

## **FINANCIAL DESTABILIZATION**

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# Financial Destabilization\*

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## Abstract

This paper uses a dynamic general equilibrium model to examine whether financial innovations destabilize an economy. Applying a neoclassical production function, we demonstrate that as financial frictions are mitigated, the economy loses stability and a flip bifurcation occurs at a certain level of financial frictions under an empirically plausible elasticity of substitution between capital and labor. Furthermore, the amplitude of fluctuations increases as financial frictions are mitigated and is maximized when the financial market approaches perfection. These outcomes imply that financial innovations are likely to destabilize an economy.

**Keywords:** Financial innovations, endogenous business cycles, financial destabilization, heterogeneous agents.

**JEL Classification Numbers:** E13, E32, E44

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# 1 Introduction

The development of modern monetary and financial systems began in the early 17th century. In the financial development process, unceasing financial innovations have enabled us to trade tremendous amounts of assets in the financial market. Under these circumstances, people have repeatedly witnessed large swings of boom-bust fluctuations, which are occasionally accompanied by credit market booms and collapses (see Boissay et al., 2016). As such, researchers and policymakers often assert that financial innovations destabilize economies (e.g., Loayza and Ranciere, 2006; Guillaumont Jeanneney and Kpodar, 2011; Rubio and Carrasco-Gallego, 2014). The purpose of this paper is to explore whether financial innovations destabilize an economy by applying a dynamic general equilibrium model with infinitely lived agents.

In our study, we assume that financial innovations directly mitigate financial frictions that entrepreneurs face. From the perspective of the 21st century, one may imagine that this kind of financial innovation is caused by financial technology combined with information technology and/or artificial intelligence. In particular, due to the development of financial technology in recent years, the amount of total debt and credit in the private sector has ballooned far beyond the scale of gross domestic product in some countries. In this paper, we investigate how the mitigation of financial frictions affects economic fluctuations.

The economy in our model is inhabited by three types of economic agents: firms, workers, and entrepreneurs. The representative firm produces general goods from capital and labor with a neoclassical production technology that exhibits positive and diminishing marginal products and is homogeneous of degree one with respect to both capital and labor. Workers inelastically supply one unit of labor to the representative firm to earn wage income in each period. They cannot borrow in the financial market, and their subjective discount factor is so small that their borrowing constraints are always binding. Therefore, they consume all their earnings in each period, i.e., they are hand-to-mouth consumers. Entrepreneurs receive idiosyncratic productivity

shocks in each period. Entrepreneurs who draw higher productivity shocks borrow in the financial market and undertake an investment project to produce capital. Those who draw lower productivity shocks store their wealth by lending in the financial market without engaging in an investment project. Because of idiosyncratic productivity shocks, investors and lenders appear endogenously in each period. Entrepreneurs face financial frictions, and they can borrow only up to a certain proportion of their net wealth. The mitigation of financial frictions promotes capital accumulation because the allocation of production resources is improved, and thus macroeconomic productivity increases.

The increase in the capital stock has two conflicting effects on capital income. On the one hand, it places upward pressure on capital income because the source of capital income (i.e., the principal) becomes greater. On the other hand, it places downward pressure on capital income because the decrease in the marginal product of capital lowers the interest rate. Therefore, the increase in the capital stock does not necessarily raise capital income. In this paper, we assume that the elasticity of the marginal product of capital in the production technology increases as the capital stock increases. In this case, at the early stage of capital accumulation, the positive effect of the increase in the capital stock on capital income surpasses the negative effect, whereas the negative effect becomes stronger than the positive effect as capital accumulates. Then, capital income increases at the early stage of capital accumulation and starts to decline when capital accumulation attains a certain threshold level. In other words, the net effect that capital accumulation has on capital income has an inverted-U shape. In our model, this inverted-U shaped effect of capital accumulation on capital income is crucial to producing endogenous business cycles.

To study how the mitigation of financial frictions affects economic fluctuations, we produce bifurcation diagrams with respect to the extent of financial frictions. Our findings are as follows. Under an empirically plausible elasticity of substitution between capital and labor, as financial frictions are mitigated, the economy loses

stability, and a flip bifurcation occurs at a certain level of financial frictions. Furthermore, the amplitude of business fluctuations increases as financial frictions are mitigated and is maximized when the financial market approaches perfection. Therefore, financial innovations are likely to destabilize the economy.

The current paper is related to the literature on (deterministic) endogenous business cycles in the dynamic general equilibrium model, which have been theoretically studied by many researchers over the past thirty years and the plausibility of which has recently been empirically supported by Beaudry et al. (2015, 2017, 2020). Examples of works that employ the overlapping generations model are Benhabib and Day (1982), Grandmont (1985), Farmer (1986), Reichlin (1986), Benhabib and Laroque (1988), Bertocchi and Wang (1995), Grandmont et al. (1998), and Rochon and Polemarchakis (2006). Whereas the models in all these works include outside money, Yokoo (2000) demonstrates that endogenous business cycles can occur in an overlapping generations model without outside money, with the elasticity of the marginal product of capital playing a crucial role in the occurrence of endogenous business cycles, as in our model. Unlike Yokoo's model, our model is inhabited by infinitely lived agents, and we investigate the destabilization of an economy caused by financial innovations. Benhabib and Nishimura (1985), Boldrin and Denecker (1990), and Nishimura and Yano (1995) employ an infinitely lived agent model in which endogenous business cycles are derived in an economy with two production sectors.

None of the abovementioned studies, however, explicitly consider financial frictions. Examples of papers that derive endogenous business cycles with financial frictions are Azariadis and Smith (1996, 1998), Favara (2012), Gokan (2011), Kikuchi (2008), Kikuchi and Stachurski (2009), Matsuyama (2007, 2013), Matsuyama et al. (2016), and Myerson (2012, 2014). All these studies employ the overlapping generations model. In contrast, Woodford (1989) constructs a dynamic general equilibrium model of infinitely lived capitalists and workers and derives endogenous business cycles. In his model, workers consume in a hand-to-mouth manner, not being allowed

to borrow and lend in the financial market, and capitalists are homogeneous, so that neither borrowing nor lending occurs among them. Although hand-to-mouth workers are also assumed in our model, entrepreneurs can borrow up to the limit of borrowing constraints as previously explained. In our model, the extent of financial frictions plays a crucial role in the instability of the economy. Aghion et al. (2004) and Pintus (2011) also derive endogenous business cycles in a model of infinitely lived agents with financial frictions, but they assume a small open economy where the world interest rate is exogenously given.<sup>1</sup> Although Kunieda and Shibata (2017) investigate how financial development affects endogenous business cycles in a dynamic general equilibrium model of infinitely lived agents, their economy does not exhibit endogenous business cycles when the financial market approaches perfection. To the best of our knowledge, no studies in the existing literature obtain the result that the amplitude of endogenous business cycles increases as the financial market approaches perfection.

