ISSN (Print) 0473-453X
Discussion Paper No. 1132
ISSN (Online) 2435-0982

PIONEER, EARLY FOLLOWER OR LATE ENTRANT: ENTRY DYNAMICS WITH LEARNING AND MARKET COMPETITION

Chia-Hui Chen
Junichiro Ishida
Arijit Mukherjee

April 2021

The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

Pioneer, Early Follower or Late Entrant: Entry Dynamics with Learning and Market Competition*

CHIA-HUI CHEN
Kyoto University

JUNICHIRO ISHIDA
Osaka University

ARIJIT MUKHERJEE
University of Nottingham

April 22, 2021

Abstract. Timing of market entry is one of the most important strategic decisions a firm must make, but its decision process becomes convoluted with information and payoff spillovers. The threat of competition pushes firms to enter earlier to preempt their rivals while the possibility of learning make them cautiously wait for others to take action. This combination amounts to a new class of timing games where first-mover advantage first emerges as in preemption games but second-mover advantage later prevails as in wars of attrition. Our model identifies under what conditions a firm becomes a pioneer, early follower or late entrant and shows that the timing of entry is excessively early (late) when there emerges a late entrant (early follower). We also argue that consumer inertia is often efficiency-enhancing in this environment, highlighting an elusive link between static market competition and dynamic entry competition.

JEL Classification Number: D82, D83, L13
Keywords: market entry, market competition, private learning, signaling, preemption, consumer inertia.

*This is a substantially revised version of the manuscript previously circulated under the title "An Entry Game with Learning and Market Competition." We thank Makoto Hanazono, Hideshi Itoh, Hülya Eraslan, Kim-Sau Chung and seminar and conference participants at Korea University, Kyoto University, Hitotsubashi University, Academia Sinica, Hong Kong Baptist University, Chinese University of Hong Kong, SAET 2018, Pan Pacific Conference in Economic Research, and Contract Theory Workshop (CTW) for helpful comments and suggestions. The first author acknowledges financial support from JSPS KAKENHI Grant Numbers JP17K13699 and JP16H03615. The second author acknowledges financial support from JSPS KAKENHI Grant Numbers JP20H05631, JP19H01468, JP18H03640, and JP16H03615 as well as the program of the Joint Usage/Research Center for Behavioral Economics at ISER, Osaka University. All errors are our own.

1. Introduction

It is often emphasized, mainly in the fields of strategic management and marketing, that timing of market entry is one of the most critical strategic decisions a firm must make whenever there is a new (geographical) market, a new product, or a new technology becoming available.[1] Market entry strategies are in fact highly complicated, as the benefit of market entry depends not only on the profitability of the market, which is often uncertain, but also on potential responses of other rival firms. Should a firm take the initiative in opening up a market and be a "pioneer," or more cautiously wait for others to take action? In case there emerges a pioneer, should a firm follow immediately or take some time to see how the market develops over time? Understanding this strategic decision process is of first-order importance, not only for potential entrants but also for policymakers, as it leads to immense welfare and policy implications: valuable resources are wasted if firms are rushed to enter a failed market while potential gains must be sacrificed if they wait too long to enter a successful one.

In a nutshell, the optimal timing of market entry comes down to the tradeoff between becoming a leader and a follower. On this tradeoff, Lieberman and Montgomery (1988)—one of the seminal articles on first-mover advantage—descriptively raise two strategic considerations as crucial forces in shaping market entry outcomes. On one hand, in the presence of market competition, there arises a benefit of *preemption*, which urges potential entrants to enter the market before their rivals do in order to seize market power (p.44-47). On the other hand, there is also a benefit of *learning from rivals* when there is uncertainty over potential benefits of market entry, as they argue "Late movers can gain an edge through resolution of market or technological uncertainty" (p.47). These two considerations generate counteracting incentives and a dynamic tradeoff of our focus.

In this paper, we build on this broad yet somewhat informal insight and develop a stylized model of market entry which sheds light on determinants and welfare consequences of entry dynamics in a tractable manner; main applications of our model include new product markets, technology adoption and foreign direct investment among others. We consider an environment with two potential entrants, each of which independently decides whether and when to enter a new market. The profitability of the market is determined by the market condition (e.g., market size, production cost, quality of labor force) which is not known to anyone initially. Each firm thus privately investigates whether the market is profitable enough over time and enters when it becomes sufficiently confident about the market.

[1] For instance, Lilien and Yoon (1990) note that "the choice of market-entry time is one of the major reasons for new product success or failure."

The strategic nature of our model is determined by two spillover effects of market entry which stem from market competition and private learning, as summarized below.

- *Payoff spillovers from market competition.* The payoff from entry (in case the market turns out to be good) is decreasing in the number of firms in the market. A firm's entry thus reduces the residual demand and makes the rival firm's subsequent entry less profitable. The payoff spillover generates a strong incentive to be the first mover to preempt the rival firm.
- *Information spillovers from private learning.* Since each firm privately collects information about the market condition, a firm's entry serves as a signal of the firm's confidence in the market. The information spillover generates a strong incentive to be the second mover to learn from the rival firm's action.

These forces generate a tradeoff where the first mover can potentially capture larger monopoly rent by entering early, but loses information that it could have obtained from its rival, thereby capturing both first-mover advantage and second-mover advantage in a unified framework. The overall benefit of becoming a market pioneer is essentially determined by the way this dynamic tradeoff resolves.

To see the workings of our model, it is important to observe that the signaling effect of market entry is generally weaker at early stages of the game when the structure of information is sufficiently symmetric. As time passes, however, it eventually reaches a point where a firm's entry reveals so much information that the rival firm immediately follows suit, thereby entirely dissipating the first-mover advantage. Due to this effect, the game is divided into two distinct phases: the *preemption phase* where the payoff spillover is the dominant concern; and the *waiting phase* where each firm has an incentive to wait for the other to make a move. Our framework provides a new class of timing games in which first-mover advantage first emerges as in preemption games while second-mover advantage later prevails as in wars of attrition.

1.1. Main results and contributions

As noted above, the timing and order of market entry are a topic of utmost concern in the strategic management and marketing literature.[2] This paper aims to contribute to this voluminous literature and offer some important implications, both positive and normative, by generating a range of (observationally distinguishable) entry patterns. Broadly

[2] Since the seminal work of Lieberman and Montgomery (1988), there is now a voluminous literature examining, mainly empirically, the extent of the first-mover advantages which is also known as the order-of-entry effect.

speaking, the main contribution of our analysis is that it incorporates both private learning (and hence signaling via market entry) and market competition in an analytically tractable manner. Despite the fact that these two factors are often regarded as the primary sources of early-mover and late-mover advantages (Lieberman and Montgomery, 1988; Mitchell et al., 1994; Luo, 1998), they have been investigated rather independently and separately from one another in the existing theoretical literature: there are many related works which incorporate either one of these elements in various ways but very few, if any, which combine them in a unified framework as we do here (see the literature review in section 1.2 for more on this point). More specifically, there are three findings which we would like to highlight here.

First, we provide a novel analytical framework—a hybrid of two widely used timing games—which allows us to pin down the temporal distribution of market entry times for a given set of parameters, thereby clarifying when and under what conditions a firm becomes a pioneer, early follower or late entrant. Although we base our analysis on a fairly simple setup, the interaction of information and payoff externalities makes the problem very difficult to handle technically, because the firms' beliefs evolve in a rather complicated manner depending on their entry strategies. We provide an analytical approach to overcome this problem, while preserving the substance of the problem, which allows us to obtain a necessary and sufficient condition for the first-mover advantage to dominate in equilibrium. When this condition is satisfied, the firms enter the market at some positive rate in the preemption phase until it reaches a "saturation point" where the amount of information revealed by a market entry becomes too much. Market entry then ceases to occur past this point, with neither firm taking any action, as the net value of entry becomes strictly negative. After a while, though, a firm that has accumulated more favorable information becomes confident enough and willing to enter the market again, even without the chance to earn the monopoly rent. Our model thus exhibits rich, on-and-off, dynamics of market entry where the firms gradually enter the market at early and late stages, with a period of no entry in between.[3]

Second, we examine the timing of entry in comparison to the cooperative benchmark (where the firms cooperatively choose the timing of entry to maximize the joint profit,

[3]Our on-and-off dynamics are reminiscent of market frenzies and crashes described by Bulow and Klemperer (1994). In an auction-like environment in which a seller has K identical units of a good, and $K+L$ buyers each wish to purchase a single unit, they show that there emerge both market frenzies in which a single purchase at a given price causes many others to offer the same price and market crashes in which it becomes common knowledge that no purchases will be made for some duration of time. The key driving force of their result is the fact that the willingness-to-pay curve necessarily becomes flatter for high-valuation buyers.

thus eliminating the externalities created by private learning and competition) to evaluate welfare implications of entry dynamics. An important observation is that the timing of entry can be either too early or too late, depending crucially on the (often unobserved) timing of first entry. Specifically, the efficiency of entry dynamics is determined by whether the first entry occurs in the preemption phase. If it does, the timing of entry is excessively early because: (i) in addition to the preemption motive which pushes the timing of entry forward, the first entrant fails to take into account the positive information externality; and (ii) the second entrant fails to take into account the negative payoff externality (or the "business-stealing effect"). If no entry occurs in the preemption phase, on the other hand, the timing of entry tends to be excessively delayed because each firm has a strong incentive to wait and see the rival's action.

This observation is useful only to the extent that the timing of first entry is observable, but the existence of a market is often not known to outside observers until its first entry actually materializes. Fortunately, though, the time lag between early and late entrants is suggestive and provides enough information, because the temporal distribution of entry times of the second mover is shaped entirely by the timing of first entry. A general rule of thumb is that when market entries are clustered in a short span of time (the case of early follower), chances are that the market was already ripe when the first entrant arrived, implying that the game has reached the waiting phase. In contrast, when entries are spaced apart in time (the case of late entrant), the first entrant was likely to be a true pioneer who entered when the market was still filled with uncertainty, or in the preemption phase. Drawing on this result, we argue, somewhat paradoxically, that the timing of entry tends to be too early when there is a late entrant while it tends to be too late when there is an early follower.

Third, we assume for most of our analysis that there is no market friction and the first-mover advantage dissipates immediately after the arrival of the follower. In reality, however, we often find instances where first-mover advantage persists over time, especially once the first mover has established its presence in the market. This form of consumer inertia can indeed arise from various factors such as brand loyalty, habit formation, switching costs, and slow diffusion of product information.[4] In an extended version of our model, we incorporate this possibility and examine how it affects entry dynamics. We argue that consumer inertia, which biases the allocation of surplus in favor of the first mover, is generally efficiency-enhancing in our environment because it raises the benefit of becoming

[4]In fact, in the field of strategic management, the impact of the timing of entry on long-run performances has been the major focus of the literature on first-mover advantage.

the first mover and facilitates information sharing between the competing firms. More broadly, this argument points to an elusive link between static market competition and dynamic entry competition that has received little attention in the literature: market competition on equal footing may be beneficial from the *ex post* point of view (once all the entry decisions are made), but it may distort the timing of market entry by limiting the benefits of becoming the first mover that are not fully internalized by potential entrants.

1.2. Related literature

Our paper is most closely related to canonical preemption games such as Reinganum (1981) and Fudenberg and Tirole (1985).[5] They consider an environment where firms independently determine the timing of technology adoption with payoff spillovers. The payoff to a firm depends on and is decreasing in the number of firms adopting the technology while the cost of doing so varies over time. Their models are pure preemption games in which there are no strategic incentives to delay adoption.[6] We extend this standard setting by incorporating state uncertainty and private learning, which create information externalities and are often considered important for market entry. This gives rise to a countervailing incentive of waiting for the rival's move (and information). This new element of private learning qualitatively changes the strategic nature of the problem and provides a new angle to address the question of when first-mover advantage prevails.

Our model also exhibits a phase which is effectively a war of attrition, and in this sense related to Chen and Ishida (2021) who analyze a war of attrition, as formulated by Fudenberg and Tirole (1986), with experimentation about the unobserved state of nature via exponential bandits.[7] In their model, each player may receive a signal indicating that the underlying state is good for sure, in which case it is optimal to stay in the game indefinitely. A crucial difference is that the state is individual-specific and independent across players, thus eliminating any possibility of learning from others.

Decamps and Mariotti (2004) consider a model of investment with a similar learning

[5] See Hopenhayn and Squintani (2016) and Bobtcheff et al. (2017) for some recent examples of preemption games. In this class of models, the game typically ends once a player takes an action, so that there is no observational learning.

[6] In canonical preemption games, the only reason to delay adoption is because the cost of doing so decrease over time; otherwise, there would only be a trivial equilibrium in which all players adopt immediately. The benefit of adoption delay is thus exogenous and non-strategic in this framework.

[7] Due to its tractability, the approach to model learning by exponential bandits, pioneered by Keller et al (2005), has becomes a workhorse specification in the literature and offered many applications such as Strulovici (2010), Bonatti and Hörner (2011), Keller and Rady (2010, 2015), Guo (2016), Che and Hörner (2018), Chen and Ishida (2018), and Margaria (2020), just to name a few.

process to ours in a model of strategic investment.[8] As in our model, they assume that the quality of the project, which is common to both players, is not known *ex ante* and gradually revealed over time via the arrival of a bad (public) signal. Although they focus mainly on the case with no market competition, they later extend the analysis to discuss the case where the follower's payoff is discounted by some fraction. Their model is one of public learning where all the information is publicly observed; as a consequence, there is again no possibility of learning from others.[9]

Several works examine the optimal timing of entry or exit with information externalities among players. To name some, Chamley and Gale (1994) consider a model in which there are N players, a random number n of whom have an investment option. Assuming that the value of investment depends positively on the random number n, there is an incentive to wait and see others' investment decisions. Grenadier (1999) analyzes an option exercise game in which each player is endowed with private information about the true value of the option and hence the timing of exercise becomes a signal for other players. Murto and Välimäki (2011) analyze an exit game with private learning where each player receives a signal in each period which partially reveals his own type. Kirpalani and Madsen (2020) analyze investment decisions with social learning where each player decides whether and when to invest in acquiring information. A common thread of those works is that they do not consider the element of market competition, or negative payoff externalities, and focus more on issues such as investment delays and waves.

In clear contrast, models of market entry in industrial organization generally focus on market competition but often assume away any dynamic learning.[10] For instance, Levin and Peck (2003) analyze a duopoly model of market entry in which each firm privately

[8]As we will detail below, we consider "no news is good news" whereby a firm observes a bad signal at some random time if the market condition is bad. Bloch et al. (2020) also incorporate private learning into a model of market entry but consider a different learning process where a firm can perfectly identify the true state of nature with some probability in each period or learn nothing at all ("no news is no news"). See Bloch et al. (2015) for a similar setup but with firm-specific entry costs.

[9]The optimal timing of investment under uncertainty is investigated actively in the real options literature. Although much of the literature focuses on a single-agent decision problem, there are some attempts to incorporate strategic interactions into the real options approach (Trigeorgis, 1991; Grenadier, 1996; Weeds, 2002; Shackleton et al., 2004; Pawlina and Kort, 2006). This strand of literature generally assumes public learning with no information asymmetry among the competing players as in Decamps and Mariotti (2004).

[10]Profit uncertainty also plays an eminent role in the context of foreign direct investment. Horstmann and Markusen (1996, 2018) consider a model in which a producer (the MNE) is unsure of the potential customer size and chooses either to contract with a local sales agent or to establish an owned local sales operation. While contracting with the local sales agent, the producer gains information about the customer size and switches to an owned sales operation if this option is found to be profitable. In their models, however, there is only one producer and hence no market competition.

observes its entry cost at the outset of the game. The market environment is similar to ours in that the first mover can earn monopoly rents until the second mover arrives. Aside from the fact that there is no learning, a crucial difference is that the cost uncertainty in their model is firm-specific and hence a firm's entry does not reveal any useful information to the other firm. Rasmusen and Yoon (2012) analyze a duopoly model of market entry which incorporate both market competition and signaling. They consider a two-period model in which one of the firms is better informed about the market size than the other, and market entry by the informed firm hence becomes a signal of its private information. As in Levin and Peck (2003), however, the information structure is exogenously fixed at the outset of the game, which rules out the possibility of learning over time.

