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CONTAGION
OF POPULIST EXTREMISM*

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Contagion of Populist Extremism*

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Abstract

To explore the propagation of undesirable policies in a form of populist extremism, we construct a social learning model featuring agency problems. Politicians in different countries sequentially implement a policy. Voters learn the incumbent politician's type and the desirable policy by observing foreign policies on top of the domestic policy. We show that populist extremism is contagious across countries through the dynamic interaction between the changing public opinion and implemented policies. This structure yields interesting long-run dynamics. First, a single moderate policy could be always enough to stop the domino effect. Second, the persistence of the domino effect depends on the correlation of the desirable policy across countries. In particular, while extremism eventually ends under the perfect correlation, it may become impossible to escape from extremism under the imperfect correlation. These results reveal a new negative aspect of decentralized policymaking.

Keywords: Political agency; Yardstick competition; Populism; Observational learning; Signaling

JEL classification: D72; D83; H73

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Let us stop the domino effect right this week, this Wednesday.
The domino effect of the wrong sort of populism winning in this world.

Mark Rutte, the Dutch prime minister (March 13, 2017)[1]

1 Introduction

Policymaking in different countries is intertwined by information propagation, resulting in policy diffusion. While successful policies naturally diffuse, undesirable policies may also diffuse across countries.[2] Nowadays, there is a growing concern about the diffusion of policies that are perceived to be undesirable among experts in a form of populist extremism. Populist extremism in one country may induce it in another country, leading to the "domino effect" (Kaltwasser 2015). During the last thirty years, Latin American countries have experienced several waves of populism. Even in Europe, concerns toward the domino effects of populism are widespread. Our epigraph exemplifies them. Following the national referendum on Brexit and the presidential election of the United States, Dutch prime minister Mark Rutte expressed a concern for the domino-like contagion of populism. Motivated by these concerns, we investigate the diffusion of policies that are perceived to be bad among experts by focusing on the diffusion of populism.

While there are various sources of the diffusion,[3] the empirical literature has recently documented that voters' response to foreign countries' populism is a key, which is called the "demonstration effect" (Kaltwasser 2015). For instance, based on the data in Europe, Ezrow, Böhmelt, and Lehrer (2019) empirically find that the diffusion is induced by the response of the anti-immigration public opinion to the electoral success of anti-immigration parties in foreign countries. However, despite its empirical importance, the theoretical foundation is limited. Developing a formal model would be needed at least for two reasons. First, it promotes our understanding of the mechanisms behind the empirical patterns, which motivates new empirical specifications. Second, constructing a model is also indispensable in characterizing and predicting the diffusion pattern as long as a reduced-form empirical model does not fully capture the causal patterns of policy diffusion. The present study provides a theoretical background for the discussion on the demonstration effect.

In modeling the diffusion of populism through learning, the canonical social learning model (e.g., Banerjee 1992; Bikchandani, Hirshleifer, and Welch 1992) seems to be a good choice.[4] How-

[1] `https://apnews.com/e995dc2fb68549fbbc1e08fd0dab0376` (Last accessed: October 11, 2019)

[2] A historical example of the diffusion of bad policy is that of temperance legislation in the early 20th century. Although there was a well-known superior system for alcohol regulation, many countries such as the United States had adopted the bad policy – the prohibition law, which eventually failed (Schrad 2010).

[3] To explain the spread of far-right extreme parties in Europe, Rydgren (2005) focuses on the adoption of the new successful master frame due to learning by politicians.

[4] Social learning drives the diffusion of mass revolution in authoritarian politics (e.g., Chen and Suen 2016; Barbara and Jackson 2019). The literature of revolution does not take agency problems into account but emphasizes a coordination problem among citizens.

ever, it is unsatisfactory for our purpose from the following two aspects. First, the key assumption in this model is that the player who chooses a policy faces uncertainty about the state of the world. However, the policymaker in representative democracy is politicians who are familiar with policy issues. Hence, the social learning model cannot explain the diffusion of policies that are perceived as bad among experts. Second, populism is a product of the domestic political process. In representative democracy, the interests of voters and politicians do not necessarily coincide. Politicians may also care about their reputation and implement policies attracting the greatest support from voters even if such policies are sub-optimal (Ashworth 2012). The canonical social learning model does not contain such a domestic agency problem.

To overcome these limitations, we construct a social learning framework nesting a political agency model that extends Acemoglu, Egorov, and Sonin's (2013) model of populism. In this setup, we first provide the characterizations of the diffusion process caused by the interesting interactions between the learning process and the agency problem. We then predict the long-run dynamics of populism different from those obtained in the canonical social learning models. The theoretical novelty of our study is to provide a new framework of social learning with agency problems.

We begin our analysis with the single-country model. There are two types of politicians: the congruent type who shares the same policy preference with (decisive) voters; and the non-congruent type who has a biased policy preference. Voters do not know the incumbent politician's type. Besides, there is information asymmetry about the state of the world. In this setting, populist extremism could arise in the presence of high reputation concerns. The radical policy that is far from the non-congruent type's ideal policy serves as a signal of the incumbent being the congruent type. Hence, when voters have a concern that most politicians are biased, they support the politician arguing for the radical policy. Given this, the congruent-type politician argues for the radical policy even if it is not voter-optimal. In line with Acemoglu, Egorov, and Sonin (2013), we interpret this as populist extremism because an undesirably extreme policy is strongly supported by voters.[5] This is also consistent with the empirical fact that supporters of populists have a concern that there is a large political difference between politicians and themselves (Akkerman, Mudde, and Zaslove, 2014).

This emergence of extremism depends on voters' belief about the state of the world. We show that extremism arises if and only if the voters' subjective probability of the optimal policy being radical exceeds a certain threshold. In other words, the more radical the public opinion, the likelier it is to induce extremism. Importantly, this threshold value could be less than a half, implying that extremism arises even when voters believe that the radical policy is unlikely to be optimal. This distinguishes our results from the pandering equilibrium of Maskin and Tirole (2004) as their results are driven by the incentive of politicians to respect voters' belief about the optimal policy.

We then extend the model to a multi-country setting wherein the incumbent's policy choice and

[5] At least conceptually, it could be the case that populism does not entail extremism. However, in reality, we often observe the strong connection between them, and our model indeed shows such a connection.

the election are sequential across countries. This structure allows voters to learn the optimal policy through the policies previously implemented in other countries. Suppose that in one country extremism arises, leading the incumbent politician to argue for the radical policy. Since some fraction of the congruent type does not care for the reputation and sincerely supports the optimal policy, the next country's voters cannot rule out the possibility that the radical policy was implemented because it is indeed the optimal policy. Hence, voters upwardly update the probability of the radical policy being desirable. This radicalization of public opinion, in turn, induces extremism in the next country. As such, policies and public opinions are distorted by each other.

Given that populist extremism is contagious, a natural next question is whether its spread ends. Our model yields two implications on the dynamics of contagion. First, the domino effect of extremism may suddenly end. In particular, a moderate policy only in one country might be always enough to stop the domino effect, independently of past histories. This indicates that stopping populism in one country can indeed have a power to end the domino effect, which can justify Mark Rutte's appeal in our epigraph. Note that this is different from the prediction in the standard learning models such that a single bounded signal is not necessarily sufficient to change belief significantly.

Second, we show that the dynamics crucially depends on the correlation of the optimal policy across countries. When the optimal policy remains the same across countries, the spread must end in the long run, which indicates that the worst scenario–the permanent propagation—is rejected. However, the domino effect might be much more serious when each country's optimal policy is only imperfectly correlated, which seems practically relevant since the optimal policy may change across countries as well as across time. We introduce imperfect correlation by assuming that the optimal policies follow a Markov process without absorbing states. In this case, unlike the case of perfect correlation, populist extremism never ends. Strikingly, we show the possibility of the convergence to the extremism, wherein it is impossible to escape from extremism. Even when the convergence does not occur, the cycles of extremism always occur. Overall, it is more difficult to stop the contagion of populist extremism under the conditions of imperfect correlation. This result also contrasts the result in the canonical social learning models.

From a broader perspective, our model is a new model of social learning under the agency problem, wherein the dynamic interaction between principals' opinions and agents' actions creates the failure of social learning. The model can also be applied to non-political issues. A prominent example is the diffusion of dividend policies across firms, which is empirically shown to exist (Adhikari and Agrawal 2018). Shareholders face two information asymmetries: the executives' types and each firm's optimal dividend policy. Our result indicates that excessively high dividend payment might be contagious across firms because the executives signal that they act in line with shareholders' interests by choosing the high dividend payment.

The remaining paper is organized as follows. Section 2 discusses the related literature. Section 3 presents our model in a single-country setting and section 4 analyzes it. In section 5, the model is extended to a multi-country setting. Section 6 analyzes a situation wherein the optimal policies are imperfectly correlated. Section 7 provides additional discussions. Section 8 outlines our conclusions. All proofs are relegated to Appendix A.

2 Related Literature

Political agency problems and reputation concerns: Politicians' reputation concerns can force congruent politicians to argue for inefficient policies—Congruent politicians pander to public opinion and implement bad policies (e.g., Canes-Wrone, Herron and Shotts 2001; Maskin and Tirole 2004; Fox and Shotts 2009; Smart and Sturm 2013). This bad reputation effect arises even in non-political contexts (Ely and Välimäki 2003). In more striking cases, politicians arguing for a sub-optimal policy might attract support from voters even if the policy is perceived to be bad by voters. This extremism rather than simple pandering cannot be explained by the pandering literature. Acemoglu, Egorov, and Sonin (2013) show that the congruent politician chooses an extreme policy, which is known to be bad, to signal that s/he is a good politician. This can be interpreted as populist extremism.[6] Fox and Stephenson (2015), Matsen, Natvik, and Torvik (2016), Kartik and Van Weelden (2019), and Kasamatsu and Kishishita (2018) apply their mechanism. In addition, in the framework of a cheap talk game, Morris (2001) also presents a similar idea.

Although Acemoglu, Egorov, and Sonin (2013) provide novel insight on populist extremism, their model has no uncertainty about the voter-optimal policy, so there is no connection between public opinion and pandering. By introducing multiple countries and uncertainty about the state of the world, we succeed in connecting the possibility of extremism and public opinion.[7] This allows us to uncover the contagion of extremism through the dynamics of public opinion.

Policy diffusion and learning: Our study is related to the literature on policy diffusion through learning. Notably, in our study, what is learned and by whom are different from most existing theoretical studies. In previous studies, the government learns the outcome of policies through other countries' experiences (e.g., Volden, Ting, and Carpenter 2008; Buera, Monge-Naranjo, and Primiceri 2011; Callander and Harstad 2015). On the contrary, in our model, politicians know the state of the world, and instead, voters learn it.[8] Furthermore, voters face multidimensional

[6]Since populism is multifaceted, each study focuses on different aspects of populism. The literature on populism (e.g., Frisell 2009; Jennings 2011; Karakas and Mitra 2017; Kishishita 2017; Buisseret and Van Weelden 2018; Guiso et al. 2018) has not analyzed the diffusion process of populism.

[7]To analyze cheap talk messages in an election, Kartik and Van Weelden (2019) consider the model wherein the state of the world follows a continuous distribution. However, as a result of the focus difference, their model does not reveal a clear relationship between the possibility of extremism and voters' beliefs about the state of the world.

[8]In the literature of preliminary elections, voters' learning and associated information cascades are often discussed (e.g., Callander 2007). However, politicians' policy choice is exogenous; voters learn something not through politicians' actions but other voters' previous voting strategies (i.e., the results of previous elections).

uncertainty: the state of the world and the incumbent politician's type.

The significance of voters' learning is supported by many empirical studies. Most notably, Pachenco (2012) shows that neighboring states' policies affect public opinion, which in turn affects electoral outcomes and induces policy diffusion. She also reveals that in explaining policy diffusion, other channels such as politicians' learning are less important than voters' learning, at least in the context of tobacco control in the United States. We theoretically investigate this mechanism in detail and show that populist extremism may be propagated. The information propagation among voters can be observed in an international context as well (e.g., Kayser and Peress 2012).

Such voters' learning is partially investigated in the literature on yardstick competition (e.g., Besley and Case 1995; Belleflamme and Hindriks 2005). In yardstick competition models, voters observe the policies of other countries. However, governments decide policies only once, so there is no sequential learning and associated dynamics, which are essential to analyzing the domino effect. The study by Hugh-Jones (2009) is related. Hence, his model incorporates the dynamics of policies after voters' social learning. However, due to the difference in the focus, his model does not include extremism or sequential political decisions. As we discuss in Section 7.4, the welfare implications of yardstick competition may be reversed in our model.

Social learning: Our model provides a new framework capturing an important aspect of observational learning under agency problems (see Chamley (2004) for the background of social learning studies). In our model, agents who are aware of the state of the world choose policies, while principals learn the state of the world by observing the past policies. Hence, players who take actions perfectly know the state. Nonetheless, learning does not work well because of the agency problem. This contrasts the existing studies wherein players who take actions face uncertainty about the state of the world and this uncertainty creates the failure of learning.[9] Studies analyzing reputation concerns are also related (e.g., Scharfstein and Stein 1990; Ottaviani and Sørensen 2001). Also in this framework, players who take actions face uncertainty. Then, they try not to undermine their reputations by arguing for a potentially wrong policy, thus resulting in herding.

This structure of our model yields two important properties absent in the canonical social learning models. First, the domino effect suddenly stops due to the discontinuous jump of voters' beliefs. In the standard models, such jump arises only when signals are unbounded (Smith and Sørensen 2000). Despite bounded signals, we show that strategic interactions create paradigm shifts through the endogenous change in signal strength. Chen and Suen (2016) consider a social learning model with a global game. Then, when players face model uncertainty, such paradigm shifts might occur because of the endogenous change in signal strength. However, the mechanism inducing the endogenous signal strength is different. As agency problems are prevalent in reality, we believe that

[9] In the analysis of riots, Lohmann (2000) reveals that players' signaling motives create information cascades, though signaling motives come not from agency problems but collective action problems.

our model enlarges the possibility of sudden paradigm shifts in social learning.

The second difference is about the case wherein the states are only imperfectly correlated in a Markovian manner. Moscarini, Ottaviani, and Smith (1998) and Nelson (2002) extend the canonical social learning model in this direction.They show that the more likely the state of the world is to change, the herding period is sustained during the shorter period. In stark contrast, we find that the opposite is the case.

