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AN EXPERIMENT ON THE WINTER DEMAND COMMITMENT BARGAINING MECHANISM

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An experiment on the Winter demand commitment bargaining mechanism

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Abstract

In this paper we experimentally compare three implementations of *Winter demand commitment bargaining mechanism*: a one-period implementation, a two-period implementation with low and with high delay costs. Despite the different theoretical predictions, our results show that the three different implementations result in similar outcomes in all our domains of investigation, namely: coalition formation, alignment with the Shapley value prediction and axioms satisfaction. Our results suggest that a lighter bargaining implementation with only one period is often sufficient in providing allocations that sustain the Shapley value as appropriate cooperative solution concept, while saving unnecessary costs in terms of time and resources.

JEL code: C71, C72, C90, D82

Keywords: Nash Program, Shapley value, Experiments, Winter mechanism

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1 Introduction

The aim of the *Nash program* (Nash, 1953) is to provide a noncooperative foundation of cooperative solution concepts. Nash started such program designing a noncooperative game which sustained as equilibrium the Nash solution of his cooperative *bargaining problem* (Nash, 1950). Since then, the Nash program had a very long history and it kept on growing thanks to many theoretical and experimental contributions (the reader is referred to Serrano, 2005, 2008, 2014, 2020, for an exhaustive literature review). Citing the first of the aforementioned surveys, and the words of Nash himself, "*The idea*" of the Nash program "*is both simple and important: the relevance of a concept [...] is enhanced if one arrives at it from different points of view*" (Serrano, 2005, pag. 220). In fact, "*it is rather significant that this different approach yields the same solution. This indicates that the solution is appropriate for a wider variety of situations*" (Nash, 1953, pag. 136).

Between the many, most of the existing papers contributing to the Nash program are devoted to sustain the noncooperative foundation of the Shapley value solution (Shapley, 1953) (see, among others, Gul, 1989; Harsanyi, 1981; Hart and Moore, 1990; Krishna and Serrano, 1995; Winter, 1994; Hart and Mas-Colell, 1996; Perez-Castrillo and Wettstein, 2001). In fact, thanks to its intuitive and desirable properties, the Shapley value has seen many applications on a variety of situations, such as cost or payoff sharing, voting power, fair division, and, most recently, on many non-economics focused contexts such as machine learning, AI models or data analysis (we can cite, between the less standard applications, the Shapley value implemented to get information about gene expression thanks to microarray games (Lucchetti et al., 2010)). As a result, the Shapley value is nowadays and undoubtedly the "most famous" axiomatic cooperative solution concept, whose appropriateness is validated by an extensive literature listing its appealing theoretical properties, and by its many applications.

In Chessa et al. (2021), we aimed to contribute to the Nash program in sustaining the Shapley value, by providing a comparison between the experimental results of a demand based (Winter, 1994) versus an offer based (*à la* Hart and Mas-Colell (1996)) mechanism. Our analysis showed that the Winter mechanism (namely, the *Winter demand commitment bargaining mechanism*) better provides allocations that reflect players' effective bargaining power, and that satisfy the axioms that characterize the Shapley value. The efficiency and frequency of grand coalition formation, instead, are not very high. This finding suggests that the Shapley value is indeed an appealing solution in all such situations in which some bargaining agents interact by expressing their demands about the share they wish to obtain from cooperation, and when highlighting players' effective bargaining power is a key point. However, in this previous work such results were obtained under an important simplification in the proposed mechanism as compared to its more generic theoretical definition, i.e., implementing a one-period version of the model. In the Winter mechanism, each player one after another becomes a proposer and makes a demand for herself, of the payoff she is willing to get from a possible collaboration. If and when at some point a compatible demand is introduced, which means that there exists a coalition for which the total demands do not exceed the value of the coalition, such coalition forms, leaves the game, and the bargaining continues with the rest of the players, till there is at least one to still have to submit a demand. Players with unsatisfied demands at the end of the first period get their individual value in the one-period version of the model.

However, real world applications of demand-based bargaining processes may provide the players more time to get an agreement. Having more time for agreeing is often costly (surely in terms of time and often as well in terms of resources), but it may be more (or less) effective. Thus, in this paper, our primarily research question is to investigate the performances of the Winter mechanism when modifying the way the players interact. In fact, we affirm that for a solution -in our case the Shapley value- to be relevant, players need to agree on it when interacting under different rules. Then, firstly, we compare a one-period implementation versus a two-period implementation. In the two-period implementation, if some players are left with unsatisfied demands after the first period, they have a second chance for cooperating, as the bargaining procedure repeats for a second time on the set of these players, by canceling their previous demands and charging them a fixed delay cost. Secondly, we compare the performances of the Winter mechanism in its two-period implementation when implementing *low* versus *high* delay costs.

The theoretical prediction expects all the three implementations to provide complete cooperation already in the first period and a power share close to the Shapley value (in average, as *ex ante* equilibrium). However, the theoretical *ex post* equilibrium payoff differs between the different implementations, in particular in terms of first mover advantage. The first mover advantage is expected to be smaller in two-period implementations than in one-period implementation. But as already observed by Fréchette et al. (2005), experiments often show that actual bargaining behavior is sometimes not as sensitive to the different bargaining rules as the theory suggests, and this is what happens also in our case.

Our results show that the three different implementations of the Winter mechanism result in similar outcomes in all our domains of investigation, namely: coalition formation, alignment with the theoretical prediction and axioms satisfaction. Moreover, we observe similar results also when observing the outcome of the first period in a two-period implementation. We interpret this finding as a robustness of the Winter mechanism in sustaining the Shapley value. Moreover, such results support the implementation of the Shapley value as an appealing cooperative solution concept in many real-world applications, both when the decisions have to be taken rapidly, or when the time for bargaining is longer. Finally, we suggest that a simpler and faster bargaining is often sufficient in providing allocations that sustain the Shapley value, as a second chance for reaching an agreement is proved to be ineffective in augmenting the chance of the players of finding an agreement, or to get closer to the predicted allocation.

The rest of the paper is organized as follows. Section 2 presents the general definition and the properties of a cooperative transferable utility (TU) game, as well as the Shapley value and its axiomatizations. Section 3 presents the Winter mechanism, describes the setting of our experiment, and presents our hypotheses. The results are presented in Section 4. Section 5 concludes.

2 Theoretical model

2.1 Cooperative TU games and solutions

Let $N = \{1, \ldots, n\}$ be a finite set of *players*. Each subset $S \subseteq N$ is called a *coalition*, and N is called the *grand coalition*. A *cooperative TU game* (from now on, *cooperative game*) consists of a couple (N, v), where N is the set of players and $v : 2^N \to \mathbb{R}$, with $v(\emptyset) = 0$, is the *characteristic function*, which assigns to each coalition $S \subseteq N$ the *worth* $v(S)$, i.e., the value that members of S can achieve by cooperation. If no ambiguity appears, we consider the set of players N fixed and we write v instead of (N, v). We denote with $\mathcal{G}^N$ the set of all games with player set N.

Players i and j are *symmetric* in $v \in \mathcal{G}^N$, if $v(S \cup \{i\}) = v(S \cup \{j\})$ for all $S \subseteq N \setminus \{i, j\}$. Player i is a *null player* in $v \in \mathcal{G}^N$ if $v(S) = v(S \setminus \{i\})$ for all $S \subseteq N$.