The remainder of this paper is organized as follows. In the following section, we develop the model and obtain the optimality conditions of entrepreneurs, workers, and the representative firm. Section 3 derives an equilibrium in which a one-dimensional dynamical system with respect to capital is obtained. In section 4, we investigate the patterns of dynamic behaviors in the economy and observe the appearance of endogenous business cycles. In section 5, we find that the mitigation of financial frictions destabilizes the economy. In this section, we also produce bifurcation diagrams with respect to the extent of financial frictions. In section 6, we conclude the paper.

## 2 The model

A closed economy continues from period 0 to  $+\infty$  in discrete time indexed by  $t$ . Although the basic setting of financial frictions is similar to that of Kunieda and Shibata

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<sup>1</sup>Orgiazzi (2008) also investigates endogenous business cycles in a small open economy, extending the model of Aghion et al. (2004), and demonstrates that the labor share plays an important role in causing instability in the model of Aghion et al. (2004).

(2016), in which there are only entrepreneurs as active agents, it differs from their model in that the economy in our model consists of a continuum of entrepreneurs, identical workers, and a representative firm. The population of entrepreneurs is normalized to one, and that of workers is normalized to  $N$ . All these economic agents are infinitely lived. The representative firm produces general goods from capital and labor. Entrepreneurs are potential capital producers: in each period, those who draw higher productivity shocks become capital producers (investors), and those who draw lower productivity shocks become lenders.

## 2.1 Entrepreneurs

### 2.1.1 Timing of events

Consider the timing of events that a certain entrepreneur, say, entrepreneur  $j \in X$  experiences from period  $t$  to period  $t + 1$ , where  $X$  is the entire set of entrepreneurs. At the beginning of period  $t$ , entrepreneur  $j$  has not yet received an idiosyncratic productivity shock for capital production. The consumption good market in period  $t$  is opened at the beginning of the period and closed before an idiosyncratic productivity shock is realized. Therefore, entrepreneur  $j$  makes a decision about consumption and saving at the beginning of period  $t$  without knowing her productivity.

At the end of period  $t$ , there are two saving methods that entrepreneur  $j$  can apply: one is lending her savings in the financial market, and the other is initiating an investment project. Because an idiosyncratic productivity shock is realized before entrepreneur  $j$  determines her saving method, she optimally chooses between these saving methods while knowing her productivity. Lending one unit of savings in the financial market in period  $t$  yields a claim to  $r_{t+1}$  units of general goods in period  $t + 1$ , where  $r_{t+1}$  is the gross real interest rate. Investing one unit of funds in a project in period  $t$  yields  $\Phi_t^j$  units of capital in period  $t + 1$ , which is sold to the representative firm at a price  $\rho_{t+1}$ . In other words, entrepreneur  $j$  is endowed with an investment technology such that  $k_{t+1}^j = \Phi_t^j \cdot i_t^j$ , where  $k_{t+1}^j$  represents capital to be used for

general goods production in period  $t + 1$ ,  $i_t^j$  is an investment, and  $\Phi_t^j$  is individual-specific productivity in capital production. Although  $\Phi_t^j$  is an idiosyncratic shock, the realization of low productivity cannot be insured against because no insurance market for idiosyncratic productivity shocks exists. We assume that  $\Phi_0^j, \Phi_1^j, \dots$  are independent and identically distributed both over time and across entrepreneurs (the i.i.d assumption), and its cumulative distribution function is given by  $G(\Phi^j)$  the support of which is given by  $[l, m]$  where  $l, m \in \mathbf{R}_+ \cup \{+\infty\}$ .  $G(\Phi^j)$  is differentiable and strictly increasing on the support.

### 2.1.2 Maximization problem

Entrepreneur  $j$  in period  $t$  maximizes the following expected lifetime utility:

$$U_t^j = E_t \left[ \sum_{\tau=t}^{\infty} \beta^{\tau-t} \ln c_{\tau}^j \right],$$

where  $\beta \in (0, 1)$  is her subjective discount factor and  $c_{\tau}^j$  is consumption in period  $\tau$ . The flow budget constraint of entrepreneur  $j$  is given by

$$i_{\tau}^j + d_{\tau}^j = (\rho_{\tau} \Phi_{\tau-1}^j i_{\tau-1}^j + r_{\tau} d_{\tau-1}^j) - c_{\tau}^j \text{ for } \tau \geq t, \quad (1)$$

where  $d_{\tau}^j$  is lending if positive and borrowing if negative. In period  $t = 0$ , the flow budget constraint is given by  $i_0^j + d_0^j = \rho_0 k_0 - c_0^j$ , where  $k_0$  is the initial capital endowment at birth and is common to all entrepreneurs.

In borrowing in the financial market, entrepreneur  $j$  faces a financial constraint, which is given by

$$d_{\tau}^j \geq -\lambda a_{\tau}^j,$$

where  $a_{\tau}^j := (\rho_{\tau} \Phi_{\tau-1}^j i_{\tau-1}^j + r_{\tau} d_{\tau-1}^j) - c_{\tau}^j$  for  $\tau \geq 1$  (or  $a_0^j := \rho_0 k_0 - c_0^j$  for period  $\tau = 0$ ) represents her savings.<sup>2</sup> Henceforth, we call  $a_t^j$  entrepreneur  $j$ 's net worth.

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<sup>2</sup>Similar financial constraints are assumed in the literature (e.g., Aghion et al., 1999; Aghion and



Note that  $\lambda \in (0, \infty)$  measures the extent of financial frictions: as  $\lambda$  increases because of financial innovations, the financial constraint is relaxed. The financial constraint disables entrepreneur  $j$  from borrowing from the financial market more than  $\lambda$  times her net worth. Because it follows from Eq. (1) that  $i_\tau^j + d_\tau^j = a_\tau^j$ , the financial constraint is rewritten as

$$d_\tau^j \geq -\mu i_\tau^j, \quad (2)$$

where  $\mu := \lambda/(1 + \lambda) \in (0, 1)$  also measures the extent of financial frictions. It is necessary to impose the nonnegativity constraint of investment as follows:

$$i_\tau^j \geq 0. \quad (3)$$

Entrepreneur  $j$  maximizes her expected lifetime utility  $U_t^j$  subject to Eqs. (1)-(3).

