Discussion Paper No. 1113

ISSN (Print) 0473-453X
ISSN (Online) 2435-0982

DEMOGRAPHIC STRUCTURE, KNOWLEDGE DIFFUSION, AND ENDOGENOUS PRODUCTIVITY GROWTH

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December 2020

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Abstract

This paper considers how increasing longevity and declining birth rates affect market entry and endogenous productivity growth in a two-country model of trade. In each country, the demographic transition to an older population induces a contraction in the labor force through a decline in the working-age population. Firm-level investment in process innovation generates productivity growth, and with imperfect knowledge diffusion the country with the larger labor force has a greater share of firms with higher productivity levels. In this framework, population aging reduces a country's labor supply, share of industry, and relative productivity. If the country with the smaller labor force experiences population aging, knowledge spillovers improve and the rate of productivity growth rises, as the level of market entry falls. Alternatively, population aging in the country with the larger labor force weakens knowledge spillovers and lowers the rate of productivity growth, but has an ambiguous affect on market entry. We show that the effects of population aging may be reversed by extending retirement age, and consider the welfare implications for demographic transition and retirement age extension arising in our framework through a quantitative analysis based on population data for the United States and Western Europe.

Key Words: Demographic Structure, Population Aging, Knowledge Diffusion, Industry Location, Productivity Growth

***JEL* Classification**: F43; J11; O30; O40

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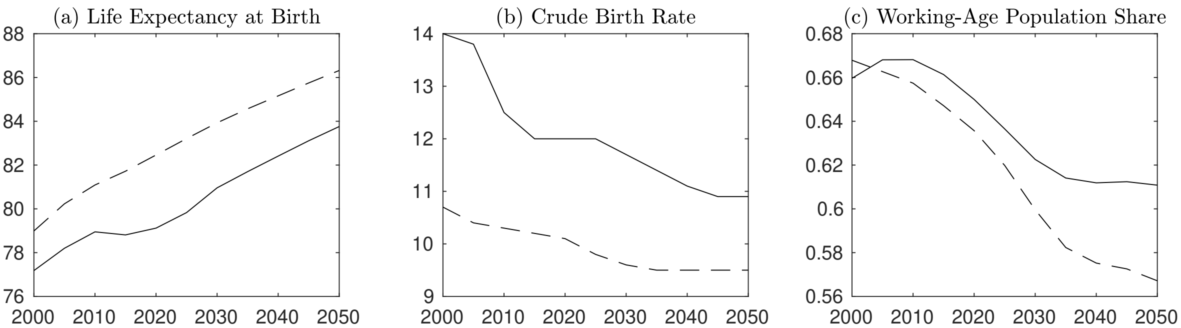
Acknowledgements: This research was financially supported by the Joint Usage/Research Center at ISER (Osaka University) and KIER (Kyoto University), and the Japan Society for the Promotion of Science: Grants-in-Aid for Scientific Research 15H05728, 16H02016, 16K03624, 19K01638, 20H05631, 20K01631, and 20K01569.

Figure 1: Projected Demographic Transition in the US and Western Europe



The solid and dashed lines plot values for the US and for Western Europe, which includes Austria, Belgium, France, Germany, Liechtenstein, Luxembourg, Monaco, the Netherlands and Switzerland. Data source: UN World Population Prospects 2019.

1 Introduction

Over the last half century, a demographic transition characterized by rising longevity and falling fertility rates has lead to a drastic decline in the per-capita labor forces of most developed countries. For example, Figure 1 shows the UN projections for life expectancy at birth, the crude birth rate (births per 1000 people), and the resulting decrease in the working-age population share of the US and Western Europe (WE) between 2000 and 2050. Importantly, the decline in labor force participation has the potential to slow economic growth through contractions in both per capital labor forces and aggregate savings (Bloom et al. 2010). A less well understood implication of population aging for economic growth, however, is the shift in industry location patterns that follows the transition in the geographic distribution of demand (Sato and Yamamoto 2005; Grafeneder-Weissteiner and Prettner 2013). Indeed, there is now a well-developed literature stressing the importance of industry location patterns for innovation-based economic growth (Baldwin and Martin 2004). But, further research is needed to understand how demographic transition affects long-run growth through adjustments in industry location patterns.

To this end, in this paper, we develop an endogenous market structure and endogenous growth framework (Smulders and van de Klundert 1995; Peretto 1996; Aghion and Howitt

1998; Laincz and Peretto 2006; Peretto 2018) to consider how population aging affects market entry and economic growth by inducing shifts in the geographic location of industry. In particular, we adapt the two-country model of Davis and Hashimoto (2015) to include individuals with heterogenous ages, following Dinopoulos and Segerstrom (1999). Importantly, national labor supply depends endogenously on the working-age share of the population, directly tying adjustments in market size with the dynamics of demographic transition. Monopolistically competitive firms assume a central role in the model, employing labor in the production of differentiated product varieties and in process innovation aimed at the reduction of future production costs. Each firm's labor productivity in process innovation depends on the weighted average of the productivities of observable technologies, with a greater weighting applied to domestically employed technologies to capture the imperfect nature of international knowledge diffusion.

In long-run equilibrium, national rates of productivity growth converge, equalizing knowledge spillovers and firm-level employment in process innovation across countries. With common innovation costs, the country with the larger labor supply (i.e., the larger country) attracts a greater share of firms, each with a higher productivity level than the firms located in the country with the smaller labor supply (i.e., the smaller country). Accordingly, the model links industry location patterns with market entry and aggregate productivity growth; that is, an increase in the concentration of more productive firms in the larger country improves knowledge spillovers and raises firm-level employment in process innovation, thereby generating a faster rate of productivity growth. The increase in employment in innovation, however, raises firm-level costs inducing exit and reducing the level of market entry.

We use the framework to study how demographic transition to an older population affects market entry and productivity growth. First, population aging in the smaller country reduces its labor supply, and thus its market size, leading to an increase in the concentration of industry in the larger country. As a result, the rate of productivity growth accelerates. In addition, the contraction of the smaller country's labor supply and the increased scale of employment in process innovation combine to ensure that the level of market entry falls. Second, population aging in the larger country causes a contraction in its market size that lowers the level of industry concentration and slows the rate of productivity growth. The level of market entry may rise or fall, however, depending on whether the negative effect of a smaller labor supply, or the positive effect of lower firm-level employment in process innovation dominates.

An often suggested policy for mitigating the effects of population aging is retirement age extension. We consider the implications of a policy of raising the retirement age proportionately with increases in life expectancy at birth. First, retirement age extension in the smaller country expands its labor force, reducing the concentration of industry in the larger country, and slowing productivity growth. With a larger labor force and lower firm-level employment in process innovation, the level of market entry rises. Second, retirement age extension in the larger country increases its labor force, raising the concentration of industry, and accelerating productivity growth. Once again, however, the level of market entry may rise or fall, depending on the balance of the positive effect of a greater labor supply and the negative effect of higher firm-level employment in process innovation

To analyze the welfare effects of changes in demographic structure, we complete a numerical analysis of our model using the UN projections for life expectancy at birth in the US and Western Europe (WE) over the period from 2000 to 2050. In our analysis, a relatively large labor force supports a greater share of firms with higher productivity levels in the US. The UN projections suggest that life expectancy will rise at similar rates in each region, implying that the impacts of declining working-age population shares in the US and WE counterbalance. The result is a rather stable relative labor supply over the period of analysis, and accordingly, the US share of firms and the international productivity differential only exhibit small upward adjustments, resulting in a negligible increase in the rate of productivity growth. In contrast, as firm-level employment in innovation is for the most part constant, the direct effect of the decrease in the working-age population shares of the US and WE is a lower level of market entry. Consequently, our numerical analysis projects that the decline in labor income associated with a contraction in the working-age population share, and the reduction in the level of market entry, becomes the key driver of falling US and WE welfare levels.

We then introduce retirement age extension as a means of reversing the negative welfare effects of population aging.[1] In particular, we consider a policy exercise where retirement age is raised proportionately with increases in expected lifespan in each region. Under such a policy setting, the comparable increases in the UN projections for the expected lifespans of the US and WE translate into similar patterns for retirement age extension in each region, with the resulting increases in the labor supplies counterbalancing another. As such, the overall impacts on the relative labor supply, the US share of firms, the international productivity differential, and the productivity growth rate are negligible. Therefore, our numerical analysis suggests that a policy of retirement age extension would generate welfare improvements in the US and WE between 2000 and 2050 by raising labor income through an expansion of the working-age share of the population, and increasing the level of market entry.

Our framework contributes to a broad literature considering the macroeconomic effects of demographic change. This literature emphasizes three channels through which population aging affects economic growth: the per capita labor force, savings and investment behavior, and educational enrollment and human capital formation (Bloom and Williamson 1998; Cutler et al. 1990; Bloom et al. 2010; Acemoglu and Restrepo 2017). Building on this literature, a burgeoning number of studies investigate the relationship between demographic change and innovation-based economic growth (Prettner 2013; Prettner and Trimborn 2017; Baldanzi et al. 2019; Gehringer and Prettner 2019; Kuhn and Prettner 2020). These studies indicate that population aging tends to expand aggregate savings, causing a fall in the interest rate that lowers the cost of innovation and raises the incentive for private investment in research and development (R&D). In Hashimoto and Tabata (2016), rising life expectancy promotes investment in human capital, raising labor productivity in R&D, while also slowing population growth and reducing the supply of researchers. As a result, their framework

[1] According to OECD (2019), the average effective male retirement age in OECD countries gradually increased from 63.1 in 2000 to 65.4 in 2018.

yields an inverted U-shaped relationship between population aging and economic growth. In a similar vein, Bucci and Prettner (2020) show that the strength of a human capital dilution effect, with faster population growth slowing the rate of human capital accumulation, determines the relationship between population aging and economic growth. In contrast, this paper contributes to the literature with the introduction of an endogenous market structure and endogenous growth framework wherein demographic change influences market entry and economic growth by inducing shifts in the geographic location of industry.[2]

In general, empirical research on the impact of population aging on economic growth has not produced an unequivocal conclusion. For example, using panel data for OECD countries, Lindh and Malmberg (1999) report a negative correlation between initial elderly population shares and subsequent growth per worker over the period from 1950 to 1990, while Aksoy et al. (2019) conclude that demographic transition to an older society led to a decline in innovation that slowed investment and output growth between 1970 and 2014. In addition, Prskawetz et al. (2007) consider data for EU-15 countries and observe that a larger old-age population had a negative affect on growth in GDP per worker between 1950 and 2005. Alternatively, using panel data for 169 countries over the period from 1990 to 2015, Acemoglu and Restrepo (2017) find no evidence of a negative relationship between population aging and per capita-GDP growth; on the contrary, the relationship is significantly positive in many specifications. Further, Gehringer and Prettner (2019) present evidence that falling death rates in OECD countries had a positive influence on both labor and total factor productivity growth between 1960 and 2011. In sum, empirical findings on the relationship between population aging and economic growth are mixed, with the impact of demographic change on economic growth varying substantially depending on the regression specification and sample coverage. In this paper, we introduce a theoretical framework that emphasizes the importance of accounting for the shifts in industry location patterns that coincide with demographic transition

[2]There are a number of multi-country overlapping generations frameworks investigating how demographic structure affects international borrowing and lending patterns (Domeiji and Floden 2006; Tosun 2008; Ito and Tabata 2010) and international trade patterns (Sayan 2005; Galor and Mountford 2006; Naito and Zhao 2009).

when estimating the size and direction of the effect of population aging on economic growth.

The paper is organized as follows. Section 2 introduces our single-sector model of trade and endogenous productivity growth. Section 3 provides a characterization of long-run equilibrium, and Section 4 investigates how changes in demographic structure and retirement age affect market entry and long-run productivity growth. The welfare implications of population aging and retirement age extension are considered in Section 5 through a numerical analysis. We conclude the paper in Section 6.