2. Model

We consider a dynamic game of market entry where two potential entrants, labeled as firms 1 and 2, contemplate to enter the market of unknown profitability. The market condition, which is common to both firms, is either good or bad, and each firm can "test the waters" before it makes an irreversible entry decision. There are two sources of information in this model: on one hand, each firm may privately observe a signal of the market condition which arrives stochastically over time; on the other hand, the entry decision of each firm is publicly observable and hence serves as an additional signal. The fact that a firm can observe the rival's entry implies a benefit of "waiting," giving rise to a second-mover advantage. The tradeoff arises, however, as the profitability of each firm depends negatively on the number of firms in the market, meaning that the first one to enter can temporarily monopolize the market (until the second one arrives).

Time is continuous and extends from zero to infinity, and each firm decides whether to enter the market at discrete points in time $0, \Delta, 2\Delta, \ldots$ by incurring the entry cost $c > 0$. Throughout the analysis, we will focus on the continuous-time limit where the length of a time interval Δ shrinks to zero and only state results in the limit without further notice.[11] As we will see below, once a firm enters, it is weakly optimal to stay in the market indefinitely, so that the firm has no further decisions to make. We assume that each firm can observe the other firm's entry decisions but not the realized payoffs (or, equivalently, the market condition).[12]

[11]We consider discrete time purely for technical reasons: in our model, there arises a situation where the best response of a firm is to enter the market *immediately* after the rival's entry, which is not well defined in continuous time.

[12]One way to justify this assumption is that the profits are often realized with some time lag. In most cases of our interest, what is immediately inferrable at time of entry is not the eventual gain of market entry but the expectation held by the entrant.

Definition 1. *A firm is called active if it has entered the market and inactive otherwise.*

Both firms start with a common prior that the market condition is good with probability p_0 and gradually acquire information via the arrival of a signal. More precisely, conditional on the market being bad, an inactive firm privately observes a signal with probability $\lambda \mathrm{d}t$ for an interval of time $[t, t + \mathrm{d}t)$.[13] Note that a signal arrives only if the market is bad and hence that the arrival of a signal indicates that the market is bad for sure. This information structure, which is often called "exponential bandits," is commonly assumed in the literature on strategic experimentation (Keller et al., 2005; Bonatti and Hörner, 2011; Che and Hörner, 2018) and is assumed here to highlight the fundamental forces with as much clarity. Later in section 6.2, we extend the analysis to incorporate "Poisson bandits"—a setup in which a signal can come from either state and no one signal is conclusive—and show that the extended version retains many key properties of this baseline model and yields analogous insights.

Definition 2. *An inactive firm is called informed if it has observed a signal and uninformed otherwise.*

According to these definitions, we can classify each firm into three distinct categories: *active*, *informed*, and *uninformed*. In particular, when we refer to a firm as either informed or uninformed, it implies that it is inactive at the moment. As will become clear below, there are no further decisions to make if a firm is either active or informed, so that we generally focus on the problem of a firm that is currently uninformed.

The net profit a firm can earn is determined by the market condition and the number of firms in the market. If the market is good, an active firm earns $(\pi + m)\Delta$ per period if it is the only firm in the market and $\pi\Delta$ if both of them are in the market. We call m the monopoly premium, which could depend on the extent of market competition, and in general assume $m > 0$. If the market is bad, on the other hand, an active firm invariably earns zero profit. The net profit for an inactive firm is also normalized at zero, implying that there is no incentive to exit from the market *ex post* even when the market condition turns out to be bad.

3. Equilibrium characterization

This section provides an equilibrium characterization of the model described above. Since the problem for the second mover (after one of the firms has entered) is rather straight-

[13] An active firm may or may not observe a signal, but this is irrelevant because there is no possibility of exit in the current setup.

forward, our analytical focus is more on the first mover who must take into account how the rival firm would react to its market entry. As we will detail below, the game has two distinct phases, called *waiting* and *preemption,* depending on the way the second mover reacts.

3.1. Belief formation

From the viewpoint of an uninformed firm, there are two unknowns: the market condition (good or bad) and the rival firm's state of knowledge (informed or uninformed). Since the rival firm is by construction uninformed if the market is good, we have three possible states of the economy as described below:

1. The market is good (state G);
2. The market is bad, and the rival firm is uninformed (state BU);
3. The market is bad, and the rival firm is informed (state BI).

An uninformed firm's belief is hence defined in two dimensions and denoted as (p_t, q_t) where: (i) p_t is the conditional probability that the market is good (state G); (ii) q_t is the conditional probability that the market is bad and the rival firm is uninformed (state BU).[14] By definition, $1 - p_t - q_t$ is the conditional probability that the market is bad and the rival firm is informed (state BI). All the probabilities are conditional on the firm being inactive and uninformed.

In what follows, we focus on symmetric perfect Bayesian equilibrium in Markov strategies (hereafter, simply equilibrium) with belief (p_t, q_t) as the state variable.[15] The use of belief (p_t, q_t) as a state variable is justified in the following way. Note first that once one of the firms has entered the market, the only relevant part of the belief is p_t,[16] and the remaining firm's strategy should be conditioned only on p_t. When both firms are inactive, on the other hand, each firm needs to infer the rival firm's state. In this case, the rival firm is uninformed with probability $p_t + q_t$, in which case the firm makes its decisions based on (p_t, q_t) which is publicly known. With the remaining probability, the firm is informed ($p_t = 0$), but this case is irrelevant because we know that the firm never enters. As such, as long as the firms adopt Markov strategies, the belief pair (p_t, q_t) provides a sufficient statistic which summarizes all the relevant information of the game.

[14]The derivation of the belief is shown later in (1) and (2), after we introduce the strategies of the model.

[15]Our model does not exclude the possibility of asymmetric equilibria. We focus on symmetric equilibrium because asymmetric equilibrium necessarily requires coordination between the firms which is extremely difficult, if not impossible, to attain in many situations of our interest.

[16]If the first entry occurs at some t', then $q_t = 1 - p_t$ for all $t > t'$, so that the belief is summarized by p_t.

3.2. Second mover

It is straightforward to derive the optimal strategy for the second mover who, with no strategic concerns, simply enters if the current belief p_t is high enough. Observe that a signal arrives only if the state is bad, and hence by Bayes' rule, the belief p_t increases monotonically over time as long as the firm observes no signal (i.e., "no news is good news").

Given current belief p, the expected profit of entering now is given by

$$p\frac{\pi}{r}-c=p\frac{\pi-rc}{r}-(1-p)c,$$

which is an increasing function of p. Alternatively, the firm may wait until the next instant.[17] Since the firm observes no signal with probability $p+(1-p)e^{-\lambda\Delta}$,[18] the expected profit of this waiting strategy is given by

$$e^{-r\Delta}\left[p\frac{\pi}{r}-[p+(1-p)e^{-\lambda\Delta}]c\right]=e^{-r\Delta}\left[p\frac{\pi-rc}{r}-(1-p)ce^{-\lambda\Delta}\right].$$

The cost of adopting this strategy is the foregone profit due to time discounting. On the other hand, by waiting, the firm can collect more information, the benefit of which is captured by $(1-p)ce^{-\lambda\Delta}$. Since a firm is less likely to observe a signal when the current belief is high, the benefit of waiting is decreasing in the belief.

It follows from these that the firm enters the market now only if

$$p\frac{\pi-rc}{r}-(1-p)c\geq e^{-r\Delta}\left[p\frac{\pi-rc}{r}-(1-p)ce^{-\lambda\Delta}\right].$$

This condition gives the cutoff belief $\overline{p}$ which converges to

$$\overline{p}=\frac{(r+\lambda)c}{\pi+\lambda c},$$

as $\Delta\to 0$, suggesting that the second mover enters the market once and for all when the belief p_t reaches the threshold $\overline{p}$. To focus on relevant cases, we assume that this threshold is smaller than one, so that a firm enters if it knows that the market condition is good almost surely.

Assumption 1. $\overline{p}<1 \iff \frac{\pi}{r}>c$.

[17]Throughout the analysis, we often say "wait for an instant (or until time t)" to refer to the following entry strategy: (i) the firm does not enter now (until t); (ii) at the next instant, the firm enters if it is still uniformed at the time.

[18]For an interval $[t,t+\delta)$, an uninformed firm observes a signal with probability $1-e^{-\lambda\Delta}$ if the state is bad. The probability of a signal arriving in $[t,t+\delta)$ is hence $(1-p)(1-e^{-\lambda\Delta})$.

Now suppose that the first mover enters the market at some time t. As this necessarily implies that the first mover has observed no signal, the second mover's belief makes a discrete jump after the entry. In the limit as $\Delta \to 0$, the updated belief becomes

$$\lim_{\Delta \to 0} p_{t+\Delta} = \phi_t := \frac{p_t}{p_t + q_t},$$

which indicates the amount of information revealed by a market entry; throughout the analysis, we often refer to ϕ_t as the *post-entry belief* for expositional clarity. Note in particular that since no firm is informed at time 0, i.e., $q_0 = 1 - p_0$ and $\phi_0 = p_0$, a firm's immediate entry reveals no additional information.

If the post-entry belief exceeds $\overline{p}$, the firm will follow immediately at the next instant, so that the first mover can appropriate almost no monopoly rent in the limit.

Lemma 1. *An uninformed firm follows immediately after observing the other firm's entry at t if and only if the post-entry belief exceeds the threshold, i.e., $\phi_t \geq \overline{p} := \frac{(\lambda+r)c}{\pi+\lambda c}$.*

3.3. First mover

We now turn to the first mover's problem which is far more complicated than the second mover's problem described above. Throughout the analysis, we often denote by (p, q) the current belief and by (p', q') the belief at the next instant when no entry will have occurred. Let σ be the (symmetric Markov) behavior strategy where $1 - e^{-\sigma(p,q)\Delta}$ is the probability of market entry in a time period, conditional on the firm being uninformed (and both firms being inactive). In what follows, we often write $s = \sigma(p, q)$ and $s' = \sigma(p', q')$ to save notation.

As mentioned, a major technical complication of our analysis arises from the fact that the evolution of the belief during this phase depends on the first mover's strategy σ. More precisely, in states G and BU, the rival firm enters the market with probability $1 - e^{-s\Delta}$; in state BI, knowing that the market is bad, the rival firm never enters. Moreover, the firm observes a signal with probability $1 - e^{-\lambda\Delta}$ if the market is bad (in states BU and BI).[19] Finally, the state changes from BU to BI when the rival firm observes a signal, which occurs also with probability $1 - e^{-\lambda\Delta}$.

Given current belief (p, q) and strategy $s = \sigma(p, q)$, the next-period belief (p', q') is

[19] Note that state BI means that the rival firm is informed. The firm in question is uninformed by construction, because the problem it faces would be trivial otherwise.

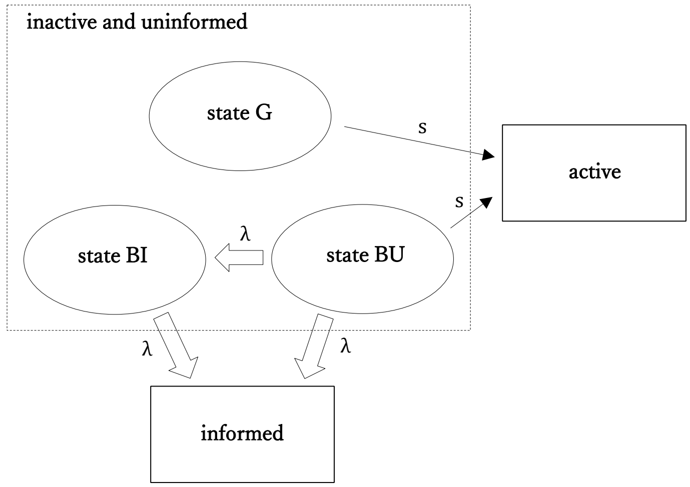

Figure 1. *State transition. There are three possible states $\{G, BI, BU\}$ when a firm is inactive and uninformed. Thick arrows indicate transition by the arrival of information; thin arrows indicate transition by the firm's strategic choice.*

computed as

$$p' = \frac{pe^{-s\Delta}}{(p + qe^{-\lambda\Delta})e^{-s\Delta} + (1-p-q)e^{-\lambda\Delta}}, \tag{1}$$

$$q' = \frac{qe^{-(s+2\lambda)\Delta}}{(p + qe^{-\lambda\Delta})e^{-s\Delta} + (1-p-q)e^{-\lambda\Delta}}, \tag{2}$$

with the initial prior given by $q_0 = 1-p_0$. Figure 1, which graphically summarizes the state-transition process, helps illustrating how we obtain these equations: recall that p and q are the probabilities of state G and BU respectively, both conditional on being inactive and uninformed (i.e., inside the dotted square). Taking the limit gives the laws of motion which clarify how the belief evolves over time:

$$\dot{p} = p\left[(1-p)\lambda - (1-p-q)s\right],$$
$$\dot{q} = -q\left[(1+p)\lambda + (1-p-q)s\right].$$

The current state of the economy is thoroughly characterized by (p, q). What is particularly crucial is the post-entry belief $\phi := \frac{p}{p+q}$ which essentially determines the amount of information revealed by an entry. Fortunately, while (p, q) may follow a quite complicated path, it is relatively straightforward to compute ϕ as it is independent of the first-mover strategy σ; from (1) and (2), we obtain

$$\phi' := \frac{p'}{p'+q'} = \frac{p}{p+qe^{-2\lambda\Delta}} > \phi,$$

which indicates that ϕ_t monotonically increases over time for any given strategy σ. This is an essential technical property of our model which enables us to simply the analysis substantially while preserving the substance of the issue at hand.

Waiting phase: $\phi \geq \overline{p}$. To characterize the first mover's optimal strategy, there are two cases we need to consider, depending on whether ϕ exceeds $\overline{p}$ or not. We start with the case where the current belief (p,q) is such that $\phi \geq \overline{p}$, so that an uninformed firm immediately follows the rival firm. Note that this is the "winner's curse range" where the first mover can monopolize the market only if the market condition is bad. As this is a phase where the second-mover advantage dominates, we call it the waiting phase.

We essentially follow the same procedure to derive the continuation equilibrium in this phase, by comparing the expected payoffs of entering now and at the next instant. Suppose first that firm 1 chooses to enter now. Conditional on the market being good, firm 2 stays inactive with probability $e^{-s\Delta}$, which allows firm 1 to monopolize the market for a period. As such, since firm 1 earns $(\pi + e^{-s\Delta}m)\Delta$ in the first period after entry and $\pi\Delta$ thereafter, the expected payoff of entering at t is

$$p\Big[\frac{\pi - rc}{r} + e^{-s\Delta}m\Delta\Big] - (1-p)c. \tag{3}$$

Now consider an alternative strategy in which firm 1 waits for an instant. In the meanwhile, firm 2 may or may not enter, but the conditional probability that the project is good, from the viewpoint of today, is still p because the expectation of posteriors coincides with the prior. Since firm 1 can monopolize the market with probability $e^{-(s+s')\Delta}$, the expected payoff of this waiting strategy is obtained as

$$e^{-r\Delta}\left[p\left(\frac{\pi - rc}{r} + e^{-(s+s')\Delta}m\Delta\right) - (1-p)ce^{-\lambda\Delta}\right], \tag{4}$$

taking s and s' as given.[20]

Comparing (3) and (4), in the limit, it is better to enter now if

$$r\Big(p\frac{\pi}{r} - c\Big) + (1 - s\Delta)s'\Delta pm > (1-p)\lambda c. \tag{5}$$

[20]Note that this is the expected payoff when firm 1 enters at the next instant regardless of whether firm 2 enters now or not, which may not hold if $\lim_{\Delta\to 0} s\Delta > 0$. For instance, if $\lim_{\Delta\to 0} s\Delta = 1$ and firm 2 does not enter, firm 1 knows for sure that the market is bad, making it optimal never to enter. As we will see below, however, the firms always enter smoothly in the waiting phase, so that this condition is always valid.