A game $v \in \mathcal{G}^N$ is said to be *monotonic* if $v(S) \leq v(T)$ for each $S \subseteq T \subseteq N$, *superadditive* if $v(S) + v(T) \leq v(S \cup T)$ whenever $S \cap T = \emptyset$, with $S, T \subseteq N$ and *convex* if $v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$, for each $S, T \subseteq N$ (*strictly convex* if the inequality holds strictly). Another equivalent definition for convexity can be stated as $v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T)$, for each $S \subseteq T \subseteq N \setminus \{i\}$. In (strictly) convex games, cooperation becomes increasingly appealing, leading to the formation of the grand coalition. We may observe that convexity ⇒ superadditivity ⇒ monotonicity.

Given a game $v \in \mathcal{G}^N$, an *allocation* is an n-dimensional vector $(x_1, \ldots, x_n) \in \mathbb{R}^N$ assigning to player i the amount $x_i \in \mathbb{R}$. For each $S \subseteq N$, we denote $x(S) = \sum_{i \in S} x_i$. The *imputation set* is defined by

$$I(v) = \{x \in \mathbb{R}^n | x(N) = v(N) \text{ and } x_i \geq v(\{i\}) \ \forall i \in N\},$$

i.e., it contains all the allocations that are *efficient* ($x(N) = v(N)$) and *individually rational* ($x_i \geq v(\{i\}) \forall i \in N$).

The core is the set of imputations that are also *coalitionally rational*, i.e.,

$$C(v) = \{x \in I(v) | x(S) \geq v(S) \ \forall S \subseteq N\}.$$

An element of the core is stable in the sense that if such a vector is proposed as an allocation for the grand coalition, no coalition will have an incentive to split off and cooperate on its own. Intuitively, the idea behind the core is analogous to that behind a (strong) Nash equilibrium of a noncooperative game: an outcome is stable if no group deviation is profitable. For the Nash equilibrium the possible deviation is for a single player, while in the core we speak about deviations of groups of players.

A *solution* is a function $\psi : \mathcal{G}^N \to \mathbb{R}^N$ that assigns an allocation $\psi(v)$ to every game $v \in \mathcal{G}^N$. The *Shapley value* is the most well-known solution concept, which is widely applied in economic models, and is defined as

$$\phi_i(v) = \sum_{S \subseteq N, i \in S} \frac{(|S|-1)!(|N|-|S|)!}{|N|!}(v(S) - v(S \setminus \{i\})) \; \forall i \in N.$$

The Shapley value assigns every player its expected marginal contribution to the coalition of players that enter before him, given that every order of entrance has equal probability. This solution concept has been defined as respecting some notion of fairness (see Section 2.2 for more discussion about its properties), but it is not, on the contrary, necessary stable. However, if the game is superadditive, the Shapley value is an imputation, and if the game is convex, is the core barycenter (and then, in particular, it belongs to it).

In our analysis, we will also consider an easier solution concept, the *equal division solution*, which distributes the worth $v(N)$ equally between the players. It is then defined as

$$ED_i(v) = \frac{v(N)}{n} \; \forall i \in N.$$

2.2 Axiomatizations of the game theoretical solutions

In the literature, we find various axiomatic characterizations of the Shapley value. The most classical one is from Shapley (1953), and it involves Efficiency, Symmetry, Additivity and Null player. The one by Young (1985) involves Efficiency, Symmetry and Strong monotonicity. The characterization by van den Brink (2002), instead, involves

Efficiency, Null player and Fairness. We list in Table 1 these axioms (for a solution $\psi : \mathcal{G}^N \to \mathbb{R}^N$).

Table 1: The axioms

Axioms	
Efficiency	for every v in $\mathcal{G}^N$, $\sum_{i \in N} \psi_i(v) = v(N)$
Symmetry	if i and j are symmetric players in game $v \in \mathcal{G}^N$, then $\psi_i(v) = \psi_j(v)$
Additivity	for all $v, w \in \mathcal{G}^N$, $\psi(v + w) = \psi(v) + \psi(w)$
Homogeneity	for all $v \in \mathcal{G}^N$ and $a \in \mathbb{R}$, $\psi(av) = a\psi(v)$
Null player	if i is a null player in game $v \in \mathcal{G}^N$, then $\psi_i(v) = 0$
Strong monotonicity	if $i \in N$ is such that $v(S \cup \{i\}) - v(S) \leq w(S \cup \{i\}) - w(S)$ for each $S \subseteq N$, then $\psi_i(v) \leq \psi_i(w)$
Fairness	if i, j are symmetric in $w \in \mathcal{G}^N$, then $\psi_i(v + w) - \psi_i(v) = \psi_j(v + w) - \psi_j(v)$ for all $v \in \mathcal{G}^N$

3 The experimental setting

3.1 The Winter mechanism

In our experiments, we implemented the bargaining model based on sequential demands for strictly convex cooperative games presented by Winter (1994). In this model, players in turns announce publicly their demands, meaning "I am willing to join any coalition yielding me a payoff of ..." and wait for these demands to be met by other players. The bargaining starts with a randomly chosen player from N, say player i. This player announces publicly her demand d_i and then points a second player who has to give her demand. Then, the game proceeds by having each player pointing at a new player to take her turn after introducing a demand herself. If and when at some point a compatible demand is introduced, which means that there exists a coalition S for which the total demand for players in S does not exceed $v(S)$, then the first player with such a demand selects a compatible coalition S. The players in S get their demands, leave the game, and the bargaining continues with the rest of the players using the same rule on v restricted on $N \backslash S$. In a one-period implementation, players with unsatisfied demands at the end of the first period get their individual value. In a T-period implementation with $|T| > 1$, T finite, instead, if some players are left with unsatisfied demands after the first period, the bargaining procedure repeats for a second time on the set of these players, by canceling their previous demands and charging them a fixed delay cost, and so on till periods are over. A more formal description of the Winter mechanism with T periods ($|T| \geq 1$, T finite) is presented in Appendix A.

For both the one-period and the T-period implementations, this mechanism has a unique subgame perfect equilibrium which assigns equal probabilities at indifference. At this equilibrium, the grand coalition forms in the first period and the *ex ante* expected equilibrium payoff coincides with the Shapley value. Regarding the *ex post* equilibrium payoff, in the one-period implementation, given a specific ordering of the players, each player's demand depends on the ordering, but only through the set of her successors and not through the way these players are ordered. In fact, each player demands the marginal contribution to the set of her successors. In the two-period implementation (which is our second case of interest), instead, at equilibrium the first player to make a demand asks her Shapley value plus the delay cost times the number of remaining players, while the other players ask their Shapley value minus once the delay cost[1]. More generally, all different implementations of the Winter mechanism present, theoretically, a first mover advantage. We stress that these theoretical results hold under some specific assumptions,

[1]The *ex-post* equilibrium of the T-period implementation with $|T| > 2$ much resembles the case $|T| = 2$.

in addition to the already mentioned strict convexity of the game. In particular, delay cost must not be too large[2].

3.2 The games

For our analysis, we considered the four four-player games shown in Table 2 together with corresponding Shapley values. The equal division payoff vector is equal to $ED(v_k) = (25, 25, 25, 25)$ when $k = 1, 2$, and $ED(v_k) = (50, 50, 50, 50)$ when $k = 3, 4$.