2 Model

This section introduces a single-sector endogenous market structure and endogenous growth model of trade between two countries, home and foreign. In each country, monopolistically competitive firms employ labor in the production of differentiated goods and in-house process innovation. There is free trade between countries, and home and foreign households have access to a perfectly integrated financial market. While home and foreign populations are assumed to grow at the same exogenous rate, national labor supplies depend on initial population size, retirement age, and demographic structures that derive from differences in individual lifespans. There is no international migration. We focus on the home country as we introduce the model setup, but similar conditions are also derived for the foreign country.

2.1 Demographic Structure and National Labor Supplies

There is a representative dynastic household residing in each country. As in Dinopoulos and Segerstrom (1999), the home-country household consists of a large number of members who, having been born on different dates, have heterogeneous ages. Each household member lives for an exogenous length of time $T > 0$, entering the labor market at age $S \in (0, Z)$, supplying one unit of labor inelastically each period from age S to age $Z \in (S, T)$, exiting the labor market at age Z (i.e., retiring), and dying at age T. Accordingly, at each moment in time t, the population $B(t)$ of the home-household includes dependent children $D(t)$, working adults $L(t)$, and retirees $B(t) - D(t) - L(t)$.

The populations of home and foreign are assumed to grow overtime at a common exogenous rate $\lambda > 0$. Thus, denoting initial population sizes by B_0 and B_0^*, the populations of home and foreign at time t are $B(t) = B_0 e^{\lambda t}$ and $B^*(t) = B_0^* e^{\lambda t}$. Although the populations grow at the same rate, we allow for differences in birth rates (β) and death rates (ζ) across countries, which stem from differences in individual lifespan (T). In home, for example, noting that the number of births $\beta B(t)$ at time t matches the number of deaths $\zeta B(t+T)$ at time $t+T$, we use $\lambda = \beta - \zeta$ to solve for the birth and death rates as

$$\beta = \frac{\lambda}{1 - e^{-\lambda T}}, \qquad \zeta = \frac{\lambda}{e^{\lambda T} - 1}, \tag{1}$$

where we have noted that $B(t+T) = B(t)e^{\lambda T}$. The birth and death rates of foreign are obtained in a similar manner: $\beta^* = \lambda/(1 - e^{-\lambda T^*})$ and $\zeta^* = \lambda/(e^{\lambda T^*} - 1)$. Given the population growth rate (λ), a rise in individual lifespan (T) leads to a slowdown in both the birth rate (β) and the mortality rate (ζ). Thus, under our specification, the country with the relatively long individual lifespan is also the country with the relatively low birth rate, matching with the UN projections for the US and WE shown in Figure 1.

At any given time, the labor force equals the number of individuals that have entered the labor market, but have yet to reach retirement age, allowing us to derive the instantaneous labor supply of home as

$$L(t) = \int_{t-Z}^{t-S} \beta B(\tau) d\tau = \varphi B(t), \tag{2}$$

where $\varphi \equiv (e^{-\lambda S} - e^{-\lambda Z})/(1 - e^{-\lambda T}) \in (0, 1)$, ensuring a positive working-age share of the population in home. The analogous instantaneous labor supply for the foreign country is $L^*(t) = \varphi^* B^*(t)$, with $\varphi^* \equiv (e^{-\lambda S^*} - e^{-\lambda Z^*})/(1 - e^{-\lambda T^*})$. An increase in individual lifespan (T) leads to a smaller population share for working adults and a larger share for retirees (i.e., population aging), whereas an increase in the retirement age (Z) leads to a larger share of working adults and a smaller share of retirees.[3] Thus, under our specification, the country with the relatively long individual lifespan and early retirement age is also the country with the smaller working-age population share and the higher old-age dependency ratio, a result that is consistent with the UN projections for the US and WE in Figure 1.

The relative labor supply assumes a key role in the analysis of how changes in demographic structure influence long-run market entry and productivity growth. As an initial result, we summarize the effects of changes in individual lifespan and retirement age on the relative labor supply in the following lemma.

Lemma 1 *The relative labor supply*

$$\ell \equiv \frac{L}{L^*} = \frac{\varphi}{\varphi^*}\frac{B_0}{B_0^*} \tag{3}$$

is decreasing in the home lifespan (T), and increasing in the home retirement age (Z).

Given the population growth rate (λ), an increase in the home lifespan (T) leads to a smaller working-age population share (φ), thereby reducing the relative labor supply (ℓ). An increase in the home retirement age (Z), however, increases the working-age population share (φ), and expands the relative labor supply (ℓ).

To maintain model tractability, we consider the case where the home and foreign populations grow at the same exogenous rate. This assumption allows us to focus on the effects of changes in demographic structure that arise purely from changes in lifespan. Of course, this specification is quite restrictive. Nevertheless, it enables us to analyze a long-run equilibrium in which production occurs in both countries and provides us with a tractable framework for analyzing the effects of population aging on industry locations patterns and economic growth.

[3] We assume that retirement age is determined independently of lifespan. In Appendix A, however, we show that under reasonable conditions, $\partial\varphi/\partial T < 0$ and $\partial\varphi/\partial Z > 0$, even when retirement age $Z(T)$ is determined as a function of individual lifespan T.

2.2 Households

The utility of the representative household in the home country is

$$U(t) = \int_t^\infty B_0 e^{-(\rho-\lambda)(\tau-t)} \ln C(\tau) d\tau, \tag{4}$$

where $C(t)$ is per capita household consumption, and $\rho > 0$ is the rate of time preference. Following Dixit and Stiglitz (1977), consumption takes the form of a constant-elasticity-of-substitution index:

$$C(t) = \left(\int_0^{N(t)} c(i,t)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

where $N(t)$ is the mass of product varieties available, $c(i,t)$ is the consumption of product variety i, and $\sigma > 1$ is the elasticity of substitution between any pair of product varieties. A large population allows the household to eliminate consumption uncertainty by providing perfect consumption insurance, with equal levels of consumption for all household members (Merz 1995; Andolfatto 1996), regardless of whether they are employed or not.

The household selects an optimal expenditure path in order to maximize lifetime utility subject to the following flow budget constraint:

$$\dot{A}(t) = r(t)A(t) + w(t)L(t) - E(t), \tag{6}$$

where $r(t)$ is the interest rate, $w(t)$ is the wage rate, $A(t)$ is asset holdings, $L(t)$ is the labor force, $E(t)$ is household expenditure in the home country, and a dot over a variable denotes differentiation with respect to time. The solution to the household's optimization problem is the standard Euler equation:

$$\frac{\dot{E}(t)}{E(t)} = \frac{\dot{E}^*(t)}{E^*(t)} = \lambda + r(t) - \rho, \tag{7}$$

where variables associated with the foreign country are indicated by an asterisk. With a perfectly integrated financial market, interest rates equalize across countries yielding common expenditure dynamics for home and foreign households.

Given per-period expenditure, we obtain the instantaneous per capita home demand for a representative product variety i as

$$c(i,t) = p(i,t)^{-\sigma} P(t)^{\sigma-1} E(t) B(t)^{-1}, \tag{8}$$

with $p(i,t)$ denoting product price. The price index associated with the consumption index (5) is denoted by

$$P(t) = \left(\int_0^{n(t)} p_i(t)^{1-\sigma} di + \int_0^{n^*(t)} p_j^*(t)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}, \tag{9}$$

where $n(t)$ and $n^*(t)$ are the masses of product varieties produced in home and foreign: $N(t) \equiv n(t) + n^*(t)$. Foreign households have symmetric preferences that generate analogous expressions for demand conditions.

2.3 Production

The production sector is characterized by monopolistic competition (Dixit and Stiglitz 1977), with each firm producing a single unique product variety to meet the demands from home and foreign households. The production technology of a home-country firm is

$$x(i,t) = \theta(i,t)(l_X(i,t) - \psi), \tag{10}$$

where $x(i,t)$ is firm-level output, $l_X(i,t)$ is labor employed in production, $\theta(i,t)$ is firm-specific productivity, and $\psi > 0$ is a fixed operating cost measured in units of labor. Substituting the instantaneous demand functions (8) into $x(i,t) = c(i,t)B(t) + c^*(i,t)B^*(t)$, firms set price equal to a constant markup over unit cost, $p(i,t) = \sigma w(t)/((\sigma-1)\theta(i,t))$, in order to maximize operating profit on sales:

$$\pi(i,t) = p(i,t)x(i,t) - w(t)l_X(i,t) = \frac{p(i,t)^{1-\sigma}}{\sigma}\frac{(E(t)+E^*(t))}{P(t)^{1-\sigma}} - w(t)\psi, \tag{11}$$

with an analogous expression for foreign firms.

2.4 Process Innovation

Firms invest in in-house R&D with the aim of lowering production costs, raising per-period profit, and increasing firm value (Smulders and van de Klundert 1995; Peretto 1996). The value of a representative firm is determined as the present value of the future profit stream:

$$V(i,t) = \int_t^\infty e^{\int_t^\tau r(\tau')d\tau'}\Pi(i,\tau)d\tau, \tag{12}$$

where per-period profit $\Pi(i,t) = \pi(i,t) - w(t)l_R(i,t)$ is equal to operating profit on sales (11) minus the cost of employing labor in process innovation.

For a home firm, investment in process innovation generates improvements in firm-level productivity according to the following technology constraint:

$$\dot{\theta}(i,t) = K(t)l_R(i,t)^\gamma, \tag{13}$$

with $\gamma \in (0,1)$ generating diminishing marginal returns to firm-level employment in R&D. Adapting the specification of Baldwin and Forslid (2000), we assume that knowledge spills over from production technologies into the R&D process. More specifically, in home,

$$K(t) = \left(\int_0^{n(t)} \theta(i,t)di + \delta\int_0^{n^*(t)} \theta^*(j,t)dj\right)N(t)^{-1}, \tag{14}$$

where $\delta \in (0,1)$ regulates the degree of international knowledge diffusion: there is no international knowledge diffusion for $\delta = 0$ and perfect knowledge diffusion for $\delta = 1$. Under this specification, labor productivity in process innovation is determined as a weighted average of the productivities of production technologies currently in use, with a stronger weighting for technologies utilized in the same country as the innovating firm.

Firms set employment in R&D ($l_R(i,t)$) to maximize firm value (12) subject to the technology constraints (13) and (14). The optimal level of employment is then captured by the following no-arbitrage condition for investment in process innovation:

$$r(t) = \frac{\gamma(\sigma-1)(\pi(i,t)+w(t)\psi)K(t)}{\theta(i,t)w(t)l_R(i,t)^{1-\gamma}} + \frac{(1-\gamma)\dot{l}_R(i,t)}{l_R(i,t)} + \frac{\dot{w}(t)}{w(t)} - \frac{\dot{K}(t)}{K(t)}, \qquad (15)$$

where we have referenced (11), and the small market shares associated with monopolistic competition lead firms to ignore the effects of price changes on national price indices and expenditure levels. This condition equates the firm's internal rate of return on investment in process innovation to the interest rate associated with investment in a risk-free asset in the integrated international financial market.[4]

2.5 Market Equilibrium

Firms respond to adjustments in firm value as they enter and exit the market. In particular, as there are no costs incurred in the creation of new product designs, firms enter the market when $V(i,t) > 0$, driving firm value downwards, and exit the market when $V(i,t) < 0$, pushing firm value upwards. This instantaneous entry and exit process ensures that $V(i,t) = 0$ at all moments in time. With firm value driven to zero in both countries, returning to the household flow-budget constraint (6), we find that expenditure is supported solely by aggregate labor income: $E(t) = w(t)L(t)$ and $E^*(t) = w^*(t)L^*(t)$. Thus, from (2) and (7), we have $\dot{E}(t)/E(t) - \lambda = \dot{E}^*(t)/E^*(t) - \lambda = \dot{w}(t)/w(t) = \dot{w}^*(t)/w^*(t) = r(t) - \rho$.

