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PECUNIARY EXTERNALITIES, BANK OVERLEVERAGE, AND MACROECONOMIC FRAGILITY

Ryo Kato
Takayuki Tsuruga

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The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

Pecuniary Externalities, Bank Overleverage, and Macroeconomic Fragility*

Ryo Kato†and Takayuki Tsuruga‡

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Abstract

Pecuniary externalities in models with financial friction justify macroprudential policies for preventing economic agents' excessive risk taking. We extend the Diamond and Rajan (2012) model of banks with the production factors and explore how a pecuniary externality affects a bank's leverage. We show that the laissez-faire banks in our model take on excessive risks compared with the constrained social optimum. Our numerical simulations suggest that the crisis probability is 2–3 percentage points higher in the laissez-faire economy than in the constrained social optimum.

JEL Classification: E3, G01, G21

Keywords: Financial crisis, Liquidity shortage, Maturity mismatch, Credit externalities, Financial regulation

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†Graduate School of Public Policy, The University of Tokyo; e-mail: rkato@pp.u-tokyo.ac.jp

‡Institute of Social and Economic Studies, Osaka University e-mail: tsuruga@iser.osaka-u.ac.jp

1 Introduction

Since the global financial crisis of 2007–08, one of the challenges for policymakers has been how to avoid systemic financial crises. Policymakers and researchers have discussed a pecuniary externality as a rationale for preventing economic agents' excessive risk taking. For example, Mendoza (2010), Korinek (2018), and Jeanne and Korinek (2019) use the small open economy model with the collateral constraint. They show that households' overborrowing takes place because households fail to internalize the general equilibrium effect of the price of the collateral on the collateral constraint.[1] However, as argued in Dávilla and Korinek (2017), it is "remarkably difficult to provide general results on the direction of inefficiency." In other words, even if a model of financial crises has a pecuniary externality, it does not guarantee that economic agents will engage in excessive risk taking. An example is the small open economy model with tradables and nontradables by Benigno, Chen, Otrok, Rebucci, and Young (BCORY, 2013). They show that, once they change the endowment economy model into the production economy model, the model of overborrowing predicts underborrowing in normal times. This example suggests that it is important to check the direction of inefficiency, namely, whether a pecuniary externality creates excessive risk taking in a model of a financial crisis.

In this paper, we take a model of bank runs to investigate the direction of inefficiency. We develop a production economy model with maturity-mismatching banks. Here the model with banks is a variant of Diamond and Rajan (2012, DR) and is discussed in Kato and Tsuruga (2016) in which a liquidity preference shock precipitates a banking crisis. We ask how a pecuniary externality affects allocations under the competitive equilibrium with the laissez-faire (LF) banks. In particular, we introduce the prices of production factors (e.g., the capital price and wages) to ask how pecuniary externalities generate inefficiency. This extension is similar to that in BCORY because we also change the endowment economy model into the production economy model. In our model, banks fail to internalize the effect of their risk taking on factor prices. Our question is whether the pecuniary externality leads to "underleverage" or "overleverage" in a model of banks.

The pecuniary externality in our model leads to inefficient allocations compared with the constrained social optimum that a social planning (SP) agent can achieve. However, our model differs from the abovementioned small open economy model with the collateral

[1] See also other related studies such as Lorenzoni (2008), Bianchi (2010, 2011), Stein (2012), Benigno, Chen, Otrok, Rebucci, and Young (2016), and Bianchi and Mendoza (2018).

constraint in a few aspects. First, the economic agent that engages in inefficient borrowing is a bank rather than a household. Second, the pecuniary externality in our model operates through the constraint on the bank's solvency rather than the collateral. The solvency constraint arises as a natural consequence of the maturity-mismatching banks that issue non-state-contingent short-term debt and invest them in illiquid assets.

Key elements of the model are (i) the bank's issuance of non-state-contingent debt and (ii) the dependence of the banks' asset value on factor prices. The first element enables banks to raise funds and to promote liquidity creation in the absence of complete markets, as discussed in Diamond and Rajan (2001a, b). However, this benefit of non-state-contingent debt comes at the cost of potential insolvency of banks. The second element is incorporated with our overlapping generations (OLG) framework that explicitly includes factor markets. In our model, banks' balance sheet directly depends on the price of capital goods. Under this dependence of banks' balance sheet on capital price, the banks fail to internalize the effect of capital prices on their solvency.

Our main finding is that banks leverage to an excessive level because of a pecuniary externality. Using numerical analysis, we compute the crisis probabilities in the competitive equilibrium. The crisis probability is inefficiently high and the magnitude of the inefficiency is not negligible. In our benchmark simulations, the crisis probability is 6.6 percent under the competitive equilibrium, compared with 4.5 percent under the constrained social optimum. Our OLG framework also generates sharp declines in economic activity in the aftermath of a crisis. When a liquidity shock precipitates a crisis, banks are required to liquidate all illiquid loans to entrepreneurs to repay depositors. The liquidation stops capital goods production and subsequently results in a significant contraction in consumption and output.

Our numerical analysis suggests that the primary source of overleverage is the underestimation of the crisis probability by the price-taking banks. The crisis probability in general equilibrium depends not only on the price of the liquidity (i.e., the interest rate) but also on all the other prices in the economy. In our model, price-taking banks miscalculate the crisis probability because they fail to internalize the general equilibrium effects stemming from factor prices. More specifically, when the LF banks raise their leverage, they underestimate the marginal increase in the crisis probability.

We quantify the importance of the pecuniary externality stemming from the banks' solvency because the banks' solvency directly influences the crisis probability. The banks' solvency depends on the capital price, which the LF banks take as given. We perform a counterfactual experiment. In the experiment, the LF banks internalize changes in the capital price that appear in the solvency constraint while ignoring all other general equilibrium effects stemming from factor prices. The result suggests that the LF banks would keep their leverage close to the constrained social optimum if they could internalize the pecuniary externality stemming from the solvency constraint.

Our paper makes a contribution to the literature by incorporating the pecuniary externality into the theory of banking. Our model extends Allen and Gale (1998) and DR in modeling banks.[2] At the same time, our model borrows the idea of the pecuniary externality from the model with collateral constraint such as Lorenzoni (2008), Bianchi (2011), and Jeanne and Korinek (2019) among others. In our model, the banks' solvency plays a crucial role in generating bank runs or financial crises. Our model further explores the possibility of inefficient financial crises via the dependence of the banks' solvency on factor prices.[3]

Our paper is also related to the macroeconomic literature on models with financial constraints. Macroeconomic models with banks have focused on how financial frictions amplify business cycles (e.g., Gertler and Karadi 2011, Gertler and Kiyotaki 2011, and Meh and Moran 2010). Our focus contrasts with these studies because we examine the efficiency of the competitive equilibrium. Angeloni and Faia (2012) incorporate banks à la Diamond and Rajan (2000, 2001a) into the dynamic stochastic general equilibrium model. While their model has the endogenous probability of bank insolvency, the probability of bank insolvency can be interpreted as a measure of individual bank fragility rather than the probability of systemic financial crises. Our model is also close to Gertler and Kiyotaki (2015) and Boissay, Collard, and Smets (2016). In Gertler and Kiyotaki (2015), whether an equilibrium with bank runs exists depends on macroeconomic fundamentals. However, financial crises per se are precipitated by self-fulfilling expectations as in Diamond and Dybvig (1983). In our model, fundamental shocks (liquidity preferences) precipitate financial crises. In this regard, our model is broadly in line with empirical findings of the business cycle view of Gorton (1988) and Allen and Gale (1998). Boissay, Collard, and Smets (2016) focus on the moral hazard in the interbank markets and successfully replicate systemic financial crises. On the other hand, we stress the fragility of the maturity mismatch of banks, as highlighted by the Gorton and Metrick (2012) "run-on-repo" view.

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[2] Our model assumes that banks issue demand deposits following DR. In our model, however, we broadly interpret banks as financial intermediaries that raise funds via short-term debt such as repo and commercial paper and transform maturities on their balance sheet. Demand deposits are an extreme case of short-term debt.

[3] Stein (2012) introduces pecuniary externalities into the macroeconomic model with banks to focus on the risks of fire sales. Ikeda (2018) studies banks in a global game context and explores banks' excessive risk taking in the general equilibrium model.

