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EFFECTS OF MONETARY POLICY IN A MODEL WITH CASH-IN-ADVANCE CONSTRAINTS ON R&D AND CAPITAL ACCUMULATION

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Effects of Monetary Policy in a Model with Cash-in-Advance Constraints on R&D and Capital Accumulation*

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Abstract

To examine the effect of monetary policy on economic growth, we formulate an endogenous growth model with cash-in-advance constraints on R&D and capital accumulation as endogenous growth engines. Within this framework, we show that the relationship between economic growth and the nominal interest rate can be an inverted-U shape. Moreover, we demonstrate that the welfare-maximizing level of the nominal interest rate is larger than the growth-rate-maximizing level of the nominal interest rate.

Keywords: Capital accumulation; CIA constraint; Endogenous growth; Monetary policy; R&D

JEL *classification*: E52; O31; O40

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1 Introduction

The relationship between economic growth and monetary policy has been one of the more important issues among economists for several years. In response, a number of theoretical studies have investigated the effect of monetary policy on economic growth and have showed that monetary policy in an economy with a cash-in-advance (CIA) constraint on investment affects the long-run growth rate. Two strands of literature are evident in these studies. The first strand employs a model in which households need to accumulate money before they consume and purchase capital; thus, in this model, there are CIA constraints on both consumption and capital accumulation (i.e., investment). For example, early work by Stockman (1981) and Abel (1985) demonstrates that the steady-state capital stock inversely relates to the rate of inflation.

The second strand develops an R&D-based growth model in which R&D firms need to borrow money before they pay their costs and households need to accumulate money before they consume. Accordingly, in this model, there are the CIA constraints on both consumption and R&D activity (i.e., investment). In particular, Chu and Cozzi (2014) show that when overinvestment in R&D takes place, the welfare-maximizing level of the nominal interest rate is positive.[1] According to these two strands, investment activities, especially capital accumulation and R&D activity, face a CIA constraint, and thus capital accumulation and R&D activity, which are important driving forces of endogenous growth, are affected by monetary policy through the CIA constraint. However, existing studies employ either the CIA constraint on capital accumulation or that on R&D activity. Therefore, the purpose of the present paper is to formulate an endogenous growth model with CIA constraints on both capital accumulation and R&D activity and to examine the effects of monetary policy on economic growth and welfare.

To formulate an endogenous growth model in which there are CIA constraints on

[1]Chu and Cozzi (2014) is the first paper to formulate an R&D-based growth model with CIA constraints on consumption and R&D activity. Subsequent to Chu and Cozzi (2014), a number of theoretical studies employ an R&D-based growth model with a CIA constraint on R&D activity. See, for example, recent work by Chen (2018), Chu, Cozzi, Furukawa, and Liao (2019) and Hori (2020).

both capital accumulation and R&D, we extend Iwaisako and Futagami's (2013) model in which there are two endogenous growth engines, one for R&D and another for capital accumulation, so that capital accumulation and R&D activity both face a CIA constraint.[2] The setting of the CIA constraint on investments draws on Chu and Cozzi (2014) (i.e., we assume that R&D firms and capital-producing firms need to borrow money from households to finance part of their costs in advance). In our model, the nominal interest rate is a monetary policy instrument, and we show that there are cases where the relationship between economic growth and the nominal interest rate is an inverted-U shape (i.e., the nominal interest rate which maximizes the growth rate is positive).

In addition, in our model we assume that the strength of the CIA constraint on the R&D sector and that on the producing capital sector differs. Therefore, an increase in the nominal interest rate stimulates investments in that sector where there is less need to borrow money. If this sector is then more productive, the growth rate increases when the nominal interest rate increases. As a result, the growth-maximizing nominal interest rate is positive; thus, we observe an inverted-U-shaped relationship between economic growth and the nominal interest rate. Moreover, conducting welfare analysis, we demonstrate that the welfare-maximizing level of the nominal interest rate is larger than the growth-rate-maximizing level of the nominal interest rate.

Several studies have already obtained an inverted-U-shaped relationship between economic growth and the nominal interest rate in an R&D-based growth model with a CIA constraint on R&D.[3] Chu, Cozzi, Furukawa, and Liao (2017) and Chu, Cozzi, Fan, Furukawa, and Liao (2019) develop a Schumpeterian growth model with endogenous entry of heterogeneous firms. In their model, only the R&D sector faces a CIA constraint. In contrast, the present paper develops an endogenous growth model with two growth engines,

[2]In Iwaisako and Futagami's (2013) model, the correlation between the growth rates of the number of intermediate goods and physical capital is negative owing to the specification of the technologies for R&D and capital production. In their model, both R&D and capital production use labor. This contrasts with Romer (1990) and Rivera-Batiz and Romer (1991), which have two growth engines, namely, a variety-expansion type of R&D and capital accumulation.

[3]For instance, Arawatari, Hori, and Mino (2017) suggest that the relationship between inflation and growth becomes nonlinear in an R&D-based model with the heterogeneous R&D abilities of agents.

R&D and capital, and both the R&D and capital-producing sectors face different strengths of the CIA constraint. Therefore, by deriving an inverted-U-shaped relationship between economic growth and the nominal interest rate through the different mechanisms, we provide a useful complement to these existing analyses. Huang, Yang, and Zheng (2018) and Zheng, Huang, and Yang (2019) develop an endogenous growth model with two R&D sectors. In their model, R&D firms in both sectors face a CIA constraint. In contrast, the present paper derives an inverted-U-shaped relationship between economic growth and the nominal interest rate in a different setting and thus provides a useful complement to Huang et al. (2018) and Zheng et al. (2019).