Enforcing symmetry across the productivity levels of firms operating in the same country, $\theta(i,t) \equiv \theta(t)$ and $\theta^*(j,t) \equiv \theta^*(t)$, we now impose the requirement that firms make an

[4]The firm's intertemporal optimization problem can be solved using a current-value Hamiltonian function $H(i,t) = \Pi(i,t) + \nu(t)K(t)l_R(i,t)^{\gamma}$, where $\nu(t)$ denotes the current shadow value of an improvement in firm-level productivity. The first-order conditions $\partial H(i,t)/\partial l_R(i,t) = 0$ and $\partial H(i,t)/\partial\theta(i,t) = r(t)\nu(t) - \dot{\nu}(t)$ then yield the no-arbitrage condition for firm-level investment in innovation (15).

optimal investment in process innovation immediately upon entering the market to obtain the current national productivity level for their production technologies. Hence, new firms enter the market in home and foreign with productivity levels $\theta(t)$ and $\theta^*(t)$. We continue, however, to allow for a productivity differential across countries: $\theta(t) \neq \theta^*(t)$.

Under these assumptions, all firms operating in a given country have the same scales of employment in production ($l_X(t)$) and process innovation ($l_R(t)$). Accordingly, the free market entry and exit conditions for home ($V(t) = 0$) and foreign ($V^*(t) = 0$) are

$$\pi(t) = w(t)l_R(t), \qquad \pi^*(t) = w^*(t)l_R^*(t), \tag{16}$$

and full employment requires that the labor markets of each country clear: $L(t) = n(t)(l_X(t) + l_R(t))$ and $L^*(t) = n^*(t)(l_X^*(t) + l_R^*(t))$.

The symmetric productivity levels associated with firms in the same country allow for the definition of a simple measure of the strength of knowledge spillovers from production into innovation in home and foreign:

$$k(\tilde{\theta}, t) \equiv \frac{K}{\theta} = s(t) + \delta s^*(t)\tilde{\theta}(t)^{-1}, \qquad k^*(\tilde{\theta}, t) \equiv \frac{K^*}{\theta^*} = \delta s(t)\tilde{\theta}(t) + s^*(t), \tag{17}$$

with the relative productivity of home firms $\tilde{\theta}(t) \equiv \theta(t)/\theta^*(t)$ describing the international productivity differential, and $s(t) \equiv n(t)/N(t)$ and $s^*(t) \equiv n^*(t)/N(t)$ measuring the shares of firms based in home and foreign.

3 Long-run Equilibrium

We characterize the long-run equilibrium of the economy through a consideration of steady states that feature constant allocations of labor across production and innovation in each country, constant shares of firms, and a constant productivity differential across countries. Hereafter, we suppress time arguments in order to simplify notation.

3.1 Process Innovation and the International Productivity Differential

We first take the time derivative of $\tilde{\theta} \equiv \theta/\theta^*$ to generate a differential equation that naturally links the dynamics of the international productivity differential with firm-level investment in process innovation in home and foreign:

$$\frac{\dot{\tilde{\theta}}}{\tilde{\theta}} = \frac{\dot{\theta}}{\theta} - \frac{\dot{\theta}^*}{\theta^*} = kl_R^\gamma - k^* {l_R^*}^\gamma, \tag{18}$$

where we have used (13) and (17). Setting this expression equal to zero yields our first steady-state condition:

$$kl_R^\gamma = k^* {l_R^*}^\gamma, \tag{19}$$

which shows that the productivity growth rates of home and foreign are equal in a long-run equilibrium with a constant productivity differential: $\dot{\theta}/\theta = \dot{\theta}^*/\theta^*$. An examination of the time derivative of (14) then shows that $\dot{\theta}/\theta = \dot{K}/K = \dot{K}^*/K^*$ in steady-state equilibrium.

Second, we derive the long-run employment levels for process innovation. Returning to the no-arbitrage condition for process innovation (15), and noting that $\dot{K}/K = \dot{\theta}/\theta$ and $\dot{K}^*/K^* = \dot{\theta}^*/\theta^*$, together the technology constraints (13) and (14), and the free entry conditions (16) yield the steady-state no-arbitrage conditions for home and foreign:

$$\rho = R \equiv \left[\gamma(\sigma-1)\left(1+\frac{\psi}{l_R}\right) - 1\right] kl_R^\gamma, \tag{20}$$

$$\rho = R^* \equiv \left[\gamma(\sigma-1)\left(1+\frac{\psi}{l_R^*}\right) - 1\right] k^* {l_R^*}^\gamma. \tag{21}$$

The righthand sides describe the steady-state internal rates of return to investment in process innovation for firms in home and foreign: R and R^*. Referring back to (19), it is clear that $l_R = l_R^*$, and hence that $k = k^*$. In turn, invoking the free entry conditions (16), we find that real operating profits also equalize across countries in the long run: $\pi/w = \pi^*/w^*$.

Third, we solve for the long-run productivity differential between home and foreign by deriving two conditions for the home share of firms (s), and then equating them. The first condition is obtained by taking the ratio of the national labor market clearing conditions $L = n(l_X + l_R)$ and $L^* = n^*(l_X^* + l_R^*)$:

$$s(\ell) = \frac{\ell}{1+\ell}, \tag{22}$$

where we have used the fact that $l_X + l_R = l_X^* + l_R^*$ in steady-state equilibrium.[5] This expression is shown as the horizontal line $s = s(\ell)$ in Figure 2a. The second condition is derived by substituting (17) into $k = k^*$ and solving for the home share of firms as

$$s(\tilde{\theta}) = \frac{1 - \delta\tilde{\theta}^{-1}}{1 - \delta\tilde{\theta} + 1 - \delta\tilde{\theta}^{-1}}. \tag{23}$$

This expression has a strictly positive slope: $ds(\tilde{\theta})/d\tilde{\theta} > 0$. As illustrated in Figure 2a, (22) and (23) combine to determine the productivity differential as a function of the relative labor supply of the home country: $\tilde{\theta} = \tilde{\theta}(\ell)$.[6] Note that (23) generates boundaries on the values of the productivity differential that are consistent with positive production shares for each country: $\tilde{\theta} \in (\delta, 1/\delta)$, with $s = 0$ for $\tilde{\theta} = \delta$, and $s = 1$ for $\tilde{\theta} = 1/\delta$. Moreover, with perfect knowledge diffusion ($\delta = 1$), the $s(\tilde{\theta})$ curve is vertical at $\tilde{\theta} = 1$ in Figure 2a, and there is no international productivity differential.

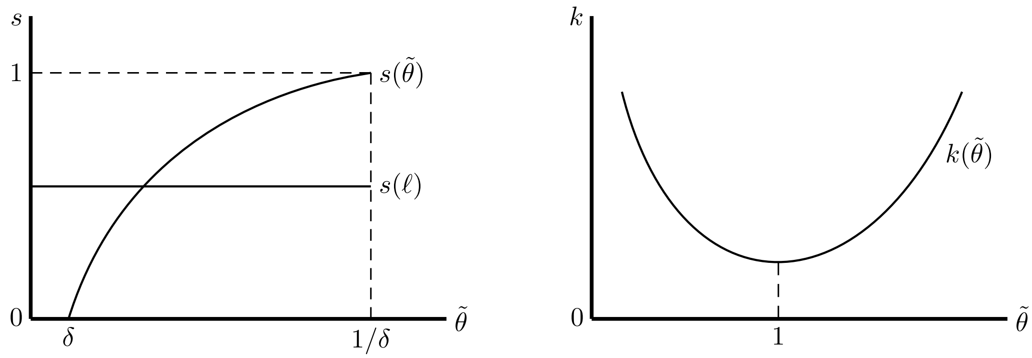
Fourth, substituting (23) into (17) and rearranging the result yields the common steady-state strength of knowledge spillovers for home and foreign:

$$k(\tilde{\theta}) = k^*(\tilde{\theta}) = \frac{1 - \delta^2}{2 - \delta\tilde{\theta} - \delta\tilde{\theta}^{-1}}. \tag{24}$$

[5] From (11), we obtain $\pi = (l_X - \sigma\psi)/(\sigma - 1)w$ and $\pi^* = (l_X^* - \sigma\psi)/(\sigma - 1)w^*$. Substituting these expressions into the free market entry conditions (16), we have $(\sigma - 1)l_R + \sigma\psi = l_X$ and $(\sigma - 1)l_R^* + \sigma\psi = l_X^*$. Since $l_R = l_R^*$ holds in the steady-state equilibrium, we can also confirm that $l_X = l_X^*$ and $l_X + l_R = l_X^* + l_R^*$.

[6] Reorganizing $s(\tilde{\theta}) = s(\ell)$ yields $\tilde{\theta}^2\delta\ell - \tilde{\theta}(1-\ell) - \delta = 0$. Hence, the explicit functional form of $\tilde{\theta}(\ell)$ is given by $\tilde{\theta} = -(1-\ell)/(2\delta\ell) \pm \sqrt{(1-\ell)^2 + 4\delta^2\ell}/(2\delta\ell)$, confirming that one root is positive and the other is negative for all values of (ℓ), and that the former is the relevant root.

Figure 2: Productivity Differential and Knowledge Spillovers



(a) Home's Share of Firms

(b) Strength of Knowledge Spillovers

As shown in Figure 2b, the steady-state strength of knowledge spillovers is convex in the international productivity differential with a minimum at $\tilde{\theta} = 1$.

In Appendix B, we investigate the local dynamics of the economy around the steady state characterized by (19), (20), (21), (22), (23), and (24), and obtain the following lemma.

Lemma 2 *The saddlepath stability of a long-run equilibrium with a balanced growth path requires that the internal rate of return to process innovation decrease with firm-level employment in R&D:*

$$\frac{\partial R}{\partial l_R} = -\left(1 - \gamma(\sigma - 1) + \frac{(1-\gamma)(\sigma-1)l_F}{l_R}\right)\gamma k l_R^{\gamma-1} < 0. \tag{25}$$

Proof: See Appendix B.

The stability condition outlined in Lemma 2 ensures a finite level of investment in process innovation for each firm. With firms setting R&D employment (l_R) to maximize firm value at each moment in time, firms increase l_R when the internal rate of return is greater than the risk-free interest rate ($R > \rho$), and decrease l_R when $R < \rho$. Accordingly, $\partial R/\partial l_R < 0$ is necessary to ensure that the internal rate of return adjusts correctly, and that firm-level investment in process innovation is finite along the balanced growth path.

An important corollary of Lemma 2 is that an increase in the strength of knowledge spillovers (k) induces firms to expand employment in process innovation (l_R). Taking the total derivative of (20), we have

$$\frac{dl_R}{dk} = -\frac{R}{k}\left(\frac{\partial R}{\partial l_R}\right)^{-1} > 0. \tag{26}$$

The increase in k raises the internal rate of return to investment in process innovation, causing firms to expand R&D employment, which then in turn reduces the internal rate of return back to equality with the risk-free interest rate ($R = \rho$), at a higher level of investment.

Having characterized the long-run equilibrium, we next consider how adjustments in the relative labor supply influence relative productivity and knowledge spillovers. Note that lifespan and retirement age only affect relative productivity and knowledge spillovers through their effects on the relative labor supply. The results are summarized in the following lemma.

Lemma 3 *An increase in the relative labor supply (ℓ) expands the international productivity differential ($\tilde{\theta}$), while decreasing the strength of knowledge spillovers (k) for $\tilde{\theta} < 1$, but increasing k for $\tilde{\theta} > 1$.*

Proof: Setting (22) and (23) equal, and taking the total derivative of the result yields

$$\frac{d\tilde{\theta}}{d\ell} = \frac{(\delta - \tilde{\theta})^2}{\delta(1 - \delta^2 + (\delta - \tilde{\theta})^2)\ell^2} > 0, \tag{27}$$

$$\frac{dk}{d\ell} = -\frac{\delta(1-\delta^2)(1-\tilde{\theta}^2)}{(2 - \delta\tilde{\theta} - \delta\tilde{\theta}^{-1})^2\tilde{\theta}^2}\frac{d\tilde{\theta}}{d\ell}\begin{cases} < 0, & \text{for } \tilde{\theta} < 1, \\ > 0, & \text{for } \tilde{\theta} > 1, \end{cases}$$

where we have used (24).

