \beta \,|\, 0 \leq H \leq \overline{H}, 0 \leq \beta \leq \overline{\beta}\}$ for some $\overline{H} > 0$ and $\overline{\beta} > 0$, and that $f(H, \beta)$ is constant across time. The last two assumptions are relaxed in Section 5 when the shareholder distribution is endogenized. But for the time-being, it is convenient to assume that $f(H, \beta)$ has reached a long-run steady-state[23] and has full support.[24]

4.3 The Repurchase Function

Now that we have the lock-in premium function $L(H, \beta)$, the shareholder density $f(H, \beta)$, and a pricing assumption, we can derive the mapping from A_t into R_t. To make this derivation straight-forward, first decompose the shareholder density $f(H, \beta)$ into $\overline{H} + 1$ sections corresponding to each possible investment horizon from 0 to $\overline{H}$, and denote these by $f^H(\beta)$. Second, designate a β value for each investment horizon, and denote these by $\beta(H)$. Finally, note that firm-value maximization entails repurchasing equity at the lowest cost, i.e.,

[22] This assumption is consistent with Ikenberry et al. (1995), which documents the *gradual* increase in share prices during a typical repurchase program: specifically, Ikenberry et al. (1995) shows that a stock's cumulative abnormal return (CAR) gradually increases over the three-year period following an open-market repurchase announcement, a time during which most repurchase activity takes place according to Stephens and Weisbach (1998). Chan et al. (2010) also documents this pattern for CARs during most of the two-year period following a repurchase announcement.

[23] As shown in Section 5, share repurchases necessarily alter the shareholder distribution by removing investors with relatively low lock-in premia (i.e., those with short investment horizons and/or few capital gains). However, a constant shareholder distribution is still possible given countervailing dynamics: a shareholder's investment horizon becomes progressively shorter as time passes, and shareholder turnover between successive repurchase programs creates a new set of shareholders with relatively low capital gains.

[24] This assumption merely ensures a continuously-differentiable mapping from A_t into R_t.

from shareholders with the lowest lock-in premia.[25] Therefore, a firm's period-t repurchase function is derived by selecting $\beta(H),\ H = 0, 1, .., \overline{H}$ to maximize:

$$R_t = \max_{\{\beta(H)\}} \left[\sum_{H=0}^{\overline{H}} \int_{\beta(H)}^{\overline{\beta}} f^H(\beta) d\beta \right] (R_t + V_{t+1}), \tag{13}$$

subject to:

$$A_t = \left[\sum_{H=0}^{\overline{H}} \int_{\beta(H)}^{\overline{\beta}} [1 + L(H, \beta)] f^H(\beta) d\beta \right] (R_t + V_{t+1}), \tag{14}$$

where V_{t+1} is the firm's continuation value. These two equations state that firms maximize the total mass of repurchased equity subject to spending A_t on the repurchase program. In the absence of lock-in premia (i.e., $L(H, \beta) = 0\ \forall\ H$ and $\forall\ \beta$) - which is typically assumed - the repurchase function collapses to $R(A_t) = A_t$. However, when lock-in premia are characterized by Equation 12, firms pay investor-specific premiums to repurchase equity, and thus, $R(A_t) \leq A_t$. This property of the repurchase function is stated in Proposition 1 along with three others.

Proposition 1 *The repurchase function has the following 4 properties for any shareholder density with full support:*

1. $R(A_t) \leq A_t$, *2.* $R'(A_t) > 0$, *3.* $R''(A_t) \leq 0$, *and* *4.* $R(A_t) \in C^1$.

See Appendix D for a proof of these results.

Proposition 1 states that $R(A_t)$ is increasing in the amount spent and below (possibly weakly) the 45° line. As more equity is repurchased, the marginal shareholder's lock-in premium increases; it follows that $R(A_t)$ is globally concave in the amount spent. Finally, the repurchase function is continuously differentiable.

With the repurchase function now derived, we return to the firm's maximization problem.

[25] Dittmar and Field (2015) provides evidence that firms typically repurchase equity in a cost-effective manner.

4.4 The Firm's Problem

Recall that firms seek to maximize their value by selecting a sequence of dividend payments, share repurchases, and capital investments from the feasible set. Formally, firms seek to:

$$\max_{\{D_s,A_s,I_s\}_{s=t}^{\infty}} V_t = \sum_{s=t}^{\infty} \left[1 + \frac{\rho}{(1-\tau_e)}\right]^{-(s-t+1)} \left(\frac{(1-\tau_d)}{(1-\tau_e)} D_s + R_s\right),$$

subject to:

- the repurchase function: $R_s = R(A_s)$,
- the per-period budget constraint: $(1-\tau_c)\pi(K_s) = D_s + A_s + I_s$,
- the law of motion for capital: $K_{s+1} = K_s + I_s$,
- and the non-negative dividend constraint: $D_s \geq 0$.

The Karush-Kuhn-Tucker necessary conditions for optimality are as follows (for any period $s \geq t$):

$$\left[1 + \frac{\rho}{(1-\tau_e)}\right]^{-(s-t+1)} \frac{(1-\tau_d)}{(1-\tau_e)} - \lambda_s^B - \lambda_s^D = 0, \tag{15}$$

$$\left[1 + \frac{\rho}{(1-\tau_e)}\right]^{-(s-t+1)} R'(A_s) - \lambda_s^B = 0, \tag{16}$$

$$\lambda_s^B (1-\tau_c)\pi'(K_s) - \lambda_s^K + \lambda_{s-1}^K = 0, \tag{17}$$

$$-\lambda_s^B - \lambda_s^K = 0, \tag{18}$$

$$\lambda_s^i \geq 0 \text{ for } i = B, K, D, \tag{19}$$

$$\lambda_s^D D_s = 0, \tag{20}$$

where λ_s^B, λ_s^K, and λ_s^D are the period-s Lagrange multipliers on the budget constraint, law of motion for capital, and non-negative dividend constraint, respectively.

4.5 Steady-State Payout

To derive the steady-state level of payout, and to show that dividends are consistent with firm-value maximization, use Equations 15 and 16 to get:

$$R'(A_s^*) = \frac{(1-\tau_d)}{(1-\tau_e)} + \lambda_s^D,$$

where $A_s^* = A^* \geq 0$ is the equilibrium amount spent repurchasing shares in period s. This equation states that equilibrium share repurchases have a marginal value equal to that of dividends plus the shadow value of relaxing the non-negative dividend constraint λ_s^D. From Equations 19 and 20, λ_s^D can take one of two qualitatively-distinct values in period s:

1. $\lambda_s^D > 0$ (when $D_s^* = 0$),
2. $\lambda_s^D = 0$ (when $D_s^* > 0$),

where D_s^* is the equilibrium level of dividends in period s.

The first of these ($\lambda_s^D > 0$) prevails whenever the marginal value of a share repurchase exceeds that of a dividend payment for the firm's entire payout. This is usually the case in perfect-information models since $R'(A_s) = 1 > (1-\tau_d)/(1-\tau_e)\ \forall\, A_s$. However, as argued in Section 4.2, the repurchase function's first derivative is not equal to unity for all A_s. Rather, it is strictly less than unity whenever a marginal shareholder has strictly-positive capital gains and a non-zero investment horizon. Furthermore, since marginal lock-in premia increase as more equity is repurchased, the marginal value of a share repurchase declines monotonically as more equity is sought (property 3 of Proposition 1). Therefore, it becomes entirely possible that $R'(A_s)$ falls below $(1-\tau_d)/(1-\tau_e)$ as A_s increases. However, management prevents this from happening in equilibrium by switching from share repurchases to dividend payments when $R'(A_s) = (1-\tau_d)/(1-\tau_e)$. This is captured by the second case above ($\lambda_s^D = 0$). In this situation, $R'(A_s^*) = (1-\tau_d)/(1-\tau_e)\ \forall\, s$, and $D_s^* = (1-\tau_c)\pi(K^*) - A_s^* > 0\ \forall\, s$, where K^* is the steady-state level of capital; thus providing the perfect-information explanation for dividends.

5 Endogenous Shareholder Distribution

This section endogenizes the shareholder distribution using numerical methods. This serves three purposes. First, it illustrates that dividends are more of a certainty in the current framework than a mere possibility for long-lived firms. Second, it establishes the relationship between equity demand curves, effective tax rates, and the firm's market value (by introducing market transactions that involve outside investors). And third, it illustrates that dividends are optimal for various types of marginal investor.

To begin with, assume that two investor types arrive at the beginning of each period: "long-term" investors with initial investment horizons of H_L; and "short-term" investors with initial investment horizons of $H_S \leq H_L$.[26] Long-term (short-term) investors are endowed with ω_L (ω_S) of wealth at the beginning of each period, and both have access to the same two investment opportunities: a risk-free security with after-tax interest rate ρ and the firm's equity. To perpetuate the shareholder distribution over time, every shareholder liquidates a fraction λ of their equity position at the beginning of a period (rebalancing their portfolios), which is sold to outside investors. The proceeds from these equity sales (and any dividend income received) are reinvested in the risk-free security until the shareholder's investment horizon is reached. Furthermore, in keeping with the model's dynamic nature, the investment horizon of every shareholder is reduced by 1 at the end of each period. Management's task is to select a sequence of dividend payments and share repurchases to maximize firm value subject to a constant level of net income (normalized to 1 in every period).[27]

As before, capital gains and losses are generated through changes in a firm's market value and through share repurchases. Unlike before, the repurchase function is now time-specific (due to the shareholder distribution's endogeneity). If we denote the period-s repurchase function by $R_s(\cdot)$, and recall that δ_s is the fraction of equity repurchased in period s (i.e., $\delta_s = R_s(A_s)/(R_s(A_s) + V_{s+1})$), then Equation 21 characterizes the period-s rate-of-return

[26] Two investor types are used here to simplify the analysis and to improve transparency, more can be added without changing the qualitative results below.

[27] Net income is exogenous in the current exercise to simplify the analysis - there is no need to specify the firm's profit function or initial capital stock. One can assume that firms have reached their efficient scale of production at a net income of 1.

from capital gains:

$$r_{g,s} = \frac{V_{s+1}}{(1-\delta_s)V_s} - 1. \tag{21}$$

The foundation for Equation 21 is developed on page 13 and is elaborated upon below (it is also explained in Appendix E). Since the period-s rate-of-return form dividends is:

$$r_{d,s} = \frac{D_s}{V_s}, \tag{22}$$

a period-t equity investment made by an investor with initial holding period H - an H-type investor for brevity - has the following after-tax value at the beginning of period $t+H$:

$$(1-\lambda)^{H-1}\left[\tau_g + (1-\tau_g)\frac{V_{t+H}}{V_t \prod_{w=0}^{H-1}(1-\delta_{t+w})}\right]$$
$$+ \lambda \sum_{h=1}^{H-1}(1+\rho)^{H-h}(1-\lambda)^{h-1}\left[\tau_g + (1-\tau_g)\frac{V_{t+h}}{V_t \prod_{w=0}^{h-1}(1-\delta_{t+w})}\right]$$
$$+ (1-\tau_d)\sum_{h=0}^{H-1}(1+\rho)^{H-h-1}(1-\lambda)^h \frac{D_{t+h}}{V_t \prod_{w=0}^{h-1}(1-\delta_{t+w})}, \tag{23}$$

where this expression accounts for: 1) the after-tax value of equity held for H periods (the first part), 2) the after-tax value of equity sold at the beginning of each period (the fraction λ) and reinvested in the risk-free security until period $t+H$ (the second part), and 3) the after-tax value of reinvested dividend income (the third part). See Appendix E for a derivation of Expression 23.

Going forward, Subsection 5.1 discusses the relationship between effective tax rates (now endogenous) and the firm's market value, Subsection 5.2 explains the dynamics of the shareholder distribution, while Subsection 5.3 presents the main result: dividends are optimal for various marginal-investor types.

5.1 Effective Tax Rates and Firm Value

This section explains the relationship between a firm's market value and the marginal investor's effective tax rate on accrued capital gains. It also explains the evolution of a firm's market value within a period.

