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CLIMATE POLICY AND WEALTH DISTRIBUTION

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Abstract

We set up a model with intergenerational bequest transfers and climate damage on the wealth of heterogeneous households. We show that, under credit market imperfections and depending on wealth distribution across households, a balanced budget climate policy may widen the wealth inequality gap between the rich and poor. Climate policy may create positive effects on the wealth of households, but these effects are asymmetric across households in terms of both magnitude and the transmission of gains from a climate policy within households. The gains of the poor from a climate policy are mainly transmitted into improving living standards and the investment in human capital due to the higher marginal return to education investment. By contrast, the gains of the rich from a climate policy are transmitted biasedly into physical capital accumulation and thereby enhance their monopolistic position in the production of intermediate inputs. We show that, for any climate policy, there exists a corresponding threshold of aggregate physical capital. When the aggregate physical capital of the economy exceeds this threshold, the corresponding climate policy may widen the intergenerational bequest transfers among heterogeneous households, thereby contributing to widening the wealth inequality gap between the rich and poor in the long run.

JEL: D62, D63, O15, Q52, Q54.

Keywords: Climate policy, balanced budget policy, credit market imperfections, intergenerational bequest transfer, wealth inequality.

1 Introduction

Two issues emerging in the twenty-first century are climate change and income/wealth inequality. The Paris Agreement under the United Nations Framework Convention on Climate Change, which aims to limit global warming to well below 2 °C above pre-industrialization levels, indeed requires strict transitions to clean economies. These transitions can be implemented through a carbon

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tax policy in which the government imposes a Pigouvian tax on the dirty production sector, and then uses the tax revenue to subsidize the clean production sector (Acemoglu et al. 2012, 2016). Such a climate policy may improve environmental quality and the living conditions of households. However, whether the effects of climate policy may be asymmetric to heterogeneous households needs to be investigated. Wealth inequality has been a topic of debate among economists and social scientists for a long time, especially since the publication of "*Capital in the twenty-first century*" by Piketty (2014). However, the link between climate policy and wealth/income inequality has not been sufficiently investigated in the literature. This paper aims to contribute a theoretical model linking climate policy and wealth inequality to identify the conditions under which climate policy may widen the wealth inequality gap between the rich and poor.

Numerous studies have considered the optimal paths for global emissions and optimal taxation strategies for the decentralization (e.g. Nordhaus 1992, 1993; Nordhaus and Boyer 2000; Pizer 1999; Acemoglu et al. 2012; Golosov et al. 2014; among others). These studies, however, have ignored the asymmetric effects of tax policy on the wealth of households in an economy. In addition, while a large body of literature focuses on the relationship between climate policy and inequality between countries, theoretical considerations of the relationship between climate policy and inequality between households remain limited. The present paper aims to fill this gap in the literature.

A sizable body of literature has also attempted to explain the relationship between wealth/income inequality and development, as well as persistent wealth/income inequality (e.g. Galor and Zeira 1993; Galor and Moav 2004, 2006; Piketty 1997; Piketty and Zucman 2014; Piketty et al. 2019; Lakner and Milanovic 2013; Liberati 2015, among others). These previous studies stress that wealth inequality can persist because of credit market imperfections and intergenerational bequest transfers within households. The presence of credit market imperfections under wealth inequality creates a long-lasting effect on investment in human capital and entrepreneurial activities, contributing to the persistence of wealth inequality.

How is the link between climate change policy and wealth/income inequality? Surprisingly, few studies, particularly from the theoretical aspect, have been conducted on the interactions between and dynamics of this link. In an alternative approach to examine climate policy under wealth inequality, Vasconcelos et al. (2014) focused on on the conflicting policies and challenges faced in achieving cooperation between rich and poor countries. Blonz et al. (2011) and William III et al. (2014) studied empirically near-term effects of climate policy on the welfare of heterogeneous households by considering the effects on populations with different age groups. Dennig et al. (2015) advanced the Dynamic Integrated Climate Economy model to stress the importance of accounting inequality within regions and point out the asymmetric effects of climate policy on the welfare of households. Ravallion et al. (2000) empirically showed that higher income inequality, both within and between countries, is associated with lower carbon emissions at given average incomes. Grunewald et al. (2011) reported a U-shaped relationship between carbon emissions and income inequality. Chancel and Piketty (2015) argued that the rich should be responsible for the cost of climate policy because, in terms of consumption-based emissions, they contribute more to global warming.

In contrast to the related literature, in this paper, we consider theoretically not only the effects of climate policy on wealth distribution, but also the conditions under which climate policy can widen the wealth gap between the rich and the poor, who cannot escape the poverty trap. We set up an overlapping generations model with heterogeneous households under credit market imperfections and a climate externality on the wealth of households. In particular, we identify an aggregate capital threshold corresponding to each climate policy. When the aggregate physical capital exceeds this threshold, such climate policy tends to widen the wealth gap between the rich and poor. This occurs because the gain from the climate policy can be transmitted intergenerationally to investment in human capital for the poor or may improve the living conditions of the poor without intergenerational transfer, whereas it is transmitted biasedly into the accumulation of physical capital for the rich. The improvement in human capital for the whole economy benefits the profits of the intermediate producing firms owned by the rich biasedly, thereby amplifying wealth inequality and enhancing the accumulation of physical capital. This increased accumulation of physical capital leads to more physical capital being allocated to the production of dirty intermediate inputs, which generates an increased burden on future climate policy.

The remainder of the paper is organized as follows. Section 2 introduces the building blocks of the model. Section 3 characterizes the equilibria and dynamics of the model. The effects of climate policy on macroeconomic variables and household wealth are presented in section 4. Section 5 provides a discussion and concludes.

2 The benchmark model

We consider a discrete time overlapping generations economy with a constant population. Following Acemoglu et al. (2012) and Dao and Edenhofer (2018), we assume that there is one homogeneous final output produced by human capital and intermediate inputs. In each period $t \in \mathbb{N}$, we define I_t as a nonempty set of adult (or working) individuals/households, i.e. each individual i who becomes an adult in period t belongs to the set I_t. Each individual $i \in I_t$ lives for two periods in childhood in $t-1$ and adulthood in t. Along with choosing the optimal education investment, individuals allocate their wealth (when they are adults), which comes from labour income, capital income and monopoly profit, between consumption and bequest transfer for their children so as to maximize their lifetime utility.

In the production sectors of this economy, we follow Dao and Edenhofer (2018) with a slight modification of aggregate human capital instead of fixed aggregate labour. In the household sector, we follow Galor and Moav (2004, 2006), in particular, the intergenerational bequest motives and education investment decisions. Differing from Galor and Moav (2004, 2006), however, we consider the effects of monopolistic profit sharing and climate damage on the dynamics of intergenerational bequest transfers.

2.1 Final goods sector

There is one homogeneous final output produced by human capital and intermediate inputs under the following (aggregate) production function[1]

$$Y_t = H_t^{1-\alpha} \left(A_c^{1-\alpha} X_{ct}^{\alpha} + A_d^{1-\alpha} X_{dt}^{\alpha} \right); \quad \alpha \in (0,1) \tag{1}$$

where Y_t is the aggregate final output and H_t is the aggregate human capital employed in the final goods production in period t, i.e.

$$H_t = \int_{I_t} h_t^i di$$

in which h_t^i is the human capital of individual $i \in I_t$.

The subscripts "c" and "d" denote "*clean*" and "*dirty*", respectively. Thus, X_{ct} and X_{dt} are the amounts of clean and dirty inputs employed in the final goods production, respectively. In addition, $A_c > 0$ and $A_d > 0$ are quality (or aggregate total factor productivity) indexes of the corresponding intermediate inputs.

Suppose that the final goods sector operates under a perfectly competitive environment. The profit maximization problem of producing firms in this sector is

$$\max_{H_t, X_{ct}, X_{dt}} H_t^{1-\alpha} \sum_{v \in \{c,d\}} A_v^{1-\alpha} X_{vt}^{\alpha} - w_t H_t - \sum_{v \in \{c,d\}} p_{vt} X_{vt}$$

given the return on human capital, w_t, and the price of intermediate input $v \in \{c,d\}$, p_{vt}, in the period t.

The first-order conditions with respect to H_t and X_{vt} give

$$w_t = \frac{1-\alpha}{H_t^{\alpha}} \sum_{v \in \{c,d\}} A_v^{1-\alpha} X_{vt}^{\alpha} \tag{2}$$

$$p_{vt} = \alpha H_t^{1-\alpha} A_v^{1-\alpha} X_{vt}^{\alpha-1}; \tag{3}$$

as the inverse demand functions of human capital and intermediate inputs.

[1] Such a final good production function is introduced extensively in Aghion and Howitt (2009). Our paper focuses on the effect of climate policy on wealth distribution rather than on the transition to a clean economy, so to keep the model as simple as possible, we adopt a production function (1) in which the intermediate inputs are completely substitutable rather than complementary. As apparent in the subsequent sections, with this production function, we can derive explicitly the rule of capital allocation for intermediate input production, which depends on the relative quality (or aggregate total factor productivity) of the intermediate inputs. The characterization of the features of the capital allocation rule is crucial for analysing the effects of climate policy on capital allocation in producing intermediate inputs, and subsequently on other macroeconomic variables such as labour income, the capital rental rate, and aggregate monopolistic profits. In their online appendix, Dao and Edenhofer (2018) provide a robustness check for a more general production function, in particular

$$Y = H^{1-\alpha} \left(A_c^{1-\alpha} X_c^{\alpha \frac{\epsilon-1}{\epsilon}} + A_d^{1-\alpha} X_d^{\alpha \frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon \in (0, +\infty)$ is a constant elasticity of substitution between intermediate inputs. With this production function, Dao and Edenhofer (2018) point out implicitly that the capital allocation rule has similar properties to the production function in this paper. In particular, physical capital will be allocated biasedly to the intermediate sector with higher aggregate total factor productivity.