The remainder of the paper proceeds as follows. Section 2 illustrates the model. In Section 3, we compare the competitive equilibrium in Section 2 with the constrained social optimum achieved by the SP banks. Section 4 discusses the numerical results and explains why the banking sector tends to be overleveraged. Section 5 concludes.

2 A Macroeconomy with Banks

2.1 Agents, Endowment, Preferences, and Technology

We consider an infinite-horizon OLG model with banks. Each generation of agents consists of households, entrepreneurs, and bankers. Each period, generation t is born at the beginning of period t and lives for two periods, t and $t+1$. Each agent is identical and constant in the population.[4]

Households are risk averse and subject to a liquidity shock that affects their preference for consumption over the two periods. The liquidity shock is an aggregate shock and the only source of uncertainty in the model. Following DR, households are endowed with a unit of consumption goods at birth and do not consume the initially endowed consumption goods at the beginning of period t. The households deposit all initial endowments at banks operating in the same generation.[5] They receive wages w_t in the competitive labor market by supplying one unit of labor in both periods, t and $t+1$.

Entrepreneurs are risk neutral and have access to capital-producing technology. They launch long-term investment projects at the beginning of period t, by borrowing households' endowments via the banks in the same generation. The investment project requires one period for gestation, and capital goods are produced in period $t+1$. We call this capital-producing technology a "project." Entrepreneurs sell the capital goods in the competitive market for the capital price q_{t+1}.

Banks raise funds from households and lend them to entrepreneurs at the beginning of period t.[6] In principle, we follow Diamond and Rajan (2001a) to model banks. Banks are risk neutral and competitive at raising and lending funds in the markets. They issue demand deposits (short-term debt) and commit to repaying the households. As the nature of

[4] We assume that an initial old generation lives for one period.

[5] We implicitly assume intraperiod perishability of endowments. More precisely, all endowments perish before the realization of the liquidity shock.

[6] We assume intragenerational banking, which effectively means that all bankers of generation t die out at the end of period $t+1$.

demand deposits, banks can provide insurance against depositors' liquidity shocks. However, when households demand repayment before the completion of the entrepreneurs' projects, banks must liquidate premature projects to meet the demand for repayment. This maturity mismatch, represented by the combination of long-term assets and short-term liabilities, leaves banks exposed to risks of a default because, depending on the amount of withdrawals in the interim period, the banks' solvency is endangered.

The technology to produce consumption goods Y_t is represented by a standard constant-returns-to-scale Cobb–Douglas production function:

$$Y_t = F\left(K_t, ZH_t\right) = K_t^{\alpha}\left(ZH_t\right)^{1-\alpha},$$

where K_t, H_t, and Z denote the capital stock, hours worked, and labor-augmenting technological progress, respectively. Demand for labor and capital satisfies

$$w_t = F_{H,t} = (1-\alpha)\left(\frac{K_t}{ZH_t}\right)^{\alpha} Z \tag{1}$$

$$q_t = F_{K,t} = \alpha\left(\frac{K_t}{ZH_t}\right)^{\alpha-1}. \tag{2}$$

In what follows, we describe each agent's decisions (consumption, withdrawal, and liquidation of the entrepreneurs' projects) *after* the liquidity shock is realized. Then, we move on to the bank's decision on its leverage *before* the realization of the liquidity shock. Table 1 summarizes the sequence of events in each generation.

2.2 Households

Under the competitive banking sector, each household accepts the banks' offer on deposit face value D_t at the beginning of period t, and observes the liquidity shock θ_t in the middle of period t. The liquidity shock is common across all households in the same generation and has the probability density function $f(\theta_t)$ with a support of $[0, 1]$. This shock represents households' preference for consumption when young and signals the need for liquidity in period t.[7]

[7] Although all households are subject to the same aggregate shock, we assume that an infinitesimally small number of households are believed to face a different θ_t from other households. This assumption ensures the existence of a Nash equilibrium, in which all households run to the banks when they believe that the banks are insolvent under the observed θ_t.

After the realization of θ_t, households make their decisions for consumption smoothing without uncertainty. Given that a crisis does not take place, households then choose withdrawal amount g_t to maximize

$$\begin{aligned} U\left(C_{1,t}, C_{2,t+1}\right) &= \theta_t \log C_{1,t} + (1-\theta_t)\log C_{2,t+1} \\ \text{s.t. } C_{1,t} &= w_t + g_t \\ C_{2,t+1} &= w_{t+1} + R_t\left(D_t - g_t\right), \end{aligned} \tag{3}$$

where $C_{1,t}$ and $C_{2,t+1}$ denote the consumption of households born in period t when young and old, respectively. Each household supplies a unit of labor in each period and receives wage income w_t in period t and w_{t+1} in period $t+1$. Here R_t denotes the one-period gross interest rate from period t to $t+1$.

The intertemporal first-order condition for consumption is

$$\frac{\theta_t}{1-\theta_t}\left(\frac{C_{1,t}}{C_{2,t+1}}\right)^{-1} = R_t. \tag{4}$$

The withdrawals can be written as

$$g_t = \theta_t\left(\frac{w_{t+1}}{R_t} + D_t\right) - (1-\theta_t)\, w_t, \tag{5}$$

which implies that large θ_t and D_t increase their withdrawals.

In our model, a financial crisis takes place endogenously, depending on θ_t and D_t. If θ_t and D_t are sufficiently low, a financial crisis does not take place and households can withdraw g_t in period t and all the remaining deposits in period $t+1$.[8] With the endogenous probability π_t, however, a financial crisis arises. When the banks cannot honor the debt, households run to the banks and their withdrawals equal the liquidation value of premature projects, $X\,(<1)$, in period t and nothing is left in period $t+1$. In the case of a crisis, their consumption ends up with $C_{1,t} = w_t + X$ and $C_{2,t+1} = w_{t+1}$.

2.3 Entrepreneurs

Entrepreneurs maximize their expected lifetime utility represented by $E\left(C^e_{1,t} + C^e_{2,t+1}\right)$, where $C^e_{1,t}$ and $C^e_{2,t+1}$ denote entrepreneurs' consumption when young and old. They use

[8] In the maximization problem of households, we assume that wage income in period t is low relative to the initial endowment, ensuring a nonnegative withdrawal g_t in the equilibrium.

a unit of consumption goods financed by banks for their capital goods production, and this production technology takes one period for gestation before its completion. In period $t+1$, the project yields a random capital goods output $\breve{\omega}$, which is distributed over $[\omega_L, \omega_H]$ with the probability density function $h(\breve{\omega})$.[9] If this project is prematurely liquidated in period t, the transformation from consumption goods into capital is incomplete. As a result, output is reduced to X units of consumption goods and is repaid fully to banks in period t. When the project is completed in period $t+1$, however, entrepreneurs can sell their output in the capital goods market for the capital price q_{t+1}.

Each entrepreneur can borrow from a bank that has, or can learn until the project matures, knowledge about an alternative, but less profitable, method to operate the project. The bank's specific knowledge allows it to generate $\gamma q_{t+1}\breve{\omega}$ from a project outcome with $\gamma < 1$. Once a bank has lent, no one else (including other banks) can learn this alternative way to operate the project. As a result, entrepreneurs accept the financing contract with each bank and repay $\gamma q_{t+1}\breve{\omega}$. They are left with $1-\gamma$ of the share of their profit and enjoy their own consumption based on their linear utility. We assume that entrepreneurs are endowed with $\underline{I}$ units of capital goods at the beginning of period $t+1$.[10] They sell this endowment capital together with the newly created capital made from the consumption goods.

2.4 Banks

Banks maximize their expected lifetime utility $E\left(C^b_{1,t} + C^b_{2,t+1}\right)$, where $C^b_{1,t}$ and $C^b_{2,t+1}$ denote consumption of banks when young and old. We borrow the microfoundation of modeling banks from Diamond and Rajan (2001a, b, 2012). Banks have no initial endowment at birth, and thus they need to raise funds from households. As the relationship lender, a bank knows how to operate the entrepreneurs' project, knowledge that cannot be transferred to households. Banks issue demand deposits (short-term debt) as a commitment device to compensate for the lack of transferability of their knowledge (i.e., collection skills).[11] The demandable nature of deposit contracts creates a collective action problem for depositors: depositors run to the banks whenever depositors anticipate that the banks cannot honor

[9] Following DR, we assume that there is no aggregate uncertainty in the project outcome.