To begin with, consider a steady-state in which $D_t = \overline{D}$ and $R_t(A_t) = \overline{R}$ for all t (the repurchase function is endogenized below).[28] Since there are two investment opportunities - the risk-free asset and the firm's equity (which is also risk-free) - the value of equity to an H-type investor - denoted by $\overline{V}_H$ accordingly - solves the following equation:

$$(1+\rho)^H = (1-\lambda)^{H-1}\left[\tau_g + (1-\tau_g)\left(\frac{\overline{R}+\overline{V}_H}{\overline{R}}\right)^H\right]$$

$$+\lambda\sum_{h=1}^{H-1}(1+\rho)^{H-h}(1-\lambda)^{h-1}\left[\tau_g + (1-\tau_g)\left(\frac{\overline{R}+\overline{V}_H}{\overline{R}}\right)^h\right]$$

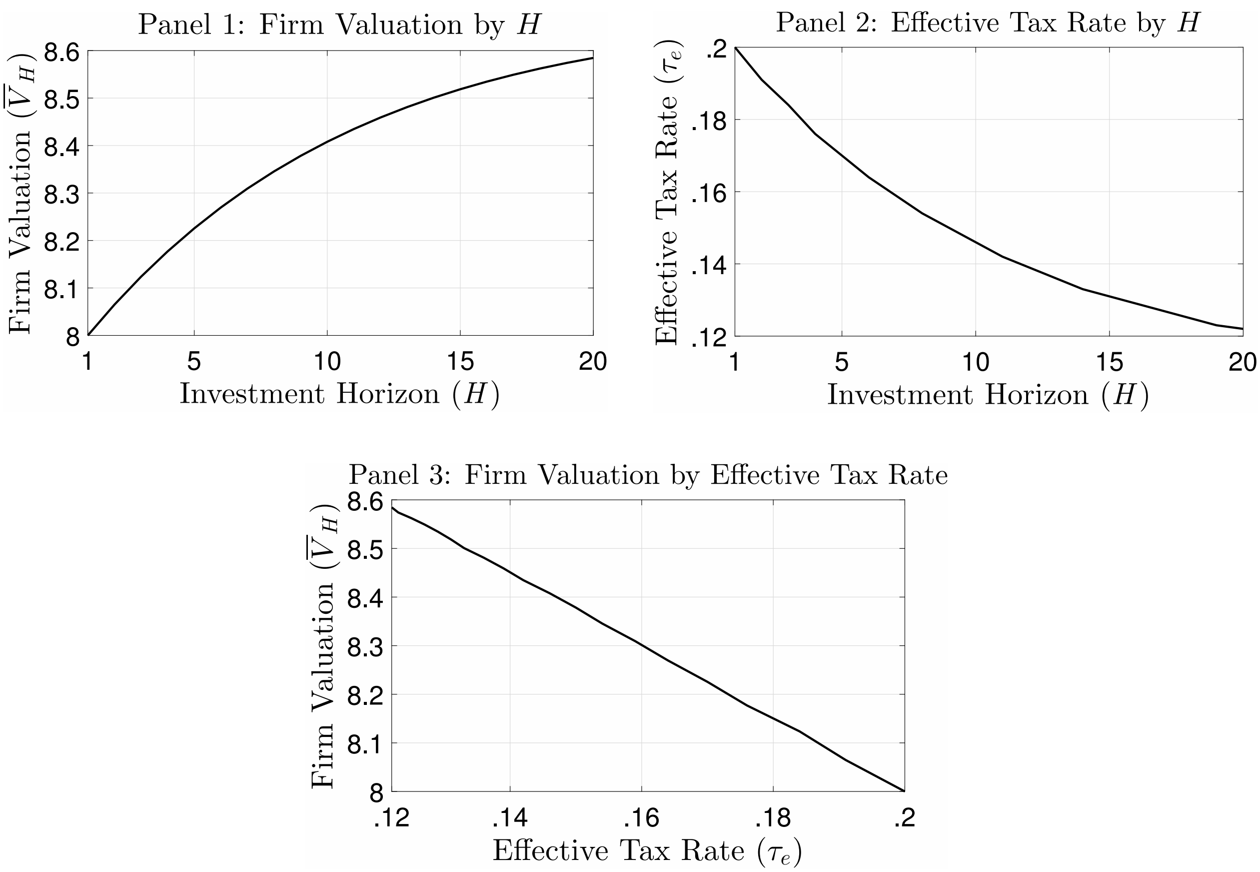
$$+(1-\tau_d)\sum_{h=0}^{H-1}(1+\rho)^{H-h-1}(1-\lambda)^h\frac{D_{t+h}}{\overline{V}_H}\left(\frac{\overline{R}+\overline{V}_H}{\overline{R}}\right)^h, \quad (24)$$

where the left-hand-side of Equation 24 is the risk-free asset's total return after H periods, while the right-hand-side is the after-tax return on an equity investment - conditional on $\overline{V}_H$ - over the same period (Expression 23 with the following substitutions: $D_t = \overline{D}$, $R_t(A_t) = \overline{R}$, and $V_t = \overline{V}_H \ \forall \ t$). This equation states that an H-type investor is willing to pay $\overline{V}_H$ for the firm's equity, and no more, since their outside option is generating an after-tax return of $(1+\rho)^H$ via the risk-free asset. To provide a concrete example, suppose that $\overline{D} = 0.25$, $\overline{R} = 0.75$, $\tau_d = \tau_g = 0.2$, $\lambda = 0.1$, and $\rho = 0.1$. Under these parameters, Panel 1 of Figure 3 plots the mapping from H into $\overline{V}_H$ and illustrates their positive relationship, which arises because longer investment horizons imply longer tax deferment.

This result is associated with an investor's "effective tax rate" on accrued capital gains: the nominal tax rate - applied to capital gains as they accrue - that equates an investor's

[28] A steady-state is used here reduce the number of variables under consideration; the results hold for any sequences $\{D_s\}_t^\infty$, $\{R_s\}_t^\infty$, and $\{V_s\}_{t+t}^\infty$ that generate capital gains.

Figure 3. Firm Valuation and Effective Tax Rates by Investment Horizon



Panel 1 plots the value of equity to an H-type investor when $\overline{D} = 0.25$, $\overline{R} = 0.75$, $\tau_d = \tau_g = 0.2$, $\lambda = 0.1$, and $\rho = 0.1$. Panel 2 plots an H-type investor's effective tax rate using the same parameters. Panel 3 plots the relationship between an H-type investors's effective tax rate and their valuation of firm equity.

after-tax return with those under a realization-based tax.[29] To derive an H-type investor's effective tax rate - denoted by $\tau_{e,H}$ accordingly - you equate Expression 23 (their after-tax return under a realization-based tax) with Expression 25 below (their after-tax return under an accrual-based tax - which is derived in Appendix F) and solve for $\tau_{e,H}$.

[29]There is some disagreement in the public-finance literature over what constitutes *the* effective tax rate. Is it the rate - applied to capital gains as they accrue - that equalizes the present value of tax revenues as in Boadway et al. (1984)? Or is it the rate that equalizes an investor's after-tax returns as in Glenday and Davis (1990)? Since the current analysis deals with investor behavior (not public finances), the latter rate is appropriate.

$$(1-\lambda)^{H-1}\prod_{w=0}^{H-1}\left(\tau_{e,H}+(1-\tau_{e,H})\frac{V_{t+w+1}}{(1-\delta_{t+w})V_{t+w}}\right)$$

$$+\lambda\sum_{h=1}^{H-1}(1+\rho)^{H-h}(1-\lambda)^{h-1}\prod_{w=0}^{h-1}\left(\tau_{e,H}+(1-\tau_{e,H})\frac{V_{t+w+1}}{(1-\delta_{t+w})V_{t+w}}\right)$$

$$+(1-\tau_d)\sum_{h=0}^{H-1}(1+\rho)^{H-h-1}(1-\lambda)^{h}\frac{D_{t+h}}{V_{t+h}}\prod_{w=0}^{h-1}\left(\tau_{e,H}+(1-\tau_{e,H})\frac{V_{t+w+1}}{(1-\delta_{t+w})V_{t+w}}\right). \quad (25)$$

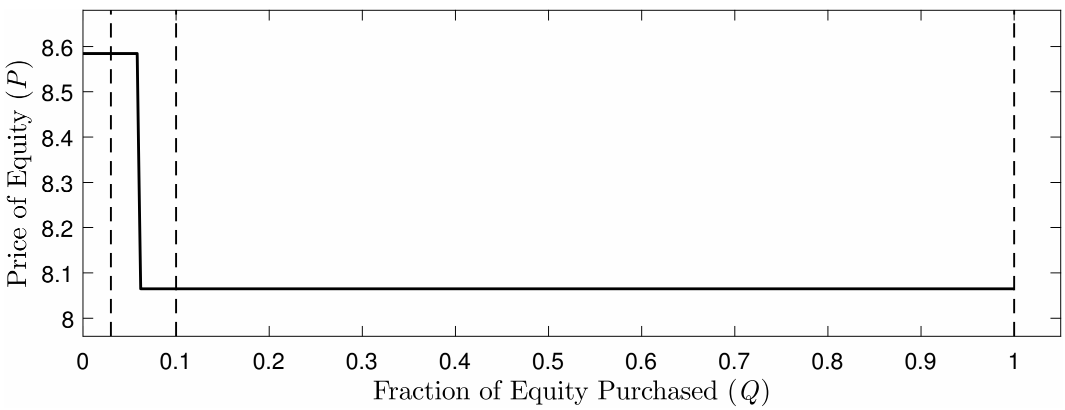
An investor's effective tax rate is monotonically decreasing in their investment horizon due to the associated tax deferment. This is illustrated by Panel 2 of Figure 3, which plots the mapping from H into $\tau_{e,H}$. In the special case of $H=1$, $\tau_{e,1}=\tau_g$ since no taxes are deferred, but $\tau_{e,H}$ becomes strictly less than τ_g for all $H>1$. The positive relationship between H and $\overline{V}_H$ (Panel 1 of Figure 3), and the negative relationship between H and $\tau_{e,H}$ (Panel 2 of Figure 3), implies the negative relationship between $\tau_{e,H}$ and $\overline{V}_H$ (plotted in Panel 3 of Figure 3).

Next, to establish the firm's market value at the beginning of a period, we require the equity demand curve and the quantity of equity sold. To determine the former, we require the investment horizon of potential investors and their aggregate wealth. Suppose that potential investors have initial investment horizons of $H_L=20$ and $H_S=2$, and aggregate wealth of $\omega_L=0.5$ and $\omega_S=\infty$. Based on these values (and $\overline{V}_H$ from Equation 24), the inverse demand curve for equity - the mapping from the fraction of equity purchased (Q) into the unit price of equity (P) - is characterized by the following function:

$$P=\begin{cases}\overline{V}_{20} & \text{if } Q\in\left[0,\frac{\omega_L}{\overline{V}_{20}}\right]\\ \frac{\omega_L}{Q} & \text{if } Q\in\left[\frac{\omega_L}{\overline{V}_{20}},\frac{\omega_L}{\overline{V}_{2}}\right]\\ \overline{V}_{2} & \text{if } Q\in\left[\frac{\omega_L}{\overline{V}_{2}},1\right]\end{cases} \implies P=\begin{cases}8.585 & \text{if } Q\in\left[0,\frac{0.5}{8.585}\right]\\ \frac{0.5}{Q} & \text{if } Q\in\left[\frac{0.5}{8.585},\frac{0.5}{8.065}\right]\\ 8.065 & \text{if } Q\in\left[\frac{0.5}{8.065},1\right],\end{cases}$$

and plotted in Figure 4. To establish the firm's market value, we also need the quantity of equity traded; three examples are depicted in Figure 4. The first (right-most dashed line at $Q=1$) is indicative of an "initial public offering" in which all equity is sold to outside investors. The firm's market value is 8.065 in this case, reflecting the marginal investor's effective tax rate ($\tau_{e,2}=0.191$) and their corresponding valuation. The second example (the dashed line at $Q = 0.1$) is indicative of a medium-sized equity sale, and here too, the market value of equity is 8.065. The third example (the dashed line at $Q = 0.03$) is indicative of a relatively-small equity sale in which long-term investors become marginal; as a result, the firm's market value increases to 8.585, reflecting the marginal investor's lower effective tax rate ($\tau_{e,20} = 0.122$) and higher valuation.

Figure 4. The Equity Demand Curve and the Firm's Market Value



This figure plots the equity-demand curve when potential investors have initial investment horizons of $H_L = 20$ and $H_S = 2$, aggregate wealth of $\omega_L = 0.5$ and $\omega_S = \infty$, and when $D_t = 0.25\ \forall\ t$, $R_t(A_t) = 0.75\ \forall\ t$, $\tau_d = \tau_g = 0.2$, $\lambda = 0.1$, and $\rho = 0.1$. The firm's equilibrium market price is determined by the marginal investor's valuation.

The preceding discussion warrants a quick note on terminology. The intrinsic/market value of equity is what *outside investors* are willing to pay for it (marginal investors) and not what firms are willing to pay during a repurchase program. If it were the latter, then firms could make themselves arbitrarily "valuable" by paying exorbitant prices during a repurchase program for small amounts of equity. This would damage the firm's financial position and reduce its true intrinsic/market value. However, since value-maximizing firms repurchase equity economically (from shareholders with the lowest lock-in premia), the distinction between maximizing V_t and the price paid during a repurchase program is immaterial, as both involve maximizing V_t.