For instance, holding the foreign labor force constant, an increase in the home labor force expands the home market through a rise in household expenditure. In Figure 2a, the $s = s(\ell)$ line shifts upward as the share of firms operating in home increases, and consequently knowledge spillovers rise temporarily in home, but fall in foreign, causing home firms to increase, and foreign firms to decrease, their investments in process innovation. As a result, the relative productivity of home firms ($\tilde{\theta}$) increases over the transition to a new steady state, at which we once again have $s(\ell) = s(\tilde{\theta})$.

Ultimately, as depicted in Figure 2b, the steady-state strength of knowledge spillovers (k) may rise or fall with an increase in the relative labor supply (ℓ), due to two opposing effects. For example, from (17), in the home country we have $dk/d\ell = (1 - \delta\tilde{\theta}^{-1})(ds/d\ell) - (\delta s^*\tilde{\theta}^{-2})(d\tilde{\theta}/d\ell)$. The first term captures the positive effect of stronger domestic knowledge spillovers as the home share of firms increases ($ds/d\ell > 0$), and the second term decribes the negative effect of weaker international knowledge spillovers as the productivity differential rises ($d\tilde{\theta}/d\ell > 0$). The negative effect dominates with $dk/d\ell < 0$ when $\tilde{\theta} < 1$; the positive effect dominates with $dk/d\ell > 0$ when $\tilde{\theta} > 1$, as illustrated in Figure 2b.

Together the results of Lemmas 2 and 3 generate a key relationship between national shares of production (s) and firm-level employment in process innovation (l_R). Because knowledge spillovers from production to innovation are weakest when production is equally distributed between home and foreign ($\ell = 1$, $s = 1/2$, and $\tilde{\theta} = 1$), and strongest when production is fully concentrated in either home ($\ell > 1$, $s = 1$, and $\tilde{\theta} > 1$) or foreign ($\ell < 1$, $s = 0$, and $\tilde{\theta} < 1$), we find that firm-level employment in R&D is increasing in the concentration of industry: $\partial l_R/\partial s < 0$ for $s < 1/2$ and $\partial l_R/\partial s > 0$ for $s > 1/2$.

3.2 Product Variety and Productivity Growth

This subsection considers long-run market entry and productivity growth. We begin with the steady-state level of market entry. As a positive rate of population growth leads to continuous market entry, we study the ratio of incumbent firms to market size as a measure of the level of product variety $\hat{N} \equiv N/(B + B^*)$, where world population is adopted as a proxy for overall market size.

Combining the labor market clearing conditions, $L = n(l_X + l_R)$ and $L^* = n^*(l_X^* + l_R^*)$, with the free market entry conditions, $l_X = (\sigma - 1)l_R + \sigma\psi$ and $l_X^* = (\sigma - 1)l_R^* + \sigma\psi$, we obtain $N = (L + L^*)/\sigma(l_R + \psi)$, which implies that the total number of firms (N) grows proportionately with the world labor supply. Hence, using (2), the steady-state measure of market entry $\hat{N} \equiv N/(B + B^*)$ is determined by

$$\hat{N} = \frac{b\varphi + b^*\varphi^*}{\sigma(l_R + \psi)}, \tag{28}$$

where $b \equiv B_0/(B_0 + B_0^*)$ and $b \equiv B_0^*/(B_0 + B_0^*)$ are the population shares of home and foreign. The long-run level of market entry $\hat{N}$ is positively related with both the home and the foreign working-age population shares (φ and φ^*).

Figure 3a illustrates the convex relationship that arises between market entry ($\hat{N}$) and the international productivity differential ($\tilde{\theta}$). Consider the effects of a change in $\tilde{\theta}$: $d\hat{N}/d\tilde{\theta} = (\partial\hat{N}/\partial l_R)(\partial l_R/\partial k)(\partial k/\partial\tilde{\theta})$. From an examination of (28), we find that an increase in firm-level employment in process innovation (l_R) reduces market entry ($\partial\hat{N}/\partial l_R < 0$), as there is a natural tension between the number and the scale of incumbent firms arising from free market exit and entry. In addition, referring to (26), improvements in knowledge spillovers induce firms to increase their investments in process innovation ($\partial l_R/\partial k > 0$). Consequently, the convex relationship between $\hat{N}$ and $\tilde{\theta}$ derives from the concave relationship between knowledge spillovers k and the product differential $\tilde{\theta}$ that is depicted in Figure 2b. An increase in $\tilde{\theta}$ expands $\hat{N}$ for $\tilde{\theta} < 1$, and contracts $\hat{N}$ for $\tilde{\theta} > 1$.

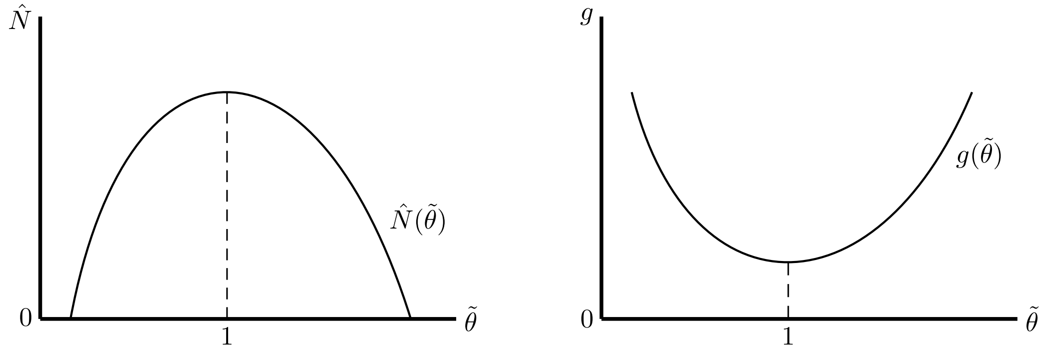
Next, returning to the steady-state no-arbitrage conditions for investment in process innovation (20) and (21), we derive the long-run rate of productivity growth as

$$g \equiv \frac{\dot{\theta}}{\theta} = kl_R^{\gamma} = \frac{\rho}{\gamma(\sigma - 1)(1 + \psi/l_R) - 1}. \tag{29}$$

An important feature of long-run productivity growth is the absence of a scale effect, with adjustments in the level of market entry fully absorbing proportionate changes in the national labor supplies (L and L^*), leaving the productivity differential unchanged.

Figure 3b illustrates the long-run relationship that arises between productivity growth (g)

Figure 3: Market Entry, Productivity Growth, and the Productivity Differential



(a) Market Entry (b) Productivity Growth Rate

and the international productivity differential ($\tilde{\theta}$). Consider once again the effects of a change in $\tilde{\theta}$: $dg/d\tilde{\theta} = (\partial g/\partial l_R)(\partial l_R/\partial k)(\partial k/\partial\tilde{\theta})$. From (29), an increase in firm-level employment in process innovation naturally accelerates the rate of productivity growth ($\partial g/\partial l_R > 0$). Then, as we have seen from (26), an increase in the strength of knowledge spillovers induces firms to raise their investment in R&D ($\partial l_R/\partial k > 0$). Thus, the concave relationship arising between $\tilde{\theta}$ and g derives from the concave relationship between $\tilde{\theta}$ and k shown in Figure 2b. An increase in the productivity differential ($\tilde{\theta}$) slows productivity growth (g) for $\tilde{\theta} < 1$, while accelerating g for $\tilde{\theta} > 1$.

Having characterized the long-run relationships among the level of market entry ($\hat{N}$), the rate of productivity growth (g), and the productivity differential ($\tilde{\theta}$), we next consider how adjustments in the relative labor supply affect productivity growth. The results are summarized in the following lemma.

Lemma 4 *An increase in the relative labor supply (ℓ) lowers the productivity growth rate (g) for $\tilde{\theta} < 1$ and raises g for $\tilde{\theta} > 1$.*

Proof: See Appendix C.

Changes in the relative labor supply affect productivity growth indirectly through adjustments in relative productivity. Returning to Lemma 3, $d\tilde{\theta}/d\ell > 0$ and an increase in the relative labor supply leads to a rightward movement along the $g(\tilde{\theta})$ curve in Figure 3b. Accordingly, the increase in the relative labor supply depresses productivity growth for $\tilde{\theta} < 1$ and accelerates it for $\tilde{\theta} > 1$. On the one hand, when home has a smaller labor force, a lower level of productivity, and a smaller share of firms ($\ell < 1$, $\tilde{\theta} < 1$, and $s < 1/2$), an increase in the relative labor supply (ℓ) reduces the concentration of industry in the relatively advanced foreign country, lowers the strength of knowledge spillovers, and thereby slows productivity growth. On the other hand, when home has a larger labor force, a higher level of productivity, and a larger share of firms ($\ell > 1$, $\tilde{\theta} > 1$, and $s > 1/2$), an increase in ℓ raises the concentration of industry in the relatively advanced home country, raises the strength of knowledge spillovers, and consequently accelerates productivity growth.

3.3 Social Welfare

In this subsection, we derive the welfare levels that arise for the home and foreign households on the balanced growth path. We first calculate the terms of trade that arise between home and foreign in the steady state. Because trade must balance at each moment in time, we have $npc^*B^* = n^*p^*cB$. Then, substituting $E = wL$, $E^* = w^*L^*$, $p = \sigma\theta/(\sigma - 1)$, and $p^* = \sigma\theta^*/(\sigma - 1)$ with (8) into the trade balance yields the home share of firms as $s = \tilde{\theta}^{1-\sigma}(w/w^*)^\sigma\ell/(1 + \tilde{\theta}^{1-\sigma}(w/w^*)^\sigma\ell)$, which given (22) indicates that we must have $w/w^* = \tilde{\theta}^{(\sigma-1)/\sigma}$. The home country's terms of trade are therefore $p/p^* = \tilde{\theta}^{-1/\sigma}$.

Next, calculating the steady-state welfare levels for the home and foreign households, we combine (4), (5), (8), and (9) with $E = wL$, $E^* = w^*L^*$, $p = \sigma\theta/(\sigma-1)$, $p^* = \sigma\theta^*/(\sigma-1)$, and $p/p^* = \tilde{\theta}^{-1/\sigma}$ to obtain

$$(\rho - \lambda)\frac{U_0}{B_0} = \ln \Theta\varphi + \ln\left(s + s^*\tilde{\theta}^{\frac{1-\sigma}{\sigma}}\right)^{\frac{1}{\sigma-1}} + \ln \hat{N}^{\frac{1}{\sigma-1}} + \frac{1}{\rho-\lambda}\left(g + \frac{\lambda}{\sigma-1}\right), \tag{30}$$

$$(\rho - \lambda)\frac{U_0^*}{B_0^*} = \ln \Theta^*\varphi^* + \ln\left(s\tilde{\theta}^{\frac{\sigma-1}{\sigma}} + s^*\right)^{\frac{1}{\sigma-1}} + \ln \hat{N}^{\frac{1}{\sigma-1}} + \frac{1}{\rho-\lambda}\left(g + \frac{\lambda}{\sigma-1}\right), \tag{31}$$

where $\Theta \equiv [(\sigma-1)/\sigma]\theta(0)(B_0+B_0^*)^{1/(\sigma-1)}$ and $\Theta^* \equiv [(\sigma-1)/\sigma]\theta^*(0)(B_0+B_0^*)^{1/(\sigma-1)}$, and we have defined $t = 0$ as the time when the economy reaches the balanced growth path. We also assume that the initial productivity of foreign firms $\theta^*(0)$ is given exogenously, such that the initial productivity of home firms is $\theta(0) = \tilde{\theta}\theta^*(0)$. The first term on the righthand side captures the direct income effect associated with changes in national labor supplies through φ and φ^*. The second term shows the welfare effect of changes in the terms of trade. The third term describes the love of variety effect as households have a higher level of utility when there are more product varieties available. The fourth term describes the benefits of growth in productivity and the mass of product varieties.