[10] For simplicity, we assume a 100 percent depreciation rate in the law of motion for capital. The introduction of the endowment of capital goods here guarantees a finite capital price in the aftermath of a financial crisis in which all projects are scrapped due to full liquidation.

[11] Diamond and Rajan (2001b) discuss the microfoundation of maturity-mismatching banks and explain why these demand deposits can promote liquidity creation under the lack of transferability of their collection skill.

the debt. The deposit contract is predetermined before observing the liquidity shock. In particular, D_t is predetermined at the beginning of period t, and a liquidity shock is realized in the middle of period t.

Each bank attracts many entrepreneurs through a competitive loan offer, resulting in an identical portfolio shared by all the symmetric banks. This assumption implies that each bank and the aggregate economy face an identical distribution of entrepreneurs. In period t, the banks receive signals ω that perfectly predict the realized value of $\breve{\omega}$ in period $t+1$. With this information ω and the households' liquidity demand observed in period t, each bank chooses one of the following options: (i) to liquidate projects in period t, obtaining X of consumption goods per project; or (ii) to collect a fraction $\gamma q_{t+1}\omega$ from a completed project in period $t+1$. The bank liquidates the project if the outcome of a project falls short of $\tilde{\omega}_{t+1}$, defined as a function of R_t/q_{t+1}:[12]

$$\tilde{\omega}_{t+1} = \frac{X}{\gamma}\frac{R_t}{q_{t+1}}. \tag{6}$$

Otherwise, the bank continues the project, and then receives repayment of $\gamma q_{t+1}\omega$ and entrepreneurs consume the remaining fraction of outcome, $(1-\gamma) q_{t+1}\omega$, per project. After repaying the full amount of the households' withdrawals, the banks consume their capital.

Let the banks' asset be $A(R_t/q_{t+1})$. The banks' assets at the beginning of period t (i.e., prior to the withdrawals) can be expressed as

$$\begin{aligned} A\left(\frac{R_t}{q_{t+1}}\right) &= \int_{\omega_L}^{\tilde{\omega}_{t+1}} Xh(\omega)\,d\omega + \frac{\gamma q_{t+1}}{R_t}\int_{\tilde{\omega}_{t+1}}^{\omega_H} \omega h(\omega)\,d\omega \\ &= L\left(\frac{R_t}{q_{t+1}}\right) + \frac{\gamma q_{t+1}}{R_t} I\left(\frac{R_t}{q_{t+1}}\right). \end{aligned} \tag{7}$$

Note that $h(\omega)$ is interchangeable with $h(\breve{\omega})$ owing to perfect signaling. The banks' assets denoted in (7) can be decomposed into two components: the values of the prematurely liquidated projects $L_t = L(R_t/q_{t+1}) \equiv \int_{\omega_L}^{\tilde{\omega}_{t+1}} Xh(\omega)\,d\omega$, which is used to meet the liquidity demand (i.e., withdrawals) from the households, and the banks' share of the investment output (measured by the present value of consumption goods) denoted as $\gamma q_{t+1} I_t/R_t$, where $I_t = I(R_t/q_{t+1}) = \int_{\tilde{\omega}_{t+1}}^{\omega_H} \omega h(\omega)\,d\omega$.

[12] Equation (6) can be reinterpreted as follows: $\gamma q_{t+1}\omega/X$ corresponds to the marginal rate of transformation (MRT) between the period-t consumption goods (i.e., liquidation) and the period-$t+1$ consumption goods (i.e., continuation of projects). The MRT is here compared with the marginal rate of substitution of the households that is observed as the interest rate, R_t.

The banks are subject to the *solvency constraint*:

$$D_t \leq A\left(R_t/q_{t+1}\right). \tag{8}$$

If ω follows a uniform distribution, $A(\cdot)$ monotonically decreases with R_t/q_{t+1}. We then define the relative price R_t^*/q_{t+1}^* that satisfies the solvency constraint with equality:

$$D_t = A\left(\frac{R_t^*}{q_{t+1}^*}\right). \tag{9}$$

We refer to R_t^* and q_{t+1}^* as the *threshold* interest rate and capital price, respectively. Hereafter, we denote a variable with an asterisk as the variable on the threshold. For the purpose of subsequent discussion, we note that given $A\left(R_t/q_{t+1}\right)$, the bank leverage $D_t/(A_t - D_t)$ is uniquely determined once D_t is chosen, and hence we refer to D_t as leverage hereafter.

2.5 Market Clearing Conditions

Four markets need to clear in the competitive equilibrium: (i) liquidity; (ii) consumption goods; (iii) capital goods; and (iv) labor. The liquidity market clearing condition is given by

$$L\left(\frac{R_t}{q_{t+1}}\right) = \theta_t\left(\frac{w_{t+1}}{R_t} + D_t\right) - (1-\theta_t)\, w_t. \tag{10}$$

Next, the market clearing condition for consumption goods is

$$Y_t + L\left(\frac{R_t}{q_{t+1}}\right) = C_{1,t} + C_{2,t} + C_{2,t}^e + C_{2,t}^b. \tag{11}$$

The left-hand side of (11) includes the supply of goods from the liquidated projects. On the right-hand side of (11), $C_{2,t}$, $C_{2,t}^e$, and $C_{2,t}^b$ are consumption when generation $t-1$ is old.

The capital goods market clearing condition is

$$K_{t+1} = \begin{cases} \underline{I} + I\left(R_t/q_{t+1}\right) & \text{at normal times} \\ \underline{I} & \text{at crises.} \end{cases} \tag{12}$$

Here the equation suggests that the capital goods supply sharply declines, in the aftermath of a crisis. Throughout the paper, we use $\underline{w}$ and $\underline{F}_H$ to denote the wage rate and the marginal product of labor evaluated at $K_{t+1} = \underline{I}$.

Finally, both young and old generations supply a unit of labor in each period. Therefore, H_t equals two for all t.

2.6 Optimal Bank Leverage

We now consider the banks' optimal leverage. The banks are competitive at issuing demand deposits, and households' endowments are scarce compared with entrepreneurs' projects. As a result of competition, the banks make a competitive offer of deposits for households, aiming to maximize household welfare (Allen and Gale 1998, 2007), while, in fact, they are maximizing their profits. Maximizing household utility via the deposit offers means that banks determine the offer taking into account the optimal behavior of the price-taking households.

In making the competitive offer to households, the banks take into account possible changes in the crisis probability π_t. To understand how the banks' choice of D_t affects π_t, we take three steps. First, we define a function R^*_{LF} as

$$R^*_{LF}(D_t) = q^*_{t+1} A^{-1}(D_t), \tag{13}$$

from (9). Second, using (10), we define a function θ^*_{LF} as

$$\theta^*_{LF}(D_t) = \frac{L\left(R^*_t / q^*_{t+1}\right) + w_t}{w_t + D_t + w^*_{t+1}/R^*_t}, \tag{14}$$

where $R^*_t = R^*_{LF}(D_t)$ and w_t, w^*_{t+1}, and q^*_{t+1} are taken as given for the LF banks. Third, we connect θ^*_t to the crisis probability π_t. Equation (13) means that any changes in D_t give rise to changes in R^*_t/q^*_{t+1}. Once R^*_t/q^*_{t+1} is determined, we can calculate the liquidity demand $L\left(R^*_t/q^*_{t+1}\right)$ in which the solvency constraint is satisfied with equality. For the liquidity market to clear, we obtain the threshold level of the liquidity shock on the brink of a financial crisis. Namely, when θ_t is strictly greater than θ^*_t, the banks turn out to be insolvent and a crisis is precipitated. Thus, the crisis probability π_t has a one-to-one relationship to θ^*_t via the probability density function $f(\theta_t)$:

$$\pi_t = \int_{\theta^*_t}^{1} f(\theta_t)\, d\theta_t. \tag{15}$$

We are now ready to set up the optimization problem for the banks to determine the level of their leverage.