3 Model Setup

There are $N \in \mathbb{N}$ countries ($i = 1, ..., N$). For each country, there is an incumbent politician and a decisive voter.[10] Hereafter, we call the incumbent politician (the decisive voter) in country i politician i (voter i). Each incumbent politician sequentially chooses a policy. At the beginning of period i, politician i chooses policy x_i from $X = \{0, 1, 2\}$ given the policies implemented before period i by the other countries $(x_1, ..., x_{i-1}) \in X^{i-1}$. Then, voter i evaluates politician i based on $(x_1, ..., x_i) \in X^i$. This evaluation is denoted by π_i.

3.1 State-Dependent Policy Rankings

The optimal policy for voters depends on the state of the world. Let $\omega_i \in \Omega \equiv \{1, 2\}$ be the state of the world in country i, which indicates the optimal policy for voter i. Politician i knows the value of ω_i, while voter i does not know its value. When $\omega_i = k$, the policy optimal for voter i is k. Hence, policy 0 is never desirable for voters, whereas the other two policies can be appropriate. We consider single-peaked preferences so that policy 1 is close to policy 0, while policy 2 is the opposite of policy 0. Hence, we refer to policy 0 as *the non-congruent policy*, policy 1 as *the moderate policy*, and policy 2 as *the radical policy*.[11]

Note that we assume that the non-congruent politicians and voters always have different tastes—That is, the ideal policy of the non-congruent type is policy 0.[12] Such a situation naturally arises when politicians come from special interest groups, such as economic elites. For example, motivated by Latin-American experience, Acemoglu, Egorov, and Sonin (2013) consider a situation wherein elites seek different policies than the average voter and elite groups might gain influence

[10] The incumbent could also be interpreted as choosing a policy platform. This interpretation is plausible because voting behavior is affected by campaign promises and breaking them are often costly. Note also that voters' heterogeneity is allowed for as long as the median voter theorem holds.

[11] The assumption that the non-congruent policy is located at the corner of the policy space is not crucial. By expanding the policy space to $X \equiv \{-2, -1, 0, 1, 2\}$, we can show that the same results hold even if the non-congruent policy is policy 0. The formal argument is available upon request.

[12] It is important to note that the asymmetry between the numbers of states and policies is not fundamental to our main conclusions. Our main contagion result appears as long as the non-congruent type and the congruent type have sufficiently different preferences (available upon request). However, asymmetry induces an interesting phenomenon of "tyranny," wherein the non-congruent type is less disciplined by the radicalization of public opinion.

over a politician through bribery. We extend this situation, so that the average voters' ideal point may change depending on economic and social conditions, but voters always deem that what elites demand is undesirable.

This situation is consistent with empirical observations of populism. For example, in designing a survey to measure populist attitudes in Europe, Akkerman, Mudde, and Zaslove (2014: 1330-1331) state "The focus of the questions is on the three core features of populism: sovereignty of the people, opposition to the elite, and the Manichean division between 'good' and 'evil.'" In our model, citizens want to choose a congruent ("good") politician representing them, rather than a non-congruent ("evil" or "elite") politician seeking a policy goal different from citizens' one.

3.2 Politicians' Types

There are two types of politicians: *the congruent type* and *the non-congruent type*. Voter i does not know the type of politician i.

The payoff of the congruent type in country i is given by $-L(|x_i - \omega_i|) + b_i \pi_i(x_i)$, where L is the loss due to policy mismatch, $\pi_i(x_i)$ is the updated belief voter i holds at the end of period i about the probability of the incumbent i being the congruent type given the implemented policy x_i.[13] The non-linearity of the utility in terms of π can be incorporated without fundamentally changing the analysis. The congruent type shares the same policy preference as the voter (i.e., the ideal policy is ω_i). In this regard, this type of the politician is a good politician. Note that L is assumed to be strictly increasing because we consider single-peaked preferences. As for normalization, $L(0) = 0$ and $L(1) = 1$. Furthermore, $L(2) = l > 2$.

On the contrary, the payoff of the non-congruent type in country i is given by $-L(|x_i|) + b_i \pi_i(x_i)$. Hence, the ideal policy for the non-congruent type is policy 0. Since policy 0 is never desirable for the voter, this politician is a bad type.

Politician i's reputation is severely undermined if their policymaking makes voter i believe that politician i is likely to be the non-congruent type and prefer policies that are undesirable to voters. The low reputation might damage the quality of the post-political life or the incumbent's soft legacy (Fong, Malhotra, and Margalit 2019). In addition, when the incumbent has a chance in the next term, his/her low reputation would prevent reelection. Such reputation concerns are added in the above payoff functions as the last term $b_i \pi_i(x_i)$. In section 7.2, we extend our model to a two-period election model and interpret reputation concerns as reelection concerns.

Here, $b_i \geq 0$ is the intensity of reputation concerns. The non-congruent types are assumed to have strong reputation concerns because they are self-interested. $b_i = b > 0$ for the non-congruent

[13] The evaluation by voter i does not depend on policies implemented after politician i's policy choice, and thus politicians are not forward-looking. This assumption is useful for tractability and often assumed in the social learning literature. Furthermore, this situation is natural because politicians typically care about the reputation in the near future, possible for the success in the coming election.

type. On the contrary, some congruent politicians might only have low reputation concerns and always implement the policy that is optimal for voter i (i.e., some politicians might be so-called statesmen). To capture this, we assume that the congruent type is divided into *the congruent type H* and *the congruent type L*. The former type has high reputation concerns similar to the non-congruent type: $b_i = b$. In contrast, the latter type is assumed to have no reputation concern $b_i = 0$. The voter does not know whether the congruent type is H or L. In country i, the incumbent is the congruent type H with probability $q_H \in (0,1)$, the congruent type L with probability $q_L \in (0,1)$, and non-congruent with probability $1 - q_H - q_L$. We define $q \equiv q_L + q_H$. $q \in (0,1)$ is also assumed. Politicians' types are independently determined across countries. Denote this type space by $T_i \equiv \{H, L, N\}$ wherein H (L) represents that politician i is the congruent type H (L) and N represents that s/he is non-congruent.

An alternative setting is to introduce the heterogeneity of reputation concerns by allowing b to be continuously distributed. The pure-strategy equilibrium in this alternative setting replicates our main qualitative predictions in mixed strategy equilibria.

3.3 Timing of the Game and Equilibrium Concept

The timing of the game is summarized as follows. In period i,

1. Nature chooses ω_i and politician i's type. Only politician i observes them.
2. Politician i chooses a policy x_i.
3. Voter i updates the belief on the incumbent's type $\pi_i(x_i)$.
4. Politician i's payoff is realized.

The equilibrium concept is a (mixed strategy) perfect Bayesian equilibrium.

4 Equilibrium: Single-Country Model

We start with the case wherein $N = 1$. This benchmark provides a useful framework to consider the mechanism that induces the spread of extremism. Let the ex-ante probability that $\omega_1 = 1$ be $p_1 \in (0,1)$. Without notational abuse, we omit the subscript "1" that represents country 1.

To focus on meaningful cases wherein extremism could arise, we suppose the following:

Assumption 1. $b \in (2, l)$.

That is, the reputation concerns of the congruent type L are sufficiently low, while those of the other types are relatively high. This is the sufficient condition to guarantee that extremism could arise.[14]

We allow players to mix actions. Let $\alpha^*(x;\omega)$ be the equilibrium probability that the congruent type H chooses x when the state is ω. Similarly, let $\beta^*(x)$ be the equilibrium probability that the non-congruent type chooses x and let $\gamma^*(x;\omega)$ be the equilibrium probability that the congruent type L chooses x when the state is ω.[15]

4.1 Equilibria

4.1.1 Characterization

We obtain the following characterization of equilibria.

Proposition 1. *(a) There is an equilibrium wherein $\alpha^*(2;\omega)=1$, $\beta^*(1)>0$, and $\beta^*(0)>0$, if and only if $p<\frac{1-q}{(b-1)q_L}$. In this equilibrium, $\beta^*(1)=\frac{(b-1)pq_L}{1-q}$; $\beta^*(0)=1-\beta^*(1)$. We refer to this as (Ex1) equilibrium.*

(b) There is an equilibrium wherein $\alpha^*(2;\omega)=1$ and $\beta^*(1)=1$, if and only if $\frac{1-q}{(b-1)q_L}\leq p\leq\frac{(b-1)(1-q)}{q_L}$. We refer to this as (Ex2) equilibrium.

(c) There is an equilibrium wherein $\alpha^*(2;2)=1$, $\alpha^*(1;1)>0$, $\alpha^*(2;1)=1-\alpha^*(1;1)$, and $\beta^*(1)=1$, if and only if $\frac{(b-1)(1-q)}{q}\leq p\leq\frac{(b-1)(1-q)}{q_L}$. In this equilibrium, $\alpha^*(1;1)=\frac{(b-1)(1-q)}{pq_H}-\frac{q_L}{q_H}$. We refer to this as (Ex3) equilibrium.

(d) There is an equilibrium wherein $\alpha^*(\omega;\omega)=1$ and $\beta^*(1)=1$, if and only if $p\geq\frac{(b-1)(1-q)}{q}$. We refer to this as (NEx) equilibrium.

(e) There is no other equilibrium.

Some of the equilibria in this proposition have an interesting feature called *populist extremism* or *extremism* (used interchangeably). To highlight this, let X^* be the set of policies that can occur with a positive probability in an equilibrium: $X^*(\omega)\equiv\{x\in X:\alpha^*(x,;\omega)+\beta^*(x)+\gamma^*(x;\omega)>0\}$.

Definition 1. *An equilibrium $(\alpha^*,\beta^*,\gamma^*,\pi^*)$ is called the (populist) extremism equilibrium if for some ω, $X^*(\omega)\setminus\{\omega\}\neq\emptyset$ and $\pi^*(x)=1$ for some $x\in X^*(\omega)\setminus\{\omega\}$.*

[14] As argued later, a reputation concern may make a congruent politician choose an inefficient but popular policy, which we define as populist extremism. Too low reputation concern kills such an incentive. Too high concern always induces such populist extremism, preventing us from meaningfully characterizing populist extremism.

[15] To be precise, we restrict our attention to equilibria wherein each player's equilibrium strategy depends on only payoff-relevant information for him/herself. Hence, β^* does not depend on ω. This is consistent with the equilibrium concept in section 5. Furthermore, Proposition 1, the key characterization, still holds even if we allow β to depend on ω. Alternatively, if we assume that the non-congruent type does not know ω, all strategies satisfy this property.

If an equilibrium is not the extremism equilibrium, we call it *the non-extremism equilibrium.* It is unsurprising that some politicians argue for extreme (i.e., undesirable) policies because they have a biased ideology. However, what we observe in the proliferation of populism is a more paradoxical situation wherein the politician who chooses extreme policies obtains strong voter support. Populist extremism represents this paradoxical phenomenon. In the extremism equilibrium, some politicians choose a policy different from the voter-optimal policy. Nonetheless, their reputations are bolstered (i.e., $\pi^* = 1$), and they get re-elected or enjoy pleasant post-political life.

(Ex1)–(Ex3) equilibria are extremism equilibria because given Proposition 1, the requirement for the extremism equilibrium is satisfied if and only if $\omega = 1$ and policy 2 is chosen with positive probability. To pretend to be the congruent type, the non-congruent type chooses policy 1, whereas the non-congruent type never chooses policy 2. Hence, in order to separate himself/herself from the non-congruent type, the congruent type H, who has high reputation concerns, implements the radical policy (i.e., policy 2), even if the radical policy is not the voter-optimal (i.e., extremism arises). Furthermore, the politician who argues for the radical policy indeed acquires high reputation because it is a good signal of the incumbent's type. As such, extremism with strong support by voters arises. This mechanism is analogous to that of Acemoglu, Egolov, and Sonin (2013).

Notably, (NEx) is the non-extremism equilibrium. In this equilibrium, the non-congruent type chooses the non-optimal policy when $\omega = 2$. In this sense, an extreme policy could be implemented. However, the politician who chooses it does not bolster his/her reputation. Furthermore, the congruent type always chooses the voter-optimal policy. Hence, nothing is paradoxical, and this equilibrium is the non-extremism one. Distinguishing from extremism, we refer to the "bad behavior" by the non-congruent type as *tyranny* based on the analogy that citizens do not wish to elect a tyrant, but they may fail to distinguish a bad politician whose support will lead them to suffer from tyranny. Using this terminology, we can say that, in the equilibrium, extremism does not occur while tyranny exists.

In summary, we have the following property.

Fact 1. *All equilibria except for the (NEx) equilibrium are extremism equilibria.*

We comment on two key assumptions that induce extremism, which we believe reflect real aspects of populist extremism. The first key assumption is the existence of the non-congruent type (i.e., corrupt politicians). In our model, voters dislike a politician with different preferences from them, which leads the congruent politicians to distort policies in order to signal their aligned preferences with voters. This situation can be interpreted that voters do not want to elect an elite politician whose preferences are very different from the common people, and they strongly support populists precisely because the populists are expected to have close preferences. This well explains the aspect of populism as anti-elitism[16] and the empirical observation that voters who distrust the established politicians support populists in Europe (e.g., Akkerman, Mudde, and Zaslove 2014).

[16]For instance, Mudde (2004: 543) defines populism as "an ideology that considers society to be ultimately separated

The second assumption is that the congruent type has reputation concerns. Without high reputation concerns, the congruent type never distorts policies—That is, extremism does not arise. This implies that populists in our model choose extreme policies for opportunistic rather than ideological reasons. While, at first glance populists' motivations seem to be ideological, some scholars have argued that populists are primarily opportunistic. For instance, Weyland (2017: 62) states that "populism tailors its appeals in opportunistic ways to maximize the leader's chances of capturing the government." That is, populists' policies are chosen in terms of how to attract voters.[17] In these aspects, the underlying mechanism of populism in our model reflects the reality.

4.1.2 Relationship with Public Opinion

The equilibrium characterization can be understood in a simple way by focusing on the relationship with public opinion. As a corollary of Proposition 1 and Fact 1, we obtain the following:

Corollary 1. *The non-extremism equilibrium exists if and only if* $p \geq \bar{p} \equiv \frac{(b-1)(1-q)}{q}$.

