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STICKY WAGES IN A WORLD OF IDEAS

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Sticky Wages in a World of Ideas*

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Abstract

The search for new ideas by profit-seeking firms and knowledge spillovers are well-known and fundamental sources of modern economic growth. This paper examines the implications of idea production and knowledge capital for monetary business cycles. We construct a sticky-wage model where workers produce goods based on firm-specific knowledge capital and researchers develop new ideas aided by the economywide stock of knowledge. As a quantitatively small group in the economy, researchers are inconsequential for the real effects of monetary shocks when the returns to research are low. However, this intuitive conclusion can be overturned when the returns to research are high. In this situation, monetary shocks can have significant real effects as long as wages are sticky for researchers, even if wages are perfectly flexible for workers, who are quantitatively dominant in the economy.

JEL Classification: E22, E24, E31, E52

Keywords: Ideas, nonrivalry, knowledge capital, sticky wages, monetary neutrality

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1 Introduction

There is long debate concerning the impact of the search for new ideas on economic growth and business cycles. The economic growth literature has long recognized that how new ideas are produced lies at the heart of economic growth. For example, Jones (2019a) argues that the shape of the idea production function is "an intriguing subject of study" in growth models including research and development (R&D) activity. In contrast, the business cycle literature has revealed that R&D activity undertaken in the search for new ideas is procyclical.[1] Such procyclicality may influence fluctuations in output and employment, suggesting the importance of idea production on output and employment in business cycles.

Unfortunately, the business cycle literature has paid little attention to how ideas are produced in considering output and employment fluctuations. For example, sticky wages are one of the standard explanations for understanding the responses of output and employment to monetary shocks in New Keynesian models.[2] However, idea production is not commonly included in standard sticky-wage models. One reason for this is that researchers is quantitatively small in terms of economic activity. Table 1 shows the shares of R&D expenditure to GDP in selected developed countries. As shown, the R&D expenditure share to GDP is only around 1 to 3 percent in these countries. R&D by researchers is also small in terms of the total labor cost of researchers relative to workers. In evidence, the US labor cost ratio of researchers to workers is only about 0.08 according to the 2018 Occupational Employment Statistics (OES). Thus, while R&D activity is potentially important for business cycles, it is often neglected in the standard business cycle literature.

To address this deficiency, this paper explores the importance of knowledge and ideas in a New Keynesian sticky-wage model. We explicitly incorporate idea production and knowledge capital into a sticky-wage model using two types of labor, namely, workers and researchers. Workers produce traditional goods based on a firm's knowledge capital. Researchers produce new ideas to add to the firm's knowledge capital with the aid of the preexisting stock of economywide knowledge. These new ideas then enhance the economywide knowledge stock, improve the productivity of all researchers, and incentivize researchers to search for other

[1] Examples include Comin and Gertler (2006), Barlevy (2007), Ouyang (2011) and Mand (2019)

[2] See Huang and Liu (2002), Huang et al. (2004), Christiano et al. (2005), and Galí (2011) among others.

Table 1: Share of R&D expenditure to GDP

Year	Canada	France	Germany	Italy	Japan	UK	US
2000	1.86	2.09	2.41	1.00	2.91	1.62	2.63
2010	1.83	2.18	2.73	1.22	3.14	1.64	2.74
2019	1.59	2.20	3.19	1.47	3.20	1.76	3.07

Source: OECD, available at https://data.oecd.org/rd/gross-domestic-spending-on-r-d.htm. Numbers are percentages.

new ideas.

We investigate whether the idea production incorporated into a sticky-wage model matters for the real effects of monetary shocks. Our calibrated parameters for knowledge capital replicate the R&D expenditure share to GDP and labor cost ratio in the US economy. Using this calibration, we simulate the sticky-wage model to evaluate the real effects of monetary shocks.

We find that the shape of the idea production function can be an important factor in generating the real effects of monetary shocks. When the returns to research are low in idea production, the quantitative impact of researchers on the real effect of monetary shocks is small. To obtain the substantial real effect of a monetary shock, the wages of workers rather than researchers must be sticky. However, when researcher wages are sticky and worker wages are flexible, money is almost neutral. Indeed, the model dynamics are effectively the same as the model without researchers. Thus, when the returns to research are low, ideas and knowledge do not have a substantial impact on the nonneutrality of money.

In contrast, when the returns to research are high, monetary shocks exert significant real effects, regardless of the source of nominal wage rigidity. Remarkably, the real effect of money is substantial, even when worker wages are flexible. In our model, the contribution of researchers to the economy is quantitatively small in terms of the R&D expenditure share to GDP and the labor cost ratio of researchers to workers. Nevertheless, we obtain the substantial real effect of money provided that researcher wages are sticky. The real effect is comparable to that when there are nominal rigidities in the wages of workers, who are quantitatively dominant in the economy.

Our findings shed new light on the conventional argument by Jones (2019a) from the viewpoint of monetary business cycle models. In the idea production function, two parameters determine how ideas are produced: the standing-on-shoulders effect and the degree of returns to research. The growth literature emphasizes the standing-on-shoulders effect as a crucial factor in understanding economic growth (Jones, 2005). By contrast, our sticky-wage model highlights the degree of returns to research, rather than the standing-on-shoulders effect, as a crucial factor for the monetary transmission mechanism. We also find that the degree of returns to research is important for both real and nominal economic variables.

The rest of the paper is organized as follows. Section 2 describes our sticky-wage model incorporating firm-specific knowledge capital generated by idea production of researchers. Section 3 presents our main results. Section 4 conducts sensitivity analysis concerning some of the important parameters. Finally, Section 5 concludes.

2 The model

The model features a continuum of households indexed by $i \in [0,1]$, each consisting of a worker and a researcher, and a continuum of firms indexed by $j \in [0,1]$, each producing a differentiated good. The labor services of workers are differentiated and imperfectly substitutable, as are those of researchers. A government conducts monetary policy.

2.1 Households

At date t, the objective of household $i \in [0,1]$ is to maximize

$$\mathrm{E_t} \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s(i)) - V_W(N_{W,s}(i)) - V_R(N_{R,s}(i)) \right], \tag{1}$$

where $\mathrm{E_t}$ denotes the conditional expectations operator, $\beta \in (0,1)$ is a subjective discount factor, $C_s(i) = \left[\int_0^1 C_s(i,j)^{(\varepsilon_P - 1)/\varepsilon_P} dj \right]^{\varepsilon_P/(\varepsilon_P - 1)}$ is the household's consumption basket, with $\varepsilon_P > 1$ and $C_s(i,j)$ representing household i's demand for goods j. Also, $N_{W,s}(i)$ and $N_{R,s}(i)$ are the household's labor supply of a worker and a researcher, respectively. The functions U, V_W, and V_R are strictly increasing and twice-continuously differentiable, with concave U and convex V_W and V_R. The household's budget constraint in period t is

$$\int_0^1 P_t(j)\, C_t(i,j)\, dj \leq W_{W,t}(i) N_{W,t}(i) + W_{R,t}(i) N_{R,t}(i) - \mathrm{E}_t[D_{t,t+1} B_{t+1}(i)] + B_t(i) + \Pi_t(i), \tag{2}$$

where $P_t(j)$ denotes good j's price, $W_{W,t}(i)$ and $W_{R,t}(i)$ are the nominal wages of worker i and researcher i, respectively, $D_{t,t+1}$ is the stochastic discount factor from date $t+1$ to t, $B_{t+1}(i)$ is the household i's holdings of one-period state-contingent nominal bonds, and $\Pi_t(i)$ is the household's claim to firm profits.

Utility maximization gives rise to household i's demand for goods j,

$$C_t(i,j) = \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon_P} C_t(i),$$

where $P_t = [\int_0^1 P_t(j)^{1-\varepsilon_P} dj]^{1/(1-\varepsilon_P)}$. The total demand for good j from all households in the economy is then given by $C_t(j) = \int_0^1 C_t(i,j) di = [P_t(j)/P_t]^{-\varepsilon_P} C_t$, where $C_t = \int_0^1 C_t(i) di$.

Households are monopolistic competitors in labor markets, where they set wages for their worker and researcher. The total demands for household i's labor are given by

$$N_{h,t}(i) = \int_0^1 N_{h,t}(i,j) dj = \left[\frac{W_{h,t}(i)}{W_{h,t}}\right]^{-\varepsilon_h} N_{h,t}, \tag{3}$$

where $N_{h,t}(i,j)$ is the demand for household i's type-h labor from firm j, as given below, and $N_{h,t} = \int_0^1 N_{h,t}(j) dj$, for $h = W, R$. Taking the corresponding labor demand schedules as given, households set wages in a randomly staggered fashion à la Calvo (1983), with constant hazard rates θ_W and θ_R when unable to adjust wages for their workers and researchers, respectively.[3] If worker (researcher) i receives the opportunity to reset its wage in period t, then the wage will be chosen to satisfy

$$W_{h,t}(i) = \frac{\varepsilon_h}{\varepsilon_h - 1} \frac{\mathrm{E}_t \sum_{s=t}^{\infty} (\beta\theta_h)^{s-t} V_h'([W_{h,t}(i)/W_{h,s}]^{-\varepsilon_h} N_{h,s}) W_{h,s}^{\varepsilon_h} N_{h,s}}{\mathrm{E}_t \sum_{s=t}^{\infty} (\beta\theta_h)^{s-t} U'(C_s(i)) W_{h,s}^{\varepsilon_h} N_{h,s}/P_s}, \quad h = W, R. \tag{4}$$

Note that households are price takers in goods and bond markets. Irrespective of whether they can set wages

[3] It is worth noting that we have fixed the measures of workers and researchers and do not allow ones to become the others as responses to shocks to the economy. In this sense, as in typical business cycle studies, we abstract from modeling occupational choice.

in labor markets, they can always choose their goods consumption and bond holdings at any given date.