Problem LF *The laissez-faire banks maximize the household expected utility*

$$\max_{D_t} \int_0^{\theta_t^*} \left\{\theta_t \ln\left(w_t + L_t\right) + \left(1-\theta_t\right) \ln\left[w_{t+1} + R_t\left(D_t - L_t\right)\right]\right\} f\left(\theta_t\right) d\theta_t$$
$$+ \int_{\theta_t^*}^1 \left[\theta_t \ln\left(w_t + X\right) + \left(1-\theta_t\right) \ln\left(\underline{w}\right)\right] f\left(\theta_t\right) d\theta_t, \tag{16}$$

subject to

$$\begin{aligned} L\left(\frac{R_t}{q_{t+1}}\right) &= \theta_t\left(\frac{w_{t+1}}{R_t} + D_t\right) - \left(1-\theta_t\right) w_t, \\ \theta_{LF}^*\left(D_t\right) &= \frac{L\left(R_t^*/q_{t+1}^*\right) + w_t}{w_t + D_t + w_{t+1}^*/R_t^*}, \end{aligned}$$

where $R_t^* = R_{LF}^*\left(D_t\right)$ *from (13) and* $\theta_t^* = \theta_{LF}^*\left(D_t\right)$ *from (14).*

The first-order condition for their leverage is

$$\begin{aligned} &\left\{\left[\theta_t^* \ln\left(\theta_t^* m_t^*\right) + \left(1-\theta_t^*\right) \ln\left(R_t^*\left(1-\theta_t^*\right) m_t^*\right)\right]\right. \\ &\left. - \left[\theta_t^* \ln\left(w_t + X\right) + \left(1-\theta_t^*\right) \ln\left(\underline{w}\right)\right]\right\} \frac{d\pi_t}{d\theta_t^*} \theta_{LF}^{*\prime}\left(D_t\right) \qquad (17) \\ = &\int_0^{\theta_t^*} \left[\frac{1}{m_t}\left(1 - \frac{w_{t+1}}{R_t^2} R_{LF}'\left(D_t, \theta_t\right)\right) + \left(1-\theta_t\right) \frac{R_{LF}'\left(D_t, \theta_t\right)}{R_t}\right] f\left(\theta_t\right) d\theta_t, \end{aligned}$$

where $m_t \equiv w_t + D_t + w_{t+1}/R_t$ is the lifetime income of households and $m_t^* \equiv w_t + D_t + w_{t+1}^*/R_t^*$, accordingly. More importantly, $\theta_{LF}^{*\prime}\left(D_t\right)$ is calculated from (14), taking capital prices and wages as given. Likewise, $R_{LF}'\left(D_t, \theta_t\right)$ is calculated from (10), taking capital prices, wages, and θ_t as given. Here we slightly abuse notations. While we use the prime for $\theta_{LF}^{*\prime}\left(D_t\right)$ and $R_{LF}'\left(D_t, \theta_t\right)$ to denote marginal changes in θ_t^* and R_t with respect to D_t, they are *not* the total derivative that internalizes the general equilibrium effect of D_t on factor prices. However, these notations will be compared in the next section.

Equation (17) provides an economic intuition. The terms in curly brackets on the left-hand side of (17) represent the loss of utility in a crisis compared with the threshold. From (15), the term outside the curly brackets indicates the marginal changes in a crisis probability with respect to bank leverage. The left-hand side of the equation is the expected loss of utility multiplied by the marginal change in the crisis probability. Simply put, the left-hand side of

(17) is the marginal cost of increasing D_t.

The right-hand side of (17) consists of the effects of increasing leverage on the expected households' utility through their lifetime income. On the one hand, the increase in D_t has a positive effect on households' income: the higher the leverage, the larger the withdrawal, allowing households to enjoy more consumption. On the other hand, the increase in D_t leads to a higher interest rate via liquidity shortage, discounting the households' labor income in period $t+1$, and reducing returns on forgoing withdrawal until period $t+1$. Therefore, as long as the positive effect on lifetime income exceeds the effect on the interest rate, the higher leverage is beneficial to households. Simply put, the right-hand side of (17) is the marginal benefit of increasing D_t.

We define the competitive equilibrium as follows.

Definition (A competitive equilibrium) *A competitive equilibrium consists of allocations and prices* $\{g_t, D_t, L_t, K_t, I_t, H_t, R_t, q_t, w_t\}_{t=0}^{\infty}$ *such that (i) withdrawal decisions are given by (5) for* $\theta_t \leq \theta_t^*$*; (ii) banks' leverage satisfies (17); (iii) banks' liquidity supply is determined by (6); and (iv) all markets clear.*

3 Social Planning Banks

We next consider the SP banks that choose their leverage as the constrained social planner. We call the allocations for the SP banks the constrained social optimum and compare the allocations with those for the LF banks. To lead off the analysis, we clarify the constraint to which the SP banks are subject. The SP banks must make all their decisions before observing θ_t. After realizing θ_t, they are left with no options. In other words, the SP banks are subject to the constraint that they cannot control households' consumption in response to a realized value of θ_t. The allocations chosen by the SP banks must be distinguished from the unconstrained, first-best optimum. The SP banks precommit to payment on their debt regardless of the states realized following their commitment. The extra ability given to the SP banks compared with the LF banks is that the former can internalize all price effects in all markets when they make decisions regarding their leverage.

Formally, Problem SP replaces q_t and w_t in Problem LF with $F_{K,t}$ and $F_{H,t}$, respectively. SP banks are faced with the solvency constraint $D_t \leq A\left(R_t/F_{K,t+1}\right)$. The newly introduced solvency constraint for the SP banks has different effects on the threshold because (9) and

(13) are now replaced with

$$
\begin{aligned}
D_t &= A\left(\frac{R_t^*}{F_{K,t+1}^*}\right), & (18)\\
R_{SP}^*(D_t) &= F_{K,t+1}^* A^{-1}(D_t). & (19)
\end{aligned}
$$

We summarize the SP banks' problem as follows.

Problem SP *The social planning banks maximize household expected utility,*

$$
\begin{aligned}
&\max_{D_t} \int_0^{\theta_t^*} \left\{\theta_t \ln\left(F_{H,t}+L_t\right)+\left(1-\theta_t\right)\ln\left[F_{H,t+1}+R_t\left(D_t-L_t\right)\right]\right\} f\left(\theta_t\right) d\theta_t \\
&+\int_{\theta_t^*}^1 \left[\theta_t \ln\left(F_{H,t}+X\right)+\left(1-\theta_t\right)\ln \underline{F}_H\right] f\left(\theta_t\right) d\theta_t,
\end{aligned}
$$

subject to

$$
\begin{aligned}
L\left(\frac{R_t}{F_{K,t+1}}\right) &= \theta_t\left(\frac{F_{H,t+1}}{R_t}+D_t\right)-\left(1-\theta_t\right)F_{H,t} & (20)\\
\theta_{SP}^*(D_t) &\equiv \frac{L\left(R_t^*/F_{K,t+1}^*\right)+F_{H,t}}{F_{H,t}+D_t+F_{H,t+1}^*/R_t^*}, & (21)
\end{aligned}
$$

where $R_t^* = R_{SP}^*(D_t)$ *from (19) and* $\theta_t^* = \theta_{SP}^*(D_t)$ *from (21).*

The first-order condition for the SP banks' leverage is

$$
\begin{aligned}
&\left\{\left[\theta_t^* \ln\left(\theta_t^* m_t^*\right)+\left(1-\theta_t^*\right)\ln\left(R_t^*\left(1-\theta_t^*\right)m_t^*\right)\right]\right.\\
&\left.-\left[\theta_t^* \ln\left(F_{H,t}+X\right)+\left(1-\theta_t^*\right)\ln\left(\underline{F}_H\right)\right]\right\}\frac{d\pi_t}{d\theta_t^*}\theta_{SP}^{*\prime}(D_t) \qquad (22)\\
=&\int_0^{\theta_t^*}\left[\frac{1}{m_t}\left(1-\frac{w_{t+1}}{R_t^2}R_{SP}'(D_t,\theta_t)\right)+\left(1-\theta_t\right)\frac{R_{SP}'(D_t,\theta_t)}{R_t}\right] f\left(\theta_t\right) d\theta_t,
\end{aligned}
$$

where $m_t \equiv F_{H,t}+D_t+F_{H,t+1}/R_t$ and $m_t^* \equiv F_{H,t}+D_t+F_{H,t+1}^*/R_t^*$. Importantly, the SP banks factor in all general equilibrium effects when they calculate $\theta_{SP}^{*\prime}(D_t)$ and $R_{SP}'(D_t,\theta_t)$.