Finally, to help with the discussion below, Figure 5 illustrates the evolution of a firm's market value *within* a period (assume a unit measure of equity).[30] The firm's market value at

[30]Strictly speaking, equity is only traded at the beginning of a period (to outside investors) and during

the beginning of period t is V_t - consistent with the marginal investor's effective tax rate ($\tau_{e,H}$) and the equilibrium sequences $\{D_s\}_t^\infty$, $\{R_s\}_t^\infty$, and $\{V_s\}_{t+1}^\infty$. The firm's cum-dividend market value at the end of period t is the sum of: the market value of dividends $(1-\tau_d)D_t/(1-\tau_{e,H})$ - recall that dividends create an immediate tax liability of $\tau_d D_t$ and reduce the capital-gains tax liability; the market value of repurchased equity R_t; and the firm's continuation value V_{t+1} - this may be smaller than, larger than, or equal to V_t depending on who the period $t+1$ marginal investor is and the equilibrium sequences $\{D_s\}_{t+1}^\infty$, $\{R_s\}_{t+1}^\infty$, and $\{V_s\}_{t+2}^\infty$ (V_{t+1} is drawn lower than V_t for convenience). As time progresses from the beginning of period t until the end (cum-dividend), the firm's market value steadily increases as the discount period shortens (i.e., the discount factor $[1-\rho/(1-\tau_{e,H})]^{-1}$ increases with time). As equity becomes ex-dividend, the firm's market value falls by $(1-\tau_d)D_t/(1-\tau_{e,H})$ - reflecting the reduction in cash balances. Since a fraction δ_t of equity is repurchased in period t, the measure of outstanding shares drops to $(1-\delta_t)$ after the repurchase program; each of these has a market value of $V_{t+1}/(1-\delta_t)$ - reflecting the firm's continuation value (V_{t+1}) and the measure of outstanding shares post-repurchase. Since the market value of all equity is identical (each share is fully fungible: having equivalent cash-flow and control rights), repurchased equity has a market value of $\delta_t V_{t+1}/(1-\delta_t)$ (i.e., the measure of equity repurchased multiplied by the unit value of equity), which equals R_t by definition of the repurchase function, and thus, the firm's market value is $R_t + V_{t+1}$ just before the repurchase program.[31] Upon completion of the repurchase program, the firm's market value falls once more to V_{t+1} - reflecting the further reduction in cash balances. This dynamic occurs in every period.

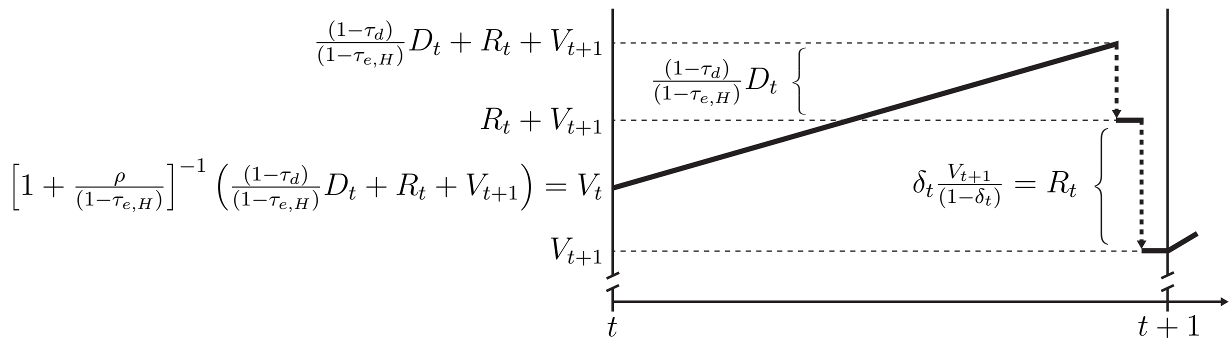
5.2 Shareholder Distribution and Lock-In Premia

This section endogenizes the shareholder distribution to explain its evolution over time; it also illustrates why firms are more likely to pay dividends as they age. The exogenous variables are now: initial investment horizon (H_L and H_S), investor wealth (ω_L and ω_S), statutory tax rates (τ_d and τ_g), the risk-free interest rate (ρ), the fraction of equity sold

a repurchase program (to the firm). A firm's within-period market value is taken to be the amount that a period-t marginal investor is *willing* to pay at each point in time.

[31]Another way to see this market-value identity is as follows. The market value of repurchased equity is $\delta_t V_{t+1}/(1-\delta_t) = R_t$ from above, while the market value of non-repurchased equity is $(1-\delta_t)V_{t+1}/(1-\delta_t) = V_{t+1}$ (i.e., the measure of non-repurchased equity multiplied by the per-unit value of equity). Their sum is $R_t + V_{t+1}$.

Figure 5. The Evolution of a Firm's Market Value Within a Period



This figure illustrates the evolution of a firm's market value within a period. The firm's market value is V_t at the beginning of period t. It steadily increases until equity is just about to become ex-dividend, at this point, it equals $[(1-\tau_d)/(1-\tau_{e,H})]D_t + R_t + V_{t+1}$ (where $\tau_{e,H}$ is the marginal investor's effective tax rate). When equity becomes ex-dividend, the firm's market value drops to $R_t + V_{t+1}$, and after the repurchase program is complete, it drops once more to V_{t+1}. Note that all end-of-period activities take place simultaneously with the beginning of period $t+1$.

at the beginning of each period (λ), and the payout sequences controlled by management ($\{D_s\}_{t=0}^{\infty}$ and $\{A_s\}_{t=0}^{\infty}$). The model is solved under the parameters reported in Table 2.[32] Table 3 reports the firm's equilibrium market value (V_t) and the fraction of equity repurchased (δ_t) in periods 1-4 and in the steady-state (these values are helpful for the discussion below). Appendix G provides a detailed discussion of the model's solution algorithm.

Table 2. Parameter Values

H_L	H_S	ω_L	ω_S	τ_d	τ_g	ρ	λ	D_t	A_t
20	2	0.5	∞	0.2	0.2	0.1	0.1	0.25 $\forall\ t$	0.75 $\forall\ t$

This table reports the parameter values used in the numerical exercise below (Section 5.2).

At the beginning of period 1, all equity is sold to outside investors (a unit measure of equity). Long-term (short-term) investors have an effective tax rate of $\tau_{e,20} = 0.118$ ($\tau_{e,2} = 0.191$) and value equity at 8.44 (8.045). Since short-term investors are marginal, the firm's market value is $V_1 = 8.045$ at the beginning of period 1. Long-term investors purchase 6.2% of the firm's equity ($\omega_L/V_1 = 0.0622$), and short-term investors purchase the remaining

[32] The wealth of each investor type is chosen to ensure that both participate in every beginning-of-period equity sale. This is maintained throughout.

Table 3. Equilibrium Values of: V_t and δ_t

	Period				
	1	2	3	4	Steady-State
V_t	8.0448	8.0431	8.0404	8.0378	7.9426
δ_t	0.0852	0.0853	0.0853	0.0854	0.0847

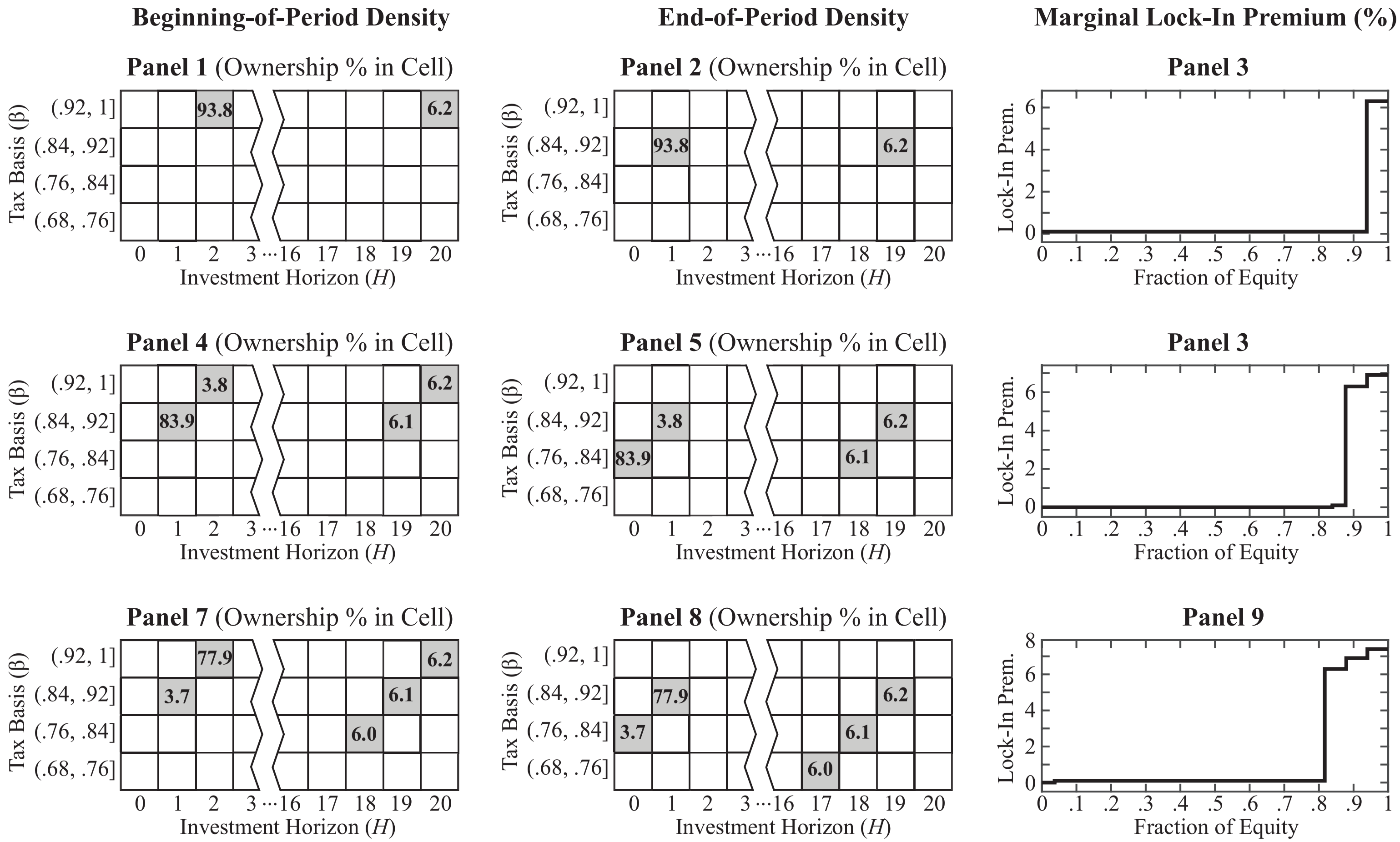
This table reports the firm's beginning-of-period market value (V_t) and the fraction of equity repurchased (δ_t) at the end of periods 1-4 and in the steady-state.

93.8%. Since all equity is purchased at the firm's market price, no capital gains are generated at the beginning of period 1 (i.e., $\beta = 1$ for all shareholders). Panel 1 of Figure 6 depicts the beginning-of-period shareholder density.

The shareholder distribution evolves in two ways during a period: investment horizons are reduced by 1, and every shareholder's β is reduced (increased) when capital gains (losses) are generated. Since a fraction $\delta_1 = 0.0852$ of equity is repurchased in period 1 (Table 3), the firm's market value increases to $V_2/(1 - \delta_1) = 8.792$ just before the repurchase program, which reduces every shareholder's proportional tax basis to $\beta = 0.915$. This fall in β, and the reduction in every shareholder's investment horizon, is reported in Panel 2 of Figure 6, which depicts the end-of-period shareholder density (just before equity is repurchased). Every shareholder requires a lock-in premium at this point given their non-zero investment horizon and accrued capital gains: short-term shareholders require a 0.1% premium, while long-term shareholders require a 6.3% premium; these values are reported in Panel 3 of Figure 6.

Since $A_1 = 0.75$ from Table 2, all equity can be repurchased from short-term shareholders at the unit price of $(1.001)V_2/(1 - \delta_1) = 8.8$ (these shareholders require a 0.1% lock-in premium from above), which enables the firm to repurchase a fraction $\delta_1 = 0.0852$ of its equity. This reduces the measure of outstanding shares to $(1 - \delta_1) = 0.915$ and produces the aforementioned capital gain. Short-term shareholders are left with 93.2% of the firm's equity after the repurchase program - their initial measure of equity (0.938) minus the measure repurchased (0.085) divided by the measure outstanding after the repurchase (0.915) - which has a total market value of $(0.932)V_2 = 7.496$, whereas long-term investors are left with 6.8% of the firm's equity with a total market value of $(0.068)V_2 = 0.547$. Finally, it

Figure 6. Shareholder Densities and Marginal Lock-In Premia



Panels 1, 4, and 7 depict the beginning-of-period shareholder density (just after equity is purchased by outside investors) in periods 1, 2, and 3, respectively. The numbers contained in each cell indicate the percentage of equity owned by each shareholder type. Panels 2, 5, and 8 depict the end-of-period shareholder density (just before equity is repurchased) in periods 1, 2, and 3, respectively. Panels 3, 6, and 9 plot the marginal shareholder's lock-in premium (as a percentage of equity's contemporaneous market value) according to the fraction of equity repurchased, in periods 1, 2, and 3, respectively.

is convenient to assume that firms split their stock at the end of each period to maintain a unit measure of equity (this has no bearing on real values).