The choice of the games has been done in order to be capable of testing the axioms we presented in Section 2.2. In Table 3 we detail for each axiom which game and thanks to which characteristics is introduced in our analysis to test it. Notice that games 1, 3 and 4 are strictly convex, while game 2 is only convex. All the four games are, by consequence, monotonic. Therefore, all but game 2 respect the assumption for the implementation of Winter mechanism. With game 2 being at least convex, however, we guessed that "strict convexity" could be relaxed and the mechanism could still provide convincing results[3].

We implemented the Winter mechanism with one period, from now on referred to as *one-period* (1p) implementation, and the mechanism with two periods and with low and high delay costs, from now on referred to as *two-period low cost* (2pL) implementation and *two-period high cost* (2pH) implementation respectively. Low delay costs are equal

[2]We refer to the original paper by Winter (1994) for the detailed description of the assumptions under which such results hold. As already mentioned, the delay cost must not be too large. Moreovoer, the results about the *ex ante* and the *ex post* equilibria hold for a discrete version of the mechanism, and when the smallest money unit approaches zero.

[3]The choice of implementing a game that is not strictly convex, is given by the wish to test the null player axiom. However, the presence of this null player raises some issues concerning the theoretical prediction of the equilibrium outcome on such game. For instance in a two-period implementation with a null player, the delay cost is always too high whatever its size, and the null player making her demand not as first mover would always prefer to ask zero and leave the game, better than paying the delay cost and incurring a negative payoff. Then, in the following of the paper and when analyzing game 2, we will allow the configuration $\{1\}$ and $\{2, 3, 4\}$ as equivalent to the grand coalition.

Table 2: The games and their Shapley values

S	1	2	3	4	1,2	1,3	1,4	2,3	2,4	3,4	1,2,3	1,2,4	1,3,4	2,3,4	N	$\phi_1(v)$	$\phi_2(v)$	$\phi_3(v)$	$\phi_4(v)$
$v_1(S)$	0	5	5	10	20	20	25	20	25	25	50	60	60	60	100	22.08	23.75	23.75	30,42
$v_2(S)$	0	20	20	30	20	20	30	45	55	60	45	55	60	100	100	0	28.33	30.83	40.83
$v_3(S)$									$= v_1(S) + v_2(S)$							22.08	52.08	54.58	71.25
$v_4(S)$									$= 2v_1(S)$							44.16	47.5	47.5	60.83

Table 3: The axioms and the four games

Axioms	**Games**
Efficiency	**all games**
Symmetry	**games 1 and 4** (symmetry of players 2 and 3)
Additivity	**games 1, 2 and 3** (game 3 is defined as the sum of games 1 and 2)
Homogeneity	**games 1 and 4** (game 4 is defined as twice game 1)
Null player	**game 2** (player 2 is a null player)
Strong monotonicity	**all games** (the marginal contributions of player 1 are always higher in game 1 than in game 2, and also higher in game 4 than in game 3)
Fairness	**games 1, 2 and 3** (game 3 is defined as the sum of games 1 and 2)

to 0.5 for games 1 and 2, and to 1 for games 3 and 4. High delay costs are equal to 2.5 for games 1 and 2, and to 5 for games 3 and 4.

3.3 The procedure

The experiment was conducted at the Institute of Social and Economic Research (ISER), Osaka University, between January 2019 and August 2019. A total of 264 students, who had never participated in similar experiments before, were recruited as subjects of the experiment. 96 for the one-period treatment, and 84 each for the two-period low and high cost treatments.[4] The experiment was computerized with z-Tree (Fischbacher, 2007) and participants were recruited using ORSEE (Greiner, 2015).

To control for potential ordering effects, each participant played all the four games twice in one of the following four orderings: 1234, 2143, 3412 and 4321. Between

[4]The difference in the number of participants between the two mechanisms is a result of variations in the show-up rate among experimental sessions.

each play of a game (called a round), players were randomly re-matched into groups of four players, and participants were randomly assigned a new role within the newly created group. At the end of the experiment, two rounds (one from the first four rounds and another from the last four rounds) were randomly selected for payments. Participants received cash reward based on the point they have earned in these two selected rounds with an exchange rate of 20 JPY = 1 points in addition to 1500 and 1900 JPY participation fee for the one-period and the two-period implementations, respectively. The experiment lasted on average 100 min for the one-period implementation and 130 min for the two-period implementations including the instruction ($\sim$ 15 min), comprehension quiz ($\sim$ 5 min), and payment.[5] The average earning was 2650 JPY, 3110 JPY, and 2960 JPY for the one-period, 2-period low costs, and two-period high costs implementations, respectively.

4 Results

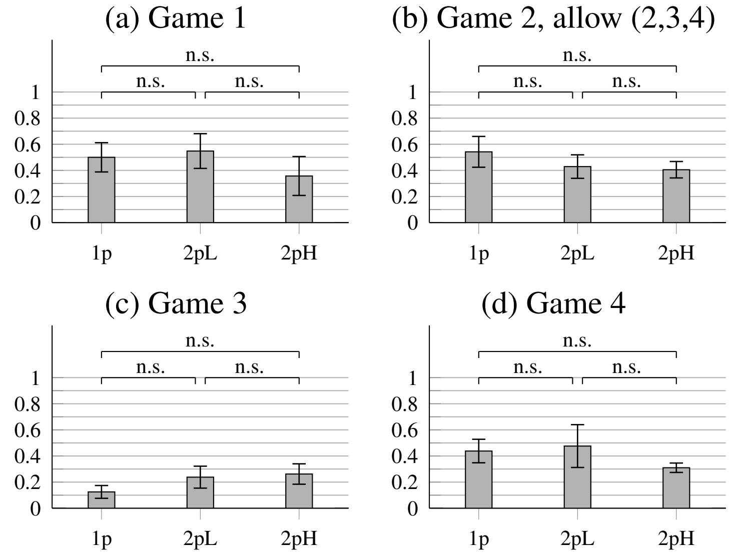
4.1 Grand coalition formation and efficiency

At first, we investigate whether the three implementations succeeded in making the players find an agreement and form the grand coalition. In Figure 1, we present the results about the grand coalition formation for the 1p, the 2pL and the 2pH implementations and for the four games.[6]

[5]Participants received a copy of instruction slides, and pre-recorded instruction movies were played. See Appendix B for English translations of the instruction slides and the comprehension quiz.

[6]The figure is created based on the estimated coefficients of the following linear regressions: $gc_i = \beta_1 1p_i + \beta_2 2pL_i + \beta_3 2pH_i + \mu_i$ where gc_i is a dummy variable that takes the value 1 if the grand coalition is formed, and zero otherwise, in group i, $1p_i$ ($2pL_i$ or $2pH$) is a dummy variable that takes value 1 if the 1p (2pL and 2pH) implementation is used, and zero otherwise. The standard errors are corrected for within session clustering effect. The statistical tests are based on the Wald test for the equality of the estimated coefficients of two treatment dummies.

Figure 1: Proportion of times the grand coalition formed.