Hence, the more strongly voters believe that the radical policy is the good policy, the more likely populist extremism is to arise. This characterization can be seen in Figure 1.[18] The intuition can be understood as follows. Suppose that sufficiently high reputations are maintained under policy 1. Then, even if policy 2 signals that the incumbent is the congruent type, the congruent type H might choose policy 1 when $\omega = 1$. Hence, whether the non-extremism equilibrium exists depends on whether high reputations are maintained under policy 1.

Whether high reputations are maintained under policy 1 is in turn dependent upon the voters' beliefs about the state of the world. To observe this, first assume the non-extremism equilibrium. In this equilibrium, the posterior of the incumbent being the congruent type given policy 1 is

$$\pi(1) = \frac{pq}{pq + 1 - q}.$$

Since the congruent type chooses policy 1 only when it is voter-optimal, this updated belief is increasing in p. Hence, when p is relatively low, choosing policy 2 is beneficial for improving reputation even when ω is actually 1, leading to an extremism equilibrium. However, note that p does not have to be less than 1/2, and thus even when voters believe that $\omega = 1$ is more likely, politicians may choose policy 2. This contrast to the pandering literature (Maskin and Tirole 2004)

into two homogeneous and antagonistic groups, 'the pure people' versus 'the corrupt elite,' and which argues that politics should be an expression of the volonté générale (general will) of the people."

[17] By quantitatively analyzing the recent U.S. elections, Gennaro, Lecce, and Morelli (2019) find that politicians including the president Donald Trump have rationally used populist arguments as a strategic vote-gaining tool, which suggests that the supply of populism is often strategic.

[18] Here, we assume that the non-extremism equilibrium arises if it exists. This is consistent with the equilibrium concept introduced in section 5.1.

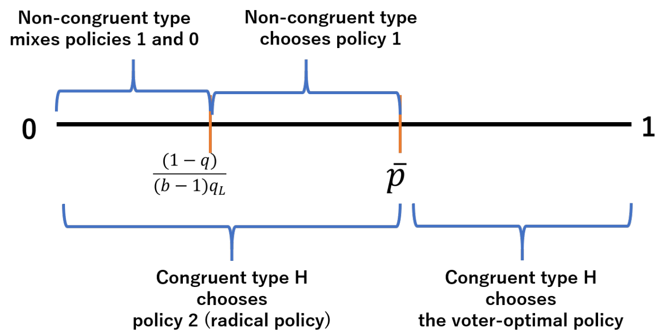

Figure 1: **Characterization of single-country equilibria.**

Notes: This is the case for $\frac{1-q}{(b-1)q_L} < \bar{p}$.

comes from the fact that the policy attracting the support by voters is always the radical policy in our model, whereas the radical policy attracts a lot of support only when $p < 0.5$ in the literature.

We remark on the equilibrium when $p = 0$. So far, we have explored the case wherein $p \in (0,1)$. However, as seen later, $p = 0$ could be the case in the dynamic model.

Lemma 1. *When $p = 0$ and $\omega = 2$, $\alpha^*(2;2) = 1$ and $\beta^*(0) = 1$ in any equilibria.*

5 Equilibrium: Multi-Country Model

We have shown that voters' beliefs regarding the state of the world determine whether extremism arises. However, this is not the entire process. The equilibrium behaviors of politicians should affect voters' beliefs. Hence, an interaction between public opinion dynamics and policy dynamics is expected. This section thus analyzes these interactions.

To this end, we consider the case wherein $N \geq 2$.[19] In addition, we suppose that the state of the world is at least imperfectly correlated across countries. To capture this, we assume the following Markovian transition of states. For all i and for each $j \in \{1,2\}$,

$$\Pr(\omega_{i+1} = j | \omega_i = j) = \theta_j \in (1/2, 1). \tag{1}$$

$\theta_1(\theta_2)$ represents the stability of the state 1 (2). The values of θ_1 and θ_2 are known to voters as well as politicians. When $\theta_1 = \theta_2 = 1$, every country's state of the world is the same, while otherwise, the state of the world is only imperfectly correlated. In this section, we suppose that $\theta_1 = \theta_2 = 1$ (i.e., the state of the world is the same across countries) to fix the idea in the simplest case. We analyze the case where ω_i varies across countries in section 6.

[19]Though we mainly consider the situation wherein each i is a different country, our model can analyze the domino effect of populist extremism overtime within one country.

5.1 Equilibrium Concept and Updates of Beliefs

First, let us formally define the equilibrium concept. Define *public history* at the beginning of period i by $h^i \equiv (x_1, ..., x_{i-1}) \in H^i \equiv X^{i-1}$. Politician i's strategy is given by $s_i : H^i \times \Omega \times T_i \to \Delta(X)$. $(p_i, 1-p_i) \in \Delta(\Omega)$ denotes the belief that voter i attaches to state 1 and 2. Our equilibrium concept is the voter-optimal Markov perfect Bayesian equilibrium:

Definition 2. $(s_i^*, \pi_i^*, p_i^*)_{i \in \{1,...,N\}}$ *constitutes an equilibrium if*

(i) They constitute a perfect Bayesian equilibrium;

(ii) For the congruent type, $s_i^* : \Delta(\Omega) \times \Omega \times T_i \to \Delta(X)$ and for the non-congruent type, $s_i^* : \Delta(\Omega) \to \Delta(X)$—That is, the equilibrium strategy of politician i depends only on their type and voter i's belief p_i, and in the case of the congruent type, it also depends on the state of the world; and

(iii) Given $p_i^* \in [0,1)$, (s_i^*, π_i^*) is the voter-optimal equilibrium in the static model.

(i) and (ii) imply the Markov perfect Bayesian equilibrium.When $p_i \geq \bar{p}$, there are multiple equilibria. (iii) implies that the equilibrium in each stage game is the voter-optimal one.[20] This selection is reasonable from the following two perspectives. First, we typically focus on the principal–optimal equilibrium in the analysis of agency problems, and the voter in our model corresponds to the principal.[21] Second, when there are multiple equilibria, the voter-optimal equilibrium is the non-extremism equilibrium. Hence, our selection is equivalent to the selection of the non-extremism equilibrium, if it exists. This is a conservative analysis of how likely extremism is to proliferate because we focus on the case wherein extremism is least likely to arise.[22]

Lastly, we assert the following assumption.

Assumption 2. $\bar{p} < 1$.

Combined with (iii) in Definition 2, this assumption guarantees that $p \in [0,1)$ exists, such that the non-extremism equilibrium arises. Without this condition, extremism by definition always arises because all equilibria are extremism equilibria.

[20]We cannot conduct the equilibrium selection using criteria such as the intuitive criterion because the multiplicity of equilibria does not occur based on off-path belief formation. Such multiplicity of equilibria sometimes arises in political agency problems with the finite action space (e.g., Fox and Shotts 2009).

[21]An example focusing on the voter-optimal equilibrium in the analysis of political agency problems is Forand (2015).

[22]The key for our results is that there exists a threshold value of p, such that the equilibrium is extremism if and only if p is less than that value. Hence, the wider class of equilibria indeed give us the almost same results. To illustrate, denote the equilibrium probability of policy 2 being implemented in country i given p_i by $R^*(p_i)$. We then define *the monotonic Markov perfect Bayesian equilibrium* by the equilibrium wherein (i) and (ii) are satisfied, while (iii') $R^*(p)$ is weakly decreasing in p (i.e., monotonicity holds). As long as we consider the monotonic Markov perfect Bayesian equilibrium, we obtain similar results.

By focusing on the voter-optimal Markov perfect equilibrium and imposing Assumption 2, the equilibrium of country i can be summarized as in Figure 1. Extremism arises if and only if the belief p_i is sufficiently small. Moreover, small p_i might also induce tyranny by making the non-congruent type take policy 0. Under $\omega = 1$, extremism and tyranny are the causes of inefficient policymaking.

Having determined which equilibrium to arise for each belief p, we can discuss voters' learning processes by observing a foreign policy. We assume that $p_1 \in (0,1)$. When $p_{i-1} \in (0,1)$, the updated belief p_i ($i \geq 2$) is given recursively based on the Bayes rule:[23]

$$p_i(x_1,...,x_{i-2},1) = \begin{cases} \frac{1+(b-1)p_{i-1}}{b} \ ((Ex1)\ equilibrium) \\ \frac{p_{i-1}(q_L+(1-q))}{p_{i-1}q_L+(1-q)} \ ((Ex2)\ equilibrium) \\ \frac{p_{i-1}}{p_{i-1}q+(1-q)} \ ((NEx)\ equilibrium) \end{cases};$$

$$p_i(x_1,...,x_{i-2},2) = \begin{cases} 1 - \frac{(1-p_{i-1})q}{(1-p_{i-1})q_L+q_H} \ ((Ex1)\ and\ (Ex2)\ equilibria) \\ 0 \ ((NEx)\ equilibrium) \end{cases};$$

$$p_i(x_1,....,x_{i-2},0) = p_{i-1} \ ((Ex1)\ equilibrium).$$

Using the above belief updating rule implies that voters learn from a foreign policy, but not from the realized welfare outcomes from it. We believe that our assumption is useful as a benchmark because determining the welfare consequence is generally difficult and, even when it is possible, takes much time. Observing the welfare outcomes would facilitate the learning of the true state and weaken the biased learning from distorted policy choices.

5.2 Spread of Extremism

We show that populist extremism is contagious due to the interaction with the public opinion. Since the radical policy induced by populist extremism is problematic when the voter-optimal policy is the moderate policy, we assume that $\omega = 1$ in this section.

Proposition 2. *Suppose $\omega = 1$.*

(a) Fix $k \in \{1,...,N-1\}$. For $i \in \{k+1,...,N\}$, $\Pr(p_i \geq \bar{p})$ is weakly increasing in p_k.

(b) Suppose that $p_1 < \bar{p}$. For $x \in \{0,1\}$ that is on-path and $i \in \{2,...,N\}$, $\Pr(p_i \geq \bar{p}|x_1 = 2) \leq \Pr(p_i \geq \bar{p}|x_1 = x)$. In addition, there exists $p_1 \in (0,\bar{p})$, such that for $x \in \{0,1\}$ that is on-path and $i \in \{3,...,N\}$, $\Pr(p_i \geq \bar{p}|x_1 = 2) < \Pr(p_i \geq \bar{p}|x_1 = x)$.

(a) states that extremism arises in subsequent countries with a higher probability as the public opinion in country k is more radical. This implies (b), indicating the domino effect of extremism.

[23] We do not present p_i for (Ex3) equilibrium because it does not occur according to our equilibrium concept.

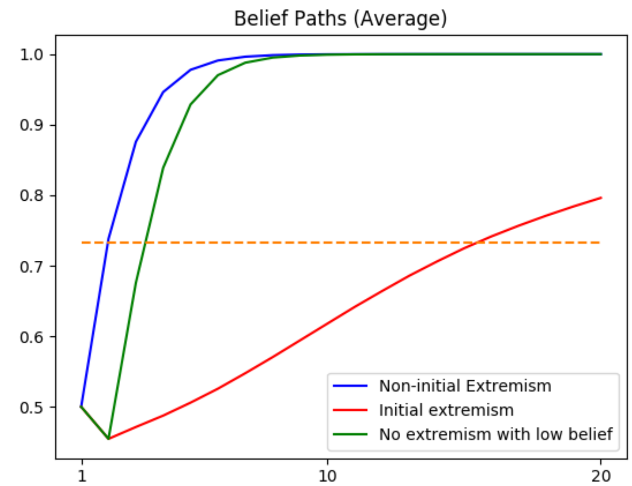

Figure 2: **Contagion of populist extremism.**

Notes: The parameter values are $l = 4$, $p_1 = 0.5$, $q_H = 0.5$, $q_L = 0.1$, $\omega = 1$, and $b = 2.1$.
The blue (red) line describes the dynamics of beliefs when $x_1 = 1(2)$. The counter-factual green line describes the dynamics of beliefs when $x_1 = 2$ and the equilibrium is always the non-extremism equilibrium independently of p. Following Chen and Suen (2016), for each case, we simulate the equilibrium for 100,000 iterations and obtain the path of p_i. We then calculate the average for each i and obtain the average path of p_i.

In particular, (b) argues that, given country 1 is in the extremism equilibrium, whether the radical policy is implemented in country 1 affects the probability of each subsequent country being in the extremism equilibrium. In particular, the implementation of the radical policy in country 1 induces extremism in subsequent countries. While we do not specify why country 1 is in the extremism equilibrium, in section 7.1, we argue that shocks to political distrust in country 1 may induce the extremism equilibrium in country 1 and leads to the contagion of extremism.

To observe contagion process in more detail, we analyze a numerical example. In Figure 2, we present the respective average path of beliefs p_i when country 1 implements policy 1 (blue) and 2 (red). It also shows the green path denoting the scenario wherein county 1 takes policy 2, but the non-extremism equilibrium is hypothetically assumed to be realized in the subsequent countries.

A comparison of the blue and red lines reveals that country 1's policy crucially affects the contagion of extremism. If the country does not implement the radical policy, the extremism equilibrium ends relatively soon. On the contrary, the implementation of the radical policy propagates throughout many countries and induces extremism. The comparison between the red and green lines shows that the effect of the radical policy in country 1 is not limited to the change in country 2's public opinion. Recall that the green line is the hypothetical scenario wherein the non-extremism equilib-

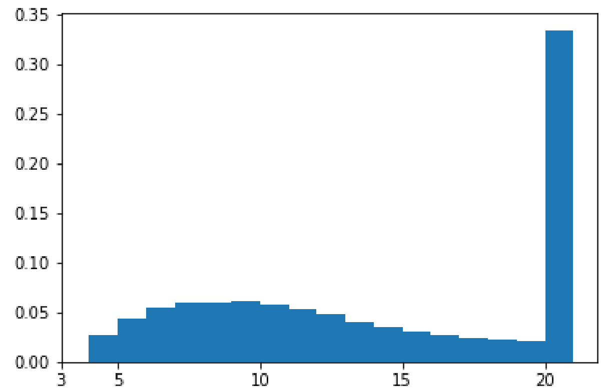

Figure 3: **When the domino stops.**

Notes: This histogram shows the frequency that the first country out of the extremism is i. If the extremism equilibrium continues for all $i \leq 20$, it shows 21. It corresponds to the red line (i.e., policy 2 in country 1) and the parameter values are $l = 4$, $p_1 = 0.5$, $q_H = 0.5$, $q_L = 0.1$, $\omega = 1$, and $b = 2.1$. We simulate the equilibrium 100,000 times to obtain the frequency.

rium is always taken, provided that the belief at the beginning of period 2 is equal to that of the red line. In this case, the spread of populism immediately ends. However, this does not happen in the actual equilibrium. When the radical policy is chosen in country 1, country 2 is also captured in the extremism equilibrium, which, in turn, induces distorted learning in the subsequent countries. Hence, the red line is far different from the green line.[24]

The severity of the contagion of extremism might be more pronounced once we recognize the possibility of a long-lasting domino effect. Figure 3 shows the country at which the extremism stops for the first time in the case of the red line in Figure 2. When the extremism occurs in all countries $i \leq 20$ and the extremism occurs for country $i = 21$, the value is shown as 21.[25] Figure 3 shows that the domino effect might be so strong that the extremism does not stop until 20 countries.