2.2 Intermediate sectors and climate policy

For the sake of simplicity, we assume that each intermediate input indexed by $v \in \{c, d\}$ is produced in period t according to the following production function

$$X_{vt} = K_{vt} \tag{4}$$

where K_{vt} is the amount of physical capital used as input in the intermediate sector $v \in \{c, d\}$. We assume that the physical capital fully depreciates in each period t of use, given that each period lasts around 30 years.[2] Thus, the cost of producing X_{vt} units of intermediate input $v \in \{c, d\}$ in period t is $r_t K_{vt}$, where r_t is the rental rate of physical capital in period t. The producer of the intermediate good vj in period t decides the quantity X_{vt} to be produced to maximize its monopolistic profit. Therefore, the monopolist profit of the entrepreneur vj in period t is

$$\pi_{vt} = \max_{k_{vjt}} (1 - \tau_{vt}) p_{vt} K_{vt} - r_t K_{vt} \tag{5}$$

given r_t and τ_{vt}, where $\tau_{vt} < 1$ is the Pigouvian tax rate (or subsidy if negative) imposed by the government on the production of each intermediate good $v \in \{c, d\}$ in period t. These tax rates represent the climate policy of the government. In this paper, we consider a climate policy in which $\tau_{dt} \in [0, 1)$ and $\tau_{ct} \leq 0$, i.e. in any period t, the government imposes a Pigouvian tax rate $\tau_{dt} \in [0, 1)$ on the production of dirty intermediate sectors and uses this tax revenue to subsidize production of the clean sector at a rate $-\tau_{ct} \geq 0$. The extreme values $(\tau_{ct}, \tau_{dt}) = (0, 0)$ imply the case that no climate policy is carried out.

2.3 Returns on factors of production and monopolistic profits

As apparent in Appendix A0, under the set-up above, the returns on physical capital and human capital, as well as the monopolistic profits, are

$$r_t = \alpha^2 \left[\frac{H_t}{K_t} \sum_{v \in \{c,d\}} (1 - \tau_{vt})^{\frac{1}{1-\alpha}} A_v \right]^{1-\alpha} \tag{6}$$

$$w_t = (1 - \alpha) \left[\sum_{v \in \{c,d\}} (1 - \tau_{vt})^{\frac{1}{1-\alpha}} A_v \right]^{1-\alpha} \left(\frac{K_t}{H_t} \right)^{\alpha} \tag{7}$$

$$\Pi_t = \sum_{v \in \{c,d\}} \pi_{vt} = \alpha(1 - \alpha) \left[H_t \sum_{v \in \{c,d\}} (1 - \tau_{vt})^{\frac{1}{1-\alpha}} A_v \right]^{1-\alpha} K_t^{\alpha} \tag{8}$$

given the aggregate physical capital K_t, the aggregate human capital H_t, and the balanced budget climate policy $(\tau_{ct}, \tau_{dt}) \in \Re_- \times [0, 1)$.

[2]In subsection 2.4, we assume the rate of decay of greenhouse gases to belong to interval $(0, 1)$, capturing the fact that physical capital and greenhouse gases evolve at different timescales. I thank an anonymous referee for pointing out this.

2.4 Dynamics of greenhouse gas stock

We assume that only the use of dirty intermediate inputs degrades the environment by emitting greenhouse gas. So the dynamics of greenhouse gas stock is characterized by

$$P_t = \bar{P} + (1-\delta)(P_{t-1} - \bar{P}) + \xi X_{dt}$$

where P_t is the greenhouse gas stock in period t, which can be viewed as the carbon concentration in the atmosphere. The term ξX_{dt} is the flow of greenhouse gases in period t emitted from dirty intermediate inputs, and $\xi > 0$ is the dirtiness coefficient of the dirty intermediate inputs. $\bar{P}$ is the natural state of greenhouse gas concentration in the atmosphere, i.e. the state of the ecological system without any human activity, and $\delta \in (0,1)$ is the decay rate of greenhouse gases which measures the convergent speed of greenhouse gas stock to the natural state $\bar{P}$. For the sake of simplicity, without losing any generality, we normalize $\bar{P} = 0$. Note that $X_{dt} = K_{dt}$, and with the allocation rule of physical capital in (21), the dynamics of greenhouse gas stock can be written as follows

$$P_t = (1-\delta)P_{t-1} + \xi K_{dt} \tag{9}$$

where $K_{dt} = \frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}} A_d}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v} K_t$ is aggregate physical capital which is allocated in the production of the dirty intermediate inputs d (see the rule of capital allocation in Appendix A0).

2.5 Individuals/households

Throughout this paper, we use the terms individual(s) and household(s) interchangeably. We assume that, in each period $t \in \mathbb{N}$, the economy is populated by a constant and continuous population of working individuals who are identical in innate talent and preference. Without losing any generality, we normalize the size of the population by 1. Each working individual in any period t has a single parent and a single offspring, and lives for two periods, say $t-1$ and t. In the first period of life, period $t-1$ (e.g. childhood), individuals spend all of their time for human capital formation. In the second period of life, period t (e.g. adulthood), they supply efficient units of labour to the market inelastically and allocate their wealth (from labour income, capital income, and probably monopolistic profit) between consumption and bequest transfers to their offspring. Human capital increases if the time for education is supplemented with capital investment in education. The human capital of adult individual $i \in I_t$, h_t^i, is

$$h_t^i = h(e_t^i)$$

where $e_t^i \geq 0$ is capital investment in education for individual $i \in I_t$, who becomes an adult in period t. The human capital formation function satisfies the following assumption 1.

Assumption 1. $h(0) = 1$, $h'(e) > 0$, $h''(e) < 0$, $\lim\limits_{e\to 0^+} h'(e) = +\infty$ and $\lim\limits_{e\to+\infty} h'(e) = 0$.

Assumption 1 implies that, without capital investment in education, each individual is naturally endowed one unit of labour. Human capital formation is a strictly increasing and concave function in education investment and satisfies Inada conditions.

The utility of individual $i \in I_t$ comes from the consumption c_t^i and the bequest to its offspring b_{t+1}^i, obeying the following utility function

$$U(c_t^i, b_{t+1}^i) = (1-\gamma)\ln c_t^i + \gamma \ln(\theta_t + b_{t+1}^i); \quad \theta_t > 0 \tag{10}$$

where $\gamma \in (0,1)$ is the preference weight towards the individual's offspring.[3]

The budget constraint of individual $i \in I_t$ is

$$c_t^i + b_{t+1}^i \leq W_t^i \phi(P_t) \tag{11}$$

where W_t^i is the wealth of individual $i \in I_t$, and $\phi(P_t) \in (0,1)$ is the damage effect of greenhouse gas stock P_t in period t and satisfies $\phi'(P_t) < 0$. We assume that the greenhouse gas stock in each period damages the wealth of individuals and acts as a negative externality. We define

$$\psi(W_t^i, P_t) = W_t^i \phi(P_t) \quad \text{and} \quad \psi(w_t, P_t) = w_t \phi(P_t)$$

respectively as *disposable wealth* and *minimal disposable wealth* of household $i \in I_t$. The $\psi(w_t, P_t)$ is constituted only by a unit of physical labour endowment of the household.

The $\ln \theta_t$, which appears in the utility function when $b_{t+1}^i = 0$, is the minimal "expected" utility derived from the offspring of household $i \in I_t$. The θ_t can be viewed as the expectation of a household of the *minimal* disposable wealth of its offspring in period $t+1$. This expectation is based on the individual's own *minimal* disposable wealth, i.e. $\psi(w_t, P_t) = w_t \phi(P_t)$. Indeed, the minimal disposable wealth $\psi(w_t, P_t)$ is what an adult household $i \in I_t$ observes in period t. Based on this observation, it assigns an expectation to the minimal disposable wealth that its offspring may own in the next period. Thus, we impose the following assumption:

Assumption 2. $\theta_t = \theta \psi(w_t, P_t)$ *and* $\theta > \frac{\gamma}{1-\gamma}$.

The parameter θ may change over time depending (endogenously) on other factors that are not captured in the model. However, θ is not a focus of the model, and hence, for simplification without lessening the power of the model, we treat θ as a constant.[4]

The household $i \in I_t$ maximizes its utility (10) subject to the budget constraint (11), given θ_t, W_t^i, and P_t. The optimal bequest transfer is

[3] We follow the utility function from Galor and Moav (2004, 2006), which allows a corner solution for bequest transfer, i.e. $b_{t+1}^i = 0$, when the wealth of individual $i \in I_t$ is too small. For simplification and consistency with Galor and Moav (2004, 2006), we abstract from consumption when old without changing the qualitative results of the households. Indeed, with the log-linear utility, consumption when old can be incorporated into the utility function, and then the dynamics of the model is altered only by a multiplicative constant.

One of the main features in analysing the heterogeneity in wealth among households is convex saving behaviour, which implies that the marginal propensity to save higher for the rich than for the poor—see Stiglitz (1969) and Bourguignon (1981). The qualitative results of our model would be valid under the convex saving function. The specific log-linear function (10) allows us to derive such the classical saving function while guaranteeing the dynamics of savings in terms of intergenerational bequest transfers to be traceable.