### 2.1.3 Optimal portfolio allocation

To consider an optimal portfolio allocation of entrepreneurs  $j$ 's net worth, we define

$$\phi_t := r_{t+1}/\rho_{t+1}. \quad (4)$$

If entrepreneur  $j$  draws her productivity such that  $\Phi_t^j > \phi_t$ , she becomes an investor. She optimally initiates an investment project with borrowing up to the limit of the financial constraint. If she draws her productivity such that  $\Phi_t^j \leq \phi_t$ , she lends her entire net worth in the financial market to acquire the gross return  $r_{t+1}$ .<sup>3</sup> Hence,  $\phi_t$  is the cutoff of the productivity shocks that divides entrepreneurs into investors and lenders in period  $t$ . Then, entrepreneur  $j$ 's optimal portfolio program is given by

$$i_t^j = \begin{cases} 0 & \text{if } \Phi_t^j \leq \phi_t \\ \frac{a_t^j}{1-\mu} & \text{if } \Phi_t^j > \phi_t, \end{cases} \quad (5)$$

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Banerjee, 2005; Aghion et al., 2005; Antras and Caballero, 2009).

<sup>3</sup>It is assumed that if entrepreneur  $j$  draws  $\Phi_t^j = \phi_t$ , she becomes a lender.

and

$$d_t^j = \begin{cases} a_t^j & \text{if } \Phi_t^j \leq \phi_t \\ -\frac{\mu}{1-\mu}a_t^j & \text{if } \Phi_t^j > \phi_t. \end{cases} \quad (6)$$

#### 2.1.4 Euler equation

By defining  $R_\tau^j := \max\{r_\tau, (\rho_\tau \Phi_{\tau-1}^j - r_\tau \mu)/(1 - \mu)\}$  and from Eqs. (5) and (6), we can rewrite the flow budget constraint (1) as follows:

$$a_\tau^j = R_\tau^j a_{\tau-1}^j - c_\tau^j. \quad (7)$$

Entrepreneur  $j$  maximizes  $U_t^j$  subject to Eq. (7). The Euler equation for all  $t \geq 0$  is given by

$$\frac{1}{c_t^j} = \beta E_t \left[ R_{t+1}^j \frac{1}{c_{t+1}^j} \right]. \quad (8)$$

## 2.2 Workers

Workers in our model are identical, being endowed with one unit of labor in each period, and supply their labor to the production sector inelastically to earn wage income. It is assumed that workers are hand-to-mouth consumers; that is, they entirely consume their current labor income. Consider a certain worker, say, worker  $h$ . Worker  $h$ 's consumption program can be given by

$$c_t^h = w_t, \quad (9)$$

for all  $t \geq 0$ , where  $c_t^h$  is worker  $h$ 's consumption.<sup>4</sup>

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<sup>4</sup>To derive Eq. (9), assume that workers' lifetime utility is given by  $U_t^h = \sum_{\tau=t}^{\infty} \tilde{\beta}^{\tau-t} c_\tau^h$  where workers' subjective discount factor is so small that  $\tilde{\beta} < 1/r_t$  for all  $t \geq 0$  in equilibrium and workers cannot borrow in the financial market. Then, workers consume all their labor income in each period. King and Leape (1998) and Guiso et al. (2003) provide empirical evidence suggesting the existence of hand-to-mouth consumers.

## 2.3 Production sector

The representative firm produces general goods with a production technology given by  $Y_t = F(K_t, N_t)$ , where  $K_t$  and  $N_t$  represent capital and labor in period  $t$ , respectively. Capital depreciates entirely in one period.  $F(\cdot, \cdot)$  is continuous and at least twice differentiable with respect to  $K_t$  and  $N_t$ . It is assumed that  $\lim_{K_t \rightarrow 0} F(K_t, N_t) = \lim_{N_t \rightarrow 0} F(K_t, N_t) = 0$ . The production technology exhibits positive and diminishing marginal products and constant returns to scale with respect to  $K_t$  and  $N_t$ . We define  $f(k_t) := F(K_t/N_t, 1)$  where  $k_t := K_t/N_t$  is per worker capital. Then, it holds that  $f'(k_t) > 0 > f''(k_t)$  with  $\lim_{k_t \rightarrow 0} f(k_t) = 0$ .

Because the capital market is competitive, capital and labor are paid their marginal products as follows:

$$\rho_t = f'(k_t), \quad (10)$$

$$w_t = f(k_t) - f'(k_t)k_t. \quad (11)$$

The elasticity of the marginal product of capital is defined as follows:

$$\eta(k_t) := -f''(k_t)k_t/f'(k_t) > 0. \quad (12)$$

For the sake of the following analysis, we prepare the following lemma.

**Lemma 1.** *Suppose that  $\epsilon_t := -(dN_t/dw_t) \cdot (w_t/N_t)$  is the elasticity of demand for labor. Then it holds that*

$$\eta(k_t) = \left( \frac{1}{\epsilon_t} \right) \left( \frac{w_t}{\rho_t k_t} \right). \quad (13)$$

*Proof.* See the Appendix.

Eq. (13) expresses the relationship among wage, capital income, the elasticity of the marginal product of capital and the elasticity of demand for labor.

### 3 Equilibrium

A competitive equilibrium is given by sequences of prices  $\{w_t, \rho_t, r_{t+1}\}$  for all  $t \geq 0$  and allocation  $\{c_t^j, a_t^j, i_t^j, d_t^j\}$  for all  $j$  and  $t \geq 0$ ,  $\{c_t^h\}$  for all  $h$  and  $t \geq 0$ , and  $\{k_t\}$  for all  $t \geq 0$ , such that (i) for each  $j$ , entrepreneur  $j$  maximizes her lifetime utility from each period  $t$  onward; (ii) for each  $h$ , worker  $h$  consumes so that  $c_t^h = w_t$  in each period; (iii) the representative firm maximizes its profits in each period; and (iv) the financial market (in section 3.2), the capital market (in section 3.3), the general goods market, and the labor market ( $N_t = N$ ) all clear.