We define the allocations chosen by the SP banks as follows.

Definition (Constrained social optimum) *Allocations and prices under the constrained social optimum are* $\{g_t, D_t, L_t, K_t, I_t, H_t, R_t, q_t, w_t\}_{t=0}^{\infty}$ *such that (i) withdrawal decisions are* *given by (5) for $\theta_t \leq \theta_t^*$; (ii) banks' leverage satisfies (22); (iii) banks' liquidity supply is determined by (6); and (iv) all markets clear.*

In general, the allocations that the SP banks achieve differ from those achieved by the LF banks because $\theta^{*\prime}_{SP}(D_t)$ and $R'_{SP}(D_t, \theta_t)$ are not necessarily the same as $\theta^{*\prime}_{LF}(D_t)$ and $R'_{LF}(D_t, \theta_t)$, respectively. In fact, because the two problems are subject to the same constraints, any discrepancy in the first-order conditions results in different allocations between the two problems. Moreover, as long as the LF banks calculate $\theta^{*\prime}_{LF}(D_t)$, taking capital prices and wages as given, the LF banks cannot achieve *the constrained social optimum* that the SP banks can achieve. We summarize the result in the following proposition.

Proposition 1 *The allocations under the competitive equilibrium are less efficient than the allocations under the constrained social optimum.*

While the presence of the inefficiency in the competitive equilibrium is clear, the above result does not necessarily imply overleverage by the LF banks. The next subsection focuses on the direction of the inefficiency, relying on the numerical simulations.

4 Numerical Results

4.1 Calibration

This section provides numerical simulations of the model to explore the inefficiency in the competitive equilibrium. We numerically consider the following questions: (i) Do the LF banks take on excessive risks? (ii) If yes, what is the primary source of overleverage? (iii) How much is the solvency constraint quantitatively important for overleverage?

Our calibration mostly follows DR. We set the value of prematurely liquidated project X at 0.95. We assume that ω follows the uniform distribution over a range of $[\omega_L, \omega_H] = [0.5, 3.5]$, similar to the original parameter values in DR. The degree of banks' special collection skills γ is set at 0.9. In addition to parameterization of DR, we need to set several other parameters. We calibrate the capital share in the production function, α, to 1/3. The capital goods endowment received by entrepreneurs, $\underline{I}$, and technological progress Z determine the size of the scarring effect of a financial crisis. Here, we set $\underline{I} = 1$ and $Z = 4$. More importantly, we assume that the liquidity shock θ_t follows the beta distribution with a mean of 0.50 and a standard deviation of 0.07. This parameterization implies a symmetric bell-shaped distribution.

To simulate the model, we numerically solve the nonlinear system of the equations consisting of the first-order conditions and resource constraints.[13] The system of equations includes equations that define the threshold variables, as well as the equations describing normal times. For example, the system includes the solvency constraint (9), which defines the relative price R_t^*/q_{t+1}^* on the brink of a crisis.

Before interpreting the numerical results, we reconfirm the economic interpretations of D_t. In the context of our model, D_t represents the gross return to bank deposits. On the other hand, D_t cannot be translated into an annual percentage rate or an interest rate *per annum*, because the model does not specify the length of each period of time (e.g., one year or ten years). To focus more clearly on the economic interpretations, D_t needs to be translated into a timeless measure such as bank leverage.

4.2 Excessive Risk Taking by LF Banks

We summarize the benchmark results in Table 2. The results suggest that the LF banks take on excessive risks in determining their leverage. Thus, the direction of inefficiency can support macroprudential policy. More specifically, the upper panel of the table reports that the LF banks choose a higher level of leverage D_t than the SP banks. As the first row shows, the LF banks set the level of D_t at 1.061, 1.2 percentage points higher than the constrained social optimum of 1.049. This overleverage of the LF banks leads to a higher crisis probability. Our calibration points to a 6.59 percent crisis probability under the competitive equilibrium compared with 4.50 percent under the constrained social optimum. These numbers may be comparable with some recent empirical studies on crises.[14] The lower panel of Table 2 compares the bank capital ratio defined as $(A_t - D_t)/A_t$ and the output of consumption goods Y_{t+1} under the competitive equilibrium and the constrained social optimum when the realized value of θ_t takes a mean of 0.5. The LF banks are undercapitalized by 1.1 percentage points compared with the constrained social optimum. Nevertheless, production does not substantially differ between the two allocations, provided that a financial crisis does not take place. The above exercise indicates that the welfare loss primarily arises from inefficiently elevated crisis probability.

It would be helpful to see our results graphically. The higher π_t in the competitive

[13] For details, see the Appendix.

[14] Based on data going back to the 1800s, Reinhart and Rogoff (2013) report that the frequencies are 7.2 percent for the advanced economies. The Basel Committee on Banking Supervision (2010, BCBS) uses multiple datasets on banking crises. In BCBS, the frequency ranges from 3.6 to 6.8 percent.

equilibrium means that the economy undergoes too frequent crises created by pecuniary externalities. To illustrate this, we run the model over 100 periods by generating liquidity shocks randomly. The upper panel of Figure 1 plots the dynamic paths of output Y_t generated by the same random shock sequences across two economies. The red lines correspond to the competitive equilibrium, while the blue lines point to the constrained social optimum.

Comparisons between the two economies reveal that the dynamic paths of the production of consumption goods are almost identical except for more frequent crises under the competitive equilibrium. Under the competitive equilibrium, crises take place in periods 5, 16, and 94, and Y_t falls sharply in each subsequent period. This simulation result indicates that the last two crises should have been prevented if the banks had taken risks at the optimal level. However, the first crisis takes place even in the constrained social optimum with the SP banks. The crisis in this allocation should not be avoided in the context of the optimal financial crises (see Allen and Gale 1998).[15]

In the model, the liquidity shock θ_t precipitates financial crises. As the lower panel of Figure 1 shows, financial crises in our model take place only when θ_t exceeds the threshold θ_t^*. For example, the liquidity shock θ_t in period 5 exceeds the curves of θ_t^* for both the LF banks and the SP banks, and the output of consumption goods falls in period 6. However, θ_t^* for the LF banks is always lower than θ_t^* for the SP banks. It implies that the financial system under the competitive equilibrium is more vulnerable to liquidity shocks than under the constrained social optimum.

The overleveraged LF banking system that results in inefficient financial crises remains a robust outcome across a range of calibrated parameters (e.g., X and distributions of ω and θ_t).[16] In addition to robustness, sensitivity analysis regarding the volatility of θ_t has noteworthy implications for the inherent fragility of the banking system. Table 3 examines how bank leverage and crisis probabilities are affected by the volatility of θ_t. In each simulation, the standard deviation of θ_t changes from the benchmark value of 0.07 to either 0.02 or 0.10, with other parameters held unchanged at the benchmark calibration. The table, together with the benchmark results, confirms that the crisis probability rises monotonically along with the increase in the standard deviation of θ_t. Note that smaller volatility of the liquidity shock does not fully wipe out the financial crisis because the banks raise their leverage under the less volatile liquidity demand. As a result, a financial crisis always remains a nonzero

[15] Their model includes a single liquidity market where the banks internalize all the effects through the single price changes. In contrast, our model includes multiple markets (e.g., the capital market), in which banks fail to internalize their general equilibrium effect.

[16] The tables of the sensitivity analysis are available upon request.

probability event.

4.3 The Primary Source of Overleverage

To understand the LF banks' overleverage in more detail, we numerically compare the optimality condition for D_t under the two allocations. Figure 2 plots the marginal cost and benefit of increasing the leverage under the two allocations. In the figure, we compute the curves based on (17) and (22).[17] The solid blue line represents the marginal cost for the SP banks (the right-hand side of (22)) while the blue dashed line is the marginal benefit for the SP banks (the left-hand side of (22)). The red solid and dashed lines are the marginal cost and benefit for the LF banks, respectively, and calculated from (17). Under the competitive equilibrium, D_t is 1.061 at the intersection of the two red lines. In Figure 2, however, the marginal benefit for the SP banks is not as high as that for the LF banks. Numerically, if we evaluate the SP banks' marginal benefits at D_t in the competitive equilibrium, the marginal benefit for the SP banks is 4.5 percent lower than that for the LF banks. By contrast, the distortion in the marginal cost is much larger than in the marginal benefit. The marginal cost for the SP banks is not as low as the marginal cost for the LF banks. Numerically, if we evaluate the SP banks' marginal cost at D_t in the competitive equilibrium, the marginal cost for the SP banks is 29.2 percent higher than that for the LF banks.