[4] $\theta > \frac{\gamma}{1-\gamma}$ holds for plausible values of θ and γ, *e.g.* $\theta \geq 1$ (i.e. households place an expectation on the minimal disposable wealth that their offspring can earn at least as much as they could) and $\gamma \simeq 1/4$ that is used in the literature (see Lagerlöf 2006 and Dao 2016). θ_t can be probably interpreted as the minimal income that the society can expect to be set by the government.

$$b_{t+1}^i = \max\left\{0, \gamma\left[\psi(W_t^i, P_t) - \frac{\theta_t(1-\gamma)}{\gamma}\right]\right\} \tag{12}$$

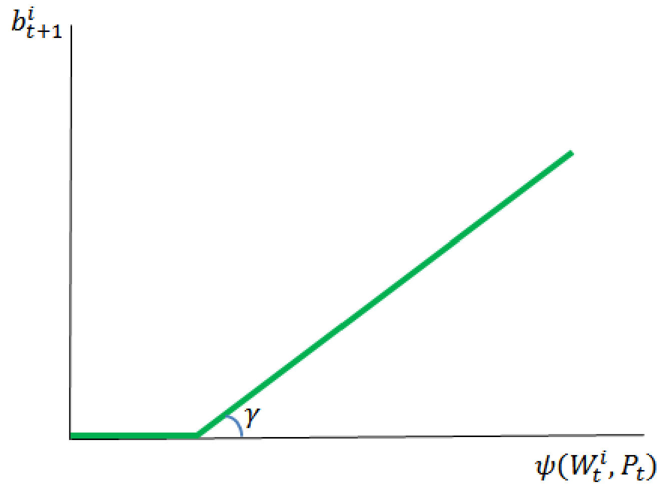

Figure 1: Bequest transfer

The intergenerational bequest transfer $b_t^i \geq 0$ that household $i \in I_t$ receives from its parent $i \in I_{t-1}$ is allocated between education investment, e_t^i, and capital saving, s_t^i, which is lent to the capital market in period t to earn capital income. Hence,

$$b_t^i = e_t^i + s_t^i$$

and the wealth of household $i \in I_t$ is[5]

$$W_t^i = w_t h(e_t^i) + (b_t^i - e_t^i)r_t + \rho_t^i \Pi_t$$

in which $w_t h(e_t^i)$ is the labour income, $(b_t^i - e_t^i)r_t$ is the capital income, and $\rho_t^i \Pi_t$ is the income from the monopolistic profits. The $\rho_t^i \in (0, 1)$ is the share of monopolistic profits assigned for household $i \in I_t$; hence, $\{\rho_t^i\}_{i \in I_t}$ holds

$$\int_{I_t} \rho_t^i di = 1$$

In the absence of borrowing constraints, the rational household $i \in I_t$ will choose e_t^i such that

$$e_t^i \in \arg\max_{e_t^i} \left[w_t h(e_t^i) + (b_t^i - e_t^i)r_t + \rho_t^i \Pi_t\right] \tag{13}$$

Note that the term $\rho_t^i \Pi_t$ in the optimization problem (13) above does not depend on e_t^i. Under assumption 1 and in the absence of the borrowing constraint, the optimal education investment is

[5]There can be a joint decision between the household $i \in I_t$ and its parent $i \in I_{t-1}$ for allocating the intergenerational bequest transfer b_t^i between investing in human capital and savings so as to maximize the wealth of household $i \in I_t$.

$$e_t^i = e_t^* \quad \forall i \in I_t \qquad \text{where} \quad h'(e_t^*) = \frac{r_t}{w_t}$$

We assume that a household cannot borrow for education investment because of the imperfection of credit markets, i.e. education investment e_t^i is limited by the bequest transfer b_t^i.

Proposition 1.

(i) Under the imperfections of credit markets, the optimal education investment e_t^i of individual $i \in I_t$ depends on the transfer bequest b_t^i that the individual receives from the parent, in particular

$$e_t^i = \min\left\{b_t^i, e_t^*\right\} \tag{14}$$

(ii) For any strictly positive aggregate stock of bequest transfer $b_t = \int_{I_t} b_t^i di > 0$, $e_t^* > \underline{e}_t$ where $\underline{e}_t \in (0, b_t)$ is a unique solution to

$$h'(e_t) = \frac{\alpha^2}{1-\alpha}\frac{h(e_t)}{b_t - e_t}$$

Moreover, $\underline{e}_t = \underline{e}(b_t)$ is an increasing function of b_t.

Proof. See Appendix A1. □

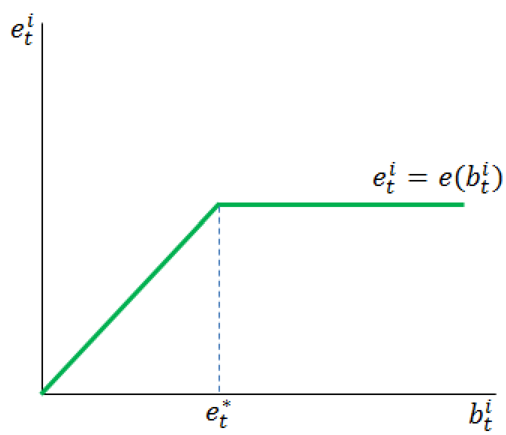

Figure 2: Education investment

Proposition 1 implies that the education investment e_t^i of household $i \in I_t$ is a bounded function of the intergenerational bequest transfer b_t^i that she/he receives from its parent $i \in I_{t-1}$. In any period t, the corresponding upper bound of education investment is e_t^*, which depends on the wage rate w_t and the rental rate of capital r_t, in particular

$$e_t^* = h'^{-1}(\frac{r_t}{w_t})$$

Under perfect credit markets, the last equation equalizes the marginal benefits of education investment and capital savings—*i.e.* $w_t h'(e_t^*) = r_t$—guaranteeing the maximal the income/wealth of household $i \in I_t$. Under imperfect credit markets, for a given wage rate $w_t > 0$ and rental rate of capital $r_t > 0$, households prioritize investing in education over savings. When the intergenerational bequest transfer is sufficiently low, particularly $b_t^i \leq e_t^*$, the household i allocates the bequest transfer for education investment, i.e. $e_t^i = b_t^i$. When the bequest transfer is sufficiently high, particularly $b_t^i > e_t^*$, the household i allocates e_t^* for education investment and the remaining $b_t^i - e_t^*$ for savings in terms of physical capital.

The statement (ii) of Proposition 1 implies that whenever the capital stock of the economy in period t is strictly positive, the demand for human capital investment of household i with intergenerational bequest transfer $b_t^i > 0$ always exists, i.e. $e_t^i > 0$. In particular, the optimal education investment under perfect credit markets is bounded from below by some strictly positive level, depending positively on the aggregate intergenerational bequest transfer. This is because, with a strictly positive stock of capital, the rental rate of capital is finite. Therefore, a household with a strictly positive bequest transfer received from its parent always can improve its potential income by allocating some amount from the bequest transfer in education investment.

We need to impose essential assumptions about the ownership of monopoly firms. We assume that in each period t, monopolists of intermediate goods represent a fraction $\lambda \in (0, 1)$ of the population. We denote I_t^λ as the set of monopolists in period t, then $I_t^\lambda \subset I_t$. The owners of the monopolistic firms own the monopolistic profits.

Assumption 3. The share of monopolistic profits is as follows

$$\rho_t^i > 0 \qquad \text{iff} \qquad b_t^i > e_t^* + \hat{k}, \qquad \text{with} \quad \hat{k} > 0$$

Assumption 3 implies that to own monopoly firms and earn monopolistic profits, the bequest transfer b_i^i that individual i receives from his or her parent must exceed a threshold at which after investing in education at level e_t^*, the individual i must have sufficient physical capital, i.e. $b_t^i - e_t^* > \hat{k}$ holds, to run a monopoly firm. This assumption admits the widely observed fact that when a potential entrepreneur accesses the capital market to borrow capital for operating a monopoly producing firm, the banks or financial intermediate institutions *always* require an initial capital capacity in addition to the entrepreneurial skills of the applicant.[6] $\hat{k}$ can be linked to the minimum equity of the firm owned by the individual in order for that individual to have rights in making decisions on production and entrepreneurial activities to obtain monopolistic profits.

3 Equilibria and dynamics

In this section, we define equilibria. Then, we characterize the dynamics of the model and obtain the reduced dynamics of intergenerational bequest transfers for any household $i \in I_t$.

Definition 1. *(**Equilibria**) Given P_{t-1} and $\{b_t^i, \rho_t^i\}_{i \in I_t}$, under the balanced budget climate policy $(\tau_{ct}, \tau_{dt}) \in \Re_- \times [0, 1)$, the competitive equilibria in period t is characterized by: (i) the determi-*

[6] In this model, we assume that individuals have identical innate talent. Hence, the difference in education investment generates a difference in entrepreneurial skills.

nations of returns on production factors in (6) and (7); (ii) the optimal choices of each household $i \in I_t$ in (12) and (14); (iii) the allocation rule of physical capital in intermediate sectors $K_{vt} = \frac{(1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}{\sum_{v' \in \{c,d\}} (1-\tau_{v't})^{\frac{1}{1-\alpha}} A_{v'}} K_t$ with $v \in \{c, d\}$; and (iv) the dynamics of greenhouse gas stock in (9).

The evolution of intergenerational bequest transfer within each family dynasty i is

$$b^i_{t+1} = \max\left\{0, \gamma\phi(P_t)\left[w_t h(e^i_t) + (b^i_t - e^i_t)r_t + \rho^i_t \Pi_t - \frac{\theta w_t(1-\gamma)}{\gamma}\right]\right\} \tag{15}$$

We study the dynamics of intergenerational bequest transfer under the following assumption

Assumption 4. $h(\underline{e}(\lambda\hat{k})) > \theta\frac{1-\gamma}{\gamma}$.

Assumption 4 implies that $w_t h(\underline{e}(\lambda\hat{k}))\phi(P_t) > \theta w_t \phi(P_t)(1-\gamma)/\gamma$, i.e. the disposable wealth of a non-monopolistic household at the education level $\underline{e}(\lambda\hat{k}) \in (0, e^*_t)$ is high enough at which the household leaves a part of wealth, as bequest transfer, for its offspring. Note that by construction,

$$\lambda\hat{k} < \inf_{t\in\mathbb{N}} \int_{I_t} b^i_t di$$

i.e. $\lambda\hat{k}$ is a lower bound of bequest stock $b_t = \int_{I_t} b^i_t di$ of the economy.

Proposition 2. *Under assumptions 1, 2, 3 and 4, there exists a threshold $\underline{b} \in (0, e^*_t)$ such that*

$$b^i_{t+1}\begin{cases} = 0 & \text{if} \quad b^i_t \in [0, \underline{b}] \\ > 0 & \text{if} \quad b^i_t > \underline{b} \end{cases} \qquad \textit{where} \quad \underline{b} = h^{-1}\left(\theta\frac{1-\gamma}{\gamma}\right)$$

Proof. See Appendix A2.

□

Proposition 2 implies that a household $i \in I_t$ will transfer to its offspring $i \in I_{t+1}$ a strictly positive bequest $b^i_{t+1} > 0$ if and only if the bequest transfer b^i_t, that it received from its parent household $i \in I_{t-1}$, is sufficiently high, in particular $b^i_t > \underline{b} = h^{-1}\left(\theta\frac{1-\gamma}{\gamma}\right) \in (0, e^*_t)$. This is because in the model set-up that focuses on the persistent effect of intergenerational bequest transfers on wealth distribution, the bequest transfer that a household receives from its parent household is the source/determinant of its wealth. Indeed, this bequest transfer is allocated for human capital formation—which generates the labour income—and capital savings—which generate the capital income and probably monopolistic profit. When the bequest transfer b^i_t that household $i \in I_t$ receives from its parent $i \in I_{t-1}$ is too small—in particular, $b^i_t \leq \underline{b}$—then household $i \in I_t$ allocates all of this bequest transfer for education investment to generate wealth $W^i_t = w_t h(b^i_t) \leq w_t h(\underline{b})$. This wealth, however, is not high enough for household $i \in I_t$ to be able to transfer optimally a strictly positive bequest $b^i_{t+1} > 0$ to its offspring household $i \in I_{t+1}$. As a consequence, $b^i_{t+1} = 0$.