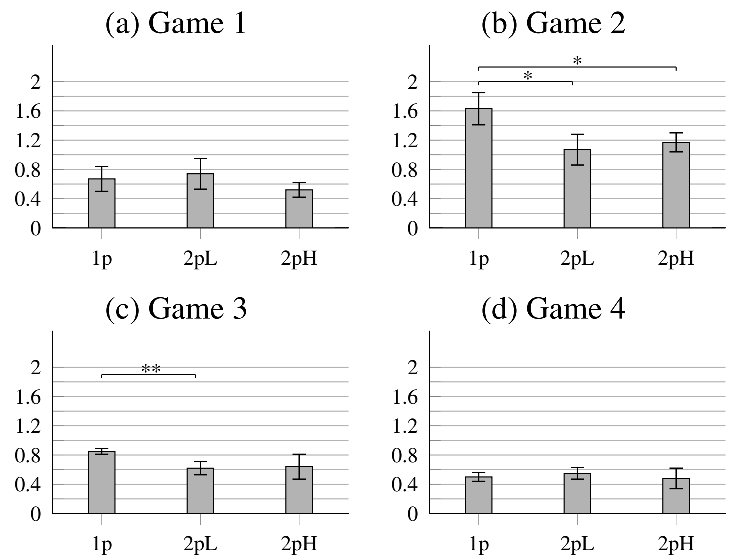
Note: Error bars show one standard error range. *** indicates the proportion of times the grand coalition formation was significantly different at the 1 % significance level (Wald test).

As game 2 is not strictly convex with the presence of the null player 1, we allow the partition $\{1\}$, $\{2,3,4\}$ as a realization of the grand coalition, as this coalition structure does not affect the total value which has to be shared between the players[7]

We may observe that, at best, the grand coalition formed slightly more (and often, much less) than 50% of the times for the four games, with no significant difference between the implementations. We may conclude that all the three implementations of the Winter mechanism equally failed in enhancing complete cooperation. Observe that, even when players were given a second chance in a two-period implementation, they did not manage to perform better in terms of grand coalition formation. In particular, Figure 2 illustrates the number of players whose demands were not met in the first loop. Surprisingly, their number is not higher (and, in same cases, is even significantly

[7]Remember that the Winter mechanism is theoretically defined for strictly convex games. Player 1, in this game, has always a zero marginal contribution and, as such, can be left out from any coalition at no cost for either him/her or the other players.

Figure 2: Average number of players whose demands were not met in the first loop.



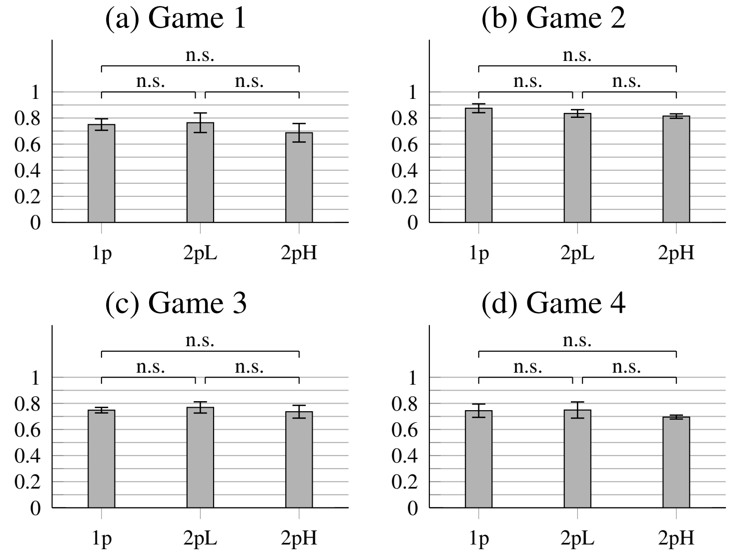
Note: Error bars show one standard error range.***, **, * significant at 1, 5, and 10% significance level, respectively (Wald test).

smaller) in the 2pL and 2pH implementations when compared to the 1p implementation. We may recall that the theoretical predictions expects the grand coalition to form in the first loop for the three implementations. However, one may expect players to demand more in the first loop of a two-period implementation, because of the second chance of forming a coalition if their requests is not met on the first try.

As a direct consequence of the failure in forming the grand coalition, we report also a failure in achieving full efficiency, with again no significant difference between the implementations, regardless of the presence of some delay costs in 2pL and 2pH (see Figure 3)[8]. Efficiency is computed as the fraction of the sum of the payoffs obtained by the four players compared to the value of the grand coalition of the considered game

[8]The figure is created based on the estimated coefficients of the following linear regressions: $Eff_i = \beta_1 1p_i + \beta_2 2pL_i + \beta_3 2pH_i + \mu_i$ where $Eff_i \equiv \frac{\sum_i \pi_i}{v(N)}$ is the efficiency measure for group i, $1p_i$ ($2pL$ and $2pH$) is a dummy variable that takes value 1 for the 1p (2pL or 2pH) treatment, and zero otherwise. The standard errors are corrected for within session clustering effect. The statistical tests are based on the Wald test for the equality of the estimated coefficients of two treatment dummies.

Figure 3: Verification of the efficiency axiom.



Note: Error bars show one standard error range. *** and ** indicate the proportion of time that verification of the efficiency axiom was significantly different at the 1 and 5% significance level (Wald test).

(100 for games 1 and 2 and 200 for games 3 and 4).

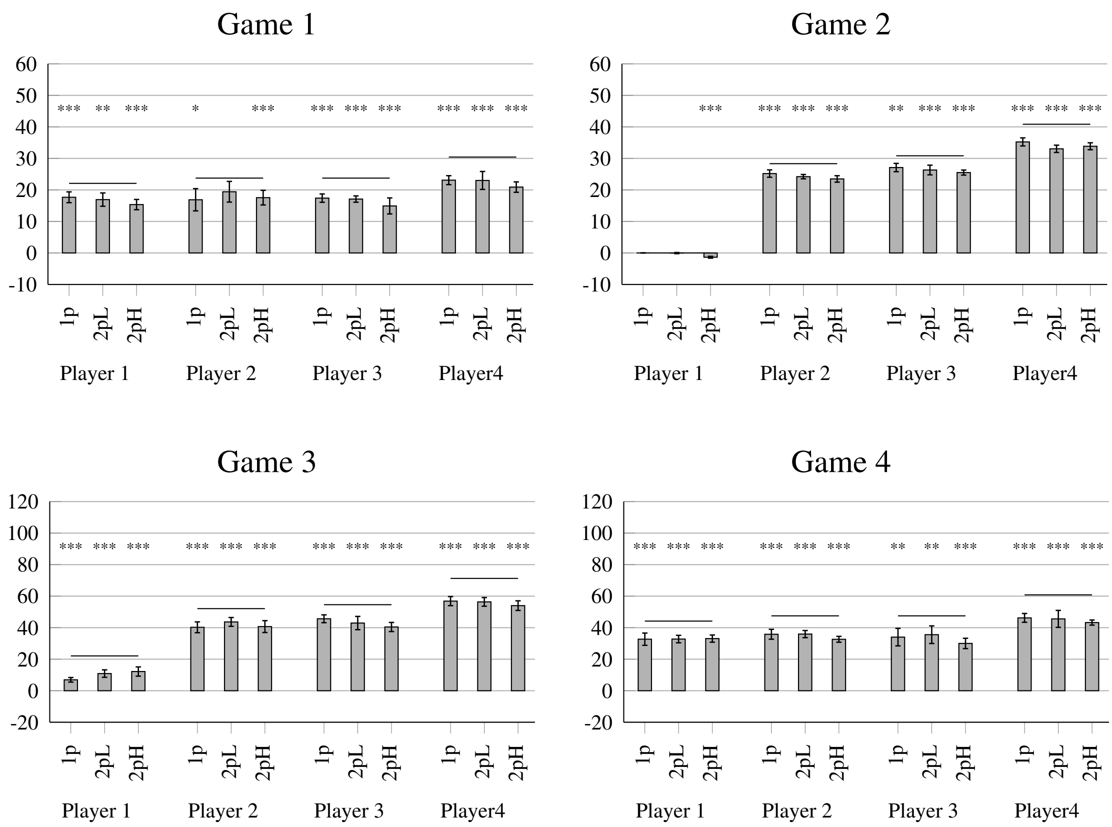
Therefore, we conclude that

Result 1. *The Winter mechanism in its one-period, two-period low cost and two-period high cost implementations provides mediocre and comparable results in terms of coalition formation and efficiency.*