From Eq. (7), it follows that  $E_t[a_{t+1}^j/c_{t+1}^j] = a_t^j E_t[R_{t+1}^j/c_{t+1}^j] - 1$ . Substituting the Euler equation (8) into this equation, we obtain  $a_t^j/c_t^j = \beta E_t[a_{t+1}^j/c_{t+1}^j] + \beta$ . From this equation and the law of iterated expectations, it follows that  $a_t^j/c_t^j = \beta^s E_t[a_{t+s}^j/c_{t+s}^j] + \beta + \beta^2 + \dots + \beta^s$ . Since the transversality condition is given by  $\lim_{s \rightarrow \infty} \beta^s E_t[a_{t+s}^j/c_{t+s}^j] = 0$ , it follows that  $a_t^j/c_t^j = \beta/(1 - \beta)$  for all  $t \geq 0$ . Substituting this equation into Eq.(7) yields

$$a_t^j = \beta R_t^j a_{t-1}^j. \quad (14)$$

#### 3.1 Aggregate net worth

Define  $\Xi_t = \{j \in X : \Phi_t^j \leq \phi_t\}$ . Then,  $\Xi_t$  is the set of entrepreneurs who draw individual-specific productivity shocks such that  $\Phi_t^j \leq \phi_t$  in period  $t$ . From Eq. (14), it follows that

$$A_t := \int_{j \in X} a_t^j dj = \int_{j \in X} \beta R_t^j a_{t-1}^j dj. \quad (15)$$

The aggregate income across all entrepreneurs is equal to the sum of the total capital income, and thus,  $\int_{j \in X} R_t^j a_{t-1}^j dj = \rho_t K_t$  where  $K_t = \int_{j \in X \setminus \Xi_t} \Phi_{t-1}^j i_{t-1}^j dj$  is the total capital used for general goods production. Therefore, the aggregate net worth across all entrepreneurs is obtained as follows:

$$A_t = \beta \rho_t K_t. \quad (16)$$

### 3.2 Financial market clearing condition

The financial market clearing condition is given by

$$\int_{j \in X} d_t^j dj = 0. \quad (17)$$

Substituting Eq. (6) into Eq. (17) yields

$$\int_{j \in \Xi_t} a_t^j dj - \frac{\mu}{1 - \mu} \int_{j \in X \setminus \Xi_t} a_t^j dj = 0. \quad (18)$$

The i.i.d. assumption computes

$$\int_{j \in \Xi_t} a_t^j dj = \int_{j \in \Xi_t} dj \cdot \int_{j \in X} a_t^j dj = G(\phi_t) A_t \quad (19)$$

and

$$\int_{j \in X \setminus \Xi_t} a_t^j dj = \int_{j \in X \setminus \Xi_t} dj \cdot \int_{j \in X} a_t^j dj = (1 - G(\phi_t)) A_t, \quad (20)$$

where we have used  $\int_{j \in \Xi_t} dj = \int_{\Phi_t^j \leq \phi_t} dG(\Phi_t^j) = G(\phi_t)$  for the second equalities in Eqs. (19) and (20). Substituting Eqs. (19) and (20) with Eq. (16) into Eq. (18) yields

$$G(\phi_t) A_t - \frac{\mu(1 - G(\phi_t))}{1 - \mu} A_t = 0. \quad (21)$$

Eq. (21) yields  $G(\phi^*) = \mu$ , from which an equilibrium cutoff can be uniquely determined as follows:

$$\phi^* = G^{-1}(\mu) =: \phi^*(\mu). \quad (22)$$

Lemma 2 below shows that  $\phi^*$  strictly increases with  $\mu$ .

**Lemma 2.**  $d\phi^*(\mu)/d\mu > 0$ .

*Proof.*  $G(\cdot)$  is differentiable and strictly increasing, and  $\mu \in (0, 1)$ . Hence,  $\phi^*$  is uniquely determined and strictly increases with  $\mu$ .  $\square$

### 3.3 Capital market clearing condition

The capital market clearing condition is given by

$$K_{t+1} = \int_{j \in X \setminus \Xi_t} \Phi_t^j \cdot i_t^j dj. \quad (23)$$

By using Eq. (5), the i.i.d. assumption rewrites Eq. (23) as

$$K_{t+1} = \frac{1}{1-\mu} \int_{j \in X \setminus \Xi_t} \Phi_t^j \cdot a_t^j dj = \frac{1}{1-\mu} \int_{\Phi_t^j > \phi_t} \Phi_t^j dG(\Phi_t^j) \cdot \int_{j \in X} a_t^j dj. \quad (24)$$

Substituting Eqs. (16) and (22) into Eq. (24) yields

$$K_{t+1} = \frac{\beta H(\phi^*)}{1-\mu} \rho_t K_t, \quad (25)$$

where  $H(\phi^*) := \int_{\Phi_t^j > \phi^*} \Phi_t^j dG(\Phi_t^j)$ . One can demonstrate that  $\lim_{\mu \rightarrow 1} H(\phi^*)/(1-\mu) = m$  by applying L'Hopital's rule and the inverse function theorem with Eq. (22), provided that  $m$  (which is the highest productivity) is a finite value. Therefore, the economy is well defined when the financial market approaches perfection. In this case, aggregate saving in the economy is used by the most productive entrepreneurs for capital production.

## 4 Dynamics

Dividing both sides of Eq. (25) by  $N$  and inserting Eqs. (10) and (22) into the resulting equation, we obtain a dynamic equation of per worker capital as follows:

$$k_{t+1} = \Omega(\mu) f'(k_t) k_t =: \Psi(k_t; \mu), \quad (26)$$

where  $\Omega(\mu) := \beta H(\phi^*(\mu))/(1-\mu)$ .

**Lemma 3.**  $\partial \Omega(\mu) / \partial \mu > 0$ .

*Proof.* See the Appendix.

From Eq. (26) and Lemma 3, it follows that  $\partial\Psi(k_t; \mu)/\partial\mu > 0$ . This implies that the configuration of Eq. (26) shifts up as financial frictions are mitigated. This is because as financial frictions are mitigated, low-productivity entrepreneurs are ruled out of capital production, and thus, productivity in the economy as a whole becomes high.

## 4.1 Steady state

Suppose that  $k^*$  is the nontrivial steady-state value of capital. From Eq. (26),  $k^*$  satisfies the following equation:

$$1 = \Omega(\mu)f'(k^*). \quad (27)$$

Assuming the existence of the nontrivial steady-state value of capital for any  $\mu \in (0, 1)$ , one notes from Eq. (27) that  $k^*$  has a one-to-one relationship with  $\mu$  because of Lemma 3 and can be written in terms of  $\mu$  as  $k^* = f'^{-1}(1/\Omega(\mu)) =: k^*(\mu)$ . In particular, we obtain the following proposition.