All shareholders liquidate a fraction $\lambda = 0.1$ of their equity at the beginning of period 2, which reduces the investment horizon on these shares to $H = 0$; short-term (long-term) shareholders retain 83.9% (6.1%) of the firm's equity (these values are reported in Panel 4 of Figure 6). Similar to period 1, long-term (short-term) investors that arrive in period 2 have effective tax rates of $\tau_{e,20} = 0.118$ ($\tau_{e,2} = 0.191$) and value equity at 8.438 (8.043). Since short-term investors are marginal, the firm's market value is $V_2 = 8.043$ at the beginning of period 2. Long-term investors purchase 6.2% of the firm's equity, whereas short-term investors purchase the remaining 3.8% (these values are reported in Panel 4 of Figure 6). Note that only 10% of the firm's equity changes hands at the beginning of period 2, since its value to a marginal investor is $V_2 = 8.043$, whereas its "locked-in" value to an existing shareholder is either: $(0.001)V_2 = 8.051$, for shareholders with $H = 1$ and $\beta = 0.915$; or $(0.063)V_2 = 8.55$, for shareholders with $H = 19$ and $\beta = 0.915$.[33]

The dynamics during period 2 are similar to those of period 1: investment horizons are reduced by 1, and every shareholder's β is reduced by 8.5% due to the capital gains that are generated because a fraction $\delta_2 = 0.0853$ of equity is repurchased. Both adjustments are reflected in Panel 5 of Figure 6. Four shareholder types emerge at the end of period 2: those with $H = 0$ and $\beta = 0.837$ (due to the compound capital gains from periods 1 and 2), these shareholders require no lock-in premium because $H = 0$; those with $H = 1$ and $\beta = 0.915$, these shareholders require a 0.1% lock-in premium; those with $H = 19$, $\beta = 0.915$, and a required lock-in premium of 6.3%; and those with $H = 18$, $\beta = 0.837$, and a required lock-in premium of 6.9% (these premia are reported in Panel 6 of Figure 6). Since 83.9% of the firm is owned by shareholders with $H = 0$ and $\beta = 0.837$, all equity can be repurchased at the market price of $V_3/(1 - \delta_2) = 8.79$; this enables a fraction $\delta_2 = 0.0853$ of equity to be repurchased when $A_2 = 0.75$.

Equity sales at the beginning of period 3 are composed of two types: a fraction $\lambda = 0.1$ of

[33] In principle, long-term investors are *willing* to pay $(0.001)V_2 = 8.051$ at the beginning of period 2, but never will, since their demand is fully satiated at the lower market price of $V_2 = 8.043$.

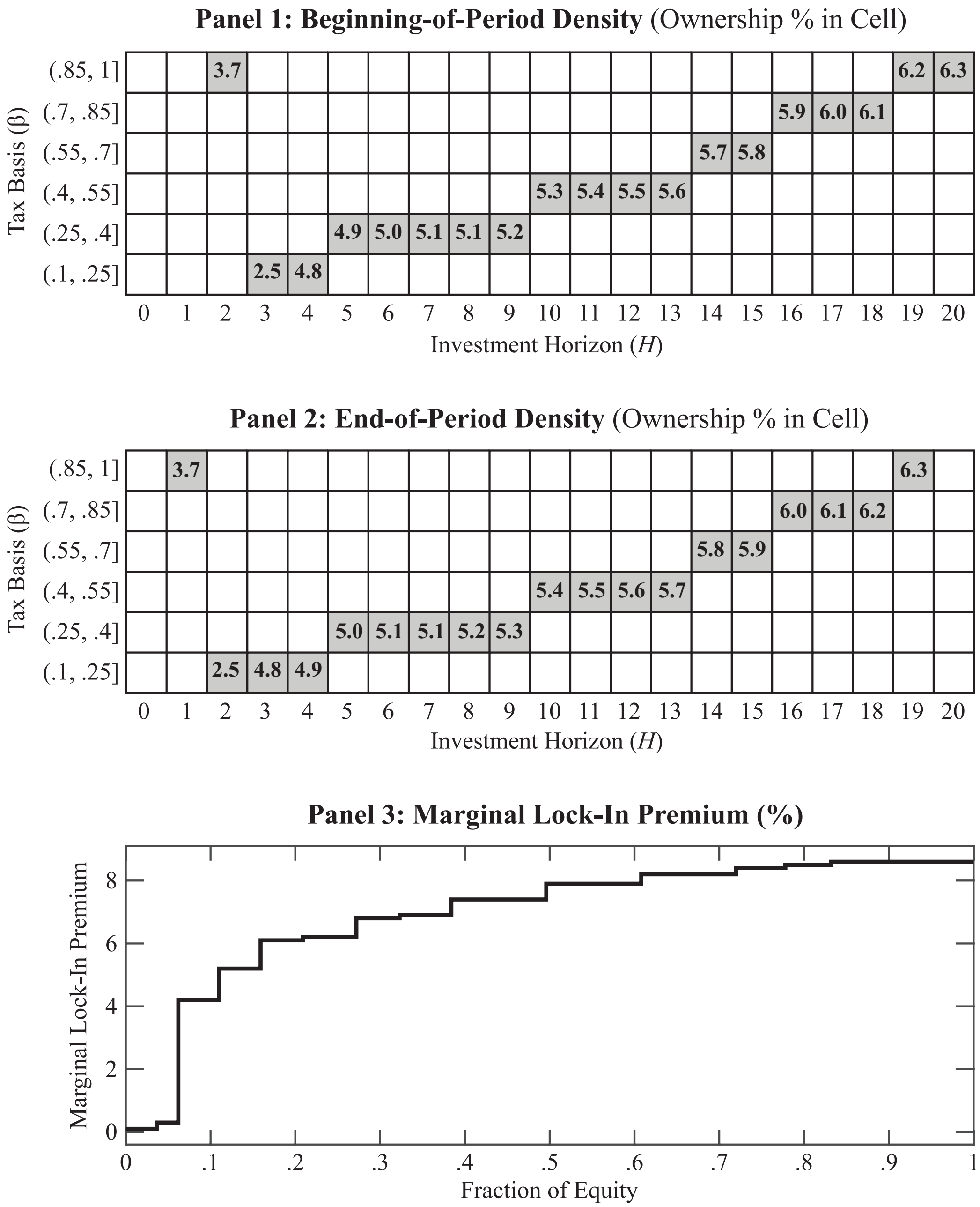
every shareholder's portfolio is liquidated, and all equity owned by shareholders with $H = 0$ (the remaining short-term shareholders from period 1) is also liquidated. The first constitutes 1.8% of the firm's total equity, whereas the latter constitutes 82.4%. Similar to periods 1 and 2, long-term investors (short-term investors) that arrive in period 3 have effective tax rates of $\tau_{e,20} = 0.118$ ($\tau_{e,2} = 0.191$) and value equity at 8.435 (8.04), which results in a market value of $V_3 = 8.04$. Long-term investors purchase 6.2% of the firm's equity, and short-term investors purchase the remaining 77.9%. These values are reported in Panel 7 of Figure 6.

By the end of period 3 there are five shareholder types according to H and β (reported in Panel 8 of Figure 6). Their required lock-in premia are: 0%, 0.1%, 6.3%, 6.9%, and 7.4% (these values are reported in Panel 9 of Figure 6). The firm can repurchase 3.7% of its equity at the market price of $V_4/(1 - \delta_3) = 8.787$ from shareholders with $H = 0$ and $\beta = 0.837$, and can repurchase the remaining 4.8% (note that $\delta_3 = 0.0853$ from Table 3) at a 0.1% premium from shareholders with $H = 1$ and $\beta = 0.915$. Taken together, the firm repurchases a fraction $\delta_3 = 0.0853$ of its equity at the end of period 3.

The model evolves according to the themes described above from period 4 onwards and reaches a stead-state by period 30. Panel 1 (Panel 2) of Figure 7 depicts the beginning-of-period (end-of-period) steady-state shareholder density, while Panel 3 plots the required lock-in premium of each shareholder type. Firms repurchase a fraction $\delta_t = 0.0847$ of their equity in each steady-state period and pay marginal lock-in premia of 4.2% (marginal shareholders are type $H = 3$ and $\beta = 0.222$).

These results illustrate why long-lived firms are more likely to pay dividends (Grullon and Michaely, 2002). Share repurchases are desirable when a large number of shareholders have relatively low lock-in premia (i.e., limited capital gains and/or short investment horizons) as in the first three periods of the model described above. However, since share repurchases target these inexpensive investors - removing them form the shareholder distribution - the posterior shareholder distribution becomes less favorable for subsequent buybacks, save for the gradual reduction in every shareholder's investment horizon and the addition of short-term investors. Furthermore, the act of repurchasing equity generates capital gains, which increases every shareholder's lock-in premium by reducing their β; this is seen by comparing

Figure 7. Shareholder Densities and Marginal Lock-In Premia: Steady State



Panel 1 depicts the beginning-of-period shareholder density (just after equity is purchased by outside investors) in the steady-state; the numbers contained within each cell indicate the percentage of equity owned by each investor type. Panel 2 depicts the end-of-period shareholder density (just before equity is repurchased) in the steady-state. Panel 3 plots the marginal shareholder's lock-in premium (as a percentage of equity's contemporaneous market value) according to the fraction of equity repurchased, in each steady-state period.

the steady-state shareholder density (in which $\beta < 0.55$ for over 60% of the firm's equity) with the shareholder density in the first three periods of the model (in which $\beta > 0.68$ for all equity). Taken together, repurchasing equity is desirable in the early stages of payout (Brav et al. 2005), whereas dividends become more desirable as time passes, ceteris paribus.

5.3 Optimal Dividends

This section presents the main result: dividends are optimal for various marginal-investor types. The parameters used here (reported in Table 4) are similar to the ones used above (Table 2), except that H_S is now variable, as are the payout sequences controlled by management.

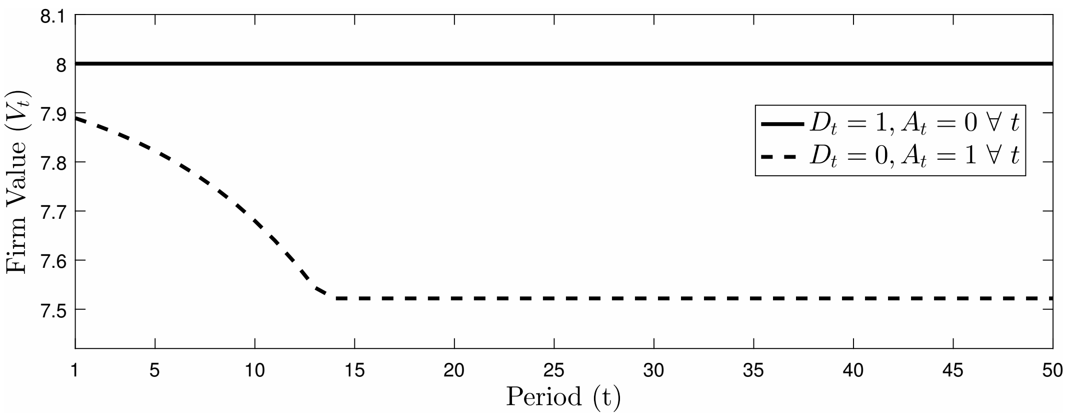
Table 4. Parameter Values

H_L	ω_L	ω_S	τ_d	τ_g	ρ	λ
20	0.5	∞	0.2	0.2	0.1	0.1

This table reports the parameter values used in the numerical exercises below (Section 5.3).

The simplest case to consider is when $H_S = 1$ since the marginal investor's effective tax rate equals the statutory rate in every period (i.e., $\tau_{e,1} = \tau_g = 0.2$), and dividends have a unit marginal value regardless of the payout mix (i.e., $(1 - \tau_d)/(1 - \tau_{e,1}) = 1$). Since the marginal value of a share repurchase is weakly less than 1 - equal to (less than) 1 in the absence (presence) of lock-in premia - excluding dividends is sub-optimal. This is illustrated by Figure 8, which plots the firm's market value under the following two policies: an all-dividend policy ($\{D_t\}_{t=1}^{\infty} = 1$, $\{A_t\}_{t=1}^{\infty} = 0$), and an all-repurchase policy ($\{D_t\}_{t=1}^{\infty} = 0$, $\{A_t\}_{t=1}^{\infty} = 1$). The firm's market value under the first policy is higher in every period since the second involves strictly-positive lock-in premia from period 13 onwards (marginal shareholders require a 9.5% premium for $\forall\, t \geq 13$, and they are type $H = 7$ and $\beta = 0.216$). Furthermore, since investors are forward looking, the payout inefficiency in later periods is capitalized into the firm's earlier-period market values: i.e., even though lock-in premia are nil for $t < 13$, the firm's market value under the all-repurchase policy is strictly lower in *every* period. These results suggest that dividends are optimal when marginal investors have no dividend-tax disadvantage.[34]

Figure 8. Firm Value when $H_S = 1$: Two Payout Policies



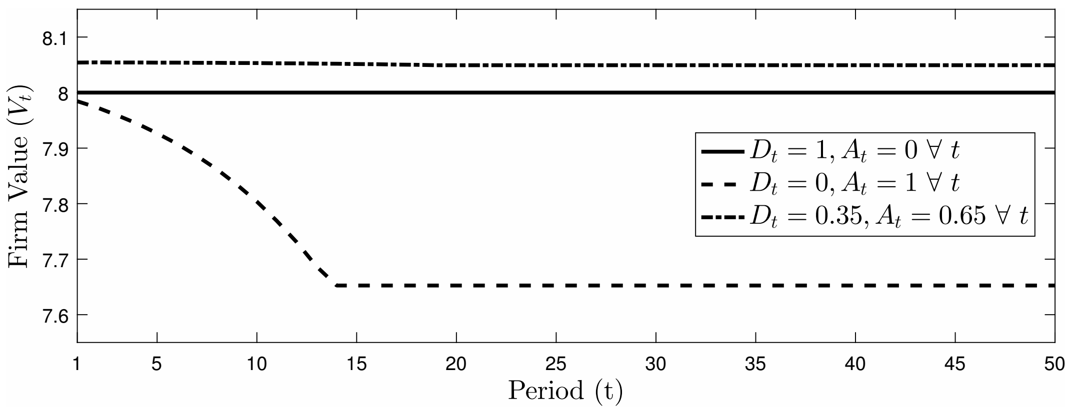
This figure plots the firm's market value at the beginning of periods 1-50 when $H_S = 1$ under the parameter values in Table 4. The solid line corresponds to the all-dividend policy ($\{D_t\}_{t=1}^{\infty} = 1$, $\{A_t\}_{t=1}^{\infty} = 0$), while the dashed line corresponds to the all-repurchase policy ($\{D_t\}_{t=1}^{\infty} = 0$, $\{A_t\}_{t=1}^{\infty} = 1$).