4.2 Payoff shares: *ex ante* theoretical prediction

According to the *ex ante* theoretical prediction, the Winter mechanism in all the three implementations is expected to provide approximately the Shapley value, in average over the different orderings of the players. We denote by $\pi^{1p}(v_k)$ a vector of payoffs obtained by the players and by implementing 1p and on game k, with $k = 1, 2, 3, 4$. Analogously, we denote by $\pi^{2pL}(v_k)$ a vector of payoffs obtained by the players by

Figure 4: Mean of the payoffs for the three mechanisms, the horizontal lines indicating the Shapley values.



Note: Error bars show one standard error range. ***, **, and * indicate the average normalized payoff being significantly different from the Shapley value at 1, 5, and 10% significance level (Wald test).

implementing 2pL and by $\pi^{2pH}(v_k)$ a vector of payoffs obtained by the players by implementing 2pH. Figure 4 shows the mean of the payoffs in the four games and for the three implementations, the horizontal lines indicating the Shapley value for each game[9].

As we may observe in Figure 4, as a consequence of the players often failing to form the grand coalition and, consequently, because of a lack of efficiency, the average

[9]The error bars are based on the standard errors that are corrected for withing session clustering effect. These standard errors are obtained running the system of linear regressions described in Section 4.4. The statistical tests are based on these regressions.

realized payoff vectors are significantly different from the Shapley value. Therefore, we focus on investigating whether the proportion of the power share, *in lieu* of the absolute payoffs, converges to the Shapley value, by considering the normalized (to the value of the grand coalition) payoff vectors. Then, in this and in the following sections we consider the normalized vectors of payoffs with components $\widetilde{\pi}_i^m(v_k) = \frac{\pi_i^m(v_k)}{\sum_{j \in N} \pi_j^{1p}(v_k)} \times v_k(N)$ for each $i = 1, 2, 3, 4$ and for each $m \in \{1p, 2pL, 2pH\}$ (remember that the value of the grand coalition is equal to 100 for games 1 and 2 and equal to 200 for games 3 and 4).

Figure 5 shows the mean of the normalized payoffs in the four games, the horizontal lines indicating the Shapley values for each game.

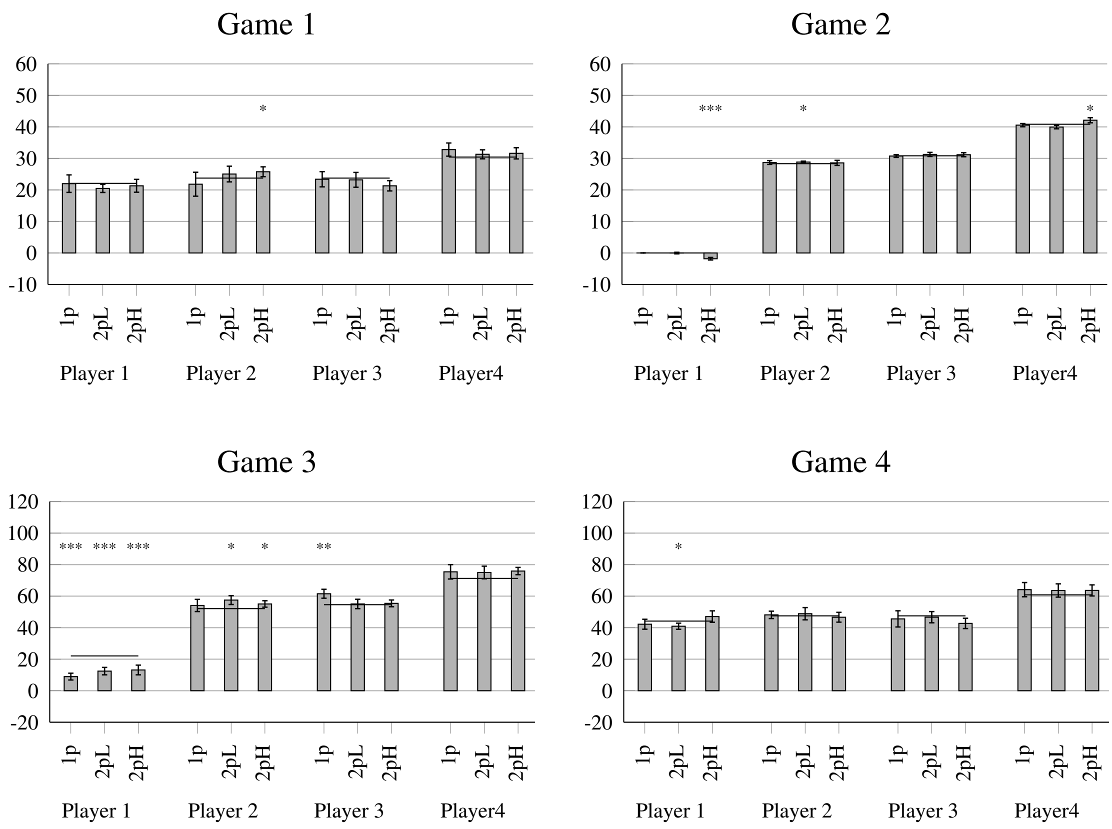
The three implementations perform well in implementing, on average, the Shapley value share. Moreover, we do not report any superiority of one of the three implementations. Thus, the following result follows.

Result 2. *The Winter mechanism in its one-period, two-period low cost and two-period high cost implementations provides good and comparable results in terms of implementation of the Shapley value power share.*

4.3 Payoff shares: *ex post* theoretical prediction and first mover advantage

The *ex post* theoretical prediction of the Winter mechanism is dependent on the ordering in which the players make their demand. In particular, a first mover advantage is predicted. Furthermore, a higher first mover advantage is expected in the 1p implementation, followed by the 2pH and, finally by the 2pL implementation. As a result, the distances between the Shapley value and the ex post theoretical predictions are largest

Figure 5: Mean of the normalized payoffs for the three mechanisms, the horizontal lines indicating the Shapley values.