Propositions 1 and 2 allow us to represent the dynamics of bequest transfers as follows

$$b_{t+1}^i = \begin{cases} 0 & \text{if} \quad b_t^i \in [0, \underline{b}) \\ \gamma\phi(P_t)\left[w_t h(b_t^i) - \frac{\theta w_t(1-\gamma)}{\gamma}\right] & \text{if} \quad b_t^i \in [\underline{b}, e_t^*) \\ \gamma\phi(P_t)\left[w_t h(e_t^*) + (b_t^i - e_t^*)r_t - \frac{\theta w_t(1-\gamma)}{\gamma}\right] & \text{if} \quad b_t^i \in [e_t^*, e_t^* + \hat{k}) \\ \gamma\phi(P_t)\left[w_t h(e_t^*) + (b_t^i - e_t^*)r_t + \rho_t^i \Pi_t - \frac{\theta w_t(1-\gamma)}{\gamma}\right] & \text{if} \quad b_t^i \geq e_t^* + \hat{k} \end{cases} \tag{16}$$

The dynamics of intergenerational bequest transfer between generations t and $t+1$ in family dynasty i can be classified into four parts, as presented in equation (16). How the bequest transfer b_{t+1}^i is determined depends on which segment that b_t^i belongs to. If $b_t^i \in [0, \underline{b})$ then, as stated in Proposition 2, $b_{t+1}^i = 0$. If $b_t^i \in [\underline{b}, e_t^*)$, then household $i \in I_t$ allocates all the bequest b_t^i—received from its parent household—in education investment to generate wealth $w_t h(b_t^i)$ by its labour income. Thereby, it transfers a bequest $b_{t+1}^i = \gamma\phi(P_t)\left[w_t h(b_t^i) - \frac{\theta w_t(1-\gamma)}{\gamma}\right]$ to its offspring household under the effect of climate damage $\phi(P_t)$. If $b_t^i \in [e_t^*, e_t^* + \hat{k})$, then household $i \in I_t$ chooses the education investment level e_t^* at which the marginal effect of education investment on improving that household's wealth equals the marginal return of capital savings, i.e. $w_t h'(e_t^*) = r_t$. The household $i \in I_t$ then allocates the remaining $b_t^i - e_t^* \in [0, \hat{k})$ as capital savings to seek capital income. In this case, the wealth of household $i \in I_t$ in period t is $w_t h(e_t^*) + (b_t^i - e_t^*)r_t$, and it transfers an amount $b_{t+1}^i = \gamma\phi(P_t)\left[w_t h(e_t^*) + (b_t^i - e_t^*)r_t - \frac{\theta w_t(1-\gamma)}{\gamma}\right]$ to its offspring household. Finally, if $b_t^i \geq e_t^* + \hat{k}$, then, differing from the last case, this bequest transfer is sufficiently high to allow household $i \in I_t$ to own monopolistic producing firms. Therefore, the current wealth of household $i \in I_t$ comes from not only the labour and capital incomes, but also the monopolistic profits. Hence, its wealth is $w_t h(e_t^*) + (b_t^i - e_t^*)r_t + \rho_t^i \Pi_t$ and the transfer to its offspring household is $b_{t+1}^i = \gamma\phi(P_t)\left[w_t h(e_t^*) + (b_t^i - e_t^*)r_t + \rho_t^i \Pi_t - \frac{\theta w_t(1-\gamma)}{\gamma}\right]$.

It would be interesting to study the evolution of bequest transfers in the space $(b_t^i, b_{t+1}^i) \subset \Re_+^2$. We refer to this *evolution* in this space as *conditional evolution* in the sense that we have to fix other variables that have their own evolution and interactions with the evolution of the bequest transfers. This simplification would be helpful for focusing on the mechanisms leading to wealth inequality. We would note, however, that this approach is valid when we focus on any two successive periods of time, t and $t+1$. Therefore, the whole evolution of bequest transfers, indeed, is the sequence of replication of what we try to do in this section. We study this *conditional evolution* under the following conditions, which guarantee the existence of multiple *conditional steady states*.

(i) $\phi(P_t) w_t \left[\gamma h(e_t^*) - \theta(1-\gamma)\right] \geq e_t^*$;

(ii) $\gamma\phi(P_t) r_t < 1$;

(iii) $\gamma\phi(P_t)\left[w_t h(e_t^*) + \hat{k} r_t - \frac{\theta w_t(1-\gamma)}{\gamma}\right] \leq e_t^* + \hat{k}$; and

(iv) $\gamma\phi(P_t)\left[w_t h(e_t^*) + \hat{k} r_t + \rho_t^i \Pi_t - \frac{\theta w_t(1-\gamma)}{\gamma}\right] > e_t^* + \hat{k} \quad \forall i \in I_t^\lambda \subset I_t$.

Under these conditions, the evolution of intergenerational bequest transfers are depicted in figures 3 and 4. These figures suggest that the model can generate multiple clusters of convergence in intergenerational bequest transfers across households. The direction of conditional convergence of bequest b_{t+1}^i, in which the household $i \in I_t$ transfers to its offspring household $i \in I_{t+1}$, depends on the bequest b_t^i that the household receives from its parent household $i \in I_{t-1}$.[7]

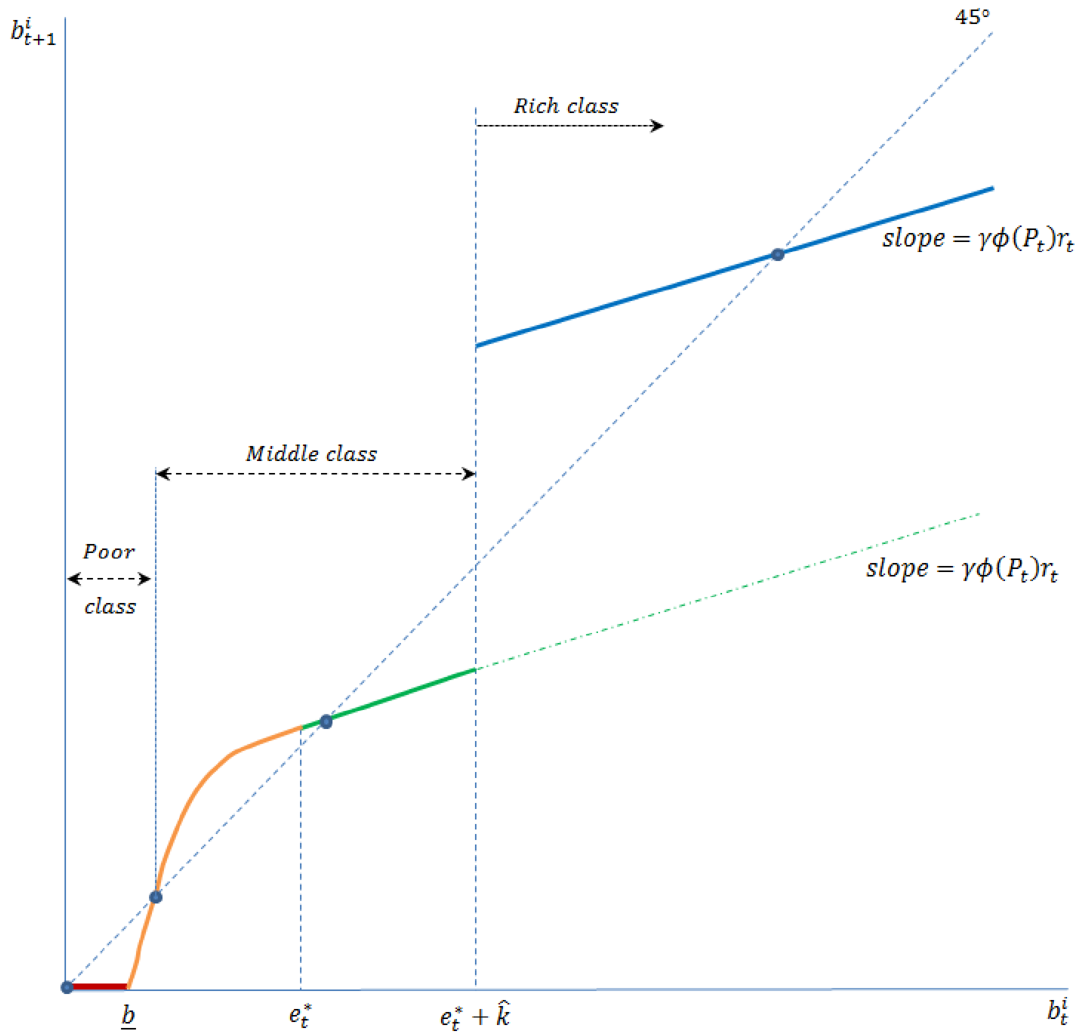

Figure 3: Conditional evolution of bequest transfers (monopoly profit shares are even across monopolists)

4 Effects of climate policy

In this section, we study the effects of balanced budget climate policy on the structure of production factors (or more precisely, the allocation of resources constituting aggregate production factors), on bequest transfers and wealth inequality across households.

Definition 2. *We define a set of balanced budget climate policies in any period t as follows:*

[7] In this model, we assume that households/individuals are identical in innate talent and preferences, and that there are no stochastic factors affecting their wealth or income. Hence, the inequality in wealth across households originates from the heterogeneity in intergenerational bequest transfers—which leads to heterogeneity in education investment and capital savings. Therefore, the dynamics of bequest transfers indicate the tendency of wealth and income inequality.