Note that from a different perspective, Proposition 2 indicates the hysteresis effect, such that country 1's politician's type affects the other subsequent countries' policy choice. In Figure 2, when country 1's policymaker is the congruent type L, the subsequent countries do not suffer from extremism. On the contrary, when the politician is the congruent type H (i.e., a populist), the subsequent countries are also likely to face populism due to the long-lasting negative externality.

The resulting spread has a substantially negative impact on welfare. Note that, in the extremism

[24]The immediate termination of extremism in the hypothetical scenario is mostly because policy 1 is taken with probability 1 when $\omega = 1$. Formally, under the hypothetical scenario, we can prove that there exists $\bar{N}$, such that for all $i \geq \bar{N}$, $\Pr(p_i \geq \bar{p} | x_1 = x) = 1$ for $x \in \{0, 1, 2\}$. That is, the implemented policy in country 1 has no long-run effect in terms of whether the belief exceeds the threshold $\bar{p}$.

[25]In this numerical example, after policy 2 was implemented in country 1, country 3 must experience extremism even if country implements policy 1. On the other hand, if country 1 implements policy 1 (i.e., the blue scenario in Figure2), country 2 does not experience extremism.

equilibrium, both populist extremism and tyranny take place and both are detrimental to welfare. In the extreme case of $p_i \simeq 0$, the congruent H type always takes policy 2, while the non-congruent type almost always takes policy 0. The contagion of populist extremism induces a malfunctioning democracy in both respects.

5.3 When the Domino Effect Stops: Paradigm Shift

The next natural question is how likely it is to stop. We show that, in contrast to standard herding models, under a certain condition, the contagion suddenly stops for *any* belief. We call this situation the "paradigm shift." We again suppose that $\omega = 1$.

Proposition 3. *Suppose $\omega = 1$. When $(b-1)b \leq \frac{q}{1-q}$, for any i and any history h^i, which can occur on the equilibrium path, such that $p_i(x_1, \ldots, x_{i-1}) < \bar{p}$, $\Pr(p_{i+1} \geq \bar{p} | p_i) > 0$.*

Hence, under certain conditions, even if the public opinion is too radical (i.e., p is small), the voter stops believing that the radical policy is optimal after observing the moderate policy (i.e., p becomes large); thus, the politicians no longer choose the radical policy. As such, extremism stops. That is, a paradigm shift from extremism to non-extremism suddenly occurs, even if the degree of extremism is severe. The paradigm shift is surprising. To observe it, let us contrast our result to the following updating process. The state space is $\Omega = \{1, 2\}$, and the voter receives a signal about the state of the world: $s \in \{1, 2\}$, where $\Pr(s = \omega) = \alpha \in (1/2, 1)$. Then, the likelihood of the posterior p' given $s = 1$ is

$$\frac{p'}{1-p'} = \frac{p}{1-p} \frac{\alpha}{1-\alpha}, \tag{2}$$

using the prior p. Hence, as $p \to 0$, $p' \to 0$, indicating that the result in Proposition 3 never holds. Our setting is similar with this process. The moderate policy is just an imperfect signal that $\omega = 1$, and the voters are Bayesian rational. In our model, even though the politician observes the true state, policy 1 is always an imperfect signal for voters. Nonetheless, we have Proposition 3.

Strategic interactions play a key role in triggering the paradigm shift. When p is sufficiently small, the non-congruent type mixes policies 1 and 0. In particular, the probability that this type chooses policy 1 is increasing in p.[26] This implies that the smaller p is, the higher the precision of policy 1 is as the signal because, in the extremism equilibrium, policy 1 is implemented either by the non-congruent type under $\omega = 1, 2$ or the congruent L under $\omega = 1$. Hence, α in (2) is decreasing in p. In particular, $\alpha \to 1$ as $p \to 0$. As a result, even if $p \to 0$, p' does not converge to 0.

[26] Intuitively, a smaller p means that the reputation that the non-congruent type acquires by implementing policy 1 is lower. Hence, the non-congruent type implements policy 0 with a higher probability.

5.4 The Domino Effect in the Long-Run: Asymptotic Learning

While extremism is contagious—at least in the short-term, such a contagion can suddenly stop due to the paradigm shift.

The following proposition shows that voters can learn the (invariant) state of the world in the long run, eliminating the possibility of perpetual extremism due to a single shock.

Proposition 4. *(a) Suppose* $\omega = 1$. *Then,* $\Pr\left(\lim_{N\to\infty} p_N = 1\right) = 1$.

(b) Suppose $\omega = 2$. *Then,* $\Pr\left(\lim_{N\to\infty} p_N = 0\right) = 1$.

Voters try to learn the state of the world only through politicians' distorted policies. Nonetheless, this proposition argues that voters eventually learn the truth. Hence, at least in the long-term, politicians' extremism does not influence voters to wrongly believe that the radical policy is good. Furthermore, since extremism does not arise when p is close to one, the spread of extremism eventually stops when the optimal policy is the moderate one. That is, the contagion of extremism does not last forever when the radical policy is not good.

The key is the existence of the congruent type L, which can be arbitrarily small. Such politicians sincerely implement the voter-optimal policy, which allows information about the state of the world to be partially transmitted to voters. Consequently, voters learn the truth asymptotically.[27] That is, the existence of politicians who sincerely implement the voter-optimal policy prevents the domino effect of populism from continuing forever. Notably, any arbitrarily small fraction of the congruent type L is enough for the asymptotic result.

The exact fraction of the congruent type L does not matter for the asymptotic result. Yet, it certainly affects the stopping time of the domino effect. To illustrate this, let us first consider the probability that the domino effect stops in country $i+1$, given p_i. This is indeed decreasing in the fraction of the congruent type L.

Fact 2. *Suppose* $\omega = 1$. *Fix* q. *For any* $p_i \in (0, \bar{p})$, $\Pr(p_{i+1} \geq \bar{p})$ *is decreasing in* q_L.

To observe further, in Table 1, we present the frequency of long-lasting extremism (i.e., extremism continues at country $i = 21$), given the radical policy in country 1. The results show that long-lasting extremism occurs much more often when q_L is smaller.[28] These results together indicate that the domino effect is less likely to end with less congruent type L politicians. It should be emphasized that the asymptotic property does not mean that the spread of extremism is irrelevant. Particularly in international contexts, the number of countries that share the same state of the world may not be large. The short-term effect is still important, as seen in Table 1.

[27] Using a different model, Goeree, Palfrey, and Rogers (2006) also find that social learning is successful in the long-term. They extend the standard social learning model à la Banerjee (1992) so that each player's payoff consists of the common value, which depends on the state of the world as well as the individual value.

[28] For various parameter values, we obtain the same relationship.

υ	0.8	0.82	0.84	0.86	0.88	0.9	0.92	0.94	0.96	0.98
Frequency	0.133	0.157	0.196	0.232	0.274	0.299	0.332	0.389	0.52	0.718

Table 1: **Frequency of long-lasting extremism.**

Notes: The table shows the frequency that extremism takes place in country 21. Let $\upsilon \in (0,1)$ be the parameter, such that $q_H = \upsilon q$ and $q_L = (1-\upsilon)q$. By changing υ, we investigate the change in q_L, keeping q fixed. The parameter values are $l = 4$, $q = 0.65$, $\omega = 1$, $b = 2.1$, and $p_1 = 0.4$. We suppose that policy 2 is implemented in country 1. We simulate the economy 100,000 times in calculating the frequency. All numbers are rounded up to three decimal places.

In contrast to standard models of social learning, learning may not improve welfare even though the true information is eventually learned. This is because we have two sources of welfare loss: the populist extremism and the tyranny. When p is too low, voters' welfare is the lowest because the voter cannot discipline the non-congruent type. However, when $\omega = 2$, p goes to zero as a result of social learning. Thus, for $\omega = 2$ and sufficiently large N, voters' utility in country N when the history is unobservable (i.e., $h^N = \emptyset$) is strictly higher than the case wherein voter N observes history. Hence, social learning is not necessarily welfare-improving.

6 Populist Extremism under Imperfect Correlation

So far, ω has been assumed to be common across countries. This is a useful simplification in investigating the nature of the spread of extremism. However, in practice, the state of the world is not necessarily the same across countries. Social and economic conditions may be different across countries. National elections are held only occasionally, implying that there is some interval in the election of country i and $i+1$. In either case, the state is likely to be correlated only imperfectly. To this end, we assume that $\theta_1, \theta_2 \in (0,1)$ (see (1)).

Then, the updated beliefs are given as follows:

$$p_{i+1}(x_1, ..., x_{i-1}) = \theta_1 p^*_{i+1} + (1-\theta_2)(1-p^*_{i+1}), \tag{3}$$

where p^*_{i+1} is defined by p_{i+1} in section 5.1.

In the analysis, we assume the following inequality to focus on meaningful cases:

Assumption 3. $1-\theta_2 < \bar{p} \leq \theta_1$ *holds.*

When θ is too low, the policy in the previous country is not informative. For instance, when $\theta_1 = \theta_2 = 1/2$, $p_{i+1} = 1/2$ independently of x_i. Assumption 3 argues that the informativeness of the previous policy should not be too low. When the previous policy x_i is the perfect signal of the previous state of the world ω_i, the previous policy should largely affect voters' belief. In particular, the following property (*) should hold: When voter $i+1$ knows that $\omega_i = 1$ (2), p_{i+1} is large (small), such that $p_{i+1} \geq \bar{p}$ ($p_{i+1} < \bar{p}$). Otherwise, the informativeness of the previous state of the world is too low, and hence whether the equilibrium is extremism becomes invariant. In such an environment, it is meaningless to analyze the contagion of extremism. Hence, we impose (*), that is, Assumption 3.

6.1 Convergence towards Extremism

In the model in section 5, voters' beliefs converge toward the truth, and thus at the limit, extremism never occurs so long as $\omega = 1$.

$$p_S \equiv \frac{1-\theta_2}{2-\theta_1-\theta_2} \in (0, \theta_1),$$

which is equal to the steady state probability of ω being 1.

Proposition 5. *(a) There exists $p_E \in (p_S, \theta_1)$, such that for any i, $p_i < \bar{p}$ implies that $p_j < \bar{p}$ holds for all $j \geq i+1$ if and only if $\bar{p} \geq p_E$.*

(b) When $\bar{p} \geq p_E$, $\lim_{N\to\infty} \Pr(p_N < \bar{p}) = 1$.

(c) p_E is strictly increasing in θ_1 and weakly increasing in q_L (while q kept fixed).

(a) and (b) argue that, when $\bar{p} \geq p_E$, the equilibrium eventually shifts into the region wherein extremism arises, and extremism then continues forever. A convergence to extremism occurs, which contrasts the result obtained in section 5. This result can be illustrated in Figure 4. In this example, the initial state is 1 and the implemented policy in country 1 is also 1. Eventually, voters' beliefs decrease to lower than $\bar{p}$, and the equilibrium never shifts outside of the extremism equilibria.[29]

Countries cannot escape from extremism once captured, because the belief p may decrease even if policy 1 is observed, as illustrated in Figure 6. The intuition is as follows. Suppose $\bar{p} > p_S$, which is the necessary condition for the convergence toward extremism. In a changing world, a policy is less informative about the state because the state might change in the next period, pushing the updated belief toward p_S, the steady state probability of $\omega = 1$. Thus, when $\bar{p} > p_S$, the belief p around $\bar{p}$ increases less when policy 1 is observed.[30] This is the first force. Furthermore, policy 1 in the extremism equilibrium is not too informative about the true state because both the congruent type L and the non-congruent type take policy 1.[31] Combined with these two forces, policy 1 becomes so uninformative that the belief approaches toward $p_S < \bar{p}$ ("negative updating" in Figure 6) even when policy 1 is observed, and hence extremism becomes unavoidable.

[29] If the states are observable, the convergence to extremism does not occur. Indeed, if $\omega_i = 1$, $p_{i+1} = 0.7 > \bar{p}$.

[30] Indeed, $p_{i+1} \lesseqgtr p^*_{i+1} \Leftrightarrow p^*_{i+1} \gtreqless p_S$.

[31] On the other hand, policy 2 in the non-extremism equilibrium is perfectly informative about the true state, since no politician takes policy 2 under $\omega = 1$. Thus, extremism can start relatively easily.

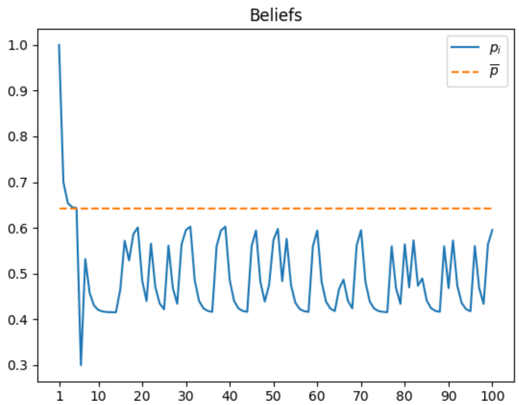

Figure 4: **Convergence to extremism.**

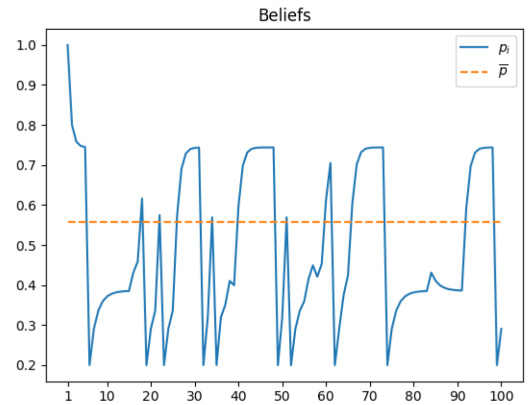

Figure 5: **Cycles of extremism.**

Notes. The left panel is a sample path when $l = 4$; $p_1 = 1$; $\omega_1 = 1; \theta_1 = \theta_2 = 0.8; q_H = 0.5; q_L = 0.2; b = 2.3$. The right panel is a sample path when $l = 4$; $p_1 = 1$; $\omega_1 = 1; \theta_1 = \theta_2 = 0.7; q_H = 0.4; q_L = 0.3; b = 2.5$. The orange dotted line is $\bar{p}$. The orange dotted line is $\bar{p}$.