**Proposition 1.** *The steady-state value of per worker capital strictly increases as financial frictions are mitigated, i.e.,  $\partial k^*(\mu)/\partial\mu > 0$ .*

*Proof.* See the Appendix.

The mitigation of financial frictions promotes the accumulation of capital stock in the steady state because productivity in the economy becomes high and output increases as financial frictions are mitigated.

## 4.2 Dynamic patterns

In this subsection, we investigate the dynamic behavior of the capital stock that is characterized by Eq. (26). From Eq. (26), it follows that

$$\Psi'(k_t; \mu) = \Omega(\mu)[f'(k_t)k_t]' = \Omega(\mu)f'(k_t)[1 - \eta(k_t)], \quad (28)$$

where  $\eta(k_t)$  is the elasticity of the marginal product defined by (12). We prepare the following assumption with respect to  $\eta(k_t)$  to explore the case in which we are interested.

**Assumption 1.**  $\eta'(k_t) > 0$ ,  $\lim_{k_t \rightarrow 0} \eta(k_t) < 1$ , and  $\lim_{k_t \rightarrow \infty} \eta(k_t) > 2$ .

Under Assumption 1, there exist unique values of  $\hat{k}$  and  $\tilde{k}$  in  $(0, \infty)$  such that  $\eta(\hat{k}) = 1$  and  $\eta(\tilde{k}) = 2$ . Then, it follows that

$$\eta(k_t) \begin{cases} \in (0, 1] & \text{for } 0 < k_t \leq \hat{k} \\ \in (1, 2] & \text{for } \hat{k} < k_t \leq \tilde{k} \\ \in (2, \infty) & \text{for } \tilde{k} < k_t. \end{cases} \quad (29)$$

Assumption 1 implies that the elasticity of the marginal product strictly increases with per worker capital. This assumption holds with a production function exhibiting a constant elasticity of substitution such as  $F(K_t, N) = (\gamma_1 N^{-\sigma} + \gamma_2 K_t^{-\sigma})^{-\frac{1}{\sigma}}$  under some parameter conditions, which is used for the numerical analysis in section 5.2.

### 4.2.1 Local dynamics

Eq. (26) can be linearized in the neighborhood of the steady state as  $k_{t+1} - k^* = \Psi'(k^*; \mu)(k_t - k^*)$ , which can be rewritten as

$$k_{t+1} - k^* = [1 - \eta(k^*)](k_t - k^*) \quad (30)$$



by using Eqs. (27) and (28). The local stability in the neighborhood of the steady state immediately follows from Eqs. (29) and (30), as summarized in Proposition 2 below.

**Proposition 2.** *Suppose that Assumption 1 holds. Then, if  $0 < k^* < \tilde{k}$ , the steady state of Eq. (26) is stable, and if  $k^* > \tilde{k}$ , the steady state of Eq. (26) is unstable.*

*Proof.* From Eqs. (29) and (30), it follows that  $|1 - \eta(k^*)| < 1$  if  $0 < k^* < \tilde{k}$ , and  $1 - \eta(k^*) < -1$  if  $k^* > \tilde{k}$ . Then, the claims hold.  $\square$

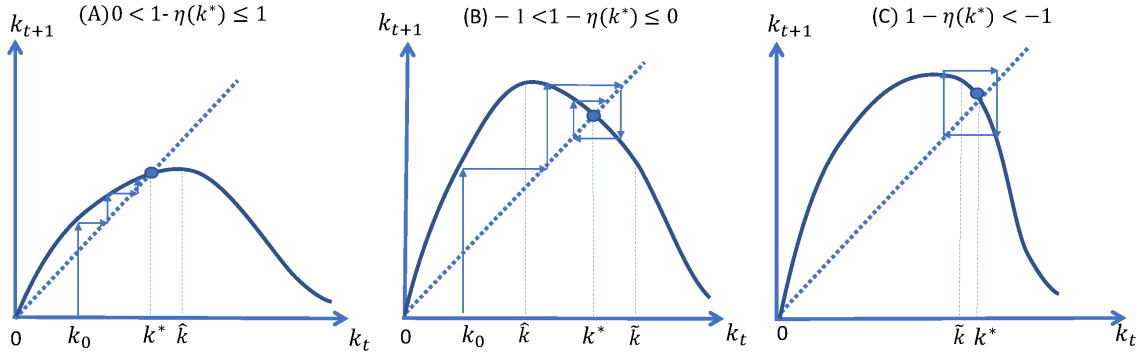


Figure 1: Dynamics of capital given by Eq. (26)

#### 4.2.2 Global dynamics

The configuration of  $\Psi(k_t; \mu)$  depends upon the sign of  $\Psi'(k_t; \mu)$  with  $\mu$  given. Since the sign of  $\Psi'(k_t; \mu)$  is the same as that of  $1 - \eta(k_t)$ , as seen in Eq. (28), we have a lemma regarding the configuration of  $\Psi(k_t)$  below.

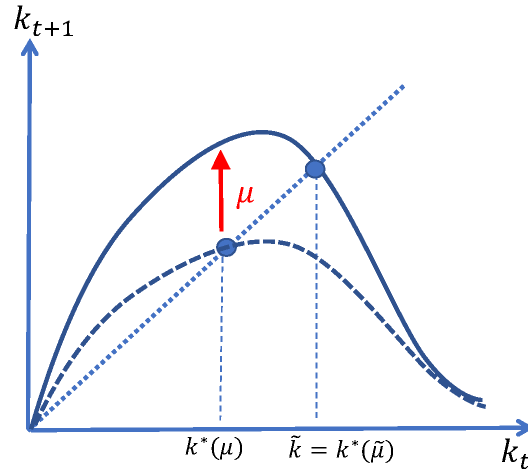
**Lemma 4.** *Suppose that Assumption 1 holds. Then,  $\Psi(\cdot; \mu)$  is a single-peaked mapping; i.e.,  $\Psi(k_t; \mu)$  increases with  $k_t \in (0, \hat{k})$  and decreases with  $k_t \in (\hat{k}, \infty)$ .*

*Proof.* Since the sign of  $\Psi'(k_t; \mu)$  is the same as that of  $1 - \eta(k_t)$ , if  $0 < k_t < \hat{k}$ , it holds that  $\Psi'(k_t; \mu) > 0$ , and if  $k_t > \hat{k}$ , it holds that  $\Psi'(k_t; \mu) < 0$ , which leads to a desired conclusion.  $\square$

To discuss the global dynamics of Eq. (26), we draw typical configurations of Eq. (26) in Figure 1. As seen in Panel A, if the steady-state capital  $k^*$  is smaller than  $\hat{k}$ , it holds that  $0 < 1 - \eta(k^*) < 1$ . In this case, the economy monotonically converges to the steady state if  $k_0 < k^*$ . If the steady-state capital  $k^*$  is greater than  $\hat{k}$  and smaller than  $\tilde{k}$ , the steady state is stable and converges to the steady state with oscillation, as seen in Panel B. If the steady-state capital stock  $k^*$  is greater than  $\tilde{k}$ , the steady state is unstable, as seen in Panel C, and the economy exhibits endogenous business cycles.