Given that the distortion is substantially large in the marginal cost, it would be useful to focus on the marginal cost as the primary source of overleverage. We are interested in the difference between the left-hand side of the equations (17) and (22). At a given level of D_t in the figure, our observation is as follows.

$$
\begin{aligned}
& \{[\theta_t^* \ln(\theta_t^* m_t^*) + (1-\theta_t^*) \ln(R_t^* (1-\theta_t^*) m_t^*)] \\
& - [\theta_t^* \ln(w_t + X) + (1-\theta_t^*) \ln(\underline{w})]\} \frac{d\pi_t}{d\theta_t^*} \theta_{SP}^{*\prime}(D_t) \\
> \; & \{[\theta_t^* \ln(\theta_t^* m_t^*) + (1-\theta_t^*) \ln(R_t^* (1-\theta_t^*) m_t^*)] \\
& - [\theta_t^* \ln(F_{H,t} + X) + (1-\theta_t^*) \ln(\underline{F}_H)]\} \frac{d\pi_t}{d\theta_t^*} \theta_{LF}^{*\prime}(D_t)
\end{aligned}
$$

All the difference between the marginal cost for the SP banks and the LF banks arises from the terms outside of the curly brackets because the marginal products replace all factor prices. Because the expression inside the curly brackets is naturally positive,[18] the above

[17] For the details of computations, see the Appendix.

[18] For the expression inside the curly brackets, note that $\theta_t^* \ln(\theta_t^* m_t^*) + (1-\theta_t^*) \ln(R_t^* (1-\theta_t^*) m_t^*)$ is the

expression can further be simplified to

$$\frac{d\pi_t}{d\theta_t^*}\theta_{SP}^{*\prime}(D_t) > \frac{d\pi_t}{d\theta_t^*}\theta_{LF}^{*\prime}(D_t). \tag{23}$$

The above equation indicates that the primary source of the overleverage in our model is the banks' evaluation of the marginal changes in the crisis probability with respect to D_t. The LF banks take other banks' decisions as given. However, the crisis probability is affected by the synchronized decisions by the banking sector as a whole. Equation (23) means that the LF banks underestimate the increase in the crisis probability when they increase their leverage. As a result, the LF banks are likely to be overleveraged. In our numerical simulations, $(d\pi_t/d\theta_t^*)\,\theta_{SP}^{*\prime}(D_t) = 1.993$ and $(d\pi_t/d\theta_t^*)\,\theta_{LF}^{*\prime}(D_t) = 1.544$. If leverage is increased by one percent from the allocation in the competitive equilibrium, each LF bank expects that the crisis probability increases up to 8.22 percent, but they are, in fact, exposed to a higher crisis probability of 8.69 percent.[19]

4.4 The Importance of the Pecuniary Externality Stemming from the Solvency Constraint

The tightness of the solvency constraint is closely related to the crisis probability. Recall that the solvency constraint of the LF banks is $D_t = A\left(R_t^*/q_{t+1}^*\right)$, or equivalently $R_{LF}^*(D_t) = q_{t+1}^* A^{-1}(D_t)$. When the LF banks determine the leverage, they internalize R_t^*. However, they take q_{t+1}^* as given because q_{t+1}^* is the price outside the liquidity market. The overestimation of the banks' solvency leads to their overleverage and the inefficiently high crisis probability.

To assess the importance of the pecuniary externality stemming from the solvency constraint, we perform a counterfactual experiment. Suppose that hypothetical banks internalize changes in q_{t+1}^* in the solvency constraint $D_t = A\left(R_t^*/q_{t+1}^*\right)$ while they do not internalize other wedges caused by the pecuniary externalities (e.g., wages). In particular, the hypothetical banks calculate the direct effects of changes in q_{t+1}^* on their solvency as the SP banks

lifetime utility on the brink of a financial crisis while $[\theta_t^* \ln(w_t + X) + (1-\theta_t^*)\ln(\underline{w})]$ is the lifetime utility at crisis evaluated at $\theta_t = \theta_t^*$. The first-order conditions are satisfied at $\theta_t = \theta_t^*$ in the former but not in the latter. As long as $X < 1$, the expression inside the curly brackets is strictly positive.

[19] These increases in the probability are obtained by transforming the marginal changes in the probability into the semielasticity of the probability: $(d\pi_t/d\theta_t^*)\,\theta_{LF}^{*\prime}(D_t) \times D_{LF,t} = d\pi_t/(dD_t/D_{LF,t}) = 1.64$ and $(d\pi_t/d\theta_t^*)\,\theta_{SP}^{*\prime}(D_t) \times D_{LF,t} = d\pi_t/(dD_t/D_{LF,t}) = 2.11$. Together with 6.58 percent of the crisis probability, the crisis probability perceived by the LF banks is $8.22\,(= 6.58 + 1.64)$ but the true probability is $8.69\,(= 6.58 + 2.11)$.

do. Apart from the effects on their solvency, they take other prices as given, as in the LF banks' decision-making. While the allocations implied by the hypothetical banks are not sustainable as an equilibrium, this counterfactual experiment helps assess the importance of the pecuniary externality stemming from the solvency constraint.

The pecuniary externality stemming from the solvency has a substantial impact on allocations. In the simulations, the banks choose their leverage at $D_t = 1.047$. The leverage in the counterfactual allocation is lower than the leverage in the competitive equilibrium (1.061). As a result, the crisis probability is also low (4.18 percent) compared with 6.58 percent in the competitive equilibrium. This level of the leverage ($D_t = 1.047$) is close to that in the constrained social optimum (1.049). Consequently, the crisis probability of 4.18 percent is close to the probability under the constrained social optimum (4.50 percent). The proximity to the constrained social optimum implies that the welfare level is also close to the constrained social optimum.

Figure 3 additionally plots the marginal cost against D_t for the hypothetical banks. For comparison, we replicate the curves for the marginal cost and the marginal benefit shown in Figure 2. The marginal benefit for hypothetical banks is not shown in the figure because it remains the same as the marginal benefit for the LF banks. All the difference from the competitive equilibrium is the internalization of the pecuniary externality stemming from the solvency constraint. In the figure, the internalization leads to the upward shift of the curve for marginal cost. The marginal cost for the hypothetical banks is 41.6 percent higher than the marginal cost for the LF banks, if we evaluate them at D_t in the competitive equilibrium.

We emphasize that our experiment is counterfactual. The LF banks have no incentive to internalize this pecuniary externality in the solvency constraint. From the households' viewpoint, $D_t = 1.047$ is less attractive to households than $D_t = 1.061$ in the competitive equilibrium. Each LF bank is selfish and lacks incentives to coordinate with each other. The competition across the LF banks ill-incentivizes banks to take on excessive risks systemically.

5 Conclusions

We extended the Diamond and Rajan (2012) model of banks with the production factors and explore how a pecuniary externality affects a bank's leverage. This extension generates a pecuniary externality because the banks do not internalize their general equilibrium effect of the factor prices on the economy. As argued in the existing literature on the pecuniary externality, however, the direction of inefficiency is model dependent. In other words, the pecuniary externality itself does not justify the macroprudential policies.

Using numerical simulations, we showed that banks in the competitive equilibrium could take on excessive risks, compared with the constrained social optimum. In the benchmark calibration, the crisis probability is 6.58 percent in the competitive equilibrium, higher than the 4.50 percent under the constrained social optimum. We numerically showed that overleverage mainly comes from the underestimation of the crisis probability when banks choose their leverage. In our simulation, when they increase their leverage by one percent, they underestimate the crisis probability to be 8.22 percent in contrast to the actual crisis probability of 8.69 percent. Our findings include the importance of the pecuniary externality stemming from the bank's solvency for generating their overleverage. We performed counterfactual analysis in which banks internalize only the pecuniary externality stemming from the solvency constraint. In this case, the chosen leverage is very close to the constrained social optimum.