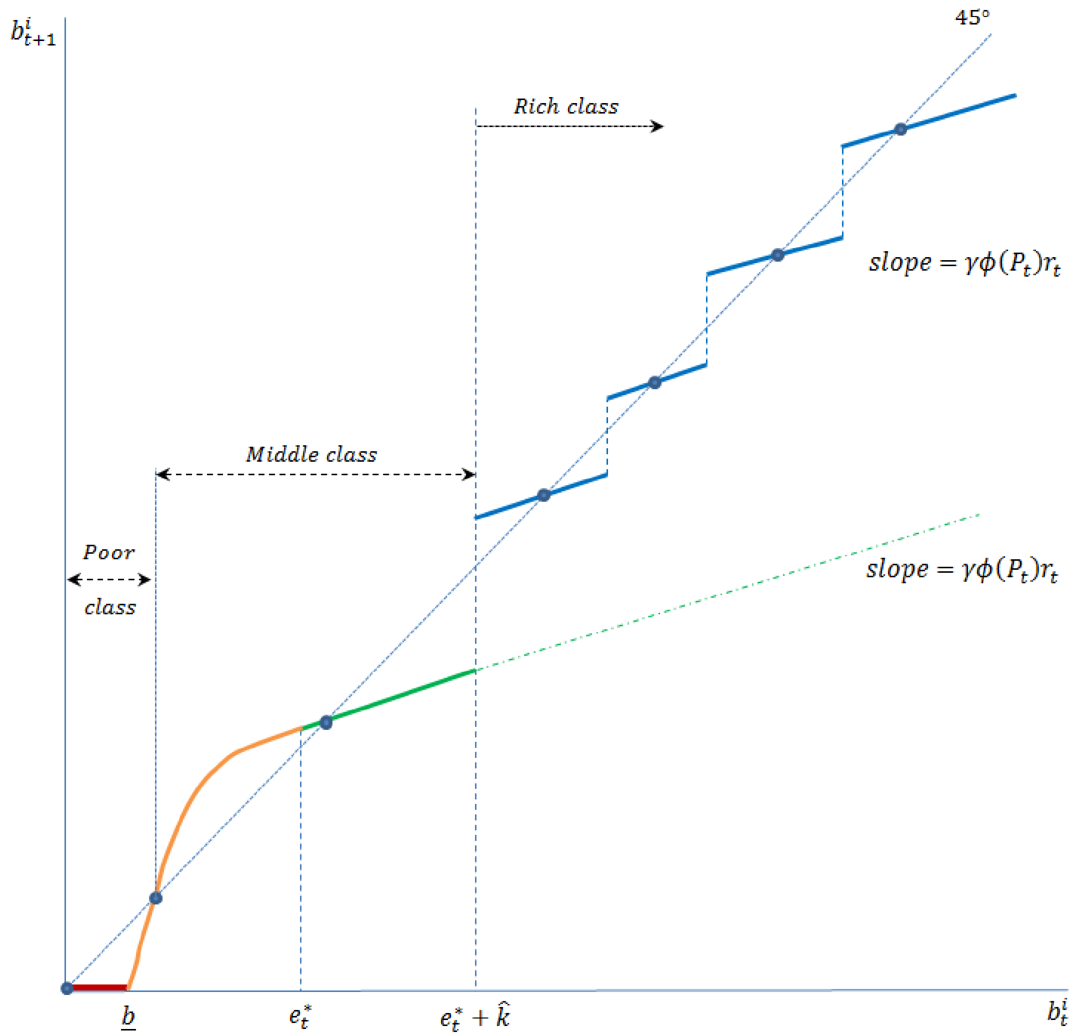

Figure 4: Conditional evolution of bequest transfers (monopoly profit shares are uneven across monopolists)

$$\mathcal{C}_t \equiv \left\{ (\tau_{ct}, \tau_{dt}) \in \Re_- \times (0,1) \text{ such that } \sum_{v \in \{c,d\}} \tau_{vt}(1-\tau_{vt})^{\frac{\alpha}{1-\alpha}} A_v = 0 \right\}$$

Proposition 3. *(**On resource allocation**) In any period t, the allocation of (bequest) resources constituting aggregate physical capital K_t and human capital $\{e^i_t, h^i_t\}_{i \in I_t}$ is independent from climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$.*

Proof. See Appendix A3. □

Proposition 3 suggests that the effects of climate policy on individual i's disposable wealth occurs through its effects on the return on human capital, the rental rate of physical capital, and the stock of greenhouse gases.

Proposition 4. *(**On macroeconomic variables**) In any period t, climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ reduces the rental rate of physical capital, the return on human capital, and the aggregate monopoly* *profit compared with the case of no climate policy* $(\tau_{ct}, \tau_{dt}) = (0,0)$. *In addition, it holds*

$$\frac{r_t(\tau_{ct}, \tau_{dt})}{r_t(0,0)} = \frac{w_t(\tau_{ct}, \tau_{dt})}{w_t(0,0)} = \frac{\Pi_t(\tau_{ct}, \tau_{dt})}{\Pi_t(0,0)} = \frac{Y_t(\tau_{ct}, \tau_{dt})}{Y_t(0,0)} = \left[\frac{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}{\sum\limits_{v\in\{c,d\}} A_v}\right]^{1-\alpha}$$

Proof. See Appendix A4.

□

The economic intuition supporting proposition 4 is as follows: the climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ distorts the optimal allocation rule of capital in producing intermediate inputs, and therefore reduces the aggregate final output. The aggregate final output, indeed, is distributed for aggregate labour income, aggregate physical capital income, and aggregate monopoly profit. The rule of distributing the aggregate final output for labour income, physical capital income and monopoly profit is indeed independent from climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$. This is because, as proven in proposition 3, K_t and H_t are independent from $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$. Hence, the aggregate labour income (and return on human capital), aggregate physical capital income (and rental rate of physical capital), and aggregate monopolistic profit are distorted in exactly the same way by the distortion in the optimal allocation rule of capital in producing intermediate inputs.

From the dynamics equation (16) of bequest transfer, we find that when $b_t^i \in [0, \underline{b}]$, then $b_{t+1}^i = 0$ regardless of the climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$. We now consider the effects of climate policy on the bequest transfer b_{t+1}^i of household $i \in I_t$ characterized by $b_t^i > \underline{b}$. Along with the equality stated in proposition 4, we can determine

$$\frac{b_{t+1}^i(\tau_{ct}, \tau_{dt})}{b_{t+1}^i(0,0)} = \left[\frac{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}{\sum\limits_{v\in\{c,d\}} A_v}\right]^{1-\alpha} \frac{\phi(P_t(\tau_{ct}, \tau_{dt}))}{\phi(P_t(0,0))} \quad \text{if} \quad b_t^i > \underline{b}$$

For exposition purposes without crucially lessening the power of the analyses, we follow Golosov et al. (2014) to specify the functional form of the climate damage as follows[8]

$$\phi(P) = \exp(-P) \quad \text{for} \quad P \geq 0$$

For this functional form of climate damage, and from the dynamics of greenhouse gas stock characterized in (9), we have that when $b_t^i > \underline{b}$, the following holds

$$\frac{b_{t+1}^i(\tau_{ct}, \tau_{dt})}{b_{t+1}^i(0,0)} = \left[\frac{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}{\sum\limits_{v\in\{c,d\}} A_v}\right]^{1-\alpha} \exp\left\{\xi\left(\frac{A_d}{\sum\limits_{v\in\{c,d\}} A_v} - \frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}} A_d}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}\right) K_t\right\} \tag{17}$$

[8] Such a damage function can also be found in Dao et al. (2017).

From the last equation, we find that the overall effect of climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ on the direction of change in bequest transfer $b^i_{t+1}(\tau_{ct}, \tau_{dt})/b^i_{t+1}(0,0)$ when $b^i_t > \underline{b}$ depends on the climate policy itself in the relation to the size of aggregate physical capital in the economy.

Proposition 5. *(**On bequest transfers and inequality**) In the economy set up above,*

(i) for any household $i \in I_t$ with $b^i_t > \underline{b}$, it holds that $b^i_{t+1}(\tau_{ct}, \tau_{dt}) > (=)(<)\, b^i_{t+1}(0,0)$ with climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ if, and only if,

$$K_t > (=)(<) \frac{1-\alpha}{\xi} \frac{\left[\tau_{ct} - \tau_{dt}\left(\frac{1-\tau_{dt}}{1-\tau_{ct}}\right)^{\frac{\alpha}{1-\alpha}}\right](\tau_{dt} - \tau_{ct})}{\left[1 - \left(\frac{1-\tau_{dt}}{1-\tau_{ct}}\right)^{\frac{1}{1-\alpha}}\right](1-\tau_{ct})\tau_{ct}\tau_{dt}} \ln\left(\frac{\tau_{ct} - \tau_{dt}\left(\frac{1-\tau_{dt}}{1-\tau_{ct}}\right)^{\frac{\alpha}{1-\alpha}}}{(\tau_{ct} - \tau_{dt})(1-\tau_{dt})^{\frac{\alpha}{1-\alpha}}}\right) = \hat{K}_t(\tau_{ct}, \tau_{dt})$$

Moreover, $\hat{K}_t(\tau_{ct}, \tau_{dt})$ is bounded for all $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$.

(ii) Climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ alters the disposable wealth $\psi(W^i_t, P_t)$ of all households $i \in I_t$ by the same multiplier, and

$$\psi(W^i_t(\tau_{ct}, \tau_{dt}), P_t(\tau_{ct}, \tau_{dt})) > (=)(<)\, \psi(W^i_t(0,0), P_t(0,0)) \quad \Longleftrightarrow \quad K_t > (=)(<)\hat{K}_t(\tau_{ct}, \tau_{dt})$$

Proof. See **Appendix A5**. □

Proposition 5 tells us that, for climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$, which is carried out in period t, if the stock of physical capital is rather high, in particular $K_t > \hat{K}_t(\tau_{ct}, \tau_{dt})$, then that climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ may increase the inequality in bequest transfers across households. This is because climate policy improves environmental quality, and thereby enhances disposable wealth among households. Under the climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ where $K_t > \hat{K}_t(\tau_{ct}, \tau_{dt})$, while the choice of bequest transfers to the offspring of the poor households is unchanged and set at 0 when the bequest the households receive from their parents does not exceed $\underline{b}$, the bequest transfers to the offspring of the rich and middle-class households increase.