This condition shows the tradeoff between entering now and waiting for an instant. By entering now, firm 1 may monopolize the market and obtain an additional payoff of $(1 - s\Delta)s'\Delta pm$ for an instant. By waiting until the next instant, the firm may receive additional information and save $(1-p)\lambda c$ by avoiding wrong entry; however, a firm also suffers a loss $r(p\frac{\pi}{r} - c)$ since the continuation payoff is realized an instant later.

The following statement establishes that equilibrium in the waiting phase is generally in mixed strategies. To see this, suppose that a firm adopts a pure strategy and enters at some t with probability one. In this case, if the firm does not enter at t, it means that the firm is informed and hence the market condition is bad for sure. Since this creates a discrete jump in the belief, the best response for the rival firm is to wait until $t + \Delta$ to gain this extra information (when Δ is sufficiently small). Clearly, this does not constitute an equilibrium because the firm also has an incentive to deviate and wait until $t + 2\Delta$ to see what the rival firm does at $t + \Delta$. As such, in equilibrium, the firms gradually enter at some positive rate to keep the belief constant at $\overline{p}$. The proposition further suggests that this is the unique symmetric continuation equilibrium in this phase.

Lemma 2 (Continuation equilibrium in the waiting phase)**.** *For $\phi_t \geq \overline{p} > p_t$ (i.e., when the game is in the waiting phase), there exists a unique symmetric continuation equilibrium in which:*

1. *Neither firm enters until the belief p_t reaches the threshold $\overline{p}$;*
2. *When p_t reaches $\overline{p}$, the two firms start entering at a rate to keep $p_t = \overline{p}$;*
3. *Once a firm enters, the other firm immediately follows at the next instant.*

Proof. See Appendix A.

If $p_0 > \overline{p}$ to begin with, we have a situation where $\phi_0 > p_0 > \overline{p}$. In this case, the firms enter with strictly positive probability at time 0 so that $\overline{p} > p_\Delta$ (see Lemma 3 in Appendix). Lemma 2 thus exhausts all the possibilities and completely characterizes the continuation equilibrium for any $\phi_t \geq \overline{p}$.

It is important to emphasize that the game in this phase is not a preemption game where each firm has an incentive to deviate slightly earlier than the rival.[21] This is a departure from the canonical preemption game such as Fudenberg and Tirole (1985) where

[21] In a preemption game, if the other firm adopts a pure strategy to enter at some time t, then the best response is to enter slightly earlier at $t - \Delta$. In the waiting phase of our model, on the other hand, the situation is the opposite as noted above: if the rival firm is to enter at t, the best response is to enter slightly later at $t + \Delta$.

there is no strategic benefit of becoming the second mover. To illustrate the key difference, it is worth emphasizing that the equilibrium identified in Lemma 2 is not a "joint-adoption equilibrium" of Fudenberg and Tirole (1985): the entry times of the two firms are arbitrarily close but not simultaneous—the type of equilibrium that does not exist in their framework (or more generally in preemption games).[22] In this phase, neither firm in fact wants to move first, because the firm must bear the risk of being the first mover to earn a benefit that is vanishingly small. The game in this phase thus resembles a war of attrition where each firm waits for the rival to move first.

Preemption phase: $\overline{p} > \phi$. The strategic nature of the problem flips if ϕ starts below this threshold $\overline{p}$. As entry in this phase is driven by the first-mover advantage and preemption motives, we call it the preemption phase. Let τ^* denote the time of the earliest possible entry in equilibrium, which is defined as

$$\tau^* := \inf\{t : \sigma_t > 0\},$$

where we assume $\tau^* > 0$ for now; a sufficient condition for this will be provided later in section 4.1. For exposition, we say that there is *pioneering entry* in equilibrium (or an equilibrium with pioneering entry) if $s > 0$ for any (p, q) such that $\overline{p} > \phi$ or, equivalently, $\overline{p} > \phi_{\tau^*}$.[23] We call it "pioneering" rather than "preemptive" because, as we will see later, it generates valuable information to the other firm and is generally socially beneficial, despite the fact that it is driven by preemption motives.

Observe that $\overline{p} > \phi$ means that if a firm enters now, the other firm's belief jumps up but is still lower than $\overline{p}$. The second mover thus will not enter immediately, and the first mover can monopolize the market for some duration of time which we denote by δ_t. Since the second mover waits until the belief reaches $\overline{p}$, δ_t is computed as

$$\delta_t = \frac{1}{\lambda} \ln \frac{\overline{p} q_t}{p_t(1-\overline{p})}.$$

Note that for a given p_0, ϕ_t depends only on t, and so is δ_t. The expected payoff of entering now is then given by

$$p\frac{\pi + (1-e^{-r\delta_t})m - rc}{r} - (1-p)c.$$

[22] Fudenberg and Tirole (1985) show that the joint-adoption equilibrium exists in continuous time but not in discrete time. This is because in discrete time, there is always an incentive to deviate and enter slightly earlier, which goes unpunished for a unit of time if the other firm cannot react immediately, generating a discrepancy between continuous-time and discrete-time equilibria. This argument of course does not apply to our setting where there is no incentive to enter earlier, so that the limit of our discrete-time equilibrium is well defined.

[23] Note that our definition of market pioneer differs from the conventional one: in the literature, it simply refers to the first entrant in a new market (Robinson and Fornell, 1985).

The equilibrium allocation becomes much more complicated when pioneering entry actually occurs on the equilibrium path with positive probability. We can nonetheless establish that any (symmetric) equilibrium must in general take the following form.

Proposition 1 (Characterization)**.** *In any symmetric equilibrium,*

(a) If there is pioneering entry in equilibrium, there exist τ^*, $\underline{\tau}$ and $\overline{\tau}$ such that the firms enter the market gradually (i.e., at some finite rate) in $(\tau^*, \underline{\tau})$ but with zero probability in $(\underline{\tau}, \overline{\tau})$, where the belief p_t reaches $\overline{p}$ at $\overline{\tau}$; after $\overline{\tau}$, the firms again enter gradually as described in Lemma 2.

(b) If there is no pioneering entry in equilibrium, the firms wait until the belief p_t reaches $\overline{p}$, and then enter gradually as described in Lemma 2.

Proof. See Appendix A.

Proposition 1 fully characterizes symmetric equilibrium of this model. If there is no pioneering entry in equilibrium, i.e., $\phi_{\tau^*} \geq \overline{p}$, the firms wait until the belief p_t reaches $\overline{p}$ and start entering at some rate past that point as described in Lemma 2. Otherwise, entry occurs in two disjoint intervals. The firms start entering at some rate from τ^*, which is by definition the earliest time of entry, but stops at $\underline{\tau}$. This is followed by an interval of no entry $(\underline{\tau}, \overline{\tau})$ where neither firm takes any action. After a while, though, the belief p_t eventually reaches $\overline{p}$ at $\overline{\tau}$, at which point the firms start entering again as in the case with no pioneering entry.

The reason why we have this period of no entry in between pertains to the amount of information revealed by an entry which increases over time. Although the firms have less private information and face more uncertainty early on, the fact that they have less private information means that there is less to learn from the rival's action, thereby making the preemption effect stronger. When this first-mover advantage dominates the cost of entering prematurely with insufficient information, the firms enter with some positive probability in the preemption phase. As each firm accumulates more information over time, however, the signaling effect of entry becomes stronger and the game reaches a point where the net payoff of becoming the first mover is negative. In any equilibrium with pioneering entry, therefore, there must be an in-between phase where market entry ceases to occur, generating on-and-off dynamics of market entry.

4. Emergence of a market pioneer

The previous section provides an equilibrium characterization and establishes that there are two forms of equilibrium, depending on whether pioneering entry occurs on the equilibrium path. Given this, we now explore under what conditions we have pioneering entry in equilibrium.

4.1. The constrained problem

To derive a precise condition for pioneering entry to occur in equilibrium, we first consider a hypothetical situation in which a firm, say firm 2, can enter only after the other firm, i.e., firm 1, enters. As it turns out, this constrained version of the problem, which excludes the possibility of entry competition, provides enough information to see when pioneering entry occurs in the original (unconstrained) problem.

Under the restriction that firm 2 must be the second mover, the problem faced by firm 1 is substantially simpler: firm 1 simply decides when to enter conditional on having observed no signal. Provided that firm 2 never enters, firm 1's belief pair at any t is given by

$$p_t = \frac{p_0}{p_0 + (1-p_0)e^{-\lambda t}}, \; q_t = \frac{(1-p_0)e^{-2\lambda t}}{p_0 + (1-p_0)e^{-\lambda t}}, \tag{6}$$

which depends only on t. As a consequence, the expected payoff of entering at t can also be written as a function of t. Let $\hat{\Pi}(t)$ denote the expected payoff of entering at t, evaluated at time 0, under the restriction that firm 2 must be the second mover. If firm 1 enters at time t, firm 2 will wait until p_t reaches $\overline{p}$. We thus obtain

$$\hat{\Pi}(t) = e^{-rt}\left[p_0 \frac{\pi + (1-e^{-r\delta_t})m - rc}{r} - (1-p_0)ce^{-\lambda t}\right]. \tag{7}$$

If $\phi_t \geq \overline{p}$, then $\delta_t = \Delta$ which converges to zero in the limit. In contrast, if $\overline{p} > \phi_t$, firm 2 must wait to collect more information, giving firm 1 some time to monopolize the market. The incentive for pioneering entry thus hinges crucially on δ_t.

Suppose first that there is no pioneering entry and $\delta_t = 0$. In this case, the earliest possible entry occurs when the belief p_t reaches $\overline{p}$. Define τ^{NP} as the time at which p_t equals $\overline{p}$ in the equilibrium with no pioneering entry, which must solve

$$e^{-\lambda\tau^{\text{NP}}} = \frac{p_0(1-\overline{p})}{\overline{p}(1-p_0)} = \frac{p_0(\pi - rc)}{(1-p_0)(\lambda + r)c}.$$

From (6), we obtain

$$\delta_t = \frac{1}{\lambda}\ln\frac{\overline{p}q_t}{p_t(1-\overline{p})} = \frac{1}{\lambda}\ln\frac{\overline{p}(1-p_0)e^{-2\lambda t}}{p_0(1-\overline{p})} = \tau^{\mathrm{NP}} - 2t,$$

meaning that $\delta_t > 0$ if and only if $\frac{\tau^{\mathrm{NP}}}{2} > t$. Let $\Pi^{\mathrm{NP}} := \max_{t\in(\frac{\tau^{\mathrm{NP}}}{2},\infty)} \hat{\Pi}(t)$ denote the expected payoff without pioneering entry, which can be written as

$$\Pi^{\mathrm{NP}} = \hat{\Pi}(\tau^{\mathrm{NP}}) = e^{-r\tau^{\mathrm{NP}}}\left[p_0\frac{\pi - rc}{r} - (1-p_0)ce^{-\lambda\tau^{\mathrm{NP}}}\right] = e^{-r\tau^{\mathrm{NP}}}\frac{p_0\lambda(\pi - rc)}{r(\lambda + r)}.$$

Now suppose that firm 1 enters in the preemption phase. If firm 1 enters at t, firm 2's belief jumps up to ϕ_t, but firm 2 still needs to wait until the belief reaches $\overline{p}$, which allows firm 1 to monopolize the market for some time δ_t. Therefore, the expected payoff of entering at $t \in [0, \frac{\tau^{\mathrm{NP}}}{2}]$ is given by

$$\hat{\Pi}(t) = e^{-rt}\left[p_0\frac{\pi + (1-e^{-r\delta_t})m - rc}{r} - (1-p_0)ce^{-\lambda t}\right].$$

Note that this problem is well defined only if $\tau^{\mathrm{NP}} > 0$. In what follows, we assume that this is indeed the case.

Assumption 2. $\tau^{\mathrm{NP}} > 0 \iff \overline{p} > p_0$.

Let τ^{P} denote the optimal timing of entry in the preemption phase, which can be found by maximizing $\hat{\Pi}$. The first-order condition is then obtained as

$$\hat{\mu}(t) := -p_0(\pi + m - rc) - p_0 m e^{-r\delta_t} + (1-p_0)(\lambda + r)ce^{-\lambda t} = 0.$$

Lemma 3. *In the limit, there exists a unique $\tau^{\mathrm{P}} \in [0, \frac{\tau^{\mathrm{NP}}}{2}]$ which maximizes $\hat{\Pi}(t)$. The optimal timing of pioneering entry, denoted by τ^{P}, is given by*

$$\tau^{\mathrm{P}} = \begin{cases} 0 & \text{if } 0 \geq \hat{\mu}(0), \\ \hat{\tau} & \text{if } \hat{\mu}(0) > 0 > \hat{\mu}(\frac{\tau^{\mathrm{NP}}}{2}), \\ \frac{\tau^{\mathrm{NP}}}{2} & \text{if } \hat{\mu}(\frac{\tau^{\mathrm{NP}}}{2}) \geq 0, \end{cases}$$

where $\hat{\tau}$ solves $\hat{\mu}(\hat{\tau}) = 0$.

Proof. See Appendix A.

Let $\Pi^{\mathrm{P}} := \max_{t \in (0, \frac{\tau^{\mathrm{NP}}}{2})} \hat{\Pi}(t)$ denote the expected payoff under the restriction that firm 2 must be the second mover, which can be written as

$$\Pi^{\mathrm{P}} = \hat{\Pi}(\tau^{\mathrm{P}}) = e^{-r\tau^{\mathrm{P}}} \left[p_0 \frac{\pi + (1 - e^{-r\delta_{\tau^{\mathrm{P}}}})m - rc}{r} - (1 - p_0)ce^{-\lambda\tau^{\mathrm{P}}} \right].$$

As the monopoly premium m becomes larger, it becomes more costly to wait and collect more information. As a consequence, the optimal timing of pioneering entry moves forward with an increase in the monopoly premium.

Finally, we have thus far assumed, somewhat loosely, that leaning is essential and $\tau^* > 0$. Assumption 2 is obviously necessary but not sufficient for $\tau^* > 0$. To ensure this, the expected payoff of entering immediately at time 0 must be smaller than Π^{NP}, i.e.,

$$\Pi^{\mathrm{NP}} \geq \hat{\Pi}(0) \iff e^{-r\tau^{\mathrm{NP}}} \frac{p_0 \lambda(\pi - rc)}{r(\lambda + r)} > p_0 \frac{\pi + (1 - e^{-r\tau^{\mathrm{NP}}})m - rc}{r} - (1 - p_0)c.$$

Note that as $p_0 \to 0$, the left-hand side converges to zero while the right-hand side dips below zero, so that this condition, as well as Assumption 2, holds if p_0 is sufficiently small.

Assumption 3. $\Pi^{\mathrm{NP}} > \hat{\Pi}(0)$.

4.2. Emergence of a market pioneer

Under the restriction that firm 2 must be the second mover, it is optimal for firm 1 to enter once and for all at τ^{P} if $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$. Clearly, though, this does not constitute an equilibrium when firm 2 is also an active player who can enter the market at any point in time. As Proposition 1 indicates, the firms must adopt mixed strategies when they compete to be the first mover. Even then, these payoffs are still useful as they provide a necessary and sufficient condition for a market pioneer to emerge in equilibrium.