The analysis in this paper can be extended in a number of directions. First, it would be useful to examine how newly introduced aggregate shocks (e.g., shocks to the asset side of banks' balance sheets) affect the economy's exposure to crisis risks. Second, the optimal macroprudential policy could be analyzed. Third, our framework may be translated into an infinitely-lived agent model for integration with quantitative business cycle studies. All of these directions would provide important avenues for future research.

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A System of Equations

Given K_t, the system of equations consists of 15 equations and 15 unknown variables D_t, π_t, R_t, K_{t+1}, q_{t+1}, w_t, w_{t+1}, m_t, H, R_t^*, θ_t^*, K_{t+1}^*, q_{t+1}^*, w_{t+1}^*, and m_t^*. Of the 15 equations, 9 specify the economy in normal times, and 6 define the threshold variables. Below, we solve the system of equations given K_t. After solving the system of equations, we can compute other endogenous variables such as output $Y_t = F(K_t, ZH_t)$ and the bank capital ratio $(A_t - D_t)/A_t$.

- Equations at normal times
  - The first-order condition for the leverage:[20]

$$\begin{aligned} & \{[\theta_t^* \ln(\theta_t^* m_t^*) + (1-\theta_t^*) \ln(R_t^*(1-\theta_t^*) m_t^*)] \\ & - [\theta_t^* \ln(w_t + X) + (1-\theta_t^*) \ln(\underline{w})]\} \frac{d\pi_t}{d\theta_t^*} \theta_{LF}^{*\prime}(D_t) \\ = & \int_0^{\theta_t^*} \left[\frac{1}{m_t} \left(1 - \frac{w_{t+1}}{R_t^2} R'_{LF}(D_t, \theta_t) \right) + (1-\theta_t) \frac{R'_{LF}(D_t, \theta_t)}{R_t} \right] f(\theta_t) \, d\theta_t, \end{aligned} \tag{24}$$

  where $\underline{w}$ is the wage in the crisis: $\underline{w} = (1-\alpha)[\underline{I}/(ZH)]^\alpha Z$. The probability density function $f(\theta_t)$ is computed from the beta distribution. We use numerical integration for the right-hand side of (24).

  - The crisis probability:

$$\pi_t = \int_{\theta_t^*}^1 f(\theta_t) \, d\theta_t, \tag{25}$$

  which implies that the marginal change in the crisis probability with respect to θ_t^* is $d\pi_t / d\theta_t^* = -f(\theta_t^*)$.

  - Liquidity market clearing condition:

$$L\left(\frac{R_t}{q_{t+1}}\right) = \theta_t \left(\frac{w_{t+1}}{R_t} + D_t \right) - (1-\theta_t) w_t. \tag{26}$$

  - Capital market clearing condition for $\theta_t \leq \theta_t^*$:

$$K_{t+1} = \underline{I} + I\left(\frac{R_t}{q_{t+1}}\right). \tag{27}$$

[20] We use the numerical integration for the right-hand side of (24).

If $\theta_t > \theta_t^*$, $K_{t+1} = \underline{I}$.

– Capital demand from firms:

$$q_{t+1} = \alpha \left(\frac{K_{t+1}}{ZH} \right)^{\alpha-1}. \tag{28}$$

– Labor demand from firms in period t and $t+1$:

$$w_t = (1-\alpha) \left(\frac{K_t}{ZH} \right)^{\alpha} Z, \tag{29}$$

$$w_{t+1} = (1-\alpha) \left(\frac{K_{t+1}}{ZH} \right)^{\alpha} Z. \tag{30}$$

– Lifetime income:

$$m_t = w_t + D_t + \frac{w_{t+1}}{R_t}. \tag{31}$$

– Labor supply:

$$H = 2. \tag{32}$$

- Equations for the threshold variables

  – The solvency constraint with equality:

$$D_t = L \left(\frac{R_t^*}{q_{t+1}^*} \right) + \frac{\gamma q_{t+1}^*}{R_t^*} I \left(\frac{R_t^*}{q_{t+1}} \right). \tag{33}$$

  – Liquidity market clearing condition:

$$L \left(\frac{R_t^*}{q_{t+1}^*} \right) = \theta_t^* \left(\frac{w_{t+1}^*}{R_t^*} + D_t \right) - (1-\theta_t^*) w_t. \tag{34}$$

  – Capital market clearing condition:

$$K_{t+1}^* = \underline{I} + I \left(\frac{R_t^*}{q_{t+1}^*} \right). \tag{35}$$

  – Capital demand from firms:

$$q_{t+1}^* = \alpha \left(\frac{K_{t+1}^*}{ZH} \right)^{\alpha-1}. \tag{36}$$

– Labor demand from firms:

$$w_{t+1}^{*}=(1-\alpha)\left(\frac{K_{t+1}^{*}}{ZH}\right)^{\alpha}Z. \tag{37}$$

– Lifetime income:

$$m_{t}^{*}=w_{t}+D_{t}+\frac{w_{t+1}^{*}}{R_{t}^{*}}. \tag{38}$$

Leverage and the crisis probability in Tables 2 and 3 are calculated under the steady-state value of $K_t = K_{t+1}$ for any t. To generate the dynamic paths of Y_t in Figure 1, we use the steady-state value of capital under the competitive equilibrium as the initial value and simulate θ_t. Once θ_t is realized, we can update the capital stock in the next period and solve the system of the equations consisting of (24)–(38). If $\theta_t \leq \theta_t^*$, $Y_{t+1} = K_{t+1}^{\alpha}(ZH)^{1-\alpha}$ where K_{t+1} is given by (27). Otherwise, $Y_{t+1} = \underline{I}^{\alpha}(ZH)^{1-\alpha}$. To plot the marginal cost and the marginal benefit in Figures 2 and 3, we compute the threshold values of K_{t+1}^* over different values of D_t. The initial value of capital is the steady-state value of K_t under the competitive equilibrium. After computing the sequence of K_{t+1}^* that corresponds to the value of D_t, we calculate q_{t+1}^*, w_{t+1}^*, and θ_t^* from (36), (37), and (34), respectively. These threshold variables allow us to compute the marginal cost given by the left-hand side of the equations (17) or (22). Regarding the marginal benefit, we log-linearize the equation for K_{t+1} around the steady state and compute R_t and w_{t+1} along with the partial derivative of R_t with respect to D_t. Given θ_t^*, we take the numerical integration over θ_t.

Table 1: Sequence of events for generation t

		Period t
1.		Households receive endowments.
2.		Banks offer deposits to households and loans to entrepreneurs.
3.		Entrepreneurs launch their projects.
4.		Households supply labor and receive wages w_t determined by that labor market conditions along with the old generation's labor supply.
5.		Liquidity shock θ_t is realized, and banks receive signals of project outcomes.
6.		Households decide the withdrawal amount g_t.
7.		Banks decide which projects to discontinue and supply liquidity L_t.
	(i)	If $g_t > L_t$, a financial crisis is precipitated and households receive repayment of X.
	(ii)	Otherwise, households can transfer their wealth into the period $t+1$.
8.		All agents consume.
		Period $t+1$
1.		Entrepreneurs receive endowments.
2.		Entrepreneurs' projects are completed, and they sell their capital goods for q_{t+1} and make repayment to banks.
3.		Households supply labor and receive wages w_{t+1} determined by that labor market conditions along with the young generation's labor supply.
4.		Households fully withdraw deposits, if any.
5.		All agents consume.