This proposition implies that for a very unequal economy in bequest transfers, the climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ may enhance inequality. For a given aggregate bequest transfer, the aggregate physical capital tends to be higher in an unequal economy than in a more equal one because when the bequest is owned too biasedly by the rich, then a higher fraction of bequest will be transformed into physical capital, making $K_t > \hat{K}_t(\tau_{ct}, \tau_{dt})$, since the education investment is bounded. Hence, in this case, the climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ enhances inequality. However, when the bequest spreads more equally to the poor and middle classes, then they are transformed more into education investment, thereby reducing the aggregate physical capital in the economy. Therefore, it is more likely for climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ to satisfy $K_t < \hat{K}_t(\tau_{ct}, \tau_{dt})$, which may reduce the inequality in bequest transfers $\{b^i_{t+1}\}_{i \in I_t}$ for the next generation. To assess more precisely the effects of

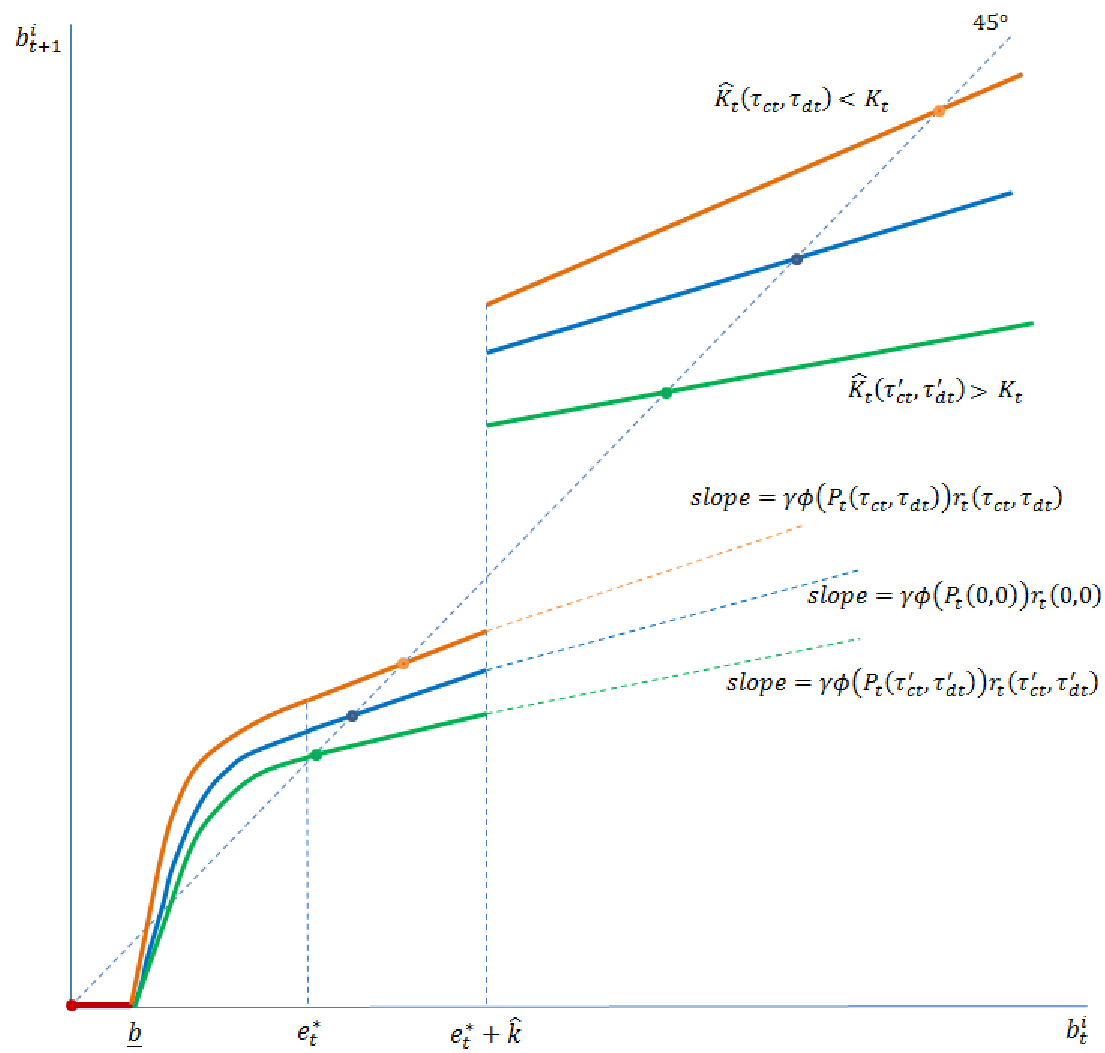

Figure 5: Effects of climate policy on bequest transfer dynamics.

climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ on the inequality in bequest transfers for generation $t+1$, we require information about the distribution of $\{b^i_t\}_{i \in I_t}$.

5 Concluding remarks

We set up an overlapping generations economy with credit market imperfections and a climate damage on the wealth of heterogeneous households to investigate theoretically the link between two emerging issues in the twenty-first century: climate change and wealth inequality. For each balanced budget climate policy, we identify a corresponding threshold of aggregate physical capital above which such a climate policy may enlarge the wealth gap between the rich and poor. This theoretical result derived from the model may be helpful for investigating the effects of climate policy on wealth inequality, as well as for designing a composite policy to improve equity and protect the environment.

The model suggests that, for a given stock of aggregate intergenerational bequest transfers, physical capital stock tends to be higher in a more unequal economy. This is because of the asymmetry in allocating bequest transfers to human capital investment and physical capital accumulation among heterogeneous households in the presence of credit market imperfections. Following the statement in Proposition 5, this makes climate policy more likely to benefit the rich in a biased manner. In addition, while climate policy may not help the poor escape the poverty trap, in the long run, the rich may become richer because of the positive effects of improving aggregate human capital on monopolistic profits. This would contribute to widening the wealth gap between the rich and poor. Hence, from the perspectives of equity, we should think about taxing monopolistic profits to subsidize the clean production sector and/or education for the poor to reduce inequality in wealth distribution.

For simplification, this model ignores the role of the research and development sector, in which the tax revenue from the dirty production sector is used to subsidy clean technology innovations (see Acemoglu et al. 2012, 2016). Introducing innovation sectors would be challenging, interesting, and promising using the present framework. The theoretical results of our model hinge specifically on a plausible assumption of credit market imperfections. Relaxing this assumption and/or introducing policies that eliminate the imperfections of credit markets would be important and interesting research. These ideas, among others, are left for further research in the future.

Appendix

A0. Monopolistic profits and returns on physical and human capital

Substituting (3) and (4) into (5), we have

$$\pi_{vt} = (1-\tau_{vt})\alpha H_t^{1-\alpha} A_v^{1-\alpha} K_{vt}^{\alpha} - r_t K_{vt} \quad \text{with} \quad K_{vt} \in \arg\max_{K_{vt}} (1-\tau_{vt})\alpha H_t^{1-\alpha} A_v^{1-\alpha} K_{vt}^{\alpha} - r_t K_{vt}$$

Hence,

$$K_{vt} = \left[\frac{\alpha^2(1-\tau_{vt})}{r_t}\right]^{\frac{1}{1-\alpha}} H_t A_v \tag{18}$$

From (18), we can compute the aggregate physical capital in period t,

$$K_t = \sum_{v\in\{c,d\}} K_{vt} = \left(\frac{\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} H_t \sum_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v \tag{19}$$

From (19), the rental rate of capital is determined by

$$r_t = \alpha^2 \left[\frac{H_t}{K_t} \sum_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v\right]^{1-\alpha} \tag{20}$$

Substituting (20) into (18) we determine the allocation rule of physical capital across intermediate sectors vj in period t

$$K_{vt} = \frac{(1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}{\sum\limits_{v' \in \{c,d\}} (1-\tau_{v't})^{\frac{1}{1-\alpha}} A_{v'}} K_t \tag{21}$$

The monopolist profit π_{vt} is determined by substituting (21) and (20) into (5), that is

$$\pi_{vt} = \alpha(1-\alpha) H_t^{1-\alpha} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v \left(\frac{K_t}{\sum\limits_{v' \in \{c,d\}} (1-\tau_{v't})^{\frac{1}{1-\alpha}} A_{v'}} \right)^{\alpha}$$

In addition, the aggregate monopolistic profit in period t is

$$\Pi_t = \sum_{v \in \{c,d\}} \pi_{vt} = \alpha(1-\alpha) \left[H_t \sum_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v \right]^{1-\alpha} K_t^{\alpha} \tag{22}$$

By substituting (21) into (2), given that $X_{vt} = K_{vt}$ for $v \in \{c,d\}$, we can determine the return on human capital,

$$w_t = (1-\alpha) \frac{\sum\limits_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{\alpha}{1-\alpha}} A_v}{\left[\sum\limits_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v \right]^{\alpha}} \left(\frac{K_t}{H_t} \right)^{\alpha} \tag{23}$$

The government balanced budget constraint requires

$$\sum_{v \in \{c,d\}} \tau_{vt} p_{vt} X_{vt} = 0 \quad \Longleftrightarrow \quad \sum_{v \in \{c,d\}} \tau_{vt} (1-\tau_{vt})^{\frac{\alpha}{1-\alpha}} A_v = 0 \tag{24}$$

Lemma 1. *Under the government balanced budget constraint of the climate policy* $(\tau_{ct}, \tau_{dt}) \in \Re_- \times [0,1)$*, the following relation holds*

$$\sum_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v = \sum_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{\alpha}{1-\alpha}} A_v$$

Proof.