The next case considers $H_S = 2$, and as such, the marginal investor's effective tax rate is strictly less than τ_g - resulting in a dividend-tax disadvantage - whenever capital gains are generated. Since the marginal value of dividends is strictly less than 1 in this case (i.e., $(1 - \tau_d)/(1 - \tau_{e,2}) < 1$), the payment of lock-in premia is now acceptable. However, as before, omitting dividends is sub-optimal. This is illustrated by Figure 9, which plots the firm's market value under the two policies described above; and once again, the all-dividend policy is superior. However, both policies are sub-optimal and dominated by the following: $\{D_t\}_{t=1}^{\infty} = 0.35$, $\{A_t\}_{t=1}^{\infty} = 0.65$. As Figure 9 illustrates, this intermediate policy results in higher market values in every period despite the payment of strictly-positive lock-in premia: which are 1.5% in every steady-state period (marginal shareholders are type $H = 1$ and $\beta = 0.229$). Taken together, these results suggest that dividends can remain optimal when marginal investors have modest dividend-tax disadvantages, but that an all-dividend policy is unlikely to be optimal.[35]

[34] The all-dividend policy is not uniquely-optimal: policies that include share repurchases are also optimal if no lock-in premia are paid (all such policies result in: $\{V_t\}_{t=1}^{\infty} = 8$). However, every optimal policy includes dividends, as evidenced by Figure 8.

[35] A reasonable conjecture is that firms should forego dividend payments altogether and only repurchase equity from shareholders when their investment horizons reach $H = 0$ - i.e, no lock-in premia - by storing

Figure 9. Firm Value when $H_S = 2$: Three Payout Policies

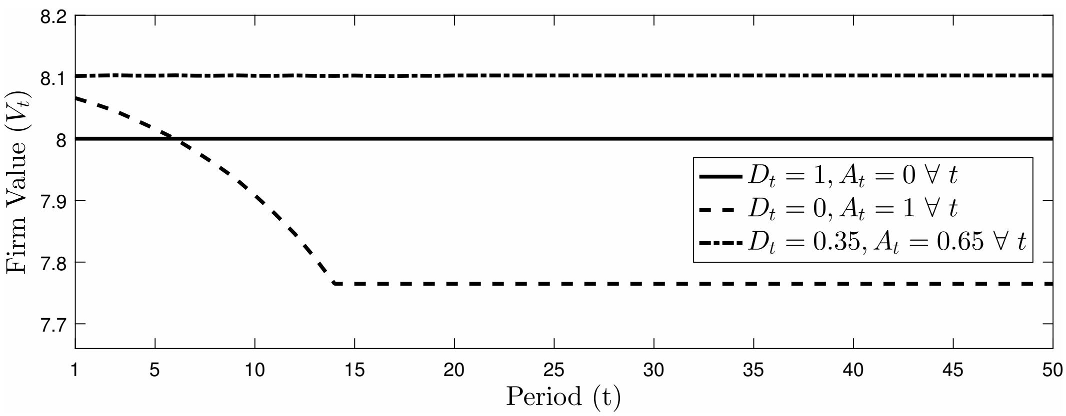


This figure plots the firm's market value at the beginning of periods 1-50 when $H_S = 2$ under the parameter values in Table 4. The solid line corresponds to the all-dividend policy ($\{D_t\}_{t=1}^{\infty} = 1$, $\{A_t\}_{t=1}^{\infty} = 0$), the dashed line corresponds to the all-repurchase policy ($\{D_t\}_{t=1}^{\infty} = 0$, $\{A_t\}_{t=1}^{\infty} = 1$), while the dotted-and-dashed line corresponds to the following policy: $\{D_t\}_{t=1}^{\infty} = 0.35$, $\{A_t\}_{t=1}^{\infty} = 0.65$.

The final case considers $H_S = 3$, and like the previous two, excluding dividends is sub-optimal. This is illustrated by Figure 10, which plots the firm's market value when dividends are excluded and when share repurchases are excluded. Unlike the previous two cases, the firm's market value under the all-repurchase policy is initially higher - due to the marginal investor's relatively-low effective tax rate ($\tau_{g,3} = 0.184$) and the favorable shareholder distribution in early periods - and then falls below that of the all-dividend policy - as the shareholder distribution becomes less favorable for buybacks. As with the previous case ($H_S = 2$), the policy $\{D_t\}_{t=1}^{\infty} = 0.35$ and $\{A_t\}_{t=1}^{\infty} = 0.65$ dominates the two extreme policies (as Figure 10 illustrates). The firm's market value under the intermediate policy is higher when $H_S = 3$ compared to when $H_S = 2$ (in every period) since capital gains become more valuable as the marginal investor's effective tax rate decreases. Furthermore, since market values are higher when $H_S = 3$, less equity is repurchased for a given amount spent (A_t), ceteris paribus, which raises every shareholder's β (compared to when $H_S = 2$) and makes repurchasing equity more desirable. Marginal lock-in premia are 0.2% in every steady-state period when $H_S = 3$, and marginal shareholders are type $H = 2$ and $\beta = 0.9258$.

excess profits in the risk-free security. However, the firm's cash holdings would grow without bound under this strategy due to the limited measure of equity with $H = 0$ in each period; this, in turn, would violate the firm's transversality condition ($\lim_{T\to\infty}[1 + \rho/(1 - \tau_e)]^{-T} V_T = 0$).

Figure 10. Firm Value when $H_S = 3$: Three Payout Policies



This figure plots the firm's market value at the beginning of periods 1-50 when $H_S = 3$ under the parameter values in Table 4. The solid line corresponds to the all-dividend policy ($\{D_t\}_{t=1}^{\infty} = 1$, $\{A_t\}_{t=1}^{\infty} = 0$), the dashed line corresponds to the all-repurchase policy ($\{D_t\}_{t=1}^{\infty} = 0$, $\{A_t\}_{t=1}^{\infty} = 1$), while the dotted-and-dashed line corresponds to the following policy: $\{D_t\}_{t=1}^{\infty} = 0.35$, $\{A_t\}_{t=1}^{\infty} = 0.65$.

Dividends remain optimal $\forall \ H_S \leq 7$ and become sub-optimal thereafter due to the marginal investor's relatively-low effective tax rate: $\tau_{e,8} = 0.155$ under the all-repurchase policy. However, if H_L and/or ω_L were to increase, then dividends could become optimal again, as both adjustments put upward pressure on lock-in premia.

These results suggest that dividends are optimal when marginal investors have no dividend-tax disadvantage ($H_S \leq 1$) and can remain optimal when they do ($H_S > 1$). However, if dividends become significantly tax disadvantaged, then excluding dividends can become optimal.

6 Conclusion

This paper developed a model of corporate payout policy to explain one aspect of the payout puzzle. Shareholders in the model have heterogeneous investment horizons and heterogeneous accrued capital gains on firm equity. It was shown that shareholder-wealth maximization - within a realization-based capital-gains tax system - can create a wedge between equity's intrinsic value and the ask price of many shareholders. This wedge - called the "lock-in premium" herein - was shown to be an increasing function of investment horizon and accrued capital gains. Paying lock-in premia is acceptable provided they remain small, but when firms repurchase large quantities of equity - especially in multiple successive years - marginal lock-in premia can become sufficiently expensive that paying tax-disadvantaged dividends becomes optimal.

Unlike most payout models that offer a solution to the payout puzzle, the current one does not rely on informational asymmetries, repurchase constraints, incomplete contracting, or irrationality. The key assumption is that capital gains are taxed upon *realization* and not *accrual*, thus providing a self-contained explanation for dividend payments within a perfect-information framework.

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A Tax Rates and Payout Variables

Dividends were taxed as ordinary income from 1972-2002 (subject to a small exclusion not exceeding \$400 from 1972-1986) according to Congressional (2014). The top federal tax rate on ordinary income was: 70% from 1972-1981, 50% from 1982-1986, 38.5% in 1987, 28% from 1988-1990, 31% from 1991-1992, 39.6% from 1993-2000, 39.1% in 2001, and 38.6% in 2002. The Jobs and Growth Tax Relief Reconciliation Act of 2003 reduced the maximum tax rate on dividends to 15%. This rate prevailed until the 2012 American Taxpayer Relief Act increased the maximum rate to 20% in 2013; this is the current rate.

Between 1972-1978 (October 31), the tax rate on capital gains (in excess of \$50,000) was 50% of the rate on ordinary income (U.S. Treasury, 1985); the maximum tax rate on ordinary income was 70% in each of these years. Between 1978 (November 1) and 1986, capital gains were taxed at 40% of the rate on ordinary income (Auten and Cordes, 1991); the maximum rate on ordinary income was 70% (50%) between 1979-1981 (1982-1986), however, realizations after June 9, 1981 were subject to a maximum ordinary-income-tax rate of 50% (instead of the 70% that prevailed in 1981) according to U.S. Treasury (1985). The Tax Reform Act of 1986 increased the maximum rate on long-term capital gains to 28% in 1987. This lasted until May 6, 1997, when the Taxpayer Relief Act of 1997 reduced the maximum rate on long-term capital gains to 20% (Auten, 1999).[36] This rate prevailed until the Jobs and Growth Tax Relief Reconciliation Act of 2003 reduced the maximum rate to 15% in 2003, which lasted until the 2012 American Taxpayer Relief Act increased the maximum rate to 20% in 2013; this is the current rate.

Note that all tax rates described above (and used in Table 1) do not incorporate: phase-out provisions, the minimum tax, alternative tax rates, income tax surcharges, the Medicare contribution, and interactions with other tax provisions. They are only meant to capture the top statutory tax rate on dividends and realized capital gains.

When calculating θ_t from Equation 1, it is assumed that marginal investors face the top statutory tax rate on dividends and long-term capital gains. An alternative characterization of θ_t would be $\sum_{j=1}^{s} w_{j,t}(1-\tau_{d,j,t})/(1-\tau_{e,j,t})$, where $\tau_{d,j,t}$ is the marginal tax rate on dividends in year t for investors in income class $j \in [1, s]$, $\tau_{e,j,t}$ is the effective tax rate on accrued capital gains in year t for investors in income class j (a proportion of the statutory rate), and the $w_{j,t}$'s are equity-ownership weights across income class and time; this approach is used in Poterba (1987) and Bernheim and Wantz (1995) and measures the *average* θ_t among shareholders . It is not clear which of the two measures is a better approximation of the marginal investor's θ_t. Conversely, Allen and Michaely (2003) point out that marginal investors may have $\tau_d = \tau_e$ (for instance, non-taxed institutional investors), and therefore, may not have a dividend-tax disadvantage. As illustrated in Section 5, dividends are still optimal when marginal investors have $\tau_d = \tau_e$ provided that *some* equity is repurchased from shareholders with a lock-in effect. Finally, it should be noted that Table 1 does not account for dynamic tax-trading strategies: i.e., when equity is traded around the ex-dividend day (by investors with different tax situations) to lower the overall tax burden; see Allen and Michaely (2003)

[36]A provision in the 2001 Tax Act reduced the maximum rate to 18% for assets purchased after 2001 and held for at least 5 years. This provision is ignored in Table 1 due to the unusually-long holding-period restriction.

for a discussion.

Table 5 describes the payout variables used in Figures 1 and 2 and reports their Compustat data codes in parentheses. These variables are similar to the ones used in Grullon and Michaely (2002) and Moser (2007), among others.

Table 5. Payout Variable Descriptions

Variable	Data Description (Compustat Variable Code)
Dividends	Ordinary dividends on common equity (DVC).
Share Repurchases	Purchases of common and preferred stock (PRSTKC) *minus* preferred stock redemption value (PSTKRV).

Description of each payout variable used in Figures 1 and 2 of Section 3.

B Derivation of Expression 8 and $\Omega(H, \beta)$

To derive the difference in after-tax wealth under the following two scenarios: 1) holding an equity position for $H > 0$ periods; and 2) selling it immediately, paying the capital-gains tax (or receiving the capital-loss offset), and reinvesting the net proceeds for $H > 0$ periods, we will first calculate the marginal after-tax wealth under each scenario and then take the difference.