2.2 Firms

A firm j hires two types of labor services from households indexed by $i \in [0,1]$: namely, workers $N_{W,t}(i,j)$ and researchers $N_{R,t}(i,j)$. The former contributes to the production of the differentiated output. The latter creates and accumulates new ideas as firm-specific knowledge. These labor services employed by firm j are aggregated by the following constant elasticity of substitution function. For $h = W, R$,

$$N_{h,t}(j) = \left[\int_0^1 N_{h,t}(i,j)^{(\varepsilon_h-1)/\varepsilon_h} di\right]^{\varepsilon_h/(\varepsilon_h-1)},$$

where $\varepsilon_h > 1$, and the wage index for labor is given by

$$W_{h,t} = \left[\int_0^1 W_{h,t}(i)^{1-\varepsilon_h} di\right]^{1/(1-\varepsilon_h)}.$$

Note that workers and researchers from household i are indifferent to working in different firms. Thus, $W_{h,t}(i)$ is independent of j.

Cost minimization gives rise to firm j's demand for worker i:

$$N_{W,t}(i,j) = \left[\frac{W_{W,t}(i)}{W_{W,t}}\right]^{-\varepsilon_W} N_{W,t}(j),$$

and for researcher i

$$N_{R,t}(i,j) = \left[\frac{W_{R,t}(i)}{W_{R,t}}\right]^{-\varepsilon_R} N_{R,t}(j).$$

The symmetry across firms implies that the total demand for household i's labor $N_{h,t}(i)$ is

$$N_{h,t}(i) = \int_0^1 N_{h,t}(i,j)dj = \left[\frac{W_{h,t}(i)}{W_{h,t}}\right]^{-\varepsilon_h} N_{h,t},$$

which is consistent with (3). While wage takers in the labor markets, firms are monopolistic competitors in the goods market, where they set prices for their products at a markup $\mu_P = \varepsilon_P/(\varepsilon_P - 1)$ over their marginal costs of production.

Workers hired by firm j, $\{N_{W,t}(i,j)\}_{i\in[0,1]}$, produce goods j, $Y_t(j)$, based on firm j's knowledge capital $K_t(j)$ according to the following goods production function:[4]

$$Y_t(j) = F(N_{W,t}(j), K_t(j)). \tag{5}$$

The function F is strictly increasing in both $N_{W,t}(j)$ and $K_t(j)$.

Knowledge can be forgotten or become obsolete over time but can also be enhanced by additional new ideas. This notion is embedded in the following law of motion for firm j's knowledge capital:

$$K_t(j) = (1-\delta)K_{t-1}(j) + X_t\,(j)\,, \tag{6}$$

where $\delta \in (0,1)$ is the rate at which existing knowledge is forgotten or becomes obsolete over time.[5]

New ideas $X_t(j)$ are produced by the labor inputs of researchers for firm j, $\{N_{R,t}(i,j)\}_{i\in[0,1]}$, with the aid of preexisting economywide knowledge K_{t-1} which itself is derived from ideas developed in the past by all researchers in the economy.[6] Thus, the idea production function is

$$X_t(j) = G(N_{R,t}(j), K_{t-1}), \tag{7}$$

where the function G is strictly increasing in $N_{R,t}(j)$ but not necessarily increasing in K_{t-1}. The dependence of $X_t(j)$ on K_{t-1} reflects that knowledge becomes nonexcludable while it is nonrival. Individual firms and researchers are so small that they take the time path of economywide knowledge as given when searching for new ideas. This modeling of ideas and knowledge capital follows the approach taken in the endogenous growth literature emphasizing the nonrivalry of ideas.[7] In (7), we assume a one-period delay until knowledge

[4]We abstract away physical and human capital from the goods production function to make the key mechanism in our model as transparent as possible. Including these forms of capital would not alter the main results of this paper or add new insights in a substantive way.

[5]See, for example, Comin and Gertler (2006) for an introduction of the possibility that knowledge can become obsolete over time and Jones (2019a) for a reflection on this possibility.

[6]Unlike workers, the labor inputs of researchers have a long-lasting impact on the economywide knowledge stock. This durability in production technology resembles the role of durable consumption goods considered by Barsky et al. (2007).

[7]See the original contribution of Romer (1990) and many subsequent works. Jones (2019a) provides a comprehensive review of this defining feature of an idea production function, which is crucial for understanding economic growth.

becomes nonexcludable, but we can easily relax this assumption. For now, we leave the functional form of $G(N_{R,t}(j), K_{t-1})$ unspecified but will discuss two parameters in the idea production function, namely, the degrees of the standing-on-shoulders effect and the returns to research.

We emphasize that knowledge capital in our model has three important differences from physical capital. First, unlike physical capital, knowledge is nonrival and nonexcludable, as emphasized in Romer (1990) and Jones (2002). Second, the idea production function $G(N_{R,t}(j), K_{t-1})$ for producing knowledge capital substantially differs from the good production function $F(N_{W,t}(j), K_t(j))$ for producing physical capital in their properties. Recent evidence provided by Bloom et al. (2020) suggests that idea production may decrease with economywide knowledge. That is, ideas become harder to find as knowledge accumulates, i.e., $\partial G(N_{R,t}(j), K_{t-1})/\partial K_{t-1} < 0$. Third, the depreciation rate of knowledge capital is often assumed to be zero, in contrast to physical capital. In the growth literature, it is standard to assume that changes in knowledge capital are equal to idea production, $\Delta K_t(j) = G(N_{R,t}(j), K_{t-1})$. In our context, this equation holds if $\delta = 0$. Therefore, throughout the paper, we assume a positive value of δ to ensure the stationarity of knowledge capital but taking an extremely small value in line with the growth literature.

Following Schmitt-Grohé and Uribe (2006, 2007), we assume that firms face a cash-in-advance (CIA) constraint for their wage payments:

$$M_t(j) = \int_0^1 W_{W,t}(i)\, N_{W,t}(i,j)\, di + \int_0^1 W_{R,t}(i)\, N_{R,t}(i,j)\, di, \tag{8}$$

where $M_t(j)$ is firm j's demand for money and the right-hand side of the equation is the wage bill paid by firm j. The firm j's profits $\Pi_t(j)$ is given by

$$\begin{aligned}\Pi_t(j) = {} & P_t(j) \int_0^1 C_t(i,j)\, di + T_t(j) \\ & + \left[M_{t-1}(j) - \int_0^1 W_{W,t-1}(i)\, N_{W,t-1}(i,j)\, di - \int_0^1 W_{R,t-1}(i)\, N_{R,t-1}(i,j)\, di \right] - M_t(j)\,. \end{aligned} \tag{9}$$

In (9), the first two terms on the right-hand side of the equation represent cash received by firm j. The firm sells its final goods to households at price $P_t(j)$ and receives monetary transfers $T_t(j)$ from the government.

The expression inside the brackets on the right-hand side of (9) represents the cash remaining at the beginning of period t. The firm must hold money to pay the wage bill at the end of period t.

Aggregating $M_t(j)$ over j yields the total demand for money M_t in this economy:

$$\begin{aligned} M_t &= \int_0^1 M_t(j)\,dj \\ &= W_{W,t}\int_0^1 N_{W,t}(j)\,dj + W_{R,t}\int_0^1 N_{R,t}(j)\,dj \\ &= W_{W,t}N_{W,t} + W_{R,t}N_{R,t}, \end{aligned} \tag{10}$$

where $N_{h,t} = \int_0^1 N_{h,t}(j)dj$ for $h = W, R$.

At date t, firm j also chooses $\{N_{W,s}(j), N_{R,s}(j), K_s(j)\}_{s\geq t}$ to maximize

$$\mathrm{E_t}\sum_{s=t}^{\infty} D_{t,s}\left[P_s(j)\,Y_s(j) - W_{W,s}N_{W,s}(j) - W_{R,s}N_{R,s}(j) + T_s(j)\right], \tag{11}$$

subject to (5)–(7).[8] Here, $D_{t,s} = \prod_{\tau=1}^{s-t} D_{t+\tau-1,t+\tau}$ denotes the s-period stochastic discount factor from s to t, for all $s > t$, with $D_{t,t} \equiv 1$. This profit-maximization problem takes into account the solution of the embodied cost-minimization problem.