4 Demographic Structure and Retirement Age

We consider the implications of changes in demographic structure for long-run productivity growth and market entry. In Subsection 4.1, we analyze the effect of increasing home lifespan (T). In Subsection 4.2, we analyze the effect of increasing home retirement age (Z).

4.1 Aging Society

We first consider the effect of increasing home lifespan (T) on productivity growth (g). As stated in Lemma 1, the relative labor supply (ℓ) is decreasing in home lifespan. Therefore, a casual examination of Lemmas 3 and 4 yields the following proposition.

Proposition 1 *An increase in home lifespan (T) raises the productivity growth rate (g) for $\tilde{\theta} < 1$ and lowers g for $\tilde{\theta} > 1$.*

Proof: See Appendix D.

An increase in the home lifespan (T) leads to a smaller working-age population share, which negatively affects the relative labor supply (ℓ). On the one hand, when home has a smaller labor force, a lower level of productivity, and a smaller share of firms ($\ell < 1$, $\tilde{\theta} < 1$, and $s < 1/2$), a decrease in the relative labor supply increases the concentration of industry in the relatively advanced foreign country, raising the strength of knowledge spillovers, and thereby increasing the rate of productivity growth. On the other hand, when home has a larger labor force, a higher level of productivity and a larger share of firms ($\ell > 1$, $\tilde{\theta} > 1$, and $s > 1/2$), a decrease in the relative labor supply reduces the concentration of industry in the relatively advanced home country, lowering the strength of knowledge spillovers, and decreasing the productivity growth rate.

Next, we consider the effect of increasing home lifespan (T) on long-run market entry ($\hat{N}$). The results are summarized in the following proposition.

Proposition 2 *An increase in home lifespan (T) has a negative effect on the level of market entry ($\hat{N}$) for $\tilde{\theta} < 1$, and an ambiguous effect on $\hat{N}$ for $\tilde{\theta} > 1$.*

Proof: See Appendix E.

From (28), an increase in home lifespan (T) affects market entry ($\hat{N}$) both directly by decreasing the working-age population-share of home ($\partial\varphi/\partial T < 0$), and indirectly by changing firm-level employment in process innovation (l_R) through adjustments in the relative labor supply ($\partial\ell/\partial T < 0$). Returning to Figure 3a, the decrease in φ shifts the $\hat{N}(\tilde{\theta})$ curve downwards, as the contraction in market size induces firms to exit, implying that the direct effect of T on $\hat{N}$ through adjustments in φ is negative regardless of the value of $\tilde{\theta}$. At the same time, the decrease in ℓ moves the economy leftward along the $\hat{N}(\tilde{\theta})$ curve, with the international productivity differential falling as firms adjust their investment levels in process innovation in response to changes in the strength of knowledge spillovers (Lemma 3). These adjustments imply that the indirect effect of T on $\hat{N}$ through adjustments in l_R is negative (resp., positive) for $\tilde{\theta} < 1$ (resp., $\tilde{\theta} > 1$). Consequently, when home has a smaller labor force, a lower level of productivity, and a smaller share of firms ($\ell < 1$, $\tilde{\theta} < 1$, and $s < 1/2$), both the direct and indirect effects of T on $\hat{N}$ are negative, and an increase in T causes a fall in $\hat{N}$. Alternatively, when home has a larger labor force, a higher level of productivity, and a larger share of firms ($\ell > 1$, $\tilde{\theta} > 1$, and $s > 1/2$), either the negative direct effect or the positive indirect effect may dominate, generating an ambiguous relationship between T and $\hat{N}$.

4.2 Retirement Age

We examine the effects of changes in the home retirement age on productivity growth and market entry. Applying the results of Lemmas 1, 3, and 4, we obtain the following proposition for the effect of increasing home retirement age (Z) on productivity growth (g).

Proposition 3 *An increase in home retirement age (Z) lowers the productivity growth rate (g) for $\tilde{\theta} < 1$ and raises g for $\tilde{\theta} > 1$.*

Proof: See Appendix F.

The intuition behind this result is similar to that of Proposition 1. An increase in the home retirement age (Z) leads to a larger working-age population share, which positively affects the relative labor supply (ℓ). When home has a smaller labor force, a lower level of productivity, and a smaller share of firms ($\ell < 1$, $\tilde{\theta} < 1$, and $s < 1/2$), an increase in the relative labor supply reduces the concentration of industry in the relatively advanced foreign country, lowers the strength of knowledge spillovers, and thereby slows productivity growth. However, when home has a larger labor force, a higher level of productivity, and a larger share of firms ($\ell > 1$, $\tilde{\theta} > 1$, and $s > 1/2$), the increase in ℓ raises the concentration of industry in the relatively advanced home country, strengthening knowledge spillovers, and thereby positively affecting the productivity growth rate.

Let us next consider the effect of an increase in the home retirement age (Z) on the level of market entry ($\hat{N}$). We obtain the following proposition.

Proposition 4 *An increase in home retirement age (Z) has a positive effect on market entry ($\hat{N}$) for $\tilde{\theta} < 1$, and an ambiguous effect on $\hat{N}$ for $\tilde{\theta} > 1$.*

Proof: See Appendix G.

The mechanism behind this result is same as that of Proposition 2. A rise in the retirement age of home (Z) influences market entry (N) directly through its effects on the working-age population share of home ($\partial\varphi/\partial Z > 0$), and indirectly through the effect of the relative labor supply adjustment ($\partial\ell/\partial Z > 0$) on firm-level investment in process innovation (l_R).

Thus, with the rise in retirement age increasing working-age share of the population φ, the $\hat{N}(\tilde{\theta})$ curve shifts upward in Figure 3a, as the expansion of market size leads to the entrance of new firms (i.e., the direct effect of Z on $\hat{N}$). In addition, the economy moves rightward along the $\hat{N}(\tilde{\theta})$ curve, in accordance with a rise in the relative productivity of home firms (Lemma 3) (i.e., the indirect effect of Z on $\hat{N}$). When home has a smaller labor force, a lower level of productivity, and a smaller share of firms ($\ell < 1$, $\tilde{\theta} < 1$, and $s < 1/2$), both the direct and indirect effects of the increase in Z are positive and the level of market entry rises. Alternatively, when home has a larger labor force, a higher level of productivity, and a larger share of firms ($\ell > 1$, $\tilde{\theta} > 1$, and $s > 1/2$), either the positive direct effect or negative indirect effect may dominate, leading to an ambiguous relationship between Z and $\hat{N}$.

5 Numerical Analysis

The nature of our framework makes a theoretical analysis of the welfare effects of changes in demographic structure and retirement age intractable. As an alternative, we use the UN projections for life expectancy at birth in the US and Western Europe (WE) over the period from 2000 to 2050 to complete a numerical analysis of our model. Following our model specifications, we assess the impacts of changes in demographic structure arising purely from changes in expected lifespan, while holding the rate of population growth constant. The main objective of the numerical analysis is not to calibrate our simple model to actual data, but rather to supplement the qualitative results of our theoretical framework. Although we have chosen parameter values carefully, the results introduced in this section should be interpreted with caution.

5.1 Model Parameterization

The home country is set as the US, and the foreign country as Western Europe (WE), with in which we include Austria, Belgium, France, Germany, Liechtenstein, Luxembourg, Monaco, the Netherlands and Switzerland, following the UN World Population Prospects 2019. Ref-

Table 1: Parameter Values

Parameter	Description	Value
B_0	Initial US population	281,710,000
B_0^*	Initial WE population	182,329,000
λ	Population growth rate	0.0015
S	Age of labor market entry	15
Z	US retirement age	64.9
Z^*	WE retirement age	60.5
ρ	Discount rate	0.024
σ	Elasticity of substitution	5
δ	Degree of knowledge diffusion	0.15
θ_0^*	Initial productivity of WE firms	1
ψ	Fixed operating cost	0.4812×10^{-6}
γ	Elasticity of labor employment in R&D	0.2

erencing population data for these regions in 2000, the initial US population (B_0) is 281.71 million and the initial WE population (B_0^*) is 182.329 million. In addition, we assume that the populations grow at a common rate of $\lambda = 0.0015$ to approximate the UN projection for the average annual population growth rate of the two regions between 2000 to 2050.[7] In the OECD statistics, the working-age population is defined as those aged 15 to 64. Applying this definition, we adopt 15 as the age of labor market entry (S).[8] The retirement ages in the US (Z) and WE (Z^*) are 64.9 and 60.5, matching with the OECD estimates for average effective male retirement ages in 2000.

Turning next to demand-side parameters, following Jones et al. (1993), we assume a value of $\rho = 0.024$ for the discount rate. The elasticity of substitution is set to $\sigma = 5$, generating a markup of $\sigma/(\sigma - 1) = 1.25$, which is consistent with the evidence presented for developed countries by Britton et al. (2000) and Gali et al. (2007). For supply-side parameters, we fix the degree of knowledge diffusion at $\delta = 0.15$ to match the mid-range estimates provided by Bloom et al. (2013). We normalize the initial productivity of WE firms (θ_0^*) to unity, and

[7]To maintain model tractability, we consider the case where the home and foreign populations grow at the same exogenous rate. The relatively similar expected population growth rates and a large volume of trade are the main reasons for selecting the US and WE for our numerical analysis.

[8]The explicit consideration of differences in labor market entry age across countries would not significantly alter the qualitative implications of our numerical analysis.

calibrate the elasticity of labor employment in R&D (γ) and the fixed operating cost (ψ) to meet the parameter conditions outlined in (25), and to generate a target value of $g = 0.02$ for the productivity growth rate. The parameter values are summarized in Table 1.

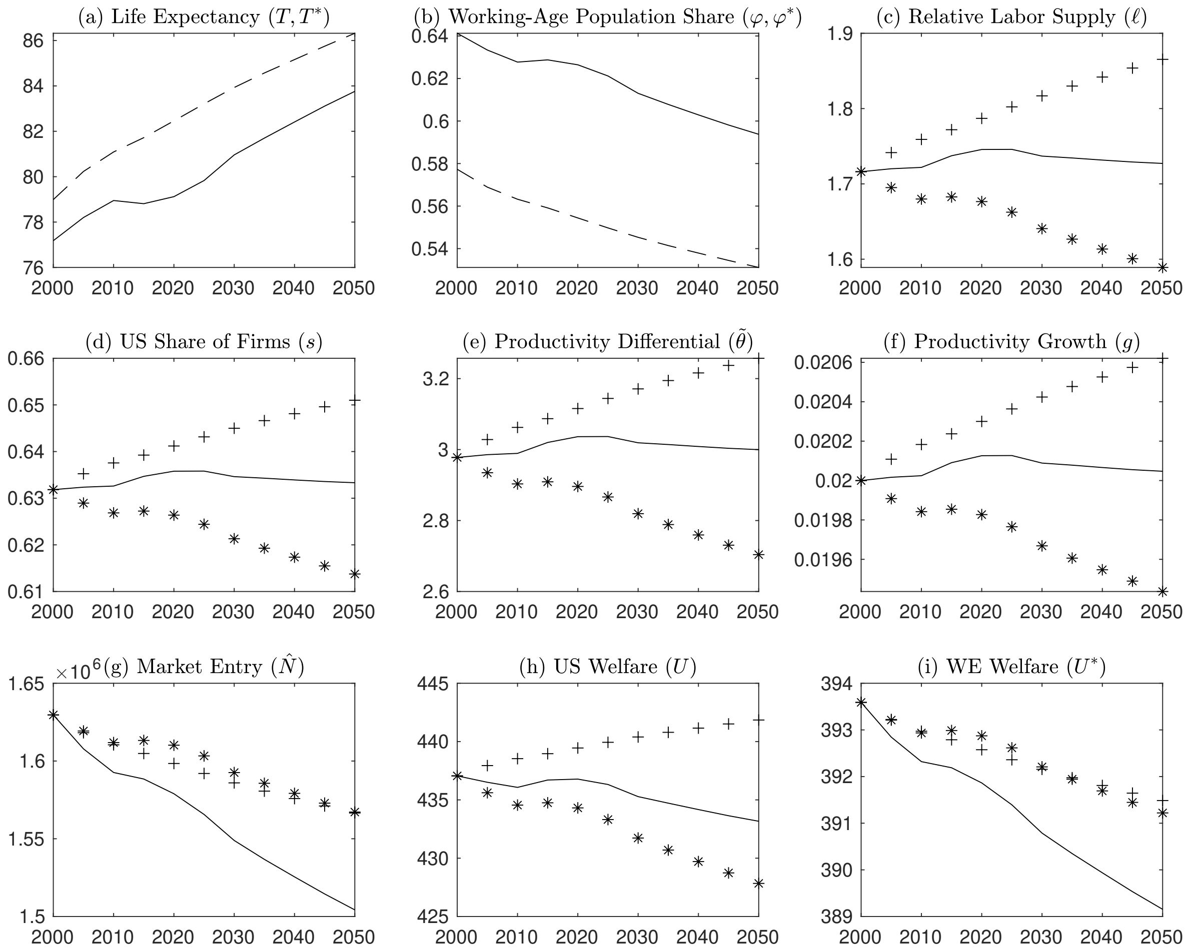
To investigate the effects of simultaneous increases in the expected lifespans of the US (T) and WE (T^*), we use the UN projections of life expectancy at birth for each region between 2000 to 2050. Figure 4a shows the evolutions of T and T^*, and Figure 4b shows the calibrated working-age population shares for the US (φ) and WE (φ^*), as calculated using (2). Under our assumptions for the population parameters, the relatively large labor force of the US ($\ell > 1$) results in a greater share of firms ($s > 1/2$) and a higher level of productivity ($\tilde{\theta} > 1$) for the US, as observed from the solid lines in Figures 4c to 4e.