## 5 Financial destabilization

In this section, we demonstrate that financial innovations can destabilize the economy.



When  $\mu$  increases from 0 to 1, a flip bifurcation occurs at  $\mu = \tilde{\mu}$ .

**Figure 2: Financial destabilization**

## 5.1 Bifurcation

The steady-state capital stock  $k^*$  increases with  $\mu$ , as see in Proposition 1, whereas  $\tilde{k}$  does not vary with  $\mu$ . To investigate bifurcations, the following assumption is imposed:

**Assumption 2.**  $\lim_{\mu \rightarrow 0} k^*(\mu) < \tilde{k} < \lim_{\mu \rightarrow 1} k^*(\mu)$ .

Again, the use of  $F(K_t, N) = (\gamma_1 N^{-\sigma} + \gamma_2 K_t^{-\sigma})^{-\frac{1}{\sigma}}$  satisfies Assumption 2 under some parameter conditions. Under Assumptions 1 and 2, there exists a unique  $\tilde{\mu}$  in  $(0, 1)$  such that  $k^*(\tilde{\mu}) = \tilde{k}$ . Then, we obtain Proposition 3 below.

**Proposition 3.** *Suppose that Assumptions 1 and 2 hold. Suppose also that  $\mu$  increases from 0 to 1. Then, a flip bifurcation occurs in the steady state at  $\mu = \tilde{\mu}$ .*

*Proof.* Since  $\eta(k^*(\tilde{\mu})) = 2$ , it follows that  $1 - \eta(k^*(\tilde{\mu})) = -1$ , and from Proposition 1 and Assumption 1, we have  $\partial(1 - \eta(k^*(\tilde{\mu}))/\partial\mu = -\eta'(k^*(\tilde{\mu}))(\partial k^*(\tilde{\mu})/\partial\mu) < 0$ . Then, a flip bifurcation occurs at  $\mu = \tilde{\mu}$ .  $\square$

The intuition regarding the occurrence of the flip bifurcation is as follows. If entrepreneurs earn higher income than that in the steady state, they invest more than the amount in the steady state. Then, the increased capital has two conflicting effects on capital income  $\rho_t k_t$ . On the one hand, the increase in  $k_t$  places upward pressure on capital income because the source of capital income becomes greater. On the other hand, the increase in  $k_t$  decreases the marginal product of capital  $\rho_t$ , which has negative impacts on capital income. When the negative effect dominates the positive one, the larger investment yields a very low return on capital income in the next period. Because of lower earnings, entrepreneurs now invest less than the amount in the steady state, and thus earn more than in the steady state in the next period. These fluctuations around the steady state become unstable as financial frictions are mitigated. One notes that when  $\mu < \tilde{\mu}$ , the steady state is stable, and the economy converges to the steady state. As  $\mu$  increases from 0 to 1,  $\Psi(k_t; \mu)$  shifts up; as seen in Figure 2, a flip bifurcation occurs in the steady state, and the

economy loses stability when  $\mu = \tilde{\mu}$ . If  $\mu$  becomes greater than  $\tilde{\mu}$ , or equivalently, if  $\eta(k^*) > 2$ , the steady state becomes locally unstable, and the economy exhibits endogenous business cycles, as illustrated in Panel C in Figure 1. This means that financial innovations can destabilize the economy.

The condition for the steady state to become locally unstable can be rewritten in terms of labor and capital income and the elasticity of labor demand by using Lemma 1. Lemma 1 rewrites  $\eta(k^*) > 2$  as follows:

$$\left(\frac{1}{\tilde{\epsilon}}\right) \left(\frac{\tilde{w}}{\tilde{\rho}\tilde{k}}\right) > 2, \quad (31)$$

where  $\tilde{\epsilon}$  is the value of  $\epsilon_t$  evaluated at  $k_t = \tilde{k}$ . The condition derived in Woodford (1989) for local instability of the steady state can be reduced to Eq. (31) provided that labor is inelastically supplied in his model. In contrast to our model, both capitalists and workers are identical in his model, and thus, neither borrowing nor lending occurs between the two groups and within each group. The distinctive features in our model relative to Woodford's model are the heterogeneity in entrepreneurs' productivity and the financial friction that they face. The heterogeneity in entrepreneurs' productivity yields the situation in which borrowing and lending occur among entrepreneurs and the financial friction allows us to investigate how the mitigation of financial frictions destabilizes the economy.

## 5.2 Numerical analysis

This subsection provides a numerical example. We produce bifurcation diagrams with respect to  $\mu$  to examine how the mitigation of financial frictions affects qualitative characteristics of dynamic behaviors in the economy.

### 5.2.1 Specification and parameterization

The functional form of the production technology is specified as

$$F(K_t, N) = (\gamma_1 N^{-\sigma} + \gamma_2 K_t^{-\sigma})^{-\frac{1}{\sigma}},$$

which exhibits a constant elasticity of substitution.  $F(K_t, N)$  can be rewritten as a per worker production function as follows:

$$f(k_t) = (\gamma_1 + \gamma_2 k_t^{-\sigma})^{-\frac{1}{\sigma}}, \quad (32)$$

where  $\gamma_1, \gamma_2 \in (0, 1)$  and  $\gamma_1 + \gamma_2 = 1$ . From Eq. (32), we have the elasticity of the marginal product of capital as follows:

$$\eta(k_t) = \frac{-f''(k_t)k_t}{f'(k_t)} = (1 + \sigma) \left( \frac{\gamma_1}{\gamma_1 + \gamma_2 k_t^{-\sigma}} \right). \quad (33)$$