The first-order conditions for $N_{W,t}(j)$, $N_{R,t}(j)$, and $K_t(j)$ are given by

$$\frac{W_{W,t}}{P_t} = F^*_{N,t}, \tag{12}$$

$$Q_t = \frac{W_{R,t}}{G_{N,t}}, \tag{13}$$

$$Q_t = P_t F^*_{K,t} + (1-\delta)\,\mathrm{E_t} D_{t,t+1} Q_{t+1}, \tag{14}$$

where we impose symmetry across firms and drop the individual firm's index j. Here Q_t denotes the Lagrange multiplier for (6) (or the shadow price of ideas). This measures the nominal marginal benefit to a firm of increasing its knowledge capital and, in equilibrium, equals its nominal marginal cost of producing new ideas given by $W_{R,t}/G_{N,t}$ where $G_{N,t} \equiv \partial G(N_{R,t}, K_{t-1})/\partial N_{R,t}$. To simplify the exposition, we also introduce two auxiliary notations, $F^*_{N,t}$ and $F^*_{K,t}$, to denote the marginal products of inputs adjusted for the price

[8] In (11), we substitute the CIA constraint (8) into the firm j's profits because (8) always holds with equality.

markup μ_P, that is, $F^*_{N,t} \equiv \mu_P^{-1} \left[\partial F\left(N_{W,t}, K_t\right)/\partial N_{W,t}\right]$ and $F^*_{K,t} \equiv \mu_P^{-1} \left[\partial F\left(N_{W,t}, K_t\right)/\partial K_t\right]$.

2.3 The government

The government finances the transfers to firms by issuing money. The budget constraint is

$$\int_0^1 T_t\left(j\right) dj = M_t^s - M_{t-1}^s.$$

The money supply M_t^s grows at a rate e^{ξ_t}:

$$M_t^s = e^{\xi_t} M_{t-1}^s, \tag{15}$$

where ξ_t is a white-noise process. Every period, the money market clears: $\int_0^1 M_t\left(j\right) dj = M_t = M_t^s$.

2.4 GDP

In our model economy, nominal GDP is defined by $P_t C_t + Q_t X_t$, where Q_t is interpreted as the imputed value of new ideas. We define real GDP as

$$Y_t^{GDP} = C_t + \frac{Q_t}{P_t} X_t. \tag{16}$$

With the inclusion of the imputed value of new ideas in GDP, we may interpret idea production by researchers in our model as the official measure of R&D in the recent redefinition of GDP where we treat R&D not as an intermediate expense but as a final investment.[9] Overall, the narrowly measured R&D included in official GDP data is only a small part of real-world innovation processes.[10] In the next section, we investigate the impact of narrowly measured R&D investment on monetary nonneutrality.

[9] We could also define real GDP as output at constant prices, as in Barsky et al. (2007). That is, $Y_t^{GDP} = PC_t + QX_t$, where P and Q are the steady-state values of P_t and Q_t, respectively. While the definition slightly differs from (16), our results are robust to the alternative definition.

[10] See, for example, Isaacson (2014), Wolfe (2014) and Jones (2019b).

3 Main results

We illustrate the main results of this paper using the model's impulse responses to a monetary shock. To generate the impulse response functions, we specify the utility and production functions and then assign values to the model's parameters. After providing the main results, we inspect the mechanism behind them. We then derive the implications of idea production for monetary business cycle models.

3.1 Functional forms and parameter values

We postulate the following functional forms

$$U\left(C_t\left(i\right)\right) = \frac{C_t\left(i\right)^{1-\sigma}}{1-\sigma}, \tag{17}$$

with $\sigma > 0$ for the period utility of consumption and

$$V_h\left(N_{h,t}\left(i\right)\right) = \frac{N_{h,t}\left(i\right)^{1+\psi_h}}{1+\psi_h}, \tag{18}$$

with $\psi_h > 0$ and $h = W, R$ for the period disutility of labor. In this case, the first-order condition (4) can be rewritten as

$$W_{h,t}(i) = \frac{\varepsilon_h}{\varepsilon_h - 1} \frac{\mathrm{E_t} \sum_{s=t}^{\infty} (\beta\theta_h)^{s-t} MRS_{h,s|t}\left(i\right) P_s \left(N_{h,s|t}(i) / \left\{\left[C_s\left(i\right)\right]^{\sigma} P_s\right\}\right)}{\mathrm{E_t} \sum_{s=t}^{\infty} (\beta\theta_h)^{s-t} \left(N_{h,s|t}(i) / \left\{\left[C_s\left(i\right)\right]^{\sigma} P_s\right\}\right)}, \tag{19}$$

where $N_{h,s|t}(i) = [W_{h,t}(i)/W_{h,s}]^{-\varepsilon_h} N_{h,s}$ and $MRS_{h,s|t}\left(i\right)$ is

$$MRS_{h,s|t}\left(i\right) = \left[C_s\left(i\right)\right]^{\sigma} \left[N_{h,s|t}\left(i\right)\right]^{\psi_h}, \tag{20}$$

for $h = W, R$.

The goods production function is specified as

$$F(N_{W,t}(j), K_t(j)) = N_{W,t}\left(j\right)^{1-\alpha} K_t\left(j\right)^{\alpha}, \tag{21}$$

where $\alpha > 0$. We assume the idea production function is given by

$$G\left(N_{R,t}(j), K_{t-1}\right) = N_{R,t}(j)^{\phi} K_{t-1}^{\lambda}, \tag{22}$$

where ϕ represents the degree of returns to research and λ captures the degree of the standing-on-shoulders effect. We impose $\phi > 0$ on the idea production function. However, as noted earlier, the sign of λ can be negative. If $\lambda < 0$, ideas become harder to find as knowledge accumulates (i.e., the so-called fishing-out effect).

We now assign values to the model's parameters. The subjective discount factor β is chosen to be consistent with an annualized steady-state real interest rate of 2 percent. We set the degree of relative risk aversion in consumption to unity, along with the inverse of the Frisch elasticities of labor supply ($\sigma = \psi_W = \psi_R = 1$). We set the elasticity of substitution between differentiated goods, ε_P, to 11, and the elasticity of substitution between differentiated labor, ε_h, to 5, for $h = W, R$. When we consider wage stickiness for workers (researchers), we set the hazard rates θ_W (θ_R) to 0.75. Thus, the average duration of newly set wages for individual workers (researchers) is four quarters.[11] When we assume that the wages of workers (researchers) are flexible, we set θ_W (θ_R) to 0, so that the average duration of newly set wages for individual workers (researchers) is one quarter. These parameter values appear reasonable choices considering the sticky-wage literature (e.g., Huang and Liu, 2002). As we demonstrate in later sensitivity analysis, the basic conclusion of this paper is robust to alternative values of these parameters within empirically plausible ranges.

The idea production function (22) has crucial implications for predictions of our model. To be specific, combine (22) with (6). Given the symmetry of $K_t(j) = K_t$ and $N_{R,t}(j) = N_{R,t}$, we have

$$\Delta K_t + \delta K_{t-1} = N_{R,t}^{\phi} K_{t-1}^{\lambda}. \tag{23}$$

[11]We treat the two hazard rates θ_W and θ_R symmetrically even if the labor types differ. The micro evidence on nominal wage adjustment suggests that the degrees of stickiness in wages do not differ substantially regardless of labor type. For example, Barattieri et al. (2014) show that differences in the frequency of wage changes are quantitatively small across occupations for the US. Le Bihan et al. (2012) and Sigurdsson and Sigurdardottir (2016) report similar results for France and Iceland, respectively. Barattieri et al. (2014) also finds that the frequency of wage changes ranges between 16.3 and 21.6 percent. The reported values suggest that our parameterization of $\theta_h = 0.75$ is not inconsistent with the data because the observed frequencies imply a hazard rate that ranges from 0.78 to 0.83.

In Romer's original contribution (i.e., Romer, 1990), the degrees of returns to research and the standing-on-shoulders effect were both set to unity. That is, $\phi = \lambda = 1$. More recent studies argue that a λ less than 1 is important for reconciling idea-based growth theory with the data.[12] A recent paper by Bloom et al. (2020) even suggests $\lambda < 0$.

We parameterize ϕ and λ as follows. Rewrite (23) to obtain the steady-state knowledge capital K as a function of the steady-state labor of researchers N_R:

$$K = \left(\frac{1}{\delta}\right)^{\eta} N_R^{\eta},$$

where $\eta = \phi/(1-\lambda)$. Bloom et al. (2020) suggest that a plausible estimate of η for the aggregate economy is 0.32. They continue to assume that $\phi = 1$ as in Romer (1990). Thus, the estimate of η can be translated into $\lambda = -2.13$. If the returns to research are decreasing ($\phi < 1$), the value of λ depends on how we calibrate ϕ. To explore the impact of idea production on business cycles, we consider two values for ϕ, namely, $\phi = 0.5$ (low returns to research) and $\phi = 1$ (high returns to research). According to these parameter values, λ is set to -0.56 and -2.13, respectively.

The other remaining parameters for technology are the returns to knowledge α in (21) and the knowledge capital depreciation rate δ in (6). To calibrate these parameters, we target the steady-state share of R&D expenditure to GDP and the steady-state ratio of labor costs of researchers to workers. The R&D expenditure share is given by

$$\frac{(Q/P)X}{C + (Q/P)X} = \frac{\alpha\delta\left[1-\beta\left(1-\delta\right)\right]^{-1}}{\mu_P + \alpha\delta\left[1-\beta\left(1-\delta\right)\right]^{-1}}, \tag{24}$$

and the labor cost ratio can be calculated as

$$\frac{W_R N_R}{W_W N_W} = \frac{\alpha}{1-\alpha}\frac{\phi\delta}{1-\beta\left(1-\delta\right)}, \tag{25}$$

where a variable without a time subscript denotes its steady-state value. To derive these expressions, we assume that $\psi_W = \psi_R$.