Under the first scenario, the after-tax value of a period-t equity position (of size 1) held until the beginning of period $t+1$ (with a period-t tax basis of β) is:

$$(1 - \tau_g)[(1 + r_g) - \beta] + \beta + (1 - \tau_d)r_d. \tag{26}$$

This is derived as follows. The position's market value at the beginning of period $t+1$ is $(1+r_g)$, and therefore, the position's total capital gain/loss is $[(1+r_g) - \beta]$. This is subject to capital-gains tax (or a capital-loss offset) at the rate τ_g, which leaves $(1-\tau_g)[(1+r_g) - \beta]$ once the tax is paid (or the refund is received). Since the tax basis (β) is not subject to capital-gains taxation, you add this back to the investor's after-tax wealth. Finally, add the after-tax dividend payment from period t (i.e., $(1-\tau_d)r_d$) to get Expression 26. This expression can be rewritten as:

$$(1-\tau_g)\left[(1+r_g+(1-\tau_d)r_d)\right]+\tau_g\left[\beta+(1-\tau_d)r_d\right].$$

which is also equal to:

$$(1-\tau_g)[(1+r_g+(1-\tau_d)r_d)^1]+\tau_g\left[\beta+(1-\tau_d)r_d\sum_{h=1}^{1}(1+r_g+(1-\tau_d)r_d)^{h-1}\right]. \quad (27)$$

Next, suppose that the position is held until the beginning of period $t+2$ (instead of period $t+1$). Its after-tax value at the beginning of period $t+2$ is:

$$(1-\tau_g)[(1+r_g)(1+r_g+(1-\tau_d)r_d)-\beta-(1-\tau_d)r_d]$$
$$+\beta+(1-\tau_d)r_d+(1-\tau_d)r_d(1+r_g+(1-\tau_d)r_d) \quad (28)$$

This is derived as follows. First, the position's market value at the beginning of period $t+1$ is composed of two parts: 1) the period-t return from capital gains - i.e., $(1+r_g)$ - plus the period-t dividend reinvestment - i.e., $(1-\tau_d)r_d$. Taken together, the position's market value at the beginning of period $t+1$ is $(1+r_g+(1-\tau_d)r_d)$. Second, since this investment generates a return from capital gains in period $t+1$ as well (the return is $(1+r_g)$, as before), the position's market value at the beginning of period $t+2$ is $(1+r_g)(1+r_g+(1-\tau_d)r_d)$. Third, calculate the investor's tax basis at the beginning of period $t+2$; this is composed of two parts: 1) the tax basis at the beginning of period t (i.e., β), and 2) the period-t dividend reinvestment (i.e., $(1-\tau_d)r_d$). Taken together, the investor's tax basis at the beginning of period $t+2$ is: $\beta+(1-\tau_d)r_d$. Fourth, given the position's market value at the beginning of period $t+2$ (from step two) and the investor's tax basis (from step three), the total capital gain/loss at the beginning of period $t+2$ is: $(1+r_g)(1+r_g+(1-\tau_d)r_d)-\beta-(1-\tau_d)r_d$. This is subject to capital-gains taxation (or a capital-loss offset), which leaves: $(1-\tau_g)[(1+r_g)(1+r_g+(1-\tau_d)r_d)-\beta-(1-\tau_d)r_d]$ once the tax is paid (or the refund is received). Fifth, add the investor's tax basis to the after-tax capital gain/loss that we just calculated (this part of the position is untaxed). And sixth, add the after-tax dividend payment received in period $t+1$, which is equal to the position's size at the beginning of period $t+1$ (i.e.,

$(1 + r_g + (1 - \tau_d)r_d)$ from step one) multiplied by the after-tax dividend yield $(1 - \tau_d)r_d$ - note that dividends in period $t+1$ are larger than in period t because the investment size is larger - this equals: $(1 - \tau_d)r_d(1 + r_g + (1 - \tau_d)r_d)$. Finally, after summing the amounts from steps four, five, and six, we get Expression 28. Note that this expression can be rewritten as follows:

$$(1 - \tau_g)[(1 + r_g + (1 - \tau_d)r_d)^2] + \tau_g \left[\beta + (1 - \tau_d)r_d \sum_{h=1}^{2} (1 + r_g + (1 - \tau_d)r_d)^{h-1}\right]. \qquad (29)$$

By examining Expressions 27 and 29, we can see that continuing this process indefinitely results in the following general expression for the position's after-tax value at the beginning of period $t + H$:

$$(1 - \tau_g) \left[(1 + r_g + (1 - \tau_d)r_d)^H\right] + \tau_g \left[\beta + (1 - \tau_d)r_d \sum_{h=1}^{H} (1 + r_g + (1 - \tau_d)r_d)^{h-1}\right],$$

which is equivalent to Expression 8 from Section 4.2.

To facilitate the derivation of $\Omega(H, \beta)$, it is helpful to note that:

$$(1 + r_g + (1 - \tau_d)r_d)^H = (r_g + (1 - \tau_d)r_d) \sum_{h=1}^{H} (1 + r_g + (1 - \tau_d)r_d)^{h-1} + 1, \qquad (30)$$

which allows us to rewrite Expression 8 as:

$$[(1 - \tau_g)r_g + (1 - \tau_d)r_d] \sum_{h=1}^{H} (1 + r_g + (1 - \tau_d)r_d)^{h-1} + (1 - \tau_g) + \tau_g\beta. \qquad (31)$$

Next, to derive the position's after-tax value under the second scenario, note that the net proceeds from the period-t equity sale are $1 - \tau_g(1 - \beta)$. After reinvesting these proceeds (in the same equity) for H periods, the investor's after-tax wealth at the beginning of period $t + H$ becomes:

$$(1 - \tau_g(1 - \beta)) \Bigg[(1 - \tau_g) \left[(1 + r_g + (1 - \tau_d)r_d)^H\right]$$

$$+ \tau_g \left[1 + (1-\tau_d) r_d \sum_{h=1}^{H} (1 + r_g + (1-\tau_d) r_d)^{h-1} \right] \Bigg]. \quad (32)$$

This expression is similar to Expression 8 above, except that equity's initial size is $1-\tau_g(1-\beta)$ instead of 1, and the initial tax basis is $1-\tau_g(1-\beta)$ instead of β. If we apply the same transformation as before (i.e., Equation 30), then Expression 32 can be rewritten as:

$$(1-\tau_g(1-\beta)) \left[[(1-\tau_g) r_g + (1-\tau_d) r_d] \sum_{h=1}^{H} (1 + r_g + (1-\tau_d) r_d)^{h-1} + 1 \right]. \quad (33)$$

Finally, subtracting Expression 33 from Expression 31 produces:

$$\Omega(H,\beta) = \tau_g(1-\beta) \left[(1-\tau_g) r_g + (1-\tau_d) r_d \right] \sum_{h=1}^{H} (1 + r_g + (1-\tau_d) r_d)^{h-1} \ \forall \ H > 0. \quad (34)$$

Furthermore, since investors with $H = 0$ desire to sell their equity immediately, it follows that $\Omega(0,\beta) = 0 \ \forall \beta$. Combining this result with Equation 34 produces Equation 11 from Section 4.2:

$$\Omega(H,\beta) = \begin{cases} \tau_g(1-\beta)[(1-\tau_g) r_g + (1-\tau_d) r_d] \sum_{h=1}^{H} (1 + r_g + (1-\tau_d) r_d)^{h-1} & \text{if } H > 0, \\ 0 & \text{if } H = 0. \end{cases}$$

C Derivation of $L(H,\beta)$

The lock-in premium is defined as the remuneration above equity's intrinsic value that creates indifference between: 1) liquidating an equity position at its intrinsic value plus this premium, and 2) retaining the position for a desired number of periods. If we denote the lock-in premium by $L(H,\beta)$ (as a percentage of equity's intrinsic value), then liquidating equity under scenario 1 provides marginal after-tax proceeds of:

$$1 + L(H,\beta) - \tau_g(1 + L(H,\beta) - \beta).$$

When these proceeds are promptly reinvested in the same stock (or identical stock), and held for $H > 0$ periods, the investor's marginal after-tax wealth becomes:

$$(1 + L(H,\beta) - \tau_g(1 + L(H,\beta) - \beta))\Big[[(1-\tau_g)r_g + (1-\tau_d)r_d]\sum_{h=1}^{H}(1 + r_g + (1-\tau_d)r_d)^{h-1} + 1\Big]. \quad (35)$$

See Appendix B for an explanation of how this expression is derived. Alternatively, the equity position's marginal after-tax value under scenario 2 is:

$$[(1-\tau_g)r_g + (1-\tau_d)r_d]\sum_{h=1}^{H}(1 + r_g + (1-\tau_d)r_d)^{h-1} + (1-\tau_g) + \tau_g\beta, \quad (36)$$

after $H > 0$ periods. Finally, equating Expressions 35 and 36, and solving for $L(H,\beta)$, produces:

$$(1-\beta)\frac{\tau_g}{1-\tau_g}\left[\frac{\Omega(H,\beta)}{\Omega(H,\beta) + \tau_g(1-\beta)}\right].$$

Combining this expression with the fact that only shareholders with capital gains require a lock-in premium (i.e., $\beta < 1$), we have Equation 12 from Section 4.2:

$$L(H,\beta) = \begin{cases} (1-\beta)\frac{\tau_g}{1-\tau_g}\left[\frac{\Omega(H,\beta)}{\Omega(H,\beta)+\tau_g(1-\beta)}\right] & \text{if } \beta < 1, \\ 0 & \text{if } \beta \geq 1. \end{cases}$$

D Properties of the Repurchase Function: Proposition 1

The firm's repurchase function is characterized by Equations 13 and 14:

$$R_t = \max_{\{\beta(H)\}}\left[\sum_{H=0}^{\overline{H}}\int_{\beta(H)}^{\overline{\beta}} f^H(\beta)d\beta\right](R_t + V_{t+1}),$$

subject to:

$$A_t = \left[\sum_{H=0}^{\overline{H}} \int_{\beta(H)}^{\overline{\beta}} [1 + L(H,\beta)] f^H(\beta) d\beta \right] (R_t + V_{t+1}).$$

These equations state that for every H, firms choose a corresponding $\beta(H)$ to maximize the mass of repurchased equity subject to spending A_t on the repurchase program. The cost of each repurchased share is equal to the firm's intrinsic value during the repurchase program (R_t+V_{t+1}) plus the shareholder-specific lock-in premium, where the firm's continuation value (V_{t+1}) is independent of the period-t payout mix: it depends on the capital stock at the beginning of period $t+1$ and the payout sequences from period $t+1$ onwards.

Recall that δ_t is the fraction of equity repurchased in period t, and therefore:

$$\sum_{H=0}^{\overline{H}} \int_{\beta(H)}^{\overline{\beta}} f^H(\beta) d\beta = \delta_t. \tag{37}$$

Furthermore, if we assume that there is a unit measure of equity at the beginning of period t (for simplicity), then the per-unit value of equity during the period-t repurchase program is:

$$R_t + V_{t+1} = \frac{V_{t+1}}{1-\delta_t}. \tag{38}$$

Given Equations 37 and 38, the period-t repurchase function can be rewritten as:

$$R_t = \max_{\{\beta(H)\}} \delta_t \frac{V_{t+1}}{(1-\delta_t)},$$

subject to:

$$A_t = \delta_t \frac{V_{t+1}}{(1-\delta_t)} + \left[\sum_{H=0}^{\overline{H}} \int_{\beta(H)}^{\overline{\beta}} L(H,\beta) f^H(\beta) d\beta \right] \frac{V_{t+1}}{(1-\delta_t)}.$$

It is trivial to show that $R(A_t)$ is weakly below the 45° line (due to $L(H,\beta) \geq 0 \ \forall \ H$ and $\forall \ \beta$) and increasing in the amount spent (given that $L(H,\beta)$ is finite for any finite H). The non-trivial task is to show that $R(A_t)$ is concave. If we take the inverse of $R(A_t)$ we have:

$$A_t = \min_{\{\beta(H)\}} \delta_t \frac{V_{t+1}}{(1-\delta_t)} + \left[\sum_{H=0}^{\overline{H}} \int_{\beta(H)}^{\overline{\beta}} L(H,\beta) f^H(\beta) d\beta \right] \frac{V_{t+1}}{(1-\delta_t)},$$

subject to:

$$R_t = \delta_t \frac{V_{t+1}}{(1-\delta_t)}.$$

The marginal change in R_t given a change in $\beta(H)$, for a particular $H \in [0, \overline{H}]$, is:

$$\frac{\partial R_t}{\partial \beta(H)} = -f^H(\beta(H)) \frac{V_{t+1}}{1-\delta_t} - \delta_t \frac{V_{t+1} f^H(\beta(H))}{(1-\delta_t)^2} = -\frac{V_{t+1} f^H(\beta(H))}{(1-\delta_t)^2},$$

As R_t increases, at least one of the $\beta(H) > 0$ must decrease. Without loss of generality, suppose that a marginal increase in R_t is made through $\beta(\widehat{H})$, where $\widehat{H} \in [0, \overline{H}]$, then:

$$\frac{\partial \beta(\widehat{H})}{\partial R_t} = -\frac{(1-\delta_t)^2}{V_{t+1} f^H(\beta(\widehat{H}))}.$$