We focus on the case in which  $\sigma > 0$ , so that Assumption 1 can hold and  $\lim_{K_t \rightarrow 0} F(K_t, N) = \lim_{N \rightarrow 0} F(K_t, N) = \lim_{k_t \rightarrow 0} f(k_t) = 0$  can hold. In this case,  $\hat{k}$  can be computed as  $\hat{k} = [\gamma_2/(\sigma\gamma_1)]^{\frac{1}{\sigma}}$ , and  $\tilde{k}$  can be computed as  $\tilde{k} = [2\gamma_2/(\gamma_1(\sigma - 1))]^{\frac{1}{\sigma}}$  only when  $\sigma > 1$ , where, as previously defined,  $\hat{k}$  and  $\tilde{k}$  satisfy  $\eta(\hat{k}) = 1$  and  $\eta(\tilde{k}) = 2$ , respectively. From Eq. (26) and  $f'(k_t)k_t = \gamma_2 k_t/(\gamma_1 k_t^\sigma + \gamma_2)^{\frac{1+\sigma}{\sigma}}$ , the dynamic equation for  $k_t$  is obtained as

$$k_{t+1} = \Psi(k_t; \mu) := \Omega(\phi^*) \frac{\gamma_2 k_t}{(\gamma_1 k_t^\sigma + \gamma_2)^{\frac{1+\sigma}{\sigma}}}. \quad (34)$$

In Eq. (34), one notes that  $\Psi(0; \mu) = 0$  for  $k_t = 0$  and  $\Psi(k_t; \mu) > 0$  for  $k_t > 0$ . It follows from Eq. (34) that

$$\Psi'(k_t; \mu) = \Omega(\phi^*) \frac{\gamma_1 \gamma_2 \sigma}{(\gamma_1 k_t^\sigma + \gamma_2)^{\frac{1+2\sigma}{\sigma}}} \left[ \frac{\gamma_2}{\sigma \gamma_1} - k_t^\sigma \right] \begin{cases} \geq 0 & \text{for } k_t \leq \hat{k} \\ < 0 & \text{for } k_t > \hat{k}, \end{cases} \quad (35)$$

which implies that  $\Psi(k_t; \mu)$  increases with  $k_t \in (0, \hat{k})$  and decreases with  $k_t \in (\hat{k}, \infty)$ . Thus,  $\Psi(\cdot; \mu)$  is a single-peaked mapping. We impose a parameter condition such that  $\Omega(\phi^*)\gamma_2^{-1/\sigma} > 1$  can hold. Under this parameter condition, Eq. (34) has a nontrivial steady state,  $k^*$ , because  $\Psi'(0) > 1 \iff \Omega(\phi^*)\gamma_2^{-1/\sigma} > 1$ .

It is assumed that  $\Phi_t^j$  follows a uniform distribution over  $[0, m]$ . Therefore, we have  $G(\phi) = \phi/m$  and  $H(\phi) = (m^2 - \phi^2)/(2m)$ .<sup>5</sup> Under the assumption of the uniform distribution, the productivity cutoff is given by

$$\phi^* = \mu m. \quad (36)$$

In the numerical analysis, bifurcation diagrams are produced by varying  $\mu$ . We set  $\beta = 0.96$  following standard real business cycles theory and  $\gamma_1 = 0.67$  and  $\gamma_2 = 0.33$  taking into account the standard income share ratio between labor and capital. We set a relatively large value for  $m$ , that is,  $m = 10$ . In this case, the average productivity of entrepreneurs is equal to  $H(0) = 5$ . Under these parameter settings, we examine the three cases of  $\sigma = 1, 1.46$  and  $2.5$ .<sup>6</sup>

### 5.2.2 Bifurcation diagrams

The bifurcation diagrams in Figure 3 depict the effects that financial constraints have on the dynamics of capital in equilibrium.

Recent empirical studies report that the elasticity of substitution between capital and labor takes a value from 0.4 to 0.6 (e.g., Klump et al., 2007; Chirinko, 2008; León-Ledesma et al., 2010). Then, we examine three cases in which the elasticity of substitution is approximately equal to 0.50, 0.41, and 0.29. As seen in the figure, when  $\sigma = 1$  (the elasticity of substitution between capital and labor is  $1/(1 + \sigma) = 0.50$ ), the economy monotonically converges to a steady state for any value of  $\mu$  because the

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<sup>5</sup>In the Appendix, we also examine the case in which  $\Phi_t^j$  follows a Pareto distribution for a robustness check and obtain a similar result to that obtained from the uniform distribution.

<sup>6</sup>Under these parameter settings, it holds that  $\Omega(\phi^*)\gamma_2^{-1/\sigma} > 1$  for  $\sigma = 1, 1.46$  and  $2.5$  and  $\mu \in (0, 1)$ , and thus, Eq. (34) has a nontrivial steady state,  $k^*$ .

steady state of Eq. (34) is stable. The capital stock in the steady state increases as  $\mu$  increases because the mitigation of financial frictions promotes capital accumulation. When  $\sigma = 1.46$  (the elasticity of substitution between capital and labor is equal to  $1/(1 + \sigma) \approx 0.41$ ), a flip bifurcation occurs at a certain value of  $\mu$  and a period-two cycle arises. One notes that as  $\mu$  increases, the amplitude of the period-two cycle increases. When  $\sigma = 2.5$  (the elasticity of substitution between capital and labor  $1/(1 + \sigma) \approx 0.29$ ), complex dynamic behaviors are obtained. In this case, whereas smaller values of  $\mu$  produce a period-four cycle, period-doubling bifurcations repeatedly occur as  $\mu$  increases. Eventually, greater values of  $\mu$  produce complex dynamics in the economy. As in the case of  $\sigma = 1.46$ , the amplitude of fluctuations increases as  $\mu$  increases.

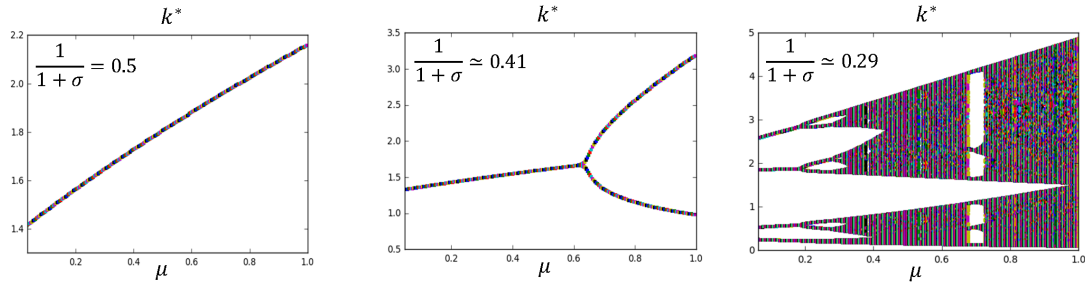


Figure 3 : Effects of financial constraints

In summary, in the case in which endogenous business cycles occur, the amplitude of fluctuations increases, and this amplitude is maximized when the financial market approaches perfection. This outcome implies that the mitigation of financial frictions is likely to destabilize the economy.