Define $\Pi(t)$ as the equilibrium payoff of entering at t evaluated at time 0 (note the difference from $\hat{\Pi}(t)$ which is the payoff when the other firm never enters first). In equilibrium, $\Pi(t)$ must be constant for $t \in (\tau^*, \underline{\tau}) \cup (\overline{\tau}, \infty)$. Two initial points, τ^* and $\overline{\tau}$, are particularly crucial. If a firm waits from τ^* to $\overline{\tau}$, the other firm enters with some probability. Suppose that the other firm enters at some τ, in which case it is optimal to wait until $\tau^{\mathrm{NP}} - \tau$. The expected payoff of becoming the second mover, when the other firm enters at τ, is hence

$$e^{-r(\tau^{\mathrm{NP}} - \tau)} \left[p_0 \frac{\pi - rc}{r} - (1 - p_0)ce^{-\lambda\tau^{\mathrm{NP}}} \right] = e^{r\tau} \Pi^{\mathrm{NP}}.$$

Given this, the expected payoff of waiting until $\overline{\tau}$ (in the continuous-time limit) is obtained as

$$\Pi(\overline{\tau}) = \int_{\tau^*}^{\overline{\tau}} e^{r\tau}\Pi^{\mathrm{NP}}\mathrm{d}F(\tau) + e^{-r(\overline{\tau}-\tau^{\mathrm{NP}})}\left(1 - \int_{\tau^*}^{\overline{\tau}} \mathrm{d}F(\tau)\right)\Pi^{\mathrm{NP}}, \tag{8}$$

where F denotes the unconditional distribution of entry times. In equilibrium, this must be equal to the expected payoff of becoming the first mover at τ^*, which can be written as

$$\Pi(\tau^*) = \hat{\Pi}(\tau^*) = e^{-r\tau^*}\left[p_0 \frac{\pi + (1 - e^{-r\delta_{\tau^*}})m - rc}{r} - (1 - p_0)ce^{-\lambda\tau^*}\right]. \tag{9}$$

Proposition 2 (Existence)**.** *There always exists a symmetric equilibrium. The equilibrium entails pioneering entry if and only if* $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$.

Proof. See Appendix A.

To see the intuition behind Proposition 2, observe that in the equilibrium with no pioneering entry, each firm waits until time τ^{NP} and then gradually enters past that point; the expected payoff in this equilibrium is hence Π^{NP} as in the constrained problem. Given this, sufficiency is obvious, because if $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$, a firm must have an incentive to deviate and enter with some positive probability in the preemption phase. On the other hand, necessity comes from the fact that the entry competition can only lower the benefit of pioneering entry while it raises the benefit of waiting due to the signaling effect (see the next section for more detail). As such, if there is no incentive to enter in the preemption phase in the constrained problem, then there is certainly no incentive to do so in the original problem.

Proposition 2 implies that as $\Pi^{\mathrm{P}} - \Pi^{\mathrm{NP}}$ becomes larger, the first-mover advantage becomes more salient, rendering pioneering entry more likely. The following proposition clarifies under what conditions a market pioneer is more likely to emerge, which offers crucial welfare and policy implications.

Proposition 3 (Determinants of pioneering entry)**.** *There exist* $\hat{m}$ *and* $\hat{p} \in (0, \overline{p})$ *such that there is an equilibrium with pioneering entry if and only if* $m > \hat{m}$ *or* $\overline{p} > p_0 > \hat{p}$.

Proof. See Appendix A.

Two factors are particularly crucial as determinants of entry dynamics. First, it is clear that the monopoly premium m, which measures the extent of market competition, has a

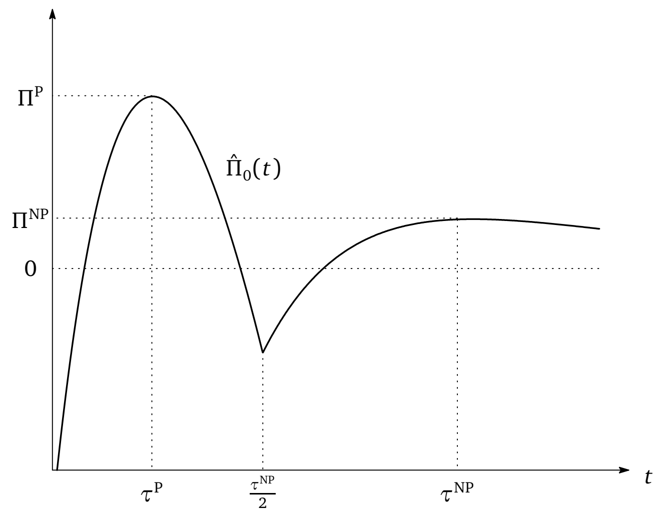

Figure 2. *The emergence of a market pioneer ($\lambda = 0.1, r = 0.1, p_0 = 0.3, c = 3, \pi = 0.5, m = 0.5$)*

decisive impact on the timing of entry. Since an increase in m only raises the value of preemption, it generally favors pioneering entry. See figures 2 and 3 which depict $\hat{\Pi}(t)$ for different values of π and m (with $\pi + m$ fixed). Second, the prior belief p_0, which measures the extent of uncertainty faced by the firms, also plays an important role in shaping entry dynamics. Although the effect of p_0 is less clear, as an increase in p_0 can raise both Π^{P} and Π^{NP}, a high p_0 tends to favor pioneering entry. To see why, observe that the expected payoff upon success is larger for the first mover; a firm can thus enter with more confidence earlier while revealing less information. From these findings, we can conclude that pioneering entry is more likely when: (i) market competition is intense; and/or (ii) there is less uncertainty regarding the eventual likelihood of success.

4.3. Pioneer, early follower and late entrant

Since the seminal work of Lieberman and Montgomery (1988), there is now a voluminous literature, mainly in the fields of marketing and strategic management, which examines the effects of the timing and order of market entry. The literature often classifies timing of entry into three broad categories: pioneer, early follower, and late entrant (Robinson and Fornell, 1985, Lambkin, 1988).

Our analysis allows us to pin down the temporal distribution of entry times for a given set of parameters and hence offers some insight for this classification. To see this more clearly, note that each realized equilibrium allocation is characterized by a pair of entry times (t_1, t_2) where t_1 (t_2) denotes the timing of first (second) entry ($t_i = \infty$ in the case

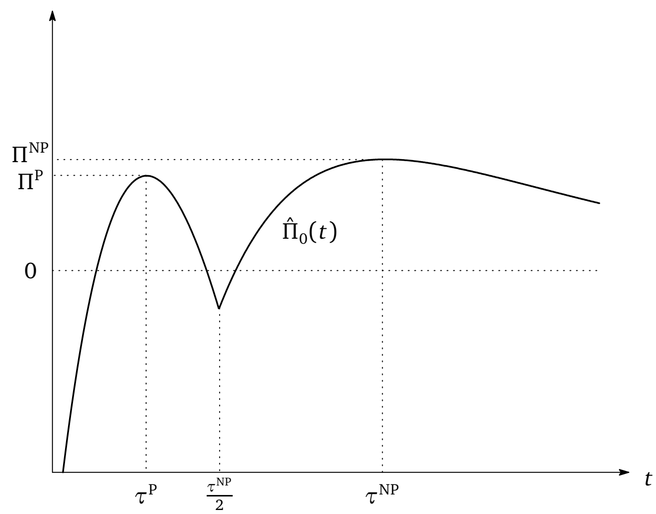

Figure 3. *No market pioneer ($\lambda = 0.1, r = 0.1, p_0 = 0.3, c = 3, \pi = 0.6, m = 0.4$)*

of no entry). Our model then generates four classes of entry dynamics that are observationally distinguishable.

1. **No entry** ($t_1 = t_2 = \infty$): Neither firm chooses to enter, and the market never materializes.
2. **Only one entry** ($t_1 < t_2 = \infty$): Only one firm enters while the other firm chooses not to follow. This is the case of premature entry.
3. **Late entrant** ($t_1 < t_2 < \infty$): A firm enters in the preemption phase and is followed by the other firm with some time lag. The two entries are spaced apart in time.
4. **Early follower** ($t_1 \approx t_2 < \infty$): A firm enters in the waiting phase and is immediately followed by the other firm. The two entries are clustered together in time.

The first two cases occur only when the market condition is bad, reflecting the obvious fact that no successful market can be monopolized forever. The latter two cases admit two entrants and are often the focus of attention in the existing literature. Note that we observe a late entrant only if there is pioneering entry. This fact suggests that even though the timing of first entry is often not observable, the time lag between entries can tell us a lot about efficiency properties of an observed entry pattern, which we will discuss next.

5. Welfare implications

5.1. Equilibrium payoff bounds

Let Π^* denote the expected equilibrium payoff for each firm where

$$\Pi^* = e^{-r\tau^*}\left[p_0 \frac{\pi + (1-e^{-r\delta_{\tau^*}})m - rc}{r} - (1-p_0)ce^{-\lambda\tau^*}\right].$$

If $\Pi^{\mathrm{NP}} > \Pi^{\mathrm{P}}$, the earliest possible entry occurs at time τ^{NP}, and $\Pi^* = \Pi^{\mathrm{NP}}$ as we have seen. If $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$, on the other hand, there is a clear gain from becoming the first mover. We in general have $\Pi^* < \Pi^{\mathrm{P}}$ (if $\Pi^{\mathrm{NP}} < \Pi^{\mathrm{P}}$) because the entry competition is self-defeating and shifts the timing of entry forward, inducing the firms to start entering before the optimal time τ^{P}. The question is then whether this competition drives the value of the first-mover advantage down to zero, i.e., $\Pi^* \to \Pi^{\mathrm{NP}}$. As it turns out, this is not the case because the second mover can benefit from the information revealed by the first mover's entry. The following result characterizes the equilibrium payoff bounds when $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$.

Proposition 4 (Equilibrium payoff bounds)**.** *Suppose that* $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$ *so that pioneering entry occurs with positive probability. Then, each firm's expected payoff is between* Π^{NP} *and* Π^{P}*, i.e.,*

$$\Pi^* = e^{-r\tau^*}\left[p_0 \frac{\pi + (1-e^{-r\delta_{\tau^*}})m - rc}{r} - (1-p_0)ce^{-\lambda\tau^*}\right] \in (\Pi^{\mathrm{NP}}, \Pi^{\mathrm{P}}).$$

Proof. See Appendix A.

To understand this result, especially why the expected payoff is not driven down to Π^{NP}, it is important to understand the roles of the two types of externality that are present in this setting. On one hand, there is a negative payoff externality via market competition which is captured by m. The payoff externality is clearly the source of the entry competition. This is most clearly seen by supposing $m = 0$, in which case

$$\hat{\Pi}(t) = e^{-rt}\left[p_0 \frac{\pi - rc}{r} - (1-p_0)ce^{-\lambda t}\right],$$

for all $t \in [0, \infty)$, suggesting that there is no preemption phase.

In contrast, as m increases, the first-mover advantage becomes more salient, giving each firm an incentive to become a market pioneer. This entry competition forces the firms to enter earlier than the optimal timing τ^{P}, which necessarily lowers the expected payoff of becoming a market pioneer. In equilibrium, this expected payoff must be driven down to the expected payoff of becoming a follower which is strictly larger than Π^{NP} because of the information externality: with the first-mover's entry providing additional information, the follower can enter earlier than τ^{NP} and hence on average achieve a higher payoff. The presence of pioneering entry thus accelerates the learning process at the industry level.

5.2. Efficient timing of entry

To derive efficiency properties of the model, we now consider a social planner who attempts to maximize the joint payoff of the firms. Since the efficient allocation under complete information is rather trivial in this setting,[24] here we focus on the situation where the social planner is subject to the same informational constraints as the firms. Specifically, we consider an environment in which the social planner specifies the entry times (τ_1, τ_2) such that firm i can enter the market at time τ_i if it is uninformed at the time.

Let $W(\tau_1, \tau_2)$ denote the joint payoff for a given pair (τ_1, τ_2). Without loss of generality, we assume $\tau_1 < \tau_2$.[25] The social planner's problem is defined as

$$\max_{(\tau_1, \tau_2)} \quad W(\tau_1, \tau_2),$$

subject to $\tau_1 < \tau_2$, where

$$W(\tau_1, \tau_2) = e^{-r\tau_1}\left[p_0 \frac{\pi + m - rc}{r} - (1-p_0)ce^{-\lambda\tau_1}\right] + e^{-r\tau_2}\left[p_0 \frac{\pi - m - rc}{r} - (1-p_0)ce^{-\lambda(\tau_1+\tau_2)}\right].$$

Two remarks are in order regarding the two types of externality in this setting. First, firm 2's belief at τ_2 is $\frac{p_0}{p_0+(1-p_0)e^{-\lambda(\tau_1+\tau_2)}}$, rather than $\frac{p_0}{p_0+(1-p_0)e^{-\lambda\tau_2}}$, because of the positive information externality of the first entry. Second, the second entry contributes only $\pi - m$ to the joint profit (while its private gain is π) due to the negative payoff externality, which corresponds to what is often referred to as the "business-stealing effect" in standard static oligopoly models.

It is also important to note that because of the payoff externality, there may arise a case where it is socially optimal to have only one firm in the market. This is the case if

$$c \geq \frac{\pi - m}{r},$$

[24] The problem is clearly trivial when the market condition is known to the social planner: if the market is good, the firms should enter immediately at time 0; if not, they should never enter.

[25] It is clearly without loss to assume $\tau_1 \leq \tau_2$. We can also show that it is never optimal to choose $\tau_1 = \tau_2$ in the limit because $W(\tau, \tau + \Delta) > W(\tau, \tau)$ for any τ. This is because firm 2 can learn from firm 1's decision at τ_1 while the social cost of slightly delaying the second entry is negligible as $\Delta \to 0$.

in which case the social planner would allow only one firm to enter the market ($t_2 = \infty$). Since this case is relatively straightforward, we restrict our attention to the case where it is socially optimal to have two firms whenever the market condition is good.

Assumption 4. $\pi - m > rc$.

Taking derivative of the joint payoff W, the first-order conditions are obtained as

$$\frac{\partial W}{\partial \tau_1} = -p_0(\pi + m - rc)e^{-r\tau_1} + (1-p_0)\left[(\lambda + r)e^{-r\tau_1} + \lambda e^{-(\lambda+r)\tau_2}\right]ce^{-\lambda\tau_1} = 0, \tag{10}$$

$$\frac{\partial W}{\partial \tau_2} = -p_0(\pi - m - rc)e^{-r\tau_2} + (1-p_0)(\lambda + r)ce^{-r\tau_2 - \lambda(\tau_1+\tau_2)} = 0. \tag{11}$$

Let $(\tau_1^{**}, \tau_2^{**})$ denote the socially optimal timing of entry and $T^{**} = \tau_1^{**} + \tau_2^{**}$. From (11), we obtain

$$e^{-\lambda T^{**}} = \frac{p_0(\pi - m - rc)}{(1-p_0)(\lambda + r)c},$$

which depends only on the primitives of the model. Since $\tau_2 \geq \tau_1$, for $\tau_1 \in [0, \frac{T^{**}}{2})$, (10) can be written as

$$\begin{aligned} p_0(\pi + m - rc) &= (1-p_0)\left[(\lambda + r)e^{-\lambda\tau_1} + \lambda e^{-\lambda T^{**} - r(\tau_2 - \tau_1)}\right]c \\ &= (1-p_0)(\lambda + r)ce^{-\lambda\tau_1} + p_0\frac{\lambda(\pi - m - rc)}{\lambda + r}e^{-r(\tau_2-\tau_1)}. \end{aligned} \tag{12}$$

If there is no $\tau_1 < \frac{T^{**}}{2}$ that can satisfy (12), the social optimal timing is "almost simultaneous" where $\tau_1 = \tau$ and $\tau_2 = \tau + \Delta$. In the limit, the first-order condition in this case is reduced to

$$p_0(\pi - rc) = \frac{1-p_0}{2}\left[\lambda + r + (2\lambda + r)e^{-\lambda\tau}\right]ce^{-\lambda\tau}, \tag{13}$$

which is independent of m. The following proposition yields some important implications regarding the timing of entry which we will discuss in depth below.