Indeed, under government balanced budget condition (24) we have

$$\sum_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v = \sum_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v + \sum_{v \in \{c,d\}} \tau_{vt} (1-\tau_{vt})^{\frac{\alpha}{1-\alpha}} A_v = \sum_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{\alpha}{1-\alpha}} A_v$$

$\square$

Under the government balanced budget constraint (24), and from lemma 1, the return on human capital (23) becomes

$$w_t = (1-\alpha)\left[\sum_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v\right]^{1-\alpha}\left(\frac{K_t}{H_t}\right)^{\alpha} \tag{25}$$

A1. Proof of Proposition 1

(i) Indeed, the existence of optimal solution e_t^i is guaranteed by the compactness of the domain set, $0 \le e_t^i \le b_t^i \ \forall i \in I_t$, and continuity in e_t^i of the objective function. When $b_t^i = 0$, it is trivial that $e_t^i = 0$. When $b_t^i \in (0, e_t^*]$, we suppose a contradiction that $e_t^i < b_t^i$ and prove that there exists $\varepsilon \in (0, b_t^i - e_t^i)$ such that

$$w_t h(e_t^i + \varepsilon) + (b_t^i - e_t^i - \varepsilon)r_t > w_t h(e_t^i) + (b_t^i - e_t^i)r_t$$

which is equivalent to

$$\frac{h(e_t^i+\varepsilon) - h(e_t^i)}{\varepsilon} > \frac{r_t}{w_t} \iff h(e_t^i+\varepsilon) - h(e_t^i) > \varepsilon h'(e_t^*)$$

since $h'(e_t^*) = r_t/w_t$. The last inequality trivially holds because: $h(e)$ is an increasing and concave function of e, and $e_t^i + \varepsilon < e_t^*$. Hence, in this case $e_t^i < b_t^i$ is not the optimal education investment. Therefore, $e_t^i = b_t^i$ when $b_t^i \in [0, e_t^*]$.

When $b_t^i > e_t^*$, then the optimal education investment is set at the level that equalizes the marginal return of education investment and marginal capital income, i.e. $w_t h'(e_t^i) = r_t$, which gives us $e_t^i = e_t^*$.

(ii) We have

$$h'(e_t) = \frac{\alpha^2}{1-\alpha}\frac{h(e_t)}{b_t - e_t} \iff G(b_t, e_t) \equiv h'(e_t) - \frac{\alpha^2}{1-\alpha}\frac{h(e_t)}{b_t - e_t} = 0 \tag{26}$$

Under assumption 1,

$$G_e(b_t, e_t) = h''(e_t) - \frac{\alpha^2}{1-\alpha}\frac{h'(e_t)(b_t - e_t) + h(e_t)}{(b_t - e_t)^2} < 0$$

i.e. $G(b_t, e_t)$ is monotonically decreasing in $e_t \in (0, b_t)$. In addition, $\lim_{e_t \to 0^+} G(b_t, e_t) = +\infty$ and $\lim_{e_t \to +\infty} G(b_t, e_t) = -\infty$. Thus, there exists a unique $\underline{e}_t \in (0, b_t)$ satisfying (26).

By applying the implicit function theorem for function $G(b_t, \underline{e}_t) = 0$, we have $\underline{e}_t$ is a function of b_t, i.e. $\underline{e}_t = \underline{e}(b_t)$, in which

$$\underline{e}'(b_t) = \frac{\alpha^2 h(\underline{e}_t)}{\alpha^2[h'(\underline{e}_t)(b_t - \underline{e}_t) + h(\underline{e}_t)] - (1-\alpha)(b_t - \underline{e}_t)^2 h''(\underline{e}_t)} > 0$$

Now we prove that $e_t^* \ge \underline{e}_t$. In effect, From (20) and (25) which determine r_t and w_t respectively, we have

$$h'(e_t^*) = \frac{r_t}{w_t} = \frac{\alpha^2}{1-\alpha}\frac{H_t}{K_t} = \frac{\alpha^2}{1-\alpha}\frac{\int_{I_t} h(e_t^i)di}{\int_{I_t}(b_t^i - e_t^i)di}$$

We suppose a contradiction that $e_t^* < \underline{e}_t$, hence

$$h(\underline{e}_t) > \int_{I_t} h(e_t^i)di \quad \text{and} \quad \underline{e}_t > \int_{I_t} e_t^i di \quad \text{since} \quad e_t^i \leq e_t^* \; \forall i \in I_t.$$

Note that, since we normalize the size of the population by 1, $\underline{e}_t$ and $h(\underline{e}_t)$ are respectively aggregate education investment and human capital when each adult individual $i \in I_t$ has an education investment $\underline{e}_t$. Thus, it would hold that

$$h'(e_t^*) = \frac{\alpha^2}{1-\alpha}\frac{\int_{I_t} h(e_t^i)di}{\int_{I_t}(b_t^i - e_t^i)di} < \frac{\alpha^2}{1-\alpha}\frac{h(\underline{e}_t)}{b_t - \underline{e}_t} = h'(\underline{e}_t) \quad \Longleftrightarrow \quad e_t^* > \underline{e}_t$$

which contradicts the assumption $e_t^* < \underline{e}_t$. Therefore, $e_t^* \geq \underline{e}_t$.

A2. Proof of Proposition 2

In effect, when $b_t^i \in (0, e_t^*)$, under assumptions 2 and 3, the disposable wealth of individual $i \in I_t$ is $\phi(P_t)w_t h(b_t^i)$. From (15), we have

$$b_{t+1}^i > (=)\, 0 \quad \Longleftrightarrow \quad \phi(P_t)w_t h(b_t^i) > (\leq)\, \frac{\theta w_t \phi(P_t)(1-\gamma)}{\gamma} \quad \Longleftrightarrow \quad h(b_t^i) > (\leq)\, \frac{\theta(1-\gamma)}{\gamma}$$

Under assumption 1, the last inequalities are equivalent to

$$b_t^i > (\leq)\, h^{-1}\left(\frac{\theta(1-\gamma)}{\gamma}\right) \equiv \underline{b}$$

Under assumption 4, we have

$$h(\underline{b}) < h(\underline{e}(\lambda\hat{k})) \quad \Longrightarrow \quad h(\underline{b}) < h(e_t^*) \quad \text{since} \quad e_t^* \geq \underline{e}(\lambda\hat{k})$$

Therefore, $\underline{b} \in (0, e_t^*)$.

A3. Proof of Proposition 3

We have in any period t, the aggregate bequest $b_t = \int_{I_t} b_t^i di$ is given. From equations (20) and (25) determining, respectively, the rental rate of physical capital and the return on human capital under climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$, we have

$$\frac{r_t(\tau_{ct}, \tau_{dt})}{w_t(\tau_{ct}, \tau_{dt})} = \frac{\alpha^2}{1-\alpha}\frac{H_t(\tau_{ct}, \tau_{dt})}{K_t(\tau_{ct}, \tau_{dt})}$$

where $r_t(\tau_{ct}, \tau_{dt})$ is the rental rate of physical capital in period t under climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$; and the analogous logic applied for other variables in period t. We suppose a negation that

$$\frac{H_t(\tau_{ct}, \tau_{dt})}{K_t(\tau_{ct}, \tau_{dt})} < \frac{H_t(0,0)}{K_t(0,0)} \quad \text{for some } (\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$$

then

$$e_t^*(\tau_{ct}, \tau_{dt}) > e_t^*(0,0)$$

Note that we prove in proposition 1 that $e_t^i(\tau_{ct}, \tau_{dt}) = \min\{b_t^i, e_t^*(\tau_{ct}, \tau_{dt})\}$, hence, the last inequality implies that

$$\begin{cases} e_t^i(\tau_{ct}, \tau_{dt}) = e_t^i(0,0) & \text{if} \quad b_t^i \in [0, e_t^*(0,0)] \\ \\ e_t^i(\tau_{ct}, \tau_{dt}) > e_t^i(0,0) & \text{if} \quad b_t^i > e_t^*(0,0) \end{cases}$$

$$\Longrightarrow \begin{cases} H_t(\tau_{ct}, \tau_{dt}) = \int_{I_t} h(e_t^i(\tau_{ct}, \tau_{dt}))di > \int_{I_t} h(e_t^i(0,0))di = H_t(0,0) \\ \\ K_t(\tau_{ct}, \tau_{dt}) = b_t - \int_{I_t} e_t^i(\tau_{ct}, \tau_{dt})di < b_t - \int_{I_t} e_t^i(0,0)di = K_t(0,0) \end{cases}$$

which leads to a contradiction that $H_t(\tau_{ct}, \tau_{dt})/K_t(\tau_{ct}, \tau_{dt}) > H_t(0,0)/K_t(0,0)$. Analogous logic can be applied to the case of supposing a negation that $H_t(\tau_{ct}, \tau_{dt})/K_t(\tau_{ct}, \tau_{dt}) > H_t(0,0)/K_t(0,0)$. Therefore, it must hold that

$$\frac{H_t(\tau_{ct}, \tau_{dt})}{K_t(\tau_{ct}, \tau_{dt})} = \frac{H_t(0,0)}{K_t(0,0)} \quad \forall (\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$$

which, along with proposition 1, trivially gives us for any climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ that

$$K_t(\tau_{ct}, \tau_{dt}) = K_t(0,0), \quad e_t^i(\tau_{ct}, \tau_{dt}) = e_t^i(0,0) \quad \text{and} \quad h_t^i(\tau_{ct}, \tau_{dt}) = h_t^i(0,0) \quad \forall i \in I_t.$$

A4. Proof of Proposition 4

Indeed, under climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ the allocation of physical capital $K_{vt}(\tau_{ct}, \tau_{dt})$, as shown in (21), is

$$K_{vt}(\tau_{ct}, \tau_{dt}) = \frac{(1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}{\sum\limits_{v' \in \{c,d\}} (1-\tau_{v't})^{\frac{1}{1-\alpha}} A_{v'}} K_t \neq \frac{A_v}{\sum\limits_{v' \in \{c,d\}} A_{v'}} K_t = K_{vt}(0,0); \quad v \in \{c,d\}$$

for given aggregate physical capital K_t.

By solving the following optimization problem, which maximizes the final output in any period t,

$$\max_{\{K_{vt}\}_{vj\in\{c,d\}\times J_t}} H_t^{1-\alpha} \sum_{v\in\{c,d\}} A_v^{1-\alpha} K_{vt}^{\alpha} \quad \text{subject to} \quad \sum_{v\in\{c,d\}} K_{vt} = K_t$$

given K_t and H_t,[9] we find that the allocation rule $K_{vt}^{*} = K_{vt}(0,0)$ is the unique optimal allocation of capital.