## 6 Conclusion

Do financial innovations destabilize economies? Many researchers and policymakers have argued that financial innovations destabilize economies. To address this issue, we

have developed a simple dynamic general equilibrium model with financial frictions. Applying the production function in which the elasticity of the marginal product of capital increases as capital accumulates, we investigate the characteristics of the dynamic behavior of the economy. According to recent empirical studies, the elasticity of substitution between capital and labor ranges from 0.4 to 0.6. Our numerical analysis shows that when it is 0.41, a flip bifurcation occurs at an intermediate extent of financial frictions, and a period-two cycle arises. Furthermore, our analysis shows that as financial frictions are mitigated, the amplitude of the cycle increases. These outcomes imply that financial innovations can destabilize economies. Our model can be extended to introduce intrinsically useless assets. The investigation into how the presence of such assets changes the destabilization property is left for future research.

## Appendix

### Proof of Lemma 1

From (11), we have  $dw_t/dN_t = -f''(k_t)k_t^2/N_t$ . Then, it follows that

$$\begin{aligned}
 \eta(k_t) &= \frac{-f''(k_t)k_t}{f'(k_t)} \\
 &= \left( \frac{N_t \frac{dw_t}{dN_t}}{k_t^2} \right) \left( \frac{k_t}{f'(k_t)} \right) \\
 &= \left( \frac{1}{\frac{w_t}{N_t} \frac{dN_t}{dw_t}} \right) \left( \frac{w_t}{f'(k_t)k_t} \right) \\
 &= \left( \frac{1}{\epsilon_t} \right) \left( \frac{w_t}{\rho_t k_t} \right). \tag{37}
 \end{aligned}$$

□



### Proof of Lemma 3

From  $\Omega(\mu) = \beta H(\phi^*(\mu))/(1 - \mu)$ , it follows that

$$\frac{\partial \Omega(\phi^*)}{\partial \mu} = \beta \left( \frac{-\phi^* G'(\phi^*) \frac{d\phi^*}{d\mu} (1 - \mu) + H(\phi^*)}{(1 - \mu)^2} \right). \quad (\text{A.1})$$

Since  $G(\phi^*) = \mu$  and  $G'(\phi^*)(d\phi^*/d\mu) = 1$ , Eq. (A.1) can be rewritten as

$$\frac{\partial \Omega(\phi^*)}{\partial \mu} = \beta \left( \frac{H(\phi^*) - \phi^*(1 - G(\phi^*))}{(1 - \mu)^2} \right). \quad (\text{A.2})$$

In Eq. (A.2), it holds that  $H(\phi^*) - \phi^*(1 - G(\phi^*)) > 0$ . Therefore,  $\partial \Omega(\phi^*)/\partial \mu > 0$ .

□

### Proof of Proposition 1

From Eq. (27), it follows that

$$\ln \Omega(\mu) = -\ln f'(k^*). \quad (\text{A.3})$$

Differentiating Eq. (A.3) with respect to  $\mu$  yields

$$\left( \frac{1}{\Omega(\mu)} \right) \left( \frac{\partial \Omega(\mu)}{\partial \mu} \right) = \left( -\frac{f''(k^*)}{f'(k^*)} \right) \left( \frac{\partial k^*}{\partial \mu} \right),$$

or equivalently,

$$\frac{\partial k^*}{\partial \mu} = \frac{f'(k^*)}{-f''(k^*)\Omega(\mu)} \left( \frac{\partial \Omega(\mu)}{\partial \mu} \right) \quad (\text{A.4})$$

In Eq. (A.4), it follows from Lemma 3 that  $\partial \Omega(\mu)/\partial \mu > 0$ . Therefore,  $\partial k^*(\mu)/\partial \mu > 0$ .

□

### Bifurcation diagrams in the case of the Pareto distribution

Suppose that  $\Phi_t^j$  follows a Pareto distribution such that

$$G(\phi) = \begin{cases} 1 - \left(\frac{\alpha}{\phi}\right)^n & \text{if } \alpha < \phi \\ 0 & \text{if } \phi \leq \alpha, \end{cases}$$

where  $\alpha > 0$  and  $n \geq 2$ . Then, we have  $H(\phi) = \phi[1 - G(\phi)]n/(n - 1)$ . It is straightforward to obtain the productivity cutoffs such that

$$\phi^* = \frac{\alpha}{(1 - \mu)^{\frac{1}{n}}}. \quad (\text{A.5})$$

To produce bifurcation diagrams, we set  $\alpha = 1$  and  $n = 2$  with other parameter values remaining the same as those in subsection 5.3. From Eq. (A.5), we obtain  $\Omega(\phi^*) = \beta H(\phi^*(\mu))/(1 - \mu) = \alpha n/[(1 - \mu)^{\frac{1}{2}}(n - 1)]$ . As seen in this equation,  $\Omega(\phi^*)$  is not well defined when  $\mu \rightarrow 1$  because  $m$  is not finite in the case of Pareto distributions. Then, we impose the upper limit of  $\mu$  at  $\mu = 0.99$  when producing the bifurcation diagrams. Figure A shows the effects of financial constraints on the dynamic behavior of capital. The results are basically the same as those of the uniform distribution.

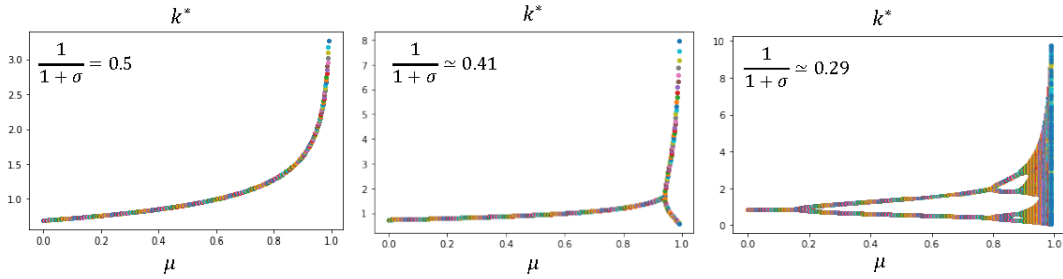


Figure A : Effects of financial constraints (Pareto distribution)

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