The present paper also relates to the literature on monetary policy and economic growth in an endogenous growth model with R&D and (physical or human) capital accumulation. As an example, Chu, Lai, and Liao (2019) develop a monetary hybrid endogenous growth model in which R&D and capital accumulation are both engines of endogenous growth. However, they consider a CIA constraint only on consumption. Elsewhere, Chu, Ning, and Zhu (2019) explore the growth and welfare effects of monetary policy in a scale-invariant Schumpeterian growth model with endogenous human capital accumulation, and consider CIA constraints on both consumption and R&D investment. Likewise, Gil and Iglésias (2020) employ an endogenous growth model with R&D and physical capital accumulation to examine the effect of monetary policy on economic growth.[4] Consequently, that work considers the CIA constraints on R&D activity and the manufacturing of intermediate goods. However, none of these studies consider CIA constraints on both capital accumulation and R&D activity.[5]

The remainder of the paper is organized as follows: Section 2 provides our model and Section 3 addresses the market equilibrium. Section 4 explains the equilibrium path and

[4] Gil and Iglésias (2020) extend Rivera-Batiz and Romer's (1991) model such that the growth rates of the number of intermediate goods and physical capital are determined proportionally. In contrast, the present paper extends Iwaisako and Futagami's (2013) model and thus the growth rates of the number of intermediate goods and physical capital exhibit a negative correlation. See also footnote 2.

[5] In Gil and Iglésias (2020), the CIA constraint on the manufacturing of intermediate goods affects capital accumulation because capital is a production factor of these intermediate goods and thus capital accumulation and R&D activity are affected by the CIA constraints. However, they do not provide an inverted-U-shaped relationship between economic growth and monetary policy.

Section 5 details the effect of the nominal interest rate. Section 6 concludes.

2 The model

To examine the relationship between economic growth and monetary policy, we develop an endogenous growth model with CIA constraints. In this model, there are two engines of endogenous growth, namely, capital accumulation and R&D, with investments in both conducted using labor. We assume that R&D firms and capital-producing firms need to borrow money from households to finance part of their costs in advance (i.e., there are CIA constraints on both R&D and capital accumulation). A continuum of intermediate goods then produces the final goods, with the varieties of intermediate goods expanding because of R&D activity. Labor and capital produce each intermediate good.

2.1 Households

In the economy, there are a fixed number of identical households. We normalize the total number of households to unity. We define L as the size of the population of each household. Each member of the household has one unit of labor and supplies her or his labor inelastically at each point of time. Then, the sum of labor supply is L at each point in time. The representative household maximizes lifetime utility over an infinite horizon given by:

$$\int_0^\infty e^{-\rho t} \log C(t) dt, \tag{1}$$

where $C(t)$ denotes the consumption of final goods and ρ is the subjective discount rate. The representative household maximizes (1) subject to the following budget constraint:

$$\dot{a}(t) + \dot{m}(t) = r(t)a(t) + i(t)b(t) + w(t)L + \tau(t) - C(t) - \pi(t)m(t), \tag{2}$$

where $a(t)$ is the real value of assets excluding money, $m(t)$ is the real money balance held by households, $b(t)$ is the amount of real money borrowed from each member of households by entrepreneurs (who engage in R&D and producing capital) to finance investment, $\tau(t)$ is a lump-sum transfer from the government, $r(t)$ is the rate of return on assets or the real interest rate, $i(t)$ is the return on lending money, $w(t)$ is the real wage rate, and $\pi(t)$ is the inflation rate. The CIA constraint is given by:

$$b(t) \leq m(t). \tag{3}$$

As there is no CIA constraint on consumption, the following familiar Euler equation characterizes the solution to the preceding dynamic maximization problem:

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho. \tag{4}$$

The dynamic optimization requires the following relationship:[6]

$$i(t) = r(t) + \pi(t). \tag{5}$$

This equation is the Fischer equation. Therefore, $i(t)$ is the nominal interest rate.

2.2 Firms producing final goods

The production function of final goods is represented by a Dixit and Stiglitz (1977) type function:

$$Y(t) = \left[\int_0^{N(t)} x(j,t)^{\alpha} dj\right]^{\frac{1}{\alpha}}, \quad 0 < \alpha < 1, \tag{6}$$

where $Y(t)$ is the output of final goods, $N(t)$ is the number of intermediate goods available, $x(j,t)$ is the input of intermediate good $j \in [0, N(t)]$, and $1/(1-\alpha) > 1$ represents the elasticity of substitution between the intermediate inputs. Given we define the final good

[6] See Appendix A for the detailed calculation.

as a numeraire, the solution of the final good firm's cost-minimizing problem is given by:

$$x(j,t) = Y(t)p(j,t)^{-\frac{1}{1-\alpha}}, \tag{7}$$

where $p(j,t)$ is the price of intermediate good j. Substituting (7) into (6) yields:

$$1 = \left[\int_0^{N(t)} p(j,t)^{-\frac{\alpha}{1-\alpha}} dj\right]^{\frac{1}{\alpha}}. \tag{8}$$

2.3 Firms producing intermediate goods

The production of each intermediate good employs labor and physical capital. The production function of intermediate good j is specified as a Cobb–Douglas form:

$$x(j,t) = Al(j,t)^{\gamma}k(j,t)^{1-\gamma}, \tag{9}$$

where $x(j,t)$ is the output of intermediate good j, and $l(j,t)$ and $k(j,t)$ are the amounts of labor and capital used for the production of intermediate good j, respectively. Solving the cost-minimizing problem yields the following unit cost function:

$$c(w(t),q(t)) = A^{-1}\gamma^{-\gamma}(1-\gamma)^{\gamma-1}w(t)^{\gamma}q(t)^{1-\gamma}, \tag{10}$$

where $q(t)$ is the rental rate of capital. From Shephard's lemma, the demand functions for labor and capital can be derived as follows:

$$l(j,t) = \frac{\partial c(w(t),q(t))}{\partial w(t)}x(j,t) = \frac{\gamma c(w(t),q(t))}{w(t)}x(j,t) \tag{11}$$

$$k(j,t) = \frac{\partial c(w(t),q(t))}{\partial q(t)}x(j,t) = \frac{(1-\gamma)c(w(t),q(t))}{q(t)}x(j,t). \tag{12}$$