Table 2: Crisis probabilities and allocations under LF banks and SP banks

	SP banks	LF banks
Leverage and crisis probabilities		
D_t	1.049	1.061
π_t (%)	4.499	6.585
Bank capital and output		
Bank capital ratio (%)	15.097	13.952
Y_{t+1}	5.459	5.457

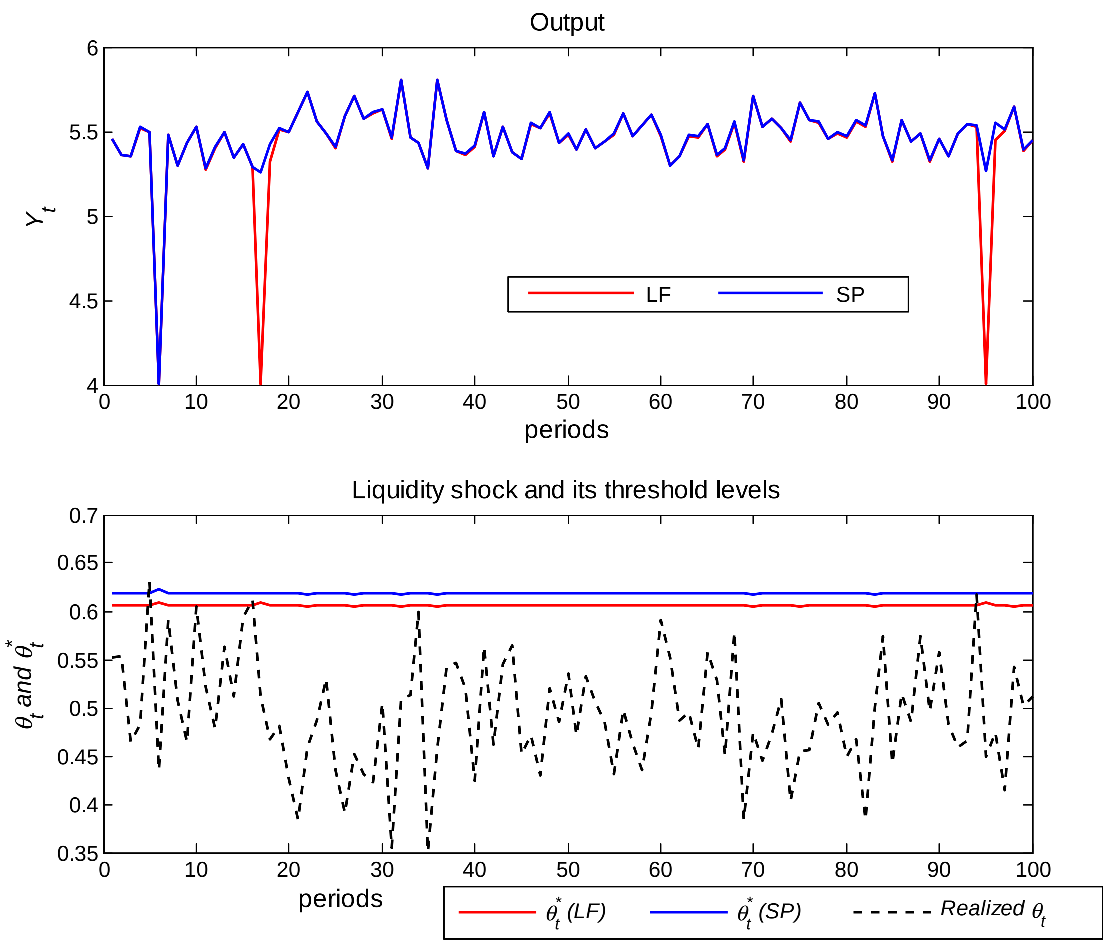
Note: Simulation results are based on the assumption that the liquidity shock θ_t follows the beta distribution. The level of bank leverage D_t and the crisis probability π_t are obtained from Problems LF and SP, respectively. The bank capital ratio is $(A_t - D_t)/A_t$.

Table 3: Financial crisis probabilities and bank leverage

	SP banks	LF banks	SP banks	LF banks
Leverage and probabilities	std(θ_t) = 0.02		std(θ_t) = 0.10	
D_t	1.129	1.132	1.014	1.030
π_t (%)	1.275	1.703	5.997	9.433

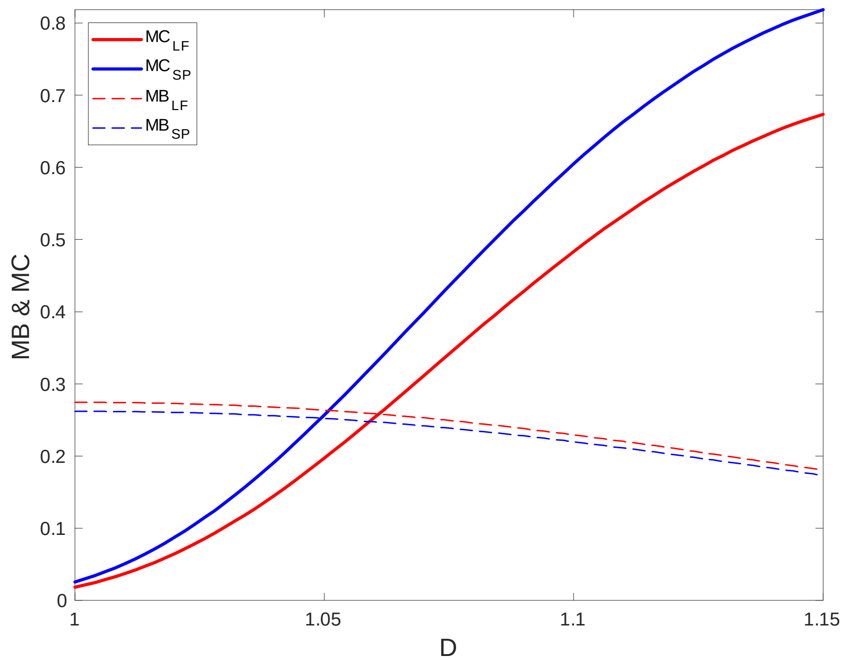
Note: Simulation results are based on various values of the standard deviation of the liquidity shock θ_t. The other details can be seen in the note for Table 2.

Figure 1: Simulated paths of output and the liquidity shock



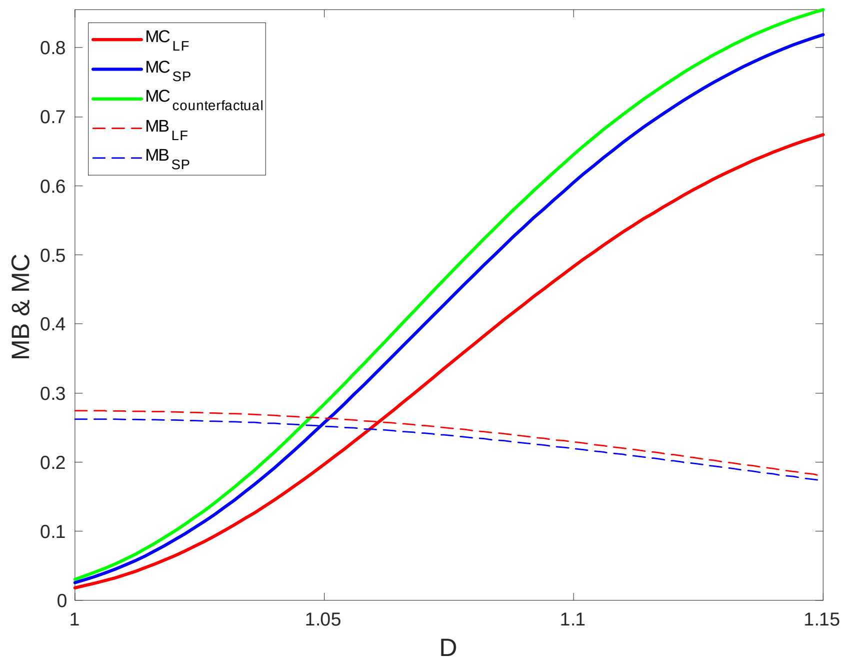
Note: The blue lines (denoted by SP) are the constrained social optimum with the SP banks. The red lines (denoted by LF) are the competitive equilibrium with the LF banks. The upper panel shows the simulated dynamic paths of output Y_t. The liquidity shock plotted as the dashed black line in the lower panel is generated from the beta distribution with a mean of 0.50 and a standard deviation of 0.07. The solid blue and red lines in the lower panel are the threshold level of the liquidity shock that satisfies the solvency constraint with equality.

Figure 2: Marginal cost and benefit against D_t



Note: The blue and red solid lines represent the marginal cost under the SP banks calculated from the left-hand side of (22) and the marginal cost under the LF banks calculated from the left-hand side of (17), respectively. The blue and red dashed lines refer to the marginal benefit under the SP banks calculated from the right-hand side of (22) and the marginal benefit under the LF banks calculated from the right-hand side of (17), respectively.

Figure 3: Marginal cost for the hypothetical banks



Note: The blue and red lines replicate the curves for marginal cost and marginal benefit in Figure 2. The newly added curve represents the marginal cost for the banks in the counterfactual experiment. The marginal benefit in the counterfactual experiment coincides with the marginal benefit for the LF banks.