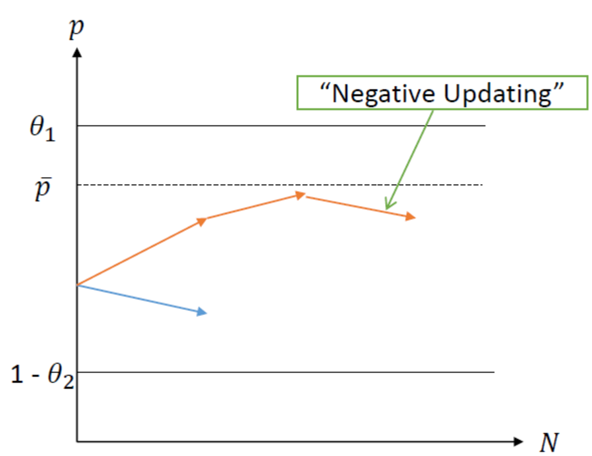

Figure 6: **Convergence to extremism: Mechanism.**

Notes. The orange arrows (blue arrow) represent the updating when the observed policy is 1 (2).

This mechanism is confirmed by the comparative static results in (c). p_E is increasing in θ_1 and q_L—that is, the higher θ_1 and q_L are, the less likelier that countries can be captured in the extremism equilibria. When θ_1 is large, the state of the world is highly stable, given that the previous state is 1. Hence, policy 1 remains sufficiently informative. In addition, when q_L is high, voters strongly believe that ω_i is likely to be 1—that is, p_{i+1} is high because the congruent type L always takes the optimal policy. On the contrary, when both θ_1 and q_L are small, the information value of policy 1 about the true state of the world is not sufficient to overturn the extremism equilibrium.

This result contrasts the conclusions of the canonical social learning model, where a changing world is less likely to sustain herding (Moscarini, Ottaviani, and Smith 1998; Nelson 2002). The primary reason is the difference in the belief under which distorted policies are implemented. In the canonical model, player i ignores the private signal and herds to the previous actions when p_i is close to either zero or one because the past actions strongly indicate that a certain policy is optimal. Hence, the changing world making p_i converge to a moderate value, p_S, prevents herding. On the contrary, in our model, with political agency problems, politician i implements the distorted policy when $p_i < \bar{p}$. Thus, when $p_S < \bar{p}$, the belief always remains moderate and herding toward extremism occurs.

This result also highlights that the extremism equilibrium is diffused differently from the non-extremism equilibrium. In our model, both of them are diffused. For instance, in the perfect correlation case, both equilibria asympotically arise. However, there is a large difference: When the state of the world is imperfectly correlated, the contagion of the extremism equilibrium could never end, while that of the non-extremism equilibrium eventually ends. That is, the diffusion effect of populism is more severe than that of non-populism.

6.2 Cycles of Extremism

Next, we consider the case wherein the convergence does not hold (i.e., $\bar{p} < p_E$). In this case, cycles of extremism are exhibited as seen in the following proposition.

Proposition 6. *Suppose $\bar{p} < p_E$.*

(a) For any integer $M \geq 1$, $\lim_{N\to\infty} \Pr(\forall i \text{ s.t. } M \leq i \leq N : p_i \geq \bar{p}) = 0$.

(b) For any integer $M \geq 1$, $\lim_{N\to\infty} \Pr(\forall i \text{ s.t. } M \leq i \leq N : p_i < \bar{p}) = 0$.

Proposition 6 shows that the probability of staying in either the non-extremism or extremism equilibrium forever is zero. Thus, in the long-term, we observe both equilibria.

Such cycles can be observed in Figure 5 wherein extremism initially arises. Although extremism spreads to around 20 countries due to the contagion mechanism, its proliferation finally ceases. After a while, extremism re-emerges because, in the case of $\omega = 2$, voters may observe policy 2 even when $p > \overline{p}$. The mechanism itself is straightforward. Since the state of the world changes across countries, voters' beliefs fluctuate highly. Hence, we obtain cycles.

6.3 Extremism and State Instability

Lastly, we investigate when the domino effect becomes serious in the sense that populism in one country induces it in many countries by focusing on the state instability. To simply analyze the role of the fluctuation of the state of the world, let us suppose that $\theta_1 = \theta_2 = \theta$ (i.e., $p_S = 0.5$). θ can be interpreted as the stability of the state of the world.[32]

We start with analyzing the condition under which the convergence to extremism occurs.

Fact 3. *p_E is strictly increasing in θ.*

Fact 3 implies that, decreasing θ triggers the convergence to extremism. This result is straightforward, since the convergence to extremism occurs due to the instability of the moderate state. This leads to the first important observation: The convergence of extremism is more likely to occur in an unstable world.

While this effect is important, it still remains that a serious domino effect may also arise even without the convergence to extremism. To illustrate, Figure 7 plots the relationship between θ and the average duration of the extremism equilibrium.[33] The left panel of the figure represents the case wherein $\bar{p} < 0.5$, whereby the extremism is not likely to arise. In this case, an increase in θ induces the longer duration of the extremism equilibrium—That is, the higher stability induces the more severe extremism once extremism arises, which is a contrast to the conjecture we obtain from Fact 3. On the contrary, we obtain the non-monotonic relationship in the right panel describing the case wherein $\bar{p} > 0.5$ (i.e., extremism is likely to arise). In short, the relationship between the stability of the state and long-lasting extremism depends on how likely extremism is to arise.

To understand this, suppose that country i is captured by the extremism equilibrium and consider country $i+1$'s public opinion. The higher stability of the state of the world implies the larger information value of country i's policy as a signal of country $i+1$'s state of the world. This has two counteracting effects. On the one hand, the lager information value of the moderate policy increases the chance to escape from extremism. On the other hand, the larger information value of the radical policy decreases the chance to escape from extremism. Hence, which dominates the other determines the total effect of higher stability. When $\bar{p}$ is low, countries can easily escape from extremism once they observe the moderate policy. Hence, the former effect is negligible and the latter effect is dominant. On the contrary, when $\bar{p}$ is high, extremism is likely to arise, meaning that extremism is always persistent as long as the radical policy is implemented. Hence, the latter effect is negligible and the former effect is dominant. This mechanism leads to Figure 7.

[32] Higher θ_1 implies the higher steady state probability of the state being moderate as well as the higher stability of the moderate state. Hence, to isolate the effect of the changes in stability, we must change the values of θ_1 and θ_2, fixing p_S. The easiest way is to assume the symmetric case. Even in the asymmetric cases, the results remain the same.

[33] Suppose that the extremism equilibrium is observed for countries 12–21, and 26–40. Then, the length of each sequence is 9 and 15, respectively. The average duration of the extremism equilibrium is calculated as (9+15)/2=12.

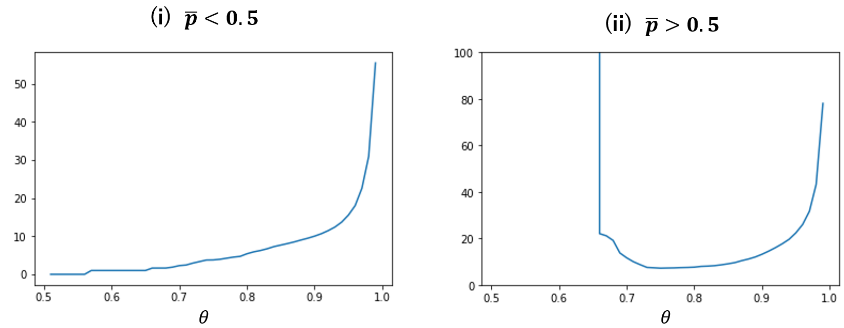

Figure 7: **Effect of θ on average duration of extremism.**

Notes. $q_H = 0.5; q_L = 0.2$. $b = 2.01$ (hence $\bar{p} = 0.432...$) in the left panel, and $b = 2.35$ (hence $\bar{p} = 0.578...$) in the right panel. The figures are calculated by simulating the model by 1,000,000 times and taking the average.

These arguments suggest various forms of populism contagion. For example, suppose that θ is negatively correlated with the geographical distance across countries. In this case, among neighborhood countries, Figure 7 suggests a long duration of the populism contagion, although the convergence to extremism is not likely to occur. On the other hand, the contagion may be prolonged even among remote countries when $\bar{p} > 0.5$. Another interpretation of θ is that it is related to the length of the interval between each country's election. In reality, there is some time interval between the election in one country and that in another. The correlation of the state of the world would be weaker as the time span between two elections expands. Our results suggest that, in the year wherein elections are held in many countries (i.e., θ is high), a serious populism contagion might occur if populism appears in one country. When the time interval between elections is long (i.e., θ is low), the convergence to extremism might occur, making the populism contagious when $\bar{p} > 0.5$. Note taht θ may also depend on what is the issue in elections. For example, sometimes immigration policy is important in many countries, while redistribution may be in another time.

7 Discussions

7.1 Distrust Shock in a Country Triggers the Domino Effect

It has been pointed out that the distrust of politics leads to populism (e.g., Akkerman, Mudde, and Zaslove 2014). Our model captures this idea. Since q, the fraction of the congruent type, represents the political trust level and $\bar{p}$ is decreasing in q, our model predicts that political distrust in country i induces populism in country i. Although political distrust is a cause of populism in one country, at first glance, political distrust in one country seems to be irrelevant to populism in another country. Our model, however, implies a catastrophic consequence of a distrust shock: The shock in only country 1 induces the domino effect of populism. Subsequent countries may face populist extremism due to voters' opinion radicalization even in the absence of direct distrust shocks. This implies that representative democracy is fragile against the distrust of politics in the sense that the political distrust shock only in one country can induce populist extremism in subsequent countries.

The mechanism is illustrated as follows. Suppose that $\omega_i = 1$ for all i and that the value of q is initially $\bar{q}$, which is relatively high so that $p_1 > \bar{p}(\bar{q})$. That is, populism does not arise in the countries. We then consider the distrust shock in country 1: Country 1's q changes from $\bar{q}$ to $\underline{q}$, while the other countries' q is $\bar{q}$. For example, the scandals about politicians' behaviors are reported in country 1, which intensify the distrust of politics in country 1. A sufficiently large shock leads to $p_1 < \bar{p}(\underline{q})$ so that country 1 is in the extremism equilibrium. Thus, country 1's congruent type H implements the radical policy, which makes country 2's opinion more radical. Consequently, it could be the case that $p_2 < \bar{p}(\bar{q})$ i.e., country 2 is also captured by extremism. That is, the distrust shock in country 1 induces populism in country 1, which in turn induces country 2's populism.

This result emphasizes that some important properties of the domino effect might be overlooked without considering voters' learning. In particular, politicians' learning, which is another important mechanism to explain the domino effect, is unlikely to predict such detrimental consequences of the distrust shock. Indeed, without the concurrent distrust shock, the shock in country 1 is irrelevant to the electoral advantage of country 2's populist parties.

7.2 Dynamic Election Model

We provide a foundation of our model as a two-period election model. In period 1, there is an incumbent politician. In each period, there is one policy issue. The policy issue in period 1 is the same as that of our basic model. In period 2, there is another policy issue y. The policy regarding this issue is chosen from $\{0,1,2\}$. Let the policy chosen by country i's policymaker in period 2 be y_i.

At the beginning of period 2, there are two candidates: the incumbent and a challenger who is the congruent type with probability q. Let the valence advantage of the incumbent be θ, which follows a uniform distribution $U[-\varepsilon,\varepsilon]$, where $\varepsilon > 0$. Voter i's utility is given by $-L(|x_i - \omega_i|) - kL(|y_i - \omega_i'|) + \mathbf{1}_i\theta$, where $\mathbf{1}_i$ is the indicator function that takes one if the incumbent is reelected. The voter's optimal policy for the issue in period 2 is $\omega_i' \in \{1,2\}$. However, since the issue is different, its relevance is also different. $k > 0$ represents the importance of the issue in period 2. The prior probability of $\omega_i' = 1$ is denoted by $r \in [0,1]$. To exclude learning about ω_i' and focus on that about ω_i, we assume that ω_i' is determined independently across countries for simplicity. The voter decides whether to reelect the incumbent based on this expected utility.[34] When the voter is

[34] This implies that the voter's strategy is not retrospective in that the voter gives the high evaluation to the incumbent who chooses the radical policy even if $p > 0.5$. Voters instead engages in prospective voting. This assumption allows us to show that populism could be contagious without assuming any kind of ad-hoc irrational behavior. Woon (2012)

indifferent between the incumbent and the challenger, the incumbent is reelected.

The congruent type's utility is $-L(|x_i-\omega_i|)+\mathbf{1}_i[\lambda_i-kL(|y_i-\omega_i'|)]$, where $\mathbf{1}_i$ is the indicator function that takes one if the politician is reelected in period 2, and $\lambda_i \geq 0$ is the office-seeking motivation. On the other hand, the non-congruent type's utility is $-L(|x_i|)+\mathbf{1}_i[\lambda_i-kL(|y_i|)]$.