As in the growth model, each intermediate good firm can monopolistically supply with a product variety expansion.[7] Therefore, the profit-maximizing price of intermediate good

[7] See the canonical model in Romer (1990) and Grossman and Helpman (1991, Ch. 3).

j is given by:[8]

$$p(j,t) = \frac{1}{\alpha} c(w(t), q(t)). \tag{13}$$

This equation implies that all intermediate goods firms charge the same price level (i.e., $p(j,t) = p(t)$). Substituting (13) into (8) yields:

$$c(w(t), q(t)) = \alpha N(t)^{\frac{1-\alpha}{\alpha}}. \tag{14}$$

Using (7), (13) and (14), we obtain the output and profit of the intermediate good as follows:

$$x(t) = \left[\frac{1}{\alpha} c(w(t), q(t))\right]^{-\frac{1}{1-\alpha}} Y(t) = Y(t) N(t)^{-\frac{1}{\alpha}} \tag{15}$$

$$\Pi(t) = \left(\frac{1}{\alpha} - 1\right) c(w(t), q(t)) x(t) = (1-\alpha) \frac{Y(t)}{N(t)}, \tag{16}$$

where we omit the goods index j because the intermediate firms are symmetrical from (13).

2.4 R&D firms

In this subsection, we characterize the R&D firm's behavior, and examine how the value of the patent, $v(t)$, is determined. $v(t)$ is equal to the sum of the discounted present value of the profit flow after period t: i.e., $v(t) = \int_t^\infty \Pi(s) \exp\left(-\int_t^s r(\nu) d\nu\right) ds$. Differentiating this equation with respect to time, t, we obtain the following equation:

$$\dot{v}(t) = r(t) v(t) - \Pi(t). \tag{17}$$

[8] This paper does not consider patent policy (i.e., patent protection is complete). A number of studies analyze the effect of patent protection in R&D-based growth models. See, for example, early work by Judd (1985), Li (2001), Goh and Olivier (2002), Iwaisako and Futagami (2003), Kwan and Lai (2003), and O'Donoghue and Zweimuller (2004).

Rewriting (17) yields the familiar no-arbitrage condition: $r(t)v(t) = \Pi(t) + \dot{v}(t)$. This equation means that the return from holding one patent, $r(t)v(t)dt$, must be equal to the sum of the monopolistic profit, $\Pi(t)dt$, and the capital gain or loss, $\dot{v}(t)dt$.

The production function for R&D is given by:

$$\dot{N}(t) = (a_N)^{-1} N(t) l_N(t), \tag{18}$$

where $l_N(t)$ is the labor input used in R&D activity, and $a_N > 0$ is constant and represents the reciprocal of the productivity of labor. In the present paper, we assume that the firm must borrow money to finance the cost of a proportion of $\eta_N \in [0, 1]$ before they invest in R&D as in Chu and Cozzi (2014).[9] η_N represents the strength of the CIA constraint on R&D. Given the firm's cost is $w(t)l_N(t)$, the amount of borrowing is $\eta_N w(t) l_N(t)$. Hence, the total cost of the firm is $w(t)l_N(t) + i(t)\eta_N w(t) l_N(t) = (1 + \eta_N i(t))w(t)l_N(t)$. Using this equation, the profit of the firm is $v(t)\dot{N}(t) - (1+\eta_N i(t))(a_N/N(t))w(t)\dot{N}(t)$. Because we assume free entry, the following free-entry (or zero-profit) condition must be satisfied:

$$v(t) = (1 + \eta_N i(t)) \frac{a_N}{N(t)} w(t). \tag{19}$$

2.5 Firms producing capital goods

In this subsection, we characterize the behavior of the firm producing capital, and examine how the value of one unit of capital stock, $v_K(t)$, is determined. $v_K(t)$ is equal to the sum of the discounted present value of the profit flow after period t: i.e.,
$v_K(t) = \int_t^\infty q(s) \exp\left(-\int_t^s r(\nu) d\nu\right) ds$. Differentiating this equation with respect to time, t, we obtain the following equation:

$$\dot{v}_K(t) = r(t) v_K(t) - q(t). \tag{20}$$

This is also the familiar no-arbitrage condition.

[9]That is, there is a CIA constraint on R&D.

The production function for producing capital is given by:

$$\dot{K}(t) = (a_K)^{-1} K(t) l_K(t), \tag{21}$$

where $l_K(t)$ is the labor input used in producing capital, and $a_K > 0$ is constant and denotes the reciprocal of the productivity of labor. We also assume that the capital-producing firm must borrow money to finance some proportion of the cost (i.e., there is a CIA constraint on producing capital). When $\eta_K \in [0, 1]$ denotes the cost of the proportion needed to be borrowed with money before the firm can produce,[10] the profit of the firm is $v_K(t)\dot{K}(t) - (1 + \eta_K i(t))(a_K/K(t))w(t)\dot{K}(t)$. Given that we assume free entry, we impose the following free-entry (or zero-profit) condition:

$$v_K(t) = (1 + \eta_k i(t)) \frac{a_K}{K(t)} w(t). \tag{22}$$

2.6 Money Authority

The monetary authority supplies money and transfers all of its (i.e., the seigniorage revenue) to each household equally in each period. Defining $M(t)$ as the nominal money supply and $P(t)$ as the final good's price, the lump-sum transfer is given by: $\tau(t) = [\dot{M}(t)/M(t)][M(t)/P(t)] = [\dot{M}(t)/M(t)]m(t)$, where $M(t)/P(t) = m(t)$ is the definition of the real money balance. We assume that monetary policy is implemented by controlling the nominal interest rate denoted by i (i.e., the nominal interest rate is the monetary policy instrument determined by the monetary authority, and thus $i(t) = i$ over time). Therefore, the growth rate of the nominal money supply, $\dot{M}(t)/M(t)$, is determined endogenously.