Proposition 5 (Efficient timing of entry)**.**

(a) $T^{**} > \tau^{\text{NP}}$.

(b) $\tau_1^{**} > \tau^{\text{P}}$ *if* $\tau^{\text{P}} > 0$.

(c) $\tau^{\text{NP}} > \tau_1^{**}$ *if* p_0 *is sufficiently small.*

Proof. See Appendix A.

Parts (a) and (b) of Proposition 5 concern the case with a late entrant and suggest that the firms enter the market too early compared to the social optimum.

- Part (a) states that the equilibrium timing of second entry is earlier than the socially optimal timing.[26] This is due to the negative payoff externality: when the second entry occurs, the firm's average net payoff when the market is good is $\pi - rc$ while its contribution to the joint profit is only $\pi - m - rc$ due to the business-stealing effect.[27] In the optimal allocation, therefore, the entry threshold is higher and the firm should wait longer to collect more information.
- Part (b) states that the equilibrium timing of first entry is also earlier than the socially optimal timing. This is mainly due to the positive information externality: if the first entry occurs later, it reveals more information and benefits the other firm. The first entrant not only ignores this benefit of signaling ($\tau_1^{**} > \tau^{\mathrm{P}}$), but in equilibrium enters even earlier so as to reveal less information to the rival firm and delay its subsequent entry ($\tau^{\mathrm{P}} > \tau^*$).

In contrast, Part (c) of the proposition concerns the case with an early follower. This case emerges when p_0 is relatively small, in which case the firms tend to enter too late. The intuition behind this is relatively clear. Once the game reaches the waiting phase, the clear winner is the one that becomes the follower as it can minimize the risk of wrong entry while losing almost no monopoly rent. This incentive to wait for the rival's action is often excessively strong, preventing the firms from entering the market at an opportune time.

Note that the timing of first entry is often not observable to the econometrician who typically lacks exact knowledge of calender time (or of time 0). Even in this case, we can make inference about whether pioneering entry occurs or not from the temporal distribution of entry times. Note that we have a late entrant only when there is pioneering entry; the fact that entries are spaced apart in time hence suggests that the first entry indeed occurs in the preemption phase. Combined with the earlier discussion in section 4.3, Proposition 5 implies a paradoxical fact which is worth emphasizing: the firms enter too early when there is a late entrant, and too late when there is an early follower.

[26] Given that the first entry occurs at some t, the second entry occurs at $\tau^{\mathrm{NP}} - t$ whereas the socially optimal timing is given by $T^{**} - t$.

[27] The celebrated excess entry theorem (Mankiw and Winston, 1986; Suzumura and Kiyono, 1987; Lahiri and Ono, 1988) generally builds on this effect and demonstrates that the number of firms in a market can be too many in static oligopoly models. Our analysis complements this literature by extending this argument to a dynamic context, showing that market entry is too early with this same effect.

6. Extensions

6.1. Benefits of consumer inertia

As we have seen, efficiency properties of the model depend crucially on the monopoly premium m which measures the extent of the payoff externality. More precisely, the possibility of business stealing generates two important forces which generally induce the firms to enter too early: first, it gives the first mover an additional strategic incentive to enter early so as to reveal less information to the other firm; second, it also induces the second mover to enter early as it fails to internalize the loss to the first mover.

In reality, however, first movers can benefit from establishing their presence early on due to consumer inertia, which could arise from various factors such as brand loyalty, habit formation, switching costs, patent protection, and slow diffusion of product information. It is hence more realistic to assume that the monopoly premium m decays only slowly over time. To capture this aspect in a simple way, we now suppose that the payoff to the first mover is $\pi + \eta m$ and to the second mover is $\pi - \eta m$, instead of both receiving π, where $\eta \in [0, 1]$ measures the extent of consumer inertia.[28]

Since detailed analysis of this extended case is out of the scope of this paper, we briefly describe important forces that are generated by consumer inertia; throughout this subsection, we restrict our attention to the equilibrium with pioneering entry. Since the expected flow payoff upon successful entry is now $\pi - \eta m$, the second mover waits until the belief reaches $p^{\mathrm{F}} := \frac{(\lambda+r)c}{\pi-\eta m+\lambda c}$, which is larger than $\overline{p}$ for any $\eta > 0$. Define τ^{F} such that

$$p_0(\pi - \eta m - rc) = (1 - p_0)(\lambda + r)ce^{-\lambda\tau^{\mathrm{F}}}.$$

If the other firm enters at τ, the expected payoff is

$$\Pi^{\mathrm{F}} := e^{-r(\tau^{\mathrm{F}}-\tau)}\left[p_0\frac{\pi - \eta m - rc}{r} - (1 - p_0)ce^{-\lambda\tau^{\mathrm{F}}}\right] = e^{-r(\tau^{\mathrm{F}}-\tau)}p_0\frac{\lambda(\pi - \eta m - rc)}{r(\lambda + r)}.$$

Given this, (8) and (9) are modified, respectively, as

$$\Pi(\overline{\tau}) = \int_{\tau^*}^{\overline{\tau}} e^{r\tau}\Pi^{\mathrm{F}}\mathrm{d}F(\tau) + e^{-r(\overline{\tau}-\tau^{\mathrm{NP}})}\left(1 - \int_{\tau^*}^{\overline{\tau}}\mathrm{d}F(\tau)\right)\Pi^{\mathrm{NP}},$$

$$\Pi(\tau^*) = \hat{\Pi}(\tau^*) = e^{-r\tau^*}\left[p_0\frac{\pi + m - (1 - \eta)me^{-r(\tau^{\mathrm{F}}-2\tau^*)} - rc}{r} - (1 - p_0)ce^{-\lambda\tau^*}\right].$$

[28] Since only the discounted sum of payoffs matters after both firms enter, η constitutes a sufficient statistic for our purpose. For instance, suppose that the payoff for the first mover when the second mover enters at some t' is $\pi + me^{-\xi(t-t')}$ for $t > t'$, in which case $\eta = \frac{r}{\xi+r}$. Our baseline model then corresponds to the limit case of this specification where $\xi \to \infty$.

Note that with the firms' strategies fixed, an increase in η (more consumer inertia) unambiguously raises $\Pi(\tau^*)$ but lowers $\Pi(\overline{\tau})$. On one hand, more consumer inertia benefits the first mover because: (i) the duopoly payoff increases from π to $\pi + \eta m$; and (ii) it reduces the payoff for the second mover and hence delays its arrival. On the other hand, more consumer inertia is clearly detrimental to the second mover. This negative effect on the second mover implies more intense entry competition and hence brings τ^* forward. The overall impact on the equilibrium payoff depends on this tradeoff. We argue, however, that the positive impact on the first mover tends to dominate the negative effect for a range of parameters values, meaning that consumer inertia is often efficiency-enhancing. This can be seen from the fact that the positive effect necessarily dominates if the probability of entry in the preemption phase, $\int_{\tau^*}^{\underline{\tau}} \mathrm{d}F(\tau)$, is sufficiently small. Observe that this is the case when Π^{P} is sufficiently close to Π^{NP}.

This argument suggests that there are possible efficiency gains from consumer inertia. As discussed, although pioneering entry is socially beneficial, the firms fail to internalize this benefit. Biasing the allocation of surplus in favor of the first mover alleviates this inefficiency by making pioneering entry more attractive. From a broader perspective, our argument offers crucial policy implications by highlighting how the extent of market competition shapes entry dynamics. Consider a regulatory authority who has policy tools to manipulate η in some ways. If the authority is concerned only about *ex post* static gains, it may be tempted to reduce consumer inertia (lower η) as much as possible, in order to intensify market competition among incumbent firms. Although these *ex post* gains, which are assumed away in our analysis, are certainly important, the extent of market competition can have huge impacts on the way firms enter a new market or adopt a new technology, thereby providing a relevant point of view that has not been discussed much in the literature.

6.2. A more general specification: Poisson bandits

Our baseline model assumes a particular information structure, often called exponential bandits in the literature, where the arrival of one signal fully resolves any underlying uncertainty. We adopt this approach, pioneered by Keller et al. (2005), because modeling learning with general jump processes can typically be quite complicated, thereby making the main messages of the paper somewhat obscure. Clearly, though, this assumption is not very realistic, nor is it meant to be so: in reality, information rarely arrives in such a stark manner. It is important to verify the key tradeoff of our focus remains in a more realistic setting.

One way to generalize our learning process is to adopt Poisson bandits, in the spirit of Keller and Rady (2010), where no one signal is conclusive about the underlying state. Formally, suppose that an inactive firm observes a signal with probability $\lambda_\theta \mathrm{d}t$ for an interval $[t, t+\mathrm{d}t)$ where $\theta = 1$ if the state is good and $\theta = 0$ if bad. Assume $\lambda_0 > \lambda_1 > 0$, so that a signal can come from either state. For clarity, let $\lambda_0 = \lambda$ and $\lambda_1 = \nu\lambda$ where $\nu \in (0,1)$.[29] The baseline model corresponds to the case with $\nu = 0$.

In the baseline model, there are only two types—either informed or uninformed—and the belief is conditional on the firm being uninformed (zero observed signals). In the Poisson-bandits model, on the other hand, a firm's type is defined by the number of signals it has observed in the past; we say that a firm is of type j if it has observed j signals. Given this, the belief space is now extended to the infinite dimension and written as $(p_t^{j,k}, q_t^{j,k})_{k=0}^{\infty}$ where $p_t^{j,k}$ ($q_t^{j,k}$) is the probability that the market is good (bad), and the rival firm has observed k signals, conditional on the firm's type j.[30] Let $p_t^j = \sum_{k=0}^{\infty} p_t^{j,k}$ and $q_t^j = \sum_{k=0}^{\infty} q_t^{j,k}$ where we must have $p_t^j + q_t^j = 1$ for all j, and s_t^j be the entry probability of type j. Observe that in the baseline model, (i) $p_t^{j,k}$ is relevant and defined only for $j = 0$; (ii) $p^{0,k} = 0$ for all $k \geq 1$; and (iii) $q_t^k = 0$ for all $k \geq 2$. As such, the belief space is reduced to $(p_t^{0,0}, q_t^{0,0}, q_t^{0,1})$ where $p_t^{0,0} + q_t^{0,0} + q_t^{0,1} = 1$ with $p_t^{0,0} = p_t$ and $q_t^{0,0} = q_t$.

Although the Poisson-bandits model is obviously very complicated, as we need to keep track of the infinite-dimension belief, and full characterization of the model is beyond the scope of this paper, there are still some properties which continue to hold in this setup; the details are relegated to Appendix B. First, given $\nu \in (0,1)$, the belief jumps downward when a signal arrives while it gradually improves otherwise. A firm thus becomes more pessimistic as it observes more signals where $p_t^j > p_t^{j+1}$ holds for all j and t regardless of the entry strategies. Second, by the same argument as in the baseline model, there is no equilibrium in which a firm enters with strictly positive probability because that would necessarily create a payoff discontinuity and an incentive to deviate. These observations imply that: (i) market entry occurs smoothly over time; and (ii) only the most optimistic type can enter the market at any point in time.

A crucial element of our information structure is that different firms have potentially different beliefs due to private learning, so that a firm can learn from the action of its rival. This feature is clearly preserved in the Poisson-bandits model. The impact of this signaling

[29]The fact that $\nu < 1$ implies that the underlying information structure is still "no-news-is-good-news," where the belief gradually improves as long as the firm observes no signal.

[30]Note that in our model, learning is private and each firm does not know how many signals the rival firm has observed. This is a crucial difference from Keller and Rady (2010) where actions and outcomes are observable, so that all players hold a common posterior belief.

effect is again captured by the post-entry belief: let ϕ_t^j be the belief for type j immediately after observing the rival firm's entry at t. Even though the belief evolves in a much more complicated way, the post-entry belief still has a simple, closed-form, representation owing to the fact that only the most optimistic type can enter at any point:

$$\phi_t^j = \frac{p_0 \nu^{j+J}}{p_0 \nu^{j+J} + (1-p_0)e^{-2(1-\nu)\lambda t}} > p_t,$$

where J is the most optimistic type at t and p_0 is the prior belief of the state being good. Note that the post-entry belief is independent of the entry strategy and monotonically increasing in t as in the baseline model. Moreover, we have $\phi_0^0 = p_0$ so that an immediate entry at time 0 reveals no additional information to the rival firm,[31] while the probability that the rival firm follows immediately, conditional on the state being good, converges to 1 as t gets larger. We can conclude from this that the first-mover advantage is more salient at early stages of the game while the second-mover advantage becomes more dominant at later stages.

In addition, it is straightforward to verify that there is a sequence of equilibria that converge to that of the baseline model as $\nu \to 0$. This can be seen from the fact that for any finite t, $\lim_{\nu \to 0} p_t^k = 0$ for all $k \geq 1$, so that the belief space is almost reduced to the one in the baseline model. Our baseline model can thus closely approximate the more general Poisson-bandits variant when ν is relatively small.

7. Conclusion

In the existing literature, the roles of pre-entry learning and subsequent market competition have been investigated extensively but almost independently, and there are very few works, if any, which combine them in a unified framework. To fill this gap and provide a more comprehensive description of the tradeoff faced by potential market entrants, this paper constructs a dynamic model of market entry which features these two elements simultaneously. We fully characterize symmetric equilibrium of this game and identify a necessary and sufficient condition for the first-mover advantage to dominate. The condition clarifies when and under what conditions a firm becomes a pioneer, early follower or late entrant. We also argue that consumer inertia is generally efficiency-enhancing, which highlights an elusive link between static market competition and dynamic entry competition and suggests some policy implications.

References

[31] Note that all firms are of type 0 at the outset of the game.

Bloch, F., Fabrizi, S. and Lippert, S., 2015, Learning and Collusion in New Markets with Uncertain Entry Costs, *Economic Theory*, 58, 273-303.

Bloch, F., Fabrizi, S. and Lippert, S., 2020, Experimentation and Entry with Common Values, mimeo, Paris School of Economics.

Bobtcheff, C., Bolte, J. and Mariotti, T., 2017, Researcher's Dilemma, *Review of Economic Studies*, 84, 969-1014.

Bonatti, A. and Hörner, J., 2011, Collaborating, *American Economic Review*, 101, 632-63.

Bulow, J. and Klemperer, P., 1994, Rational Frenzies and Crashes, *Journal of Political Economy*, 102, 1-23.

Chamley, C. and Gale, D., 1994, Information Revelation and Strategic Delay in a Model of Investment, *Econometrica*, 62, 1065-85.

Che, Y.-K. and Hörner, J., 2018, Recommender Systems as Mechanisms for Social Learning, *Quarterly Journal of Economics*, 133, 871-925.

Chen, C.-H. and Ishida, J., 2018, Hierarchical Experimentation, *Journal of Economic Theory*, 177, 365-404.

Chen, C.-H. and Ishida, J., 2021, A War of Attrition with Experimenting Players, *Journal of Industrial Economics*, forthcoming.

Decamps, J.-P. and Mariotti, T., 2004, Investment Timing and Learning Externalities, *Journal of Economic Theory*, 118, 80-102.

Fudenberg, D. and Tirole, J., 1986, A Theory of Exit in Duopoly, *Econometrica*, 54, 943-60.