Under this setting, the reelection probability of the incumbent is equal to

$$\Pr(\theta \geq (q-\pi)k[r+l(1-r)]) = \frac{1}{2}-\frac{qk}{2\varepsilon}[r+l(1-r)]+\frac{\pi_i(x_i)k}{2\varepsilon}[r+l(1-r)].$$ [35]

under the assumption that $\varepsilon \geq (1-q)k[r+l(1-r)]$. Hence, ignoring constants, the incumbent's objective at the beginning of period 1 is to maximize

$$-L(|x_i-x_i^*|)+\frac{\lambda_i k}{2\varepsilon}[r+l(1-r)]\pi_i(x_i),$$

where $x_i^*=\omega_i$ for the congruent type, while $x_i^*=0$ for the non-congruent type. Hence, by defining $b_i \equiv \frac{\lambda_i k}{2\varepsilon}[r+l(1-r)]$, the present dynamic election model is reduced to the original model.

7.3 Empirical Implications

Motivated by reduced-form empirical results (e.g., Ezrow, Böhmelt, and Lehrer 2019), we provided a foundation for the diffusion of populism. Our model, in turn, motivates new empirical specifications. We suggest two modifications of the reduced-form empirical approaches to accommodate key driving forces of our model.

First, our theory suggests that controlling for voters' preferences, mainly the median voters' position, may cause an over-control problem in empirically detecting the diffusion effect. To observe this point, let y_{it} denote a policy position in country i at time t. A canonical regression equation in the policy diffusion literature (Böhmelt et al. 2016, Equation 1) is

$$y_{it} = \phi y_{it-1}+\beta X_{it-1}+\rho W y_{e-1}+\varepsilon_{it}, \tag{4}$$

where X_{it-1} are the lagged control variables, W is the spatial weighting matrix summarizing the importance of party $j \neq i$ to party i, and y_{e-1} is each party's position in the year before the last election held in its country before time t. The parameter of interest is ρ, which captures the effect of foreign parties' positions on party i's position.

Now, suppose that X_{it-1} includes the median voters' position. Then, given that it is influenced by foreign party's positioning, X_{it-1} can be written as a function of Wy_{e-1}, implying that the term

also experimentally supports the prospective voting behavior in a simple election model.

[35] Since politicians are unaccountable in period 2, the expected payoff when the voter reelects the incumbent is $-(1-\pi_i(x_i))k[r+l(1-r)]$, while that when the voter elects the challenger is $-(1-q)k[r+l(1-r)]$. The former is larger than or equal to the latter if and only if $\theta \geq (q-\pi)k[r+l(1-r)]$.

βX_{it-1} absorbs parts of the effects that should be captured by the coefficient ρ. Thus, as long as party policy diffusion through voters' updating is an effect of interest, control variables regarding voters' preference should be carefully chosen to avoid over-controlling for these effects.

Second, from section 7.1, shocks to the political distrust in the foreign country may impact domestic policy-making. The following modification to (4) would be informative:

$$y_{it} = \phi y_{it-1} + \beta X_{it-1} + \gamma d_{i,t-1} + \rho_d W d_{e-1} + \varepsilon_{it},$$

where d denotes the level of political distrust. As long as the diffusion effect enters through political distrust, the distrust level in foreign countries would have an explanatory power even after controlling for own distrust level.[36]

7.4 Diffusion in a Specific Issue: An Application to Yardstick Competition

Empirically, various policies have been found to be correlated across space. We discuss the implications for yardstick competition in personal income taxation. Besley and Case (1995) show that excessive spending is curbed when citizens observe the political outcomes of other jurisdictions. Our model, on the other hand, reveals a new disadvantage of voters' benchmarking in elections.

Suppose that when $\omega = 1(2)$, jurisdiction experiences moderate (low) fiscal needs. Policy 1 is a moderate tax rate, policy 2 is a low tax rate, and policy 0 is a high tax rate. Here, following Besley and Case (1995), we assume that policy 0 is preferred by a "Leviathan" who seeks fiscal control and is unambiguously harmful to voters. The utility loss of voters depends on the difference between their fiscal needs and the implemented tax rate.

Our propositions predict that sub-optimally low tax rates may be contagious even when fiscal needs are not significantly low. Thus, information flow between jurisdictions may induce excessively low tax rates, leading to the collapse of the welfare state. The congruent type H sets a lower tax rate than the country's fiscal needs to signal that they are not a "Leviathan". Moreover, the excessively low tax rate in one jurisdiction can propagate because voters in other jurisdictions now believe more strongly that the low tax rate is optimal. This result reveals an important side-effect of yardstick competition. While yardstick competition is typically regarded as beneficial by disciplining politicians, it may also distort policies by facilitating the propagation of wrong information. This drawback of yardstick competition is indeed consistent with Shigeoka and Watanabe (2019) showing that inefficient childcare policy diffused due to election concerns.

[36] In this regression, the foreign policy positions are not included since the effect of distrust works only through foreign policy positions. Theoretically, the inclusion would make $\rho_d = 0$.

8 Concluding Remarks

This paper investigated the diffusion of undesirable policies in the form of populist extremism. To this end, we constructed a novel social learning model with agency problems. In the single-country model, we showed that populist extremism can arise depending on voters' beliefs about the state of the world. We then analyzed the dynamic multi-country model. We first found that populist extremism is contagious across countries through the dynamic interaction between the public opinion and implemented policies. The long-run dynamics have unique features absent in the canonical social learning models. First, populist extremism could suddenly stop even if the public opinion is quite radical, because of the discontinuous jump of the public opinion. Second, the long-run dynamics depends on the correlation of the state of the world across countries. We showed the possibility of never-ending populist extremism under the imperfect correlation, while extremism eventually stops spreading under the perfect correlation.

Before concluding this paper, we point out the remaining challenges for future research. First, examining whether similar patterns of the propagation occur for other aspects of populism may be worthwhile. Second, studying learning patterns in more complex networks may also be beneficial. Given recent developments on these issues, it may be promising to introduce a network structure. Third, although our model assumes Bayesian rationality to focus on the fundamental contagion mechanism, voters might not be Bayesian rational in reality. These issues are left to future work.

A Omitted Proofs

For the analysis in the single country model, the following lemma is sometimes invoked:

Lemma A.1. *(i) There is no equilibrium wherein $\beta^*(0) = 1$. In addition, (ii) there is no equilibrium wherein $\beta^*(2) > 0$, (iii) in any equilibrium, $\alpha^*(2;2) = 1$, (iv) there is no equilibrium wherein $\alpha^*(0;\omega) > 0$ for some ω, and (v) in any equilibrium, $\gamma^*(\omega;\omega) = 1$.*

Proof. **(i):** Suppose that there exists such an equilibrium. In this equilibrium, $\pi(1) = 1$ holds since the non-congruent type never chooses 1 (see (v)). Given this, consider the deviation incentive of the non-congruent type. S/he deviates from 0 to 1 if $-1+b \geq 0$. This holds since $b > 2$. Hence, there is no such equilibrium.

(ii): The non-congruent type loses l by taking 2 instead from 0. Since $b < l$, it never does so whatever belief the voter holds.

(iii): From (ii), $\pi(2) = 1$. Given this, $\alpha^*(2;2) = 1$ must hold because it is the ideal policy for the congruent type, and it ensures the high reputation.

(iv): From (ii), $\pi(2) = 1$. Given this, $\alpha^*(0;\omega) > 0$ can be the case only when $\pi(0) = 1$. Note that taking 2 also brings the same utility loss of 1 and ensures the high reputation. However, when $\pi(0) = 1$, the non-congruent type chooses 0 so that $\pi(0) \neq 1$. This is contradiction.

(v): The congruent type loses at least one by taking the policy different from ω. Since $b_L < 1$, this is not optimal for the congruent type L.

$\square$

A1 Proof of Proposition 1

(a) First of all, from Lemma A.1, $\beta^*(2;\omega) = 0$.

The non-congruent type mixes 1 and 0 only when s/he is indifferent between these two policies i.e.,

$$-1 + b\frac{pq_L}{pq_L + \beta^*(1)(1-q)} = 0 \Leftrightarrow \beta^*(1) = \frac{(b-1)pq_L}{1-q}. \tag{5}$$

This $\beta^*(1) < 1$ if and only if

$$p < \frac{1-q}{(b-1)q_L}.$$

Given this, examine whether the congruent type H has a deviation incentive. The congruent type H has no incentive to deviate from 2 to 1 if and only if

$$-1 + b \geq b\frac{pq_L}{pq_L + \beta^*(1)(1-q)} \Leftrightarrow b \geq 2,$$

which holds since $b > 2$.

(b) First, observe that from this equilibrium strategy,

$$\pi(2) = 1; \;\; \pi(1) = \frac{q_L p}{q_L p + (1-q)}. \tag{6}$$

Given this, the congruent type H has no deviation incentive from 2 to 1 when $\omega = 1$ if and only if

$$-1 + b \geq b\frac{q_L p}{q_L p + (1-q)} \Leftrightarrow p \leq \frac{(b-1)(1-q)}{q_L}. \tag{7}$$

Note that the congruent type H obviously has no deviation incentive when $\omega = 2$.

Next, consider the non-congruent type's incentive. S/he has no incentive to deviate from 1 to 0 if and only if

$$-1 + b\frac{q_L p}{q_L p + (1-q)} \geq \pi(0).$$

The right-hand side is minimized when $\pi(0) = 0$. Hence, there is an off-path belief for which the non-congruent type has no deviation incentive if and only if

$$-1 + b\frac{q_L p}{q_L p + (1-q)} \geq 0 \Leftrightarrow p \geq \frac{1-q}{(b-1)q_L}. \tag{8}$$

Note that the non-congruent type has no incentive to choose 2 from Lemma A.1.

Combining (7) and (8) yield the condition.

(c) In this equilibrium,

$$\pi(2) = 1; \;\; \pi(1) = \frac{pq_L + p\alpha^*(1;1)q_H}{pq_L + p\alpha^*(1;1)q_H + (1-q)}.$$

Given this, the congruent type H mixes 1 and 2 when $\omega = 1$ if and only if

$$b\frac{pq_L + p\alpha^*(1;1)q_H}{pq_L + p\alpha^*(1;1)q_H + (1-q)} = -1 + b \Leftrightarrow \alpha^*(1;1) = \frac{(b-1)(1-q)}{pq_H} - \frac{q_L}{q_H}. \tag{9}$$

Here, the derived $\alpha^*(1;1)$ is less than one if and only if

$$p \geq \frac{(b-1)(1-q)}{q}. \tag{10}$$

In addition, it is larger than zero if and only if

$$p \leq \frac{(b-1)(1-q)}{q_L}. \tag{11}$$

Combining these two inequalities, we have the condition.

Lastly, examine the non-congruent type's incentive. Since the deviation incentive of the non-congruent type is minimized when $\pi(0) = 0$, s/he has no deviation incentive when

$$-1 + b\frac{pq_L + p\alpha^*(1;1)q_H}{pq_L + p\alpha^*(1;1)q_H + (1-q)} \geq 0 \Leftrightarrow b \geq 2,$$

which holds since $b > 2$.

(d) First, the equilibrium belief is given by

$$\pi(2) = 1; \;\; \pi(1) = \frac{pq}{pq + (1-q)}.$$

Given this belief, the congruent type H has no incentive to deviate from 1 to 2 when $\omega = 1$ if and only if

$$-1 + b \leq b\frac{pq}{pq + (1-q)} \Leftrightarrow p \geq \frac{(b-1)(1-q)}{q}. \tag{12}$$

Note that the congruent type H obviously has no deviation incentive when $\omega = 2$.

Next, consider the non-congruent type's deviation incentive. S/he has no incentive to deviate from 1 to 0 if and only if

$$-1 + b\frac{pq}{pq + (1-q)} \geq 0 \Leftrightarrow p \geq \frac{1-q}{(b-1)q}. \tag{13}$$

This is because the deviation incentive is minimized when $\pi(0) = 0$.

Combining (12) and (13), we have the lemma. Note that because $b > 2$, $(b-1)(1-q)/q > (1-q)/[(b-1)q]$.

(e) Denote the set of policies chosen by the congruent type H with a positive probability given the state

ω by $X_C^*(\omega) \equiv \{x \in \{0,1,2\} : \alpha^*(x;\omega) > 0\}$, and denote the element of $X_C^*(\omega)$ by $x_C^*(\omega)$. Similarly, define X_N^* and x_N^* for those of the non-congruent type corresponding to the above notions.

Step 1. We start by investigating the conditions under which equilibria fully separate, such that $X_C^*(\omega) \cap X_N^* = \emptyset$ for all ω. Prove that there is no separating equilibrium except for (Ex1) and (Ex2) equilibria. From Lemma A.1 (i) and (ii), if such an equilibrium exists, either (I) $\beta^*(1) = 1$, or (II) $\beta^*(1) + \beta^*(0) = 1$, $\beta^*(1) \in (0,1)$, and $\beta^*(0) \in (0,1)$. Then, from Lemma A.1 (iv), $\alpha^*(2;\omega) = 1$ holds in any fully separating equilibrium.

Step 2. Next, we explore semi-separating equilibria, such that $X_C^*(\omega) \cap X_N^* \neq \emptyset$ for some ω, but $X_C^*(\omega) \neq X_N^*$ for some ω. Prove that there is no separating equilibrium except for (Ex3) and (NEx) equilibria.

From Lemma A.1 (i) and (ii), if such an equilibrium exists, either (I) $\beta^*(1) = 1$, or (II) $\beta^*(1) + \beta^*(0) = 1$, $\beta^*(1) \in (0,1)$, and $\beta^*(0) \in (0,1)$.

Case (I). $\alpha^*(1;1) > 0$ must hold in semi-separating equilibria because $\alpha^*(2;2) = 1$ from Lemma A.1 (iii). When $\alpha^*(1;1) = 1$, this is the equilibrium in (a). When the congruent type mixes 1 and 2, that is the equilibrium in (b).

Case (II). As in case (I), $\alpha^*(1;1) > 0$ must hold. Consider the case where $\alpha^*(1;1) = 1$ and the case where the congruent type H takes a mixed strategy one by one.

Case (II-1). $\alpha^*(1;1) = 1$. The non-congruent type mixes 0 and 1 if and only if

$$-1 + b\frac{pq}{pq + \beta^*(1)(1-q)} = 0 \Leftrightarrow \beta^*(1) = \frac{(b-1)pq}{1-q}.$$

Given this, the congruent type H has no incentive to deviate from 1 to 2 when $\omega = 1$ if and only if

$$b\frac{pq}{pq + \beta^*(1)(1-q)} \geq -1 + b \Leftrightarrow b \leq 2,$$

which does not hold. Hence, there is no such an equilibrium.