[10]Given the firm's cost is $w(t)l_K(t)$, the amount of borrowing is $\eta_K w(t)l_K(t)$. Thus, the total cost is $w(t)l_K(t) + \eta_K i(t)w(t)l_K(t)$.

3 Market equilibrium

The households inelastically supply L units of labor in each period. Firms producing intermediate goods, firms engaging in R&D, and firms producing capital goods all demand labor. Therefore, the equilibrium condition for the labor market is given by: $\int_0^{N(t)} l(j,t)dj + l_N + l_K = L$. Substituting (14) and (15) into (11), we obtain the following labor demand function of a representative firm producing one intermediate good:

$$l(t) = \alpha\gamma Y(t)/[w(t)N(t)]. \tag{23}$$

Using this equation, (18), (21) and the equilibrium condition for the labor market, we obtain:

$$\alpha\gamma\frac{Y(t)}{w(t)} + a_K\frac{\dot{K}(t)}{K(t)} + a_N\frac{\dot{N}(t)}{N(t)} = L. \tag{24}$$

The equilibrium condition for the capital market is: $\int_0^{N(t)} k(j,t)dj = K(t)$ because capital is only used for the production of intermediate goods. Substituting (14) and (15) into (12), we obtain: $k(t) = \alpha(1-\gamma)Y(t)/[q(t)N(t)]$. Moreover, substituting this equation into the equilibrium condition for the capital market yields:

$$\alpha(1-\gamma)\frac{Y(t)}{q(t)} = K(t). \tag{25}$$

The firms providing R&D and those producing capital borrow money. The R&D firm borrows $\eta_N w(t)l_N(t)$ and the capital-producing firm borrows $\eta_K w(t)l_K(t)$. As the households lend $b(t)$ unit of money, the market for borrowing is given by $\eta_N w(t)l_N(t)+\eta_K w(t)l_K(t) = b(t)$. Combining this equation, (18) and (21), we obtain:

$$\eta_K w(t)a_K\frac{\dot{K}(t)}{K(t)} + \eta_N w(t)a_N\frac{\dot{N}(t)}{N(t)} = b(t). \tag{26}$$

From Appendix A, the households use all of their money held for lending to entrepreneurs. Therefore, in the equilibrium, the following equation is satisfied:

$$b(t) = m(t). \tag{27}$$

The final goods market-clearing condition is given by:

$$Y(t) = C(t). \tag{28}$$

4 Equilibrium path

We define $V(t) \equiv [v(t)N(t)]/(1 + \eta_N i)$ and $V_K(t) \equiv [v_K(t)K(t)]/(1 + \eta_K i)$. From the free-entry conditions for R&D, (19), and producing capital, (22), the equilibrium wage is given by:

$$w(t) = \frac{V(t)}{a_N} = \frac{V_K(t)}{a_K}. \tag{29}$$

Using (25), we obtain the aggregate rental, $Q(t) \equiv q(t)K(t)$, as follows:

$$Q(t) = \alpha(1 - \gamma)Y(t). \tag{30}$$

We substitute (29) into (24) to obtain:

$$\mathrm{g}_N(t) = \frac{L}{a_N} - \alpha\gamma\frac{a_K}{a_N}\frac{1}{V_K^Y(t)} - \frac{a_K}{a_N}\mathrm{g}_K(t), \tag{31}$$

where $g_N(t) \equiv \dot{N}(t)/N(t)$, $g_K(t) \equiv \dot{K}(t)/K(t)$ and $V_K^Y(t) \equiv V_K(t)/Y(t)$[11]. Substituting the profit flow, (16), and (29) into the no-arbitrage condition, (17), yields:

$$\frac{\dot{v}(t)}{v(t)} = r(t) - (1-\alpha)\frac{a_K}{a_N}\frac{1}{(1+\eta_N i)V_K^Y(t)}. \tag{32}$$

Moreover, substituting the aggregate rental, (30), into the no-arbitrage condition, (20), we obtain:

$$\frac{\dot{v}_K(t)}{v_K(t)} = r(t) - \alpha(1-\gamma)\frac{1}{(1+\eta_K i)V_K^Y(t)}. \tag{33}$$

The nominal interest rate is the constant policy parameter because we assume that it is the policy instrument of the monetary authority. Using (30) to (33), and i is constant, we obtain the growth rates of $V(t)$ and $V_K(t)$ as follows:

$$\frac{\dot{V}(t)}{V(t)} = g_N(t) + \frac{\dot{v}(t)}{v(t)} = r(t) - (1-\alpha)\frac{a_K}{a_N}\frac{1}{(1+\eta_N i)V_K^Y(t)} + \frac{L}{a_N} - \alpha\gamma\frac{a_K}{a_N}\frac{1}{V_K^Y(t)} - \frac{a_K}{a_N}g_K(t), \tag{34}$$

$$\frac{\dot{V}_K(t)}{V_K(t)} = g_K(t) + \frac{\dot{v}_K(t)}{v_K(t)} = r(t) - \alpha(1-\gamma)\frac{1}{(1+\eta_K i)V_K^Y(t)} + g_K(t). \tag{35}$$