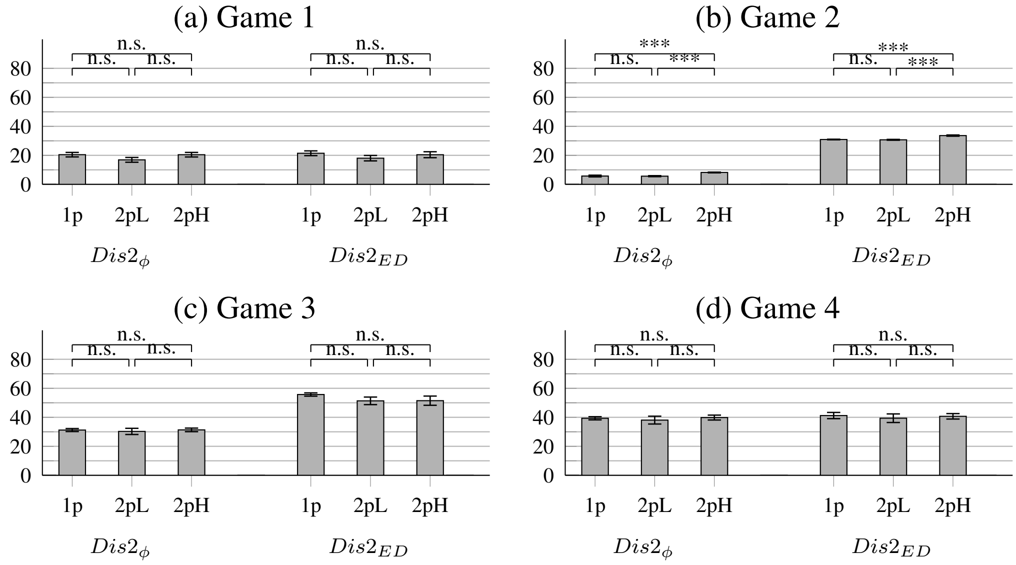
Note: Error bars show one standard error range. ***, **, and * indicate the average normalized payoff being significantly different from the Shapley value at 1, 5, and 10% significance level (Wald test).

in 1p implementation, followed by 2pH and then 2pL.

Figure 6 shows the mean of the euclidean distances of the normalized payoff vectors from the Shapley value, as well as from the equal division solution, for the four games. Such distances are computed as $Dis2^m_\phi = \sqrt{\sum_i(\widetilde{\pi}_i - \phi_i(v))^2}$ and $Dis2_{ED} = \sqrt{\sum_i(\widetilde{\pi}_i - ED_i)^2}$ where $\phi_i(v)$ denotes the Shapley value for player i in game v.[10]

[10]The figure is created based on the estimated coefficients of the following linear regressions: $Dis_i = \beta_1 1p_i + \beta_2 2pL_i + \beta_3 2pH_i + \mu_i$ where Dis_i is the relevant distance measure for group i, $1p_i$ ($2pL_i$ and $2pH$) is a dummy variable that takes value 1 if 1p (2pL or 2pH) treatment is used, and zero otherwise. The standard errors are corrected for within session clustering effect. The statistical tests are based on the

Figure 6: Mean of the distances of the normalized payoff vectors from the SPNEs and the equal division solutions.



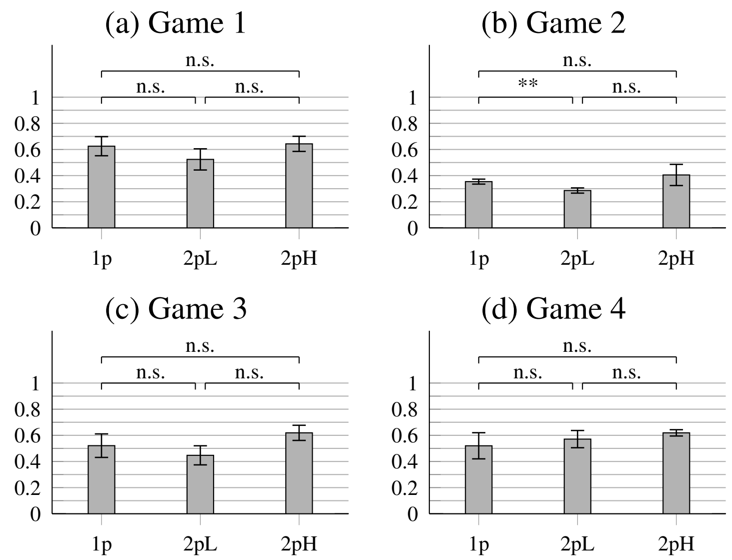
Note: Error bars show one standard error range. ***, **, and * indicate the distance of the normalized payoff vectors from the Shapley or from the equal division solution was significantly different at 1, 5, and 10% significance level, respectively (Wald test).

We may observe that, contrary to the ex post theoretical predictions, the distance of the realized normalized payoffs to the Shapley value in the three implementations are not significantly different in three out of four games. Only in Game 2, $Dis2_{\phi}$ was significantly larger in 2pH compared to 1p and 2pL. We also observe that the three implementations perform similarly in terms of distance to the equal division solution in three out of four games. The only exception is, again, Game 2, in which both 1p and 2pL are significantly closer to the equal division solution than 2pH. Therefore, we conclude that

Result 3. *The Winter mechanism in its three implementations provide similar results in terms of the distance between the realized normalized payoffs and the Shapley value as well as the equal division solution.*

Wald test for the equality of the estimated coefficients of two treatment dummies.

Figure 7: Proportion of times first mover advantage is verified according to the normalized payoff vectors.

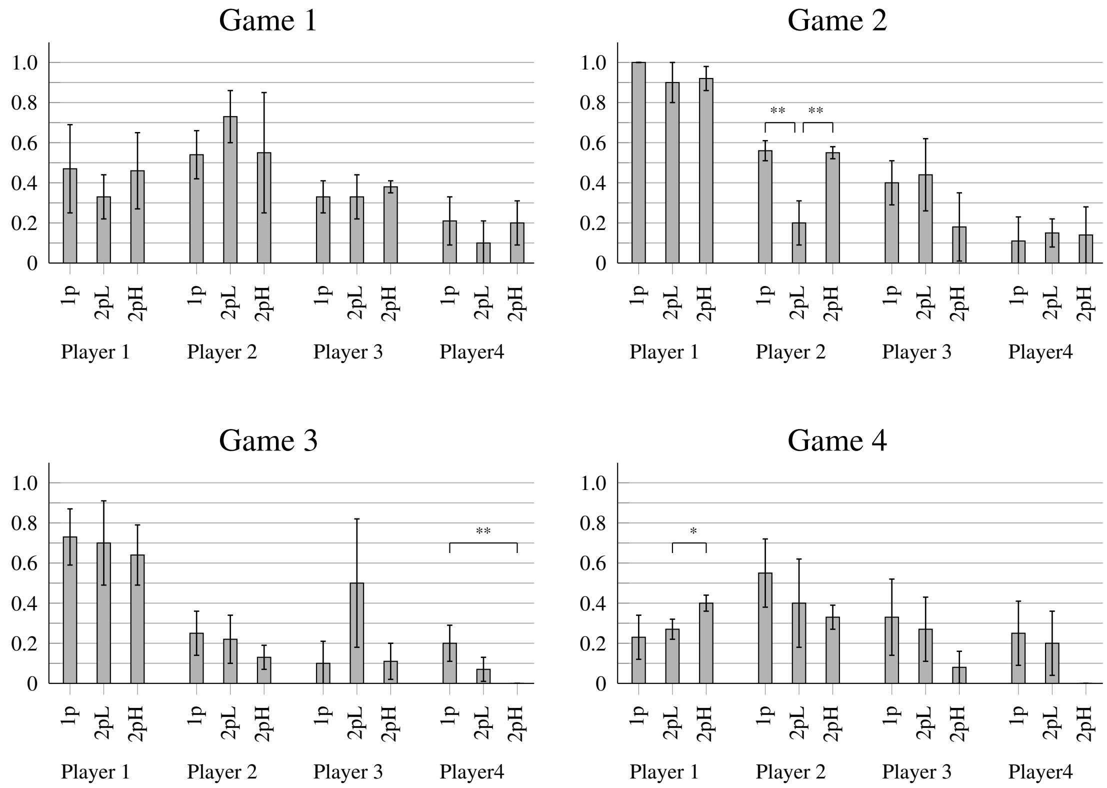


Note: Error bars show one standard error range.***, **, * significant at 1, 5, and 10% significance level, respectively (Wald test)