Case (II-2). Mixed strategy. The congruent type H mixes 1 and 2 when $\omega = 1$ if and only if

$$b\frac{pq_L + \alpha^*(1;1)q_H}{pq_L + \alpha^*(1;1)pq_H + \beta^*(1)(1-q)} = -1 + b. \tag{14}$$

Similarly, the non-congruent type mixes 0 and 1 if and only if

$$-1 + b\frac{pq_L + \alpha^*(1;1)q_H}{pq_L + \alpha^*(1;1)pq_H + \beta^*(1)(1-q)} = 0. \tag{15}$$

(14) and (15) simultaneously hold only when $b = 2$. Hence, this equilibrium does not exist.

Step 3. Lastly, there is no equilibrium wherein for any ω, $X_C^*(\omega) = X_N^*$ because $\alpha^*(2;2) = 1$ but $\beta^*(2) = 0$ (Lemma A.1).

From steps 1-3, we have (e). $\square$

A2 Proof of Lemma 1

Step 1. Prove that there is an equilibrium, such that $\alpha^*(2;2) = 1$ and $\beta^*(0) = 1$. When $\pi(1) = 0$, it is straightforward that no one has deviation incentive.

Step 2. Prove that there exist no other equilibria. From Lemma A.1, $\alpha^*(2;2) = 1$ and $\beta^*(2) = 0$. Hence, the candidate of other equilibria is that $\beta^*(1) > 0$. Prove by contradiction. When $\beta^*(1) > 0$, $\pi(1) = 0$ since $p = 0$. Thus, the non-congruent type has no incentive to choose 1, which contradicts $\beta^*(1) > 0$. □

A3 Proof of Proposition 2

We first prove several lemmas. The first lemma is about the equilibrium in country i given p_i. As an immediate consequence of Proposition 1 and the definition of equilibria, the equilibrium in country i is determined as follows:

Lemma A.2. *Suppose first that $\frac{1-q}{(b-1)q_L} < \frac{(b-1)(1-q)}{q}$. Then,*

(i) When $p < \frac{1-q}{(b-1)q_L}$, (Ex1) equilibrium arises.

(ii) When $\frac{1-q}{(b-1)q_L} \leq p < \frac{(b-1)(1-q)}{q}$, (Ex2) equilibrium arises.

(iii) When $p \geq \frac{(b-1)(1-q)}{q}$, (NEx) equilibrium arises.

Next, suppose that $\frac{1-q}{(b-1)q_L} > \frac{(b-1)(1-q)}{q}$. Then,

(i) When $p < \frac{(b-1)(1-q)}{q}$, (Ex1) equilibrium arises.

(ii) When $p \geq \frac{(b-1)(1-q)}{q}$, (NEx) equilibrium arises.

We next show the useful fact about belief updating.

Lemma A.3. *For each $\tilde{p} \in (0,1]$, $\Pr(p_{i+1} < \tilde{p}|p_i)$ is weakly decreasing in p_i.*

Proof. From Lemma A.2, $p_{i+1}(p_i,2)$, $p_{i+1}(p_i,1)$, and $p_{i+1}(p_i,0)$ are increasing in p_i and $p_{i+1}(p_i,2) < p_{i+1}(p_i,0) < p_{i+1}(p_i,1)$.[37] Note that both $\Pr(x_i = 2|p_i)$ and $\Pr(x_i = 0|p_i)$ are weakly decreasing in p_i.

Fix $\tilde{p}$. Depending on the value of p_i, we can potentially have the following cases. The cases are ordered so that a case with a smaller number corresponds to that under smaller p_i.

Case 1. $p_{i+1}(p_i,1) < \tilde{p}$: In this case, $\Pr(p_{i+1} < \tilde{p}|p_i) = 1$.

Case 2. $p_{i+1}(p_i,0) < \tilde{p} \leq p_{i+1}(p_i,1)$: In this case, $p_{i+1} < \tilde{p}$ if and only if $x_i = 0$ or 2. Hence, $\Pr(p_{i+1} < \tilde{p}|p_i) < 1$. In addition, since both $\Pr(x_i = 2|p_i)$ and $\Pr(x_i = 0|p_i)$ are weakly decreasing in p_i, $\Pr(p_{i+1} < \tilde{p}|p_i)$ is weakly decreasing in p_i.

Case 3. $p_{i+1}(p_i,2) < \tilde{p} \leq p_{i+1}(p_i,0)$: In this case, $p_i < \tilde{p}$ if and only if $x_i = 2$. Hence, $\Pr(p_{i+1} < \tilde{p}|p_i)$ is smaller than that of Case 2. In addition, since $\Pr(x_i = 2|p_i)$ is weakly decreasing in p_i, $\Pr(p_{i+1} < \tilde{p}|p_i)$ is weakly decreasing in p_i.

Case 4. $\tilde{p} \leq p_{i+1}(p_i,2)$: In this case, $\Pr(p_{i+1} < \tilde{p}|p_i) = 0$ regardless of the value of p_i.

From these cases, $\Pr(p_{i+1} < \tilde{p}|p_i)$ is weakly decreasing in p_i. □

[37] When $x_i = 0$ is the off-equilibrium path, we ignore the case where $x_i = 0$.

It means that the future value of p is higher (in the probabilistic sense) if the current p is higher. Given this, we prove the proposition.

(a) Without loss of generality, we focus on the case where $k = 1$. Note that $\Pr(p_2 \geq \bar{p})$ is weakly increasing in p_1.

For each $\tilde{p} \in (0,1]$, $\Pr(p_2 < \tilde{p}|p_1)$ is weakly decreasing in p_1. Here,

$$\Pr(p_3 \geq \tilde{p}|p_1) = \sum_{p_2' \in \mathrm{Supp}(p_2|p_1)} \left[\Pr(p_3 \geq \tilde{p}|p_2 = p_2') \cdot \Pr(p_2 = p_2'|p_1)\right], \tag{16}$$

where for $i \geq 2$, $\mathrm{Supp}(p_i|p_1) \equiv \{p_i \in [0,1] | \Pr(p_i = p_i'|p_1) > 0\}$.[38] In addition, since $\Pr(p_3 \geq \tilde{p}|p_2 = p_2')$ is non-decreasing in p_2', for each $\tilde{p} \in (0,1]$, $\Pr(p_3 < \tilde{p}|p_2)$ is weakly decreasing in p_2. Compare $p_1 = p_H$ and $p_1 = p_L$ where $p_H > p_L$. We have $\Pr(p_2 < \tilde{p}|p_1 = p_H) \leq \Pr(p_2 < \tilde{p}|p_1 = p_L)$ for each $\tilde{p} \in (0,1]$, meaning that the distribution of p_2 under $p_1 = p_H$ first-order stochastically dominates that under $p_1 = p_L$. Hence, from the property of the first-order stochastic dominance, (16) is weakly increasing in p_1.[39] Thus, we have proven the assertion for $i = 2$.

The result for $i = 2$ directly implies that $\Pr(p_3 < \tilde{p}|p_1)$ is weakly decreasing in p_1. Having this result at hand, we can repeat the same argument for all $i \geq 3$ to show that

$$\Pr(p_{i+1} \geq \tilde{p}|p_1) = \sum_{p_i' \in \mathrm{Supp}(p_i|p_1)} \left[\Pr(p_{i+1} \geq \tilde{p}|p_i = p_i') \cdot \Pr(p_i = p_i'|p_1)\right] \tag{17}$$

is weakly increasing in p_1.

(b) From (a), $\Pr(p_i \leq \bar{p}|p_2)$ is weakly increasing in p_2. Furthermore, $p_2(2) < p_2(1)$ and $p_2(2) < p_2(0)$. Hence, we have the first part of (b). Next, prove the second part.

Case 1. $\frac{1-q}{(b-1)q_L} \leq \bar{p}$: Consider $p_1 \in (0,\bar{p})$ that is sufficiently close to $\bar{p}$. Since $x_1 = 0$ is an off-equilibrium path, it suffices to show that $\Pr(p_i \geq \bar{p}|x_1 = 2) < \Pr(p_i \geq \bar{p}|x_1 = 1)$. For such p_1, $p_2(2,p_1) < \bar{p} < p_1(1,p_1)$. Thus, $\Pr(p_i \geq \bar{p}|x_1 = 2) < 1 = \Pr(p_i \geq \bar{p}|x_1 = 1)$.

Case 2. $\frac{1-q}{(b-1)q_L} > \bar{p}$: In this case, for any $p \in (0,\bar{p})$, (Ex1) equilibrium is realized. Hence, we need to prove both $\Pr(p_i \geq \bar{p}|x_1 = 2) < \Pr(p_i \geq \bar{p}|x_1 = 1)$ and $\Pr(p_i \geq \bar{p}|x_1 = 2) < \Pr(p_i \geq \bar{p}|x_1 = 0)$.

(i) $\Pr(p_i \geq \bar{p}|x_1 = 2) < \Pr(p_i \geq \bar{p}|x_1 = 1)$. Consider $p_1 \in (0,\bar{p})$ that is sufficiently close to $\bar{p}$. Then, as in Case 1, $\Pr(p_i \geq \bar{p}|x_1 = 2) < 1 = \Pr(p_i \geq \bar{p}|x_1 = 1)$.

(ii) $\Pr(p_i \geq \bar{p}|x_1 = 2) < \Pr(p_i \geq \bar{p}|x_1 = 0)$. Again, consider $p_1 \in (0,\bar{p})$ that is sufficiently close to $\bar{p}$. First, derive the upper bound of $\Pr(p_i \geq \bar{p}|x_1 = 2)$. There are three cases depending on the value of x_2. $\Pr(p_i \geq \bar{p}|x_1 = 2, x_2 = 1) \leq 1$. $\Pr(p_i \geq \bar{p}|x_1 = 2, x_2 = 0) = \Pr(p_{i-1} \geq \bar{p}|x_1 = 2)$ because $p_2(x_1 = 2) = p_3(x_1 = 2, x_2 = 0)$. Furthermore, $\Pr(p_i \geq \bar{p}|x_1 = 2, x_2 = 2) \leq \Pr(p_i \geq \bar{p}|x_1 = 2, x_2 =$

[38] This is a finite set.

[39] In our model, the distribution of p_2 is a discrete distribution. However, even in a discrete case, the property holds (Courtault, Crettez, and Hayek 2006).

$0) = \Pr(p_{i-1} \geq \bar{p} | x_1 = 2)$. Here, the first inequality comes from (a). By combining these three cases, we have

$$\Pr(p_i \geq \bar{p}|x_1 = 2) \leq \Pr(x_2 = 1|x_1 = 2) + [1 - \Pr(x_2 = 1|x_1 = 2)] \cdot \Pr(p_{i-1} \geq \bar{p}|x_1 = 2). \tag{18}$$

Next, derive the lower bound of $\Pr(p_i \geq \bar{p}|x_1 = 0)$. There are three cases depending on the value of x_2. $\Pr(p_i \geq \bar{p}|x_1 = 0, x_2 = 1) = 1$ because $p_2 (= p_1)$ is sufficiently close to $\bar{p}$. $\Pr(p_i \geq \bar{p}|x_1 = 0, x_2 = 0) = \Pr(p_{i-1} \geq \bar{p}|x_1 = 0) \geq \Pr(p_{i-1} \geq \bar{p}|x_1 = 2)$. Here, the second inequality comes from the fact that $p_{i-1} \geq \bar{p}$ implies $p_i \geq \bar{p}$. In addition, $\Pr(p_i \geq \bar{p}|x_1 = 0, x_2 = 2) = \Pr(p_{i-1} \geq \bar{p}|x_1 = 2)$. By combining these three cases, we have

$$\Pr(p_i \geq \bar{p}|x_1 = 0) \geq \Pr(x_2 = 1|x_1 = 0) + [1 - \Pr(x_2 = 1|x_1 = 0)] \cdot \Pr(p_{i-1} \geq \bar{p}|x_1 = 2). \tag{19}$$

Now, country 2 is in (Ex1) equilibrium so that $p_2(2) < p_2(1)$ implies $\Pr(x_2 = 1|x_1 = 2) < \Pr(x_2 = 1|x_1 = 0)$. Hence, the right-hand side of (18) is strictly smaller than that of (19). That is, $\Pr(p_i \geq \bar{p}|x_1 = 2) < \Pr(p_i \geq \bar{p}|x_1 = 0)$.

From cases 1 and 2, we obtain the second part of (b). □

A4 Proof of Proposition 3

First of all, $p_i > 0$ since $\omega = 1$. Thus, when $p_i(x_1, ..., x_{i-1}) < \bar{p}$, $p_{i+1}(x_1, ..., x_{i-1}, 1) = [1 + (b-1)p_i]/b > 1/b$. Here, when $(b-1)b \leq q/(1-q)$, $1/b > \bar{p}$. Hence, if the probability that $x_i = 1$ is positive for any $p_i \in (0, 1]$, the proposition holds.[40] Indeed, when $\omega = 1$, this probability is always positive because the congruent type L chooses 1; thus, the probability of $x_i = 1$ is greater than or equal to $q_L > 0$. □

A5 Proof of Proposition 4

Our starting point is showing that voters' beliefs almost surely converge to a single point. By applying Martingale Convergence Theorem to our scenarios, we can show that there exists $p^* \in [0, 1]$, such that $\Pr(\lim_{N\to\infty} p_N = p^*) = 1$ (Chamley 2004: Proposition 2.7).

(a) Prove by contradiction. Suppose that there exists $p^* \neq 1$, such that $\Pr(\lim_{N\to\infty} p_N = p^*) = 1$. If this gives us a contradiction, $\Pr(\lim_{N\to\infty} p_N = 1) = 1$.

Case 1. $p^* \in (0, 1)$: Since almost sure convergence implies convergence in probability, the following must hold: $\forall\varepsilon > 0, \forall\delta > 0, \exists N^*(\varepsilon, \delta)$ s.t.

$$\forall N \geq N^*(\varepsilon, \delta) \;\; \Pr(|p_N - p^*| \geq \varepsilon) < \delta. \tag{20}$$

[40] The belief p_{i+1} is also larger than $1/b$ when the equilibrium is (Ex1). To observe this, note that the belief when $p_i = \frac{1-q}{(b-1)q_L}$ is larger than $1/b$ and p_{i+1} is increasing in p_i in (Ex1) equilibrium.