From (29), $\dot{V}(t)/V(t) = \dot{V}_K(t)/V_K(t)$ holds for all t. Combining (34) and (35), we can derive the growth rate of capital:

$$g_K(t) = \frac{1}{a_K + a_N}\left\{L + \frac{\alpha}{V_K^Y(t)}\left[\frac{1-\gamma}{1+\eta_K i}a_N - a_K\left(\frac{(1-\alpha)/\alpha}{1+\eta_N i} + \gamma\right)\right]\right\}. \tag{36}$$

From the market-clearing condition of the final good, it follows that $\dot{C}(t)/C(t) = \dot{Y}(t)/Y(t)$. Substituting this into the Euler equation (4) yields $g_Y(t) \equiv \dot{Y}(t)/Y(t) = r(t) - \rho$. we use

[11] Although the growth rates of variety and capital may be negatively related, it is not surprising. In our model, the labor force input determines these rates. Therefore, if the labor input is concentrated in the R&D (capital) sector, it correspondingly decreases in the capital (R&D) sector. This is consistent with the argument in Iwaisako and Futagami (2013). In fact, the above case may arise in the equilibrium.

this equation, (35) and (36) to get:

$$\dot{V}_K^Y(t) = \left(\rho + \frac{L}{a_K + a_N}\right) V_K^Y(t) - \frac{a_K}{a_K + a_N}\left[\frac{1-\alpha}{1+\eta_N i} + \alpha\left(\frac{1-\gamma}{1+\eta_K i} + \gamma\right)\right]. \tag{37}$$

From this equation, we obtain the following lemma.

Lemma 1. *The path initially jumping to a unique balanced growth path (BGP) is the market equilibrium path.*

Proof. See Appendix B. □

This lemma implies that if policy changes unexpectedly, the market equilibrium path has no transition dynamics because it immediately jumps to the unique BGP.

5 Effects of the nominal interest rate

In this section, we examine the effects of the nominal interest rate on the BGP, which is the market equilibrium path. We first derive the equilibrium value of $V_K^Y(t)$. Combining $\dot{V}_K^Y(t) = 0$ and (37) yields:

$$V_K^Y = a_K \frac{\frac{1-\alpha}{1+\eta_N i} + \alpha\left(\gamma + \frac{1-\gamma}{1+\eta_K i}\right)}{L + (a_N + a_K)\rho}, \tag{38}$$

where V_K^Y is the equilibrium value of $V_K^Y(t)$. Hereafter, we define the variable omitted t as the equilibrium values (or the values on the BGP).

To start, we analyze the effects of the nominal interest rate on the rate of growth of the number of intermediate goods and capital. Using (31), (36), and (38), we obtain the growth rate of the number of intermediate goods and capital on the BGP as follows[12]:

$$\mathrm{g}_N = \frac{1}{a_N}\frac{1-\alpha}{1+\eta_N i}\frac{L+(a_N+a_K)\rho}{\frac{1-\alpha}{1+\eta_N i}+\alpha\left(\gamma+\frac{1-\gamma}{1+\eta_K i}\right)}-\rho, \tag{39}$$

$$\mathrm{g}_K = \frac{\alpha}{a_K}\frac{1-\gamma}{1+\eta_K i}\frac{L+(a_N+a_K)\rho}{\frac{1-\alpha}{1+\eta_N i}+\alpha\left(\gamma+\frac{1-\gamma}{1+\eta_K i}\right)}-\rho. \tag{40}$$

From these equations, we obtain the following lemma.

Lemma 2. *Suppose that the nominal interest rate, $i \geq 0$, is sufficiently small. An increase in the nominal interest rate then enhances the growth rate of the number of intermediate goods if $(\eta_N/\eta_K) < 1-\gamma$. Moreover, an increase in the nominal interest rate enhances the growth rate of capital if $(\eta_N/\eta_K) > 1 + [(\alpha\gamma)/(1-\alpha)]$.*

Proof. Differentiating (39) and (40) with respect to the nominal interest rate, i, we obtain

$$\frac{d\mathrm{g}_N}{di} = -\frac{(1-\alpha)[L+(a_N+a_K)\rho]}{a_N}\frac{\gamma\eta_N+(1-\gamma)\frac{\eta_N-\eta_K}{(1+\eta_K i)^2}}{\left\{1-\alpha+\alpha\left[\gamma(1+\eta_N i)+\frac{1+\eta_N i}{1+\eta_K i}(1-\gamma)\right]\right\}^2}, \tag{41}$$

$$\frac{d\mathrm{g}_K}{di} = -\frac{\alpha(1-\gamma)[L+(a_N+a_K)\rho]}{a_K}\frac{\alpha\gamma\eta_K+(1-\alpha)\frac{\eta_K-\eta_N}{(1+\eta_N i)^2}}{\left\{\frac{1+\eta_K i}{1+\eta_N i}(1-\alpha)+\alpha\left[\gamma(1+\eta_K i)+1-\gamma\right]\right\}^2}. \tag{42}$$

If each condition is satisfied, the above equations are positive when $i = 0$ (i.e., $d\mathrm{g}_N/di|_{i=0} > 0$ if $(\eta_N/\eta_K) < 1-\gamma$ and $d\mathrm{g}_K/di|_{i=0} > 0$ if $(\eta_N/\eta_K) > 1 + [(\alpha\gamma)/(1-\alpha)]$). □

We interpret Lemma 2 as follows. An increase in the nominal interest rate affects the growth rates of the number of intermediate goods (capital) through the following two channels. First, a higher nominal interest rate makes R&D (capital-producing) activity more costly through the CIA constraint and thus stifles investment in the R&D sector

[12]Equations (39) and (40) imply that the growth rates of the number of intermediate goods and capital differ. This is not consistent with Gil and Iglesias (2020) and Howitt and Aghion (1998) given the difference in the specification of the investment technologies in the R&D and capital-producing sectors. In their model, R&D and capital production are through inputting final goods under a linear technology. Therefore, the growth rates of the final goods, the number of intermediate goods, and capital are identical in the BGP. However, in our model, R&D and capital production are through inputting labor. As a result, in the BGP in our model, the number of labor inputs for each sector can differ. Therefore, the rate of growth of the number of intermediate goods and capital can be different, as in Chu, Lai, and Liao (2019). See also footnote 4.