Furthermore, the change in A_t from a marginal change in $\beta(\widehat{H})$ is:

$$\frac{\partial A_t}{\partial \beta(\widehat{H})} = -\frac{V_{t+1} f^H(\beta(\widehat{H}))}{(1-\delta_t)^2} - \frac{L(\widehat{H}, \beta(\widehat{H})) f^H(\beta(\widehat{H})) V_{t+1}}{(1-\delta_t)} - \left[\sum_{H=0}^{\overline{H}} \int_{\beta(H)}^{\overline{\beta}} L(H,\beta) f^H(\beta) d\beta \right] \frac{V_{t+1} f^H(\beta(\widehat{H}))}{(1-\delta_t)^2}.$$

Therefore, we have:

$$\frac{\partial A_t}{\partial R_t} = 1 + L(\widehat{H}, \beta(\widehat{H}))(1-\delta_t) + \sum_{H=0}^{\overline{H}} \int_{\beta(H)}^{\overline{\beta}} L(H,\beta) f^H(\beta) d\beta,$$

which implies that A_t is an increasing function of R_t (when the increase is made through $\beta(\widehat{H})$), since $\delta_t < 1$, and $L(H,\beta) \geq 0 \ \forall \ H \ \& \ \beta$. The derivative of $\partial A_t / \partial R_t$ with respect

$\beta(\widehat{H})$ is:

$$\frac{\partial^2 A_t}{\partial R_t \partial \beta(\widehat{H})} = L'_\beta(\widehat{H}, \beta(\widehat{H}))(1 - \delta_t),$$

where $L'_\beta(\widehat{H}, \beta(\widehat{H}))$ is the first derivative of $L(H, \beta)$ with respect to β evaluated at $\widehat{H}$ and $\beta(\widehat{H})$. This, in turn, equals:

$$L'_\beta(\widehat{H}, \beta(\widehat{H})) = -\frac{\tau_g}{1 - \tau_g} \left[\frac{\Omega(\widehat{H}, \beta(\widehat{H})) - \Omega'_\beta(\widehat{H}, \beta(\widehat{H}))(1 - \beta)^2}{[\Omega(\widehat{H}, \beta(\widehat{H})) + \tau_g(1 - \beta)]^2} \right] < 0,$$

where $\Omega'_\beta(\widehat{H}, \beta(\widehat{H})) < 0$ is the first derivative of Equation 11 with respect to β evaluated at $\widehat{H}$ and $\beta(\widehat{H})$. Therefore:

$$\frac{\partial^2 A_t}{\partial R_t^2} = -\frac{L'_\beta(\widehat{H}, \beta(\widehat{H}))(1 - \delta_t)^3}{V_{t+1} f^{\widehat{H}} \beta(\widehat{H})} > 0,$$

which implies that $A(R_t)$ is a convex function using any $\beta(H) > 0$ to expand the set of repurchased equity, and thus, the inverse of this function ($R(A_t)$) is concave. Furthermore, the continuity of $R(A_t)$ and $R'(A_t)$ follows from the continuity of $L(H, \beta)$ and δ_t with respect to β.

E Deriving the Return on Equity

The after-tax return on equity for an H-type investor has three components: 1) the return from equity held for H periods, 2) the return from equity sold at the beginning of each period and reinvested in the risk-free asset, and 3) the return from dividends that are reinvested in the risk-free asset.

As mentioned above, capital gains/losses are generated in two ways: through changes in the firm's market value, and through share repurchases. If we assume a unit measure of equity at the beginning of period t (for simplicity), then when a fraction δ_t of equity is repurchased, the measure of outstanding shares becomes $(1 - \delta_t)$ post-repurchase. Since the market value of equity at the beginning of period $t + 1$ is V_{t+1} by definition, the per-unit value of non-repurchased equity is $V_{t+1}/(1 - \delta_t)$. Furthermore, since the market value of repurchased equity and non-repurchased equity must be the same, the firm's total market value just before to the repurchase program is the sum of: $\delta_t V_{t+1}/(1-\delta_t)$ - the measure of repurchased equity multiplied by the unit value of equity - plus $(1-\delta_t)V_{t+1}/(1-\delta_t)$ - the measure of non-repurchased equity multiplied by the unit value of equity. Therefore, the firm's total market value just before the repurchase program is $V_{t+1}/(1-\delta_t)$. If we divide this by V_t (the firm's market value at the beginning of period t), we get the return from capital gains in period t (Equation 21 rearranged):

$$1 + r_{g,t} = \frac{V_{t+1}}{V_t(1-\delta_t)}.$$

Furthermore, recall that the rate-of-return from dividends in period t is equal to (Equation 22):

$$r_{d,t} = \frac{D_t}{V_t}.$$

To derive an H-type investor's total return from equity, we will derive their after-tax return from each component above (Components 1 - 3) in turn. With regard to the first component, when the investment horizon is $H = 1$, the after-tax return from capital gains on a marginal investment (of size 1) from period t to period $t+1$ is:

$$\tau_g + (1-\tau_g)\frac{V_{t+1}}{V_t(1-\delta_t)}. \tag{39}$$

This is derived as follows. The rate-of-return from capital gains is $r_{g,t} = V_{t+1}/V_t(1-\delta_t) - 1$ from Equation 21, which results in an after-tax rate-of-return equal to $(1-\tau_g)[V_{t+1}/V_t(1-\delta_t) - 1]$ and an after-tax return of $1 + (1-\tau_g)[V_{t+1}/V_t(1-\delta_t) - 1]$ (by adding 1 - the tax basis); this equals Expression 39 above.

Next, derive Component 1 when the investment horizon is $H = 2$. The compound return from capital gains in periods t *and* $t+1$ is:

$$\left(\frac{V_{t+1}}{V_t(1-\delta_t)}\right)\left(\frac{V_{t+2}}{V_{t+1}(1-\delta_{t+1})}\right) = \frac{V_{t+2}}{V_t\prod_{w=0}^{1}(1-\delta_{t+w})},$$

which results in an after-tax return at the beginning of period $t+2$ equal to:

$$(1-\tau_g)\left[\frac{V_{t+2}}{V_t\prod_{w=0}^{1}(1-\delta_{t+w})}-1\right]+1,$$

which can be rearranged as follows:

$$\tau_g+(1-\tau_g)\frac{V_{t+2}}{V_t\prod_{w=0}^{1}(1-\delta_{t+w})}. \tag{40}$$

Furthermore, since the investor sells a fraction λ of her equity position at the beginning of period $t+1$ (since her investment horizon is reached at the beginning of period $t+2$), Expression 40 is applied to a fraction $(1-\lambda)$ of the original investment, resulting in:

$$(1-\lambda)\left[\tau_g+(1-\tau_g)\frac{V_{t+2}}{V_t\prod_{w=0}^{1}(1-\delta_{t+w})}\right].$$

Using a similar process to that above, the after-tax return from a marginal investment made in period t and held until period $t+3$ (i.e., $H=3$) is:

$$\tau_g+(1-\tau_g)\frac{V_{t+3}}{V_t\prod_{w=0}^{2}(1-\delta_{t+w})},$$

which is applied to a fraction $(1-\lambda)^2$ of the marginal investment, since a fraction λ of the position is sold at the beginning of period $t+1$ *and* period $t+2$, resulting in the following expression for Component 1 when $H=3$:

$$(1-\lambda)^2\left[\tau_g+(1-\tau_g)\frac{V_{t+3}}{V_t\prod_{w=0}^{2}(1-\delta_{t+w})}\right].$$

Continuing this process indefinitely results in the following general expression for an H-type investor's Component 1:

$$(1-\lambda)^{H-1}\left[\tau_g+(1-\tau_g)\frac{V_{t+H}}{V_t\prod_{w=0}^{H-1}(1-\delta_{t+w})}\right]. \tag{41}$$

Moving on to Component 2. When the investment horizon is $H=1$, this component is equal to zero, since the entire equity position is liquidated at the beginning of period $t+1$. When the investment horizon is $H = 2$, this component equals:

$$(1+\rho)\lambda \left[\tau_g + (1-\tau_g)\frac{V_{t+1}}{V_t(1-\delta_t)}\right]. \tag{42}$$

This is derived as follows. At the beginning of period $t+1$ the marginal investment has appreciated in value to $V_{t+1}/V_t(1-\delta_t)$. When a fraction λ of the position is sold, the after-tax proceeds are $(1-\tau_g)[\lambda V_{t+1}/V_t(1-\delta_t) - \lambda] + \lambda$, since the tax basis on this portion of equity is λ. This can be rewritten as:

$$\lambda\tau_g + (1-\tau_g)\lambda\frac{V_{t+1}}{V_t(1-\delta_t)}. \tag{43}$$

Furthermore, since the investment horizon is reached at the beginning of period $t+2$, and this sale happens at the beginning of period $t+1$, the after-tax proceeds are reinvested for one period (in the risk-free asset). Therefore, Expression 43 is multiplied by $(1+\rho)$ to arrive at Expression 42.

When the investment horizon is $H = 3$, the equity sale at the beginning of period $t+1$ is reinvested for two periods (from the beginning of period $t+1$ until the beginning of period $t+3$), and therefore, you multiply Expression 43 by $(1+\rho)^2$ to arrive at the period $t+3$ value of the period $t+1$ equity sale, which is:

$$(1+\rho)^2\lambda \left[\tau_g + (1-\tau_g)\frac{V_{t+1}}{V_t(1-\delta_t)}\right]. \tag{44}$$

In addition to the period $t+1$ equity sale, a fraction λ of the remaining position is also sold at the beginning of period $t+2$ (the investment horizon is reached at the beginning of period $t+3$). Since the remaining equity at the beginning of period $t+2$ is a fraction $(1-\lambda)$ of the original position, due to the fraction λ sold at the beginning of period $t+1$, $\lambda(1-\lambda)$ of the original position is sold at the beginning of period $t+2$. The after-tax proceeds of this sale are:

$$\lambda(1-\lambda) \left[\tau_g + (1-\tau_g)\frac{V_{t+2}}{V_t\prod_{w=0}^{1}(1-\delta_{t+w})}\right], \tag{45}$$

since, as before, the compound return from capital gains in periods t and $t+1$ is:

$$\frac{V_{t+2}}{V_t \prod_{w=0}^{1}(1-\delta_{t+w})}.$$

Since the after-tax proceeds from this sale are reinvested for one period (until the beginning of period $t+3$), you multiply Expression 45 by $(1+\rho)$ to arrive at the period $t+3$ value of the period $t+2$ equity sale, i.e.:

$$(1+\rho)\lambda(1-\lambda)\left[\tau_g + (1-\tau_g)\frac{V_{t+2}}{V_t \prod_{w=0}^{1}(1-\delta_{t+w})}\right]. \tag{46}$$

Finally, Component 2 for this investor is the period $t+3$ value of both beginning-of-period equity sales (Expression 44 plus Expression 46), which equals:

$$\lambda \sum_{S=1}^{2}(1+\rho)^{3-S}(1-\lambda)^{S-1}\left[\tau_g + (1-\tau_g)\frac{V_{t+S}}{V_t \prod_{w=0}^{S-1}(1-\delta_{t+w})}\right].$$

Continuing this process indefinitely results in the following general expression for an H-type investor's Component 2:

$$\lambda \sum_{S=1}^{H-1}(1+\rho)^{H-S}(1-\lambda)^{S-1}\left[\tau_g + (1-\tau_g)\frac{V_{t+S}}{V_t \prod_{w=0}^{S-1}(1-\delta_{t+w})}\right]. \tag{47}$$

Moving on to Component 3. A marginal investment at the beginning of period t results in a fractional ownership of $1/V_t$ of the firm's equity. This entitles the owner to D_t/V_t of the period-t dividend, which has an after-tax value of:

$$(1-\tau_d)\frac{D_t}{V_t}, \tag{48}$$

and constitutes Component 3 when the investment horizons is $H=1$ (since all equity is sold at the beginning of period $t+1$).

When the investment horizon is $H=2$, a marginal investment also receives the after-tax dividend from Expression 48. However, since the investment horizon is reached at the beginning of period $t+2$, these dividends are reinvested in the risk-free asset for one period, producing the following value at the beginning of period $t+2$:

$$(1+\rho)(1-\tau_d)\frac{D_t}{V_t}. \tag{49}$$

In addition to receiving period-t dividends, the investor also receives a dividend payment in period $t+1$. Since the firm repurchases a fraction $(1-\delta_t)$ of its equity in period t, a marginal investment made at the beginning of period t confers ownership rights to a fraction $1/V_t(1-\delta_t)$ of the firm's equity at the beginning of period $t+1$. However, since a fraction λ of this position is sold (by the investor) at the beginning of period $t+1$, the investor's ownership fraction declines to $(1-\lambda)/V_t(1-\delta_t)$ before dividends are paid in period $t+1$. Taken together, the owner is entitled to $(1-\lambda)D_{t+1}/V_t(1-\delta_t)$ of the period $t+1$ dividend, which has an after-tax value of:

$$(1-\tau_d)(1-\lambda)\frac{D_{t+1}}{V_t(1-\delta_t)}. \tag{50}$$

Therefore, when the investment horizon is $H=2$, Component 3 is the sum of Expression 49 and 50, which equals:

$$(1-\tau_d)\sum_{S=0}^{1}(1+\rho)^{1-S}(1-\lambda)^S\frac{D_{t+S}}{V_t\prod_{w=0}^{S-1}(1-\delta_{t+w})},$$

where $\prod_{w=0}^{-1}(1-\delta_{t+w})=1$ (for the case of period-t dividends).