Let

$$\theta_1(\varepsilon) \equiv \min\left\{\frac{1+(b-1)(p^*-\varepsilon)}{b}, \frac{(p^*-\varepsilon)(q_L+(1-q))}{(p^*-\varepsilon)q_L+(1-q)}, \frac{p^*-\varepsilon}{(p^*-\varepsilon)q+(1-q)}\right\}.$$

Fix $\delta = \frac{q_L}{1+q_L}$ and ε, such that $\theta_1(\varepsilon) \geq p^* + \varepsilon$.[41] Consider $N \geq N^*(\varepsilon, \delta)$. Then, Lemma A.2 implies that when $|p_N - p^*| < \varepsilon$, the updated belief is $p_{N+1}(1) > \theta_1(\varepsilon) \geq p^* + \varepsilon$. Since the probability that policy 1 is realized is at least q_L, $\Pr(|p_{N+1} - p^*| \geq \varepsilon) \geq (1-\delta)q_L \geq \delta$, which contradicts (20).

Case 2. $p^* = 0$: First, observe that when $\omega = 1$, $p_N > 0$ always holds. Hence, for $N \geq 2$, $p_N(1) > 0$. In particular, $p_N(1) > 1/b$ from the proof of Proposition 3. Furthermore, at least with probability q_L, $x_N = 1$ for any $p_N \in (0,1]$. Hence, $\Pr(p_N > 1/b) \geq q_L$ for all N, meaning that $\Pr(\lim_{N\to\infty} p_N \geq 1/b) > 0$.

From cases 1 and 2, $p^* \neq 1$ leads to a contradiction. That is, $p^* = 1$.

(b) Prove by contradiction. Suppose that there exists $p^* \neq 0$, such that $\Pr(\lim_{N\to\infty} p_N = p^*) = 1$.

Case 1. $p^* \in (0,1)$: As in case 1 of (a), the following must hold: $\forall\varepsilon > 0, \forall\delta > 0, \exists N^*(\varepsilon,\delta)$ s.t. (20).

Let

$$\theta_2(\varepsilon) \equiv \max\left\{1 - \frac{(1-(p^*+\varepsilon))q}{(1-(p^*+\varepsilon))q_L + q_H}, 1 - \frac{(1-(p^*+\varepsilon))q}{q-(b-1)(1-q)}, 0\right\} > 0.$$

Fix $\delta = \frac{q_L}{1+q_L}$ and ε, such that $\theta_2(\varepsilon) \leq p^* - \varepsilon$.[42] Consider $N \geq N^*(\varepsilon, \delta)$. Then, Lemma A.2 implies that when $|p_N - p^*| < \varepsilon$, the updated belief is $p_{N+1}(2) < \theta_2(\varepsilon) < p^* - \varepsilon$ i.e., $|p_{N+1}(2) - p^*| > \varepsilon$. Since the probability that policy 2 is realized is at least q_L, $\Pr(|p_{N+1} - p^*| \geq \varepsilon) \geq (1-\delta)q_L \geq \delta$, which contradicts (20).

Case 2. $p^* = 1$: As in case 1 of (a), the following must hold: $\forall\varepsilon > 0, \forall\delta > 0, \exists N^*(\varepsilon,\delta)$ s.t. (20). Fix $\varepsilon = 1 - \bar{q}$ and $\delta = \frac{q_L}{1+q_L}$. Consider $N \geq N^*(\varepsilon,\delta)$. Then, Theorem 1 implies that when $|p_N - 1| < \varepsilon$, $p_{N+1} = 0$ with at least probability q_L. This is because $p_N \geq \bar{q}$ and thus the congruent type L takes policy 2. Hence, $\Pr(|p_{N+1} - 1| \geq \varepsilon) \geq (1-\delta)q_L \geq \delta$, which contradicts (20).

From cases 1 and 2, $p^* \neq 0$ leads to a contradiction. That is, $p^* = 0$. □

A6 Proof of Proposition 5

(a) First, observe that (3) is increasing in p_i because p^*_{i+1} is increasing in p_i when $p_i < \bar{p}$. Hence, the necessary and sufficient condition for (a) is equivalent that when $p_i = \bar{p}$, $p_{i+1} \leq \bar{p}$ holds.

Case 1. $\frac{1-q}{(b-1)q_L} \leq \bar{p}$: $p_{i+1} \leq \bar{p}$ can be rewritten as

$$p_{i+1}(\bar{p},1) \leq \bar{p} \Leftrightarrow \frac{\bar{p}q_L + \bar{p}(1-q)}{\bar{p}q_L + 1 - q}(\theta_1+\theta_2-1) + 1 - \theta_2 \leq \bar{p}$$
$$\Leftrightarrow q_L\bar{p}^2 + [(2-\theta_1-\theta_2)(1-q) - \theta_1 q_L]\bar{p} - (1-q)(1-\theta_2) \geq 0. \qquad (21)$$

[41] Such $\varepsilon > 0$ exists because $\theta_1(\varepsilon)$ is continuous with respect to ε and $\theta_1(0) > p^*$.
[42] Such $\varepsilon > 0$ exists because $\theta_2(\varepsilon)$ is continuous with respect to ε and $\theta_2(0) < p^*$.

Here, the left-hand side of (21) is

$$q_L \frac{1-\theta_2}{2-\theta_1-\theta_2}\left(\frac{1-\theta_2}{2-\theta_1-\theta_2}-\theta_1\right) = q_L \frac{1-\theta_2}{(2-\theta_1-\theta_2)^2}(\theta_1+\theta_2-1)(\theta_1-1) < 0$$

when $\bar{p} = p_S$ while it is $-(\theta_1+\theta_2-1)(\theta_1-1) > 0$ when $\bar{p} = \theta_1$. Hence, there exists $p_E \in (p_S, \theta_1)$, such that if and only if $\bar{p} \geq p_E$, (21) holds.

Case 2. $\frac{1-q}{(b-1)q_L} > \bar{p}$: The condition $p_{i+1} \leq \bar{p}$ can be rewritten as

$$\begin{aligned} p_{i+1}(\bar{p},1) \leq \bar{p} &\Leftrightarrow \frac{1+(b-1)\bar{p}}{b}(\theta_1+\theta_2-1)+1-\theta_2 \leq \bar{p} \\ &\Leftrightarrow \bar{p} \geq p_E \equiv \frac{\theta_1+\theta_2-1+b(1-\theta_2)}{\theta_1+\theta_2-1+b(2-\theta_1-\theta_2)}. \end{aligned} \tag{22}$$

Here, we can easily verify that $p_E \in (p_S, \theta_1)$.

From cases 1 and 2, we have (a).

(b) From (a), when $p < \bar{p}$, $\Pr(p_N < \bar{p}) = 1$ for all N.

Next, consider $p \geq \bar{p}$. From (a), $\Pr(p_N < \bar{p}) = \Pr(\exists i \leq N : p_i < \bar{p})$. Hence, it suffices to show that $\lim_{N\to\infty} \Pr(\forall i \leq N : p_i \geq \bar{p}) = 0$. Observe that when $p_i \geq \bar{p}$ and $x_i = 2$, $p_{i+1}(2) = 1-\theta_2 < p_S < \bar{p}$. In addition, $x_i = 2$ at least with probability $(1-\theta_1)q_L$. Thus,

$$\Pr(\forall i \leq N : p_i \geq \bar{p}) \leq [1-(1-\theta_1)q_L]^N.$$

This goes to zero as $N \to \infty$. Hence, $\lim_{N\to\infty} \Pr(\forall i \leq N : p_i \geq \bar{p}) = 0$.

(c) (i). Prove that $\frac{\partial p_E}{\partial \theta_1} > 0$. Consider case 1, first. Applying the implicit function theorem to (21) with equality yields

$$\frac{\partial p_E}{\partial \theta_1} = -\frac{-q_H p_E}{2q_L p_E + (2-\theta_1-\theta_2)(1-q) - \theta_1 q_L}.$$

Here, since $p_E[q_L p_E + (2-\theta_1-\theta_2)(1-q) - \theta_1 q_L] = (1-q)(1-\theta_2)$ holds, the denominator is positive. In addition, the numerator is negative. Hence, $\frac{\partial p_E}{\partial \theta_1} > 0$.

Next, consider case 2.

$$\frac{\partial p_E}{\partial \theta_1} = \frac{b(1-\theta_1)}{[\theta_1+\theta_2-1+b(2-\theta_1-\theta_2)]^2} > 0.$$

From cases 1 and 2, we have $\frac{\partial p_E}{\partial \theta_1} > 0$.

(ii). Prove that p_E is weakly increasing in q_L. $\frac{1-q}{(b-1)q_L}$ is decreasing in q_L so that there exists $\bar{q}_L$, such that case 2 holds for $q_L < \bar{q}_L$, while case 1 holds for $q_L \geq \bar{q}_L$. Furthermore, p_E under case 2 when $q_L \to \bar{q}_L$ is equal to p_E under case 1 when $q_L = \bar{q}_L$.

Hence, it suffices to prove that $\frac{\partial p_E}{\partial q_L} \geq 0$ for $q_L \geq \bar{q}_L$. Applying the implicit function theorem to (21) with equality yields

$$\frac{\partial p_E}{\partial q_L} = -\frac{p_E(p_E-\theta_1)}{2q_L p_E + (2-\theta_1-\theta_2)(1-q) - \theta_1 q_L}.$$

Here, the denominator is positive. In addition, the numerator is negative since $p_E < \theta_1$ from (a). Hence, $\frac{\partial p_E}{\partial q_L} > 0$ for case 1. $\square$

A7 Proof of Proposition 6

(a) Observe that when $p_i \geq \bar{p}$ and $x_i = 2$, $p_{i+1}(2) = 1 - \theta_2 < \bar{p}$. In addition, $x_i = 2$ at least with probability $(1-\theta_1)q_L$. Thus,

$$\Pr(\forall i \text{ s.t. } M \leq i \leq N : p_i \geq \bar{p}) \leq [1-(1-\theta_1)q_L]^{N-M}.$$

This goes to zero as $N \to \infty$. Hence, we have (a).

(b) Case 1. $\frac{1-q}{(b-1)q_L} > \bar{p}$. Consider how many times $x = 1$ must be observed at most to reach $p_i \geq \bar{p}$. Define

$$p_C \equiv \frac{b[\bar{p}-(1-\theta_2)]}{(b-1)(\theta_1+\theta_2-1)} - \frac{1}{b-1},$$

Note that $p_C < \bar{p}$ holds since $\bar{p} < p_E$ by the assumption.

(i) Suppose that $p_i \geq p_C$. Then, $p_{i+1}(1) \geq \bar{p}$.

(ii) Suppose that $p_i < p_C$. Then,

$$\begin{aligned} p_{i+1} - p_i =& \frac{\theta_1+\theta_2-1+b(1-\theta_2)-p_i[\theta_1+\theta_2-1+b(2-\theta_1-\theta_2)]}{b} \\ \geq& \frac{\theta_1+\theta_2-1+b(1-\theta_2)-\bar{p}[\theta_1+\theta_2-1+b(2-\theta_1-\theta_2)]}{b} > 0. \end{aligned}$$

Here, the first inequality comes from the fact that the function is decreasing in p_i and the last inequality comes from the assumption that $\bar{p} < p_E$. Thus, when $(x_i, ..., x_{i+K^*-1}) = (1, ..., 1)$, $p_{i+K^*} \geq \bar{p}$ holds, where K^* is the smallest integer K satisfying

$$K \frac{\theta_1+\theta_2-1+b(1-\theta_2)-\bar{p}[\theta_1+\theta_2-1+b(2-\theta_1-\theta_2)]}{b} > \bar{p}.$$

Hence, when $(x_i, ..., x_{i+K^*-1}) = (1, ..., 1)$, $p_{i+K^*} \geq \bar{p}$.

From (i) and (ii), $p_{i+K^*} \geq \bar{p}$ holds for all $p_i \in (0,1)$ when $(x_i, ..., x_{i+K^*-1}) = (1, ..., 1)$.

Here, divide $\{M, ..., N\}$ into subgroups $\{M, .., M+K^*-1\}, \{M+K^*, ..., M+2K^*-1\}, ..., \{M+(L+1)K^*, ..., N\}$, where L is the quotient when $N-M+1$ is divided by K^*. Then, from the above discussion,

$$\begin{aligned} &\Pr(p_i < \bar{p} \; \forall i \text{ s.t. } M \leq i \leq N) \\ &\leq \Pr(\forall k \in \{0, ..., L-1\}, \exists i \in \{kK^*+M, ..., M+(k+1)K^*-1\}; x_i \neq 1) \\ &\leq (1-(1-\theta_2)q_L^{K^*})^L. \end{aligned} \tag{23}$$

The first inequality comes from the fact that when $(x_i, ..., x_{i+K^*-1}) = (1, ..., 1)$, $p_{i+K^*} \geq \bar{q}$. The second inequality comes from the fact that at least with probability $(1-\theta_2)q_L$, $x_i = 1$.

Therefore, $\lim_{N\to\infty} \Pr(\forall i \text{ s.t. } M \le i \le N : p_i < \bar{p}) = 0$ because $L \to \infty$.

Case 2. $\frac{1-q}{(b-1)q_L} \le \bar{p}$. Similarly, we have $\lim_{N\to\infty} \Pr(\forall i \text{ s.t. } M \le i \le N : p_i < \bar{p}) = 0$. □

A8 Proof of Fact 3

Prove that $\frac{\partial p_E}{\partial \theta} > 0$. Consider case 1 in the proof of Proposition 5, first. Applying the implicit function theorem to (21) with equality yields

$$\frac{\partial p_E}{\partial \theta} = -\frac{-[2(1-q)+q_L]p_E + 1 - q}{2q_L p_E + 2(1-\theta)(1-q) - \theta q_L}.$$

Here, since $p_E[q_L p_E + 2(1-\theta)(1-q) - \theta q_L] = (1-q)(1-\theta)$ holds, the denominator is positive. In addition, the numerator is negative since it can be rewritten as $(1-q)(1-2p_E) - p_E q_L$ and $p_E > 1/2$. Hence, $\frac{\partial p_E}{\partial \theta} > 0$ in case 1.

Next, consider case 2.

$$\frac{\partial p_E}{\partial \theta} = \frac{b}{[2\theta - 1 + 2b(1-\theta)]^2} > 0.$$

□

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