(the capital-producing sector). Hence, a higher nominal interest rate drives down the growth rates of the number of intermediate goods (capital). This negative effect positively correlates with the strength of the CIA constraint on R&D (capital-producing) activity. Second, a higher nominal interest rate also makes capital-producing (R&D) activity more costly through the CIA constraint and thus stifles investment in the capital-producing sector (the R&D sector). This brings about a reduction in labor demand in the capital-producing sector (the R&D sector). Labor is then reallocated from the capital-producing (R&D) sector to both the intermediate goods production sector and the R&D (capital-producing) sector. This is because R&D and capital-producing activity and the production of intermediate goods all use labor. Hence, a higher nominal interest rate drives up the growth rates of the number of intermediate goods (capital) through labor reallocation.[13] This positive effect positively correlates with the strength of the CIA constraint on capital-producing (R&D) activity. If the CIA constraint in the R&D (capital-producing) sector is relatively weaker than that in the capital-producing (R&D) sector, the second positive effect of the nominal interest rate on the growth rates of the number of intermediate goods (capital) can dominate the first negative effect. Hence, an increase in the nominal interest rate stimulates the growth rate of the number of intermediate goods (capital) when the nominal interest rate is sufficiently low. Lemma 2 implies that an increase in the nominal interest rate can bring about resource reallocation from the investment sector with a relatively stronger CIA constraint to the investment sector with a relatively weaker CIA constraint.

Next, we analyze the effect of the growth rate of output. Output can be represented by the unit cost function of the intermediate firms, (10). Using (15), we obtain:

$$\begin{aligned} Y(t) &= \left[\int_0^{N(t)} \left(\frac{1}{\alpha} c(w(t), q(t)) \right)^{-\frac{\alpha}{1-\alpha}} Y(t)^{\alpha} dj \right]^{\frac{1}{\alpha}} \\ &= [\alpha A \gamma^{\gamma} (1-\gamma)^{1-\gamma} w(t)^{-\gamma} Q(t)^{\gamma-1} K(t)^{1-\gamma}]^{\frac{1}{1-\alpha}} Y(t) N(t)^{\frac{1}{\alpha}}. \end{aligned} \tag{43}$$

[13]This positive effect comes from the specification of the technologies for R&D and capital production. See footnote 11.

Substituting (29) and (30) into (43), we obtain the value of output as follows[14]:

$$Y(t) = A\,(\alpha\gamma a_K)^{\gamma}\,(V_K^Y)^{-\gamma}K(t)^{1-\gamma}N(t)^{\frac{1-\alpha}{\alpha}}. \tag{44}$$

Given V_K^Y does not change on the BGP, the growth rate of output, $\mathrm{g}_Y \equiv \dot{Y}(t)/Y(t)$, is given by $\mathrm{g}_Y = (1-\gamma)\mathrm{g}_K + [(1-\alpha)/\alpha]\mathrm{g}_N$ from (44). Substituting the growth rates, (39) and (40), we obtain:

$$\begin{aligned}
\mathrm{g}_Y &= \left[\frac{\alpha}{a_K}\frac{(1-\gamma)^2}{1+\eta_K i} + \frac{(1-\alpha)^2}{\alpha}\frac{1}{a_N}\frac{1}{1+\eta_N i}\right]\frac{L+(a_N+1)\rho}{\frac{1-\alpha}{1+\eta_N i} + \alpha\left(\gamma + \frac{1-\gamma}{1+\eta_K i}\right)} + \textit{constant term}\\
&= \left[\frac{1}{a_K}\frac{(1-\gamma)^2}{1+\eta_K i} + \left(\frac{1-\alpha}{\alpha}\right)^2\frac{1}{a_N}\frac{1}{1+\eta_N i}\right]\frac{L+(a_N+1)\rho}{\frac{(1-\alpha)/\alpha}{1+\eta_N i} + \gamma + \frac{1-\gamma}{1+\eta_K i}} + \textit{constant term},
\end{aligned} \tag{45}$$

where *constant term* is the term that does not depend on the nominal interest rate. From this equation, we obtain the following proposition:

Proposition 1. *If*

$$\frac{(1-\gamma)^2}{a_K} - \frac{1-\alpha}{\alpha}\frac{1}{a_N} > (<)0 \tag{46}$$

and

$$\frac{\eta_N}{\eta_K} > (<)1 + \frac{\alpha\gamma}{1-\alpha}\frac{\left(\frac{1-\alpha}{\alpha}\right)^2\frac{1}{a_N} + \frac{(1-\gamma)^2}{a_K}}{\frac{(1-\gamma)^2}{a_K} - \frac{1-\alpha}{\alpha}\frac{1}{a_N}} \tag{47}$$

are satisfied, the nominal interest rate maximizing the growth rate of output is positive.