[12]See Jones (1995, 1999, 2002, 2005) and Fernald and Jones (2014) for details.

Regarding the share of R&D expenditure to GDP, we set the share at 3 percent based on the US data in 2019 shown in Table 1. We set the labor cost ratio at 0.08 using data from the 2018 OES compiled by the Bureau of Labor Statistics. The OES reports the employment and annual mean wages of various occupations. Our definition of "researchers" includes some research-related managers, computer and mathematical occupations, engineers, scientists and researchers, and doctors.[13] All remaining occupations are considered "workers." Using this classification, we obtain the mean annual wage and the number of employed. As a result, the calibrated parameter values are $(\alpha, \delta) = (0.79, 2.26 \times 10^{-4})$ when $\phi = 0.5$ and $(\alpha, \delta) = (0.58, 3.15 \times 10^{-4})$ when $\phi = 1$.[14] Interestingly, the parameter values of δ are positive but close to zero, consistent with our assumption (e.g., 3.15×10^{-4}).

We emphasize that both the share of R&D expenditure to GDP and the labor cost ratio are too low to generate significant aggregate effects on output. Below we show that the seemingly unimportant researchers can have a substantial impact on real effect of monetary shocks, depending on the shape of the idea production function.

3.2 Responses of real GDP

We investigate the real effect of a monetary shock in the sticky-wage model using the two levels of returns to research ($\phi = 0.5$ and 1) in idea production. Figure 1 plots the impulse responses of real GDP to a one-percent increase in the money supply under different configurations of nominal wage rigidities. The dotted line represents the responses of real GDP when the wages of both workers and researchers are sticky ("W-sticky/R-sticky," $\theta_W = \theta_R = 0.75$). The dashed line corresponds to the responses when worker wages are sticky, but researcher wages are flexible ("W-sticky/R-flexible," $\theta_W = 0.75$, $\theta_R = 0$). However, we have a particular interest in the configuration where researcher wages are sticky, but worker wages are flexible ("W-flexible/R-sticky," $\theta_W = 0$, $\theta_R = 0.75$).

[13] The Appendix provides the details of all occupations included in our definition of "researchers." Admittedly, classifying occupations in the OES into our definition of "researchers" is somewhat arbitrary. For example, doctors may produce knowledge, but their knowledge may not necessarily contribute to the production of consumption goods. However, we confirm that excluding doctors from researchers leads to only a minor impact on our main results.

[14] Jones (1995) also includes physical capital in a goods production function. We can do the same here without altering our main conclusion in any substantial way. We can also insert in (21) a term for physical capital that is kept constant at the business cycle frequency (an assumption often made in the monetary business cycle literature) without changing any of our results.

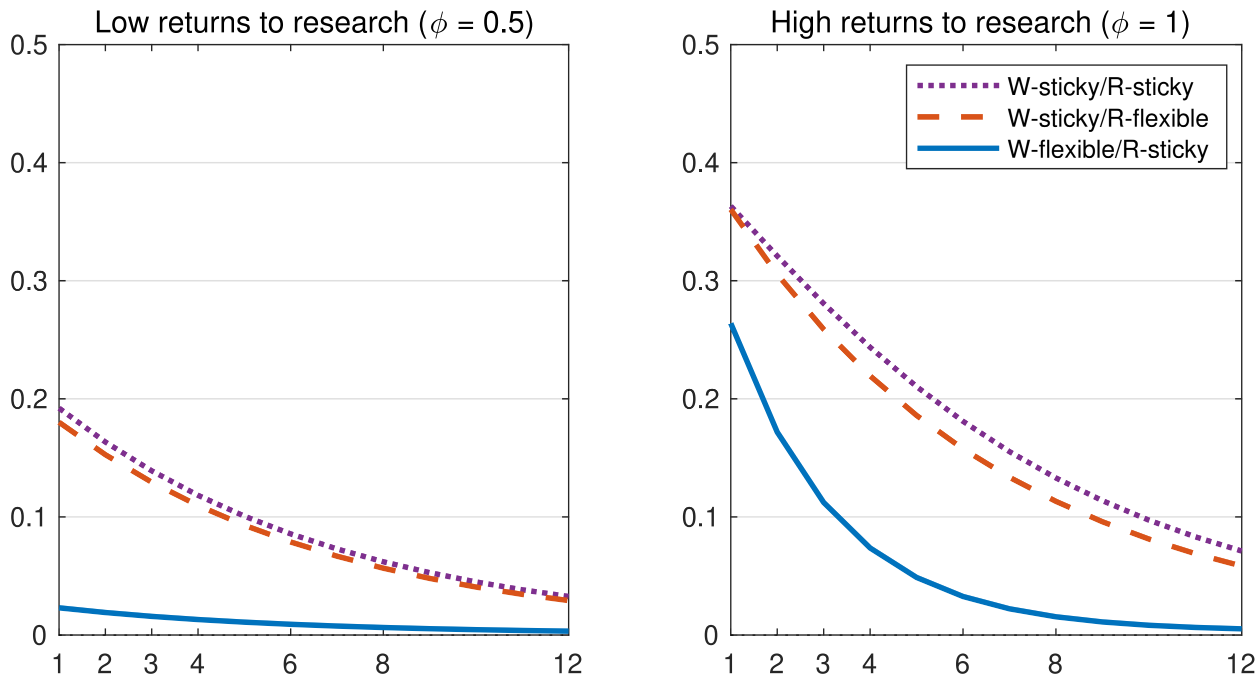

Figure 1: Responses of real GDP to a monetary shock

Notes: The left panel depicts the responses of real GDP to a one-percent increase in the money supply when the returns to research are low in idea production ($\phi = 0.5$). The right panel presents those when the returns to research are high ($\phi = 1$). In each panel, dotted lines represent the responses of real GDP when the wages of both workers and researchers are sticky; dashed lines are when worker wages are sticky, but researcher wages are flexible; solid lines are when worker wages are flexible, but researcher wages are sticky. Vertical axes measure percentage deviations from the steady state. Time is given in quarters on the horizontal axes.

The left panel of Figure 1 depicts the impulse responses under $\phi = 0.5$. In the cases of W-sticky/R-sticky (the dotted line) and W-sticky/R-flexible (the dashed line), we confirm that the real effects of a monetary shock are substantial because of sticky wages of workers. Indeed, the initial responses of real GDP are 0.19 and 0.18 percent, respectively. The effects of the monetary shock on real GDP take more than a year to return to the steady state of zero. Comparing the responses in these configurations, we also find that they are similar. When researcher wages are sticky, but worker wages are flexible (i.e., W-flexible/R-sticky, being the solid line), the real effect of a monetary shock on real GDP is small. The response is 0.02 percent on impact and extremely insignificant compared to those in the other two configurations.

The reason for these results is straightforward. In this model, researchers are a quantitatively small group in the economy. In our model, steady-state R&D expenditure accounts for only 3 percent of all expenditures and the steady-state labor cost ratio of researchers to workers is only 0.08. Therefore, the presence of researchers in the model does not strongly affect the responses of real GDP. As a result, the responses of real GDP under W-sticky/R-sticky and W-sticky/R-flexible will be similar. The extremely small responses of real GDP under W-flexible/R-sticky are also easy to understand because nominal rigidities are present only in the wages of researchers, who are quantitatively small in the economy. Thus, this observation suggests that, not surprisingly, nominal rigidities in the wages of workers, who are quantitatively dominant in the economy, are necessary for the strong real effects of a monetary shock.

However, when the returns to research are high (i.e., $\phi = 1$), this result can be overturned. In the right panel of Figure 1, we again plot the impulse responses of output, but in the economy with $\phi = 1$. The solid line in the right panel shows that the real effect can be strong under W-flexible/R-sticky. More specifically, even when nominal rigidities are present only in the wages of researchers, who are quantitatively small in the economy, real GDP increases by a large amount (e.g., 0.26 percent on impact). The positive responses of real GDP are visible and comparable to those in the other configurations.[15] Although the real effect converges to zero more quickly than in the other two configurations, the initial response of 0.26 percent under W-flexible/R-sticky is not substantially lower than the 0.36 percent under W-sticky/R-sticky and W-sticky/R-flexible.

A key message is that the shape of the idea production function critically affects the real effect of a monetary shock. When the returns to research are low, researchers are insignificant in generating real effects. However, when the returns to research are high, we find that researchers play a nonnegligible role in generating the real effects of a monetary shock. Importantly, even if wages are fully flexible in most labor markets, stickiness only in the wages of researchers can generate strong real effects of monetary shocks to an extent comparable to the case where nominal wages are sticky in all labor markets.

3.3 Inspecting the mechanism

In this section, we inspect the transmission mechanism of monetary shocks. Using the goods and idea production functions, real GDP is given by $C_t + (Q_t/P_t)X_t = F\left(N_{W,t}, K_t\right) + (Q_t/P_t)G\left(N_{R,t}, K_{t-1}\right)$. Thus,

[15] We note that the calibrated values of returns to knowledge in (21) differ between the two panels. We calibrate α at 0.79 in the left panel and 0.58 in the right panel. However, even if we use the same α for the right panel, the responses of real GDP in the right panel do not substantially differ.