By substituting the physical capital allocation rules $K_{vt}(\tau_{ct}, \tau_{dt})$ under $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ and $K_{vt}(0,0)$ into the final good production function, along with the government balanced budget constraint, as mentioned in lemma 1, we have

$$Y_t(\tau_{ct}, \tau_{dt}) = \left[H_t \sum_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v \right]^{1-\alpha} K_t^{\alpha} < \left[H_t \sum_{v\in\{c,d\}} A_v \right]^{1-\alpha} K_t^{\alpha} = Y_t(0,0)$$

Hence, with $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$, it holds that

$$\sum_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v < \sum_{v\in\{c,d\}} A_v$$

Now, it is quite straightforward from equations (20), (22) and (25), which determine, respectively, the rental rate of physical capital, the aggregate monopolistic profit and the return on human capital, and the final good production, that, with balanced budget climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$, they hold

$$\frac{r_t(\tau_{ct}, \tau_{dt})}{r_t(0,0)} = \frac{w_t(\tau_{ct}, \tau_{dt})}{w_t(0,0)} = \frac{\Pi_t(\tau_{ct}, \tau_{dt})}{\Pi_t(0,0)} = \frac{Y_t(\tau_{ct}, \tau_{dt})}{Y_t(0,0)} = \left[\frac{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}{\sum\limits_{v\in\{c,d\}} A_v} \right]^{1-\alpha} < 1.$$

A5. Proof of Proposition 5

(i) The proof for this statement is fairly straightforward from equation (17) when we evaluate the bequest ratio $b_{t+1}^{i}(\tau_{ct}, \tau_{dt})/b_{t+1}^{i}(0,0)$ in comparison with 1. Indeed,

$$\frac{b_{t+1}^{i}(\tau_{ct}, \tau_{dt})}{b_{t+1}^{i}(0,0)} > (=)(<)\, 1$$

if, and only if

$$(1-\alpha)\ln\left(\frac{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}{\sum\limits_{v\in\{c,d\}} A_v} \right) + \xi \left(\frac{A_d}{\sum\limits_{v\in\{c,d\}} A_v} - \frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}} A_d}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v} \right) K_t > (=)(<)\, 0$$

[9] Note that in the final good production function, we replace X_{vt} by K_{vt} because of the production function $X_{vt} = K_{vt}$ in the intermediate sector $v \in \{c,d\}$.

which is equivalent to

$$K_t > (=)(<) \frac{1-\alpha}{\xi} \left(\frac{A_d}{\sum\limits_{v\in\{c,d\}} A_v} - \frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}} A_d}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v} \right)^{-1} \ln \left(\frac{\sum\limits_{v\in\{c,d\}} A_v}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v} \right) > 0$$

Using the balanced government budget condition $\sum\limits_{v\in\{c,d\}} \tau_{vt}(1-\tau_{vt})^{\frac{\alpha}{1-\alpha}} A_v = 0$ as mentioned in (24) and substituting it into the right-hand side of the last inequality, we obtain $\hat{K}_t(\tau_{ct}, \tau_{dt})$.

We have the balanced budget condition

$$\tau_{ct}(1-\tau_{ct})^{\frac{\alpha}{1-\alpha}} A_c + \tau_{dt}(1-\tau_{dt})^{\frac{\alpha}{1-\alpha}} A_d = 0$$

By applying the implicit function theorem for the last equation with respect to τ_{ct} and τ_{dt}, we have τ_{ct} as function of τ_{dt}, in which $\lim\limits_{\tau_{dt}\to 0^+} \tau_{ct} = \lim\limits_{\tau_{dt}\to 1^-} \tau_{ct} = 0$, and

$$\lim_{\tau_{dt}\to 0^+} \frac{\partial \tau_{ct}}{\partial \tau_{dt}} = -\frac{A_d}{A_c}$$

We have

$$\hat{K}_t(\tau_{ct}, \tau_{dt}) = \frac{1-\alpha}{\xi} \left(\frac{A_d}{\sum\limits_{v\in\{c,d\}} A_v} - \frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}} A_d}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v} \right)^{-1} \ln \left(\frac{\sum\limits_{v\in\{c,d\}} A_v}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v} \right)$$

Since $\hat{K}_t(\tau_{ct}, \tau_{dt}) > 0$ for all $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$, then we only need to prove that $\hat{K}_t(\tau_{ct}, \tau_{dt})$ is bounded from above. Suppose a negation that there existed $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ such that $\hat{K}_t(\tau_{ct}, \tau_{dt})$ could approach $+\infty$. We have the term $\ln \left(\sum\limits_{v\in\{c,d\}} A_v / \sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v \right) > 0$, which is always bounded from above because $0 < \sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v < \sum\limits_{v\in\{c,d\}} A_v < +\infty$. Hence, for $\hat{K}_t(\tau_{ct}, \tau_{dt})$ to approach $+\infty$, the only possibility is that $\frac{A_d}{\sum_{v\in\{c,d\}} A_v} - \frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}} A_d}{\sum_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}$ approaches 0 from the right. This possibility occurs only if τ_{dt} approaches 0.[10] However,

$$\tau_{dt} \to 0 \quad \Longrightarrow \quad \ln \left(\frac{\sum\limits_{v\in\{c,d\}} A_v}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v} \right) \to 0$$

[10] It is straightforward to show that $\frac{A_{dt}}{\sum_{v\in\{c,d\}} A_{vt}} - \frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}} A_{dt}}{\sum_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_{vt}} > 0 \ \forall (\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$. Because it is equivalent to $\frac{1-\tau_{ct}}{1-\tau_{dt}} > 1$ $\forall (\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$, which trivially holds.

Therefore, by applying l'Hopital rule, and noting that $\lim\limits_{\tau_{dt}\to 0^+} \frac{\partial \tau_{ct}}{\partial \tau_{dt}} = -\frac{A_d}{A_c}$, we have

$$\lim_{\tau_{dt}\to 0^+} \hat{K}_t(\tau_{ct}, \tau_{dt}) = \frac{1-\alpha}{\xi} \lim_{\tau_{dt}\to 0^+} \frac{\left[A_d + \frac{\partial \tau_{ct}}{\partial \tau_{dt}} A_c\right] \sum\limits_{v\in\{c,d\}} A_v}{A_d \sum\limits_{v\in\{c,d\}} A_v - A_d \left[A_d + \frac{\partial \tau_{ct}}{\partial \tau_{dt}} A_c\right]} = 0$$

That is to say, $\hat{K}_t(\tau_{ct}, \tau_{dt})$ is bounded.

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6 References

Acemoglu, D. et al. (2012). The Environment and Directed Technical Change. *American Economic Review* 102(1), pp. 131–166.

Acemoglu, D. et al. (2016). Transition to Clean Technology. *Journal of Political Economy* 124(1), 52-104.

Aghion, Ph. and P. Howitt (2009). *The Economics of Growth*. MIT Press.

Blonz, J. et al. (2011). How Do the Costs of Climate Policy Affect Households? The Distribution of Impacts by Age, Income, and Region. *RFF Discussion Paper* 10 - 55.

Bourguignon, F. (1981). Pareto Superiority of Unegalitarian Equilibria in Stiglitz' Model of Wealth Distribution with Convex Saving Function. *Econometrica* 49 (6), 1469 - 1475.

Chancel, L. and Th. Piketty (2015). Carbon and inequality: from Kyoto to Paris. *Paris School of Economics Discussion Paper* 11.2015.

Dao, N.T. (2016). From agriculture to manufacturing: How does geography matter? *Cliometrica* 10(3), 277 - 309.

Dao, N.T. et al. (2017). Self-Enforcing Intergenerational Social Contracts for Pareto Improving Pollution Mitigation. *Environmental and Resource Economics* 68, 129–173.

Dao, N.T. and O. Edenhofer (2018). On the fiscal strategies of escaping poverty-environment traps towards sustainable growth. Journal of Macroeconomics 55, 253 - 273.

Dennig, F. et al. (2015). Inequality, climate impacts on the future poor, and carbon prices. *PNAS* 112 (52), 15827 - 15832.

Galor, O. and O. Moav (2004). From Physical to Human Capital Accumulation: Inequality and the Process of Development. *Review of Economic Studies* 71(4), 1001-1026.

Galor, O. and O. Moav (2006). Das Human-Kapital: A Theory of the Demise of the Class Structure. *Review of Economic Studies* 73, 85-117

Galor, O. and J. Zeira (1993). Income Distribution and Macroeconomics. *Review of Economic Studies* 60 (1), 35-52.

Golosov et al. (2014). Optimal Taxes on Fossil Fuel in General Equilibrium. *Econometrica* 82 (1), 44 - 88.

Lagerlöf NP (2006) The Galor–Weil model revisited: a quantitative exercise. *Review of Economic Dynamics* 9(1):116–142

Lakner, C. and B. Milanovic (2013). Global Income Distribution: From the Fall of the Berlin Wall to the Great Recession. *World Bank Policy Research Working Paper* 6719.

Liberati, P. (2015). The World Distribution of Income and Its Inequality, 1970-2009. *Review of Income and Wealth* 61(2), 248–73.

Nordhaus, W.D. (1992). An Optimal Transition Path for Controlling Greenhouse Gases. *Science* 20, 1315 - 1319.

Nordhaus, W.D. (1993). Optimal Greenhouse-Gas Reductions and Tax Policy in the DICE Model. *American Economic Review* 83(2), 313-317.

Nordhaus, W. and J. Boyer (2000). *Warming the World: Economic Modeling of Global Warming*. MIT Press.

Piketty, Th. (1997). The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing. *Review of Economic Studies* 64 (2), 173–189.

Piketty, Th. (2014). *Capital in the twenty-first century*. Harvard University Press.

Piketty, Th. et al. (2019). Capital Accumulation, Private Property, and Rising Inequality in China, 1978–2015: Dataset. *American Economic Review* 109 (7): 2469 - 2496.

Piketty, Th. and G. Zucman (2014). Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2010. *Quarterly Journal of Economics* 129 (3): 1255–310.

Pizer, W. (1999). The Optimal Choice of Climate Change Policy in the Presence of Uncertainty. *Resource and Energy Economics* 21, 255–287.

Ravallion, M. et al. (2000). Carbon emission and income inequality. *Oxford Economics Papers* 52, 651 - 669.

Stiglitz, J.E. (1969). Distribution of Income and Wealth among Individuals. *Econometrica* 37 (3), 382 - 397.

Vasconcelos, V.V. et al. (2014). Climate policies under wealth inequality. *PNAS* 111 (6), 2212 - 2216.

Williams, R.C III et al. (2014). The initial incidence of a carbon tax across income groups. *National Tax Journal* 68(1), 195-214.