5.2 Changes in Demographic Structure

We investigate how changes in demographic structure affect productivity growth (g), the level of market entry ($\hat{N}$), and welfare (U and U^*) by simulating the long-run effects of predicted increases in expected lifespan in the US (T) and WE (T^*) between 2000 and 2050. In Figures 4c to 4i, we plot the numerical results for the key macroeconomic variables of our framework under three cases. The solid lines track values for which both T and T^* adjust following the UN projections. Then, for comparison, the "$*$" lines depict values for which the US lifespan is allowed to adjust, while the WE lifespan is fixed at the initial value $T^* = 78.94$. Similarly, the "$+$" lines show values for which the US lifespan is set at the initial value $T = 77.18$, and the WE lifespan is allowed to adjust. In the following explanations, we focus on the results of the solid lines, but other cases are mentioned briefly for comparison.

The UN projections suggest that life expectancy will rise at similar rates in each region (Figure 4a), implying that the impacts of declining working-age population shares in the US and WE (Figure 4b) counterbalance. The result is a rather stable relative labor supply (ℓ) over the period of analysis, and accordingly, the US share of firms (s), the international productivity differential ($\tilde{\theta}$), and the productivity growth rate (g) only exhibit small upward

Figure 4: The Effects of Increasing Lifespan (T, T^*)



In Panels (a) and (b), the solid and dashed lines plot values for the US and WE. In Panels (c) through (i), the solid lines plot values for which T and T^* adjust, the "$*$" lines plot values for which T adjusts, and the "$+$" lines plot values for which T^* adjusts.

adjustments, as observed from the solid lines in Figures 4c, 4d, 4e and 4f. In contrast, as firm-level employment in innovation is for the most part constant, the direct effect of the declines in working-age population shares is a fall in the level of market entry ($\hat{N}$), as shown by the solid line in Figure 4g. As summarized in Table 2, our framework predicts a 0.0047 percentage point (pp) increase in the productivity growth rate, and a -7.7054% reduction in the level of market entry between 2000 and 2050. In particular, the negligible effect of population aging on long-run productivity growth, driven by the counter-opposing influences of contracting labor forces in the US and WE, may help to explain the ambiguous relationship

Table 2: Quantitative Effects of Changes in Demographic Structure

Variables	Results
Change in productivity growth: $g_{2050} - g_{2000}$	0.0047pp
Change in market entry: $(\hat{N}_{2050} - \hat{N}_{2000})/\hat{N}_{2000}$	-7.7054%
Welfare change in the US: $(U_{2050} - U_{2000})/U_{2000}$	-0.8892%
Welfare change in WE: $(U^*_{2050} - U^*_{2000})/U^*_{2000}$	-1.1196%

between demographic transition and economic growth found in the empirical literature.

We now consider how changes in demographic structure affect the welfare levels of US and WE households. Given the relatively small adjustments in the US share of firms and the international productivity differential, the terms of trade and the productivity growth rate remain stable over the period of analysis, and the direct labor income effects associated with falling working-age population shares and a contracting level of market entry become the key drivers of changes in welfare levels. As a result, the welfare levels of the US and WE decrease between 2000 and 2050, as presented by the solid lines in Figures 4h and 4i. Table 2 reports the overall reductions in welfare: -0.8892% for the US and -1.1196% for WE.

Lastly, we reflect on the cross-country spillover effects of demographic transition. First, the decrease in the working-age population share of WE (φ^*) affects US welfare (U) negatively by lowering market entry ($\hat{N}$), but has a positive impact on US welfare through an improvement in the US terms of trade and faster productivity growth (g). Overall, the positive effects dominate and the fall in φ^* tends to improve US welfare, as shown by the "+" line in Figure 4h. Second, the US decline in the working-age population share (φ) affects WE welfare (U^*) both positively through an improvement in the WE terms of trade, and negatively through a fall in market entry and slower productivity growth. Overall, the negative effects dominate and the fall in φ tends to lower WE welfare, as seen from the "$*$" line in Figure 4i. The asymmetric nature of the cross-country spillover effects of demographic transition, with population aging in the US causing a deterioration in WE welfare and population aging in WE improving US welfare, explains in part the smaller welfare reduction for the US over the period of analysis, as summarized in Table 2.

5.3 Changes in Retirement Age

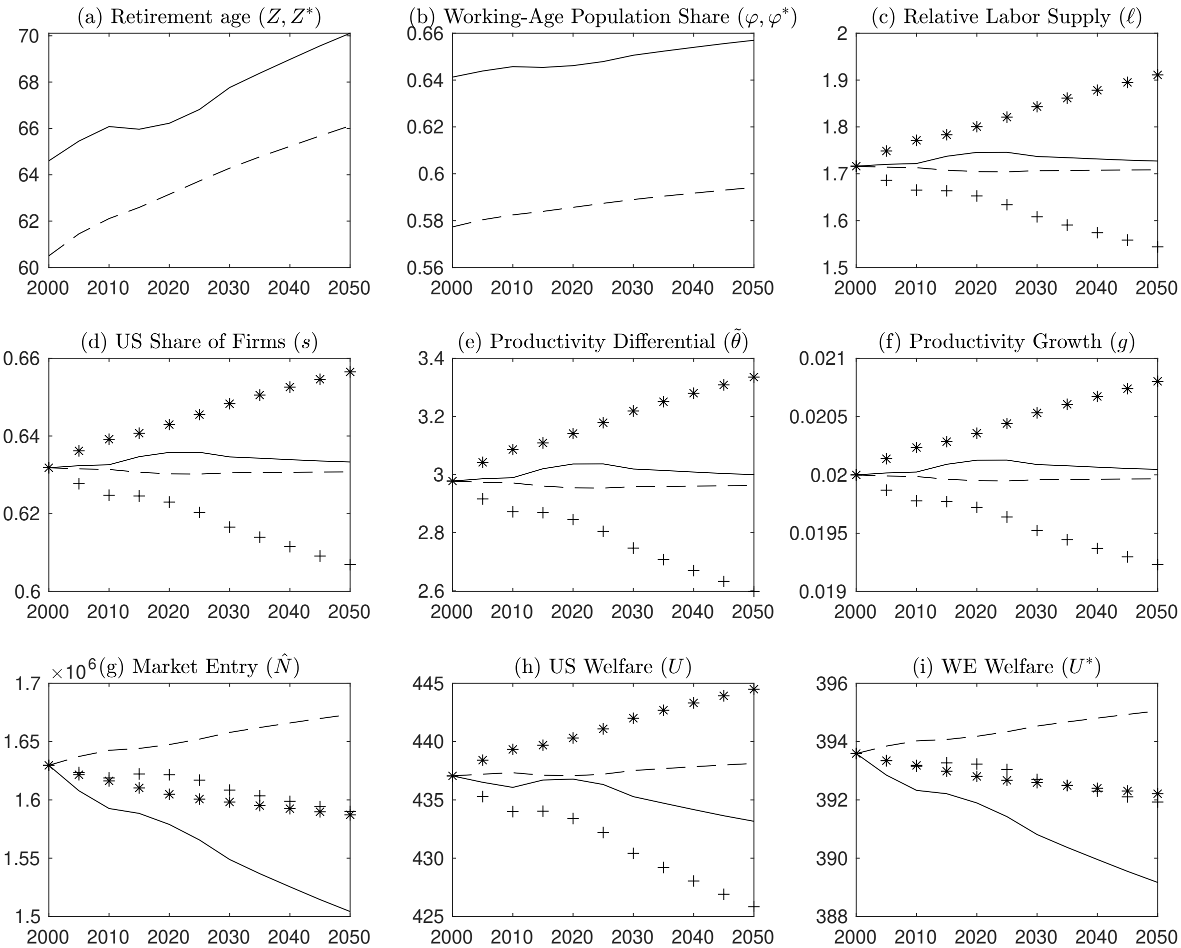
To study the implications of retirement age extensions for welfare in an aging economy, we consider the case where the retirement age is raised proportionately with increases in expected lifespan, thereby maintaining the retirement age-lifespan ratio at its initial value in each region ($Z/T = Z_0/T_0$ and $Z^*/T^* = Z_0^*/T_0^*$). Under this policy setting, with $(Z-S)/T$ increasing in T, the length of the employment period expands at a faster rate than the rise in expected lifespan ensuring an increase in the region's labor supply. Figure 5a plots the evolution of the US and WE retirement ages (Z and Z^*) corresponding to the UN projections for expected lifespan (T and T^*). And, Figure 5b shows the subsequent increases in the working-age population shares (φ and φ^*), as derived using (2).[9] Contrary to the results in Figure 4b, with proportionate extensions of the retirement age, the calibrated working-age share of the population increases along with expected lifespan.

We present the numerical results for the key macroeconomic variables of our framework in Figures 5c to 5i. The dashed lines plot values corresponding to the policy setting outlined above, with retirement ages (Z and Z^*) adjusting proportionately in response to changes in projected lifespans (T and T^*). Then, for comparison, the solid lines reproduce our results from Figure 4 with retirement age held constant as lifespan adjusts. In addition, the "$*$" lines now track values for which the US retirement age adjusts with lifespan, while the WE retirement age is fixed at $Z^* = 60.5$. And, the "+" lines show values for which the US retirement age is set at $Z = 64.6$, as the WE retirement age adjusts with lifespan. In the following analysis, we focus on a comparison of the results of the dashed lines with those of the benchmark solid lines, but other cases are also mentioned briefly.

Consistent with our previous numerical results, we find that the comparable increases in the UN projections for the expected lifespans of the US and WE translate into similar patterns

[9] Allowing retirement age to increase proportionately with projections for lifespan, the US retirement age increases from 64.6 to 70.11 and the WE retirement age increases from 60.50 to 66.11 between 2000 and 2050.