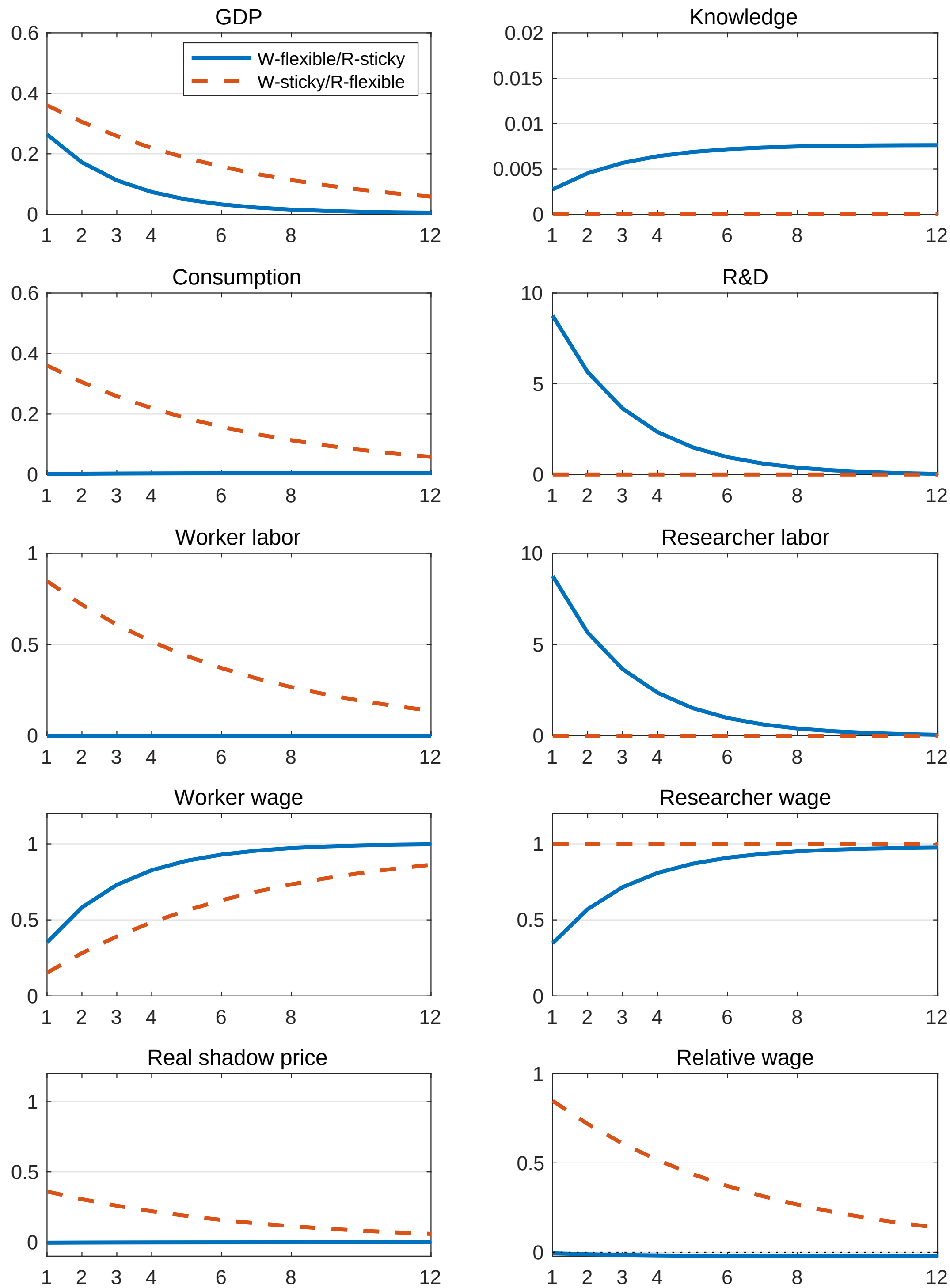

Figure 2: Responses to a monetary shock

Notes: Each panel depicts the responses of the corresponding variables in our sticky-wage model to a one-percent increase in the money supply given the two alternative configurations for wage stickiness. Solid lines represent the case under W-flexible/R-sticky. Dashed lines are for W-sticky/R-flexible. In the figure, the returns to research are high ($\phi = 1$). Vertical axes measure percentage deviations from the steady state. Time is given in quarters on the horizontal axes.

the real effect of a monetary shock on real GDP is determined by the responses of four variables: the labor services of workers, $N_{W,t}$, knowledge capital, K_t, the real shadow price of ideas, Q_t/P_t, and the labor services of researchers, $N_{R,t}$. Figure 2 plots the impulse responses of these variables to a monetary injection as well as other variables of interest under our baseline parameterization. In the figure, the returns to research are parameterized to be high (i.e., $\phi = 1$). The figure compares the responses under the most important configuration of W-flexible/R-sticky to those under W-sticky/R-flexible. As shown in Figure 2, $N_{W,t}$, K_t, and Q_t/P_t are almost unchanged under W-flexible/R-sticky. In what follows, we explain why this is the case. Then, we explore how increases in $N_{R,t}$ lead to the real effect of monetary shocks.

We first consider knowledge capital K_t. If the depreciation rate δ is low, the flow–stock ratio is so low that even large changes in idea production have small effects on the stock of knowledge capital. Therefore, as shown in the first row of Figure 2, a monetary injection does not produce a sizable increment in the stock of knowledge capital, independent of the configurations of nominal rigidities. Consequently, it is helpful to treat knowledge capital as roughly constant: $K_t \simeq K$.

Second, we consider the labor services of workers $N_{W,t}$. In equilibrium, wage markups for workers $\mu_{W,t}$ equal the gap between the marginal product of workers $F^*_{N,t}$ in (12) and the marginal rate of substitution of their labor for consumption $V'_W(N_{W,t})/U'(C_t)$. This relationship can be expressed as

$$\mu_{W,t} = F^*_N(N_{W,t}, K_t) \Bigg/ \left\{ \frac{V'_W(N_{W,t})}{U'\left[F(N_{W,t}, K_t)\right]} \right\}, \tag{26}$$

where we used $C_t = F(N_{W,t}, K_t)$. Equation (26) suggests that $N_{W,t}$ effectively has a one-to-one relationship to wage markups for workers $\mu_{W,t}$ given the near constancy of K_t. Thus, $N_{W,t}$ does not respond to a monetary injection if worker wages are flexible (i.e., $\mu_{W,t}$ is constant for all t). Similarly, consumption exhibits extremely small movement due to $C_t = F(N_{W,t}, K_t)$ along with the muted responses of K_t and $N_{W,t}$ to the monetary shock.[16] Therefore, as the solid lines in the second and third rows of Figure 2 show, the responses of $N_{W,t}$ and C_t are almost zero under W-flexible/R-sticky.[17]

[16] The validity of our analysis and the basic conclusion does not depend on how we parameterize preferences and technology. In fact, it is sufficient to assume $U' > 0$, $U'' \leq 0$, $V'_W > 0$, $V''_W \geq 0$, $F^*_N > 0$, $F^*_K \geq 0$, $F^*_{NK} \geq 0$, and $F^*_{NN} \leq 0$.

[17] Under W-sticky/R-flexible, $\mu_{W,t}$ decreases in response to a positive monetary shock. As a result, the labor services of

Finally, consider the real shadow price of ideas Q_t/P_t. To this end, we rewrite (14) as

$$\begin{aligned} \frac{Q_t}{P_t} &= F_K^*\left(N_{W,t}, K_t\right) + \beta\left(1-\delta\right)\mathrm{E_t}\left[\frac{U'(C_{t+1})}{U'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right] \\ &\simeq F_K^*\left(N_{W,t}, K_t\right) + \beta\left(1-\delta\right)\mathrm{E_t}\left[\frac{Q_{t+1}}{P_{t+1}}\right], \end{aligned} \tag{27}$$

where the approximate equality in the second line results from the fact that $C_t = F(N_{W,t}, K_t)$ is almost constant under W-flexible/R-sticky. Equation (27) implies that Q_t/P_t is stable over time when worker wages are flexible because $N_{W,t}$ and K_t are all near constant in response to a monetary shock. Indeed, Q_t/P_t shown in the solid line of the bottom row of Figure 2 indicates that the responses of the shadow price of ideas are again almost zero under W-flexible/R-sticky.

All the above observations imply that the real effect of a monetary shock under W-flexible/R-sticky depends almost entirely on how $N_{R,t}$ responds. Note that wage markups for researchers $\mu_{R,t}$ equal the gap between their marginal revenue product $(Q_t/P_t)G_{N,t}$ and the marginal rate of substitution of their labor for consumption $V_R'\left(N_{R,t}\right)/U'\left(C_t\right)$. This relationship can be expressed as

$$\mu_{R,t} = \left[\frac{Q_t}{P_t}G_N(N_{R,t}, K_{t-1})\right] \Big/ \left\{\frac{V_R'(N_{R,t})}{U'[F(N_{W,t}, K_t)]}\right\}, \tag{28}$$

where we replace C_t in $U'(C_t)$ by $F(N_{W,t}, K_t)$. In the right-hand side of (28), K_t, $N_{W,t}$, and Q_t/P_t are all near constant. The stability of these variables implies that $N_{R,t}$ has a one-to-one relationship with $\mu_{R,t}$.