Figure 7 shows the proportion of times first mover advantage appears, i.e., when the first mover obtains a higher normalized payoff than her Shapley value. The three implementations perform in a similar way, and they do not show any first mover advantage effect (notice that the first mover takes in average more than predicted by her Shapley value half of the time, and, consequently, less than predicted the other half of the times). Figure 8 details these results for each single player and Figure 9 presents the mean of the amount demanded as first mover. Again, we may observe that the three implementations perform in a similar way, regardless of the differences of the theoretical prediction. We can now state the following result.

Result 4. *The Winter mechanism in its one-period, two-period low cost and two-period high cost implementations do not show any first mover advantage effect.*

Figure 8: Frequency of the first mover demanding more than SV

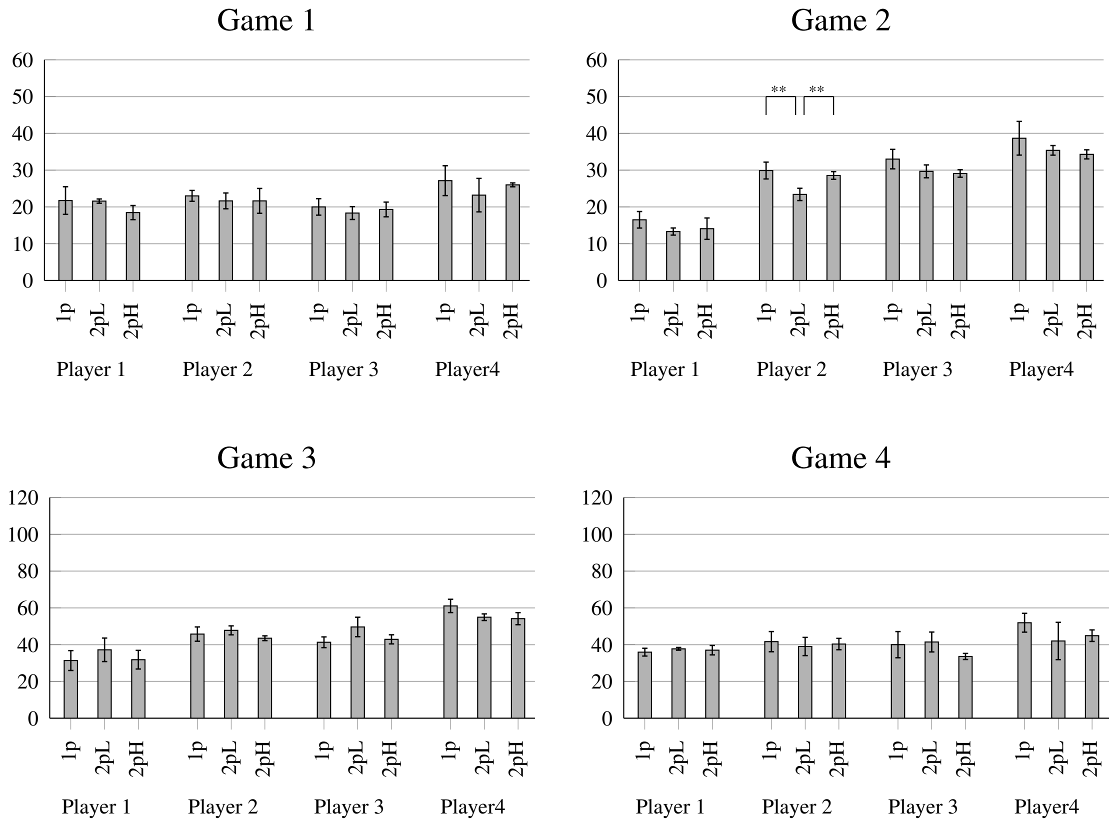


Note: Error bars show one standard error range. ** and * indicate significant difference between treatments at 5% and 10% significance level (Wald test).

4.4 Testing for the axioms

From Figure 5 we may notice that both the one-period and the two-period low cost implementations verify the null player property. In particular, the one-period implementation satisfies it fully (100% of the times), always assigning a payoff equal to 0 to the null player 1, in game 2. Instead, the other two implementations assign in average a negative payoff to the null player, as a consequence of the delay costs. As expected, small delay costs, as in 2pL, let the mean be close to 0, while on the other side, large delay costs, as in 2pH, let the final average payoff for player 1 in game 2 be significantly

Figure 9: Mean of the amount demanded as the first mover



Note: Error bars show one standard error range. ** indicates significant difference between treatments 5% significance level (Wald test).

different from 0.

In order to test symmetry, additivity, homogeneity, strong monotonicity and fairness, we perform a set of OLS regressions for the following system of equations, with dependent variables the average normalized payoffs $\widetilde{\pi}_i$, and as independent variables g_1,

g_2, g_3, g_4 and without constant:

$$\widetilde{\pi}_1 = a_1 g_1 + a_2 g_2 + a_3 g_3 + a_4 g_4 + u_1$$
$$\widetilde{\pi}_2 = b_1 g_1 + b_2 g_2 + b_3 g_3 + b_4 g_4 + u_2$$
$$\widetilde{\pi}_3 = c_1 g_1 + c_2 g_2 + c_3 g_3 + c_4 g_4 + u_3$$
$$\widetilde{\pi}_4 = d_1 g_1 + d_2 g_2 + d_3 g_3 + d_4 g_4 + u_4$$

where g_i is a dummy variable that takes value 1 if the game i is played, and zero otherwise. Various axioms are tested based on the estimated coefficients of these regressions. Symmetry requires $b_1 = c_1$ and $b_4 = c_4$. Additivity and Homogeneity require $x_3 = x_1 + x_2$ and $x_4 = 2x_1$ for $x \in \{a, b, c, d\}$, respectively. Strong monotonicity requires $a_1 > a_2$ and $a_4 > a_3$. Finally, fairness requires, $b_3 - b_2 = c_3 - c_2$. In Table 5 in Appendix C, we present the results of Wald test of the verification of these axioms, together with the null hypothesis (H_0).