Figure 5: The Effects of Increasing Retirement Age (Z, Z^*)



In Panels (a) and (b), the solid and dashed lines plot values for the US and WE. In Panels (c) through (i), the dashed lines plot values for which Z and Z^* adjust, the "$*$" lines plot values for which Z adjusts, and the "$+$" lines plot values for which Z^* adjusts. The solid lines reproduce the plots of Figure 4.

for retirement age extension in each region (Figure 5a), with the resulting expansions in the labor supplies (Figure 5b) counterbalancing another. As such, the overall impacts on the relative labor supply (ℓ), the US share of firms (s), the international productivity differential ($\tilde{\theta}$), and the productivity growth rate (g) are small, as illustrated by the dashed lines in Figures 5c, 5d, 5e and 5f. In particular, we observe that the dashed lines for these variables are slightly negative and below the benchmark solid lines, resulting in a -0.0034 percentage point (pp) decrease in the productivity growth rate over the period of analysis, as shown in Table 3. With labor supplies now expanding in each region, however, given the relatively constant scale of

Table 3: Quantitative Effects of Changes in Retirement Age

Variables	Results
Change in productivity growth: $g_{2050} - g_{2000}$	-0.0034pp
Change in market entry: $(\hat{N}_{2050} - \hat{N}_{2000})/\hat{N}_{2000}$	2.6811%
Welfare change in the US: $(U_{2050} - U_{2000})/U_{2000}$	0.2440%
Welfare change in WE: $(U^*_{2050} - U^*_{2000})/U^*_{2000}$	0.3713%

firm-level employment in innovation, the level of market entry ($\hat{N}$) rises by 2.6811%, and lies above the benchmark solid line, as observed in the dashed line in Figure 5g. The results indicate that the impacts of retirement age extensions in the US and WE on productivity growth counterbalance. Consequently, the overall quantitative effects of retirement age extensions on productivity growth are small, while the level of market entry increases substantially.

Turning now to a discussion of the implications of retirement age extensions for the welfare levels of the US (U) and WE (U^*), we find that with relatively small changes in the US share of firms and the international productivity differential, the terms of trade and the rate of productivity growth do not exhibit large adjustments, implying that welfare effects are primarily driven by changes in the labor supplies and the level of market entry. More specifically, the direct positive labor income effect associated with increases in working-age population shares and the expansion in the level of market entry generate welfare improvements in both regions over the period of analysis. As presented in Table 3, US welfare rises by 0.2444% and WE welfare by 0.3713%. Therefore, a policy of retirement age extension is clearly beneficial, as demonstrated by a comparison of the dashed and solid lines in Figures 5h and 5i, in which the dashed lines always lie above the benchmark solid lines.

We briefly consider the cross-country spillover effects of retirement age extension. First, an extension of the retirement age in WE (Z^*) affects US welfare (U), both negatively through a deterioration of the US terms of trade and slower rate of productivity growth (g), and positively through an increase in market entry ($\hat{N}$). The negative effect dominates, however, and consequently retirement age extension in WE hurts US welfare, as observed from a comparison of the “$+$” line with the solid line in Figure 5h. Second, a retirement age extension in the US (Z) influences WE welfare (U^*) both positively by increasing market entry and raising the rate of productivity growth, and negatively through a deterioration in the WE terms of trade. Overall, the positive effect dominates and retirement age extension in the US leads to an improvement in WE welfare, as shown by a comparison of the “$*$” line with the solid line in Figure 5i.

Summarizing, our numerical results indicate that retirement age extension in the US has a positive effect on the welfare of the WE, whereas retirement age extension in the WE has a negative effect on the welfare of the US. Due to the asymmetric natural of the cross-country spillover effects of retirement age extensions, as presented in Figures 5h and 5i of our numerical analysis, the largest improvement in US welfare (U) is achieved when the US retirement age is raised proportionately with life expectancy while holding the WE retirement age constant (i.e., the “$*$” lines), whereas the largest improvement in WE welfare (U^*) is achieved when both the US and WE retirement ages are raised proportionately with rising lifespans (i.e., the dashed lines).

6 Conclusion

In this paper, we have investigated how rising life expectancy and declining birth rates affect market entry and productivity growth in a two-country model of industry location and international trade. In each country, the demographic transition to an older population induces a contraction in the labor force through a reduction of the working-age population. Productivity growth is driven by firm-level investment in process innovation, and imperfect international knowledge diffusion ensures that the country with the larger labor force, and thus the larger market, hosts a greater share of relatively productivity firms. In this setting, an increase in the geographic concentration of industry improves knowledge spillovers with firms expanding employment in innovation. Therefore, a key feature of the model is that greater industry concentration in the country with the larger labor force results in faster productivity growth and a lower level of market entry.

Using the model to study the effects of population aging, we find that population aging in the country with the smaller labor force reduces its market size, increasing the concentration of industry in the country with the larger labor force. As a result, productivity growth accelerates. In addition, the contraction of the national labor supply combines with an expansion in firm-level employment in innovation to reduce the level of market entry. Alternatively, population aging in the country with the larger labor force leads to a fall in the concentration of industry, thereby slowing productivity growth. While the labor-supply contraction tends to reduce market entry, however, the reduction in firm-level employment in innovation tends to induce market entry. Accordingly, the relationship between population aging and market entry is ambiguous for the country with the larger labor force.

A simple calibration of the model using population projections for the US and Western Europe (WE) shows that the overall effects of changes in population aging on productivity growth are small. As such, our framework may help to explain the ambiguous evidence associated with the relationship between demographic structure and economic growth in the empirical literature. Our numerical analysis also suggests that the key driver of the negative welfare adjustments associated with population aging is the reduced product variety that coincides with a lower level of market entry. We consider retirement age extension as a means of reversing the effects of population aging, and show that the negative welfare effects of demographic transition in the US and WE can be counterbalanced through a policy that extends retirement age proportionately with increases in life expectancy.

The numerical framework can also be used to analyze the cross-country spillover effects of retirement age extension. We show that retirement age extension in the US has a positive effect on the welfare of WE, whereas retirement age extension in WE has a negative effect on the welfare of the US. Given the asymmetric nature of the spillover effects associated with a policy of retirement age extension, our numerical analysis suggests that the largest improvement in US welfare is achieved when the US retirement age is increased proportionately with expected lifespan, while holding WE retirement age constant. In contrast, the largest improvement in WE welfare is achieved when both the US and WE retirement ages are increased proportionately with rising expected lifespans.

Appendix A: Retirement Age as a Function of Lifespan

This appendix considers the case when retirement age depends on individual lifespan: $Z = Z(T)$. The national labor supply becomes $L(t,T) = (1 - e^{-\lambda Z(T)})B(t)/(1 - e^{-\lambda T})$, and the effect of a rise in T can be expressed as follows:

$$\frac{d\ell(T)}{dT} = -\left(1 - Z'(T)\frac{e^{\lambda T} - 1}{e^{\lambda Z(T)} - 1}\right)\frac{\lambda e^{-\lambda T} L(T)}{1 - e^{-\lambda T}}.$$

Then, we have

$$\begin{aligned} \frac{dL(T)}{dT} &< 0 \ \text{ if } \ Z'(T) < \frac{e^{\lambda Z(T-S)} - 1}{e^{\lambda T} - 1} \in [0,1], \\ \frac{dL(T)}{dT} &\geq 0 \ \text{ if } \ Z'(T) \geq \frac{e^{\lambda Z(T-S)} - 1}{e^{\lambda T} - 1} \in [0,1]. \end{aligned}$$

Therefore, even if retirement age depends on individual lifespan, the results of this paper are still obtained if retirement age is not very elastic with respect to lifespan ($dL(T)/dT < 0$).

Appendix B: Stability Analysis

We provide a localized stability analysis of the steady state described by (19), (20), (21), (22), (23), and (24) in order to show that $\partial R/\partial l_R = \partial R^*/\partial l_R^* < 0$ is a necessary condition for the saddlepath stability of long-run equilibrium. As a first step, we take the ratio of the labor market clearing conditions $L = n(l_X + l_R)$ and $L^* = n^*(l_X^* + l_R^*)$ to obtain the national shares of firms as

$$s = \frac{(l_R^* + \psi)\ell}{(l_R + \psi) + (l_R^* + \psi)\ell}, \qquad s^* = \frac{(l_R + \psi)}{(l_R + \psi) + (l_R^* + \psi)\ell}, \tag{B1}$$

where we have used the free entry conditions $l_X = (\sigma - 1)l_R + \sigma\psi$ and $l_X^* = (\sigma - 1)l_R^* + \sigma\psi$. Therefore, (B1) allows us to set $k = k(\tilde{\theta}, l_R, l_R^*)$ and $k^* = k^*(\tilde{\theta}, l_R, l_R^*)$, and the dynamics of relative productivity (18) are determined as a function $\tilde{\theta}$, l_R, and l_R^*. Next, using $K = k(\tilde{\theta}, l_R, l_R^*)\theta$, $K^* = k^*(\tilde{\theta}, l_R, l_R^*)\theta^*$, and $\dot{w}/w = \dot{w}^*/w^* = r - \rho$, in the no-arbitrage conditions (15) for home and foreign, we obtain

$$\rho = \bar{R} + \left(1 - \gamma + \frac{ss^*(1 - \delta\tilde{\theta}^{-1})l_R}{k(l_R + \psi)}\right)\frac{\dot{l}_R}{l_R} - \frac{ss^*(1 - \delta\tilde{\theta}^{-1})l_R^*}{k(l_R + \psi)}\frac{\dot{l}_R^*}{l_R^*} - \frac{\delta s^*\tilde{\theta}^{-1}}{k}\frac{\dot{\tilde{\theta}}}{\tilde{\theta}},$$

$$\rho = \bar{R}^* + \left(1 - \gamma + \frac{ss^*(1 - \delta\tilde{\theta})l_R^*}{k^*(l_R^* + \psi)}\right)\frac{\dot{l}_R^*}{l_R^*} - \frac{ss^*(1 - \delta\tilde{\theta})l_R}{k^*(l_R^* + \psi)}\frac{\dot{l}_R}{l_R} + \frac{\delta s\tilde{\theta}}{k^*}\frac{\dot{\tilde{\theta}}}{\tilde{\theta}}.$$

These two expressions are solved for the dynamics of l_R and l_R^*, and thus the dynamic system is reduced to the following three differential equations in $\tilde{\theta}$, l_R, and l_R^*:

$$\frac{\dot{\tilde{\theta}}}{\tilde{\theta}} = kl_R - k^*l_R^*, \tag{B2}$$

$$\begin{aligned}\frac{\dot{l}_R}{l_R} = {} & \frac{\left(1 - \gamma + \frac{ss^*(1-\delta\tilde{\theta})l_R^*}{k^*(l_R^*+\psi)}\right)\left[\rho - \left(\frac{\gamma(\sigma-1)(l_R+\psi)}{\sigma l_R} - 1\right)kl_R^{\gamma} - \frac{\delta s^*\tilde{\theta}^{-1}}{k}\frac{\dot{\tilde{\theta}}}{\tilde{\theta}}\right]}{(1-\gamma)(1-\gamma+\Omega)} \\ & + \frac{\frac{ss^*(1-\delta\tilde{\theta}^{-1})l_R}{k(l_R+\psi)}\left[\rho - \left(\frac{\gamma(\sigma-1)(l_R^*+\psi)}{\sigma l_R^*} - 1\right)k^*{l_R^*}^{\gamma} + \frac{\delta s\tilde{\theta}}{k^*}\frac{\dot{\tilde{\theta}}}{\tilde{\theta}}\right]}{(1-\gamma)(1-\gamma+\Omega)},\end{aligned} \tag{B3}$$

$$\begin{aligned}\frac{\dot{l}_R^*}{l_R^*} = {} & \frac{\left(1 - \gamma + \frac{ss^*(1-\delta\tilde{\theta}^{-1})l_R}{k(l_R+\psi)}\right)\left[\rho - \left(\frac{\gamma(\sigma-1)(l_R^*+\psi)}{\sigma l_R^*} - 1\right)k^*{l_R^*}^{\gamma} + \frac{\delta s\tilde{\theta}}{k^*}\frac{\dot{\tilde{\theta}}}{\tilde{\theta}}\right]}{(1-\gamma)(1-\gamma+\Omega)} \\ & + \frac{\frac{ss^*(1-\delta\tilde{\theta})l_R^*}{k^*(l_R^*+\psi)}\left[\rho - \left(\frac{\gamma(\sigma-1)(l_R+\psi)}{\sigma l_R} - 1\right)k{l_R}^{\gamma} - \frac{\delta s^*\tilde{\theta}^{-1}}{k}\frac{\dot{\tilde{\theta}}}{\tilde{\theta}}\right]}{(1-\gamma)(1-\gamma+\Omega)},\end{aligned} \tag{B4}$$

where $\Omega = ss^*(1 - \delta\tilde{\theta}^{-1})l_R/(k(l_R + \psi)) + (ss^*(1 - \delta\tilde{\theta})l_R^*)/(k^*(l_R^* + \psi)) > 0$. Evaluating a linear expansion of the system at the steady state given by (19), (20), (21), (22), (23), and