Grenadier, S.R., 1996, The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets, *Journal of Finance*, 51, 1653-79.

Grenadier, S.R., 1999, Information Revelation Through Option Exercise, *Review of Financial Studies*, 12, 95-129.

Guo, Y., 2016, Dynamic Delegation of Experimentation, *American Economic Review*, 106, 1969-2008.

Hopenhayn, H.A. and Squintani, F., 2016, Patent Rights and Innovation Disclosure, *Review of Economic Studies*, 83, 199-230.

Horstmann, I.J. and Markusen, J.R., 1996, Exploring New Markets: Direct Investment, Contractual Relations and the Multinational Enterprise, *International Economic Review*, 37, 1-19.

Horstmann, I.J. and Markusen, J.R., 2018, Learning to Sell in New Markets: A Preliminary Analysis of Market Entry by a Multinational Firm, *Review of International Economics*, 26, 1040-52.

Kirpalani, R. and Madsen, E., 2020, Strategic Investment Evaluation, mimeo, New York University.

Keller, G. and Rady, S., 2010, Strategic Experimentation with Poisson Bandits, *Theoretical Economics*, 5, 275-311.

Keller, G. and Rady, S., 2015, Breakdowns, *Theoretical Economics*, 10, 175-202.

Keller, G., Rady, S. and Cripps, M., 2005, Strategic Experimentation with Exponential Bandits, *Econometrica*, 73, 39-68.

Lahiri, S. and Ono, Y., 1988, Helping Minor Firms Reduces Welfare, *Economic Journal*, 98, 1199-202.

Levin, D. and Peck, J., 2003, To Grab for the Market or to Bide One's Time: A Dynamic Model of Entry, *RAND Journal of Economics*, 34, 536-56.

Lieberman, M.B. and Montgomery, D.B., 1988, First-Mover Advantages, *Strategic Management Journal*, 9, 41-58.

Lilien, G.K. and Yoon, E., 1990, The Timing of Competitive Market Entry: An Exploratory Study of New Industry Products, *Management Science*, 36, 568-85.

Luo, Y., 1998, Timing of Investment and International Expansion Performance in China, *Journal of International Business Studies*, 29, 391-407.

Mankiw, N.G. and Whinston, M.D., 1986, Free Entry and Social Efficiency, *RAND Journal of Economics*, 17, 48-58.

Margaria, C., 2020, Learning and Payoff Externalities in an Investment Game, *Games and Economic Behavior*, 119, 234-50.

Mitchell, W., Shaver, J.M. and Yeung, B., 1994, Foreign Entrant Survival and Foreign Market Share: Canadian Companies' Experience in United States Medical Sector Markets, *Strategic Management Journal*, 15, 555-67.

Murto, P. and Välimäki, J., 2011, Learning and Information Aggregation in an Exit Game, *Review of Economic Studies*, 78, 1426-61.

Pawlina, G. and Kort, P.M., 2006, Real Options in an Asymmetric Oligopoly: Who Benefits from Your Competitive Disadvantage? *Journal of Economics and Management Strategy*, 15, 1-35.

Rasmusen, E. and Yoon, Y.-R., 2012, First versus Second Mover Advantage with Information Asymmetry about the Profitability of New Markets, *Joural of Industrial Economics*, 60, 374-405.

Robinson, W.T. and Fornell, C., 1985, Sources of Market Pioneer Advantages in Consumer Goods Industries, *Journal of Marketing Research*, 22, 305-17.

Shackleton, M.B., Tsekrekos, A.E., and Wojakowski, R., 2004, Strategic Entry and Market Leadership in a Two-Player Real Options Game, *Journal of Banking and Finance*, 28, 179-201.

Shaver, J.M., Mitchell, J. and Yeung, B., 1997, The Effect of Own-Firm and Other-Firm Experience on Foreign Direct Investment Survival in the United States, 1987-92, *Strategic Management Journal*, 18, 811-24.

Strulovici, B., 2010, Learning While Voting: Determinants of Collective Experimentation, *Econometrica*, 78, 933-71.

Suzumura, K. and Kiyono, K., 1987, Entry Barriers and Economic Welfare, *Review of Economic Studies*, 54, 157-67.

Trigeorgis, L., 1991, Anticipated Competitive Entry and Early Preemptive Investment in Deferrable Projects, *Journal of Business and Economics*, 43, 143-56.

Weeds, H., 2002, Strategic Delay in a Real Options Model of R&D Competition, *Review of Economic Studies*, 69, 729-47.

Zachary, M.A., Gianiodis, P.T., Payne, G.T., and Markman, G.D., Entry Timing: Enduring Lessons and Future Directions, *Journal of Management*, 41, 1388-415.

Appendix A: Proofs

Proof of Lemma 2. To prove the proposition, we first establish the following facts.

Fact 1. $\lim_{\Delta\to 0} s\Delta < 1$ *for any* $\phi \geq \overline{p}$.

Proof. Suppose that $\lim_{\Delta\to 0} s\Delta = 1$ for some (p,q). This means that each firm earns $p\frac{m\pi}{r} - c$ by entering now. Now suppose that a firm deviates and delays entry until the next instant. In this case, if the other firm does not enter now, the firm knows for sure that the market is bad, and the belief drops to zero, making it optimal never to enter. Taking this into account, the expected profit is

$$e^{-r\Delta}\left[p\frac{\pi - rc}{r} - qce^{-\lambda\Delta}\right],$$

which is larger than $p\frac{\pi-rc}{r} - (1-p)c$ if Δ is sufficiently small, a contradiction. ■

Fact 2. $s = 0$ *if* $\phi \geq \overline{p} > p$.

Proof. Suppose otherwise, i.e., $s > 0$. This is the case if

$$(1-s\Delta)s'\Delta pm > (1-p)\lambda c - r\left[p\frac{\pi - rc}{r} - (1-p)c\right].$$

Note that the right-hand side is strictly positive by definition when $\overline{p} > p$, which implies that $\lim_{\Delta\to 0} s'\Delta > 0$ and $p > p'$. Given this, we can apply the same argument to show that $\lim_{\Delta\to 0} s_t\Delta > 0$ for all future periods. Given $\overline{p} > p$, we can take an arbitrarily small ε and define

$$N_\varepsilon(\Delta) := \max\{k \in \mathbb{N} \mid \varepsilon > k\Delta\},$$

for some ε. Then, we must have $\lim_{\Delta\to 0} \Pi_{k=0}^{N_\varepsilon(\Delta)}(1 - s_{t+k\Delta}\Delta) = 0$. This means that if the other firm does not enter by $t + N_\varepsilon(\Delta)\Delta$, the firm's belief will drop almost to zero. By the same argument as in Fact 1, it is strictly better to wait until $t + N_\varepsilon(\Delta)\Delta$, and hence $s = 0$ which is a contradiction. ■

Fact 3. *For* $\phi \geq \overline{p}$,

1. $s > 0$ *if* $p \geq \overline{p}$;

2. $\lim_{\Delta\to 0} s\Delta > 0$ *such that* $\overline{p} > \lim_{\Delta\to 0} p'$ *if* $p > \overline{p}$*, where* p' *indicates the belief at the next instant (in case of no entry).*

Proof. We first show that $s > 0$. Suppose that $s = 0$. Then $p' > p$, and the continuation payoff is weakly lower than

$$e^{-r\Delta}\left[p\left(\frac{\pi - rc}{r} + m\Delta\right) - (1-p)ce^{-\lambda\Delta}\right],$$

which is in turn lower than $p(\frac{\pi - rc}{r} + m\Delta) - (1-p)c$, the expected payoff of entering now, if $p \geq \overline{p}$.

To prove the second statement, note that

$$(1-p)\lambda c - r\left[p\frac{\pi - rc}{r} - (1-p)c\right] < 0,$$

so that (5) must be satisfied. As such, it is strictly better to enter now than at the next instant. By Fact 1, however, a firm, say firm 1, waits for an instant with positive probability. This implies that firm 1's entry decision at the next instant must depend on what firm 2 does now, for otherwise there would be no incentive for firm 1 to wait. The next-period belief must hence be low enough to satisfy $\overline{p} > p'$ so that firm 1 enters at the next instant if and only if firm 2 enters now. ∎

Fact 4. *For* $\phi \geq \overline{p}$*,* $s \in (0, \infty)$ *such that* $p' = \overline{p}$ *if* $p = \overline{p}$*.*

Proof. From Facts 1 and 2, we know that a firm must play a mixed strategy if $p \geq \overline{p}$. Suppose first that s is small enough and $p' > p = \overline{p}$. Then, $s'\Delta > 0$ by Fact 3, which implies that (5) strictly holds given $p = \overline{p}$. This implies, however, that it is strictly better to enter now than at the next instant, which is a contradiction. Now suppose that s is large enough and $p' < \overline{p}$. Then, $s' = 0$ by Fact 2, so that

$$e^{-s\Delta}(1 - e^{-s'\Delta})pm = (1-p)\lambda c - r\left[p\frac{\pi - rc}{r} - (1-p)c\right] = 0.$$

This means that the expected payoff of entering now is equal to that of entering at the next instant regardless of the other firm's action. Given $p' < \overline{p}$, however, the optimal strategy at the next instant is to enter if and only if the other firm enters now. Therefore, a firm can earn a higher payoff by waiting for an instant, which is a contradiction. This proves that the belief must be kept at $\overline{p}$ once it reaches this level. ∎

These results show that starting from some $p_t < \overline{p}$, no firm enters until p_t reaches $\overline{p}$. Once the belief reaches $\overline{p}$, then the firms start randomizing so as to keep $p_t = \overline{p}$ as suggested by Lemmas 3 and 4. ■

Proof of Proposition 1. We start with part (a) of the proposition. Let $s_t = \sigma(p_t, q_t)$. Since ϕ_t is strictly increasing, it will reach $\overline{p}$ sooner or later, and the firms enter at some positive rate as shown in Lemma 2. This means that there must be a period in which $s_t = 0$, followed by a period in which $s_t \in (0, \infty)$. This means that we just need to show that $s_t \in (0, \infty)$ for $t \in (\tau^*, \underline{\tau})$.

Suppose first that the firms enter with strictly positive probability, i.e., $s_t = \infty$, at some $t \in (\tau^*, \underline{\tau})$. This means that the expected payoff of entering at t drops discretely, because the firms may enter at the same time with positive probability. Then, we must also have $s_{t-\Delta} = \infty$, for otherwise it is not optimal to have $s_t > 0$. This process continues until $t = 0$, but then it violates the assumption that learning is essential and $\tau^* > 0$.

Given this, we now show that $s_t > 0$ for all $t \in (\tau^*, \underline{\tau})$. Suppose that there exists an interval $(\underline{a}, \overline{a})$ such that $s_t = 0$ for $t \in (\underline{a}, \overline{a})$ but $s_t > 0$ for $t \in (\underline{a} - \varepsilon, \underline{a}) \cup (\overline{a}, \overline{a} + \varepsilon)$ where ε is some positive number. This implies that a firm obtains a higher payoff by entering at $\underline{a}$ or at $\overline{a}$ than at any time in $(\underline{a}, \overline{a})$. Given that $s_t = 0$ for $t \in (\underline{a}, \overline{a})$, by entering at $t + dt$ instead of entering at t, the firm's expected payoff increases by

$$(r + \lambda)c - p_t \left[\pi + (1 + e^{-r\delta_t})m + \lambda c \right],$$

which is decreasing in t. This means that the expected payoff is concave in $(\underline{a}, \overline{a})$, and hence we cannot have the payoff maximized at $\underline{a}$ and $\overline{a}$ in this interval. Therefore, $s_t \in (0, \infty)$ for $t \in (\tau^*, \underline{\tau})$.

Finally, if $\phi_t = \overline{p} > p_t$, it is strictly optimal for a firm to wait until p_t reaches $\overline{p}$. Therefore, shortly before ϕ_t reaches $\overline{p}$, it is still optimal to wait, meaning that $\phi_{\underline{\tau}} < \overline{p}$. From Lemma 2, the firms wait until the belief p_t reaches $\overline{p}$ at $\overline{\tau}$ so that $s_t = 0$ for $t \in (\underline{\tau}, \overline{\tau})$.

Part (b) of the proposition follows immediately from Lemma 2. Once the belief p_t reaches $\overline{p}$, it is optimal to enter even if it is immediately followed by the rival firm. The firms thus enter at some positive rate to keep p_t at $\overline{p}$. ■

Proof of Lemma 3. It is straightforward to verify that $\hat{\mu}$ is strictly decreasing in t for $t \in [0, \frac{\tau^{\text{NP}}}{2}]$, meaning that there exists at most one $\hat{\tau}$ such that $\hat{\mu}(\hat{\tau}) = 0$. In the limit, the optimal timing of pioneering entry, denoted by τ^{P}, is given by

$$\tau^{\mathrm{P}} = \begin{cases} 0 & \text{if } 0 \geq \hat{\mu}(0), \\ \hat{\tau} & \text{if } \hat{\mu}(0) > 0 > \hat{\mu}(\frac{\tau^{\mathrm{NP}}}{2}), \\ \frac{\tau^{\mathrm{NP}}}{2} & \text{if } \hat{\mu}(\frac{\tau^{\mathrm{NP}}}{2}) \geq 0, \end{cases}$$

where $\hat{\tau}$ solves $\hat{\mu}(\hat{\tau}) = 0$. ■

Proof of Proposition 2. We first show that pioneering entry occurs if and only if $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$. We then construct an equilibrium to show its existence.

Necessary and sufficient condition: The sufficiency is obvious. If $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$, there is an incentive for a firm to enter when $\overline{p} > \phi_t$. Pioneering entry must occur with some probability.

To establish the necessity, suppose that pioneering entry occurs. Then, if a firm enters at τ^*, the expected payoff is

$$p_{\tau^*}\frac{\pi + (1 - e^{-r\delta_{\tau^*}})m - rc}{r} - (1 - p_{\tau^*})c,$$

which equals to the payoff of waiting until any $t \in (\tau^*, \underline{\tau}) \cup (\overline{\tau}, \infty)$. Now suppose that the firm instead waits until $\tau^{\mathrm{NP}} - \tau^*$ regardless of what the other firm does, in which case the expected payoff is

$$e^{-r(\tau^{\mathrm{NP}} - \tau^*)}\left[p_{\tau^*}\frac{\pi - rc}{r} - (1 - p_{\tau^*})ce^{-\lambda(\tau^{\mathrm{NP}} - \tau^*)}\right].$$

Since this is the payoff when a firm does not utilize any information from the other firm, we must have

$$p_{\tau^*}\frac{\pi + (1 - e^{-r\delta_{\tau^*}})m - rc}{r} - (1 - p_{\tau^*})c > e^{-r(\tau^{\mathrm{NP}} - \tau^*)}\left[p_{\tau^*}\frac{\pi - rc}{r} - (1 - p_{\tau^*})ce^{-\lambda(\tau^{\mathrm{NP}} - \tau^*)}\right].$$

Note that the right-hand side can be written as

$$e^{-r(\tau^{\mathrm{NP}} - \tau^*)}\left[p_{\tau^*}\frac{\pi - rc}{r} - (1 - p_{\tau^*})ce^{-\lambda(\tau^{\mathrm{NP}} - \tau^*)}\right] = e^{r\tau^*}\frac{p_{\tau^*}}{p_0}\Pi^{\mathrm{NP}}.$$

Since

$$\Pi^{\mathrm{P}} \geq e^{-r\tau^*}\frac{p_0}{p_{\tau^*}}\left[p_{\tau^*}\frac{\pi + (1 - e^{-r\delta_{\tau^*}})m - rc}{r} - (1 - p_{\tau^*})c\right],$$

it follows that $\Pi^{\text{P}} > \Pi^{\text{NP}}$.