Symmetry (according to which H_0 should not be rejected) is fully confirmed by the 1p and the 2pL implementations, and only for game 4 (then, half of the times) by the 2pH implementation. The additivity and homogeneity (according to which H_0 should not be rejected) are almost always confirmed by the three implementations. The strong monotonicity (according to which H_0 *should be rejected*) is confirmed by the three implementations. The fairness (according to which H_0 should not be rejected) is rejected by the 1p and the 2pH implementations, but confirmed by the 2pL implementation. Table 4 summarizes whether each axiom is satisfied in average (+) or not (–) for the three implementations. To conclude this section, we can state as follows:

Result 5. *The Winter mechanism in its one-period, two-period with low costs and two-period with highs cost implementations provides good and comparable results in terms*

Table 4: Winter mechanisms, axioms

Axiom	1p	2pL	2pH
Efficiency	–	–	–
Symmetry	+	+	–
Additivity	+	+	+
Homogeneity	+	+	+
Null player property	+	+	–
Strong monotonicity	+	+	+
Fairness	–	+	–

of satisfaction of the axioms.

5 Conclusions

In this paper we provide an experimental comparison of three different implementations of the Winter demand commitment bargaining mechanism: one-period, two-period with low costs and two-period with high costs. These three implementations predict the same *ex-ante* outcomes, while they differ in terms of *ex-post* outcomes. Our experiment shows that, however, these three different implementations provide comparable results for both ex-ante and ex-post outcomes. No significant difference appeared in any of our domains of investigation: coalition formation, alignment with the theoretical prediction and axioms satisfaction.

An example we borrow by Winter (1994) on the bargaining over government formation may help presenting the implications of our results. Bargaining over government formation is a process that naturally resembles the demand commitment model with more than one period. Parliamentarian negotiations are usually based on demands rather than proposals, and often these demands are not compatible in the first period, and at least the second round of requests is implemented to find an agreement. A second round may be costly in terms of time, and it can make the bargaining process unnecessarily slow and cumbersome. However, lengthening the bargaining process is often considered as essential and crucial for parties to match and a coalition to form.

Surprisingly, our experimental results suggest that this could not be always the case. In fact, our three implementations resulted in similar outcomes in all our domains of investigation. The take-off message of our paper suggests a mechanism designer to implement, whenever possible, the lightest possible mechanism for bargaining, as refinements may turn out to be costly to implement, but ineffective in terms of quality of the performances. In fact, players converge to similar outcomes (e.g., total or partial cooperation) without taking advantage of any second chance, and regardless of the different theoretical predictions, differences that are not matched behaviorally.

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A The Winter Mechanism

We present here more formally the T period Winter mechanism as defined by Winter (1994). A decision point position m at period t is given by the vector $(S_1^{t,m}, S_2^{t,m}, d_{S_2^{t,m}}, j)$, where:

$0 \leq t \leq T$ is the current period of the bargaining,

$S_1^{t,m} \subseteq N$ is the set of players remaining in the game,

$S_2^{t,m} \subset S_1^{t,m}$ is the set of players who have submitted demands which are not yet met,

$d_{S_2^{t,m}} = (d_i)_{i \in S_2^{t,m}}$ is the vector of demands submitted by players in $S_2^{t,m}$, ($0 \leq d_i \leq \max_{S \subseteq N} v(S)$), and

$j \in S_1^{t,m} \setminus S_2^{t,m}$ is the player taking the decision by introducing a demand d_j. Her demand d_j is said to be *compatible* if there exists some $S \subseteq S_2^{t,m}$ with $v(S \cup \{j\}) - \sum_{i \in S} d_i \geq d_j$. Otherwise d_j is not compatible.

With j's decision, the game proceeds now in the following way:

1) If d_j is compatible, then j specifies a compatible coalition S, each player $i \in S \cup \{j\}$ is paid $d_i - tc$ (c is the delay cost per period), and nature chooses randomly a player $k \neq j$ from $S_1^{t,m} \setminus S_2^{t,m}$. The new position is now given by $(t, S_1^{t,m+1}, S_2^{t,m+1}, d_{S_2^{t,m+1}}, k)$, with $S_1^{t,m+1} = S_1^{t,m} \setminus (S \cup \{j\})$ and $S_2^{t,m+1} = S_2^{t,m} \setminus (S \cup \{j\})$ (we remain at time t and we increment the position from m to $m + 1$).

2) If d_j is non-compatible, then two cases are distinguished:

2_a) if $S_2^{t,m} = S_1^{t,m} \setminus \{j\}$ (j is the last player to demand in the current period), then a new player k is chosen randomly from $S_1^{t,m}$ and the new position is given by $(S_1^{t+1,1}, \emptyset, k)$ (we increment the period from t to $t+1$ and we start back at period $m = 1$);

2_b) if $S_2^{t,m} \subset S_1^{t,m} \setminus \{j\}$, then j specifies a new player $k \neq j$ in $S_1^{t,m} \setminus S_2^{t,m}$ and the new position is $(S_1^{t,m+1}, S_2^{t,m+1}, d_{S_2^{t,m+1}}, k)$, with $S_1^{t,m+1} = S_1^{t,m}$ and $S_2^{t,m+1} = S_2^{t,m} \cup \{j\}$ (we remain at time t and we increment the position from m to $m+1$).

The game starts with randomly chosen a player $j \in N$. Then the initial position is set to be $(N, \emptyset, d_\emptyset, j)$. It terminates either when there are no more players in the game (see point 1 above), or when $t = T$ and $S_1^{t,m} \cup \{j\} = S_2^{t,m}$. In the second case, each $i \in S_1^{t,m} \cup \{j\}$ is paid $v(\{i\}) - Tc$.

B Translated instructions and comprehension quiz

An English translation of the instruction handout can be downloaded from

- `https://bit.ly/33IzgMM` for Winter 1 period implementation
- `http://bit.ly/3avJPoq` for Winter 2 period implementation

An English translation of the comprehension quiz can be downloaded from `http://bit.ly/3oOMVsL`

C Verification of Axioms

Table 5: Winter mechanisms, Wald tests for the verification of the symmetry, additivity, homogeneity, strong monotonicity and fairness axioms

		1p		2pL		2pH	
Axiom	H_0	χ^2	p-value	χ^2	p-value	χ^2	p-value
Symmetry	$b_1 = c_1$	0.08	0.781	0.15	0.694	12.91	0.0003
	$b_4 = c_4$	0.14	0.712	0.11	0.742	0.30	0.581
Additivity	$a_3 = a_1 + a_2$	7.25	0.007	5.51	0.019	1.77	0.184
	$b_3 = b_1 + b_2$	0.65	0.422	1.70	0.193	0.05	0.815
	$c_3 = c_1 + c_2$	2.54	0.111	0.03	0.862	1.37	0.243
	$d_3 = d_1 + d_2$	0.35	0.555	0.50	0.479	0.31	0.579
Homogeneity	$a_4 = 2a_1$	0.06	0.805	0.00	0.979	1.61	0.204
	$b_4 = 2b_1$	0.37	0.542	0.10	0.756	29.04	0.000
	$c_4 = 2c_1$	0.02	0.892	0.02	0.892	0.00	0.991
	$d_4 = 2d_1$	0.35	0.552	0.03	0.870	0.01	0.922
Strong monotonicity	$a_1 = a_2$	62.74	0.000	221.37	0.007	196.89	0.000
	$a_4 = a_3$	147.12	0.000	44.21	0.000	25.57	0.000
Fairness	$b_3 - b_2 = c_3 - c_2$	7.53	0.006	1.52	0.217	4.09	0.043