The transmission mechanism of a monetary injection under W-flexible/R-sticky is as follows. When M_t increases, it relaxes the CIA constraint of firms and increases the nominal demand for labor inputs. Under W-flexible/R-sticky, a monetary injection lowers the wage markups only for researchers. In response to the decline in the wage markups for researchers, $N_{R,t}$ increases.[18] The increase in $N_{R,t}$ leads to an increase in idea production X_t. An increase in X_t results in an increase in real GDP given by $C_t + (Q_t/P_t)X_t$ while C_t and Q_t/P_t remain unchanged.

workers $N_{W,t}$ and thus the consumption C_t increase (see the dashed lines in Figure 2). Given a large expenditure share of consumption in GDP, the sticky wages of workers play a dominant role in generating the significant real effects of a monetary shock.

[18]We can confirm this inverse relationship from the concavity of G with respect to $N_{R,t}$ and the convexity of V_R in (28).

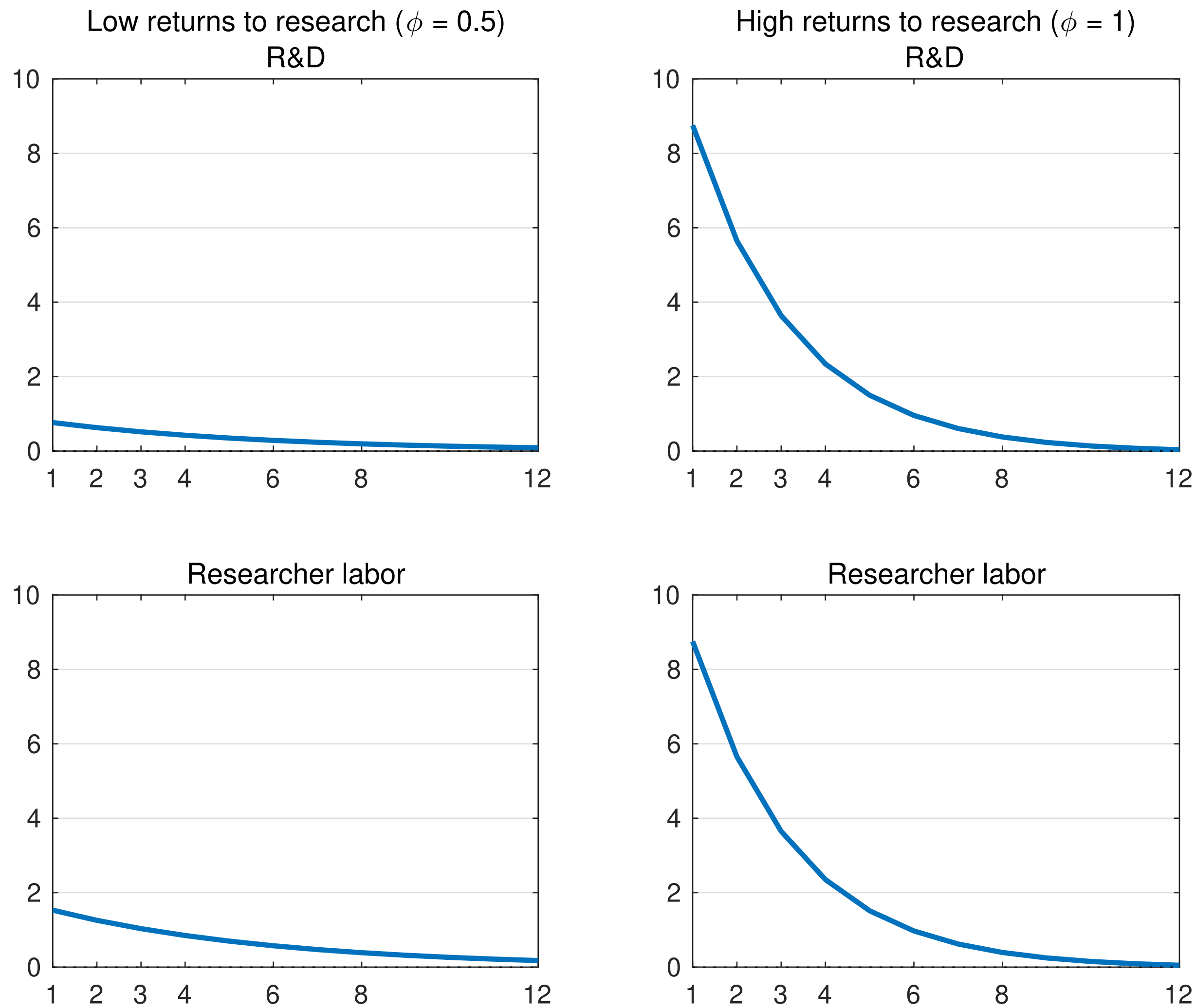

Figure 3: Responses of R&D and researcher labor services to a monetary shock

Notes: Top panels depict the responses of R&D to a one-percent increase in the money supply, and the bottom panels those of researcher labor services. Left panels exhibit the case when the returns to research are low ($\phi = 0.5$), while the right panels correspond to those when the returns to research are high ($\phi = 1$). For the configuration of nominal wage rigidities, worker wages are flexible, but researcher wages are sticky (W-flexible/R-sticky). Vertical axes measure percentage deviations from the steady state. Time is given in quarters on the horizontal axes.

3.4 Importance of the returns to research

In the previous subsection, we showed that the output responses to a monetary shock critically depend on responses of idea production. This subsection discusses the importance of returns to research in idea production for the real effects of monetary shocks.

Figure 3 compares the responses of idea production (R&D) and those of researcher labor services to a monetary shock with different degrees of returns to research ($\phi = 0.5$ on the left panels and $\phi = 1$ on the right panels). As the upper panels of the figure indicate, the responses of X_t under $\phi = 1$ are substantially larger than those under $\phi = 0.5$. In particular, a one-percent increase in the money supply leads to an increase of X_t by 8.75 percent at impact under $\phi = 1$, but the increase is only 0.76 percent under $\phi = 0.5$. Turning to the lower panels, the initial response of $N_{R,t}$ is 8.75 percent under $\phi = 1$ while it is only 1.53 percent under $\phi = 0.5$. Again, there is a substantial difference between the cases with high and low returns to research.

When returns to research are high, there is a strong incentive to hire researchers in response to a monetary injection. This incentive is reinforced by the strong demand for ideas because knowledge is a durable input that lasts many periods. As the returns to research are not decreasing in $N_{R,t}$, the large increase in researcher labor services $N_{R,t}$ is transmitted to idea production X_t. Thus, we obtain a substantial real effect of money even though researchers are quantitatively small in the economy. By contrast, when the returns to research are low, the incentive to hire researchers is weak. Given that the productivity of research decreases with $N_{R,t}$, the increase in $N_{R,t}$ is further weakly transmitted to X_t. Consequently, the output response to a monetary shock is near zero.

Different degrees of returns to research affect (i) the sensitivity of X_t to $N_{R,t}$ and (ii) the sensitivity of $N_{R,t}$ to $\mu_{R,t}$. Using our specifications for the idea production function $G(N_{R,t}, K_{t-1}) = N_{R,t}^{\phi} K_{t-1}^{\lambda}$ and the period disutility function $V(N_{R,t}) = N_{R,t}^{1+\psi_R}/(1+\psi_R)$, we can approximate (22) and (28) as

$$\frac{d\widehat{X}_t}{d\widehat{N}_{R,t}} \simeq \phi, \tag{29}$$

$$\frac{d\widehat{N}_{R,t}}{d\widehat{\mu}_{R,t}} \simeq \frac{-1}{1+\psi_R-\phi}, \tag{30}$$

where hatted variables represent the log deviation of the corresponding variables from the steady state. In deriving these equations, we assume that K_t, $N_{W,t}$, and Q_t/P_t are approximately constant due to W-flexible/R-sticky. It is important to note that, as the returns to research ϕ increase from 0.5 to 1, the sensitivity of both $\widehat{X}_t$ to $\widehat{N}_{R,t}$ and $\widehat{N}_{R,t}$ to $\widehat{\mu}_{R,t}$ increase.

We emphasize that the main driver of idea production in this monetary business cycle model is the labor services of researchers rather than knowledge capital. Because K_t is near constant in response to a monetary shock, the role of knowledge capital in idea production in business cycle frequency is limited. Variations in idea production mostly stem from changes in the labor services of researchers.

Therefore, our findings shed new light on the discussion regarding the shape of the idea production function. Jones (2005, 2019a) emphasize that the standing-on-shoulders effect determines the long-run trend in economic growth. We find instead that the returns to research determine the real effect of monetary shocks in business cycle frequency. In particular, the parameter ϕ capturing the returns to research in $G(N_{R,t}, K_{t-1}) = N_{R,t}^{\phi} K_{t-1}^{\lambda}$ strongly influences idea production through changes in labor services of researchers. In contrast to ϕ, the parameter λ capturing the standing-on-shoulders effect plays only a minor role in the real effect of monetary shocks because of the near constant knowledge capital. Nevertheless, the shape of the idea production function matters in generating the real effects of monetary shocks. In this sense, our study complements Jones (2005, 2019a).

3.5 Responses of prices and wages

Our monetary business cycle model allows us to further explore the implications of the shape of the idea production function for nominal variables. Figure 4 compares the responses of nominal prices and wages to a monetary shock between high and low returns to research. In the figure, the configuration of nominal rigidities remains W-flexible/R-sticky.