(24) yields the following Jacobian matrix:

$$J = \begin{bmatrix} \frac{\partial \dot{\tilde{\theta}}}{\partial \tilde{\theta}} & \frac{\partial \dot{\tilde{\theta}}}{\partial l_R} & \frac{\partial \dot{\tilde{\theta}}}{\partial l_R^*} \\ \frac{\partial \dot{l_R}}{\partial \tilde{\theta}} & \frac{\partial \dot{l_R}}{\partial l_R} & \frac{\partial \dot{l_R}}{\partial l_R^*} \\ \frac{\partial \dot{l_R^*}}{\partial \tilde{\theta}} & \frac{\partial \dot{l_R^*}}{\partial l_R} & \frac{\partial \dot{l_R^*}}{\partial l_R^*} \end{bmatrix},$$

with

$$\frac{\partial \dot{\tilde{\theta}}}{\partial \tilde{\theta}} = -\delta(s + s^* \tilde{\theta}^{-2}) \tilde{\theta} l_R^{\gamma},$$

$$\frac{\partial \dot{\tilde{\theta}}}{\partial l_R} = -\frac{\partial \dot{\tilde{\theta}}}{\partial l_R^*} = (\gamma - \Omega) k \tilde{\theta} l_R^{\gamma - 1},$$

$$\frac{\partial \dot{l}_R}{\partial \tilde{\theta}} = \frac{\delta l_R \left((1-\gamma) s^* \tilde{\theta}^{-2} - (s^2 - s^{*2} \tilde{\theta}^{-2}) \Omega \right)}{k(1-\gamma)(1-\gamma+\Omega)} \left(\rho - \frac{\partial \dot{\tilde{\theta}}}{\partial \tilde{\theta}} \right),$$

$$\frac{\partial \dot{l}_R}{\partial l_R} = \frac{l_R(1-\gamma+s^*\Omega)}{(1-\gamma)(1-\gamma+\Omega)} \left[\frac{\rho s \Omega}{l_R} - \frac{\delta s^* \tilde{\theta}^{-2}}{k} \frac{\partial \dot{\tilde{\theta}}}{\partial l_R} - \frac{\partial R}{\partial l_R} + \frac{s\Omega}{(1-\gamma+s^*\Omega)} \left(\frac{\rho s^* \Omega}{l_R} + \frac{\delta s}{k} \frac{\partial \dot{\tilde{\theta}}}{\partial l_R} \right) \right],$$

$$\frac{\partial \dot{l}_R}{\partial l_R^*} = -\frac{l_R(1-\gamma+s^*\Omega)}{(1-\gamma)(1-\gamma+\Omega)} \left[\frac{\rho s \Omega}{l_R} + \frac{\delta s^* \tilde{\theta}^{-2}}{k} \frac{\partial \dot{\tilde{\theta}}}{\partial l_R^*} - \frac{s\Omega}{(1-\gamma+s^*\Omega)} \left(\frac{\rho s^* \Omega}{l_R} + \frac{\delta s}{k} \frac{\partial \dot{\tilde{\theta}}}{\partial l_R^*} - \frac{\partial R^*}{\partial l_R^*} \right) \right],$$

$$\frac{\partial \dot{l}_R^*}{\partial \tilde{\theta}} = -\frac{\delta l_R \left((1-\gamma) s + (s^2 - s^{*2} \tilde{\theta}^{-2}) \Omega \right)}{k(1-\gamma)(1-\gamma+\Omega)} \left(\rho - \frac{\partial \dot{\tilde{\theta}}}{\partial \tilde{\theta}} \right),$$

$$\frac{\partial \dot{l}_R^*}{\partial l_R} = -\frac{l_R(1-\gamma+s\Omega)}{(1-\gamma)(1-\gamma+\Omega)} \left[\frac{\rho s^* \Omega}{l_R} - \frac{\delta s}{k} \frac{\partial \dot{\tilde{\theta}}}{\partial l_R} - \frac{s^*\Omega}{(1-\gamma+s^*\Omega)} \left(\frac{\rho s \Omega}{l_R} - \frac{\delta s^* \tilde{\theta}^{-2}}{k} \frac{\partial \dot{\tilde{\theta}}}{\partial l_R} - \frac{\partial R}{\partial l_R} \right) \right],$$

$$\frac{\partial \dot{l}_R^*}{\partial l_R^*} = \frac{l_R(1-\gamma+s\Omega)}{(1-\gamma)(1-\gamma+\Omega)} \left[\frac{\rho s^* \Omega}{l_R} + \frac{\delta s}{k} \frac{\partial \dot{\tilde{\theta}}}{\partial l_R^*} - \frac{\partial R^*}{\partial l_R^*} - \frac{s^*\Omega}{(1-\gamma+s^*\Omega)} \left(\frac{\rho s \Omega}{l_R} - \frac{\delta s^* \tilde{\theta}^{-2}}{k} \frac{\partial \dot{\tilde{\theta}}}{\partial l_R^*} \right) \right].$$

Noting that $\partial R / \partial l_R = \partial R^* / \partial l_R$ in the steady-state, we solve for the determinant of J as

$$|J| = \frac{\gamma(\sigma-1)\psi\delta(s\tilde{\theta} + s^*\tilde{\theta}^{-1}) k l_R^{2\gamma}}{(1-\gamma)^2(1-\gamma+\Omega)} \frac{\partial R}{\partial l_R},$$

and the trace of J as

$$tr(J) = -\frac{\delta(s\tilde{\theta} + s^*\tilde{\theta}^{-1})kl_R^\gamma}{1-\gamma+\Omega} - \frac{(2(1-\gamma)+\Omega)}{(1-\gamma)(1-\gamma+\Omega)}\frac{\partial R}{\partial l_R} + \frac{\rho\Omega}{1-\gamma+\Omega}.$$

The system includes one state variable ($\tilde{\theta}$), and two control variables (l_R) and (l_R^*), and we therefore require one positive and two negative eigenvalues for the saddlepath stability of the system. As such, we require $|J| < 0$, and $\partial R/\partial l_R < 0$ becomes a necessary condition. With $|J| < 0$, the steady state is saddlepath stable when $tr(J) > 0$, which will be the case if $\partial R/\partial l_R$ is sufficiently negative.

Appendix C: Proof of Lemma 4

The total derivative of (29) with respect to ℓ is

$$\frac{dg}{d\ell} = \underbrace{\frac{\partial g}{\partial l_R}}_{(+)}\underbrace{\frac{\partial l_R}{\partial k}}_{(+)}\frac{\partial k}{\partial \tilde{\theta}}\underbrace{\frac{\partial \tilde{\theta}}{\partial \ell}}_{(+)}\begin{cases} < 0, & \text{for } \tilde{\theta} < 1, \\ > 0, & \text{for } \tilde{\theta} > 1. \end{cases}$$

From (26), (27) and (29), we obtain $\partial g/\partial l_R > 0$, $\partial l_R/\partial k > 0$ and $\partial\tilde{\theta}/\partial\ell > 0$. Thus, $dg/d\ell$ can be signed using (24).

Appendix D: Proof of Proposition 1

The total derivative of (29) with respect to T is

$$\frac{dg}{dT} = \underbrace{\frac{\partial g}{\partial l_R}}_{(+)}\underbrace{\frac{\partial l_R}{\partial k}}_{(+)}\frac{\partial k}{\partial \tilde{\theta}}\underbrace{\frac{\partial \tilde{\theta}}{\partial \ell}}_{(+)}\underbrace{\frac{\partial \ell}{\partial T}}_{(-)}\begin{cases} > 0, & \text{for } \tilde{\theta} < 1, \\ < 0, & \text{for } \tilde{\theta} > 1. \end{cases}$$

From (26), (27), (29) and Lemma 1, we obtain $\partial g/\partial l_R > 0$, $\partial l_R/\partial k > 0$, $\partial\tilde{\theta}/\partial\ell > 0$ and $\partial\ell/\partial T < 0$. Therefore, dg/dT can be signed referring to (24).

Appendix E: Proof of Proposition 2

The total derivative of (28) with respect to T is

$$\frac{d\hat{N}}{dT} = \underbrace{\frac{\partial \hat{N}}{\partial l_R}}_{(-)} \underbrace{\frac{\partial l_R}{\partial k}}_{(+)} \frac{\partial k}{\partial \tilde{\theta}} \underbrace{\frac{\partial \tilde{\theta}}{\partial \ell}}_{(+)} \underbrace{\frac{\partial \ell}{\partial T}}_{(-)} + \underbrace{\frac{\partial \hat{N}}{\partial \varphi}}_{(+)} \underbrace{\frac{\partial \varphi}{\partial T}}_{(-)} < 0, \qquad \text{for } \tilde{\theta} < 1.$$

From (26), (27), (28), (29) and Lemma 1, we obtain $\partial \hat{N}/\partial l_R < 0$, $\partial l_R/\partial k > 0$, $\partial \tilde{\theta}/\partial \ell > 0$ and $\partial \ell/\partial T < 0$. As such, the first term on the right hand side can be signed using (24). Moreover, referring to (2) and (28), we have $\partial \hat{N}/\partial \varphi > 0$ and $\partial \varphi/\partial T < 0$. Thus, the second term on the right hand is negative.

Appendix F: Proof of Proposition 3

The total derivative of (29) with respect to Z is

$$\frac{dg}{dZ} = \underbrace{\frac{\partial g}{\partial l_R}}_{(+)} \underbrace{\frac{\partial l_R}{\partial k}}_{(+)} \frac{\partial k}{\partial \tilde{\theta}} \underbrace{\frac{\partial \tilde{\theta}}{\partial \ell}}_{(+)} \underbrace{\frac{\partial \ell}{\partial Z}}_{(+)} \begin{cases} < 0, & \text{for } \tilde{\theta} < 1, \\ > 0, & \text{for } \tilde{\theta} > 1. \end{cases}$$

From (26), (27), (29) and Lemma 1, we obtain $\partial g/\partial l_R > 0$, $\partial l_R/\partial k > 0$, $\partial \tilde{\theta}/\partial \ell > 0$ and $\partial \ell/\partial Z > 0$. Therefore, dg/dZ can be signed using (24).

Appendix G: Proof of Proposition 4

The total derivative of (28) with respect to Z is

$$\frac{d\hat{N}}{dZ} = \underbrace{\frac{\partial \hat{N}}{\partial l_R}}_{(-)} \underbrace{\frac{\partial l_R}{\partial k}}_{(+)} \frac{\partial k}{\partial \tilde{\theta}} \underbrace{\frac{\partial \tilde{\theta}}{\partial \ell}}_{(+)} \underbrace{\frac{\partial \ell}{\partial Z}}_{(+)} + \underbrace{\frac{\partial \hat{N}}{\partial \varphi}}_{(+)} \underbrace{\frac{\partial \varphi}{\partial Z}}_{(+)} > 0, \qquad \text{for } \tilde{\theta} < 1.$$

From (26), (27), (28), (29) and Lemma 1, we obtain $\partial \hat{N}/\partial l_R < 0$, $\partial l_R/\partial k > 0$, $\partial \tilde{\theta}/\partial \ell > 0$ and $\partial \ell/\partial Z > 0$. Thus, the first term on the right hand side can be signed using (24). In addition, from (2) and (28), we have $\partial \hat{N}/\partial \varphi > 0$ and $\partial \varphi/\partial Z > 0$. Hence, the second term on the right hand is positive.

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