Equilibrium existence: If a firm enters now, the expected payoff is

$$p\frac{\pi + e^{-s\Delta}(1-e^{r\delta_t})m - rc}{r} - (1-p)c.$$

If the firm waits for an instant, on the other hand, the other firm may enter with some probability, in which case it follows at $\tau^{\text{NP}} - t$ if it is uninformed; otherwise, the firm enters at the next instant if it is uninformed. The expected payoff of this waiting strategy is given by

$$e^{-r\Delta}\left[pe^{-s\Delta}\frac{\pi + e^{-s'\Delta}(1-e^{-r\delta_{t+\Delta}})m - rc}{r} - [1-p-q(1-e^{-s\Delta})]ce^{-\lambda\Delta}\right]$$
$$+(1-e^{-s\Delta})e^{-r\delta_t}\left(p\frac{\pi - rc}{r} - qce^{-\lambda\delta_t}\right).$$

As $\Delta \to 0$, the indifference condition is obtained as

$$s\left[p\frac{\pi + (1-e^{-r\delta_t})m - rc}{r} - qc - e^{-r\delta_t}\left(p\frac{\pi - rc}{r} - qce^{-\lambda\delta_t}\right)\right] = \mu(p,t), \tag{14}$$

where

$$\mu(p,t) := -p(\pi + m - rc) - pme^{-r\delta_t} + (1-p)(\lambda + r)c.$$

Since

$$\delta_t = \frac{1}{\lambda}\ln\frac{\overline{p}q}{p(1-\overline{p})},$$

this gives us a Markov strategy for any given (p,q).

Observe that at τ^*, (14) is reduced to

$$se^{r\tau^*}\left[\hat{\Pi}(\tau^*) - \Pi^{\text{NP}}\right] = \hat{\mu}(\tau^*).$$

For any $\tau^* < \tau^{\text{P}}$, $\hat{\mu}(\tau^*) > 0$. This means that the firms enter at positive rate only if the left-hand side is positive. By definition, this is the case for any $\tau^* \in \Gamma$. For any arbitrary choice of $\tau^* < \tau^{\text{P}}$, we can drive a sequence of strategies $\{s_t\}$ and the consequent belief path $\{p_t\}$. At $\underline{\tau}$, we must have

$$\mu(p_{\underline{\tau}}, \underline{\tau}) = 0.$$

To see this, if $\mu(p_{\underline{\tau}}, \underline{\tau}) < 0$, it is strictly better to enter now. If $\mu(p_{\underline{\tau}}, \underline{\tau}) > 0$, on the other hand, the expected payoff increases for $t > \underline{\tau}$, so it is strictly better to wait. Therefore, we can pin down $\underline{\tau}$ and $\overline{\tau}$ as a function of τ^*.

Note that $\underline{\tau}(\tau^{\mathrm{P}}) = \tau^{\mathrm{P}}$ by definition. On the other hand, we have

$$\lim_{\tau^* \to \inf \Gamma} \Pi(\tau^*) = \lim_{\tau^* \to \inf \Gamma} \hat{\Pi}(\tau^*) < \Pi^{\mathrm{NP}}.$$

Since $\Pi(t)$ is continuous in t, there must be some $\tau^* \in \Gamma$ such that $\Pi(\tau^*) = \Pi(\underline{\tau}(\tau^*)) = \Pi(\overline{\tau}(\tau^*))$. ■

Proof of Proposition 3. Note that Π^{NP} and Π^{P} as functions of p_0 are given by

$$\Pi^{\mathrm{NP}}(p_0) = e^{-r\tau^{\mathrm{NP}}}\left[p_0 \frac{\pi - rc}{r} - (1-p_0)ce^{-\lambda\tau^{\mathrm{NP}}}\right], \tag{15}$$

$$\Pi^{\mathrm{P}}(p_0) = e^{-r\tau^{\mathrm{P}}}\Big[p_0 \frac{\pi + (1-e^{-r\delta_{\tau^{\mathrm{P}}}})m - rc}{r} - (1-p_0)ce^{-\lambda\tau^{\mathrm{P}}}\Big]. \tag{16}$$

It is clear that (16) can only increase with m by the envelope theorem while (15) is independent of it. Therefore, there must be a threshold $\hat{m}$ such that the condition for pioneering entry is satisfied if and only if $m > \hat{m}$.

For the effect of p_0, define $p_0'' := \frac{p_0'}{p_0' + (1-p_0')e^{-\lambda\Delta}}$ for a given $p_0' < \overline{p}$ where Δ is assumed to be arbitrarily small. Also, we write τ^{NP} and τ^{P} both as functions of p_0. Since

$$\Pi^{\mathrm{NP}}(p_0) = e^{-r\tau^{\mathrm{NP}}(p_0)} \frac{p_0}{\overline{p}} \left(\overline{p}\frac{\pi}{r} - c\right),$$

we have

$$\Pi^{\mathrm{NP}}(p_0'') = \frac{e^{-r\tau^{\mathrm{NP}}(p_0'')} p_0''}{e^{-r\tau^{\mathrm{NP}}(p_0')} p_0'} \Pi^{\mathrm{NP}}(p_0') = e^{r\Delta} \frac{p_0''}{p_0'} \Pi^{\mathrm{NP}}(p_0'), \tag{17}$$

for any $\overline{p} > p_0'' > p_0'$.

As for Π^{P}, observe first that by Lemma 3,

$$\begin{aligned} \lim_{p_0 \uparrow \overline{p}} \hat{\mu}(0) &= -p_0(\pi + m - rc) - p_0 m e^{-r\tau^{\mathrm{NP}}} + (1-p_0)(\lambda + r)c \\ &= -\frac{(\lambda + r)c}{\pi + \lambda c}(\pi + m - rc) - \frac{(\lambda + r)c}{\pi + \lambda c} m e^{-r\tau^{\mathrm{NP}}} + \frac{\pi - rc}{\pi + \lambda c}(\lambda + r)c < 0, \end{aligned}$$

suggesting that there exists some threshold $\tilde{p} < \overline{p}$ such that $\tau^{\mathrm{P}} = 0$ if and only if $p_0 \in [\tilde{p}, \overline{p})$. Also, define $p^+ := \frac{p_0'}{p_0' + (1-p_0')e^{-\lambda\tau^{\mathrm{P}}(p_0')}}$.

Observe that $p_0 < \tilde{p}$, or alternatively $p^+ \geq p_0''$ and $\tau^{\mathrm{P}}(p_0') \geq \Delta$ by assumption. We then have

$$\begin{aligned}\Pi^{\mathrm{P}}(p_0') &= e^{-r\tau^{\mathrm{P}}(p_0')}\left[p_0'\frac{\pi+(1-e^{-r(\tau^{\mathrm{NP}}(p_0')-2\tau^{\mathrm{P}}(p_0'))})m-rc}{r}-(1-p_0')ce^{-\lambda\tau^{\mathrm{P}}(p_0')}\right]\\ &= e^{-r\tau^{\mathrm{P}(p_0')}}\frac{p_0'}{p^+}\left[p^+\frac{\pi+(1-e^{-r(\tau^{\mathrm{NP}}(p_0')-2\tau^{\mathrm{P}}(p_0'))})m-rc}{r}-(1-p^+)c\right].\end{aligned}$$

Similarly,

$$\begin{aligned}\Pi^{\mathrm{P}}(p_0'') &= e^{-r\tau^{\mathrm{P}}(p_0'')}\left[p_0''\frac{\pi+(1-e^{-r[\tau^{\mathrm{NP}}(p_0'')-2\tau^{\mathrm{P}}(p_0'')]})m-rc}{r}-(1-p_0'')ce^{-\lambda\tau^{\mathrm{P}}(p_0'')}\right]\\ &> e^{-r(\tau^{\mathrm{P}}(p_0')-\Delta)}\frac{p_0''}{p^+}\left[p^+\frac{\pi+(1-e^{-r[\tau^{\mathrm{NP}}(p_0'')-2(\tau^{\mathrm{P}}(p_0')-\Delta)]})m-rc}{r}-(1-p^+)c\right]\\ &> e^{-r(\tau^{\mathrm{P}}(p_0')-\Delta)}\frac{p_0''}{p^+}\left[p^+\frac{\pi+(1-e^{-r[\tau^{\mathrm{NP}}(p_0')-2\tau^{\mathrm{P}}(p_0')]})m-rc}{r}-(1-p^+)c\right]\\ &= e^{r\Delta}\frac{p_0''}{p_0'}\Pi^{\mathrm{P}}(p_0').\end{aligned}$$

Here, the second line shows the payoff when the firm enters at $\tau^{\mathrm{P}}(p_0')-\Delta$ at which point the belief reaches p^+. From the second line to the third, we use the fact that

$$\tau^{\mathrm{NP}}(p'')+2\Delta > \tau^{\mathrm{NP}}(p'')+\Delta = \tau^{\mathrm{NP}}(p_0').$$

It follows from above that if $\Pi^{\mathrm{NP}}(p_0') = \Pi^{\mathrm{P}}(p_0')$, then $\Pi^{\mathrm{NP}}(p_0'') < \Pi^{\mathrm{P}}(p_0'')$. This suggests that Π^{NP} and Π^{P} intersect at most twice for $p_0 \in (0, \overline{p}]$. Note also that $\overline{p} > \tilde{p}$ and $\Pi^{\mathrm{NP}}(\overline{p}) = \Pi^{\mathrm{P}}(\overline{p})$. This proves that there is a threshold $\hat{p} \in (0, \overline{p})$ such that the condition for pioneering entry is satisfied if and only if $p_0 \in (\hat{p}, \overline{p})$. ■

Proof of Proposition 4. In the proof of Proposition 2, we observe that

$$p_{\tau^*}\frac{\pi+(1-e^{-r\delta_{\tau^*}})m-rc}{r}-(1-p_{\tau^*})c > e^{r\tau^*}\frac{p_{\tau^*}}{p_0}\Pi^{\mathrm{NP}}.$$

Since $\frac{1-p_{\tau^*}}{p_{\tau^*}} = \frac{1-p_0}{p_0}e^{-\lambda\tau^*}$, we obtain

$$e^{-r\tau^*}\frac{p_0}{p_{\tau^*}}\left[p_{\tau^*}\frac{\pi+(1-e^{-r\delta_{\tau^*}})m-rc}{r}-(1-p_{\tau^*})c\right] = \Pi^* > \Pi^{\mathrm{NP}}.$$

We can also show that the firms start entering earlier than τ^{P}, i.e., $\tau^{\mathrm{P}} > \tau^*$, if $\Pi^{\mathrm{P}} > \Pi^{\mathrm{NP}}$. To see this, suppose on the contrary that $\tau^* \geq \tau^{\mathrm{P}}$. Let (p,q) be the belief at $\tau^* - \Delta$. If a firm, say firm 1, deviates and enters unilaterally at $\tau^* - \Delta$, the expected payoff is

$$p\frac{\pi + (1 - e^{-r(\delta^* + 2\Delta)})m - rc}{r} - (1-p)c. \tag{18}$$

where $\delta^* := \tau^{\mathrm{NP}} - 2\tau^*$. Now suppose that the firm waits for a period and enters at τ^* (if it is still uninformed), for which the expected payoff is computed as

$$e^{-r\Delta}\left[p\frac{\pi + e^{-s'\Delta}(1 - e^{-r\delta^*})m - rc}{r} - (1-p)ce^{-\lambda\Delta}\right]. \tag{19}$$

By comparing (18) and (19), it is better to deviate if

$$p\frac{\pi + [1 - e^{-r(\delta^* + 2\Delta)}]m - rc}{r} - (1-p)c > e^{-r\Delta}\left[p\frac{\pi + e^{-s'\Delta}(1 - e^{-r\delta^*})m - rc}{r} - (1-p)ce^{-\lambda\Delta}\right].$$

As $\Delta \to 0$, this reduces to

$$p\frac{s'(1 - e^{-r\delta^*})m}{r} > (1-p)(\lambda + r)c - pme^{-r\delta^*} - p(\pi + m - rc).$$

Observe that the right-hand side is proportional to $\hat{\mu}(t)$ and takes a non-positive value for $t \geq \tau^{\mathrm{P}}$ if $\frac{\tau^{\mathrm{NP}}}{2} > \tau^{\mathrm{P}}$. This means that firm 1 always has an incentive to deviate by entering at $\tau^* - \Delta$, a contradiction. This means that $\tau^{\mathrm{P}} > \tau^*$ and hence $\Pi^* < \Pi^{\mathrm{P}}$, giving the payoff bounds. ■

Proof of Proposition 5. (i) Note that τ^{NP} must solve

$$p_0(\pi - rc) = (1 - p_0)(\lambda + r)ce^{-\lambda\tau^{\mathrm{NP}}}.$$

The only difference from (11) is that the left-hand side is $p_0(\pi - rc)$ instead of $p_0(\pi - m - rc)$, which implies $T^{**} > \tau^{\mathrm{NP}}$.

(ii) Since $\frac{T^{**}}{2} > \frac{\tau^{\mathrm{NP}}}{2} \geq \tau^{\mathrm{P}}$, $\tau_1^{**} > \tau^{\mathrm{P}}$ holds if $\tau_1^{**} \geq \frac{T^{**}}{2}$. This means that we can focus on $\tau_1^{**} < \frac{T^{**}}{2}$ (with an interior optimum satisfying the second-order condition). In this case, τ_1^{**} must solve (12), which implies that

$$e^{-\lambda\tau_1^{**}} < \frac{p_0(\pi + m - rc)}{(1 - p_0)(\lambda + r)c}.$$

On the other hand, if $\tau^{\mathrm{P}} > 0$, it must satisfy

$$p_0(\pi + m - rc) = (1 - p_0)(\lambda + r)ce^{-\lambda\tau^{\mathrm{P}}} - p_0me^{-r(\tau^{\mathrm{NP}} - 2\tau^{\mathrm{P}})}.$$

As this implies that

$$e^{-\lambda\tau^{\mathrm{P}}} > \frac{p_0(\pi+m-rc)}{(1-p_0)(\lambda+r)c},$$

we have $\tau_1^{**} > \tau^{\mathrm{P}}$.

(iii) Observe that $\tau_1^{**} \geq \frac{T^{**}}{2}$ if

$$p_0(\pi+m-rc) < (1-p_0)(\lambda+r)ce^{-\lambda\tau_1} + p_0\frac{\lambda(\pi-m-rc)}{\lambda+r}e^{-r(T^{**}-2\tau_1)},$$

for all $\tau_1 \in [0, \frac{T^{**}}{2})$. This obviously holds if p_0 is sufficiently small, in which case $\tau_1 = \tau$ and $\tau_2 = \tau + \Delta$, where τ must solve

$$p_0(\pi - rc) = \frac{1-p_0}{2}\left[\lambda + r + (2\lambda+r)e^{-\lambda\tau}\right]ce^{-\lambda\tau},$$

as $\Delta \to 0$. Plugging $\tau_1 = \tau^{\mathrm{NP}}$ into this, the right-hand side becomes

$$\frac{1-p_0}{2}\left[(\lambda+r) + (2\lambda+r)\frac{p_0(\pi-rc)}{(1-p_0)(\lambda+r)c}\right]ce^{-\lambda\tau^{\mathrm{NP}}}.$$

This is smaller than $(1-p_0)(\lambda+r)ce^{-\lambda\tau^{\mathrm{NP}}} = p_0(\pi - rc)$ if p_0 is small enough to satisfy $\frac{p_0(\pi-rc)}{(1-p_0)(\lambda+r)c} < \frac{\lambda+r}{2\lambda+r}$. We then have

$$2p_0(\pi-rc) > (1-p_0)\left[(\lambda+r) + (2\lambda+r)e^{-\lambda\tau_1}\right]ce^{-\lambda\tau_1},$$

for all $\tau_1 \geq \tau^{\mathrm{NP}}$ and hence $\tau_1^{**} < \tau^{\mathrm{NP}}$. ■

Appendix B: Poisson bandits

In this appendix, we develop a generalized version of the baseline model—the Poisson-bandits model—where a signal can come from either state and no one signal is conclusive. The purpose of this appendix is to provide a heuristic argument to illustrate that this extended version exhibits many important qualitative properties and analogous insights.