The case with high returns to research delivers interesting implications for wages and prices. Under W-flexible/R-sticky, all prices other than researcher wages (P_t, $W_{W,t}$, Q_t) are flexible. Interestingly, Figure 4 indicates that the adjustment of these prices to the new steady state slows as ϕ increases from $\phi = 0.5$ to $\phi = 1$. When the returns to research are low ($\phi = 0.5$), the consumption good price, the shadow price of ideas, and worker wages adjust immediately to a value close to one percent in response to a one-percent increase in the money supply. By contrast, when the returns to research are high, the initial responses are only about 0.35 percent. In other words, higher returns to research generate a slower adjustment of prices, even if they are completely flexible.

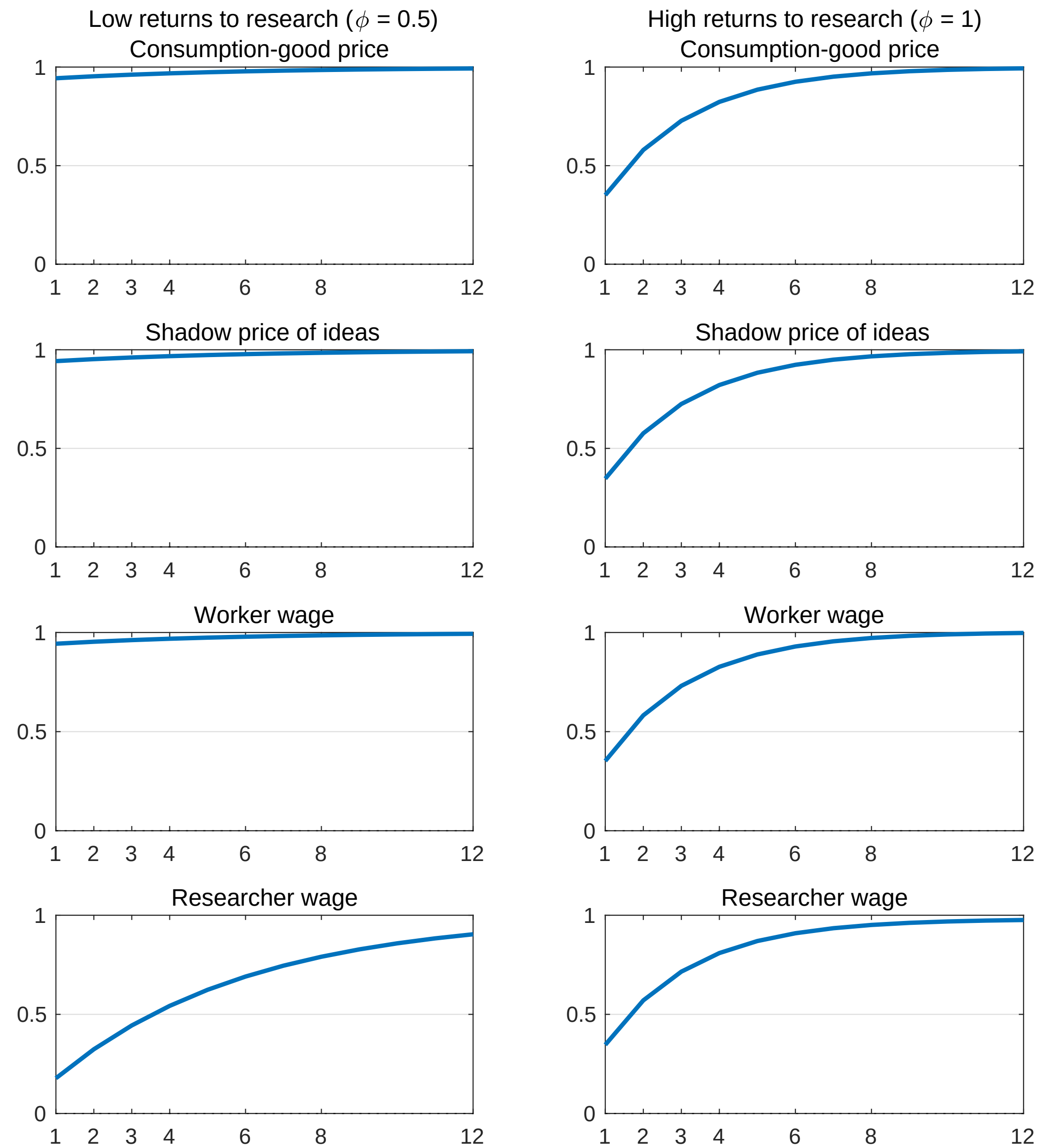

Figure 4: Responses of prices and wages to a monetary shock

Notes: Each panel depicts the responses of the corresponding variables in our sticky-wage model to a one-percent increase in the money supply. Left panels show the case when the returns to research are low ($\phi = 0.5$). Right panels correspond to the case when the returns to research are high ($\phi = 1$). For the configuration of nominal wage rigidities, worker wages are flexible, but researcher wages are sticky (W-flexible/R-sticky). Vertical axes measure percentage deviations from the steady state. Time is given in quarters on the horizontal axes.

This slower adjustment results from complementary nature across prices and wages in our model. When $\phi = 0.5$, the responses of P_t, $W_{W,t}$, and Q_t are close to each other, and they substantially differ from those of $W_{R,t}$. However, when $\phi = 1$, these flexible prices move in tandem with sticky researcher wages.

We can easily observe this synchronization with sticky researcher wages from (12) and (13). These equations imply

$$\frac{W_{R,t}}{W_{W,t}} = \frac{Q_t}{P_t}\frac{G_N(N_{R,t}, K_{t-1})}{F_N^*(N_{W,t}, K_t)}. \tag{31}$$

As discussed in Section 3.3, Q_t/P_t and $F_N^*(N_{W,t}, K_t)$ are stable over time under W-flexible/R-sticky. Thus, the response of the relative wage critically depends on $G_N(N_{R,t}, K_{t-1}) = \phi N_{R,t}^{\phi-1} K_{t-1}^{\lambda}$, which is almost unchanged when $\phi = 1$. Thus, the relative wage is nearly constant.[19] The nearly constant relative wage implies that flexible $W_{W,t}$ synchronizes with sticky $W_{R,t}$. It immediately follows from (12) and the near constancy of Q_t/P_t that $W_{W,t} \simeq P_t \simeq Q_t$.

The above discussion reinforces the importance of the returns to research in idea production. The results in Section 3.4 highlight the role of ϕ in terms of real variables. Our results in this subsection also suggest that the returns to research matter for nominal variables. The shape of the idea production function can be an important factor in generating the slow adjustment of nominal variables as well as the real effects of a monetary shock.

4 Sensitivity analysis

In this section, we check whether the results for $\phi = 0.5$ and $\phi = 1$ are robust to alternative parameter values. We consider the following parameters for the sensitivity analysis: the degree of relative risk aversion (σ); the inverse of the Frisch elasticities of labor supply (ψ_R and ψ_W); the degree of stickiness in researcher wages (θ_R); the depreciation rate of knowledge capital (δ); and the returns to knowledge in the goods production function (α). The comparisons depicted in the figures in this section are for two values of the returns to research: low returns to research (left panel) versus high returns to research (right panel). In both panels, worker wages are fully flexible and researcher wages are sticky. Once again, researchers are quantitatively small in the economy. Nevertheless, if the returns to research are high, sticky researcher wages can generate substantial real effects of monetary shocks. We also make some remarks on the robustness to alternative functional forms of the idea and goods production functions.

[19] We can reconfirm the responses of relative wages from the bottom row of Figure 2.