[14] The reason that V_K^Y is included in (44) is as follows. In our model, the intermediate goods are produced by inputting not only capital but also labor. Therefore, the production of the intermediate goods decreases if the input of labor is decreased by an increase in wage. A decrease in the production of intermediate goods reduces the output of the final goods. The term $V_K^Y = V_K(t)/Y(t) = a_K w(t)/Y(t)$ in (44) captures this channel.

Proof. Differentiating (45) with respect to the nominal interest rate, i, we obtain:

$$\frac{d\mathrm{g}_Y}{di} = \frac{L + (a_K + a_N)\rho}{\left\{\left[\frac{(1-\alpha)/\alpha}{1+\eta_N i} + \gamma + \frac{1-\gamma}{1+\eta_K i}\right](1+\eta_K i)(1+\eta_N i)\right\}^2}$$
$$\times \left\{ \frac{1-\alpha}{\alpha}(1-\gamma)\left(\frac{1-\gamma}{a_K} - \frac{1-\alpha}{\alpha}\frac{1}{a_N}\right)(\eta_N - \eta_K) \right.$$
$$\left. - \gamma\left[(1-\gamma)^2\frac{\eta_K}{a_K}(1+\eta_N i)^2 + \left(\frac{1-\alpha}{\alpha}\right)^2\frac{1}{a_N}\eta_N(1+\eta_K i)^2\right]\right\}. \tag{48}$$

If conditions, (46) and (47), are satisfied, (48) is positive when $i = 0$. Furthermore, it is easy to prove $d^2\mathrm{g}_Y/di^2 < 0$. Therefore, the nominal interest rate maximizing the growth rate of output is positive. □

Proposition 1 implies that there is an inverted-U-shaped relationship between economic growth and monetary policy (i.e., the nominal interest rate). We interpret Proposition 1 as follows. From (48), the necessary conditions for the nominal interest rate maximizing the growth rate of output becoming positive are $[(1-\gamma)/a_K] - [(1-\alpha)/\alpha](1/a_N) > (<)0$ and $\eta_N > (<)\eta_K$. First, we consider the case where $[(1-\gamma)/a_K] - [(1-\alpha)/\alpha](1/a_N) > 0$ and $\eta_N > \eta_K$.[15] This condition is satisfied when γ and a_K are smaller. This means that there is advantage to producing capital because the marginal productivity of capital in the final good sector is high and the capital-producing cost in this case is low. Therefore, when $[(1-\gamma)/a_K] - [(1-\alpha)/\alpha](1/a_N) > 0$, increasing capital production makes the growth rate of output increase. From (42), if $\eta_N > \eta_K$, there is a possibility of an increase in the nominal interest rate increasing the growth rate of capital. Therefore, there exists a case in which increasing the nominal interest rate increases the growth rate of output. Similarly, we obtain the reason why the growth rate of output increases when $[(1-\gamma)/a_K] - [(1-\alpha)/\alpha](1/a_N) < 0$. When there is an advantage to R&D, increasing R&D makes the growth rate of output increase. From (41), if $\eta_N < \eta_K$, there is also

[15] $\eta_N(\eta_K)$ is the strength of the CIA constraint on R&D (capital accumulation). According to Chu and Cozzi (2014), empirical studies (e.g., Brown and Petersen 2009; Brown, Martinsson, and Petersen 2012; Brown and Petersen 2015) support the argument that R&D investment is even more severely affected by liquidity requirements than physical capital accumulation. Hence, in practice, $\eta_N > \eta_K$ is likely to hold.

the possibility that increasing the nominal interest rate increased the growth rate of the number of intermediate goods. Then, the growth rate of output increases.

In the last part of this section, we discuss the welfare effects. Given $C(t) = Y(t)$, lifetime utility is given by:

$$W \equiv \int_0^\infty e^{-\rho t} \log Y(t) dt. \tag{49}$$

As the BGP is the market equilibrium path, $Y(t) = Y(0)e^{\mathrm{g}_Y t}$. Therefore, we rewrite (49):

$$W = \left(\frac{1}{\rho}\right)^2 \left[\mathrm{g}_Y - \rho\gamma \log V_K^Y\right] + \textit{constant term}, \tag{50}$$

where "*constant term*" does not depend on i. Differentiating (38) with respect to i yields $dV_K^Y/di < 0$. Therefore, "$-\rho\gamma \log V_K^Y$" is increasing in the nominal interest rate, i. This implies that the welfare-maximizing level of the nominal interest rate is larger than the growth-rate-maximizing level of the nominal interest rate. The reason is as follows. A higher nominal interest rate increases the costs of these two types of investment through the CIA constraints, and thus leads to a lower level of investment and lower labor inputs into these investments. This makes it possible to reallocate labor from investment to production. As a result, a higher nominal interest rate increases the production level and thus improves welfare. The existence of the effect as we explained brings about the result that the welfare-maximizing level of the nominal interest rate is larger than the growth-rate-maximizing level of the nominal interest rate.

6 Conclusion

By formulating a simple endogenous growth model with CIA constraints on both capital accumulation and R&D activity, we have shown that the relationship between economic growth and the nominal interest rate as the monetary policy instrument can be an inverted-U shape depending on the strength of the CIA constraints. In the realistic case where capital accumulation and R&D activity, which are important driving force of endogenous growth, face CIA constraints, the growth-rate-maximizing level of the nominal interest rate can be positive. This implies that monetary authority policymakers should consider the strength of the CIA constraints on all investment sectors to maximize the growth rate.