B.1. Belief updating

Suppose that the entry probability of type j is given by s^j and the most optimistic type is currently J so that $p^{j,k} = q^{j,k} = 0$ for $k < J$. Given current belief $(p^{j,k}, q^{j,k})_{k=0}^{\infty}$ and strategy $(s^j)_{j=0}^{\infty}$, the next-period belief is $(p^{j,k\prime}, q^{j,k\prime})_{k=0}^{\infty}$ if the firm does not observe a signal, where

$$p^{j,k\prime} = \frac{e^{-\nu\lambda\Delta}[p^{j,k}e^{-(s^k+\nu\lambda)\Delta} + p^{j,k-1}e^{-s^{k-1}\Delta}(1-e^{-\nu\lambda\Delta})]}{e^{-\nu\lambda\Delta}\sum_{\ell=J}^{\infty} p^{j,\ell}e^{-s^\ell\Delta} + e^{-\lambda\Delta}\sum_{\ell=J}^{\infty} q^{j,\ell}e^{-s^\ell\Delta}},$$

$$q^{j,k\prime} = \frac{e^{-\lambda\Delta}[q^{j,k}e^{-(s^k+\lambda)\Delta} + q^{j,k-1}e^{-s^{k-1}\Delta}(1-e^{-\lambda\Delta})]}{e^{-\nu\lambda\Delta}\sum_{\ell=J}^{\infty} p^{j,\ell}e^{-s^\ell\Delta} + e^{-\lambda\Delta}\sum_{\ell=J}^{\infty} q^{j,\ell}e^{-s^\ell\Delta}},$$

If the firm observes a signal, the next-period belief is $(p^{j+1,k'}, q^{j+1,k'})_{k=0}^{\infty}$, where

$$p^{j+1,k'} = \frac{(1-e^{-\nu\lambda\Delta})[p^{j,k}e^{-(s^k+\nu\lambda)\Delta} + p^{j,k-1}(1-e^{-(s^{k-1}+\nu\lambda)\Delta})]}{(1-e^{-\nu\lambda\Delta})\sum_{\ell=J}^{\infty} p^{j,\ell}e^{-s^\ell\Delta} + (1-e^{-\lambda\Delta})\sum_{\ell=J}^{\infty} q^{j,\ell}e^{-s^\ell\Delta}},$$
$$q^{j+1,k'} = \frac{(1-e^{-\lambda\Delta})[q^{j,k}e^{-(s^k+\lambda)\Delta} + q^{j,k-1}(1-e^{-(s^{k-1}+\lambda)\Delta})]}{(1-e^{-\nu\lambda\Delta})\sum_{\ell=J}^{\infty} p^{j,\ell}e^{-s^\ell\Delta} + (1-e^{-\lambda\Delta})\sum_{\ell=J}^{\infty} q^{j,\ell}e^{-s^\ell\Delta}}.$$

Let $p^j := \sum_{k=J}^{\infty} p^{j,k}$ and $q^j := \sum_{k=J}^{\infty} q^{j,k}$ where p^j is the probability of the market being good condition on having observed j signals. The belief p^j rises to p_+^j if the firm observes no signal and declines to p_-^j if it observes a signal, where

$$p_+^j = \frac{e^{-\nu\lambda\Delta}\sum_{k=J}^{\infty} p^{j,k}e^{-s^k\Delta}}{e^{-\nu\lambda\Delta}\sum_{k=J}^{\infty} p^{j,k}e^{-s^k\Delta} + e^{-\lambda\Delta}\sum_{k=J}^{\infty} q^{j,k}e^{-s^k\Delta}}, \tag{20}$$
$$p_-^j = \frac{(1-e^{-\nu\lambda\Delta})\sum_{k=J}^{\infty} p^{j,k}e^{-s^k\Delta}}{(1-e^{-\nu\lambda\Delta})\sum_{k=J}^{\infty} p^{j,k}e^{-s^k\Delta} + (1-e^{-\lambda\Delta})\sum_{k=J}^{\infty} q^{j,k}e^{-s^k\Delta}}, \tag{21}$$

In general, as in the baseline model, market entry must occur smoothly over time in any equilibrium because there would be a payoff discontinuity otherwise, giving an incentive to deviate. This means that the most optimistic type must be indifferent between entering and waiting while all other types strictly prefer to wait. As a consequence, (20) and (21) can be written as

$$p_+^j = \frac{e^{-\nu\lambda\Delta}[p^j - p^{j,J}(1-e^{-s^J\Delta})]}{e^{-\nu\lambda\Delta}[p^j - p^{j,J}(1-e^{-s^J\Delta})] + e^{-\lambda\Delta}[q^j - q^{j,J}(1-e^{-s^J\Delta})]},$$
$$p_-^j = \frac{(1-e^{-\nu\lambda\Delta})[p^j - p^{j,J}(1-e^{-s^J\Delta})]}{(1-e^{-\nu\lambda\Delta})[p^j - p^{j,J}(1-e^{-s^J\Delta})] + (1-e^{-\lambda\Delta})[q^j - q^{j,J}(1-e^{-s^J\Delta})]}.$$

In particular, if there is no market entry, they are further reduced to

$$p_+^j = \frac{e^{-\nu\lambda\Delta}p^j}{e^{-\nu\lambda\Delta}p^j + e^{-\lambda\Delta}(1-p^j)}, \; p_-^j = \frac{(1-e^{-\nu\lambda\Delta})p^j}{(1-e^{-\nu\lambda\Delta})p^j + (1-e^{-\lambda\Delta})(1-q^j)}.$$

Observe that $\lim_{\Delta\to 0} p_+^j = p^j > \lim_{\Delta\to 0} p_-^j$, so that there is a downward jump upon observing a signal as in the baseline model.

Let J be the most optimistic type, and a type j firm observes the rival firm's entry. The firm's belief then jumps up to the post-entry belief ϕ^j given by

$$\phi^j = \frac{p^{j,J}}{p^{j,J} + q^{j,J}} = \frac{1}{1 + L^{j,J}},$$

where $L^{j,J} := \frac{q^{j,J}}{p^{j,J}}$ is simply the likelihood ratio. Since $p^{j,J-1} = q^{j,J-1} = 0$ by definition, the post-entry belief evolves according to

$$\begin{aligned}\phi^{j\prime} &= \frac{e^{-\nu\lambda\Delta} p^{j,J} e^{-(s^J+\nu\lambda)\Delta}}{e^{-\nu\lambda\Delta} p^{j,J} e^{-(s^J+\nu\lambda)\Delta} + e^{-\lambda\Delta} q^{j,J} e^{-(s^J+\lambda)\Delta}} \\ &= \frac{p^{j,J}}{p^{j,J} + q^{j,J} e^{-2(1-\nu)\lambda\Delta}} > \phi^j.\end{aligned}$$

The post-entry belief gradually increases over time independently of the entry strategy and hence behaves in a manner quite similar to that in the baseline model.

Although the dynamics of the whole belief system is quite complicated, the post-entry belief has a simple, closed-form, representation as in the baseline model. As we have seen above, the post-entry belief is independent of the entry strategy, and is pinned down by the likelihood ratio $L^{j,J}$. Observe that since the number of observed signals follows a Poisson distribution and is conditionally independent, the conditional probability that a firm has observed j signals and the other firm has observed k signals at any t is

$$P(j,k \mid \theta, t) = \frac{(\lambda_\theta t)^j}{j!} e^{-\lambda_\theta t} \times \frac{(\lambda_\theta t)^k}{k!} e^{-\lambda_\theta t}.$$

Although this probability does not take into account the possibility of market entry, this event happens equally across the two states (as it depends only on j) and hence is exactly canceled out. The likelihood ratio thus equals to

$$L_t^{j,k} = \frac{q^{j,k}}{p^{j,k}} = \frac{(1-p_0)\frac{(\lambda t)^j}{j!} e^{-\lambda t} \times \frac{(\lambda t)^k}{k!} e^{-\lambda t}}{p_0 \frac{(\nu\lambda t)^j}{j!} e^{-\nu\lambda t} \times \frac{(\nu\lambda t)^k}{k!} e^{-\nu\lambda t}} = \frac{(1-p_0) e^{-2(1-\nu)\lambda t}}{p_0\, \nu^{j+k}},$$

for any (j,k), where p_0 denotes the prior probability of the state being good, and the post-entry belief is given by

$$\phi_t^j = \frac{1}{1 + L_t^{j,J}} = \frac{p_0\, \nu^{j+J}}{p_0\, \nu^{j+J} + (1-p_0) e^{-2(1-\nu)\lambda t}}. \tag{22}$$

The monotone likelihood ratio property of the Poisson distribution implies $\phi_t^j < \phi_t^{j-1}$ for all t and $j \geq 1$ if $\nu \in (0,1)$, which can be easily confirmed from the formula.

Given this, it is easy to verify that the post-entry belief is strictly higher than the current belief p^j, i.e.,

$$\phi^j = \frac{p^{j,J}}{p^{j,J} + q^{j,J}} > p^j.$$

Alternatively, this condition can be written as

$$p^{j,J} > (p^{j,J} + q^{j,J})p^j.$$

Summing over $k = J, J+1, \ldots$, we obtain

$$\sum_{k=J}^{\infty} p^{j,k} = \sum_{k=J}^{\infty} (p^{j,k} + q^{j,k})p^j,$$

which holds because $\sum_{k=J}^{\infty} p^{j,k} = p^j$ and $\sum_{k=J}^{\infty}(p^{j,k} + q^{j,k}) = 1$ by definition. Suppose on the contrary that $p^j \geq \phi^j$ or, equivalently, $(p^{j,J} + q^{j,J})p^j \geq p^{j,J}$. Since $L^{j,k} > L^{j,k-1}$, we then have $(p^{j,k} + q^{j,k})p^j > p^{j,k}$ for all $k > J$. This implies

$$\sum_{k=J}^{\infty} p^{j,k} < \sum_{k=J}^{\infty} (p^{j,k} + q^{j,k})p^j,$$

which is a contradiction. This shows that a firm's entry is always a good signal in equilibrium.

B.2. Entry dynamics

Suppose that the first mover enters at t. Then, the second mover, which must be of type $j \geq J$, updates the belief to ϕ_t^j and enters the market at τ_t^j. As such, each market entry induces a countably infinite set of deterministic entry times $\{\tau_t^J, \tau_t^{J+1}, \ldots\}$ where $\tau_t^{j+1} \geq \tau_t^j \geq t$ for each j; let $\tau_t^j = t$ if the second mover follows immediately and $\tau_t^j = \infty$ if it will never enter. Notice the difference from the baseline model where there are only two types, either $j = 0$ or $j = 1$, and $\tau_t^1 = \infty$ for any t. In contrast, in the Poisson-bandits model, the belief is always bounded away from 0, and hence $\tau_t^j < \infty$ for any finite j.

The fact that market entry must occur smoothly and only the most optimistic type can enter at any point in time suggests that any equilibrium must have a sequence of intervals $(T^0, T^1, \ldots)$, where $T^j := (\underline{\tau}^j, \overline{\tau}^j)$, such that only type j enters at a (weakly) positive rate for $t \in T^j$. In each interval T^j, type j is the most optimistic type and both firms must be of type $k \geq j$. Notice that those intervals are disjoint, i.e., $\overline{\tau}^{j-1} < \underline{\tau}^j$ for all j, because different types have different beliefs that are bounded away from each other. More precisely, suppose $\overline{\tau}^{j-1} = \underline{\tau}^j$ and consider $\underline{\tau}^j - \varepsilon$ and $\underline{\tau}^j + \varepsilon$. We then have

$$\phi^k_{\underline{\tau}^j - \varepsilon} = \frac{1}{1 + L^{k,j-1}} > \phi^k_{\underline{\tau}^j + \varepsilon} = \frac{1}{1 + L^{k,j}},$$

for any arbitrarily small $\varepsilon > 0$ and $k \geq j$ (which follows from the monotone likelihood ratio property). This means that the waiting time is strictly longer, making it strictly better for the first mover to enter at $\underline{\tau}^j + \varepsilon$ than at $\underline{\tau}^j - \varepsilon$.

It is also worth mentioning that the baseline model is a special case where $T^0 = (\tau^*, \infty)$. Later, we will show that as $\nu \to 0$, $\overline{\tau}^0 \to \infty$ and the equilibrium converges to that of the baseline model.

B.3. Tradeoff between entering early and late

Since different types have different best responses, the Poisson-bandits model exhibits no sharp distinction between the preemption phase and the waiting phase as in the baseline model. Even then, the benefit of becoming the first mover is still determined by the expected waiting time of the second mover although it now varies across types. Let $I_t := \{j : \tau_t^j = t\}$ be the set of types that would immediately follow upon observing the rival firm's entry. The extent of the first-mover advantage can roughly be measured by the probability that the rival firm follows immediately when the state is good; let $P_t := \sum_{j \in I_t} p_t^j$ be the probability of that happening. This probability is either 0 or 1 in the baseline model, thereby giving rise to the sharp distinction between the two phases, whereas it can take any value between 0 and 1 in the Poisson-bandits model.

To quantify the first-mover advantage, it is instructive to examine (22) and see how the post-entry belief evolves over time. There are three observations we can make. First, at time 0, all firms are of type 0, and $\phi_0^0 = p_0$, i.e., a market entry at time 0 reveals no information. Second, the post-entry belief is monotonically increasing in t and converges to 1 for any j and J. These are the properties that are present in the baseline model and continue to hold in the Poisson-bandits model. Third, since $\phi_t^j < \phi_t^{j-1}$, there is a threshold type $\overline{j}_t$ for each t such that $j \in I_t$ if and only if $j \leq \overline{j}_t$.

These properties suggest that the amount of information revealed by a firm's entry is arbitrarily small at early stages of the game. Since the benefit of market entry is discretely lower for the second mover, this means that $P_t = 0$ if t is sufficiently small, giving rise to the first-mover advantage. As time passes, however, the game eventually reaches a point where a market entry reveals sufficient information and is followed by the rival firm with sufficiently high probability. This can be seen from the fact that for any j, J and ε, there is $\overline{t}^j$ such that $\phi^j(t;\nu) > 1 - \varepsilon$ for all $t > \overline{t}^j$. Moreover, as t gets arbitrarily large, the distributions of types degenerate towards the means: $\nu\lambda t$ when the state is good and λt when the state is bad. Given these facts, therefore, P_t must approach 1 as t tends to infinity, thereby rendering the first-mover advantage dissipate at some point. The Poisson-bandits model is hence characterized by the same tradeoff as in the baseline model, where the first-mover advantage emerges at early stages of the game while the second-mover advantages prevails at later stages.

B.4. Limit results

Now consider a limit case where $\nu \to 0$. Note that for any t and $j + J \geq 1$, we have $\lim_{\nu\to 0} \phi^j(t; \nu) = 0$. This in turn means that $\tau_t^j \to \infty$ for any t and $j+J \geq 1$. The strategic effects of all types $j \geq 1$ become negligibly small and can be excluded from consideration as ν becomes arbitrarily small. The firm's optimization problems thus converge to those in the baseline model, yielding arbitrarily close solutions. Note also that if the state is good, a type 0 firm remains type 0 almost surely for any t, no matter how large it is, and a strictly positive measure of them always stay in the game, so that $\overline{\tau}^0 \to \infty$. Our baseline model thus closely approximates the equilibrium allocation when ν is relatively small.