When the investment horizon is $H=3$, the after-tax dividends received in period t are reinvested for two periods, so Expression 48 is multiplied by $(1+\rho)^2$, while the after-tax dividends received in period $t+1$ are reinvested for one period, so Expression 50 is multiplied by $(1+\rho)$. Finally, the after-tax dividend received in period $t+2$ is:

$$(1-\tau_d)(1-\lambda)^2\frac{D_{t+2}}{V_t\prod_{w=0}^{1}(1-\delta_{t+w})},$$

since a fraction λ of the position sold at the beginning of period $t+1$ and $t+2$, and furthermore, the firm repurchases a fraction δ_t (δ_{t+1}) of its equity in period t $(t+1)$. Taken together, when the investment horizon is $H = 3$, Component 3 is equal to:

$$(1-\tau_d)\sum_{S=0}^{2}(1+\rho)^{2-S}(1-\lambda)^S\frac{D_{t+S}}{V_t\prod_{w=0}^{S-1}(1-\delta_{t+w})}.$$

Continuing this process indefinitely results in the following general expression for an H-type investor's Component 3:

$$(1-\tau_d)\sum_{S=0}^{H-1}(1+\rho)^{H-S-1}(1-\lambda)^S\frac{D_{t+S}}{V_t\prod_{w=0}^{S-1}(1-\delta_{t+w})}. \tag{51}$$

Now that Components 1, 2, and 3 are derived for an H-type investor (Expressions 41, 47, and 51, respectively), we can take their sum to arrive at Expression 23 - the total after-tax return on a marginal equity investment made at the beginning of period t.

F Derivation of Expression 25

Recall that capital gains have a period-t rate-of-return equal to:

$$r_{g,t} = \frac{V_{t+1}}{(1-\delta_t)V_t} - 1, \tag{52}$$

from Equation 21. When you apply an effective tax rate to these gains (levied upon accrual), an H-type investor's after-tax rate-of-return is:

$$(1-\tau_{e,H})r_{g,t} = (1-\tau_{e,H})\left(\frac{V_{t+1}}{(1-\delta_t)V_t} - 1\right),$$

where $\tau_{e,H}$ is the investor's effective tax rate. The total return is therefore:

$$1+(1-\tau_{e,H})r_{g,t} = 1+(1-\tau_{e,H})\left(\frac{V_{t+1}}{(1-\delta_t)V_t} - 1\right) = \tau_{e,H} + (1-\tau_{e,H})\frac{V_{t+1}}{(1-\delta_t)V_t}, \tag{53}$$

by adding 1. Given Equation 53, the compound after-tax return for an H-type investor from period t to period $t+S$, based on an accrual-based tax, is therefore:

$$\prod_{w=0}^{S-1}\left(\tau_{e,H}+(1-\tau_{e,H})\frac{V_{t+w+1}}{(1-\delta_{t+w})V_{t+w}}\right). \tag{54}$$

Recall from Expression 23 that the after-tax compound return from capital gains under a realization-based tax (from period t to period $t+S$) is:

$$\tau_g+(1-\tau_g)\frac{V_{t+S}}{V_t\prod_{w=0}^{S-1}(1-\delta_{t+w})}. \tag{55}$$

Therefore, substitute Expression 54 for Expression 55 in Expression 23 to convert the after-tax return from a realization-based tax to an accrual-based tax (one of these substitutions is for $t+H$ (the first line of Expression 23) and the other is for $t+S$ (the second line of Expression 23)).

Next, to convert the return from dividends based on a realization-based tax to that of an accrual-based tax, note that the *pre-tax* compound return from capital gains from period t to period $t+S$ under a realization-based tax is:

$$\frac{V_{t+S}}{V_t\prod_{w=0}^{S-1}(1-\delta_{t+w})}, \tag{56}$$

from Equation 52. Expression 56, in conjunction with the pre-tax rate-of-return from dividends in period t (i.e., Equation 22):

$$r_{d,t}=\frac{D_t}{V_t},$$

produces the following component of Expression 23:

$$\frac{D_{t+S}}{V_{t+S}}\frac{V_{t+S}}{V_t\prod_{w=0}^{S-1}(1-\delta_{t+w})}=\frac{D_{t+S}}{V_t\prod_{w=0}^{S-1}(1-\delta_{t+w})}. \tag{57}$$

To convert this component of Expression 23 into one based on an accrual-based tax, you substitute Expression 54 for Expression 56 in Equation 57 to arrive at:

$$\frac{D_{t+S}}{V_{t+S}} \prod_{w=0}^{S-1} \left(\tau_{e,H} + (1-\tau_{e,H}) \frac{V_{t+w+1}}{(1-\delta_{t+w})V_{t+w}} \right). \tag{58}$$

Finally, substitute Expression 58 for Equation 57 in Expression 23 to arrive at Expression 25 (along with the previous two substitutions, i.e., Expression 54 for Expression 55 in Expression 23 - twice).

G Solution Algorithm

This section describes the model's solution algorithm.

Step 1) Select the parameter values, which are:

- Statutory tax rates on dividends and realized capital gains: τ_d and τ_g, respectively.
- The after-tax rate-of-return on the risk-free asset: ρ.
- The fraction of equity sold by shareholders at the beginning of each period: λ.
- The initial investment horizon of new investors: H_L and H_S.
- The wealth of new investors in each period: ω_L and ω_S.
- The number of periods to analyze: N.

Step 2) Select the payout sequences controlled by management:

- Dividends: $\{D_t\}_{t=1}^N$.
- The amount spent repurchasing equity: $\{A_t\}_{t=1}^N$.

Step 3) Make an initial guess regarding:

- The sequence of firm values: $\{V_t\}_{t=1}^N$.
- The fraction of equity repurchased in each period: $\{\delta_t\}_{t=1}^N$.

Step 4) With the sequences $\{V_t\}_{t=1}^N$, $\{D_t\}_{t=1}^N$, and $\{\delta_t\}_{t=1}^N$, and the parameters τ_d, τ_g, λ, and ρ, determine each shareholder's required lock-in premium in each period. This is done in three sub-steps:

1. Calculate the buy-and-hold after-tax return of each shareholder with an initial investment horizon H that purchases equity at the beginning of period t (Expression 23):

$$(1-\lambda)^{H-1}\left[\tau_g + (1-\tau_g)\frac{V_{t+H}}{V_t \prod_{w=0}^{H-1}(1-\delta_{t+w})}\right]$$
$$+\lambda\sum_{S=1}^{H-1}(1+\rho)^{H-S}(1-\lambda)^{S-1}\left[\tau_g + (1-\tau_g)\frac{V_{t+S}}{V_t \prod_{w=0}^{S-1}(1-\delta_{t+w})}\right]$$
$$+(1-\tau_d)\sum_{S=0}^{H-1}(1+\rho)^{H-S-1}(1-\lambda)^{S}\frac{D_{t+S}}{V_t \prod_{w=0}^{S-1}(1-\delta_{t+w})}, \quad (59)$$

where the first part of this expression is the future after-tax return from equity retained in the firm, the second is the future after-tax return from all beginning-of-period equity sales (the fraction λ of the position sold at the beginning of each period) that are reinvested in the risk-free asset, and the third is the future after-tax return from all dividend income reinvested in the risk-free asset.[37]

2. Calculate the same investor's after-tax return when their position is sold at the end of period $t+b$ (where $b \in \{B \in \mathbb{N} | 0 \leq B \leq H-1\}$) for the amount $(1+\tilde{L})V_{t+b+1}/(1-\delta_{t+b})$ (where $\tilde{L}$ is the multiplicative repurchase premium that we are solving for), and the proceeds of this equity sale are reinvested in the risk-free asset for the remaining $H-b-1$ periods. This equals:

$$(1+\rho)^{H-b-1}\Bigg\{(1-\lambda)^{b}\left[\tau_g + (1-\tau_g)\frac{(1+\tilde{L})V_{t+b+1}}{V_t \prod_{w=0}^{b}(1-\delta_{t+w})}\right]$$

[37] To calculate lock-in premia we require both contemporary and future values of: D_t, V_t and δ_t. Specifically, to calculate premia for periods 1 through N (and for all shareholders) we require V_t (D_t and δ_t) over the periods 1 to $N+H_L$ (1 to $N+H_L-1$) - see Expressions 59 and 60. As such, for all $t>N$, V_t is set to V_N, D_t is set to D_N, and δ_t is set to δ_N.

$$+\lambda\sum_{S=1}^{b}(1+\rho)^{b+1-S}(1-\lambda)^{S-1}\left[\tau_g+(1-\tau_g)\frac{V_{t+S}}{V_t\prod_{w=0}^{S-1}(1-\delta_{t+w})}\right]$$

$$+(1-\tau_d)\sum_{S=0}^{b}(1+\rho)^{b-S}(1-\lambda)^{S}\frac{D_{t+S}}{V_t\prod_{w=0}^{S-1}(1-\delta_{t+w})}\Bigg\},\quad (60)$$

where the part of this expression inside of the large curly brackets is the investor's after-tax return at the end of period $t+b$ (after selling their equity back to the firm), which is reinvested in the risk-free asset for $H-b-1$ periods (the risk-free asset's return is $(1+\rho)$).

For example, suppose that the investor's initial holding period is $H=3$, and that $b=1$. In this case, the equity position is held for 2 periods (from the beginning of period t until the end of period $t+1$), the relevant market value is V_{t+2} (i.e., the firm's market value at the very end of period $t+1$ is the same as the market value at the beginning of period $t+2$ - both occur simultaneously). Finally, since the investor's initial holding period is $H=3$, the proceeds from this equity sale (and the amount already invested in the risk-free asset) are reinvested for 1 period at the rate-of-return ρ.

3. Finally, solve for the value of $\tilde{L}$ that equates Expression 59 with Expression 60. This is the shareholder's required lock-in premium (as a fraction of equity's market value).

This procedure is carried out for each shareholder in each cohort (periods 1 to N) in each potential period of ownership. For example, shareholders with an initial investment horizon of H who purchase equity at the beginning of period t *may* own equity at the end of periods t through $t+H-1$; their investment horizon becomes $H=0$ afterwards (at the beginning of period $t+H$), and thus, no equity is retained afterwards.

Step 5) Determine the evolution of the shareholder distribution starting with the first period. This is done in four sub-steps:

1. Determine the fraction of equity sold at the beginning of a period, which is composed of two types:

   - Equity sold by shareholders with an investment horizon of $H=0$.

- A fraction λ of every shareholder's equity position.

Remove this equity from the shareholder distribution (i.e., remove a fraction λ of equity from all shareholders, and remove all equity from shareholders with $H = 0$). Next, given the firm's beginning-of-period market value (from the sequence $\{V_t\}_{t=1}^N$) and the wealth of long-term investors (i.e., ω_L), determine how much equity they can afford - this constitutes their initial ownership share. Next, allocate the remaining equity (sold at the beginning of a period) to the group of short-term investors - this constitutes their initial ownership share. Since all equity is purchased at the firm's market price, every shareholder's proportional tax basis is initially $\beta = 1$.

2. Right before the repurchase program commences, reduce every shareholder's investment horizon by one period. Also, reduce every shareholder's β by an amount commensurate with the capital gains generated in that period (recall that period-t capital gains are $V_{t+1}/(1-\delta_t) - V_t$).

3. Sort all shareholders by their required lock-in premium (determined in Step 4) from lowest to highest, and remove equity from shareholders in that order until the total fraction of equity removed equals the fraction of equity repurchased - according to the sequence $\{\delta_t\}_{t=1}^N$.

4. Re-normalize the total mass of equity to 1 (after the repurchase) by dividend the mass of every shareholder's remaining equity position by $(1-\delta_t)$. This constitutes the shareholder distribution at the beginning of the subsequent period (just before new investors arrive).

Note, the four sub-steps above are repeated for all periods (from 1 to N) to determine the shareholder-distribution's evolution.

Step 6) Now that we know which shareholders have their equity repurchased in each period (from Step 5) and the cost of that equity (from Step 4), we can calculate the cost of repurchasing a fraction δ_t of the firm in period t. When the cost of this repurchased equity is greater than (less than) A_t, decrease (increase) the value of δ_t to arrive at a new sequence $\{\delta_t\}_{t=1}^N$.

Step 7) With the new sequence $\{\delta_t\}_{t=1}^{N}$ (from Step 6) and the original sequence $\{D_t\}_{t=1}^{N}$, calculate the firm's market value in each period (based on the marginal shareholder's valuation). This is done using backward induction starting with the last period, where D_t, V_t and δ_t are held constant from period N onwards.

Step 8) With the updated sequences $\{V_t\}_{t=1}^{N}$ (from Step 7) and $\{\delta_t\}_{t=1}^{N}$ (from Step 6), and the original sequence $\{D_t\}_{t=1}^{N}$, repeat Steps 4-7 until $\{V_t\}_{t=1}^{N}$ and $\{\delta_t\}_{t=1}^{N}$ converge. This produces the model's dynamic equilibrium - all other variables/distributions/functions can be obtained afterwards.