Appendix

A The optimization of the household

We denote the Hamiltonian as follows:

$$\begin{aligned} H = & \ln C(t) + \lambda(t)(r(t)a(t) + i(t)b(t) + w(t)L + \tau(t) - C(t) - \pi(t)m(t) - I_m(t)) + \kappa(t)I_m(t) \\ & + \phi(t)(m(t) - b(t)), \end{aligned} \tag{A.1}$$

where $\lambda(t)$ and $\kappa(t)$ are the co-state variables of assets and money, respectively, $\dot{m}(t) \equiv I_m(t)$, and $\phi(t)$ is the Lagrangian multiplier of the CIA constraint. The optimality conditions are given by:

$$C(t):\ \frac{1}{C(t)} - \lambda(t) = 0 \tag{A.2}$$

$$I_m(t):\ -\lambda(t) + \kappa(t) \gtreqless 0 \tag{A.3}$$

$$b(t):\ i(t)\lambda(t) - \phi(t) \gtreqless 0 \tag{A.4}$$

$$a(t):\ \dot{\lambda}(t) - \rho\lambda(t) = -r(t)\lambda(t) \tag{A.5}$$

$$m(t):\ \dot{\kappa}(t) - \rho\kappa(t) = \pi(t)\lambda(t) - \phi(t) \tag{A.6}$$

$$\phi(t):\ \phi(t)(m(t) - b(t)) = 0 \ and \ \phi(t) \geq 0 \tag{A.7}$$

$$\text{Transversality condition (TVC)} \ \lim_{t\to\infty} e^{-\rho t}\lambda(t)a(t) = \lim_{t\to\infty} e^{-\rho t}\kappa(t)m(t) = 0 \tag{A.8}$$

From (A.2) and (A.5), we obtain the Euler equation (4). From (A.3), if the households hold both assets and money, we obtain:

$$\lambda(t) = \kappa(t). \tag{A.9}$$

From (A.9), we get:

$$\frac{\dot{\kappa}(t)}{\kappa(t)} = \frac{\dot{\lambda}(t)}{\lambda(t)}. \tag{A.10}$$

Using (A.5), (A.6), (A.9) and (A.10), we obtain:

$$\phi(t) = (r(t) + \pi(t))\lambda(t) \tag{A.11}$$

Substituting (A.4) into (A.11), and if $b(t) > 0$, $[i(t) - (r(t) + \pi(t))]\lambda(t) = 0$. If assets are held, $\lambda(t) \neq 0$. Therefore, we get (5).

B Proof of Lemma 1

First, we show that the BGP is unique. The differential equation given by (37) has a unique but unstable nonzero steady state given the following inequalities hold:

$$i \geq 0, \quad \rho + \frac{L}{a_K + a_N} > 0 \quad \text{and} \quad \frac{a_K}{a_K + a_N}\left[\frac{1-\alpha}{1+\eta_N i} + \alpha\left(\frac{1-\gamma}{1+\eta_K i} + \gamma\right)\right] > 0. \tag{B.1}$$

This means that there is a unique BGP.

Second, we show that the path initially jumping to the BGP is the market equilibrium path. Given all assets are held by households and (29) must be satisfied in equilibrium,

$$a(t) = V_K(t) + V(t) = \left(1 + \frac{a_N}{a_K}\right) V_K(t). \tag{B.2}$$

Using this equation, $C(t) = Y(t)$, and (A.2), we rewrite the transversality condition (TVC) as follows:

$$\lim_{t\to\infty} e^{-\rho t}\frac{1}{C(t)}\left(1+\frac{a_N}{a_K}\right)V_K(t) = \lim_{t\to\infty} e^{-\rho t}V_K^Y(t)\left(1+\frac{a_N}{a_K}\right) = 0. \tag{B.3}$$

At first, we consider the case in which $V_K^Y < V_K^Y(t)$, where V_K^Y is the steady-state value of $V_Y^K(t)$. In this case, $V_K^Y(t)$ diverges to positive infinity. From (37), the growth rate of $e^{-\rho t}V_K^Y(t)[1+(a_N/a_K)]$ is given by:

$$-\rho + \frac{\dot{V}_K^Y(t)}{V_K^Y(t)} = \frac{L}{a_K+a_N} - \frac{a_K}{V_K^Y(t)(a_K+a_N)}\left[\frac{1-\alpha}{1+\eta_N i} + \alpha\left(\frac{1-\gamma}{1+\eta_K i}+\gamma\right)\right] \tag{B.4}$$

In the case in which $V_K^Y < V_K^Y(t)$, it follows that $\lim_{t\to\infty} V_K^Y(t) = \infty$. Therefore, the path along which $V_K^Y(t)$ diverges to positive infinity violates the TVC. Next, we consider the case in which $V_K^Y > V_K^Y(t)$. In this case, $V_K^Y(t)$ reaches zero in a finite period. As $V_K^Y(t)$ cannot take a negative value, the path along which $V_K^Y(t)$ approaches zero deviates from feasibility. Finally, we consider the path along which $V_K^Y = V_K^Y(t)$. This case satisfies the TVC, $\lim_{t\to\infty} e^{-\rho t}V_K^Y[1+(a_N/a_K)] = 0$, because the growth rate of $e^{-\rho t}V_K^Y(t)[1+(a_N/a_K)]$ is negative.

In the rest of this section, we check whether the BGP satisfies the other TVC:

$$\lim_{t\to\infty} e^{-\rho t}\kappa(t)m(t) = 0. \tag{B.5}$$

On the BGP, from (26), (27) and (29), we obtain $m(t) = [V_K(t)/a_K](\eta_K a_K g_K + \eta_N a_N g_N)$. Using this, (A.9), (A.2) and (B.5), we obtain

$$\lim_{t\to\infty} e^{-\rho t}\frac{V_K^Y}{a_K}(\eta_K a_K g_K + \eta_N a_N g_N) = 0. \tag{B.6}$$

Therefore, we find that the BGP satisfies the TVC; thus, the path initially jumping to the BGP is the market equilibrium path.

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