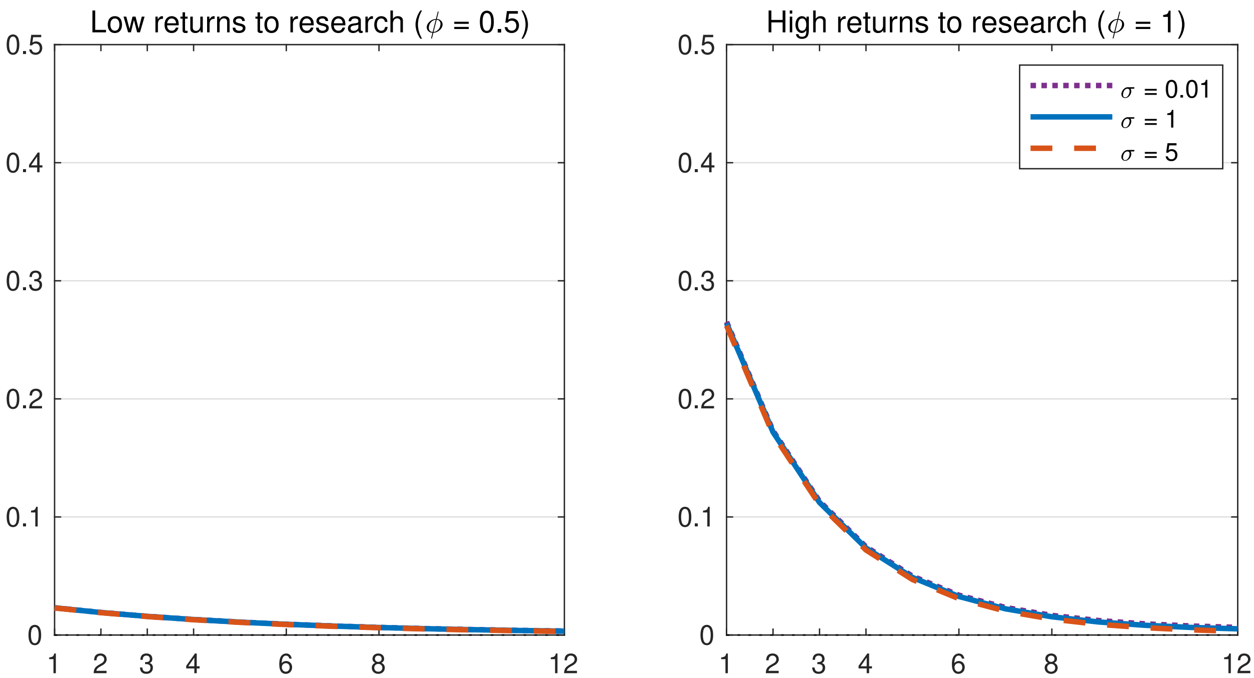

Figure 5: Responses of real GDP for different degrees of relative risk aversion

Notes: The left panel depicts the responses of real GDP to a one-percent increase in the money supply when the returns to research are low in idea production ($\phi = 0.5$). The right panel is when the returns to research are high ($\phi = 1$). For the configuration of nominal wage rigidity, worker wages are flexible, but researcher wages are sticky (W-flexible/R-sticky). In each panel, dotted lines represent the responses of real GDP when $\sigma = 0.01$; dashed lines correspond to when $\sigma = 1$; solid lines are when $\sigma = 5$. Vertical axes measure percentage deviations from the steady state. Time is given in quarters on the horizontal axes.

4.1 Degree of relative risk aversion

We first show that our results are robust to the degree of relative risk aversion σ. In the baseline parameterization, we set σ to unity. However, a higher value of σ (i.e., more inelastic intertemporal substitution in consumption) strengthens the consumption smoothing motive of households. This may result in smaller fluctuations in aggregate demand following monetary shocks and weaker real effects of monetary shocks, even under $\phi = 1$.

Figure 5 displays the impulse responses of real GDP to a one-percent increase in the money supply for three different values of σ, with dotted lines for $\sigma = 0.01$, solid lines for $\sigma = 1$, and dashed lines for $\sigma = 5$. As both panels of Figure 5 show, the responses of real GDP to monetary shocks are not essentially affected by the value of σ. The intuition is simple. When worker wages are flexible, monetary shocks do not substantially change C_t. If consumption is constant, the parameter for the curvature of the period utility does not matter

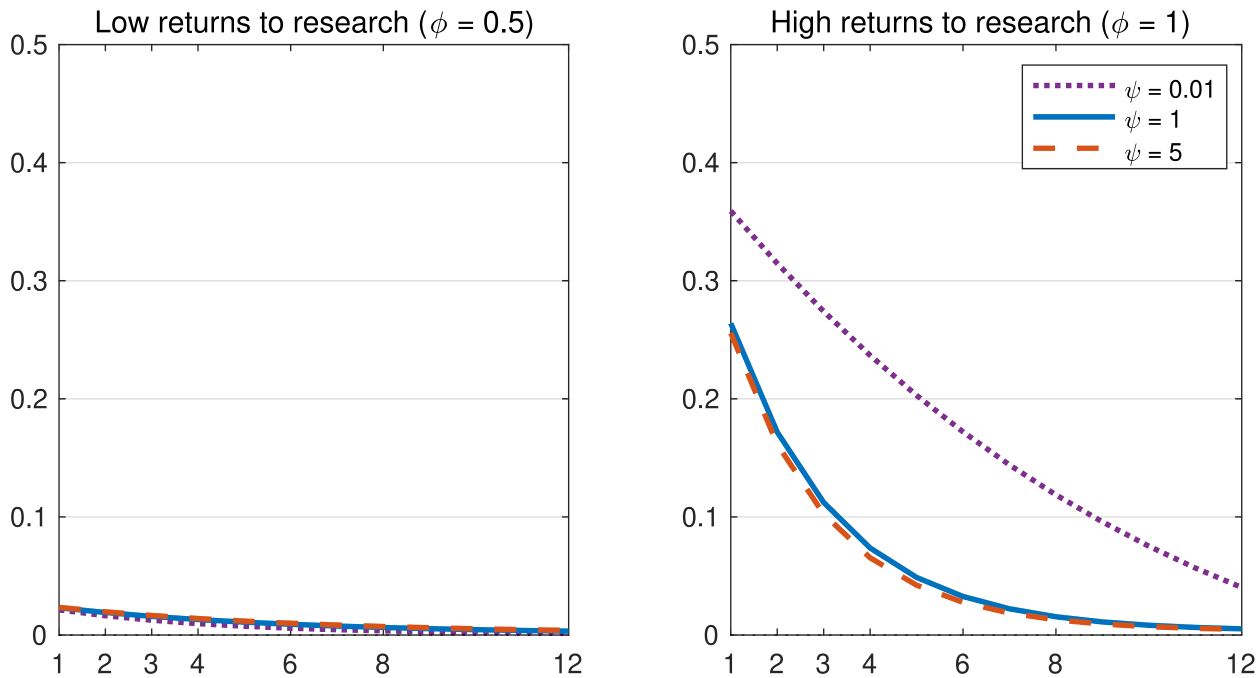

Figure 6: Responses of real GDP with different values of ψ

Notes: The left panel depicts the responses of real GDP to a one-percent increase in the money supply when the returns to research are low in idea production ($\phi = 0.5$). The right panel is when the returns to research are high ($\phi = 1$). For the configuration of nominal wage rigidity, worker wages are flexible, but researcher wages are sticky (W-flexible/R-sticky). We assume $\psi_W = \psi_R = \psi$. In each panel, dotted lines represent the responses of real GDP when $\psi = 0.01$; dashed lines correspond to when $\psi = 1$; solid lines are when $\psi = 5$. Vertical axes measure percentage deviations from the steady state. Time is given in quarters on the horizontal axes.

for the model's dynamics.

4.2 The inverse of the Frisch elasticity of labor supply

We next consider the robustness of our results to the inverse of the Frisch elasticities of labor supply (ψ_W and ψ_R), which are set to unity in the baseline parameterization. A large ψ_W and ψ_R may also strengthen the motive for smoothing labor supply. Moreover, unlike the case of σ, (30) indicates that the value of ψ_R directly affects the sensitivity of $N_{R,t}$ to their wages. Thus, the sensitivity analysis to the inverse of the Frisch elasticity of labor supply is more important than that to σ. Below, we compute the responses of real GDP to a monetary shock, maintaining the assumption of $\psi_W = \psi_R$ in the robustness analysis.

Figure 6 displays the impulse responses of real GDP for three values of ψ $(= \psi_W = \psi_R)$, with dotted lines for $\psi = 0.01$, solid lines for $\psi = 1$, and dashed lines for $\psi = 5$. The left panel of the figure confirms that when the returns to research are low, the real effects of a monetary shock are weak for all three values of ψ. When ϕ is low, the increase in $N_{R,t}$ is weakly transmitted to X_t via the idea production function. The right panel of the same figure corresponds to the case of $\phi = 1$. When the returns to research are high, the increase in real GDP is only slightly lower under $\psi = 5$ (the dashed line) than $\psi = 1$ (the solid line). In our simulations, the impact of increasing ψ from 1 to 5 on the response of real GDP is small. On the contrary, if we reduce ψ from 1 to 0.01, the real effect of a monetary shock becomes substantially larger.[20] Therefore, the real effect of a monetary shock under W-flexible/R-sticky remains robust to alternative values of ψ.

4.3 Degree of stickiness in researcher wages

In the third sensitivity analysis, we directly vary the degree of stickiness in researcher wages to observe how changes in θ_R affect our main conclusion. We find that it has little effect. To illustrate, we consider three values for θ_R, $\theta_R = 0.5$ (dotted lines), $\theta_R = 0.75$ (solid lines), and $\theta_R = 0.875$ (dashed lines), corresponding to a two, four, and eight quarter average duration of newly set wages of researchers, respectively.

Figure 7 illustrates the results of the sensitivity analysis. When the returns to research are low (the left panel), the real effects of a monetary shock remain weak, even under the higher degree of stickiness in researcher wages. Qualitatively, increasing the degree of stickiness in researcher wages strengthens the real effects of the monetary shock. However, the quantitative differences are small when the returns to research are low and large when the returns to research are high. That is, an increase in the degree of wage stickiness amplifies the real effect when the returns to research are high. Even when $\theta_R = 0.5$, the initial response of real GDP is 0.15 percent, which is much larger than any initial response of real GDP in the right panel. Thus, the shape of the idea production function is much more important than the degree of nominal wage stickiness in generating the real effect of monetary shocks.

[20] This asymmetry is because of the effect of ψ_R appearing in the denominator on the right-hand side of (30). That is, the sensitivity of the labor services of researchers to wage markups more strongly increases as ψ_R becomes smaller. In particular, when the decline in ψ_R is from 5 to 1, the increase in $d\widehat{N}_{R,t}/d\widehat{\mu}_{R,t}$ is from 0.2 (= 1/5) to 1 (= 1/1). Alternatively, when the decline is from 1 to 0.01, it increases $d\widehat{N}_{R,t}/d\widehat{\mu}_{R,t}$ from 1 (=1/1) to 100 (=1/0.01).

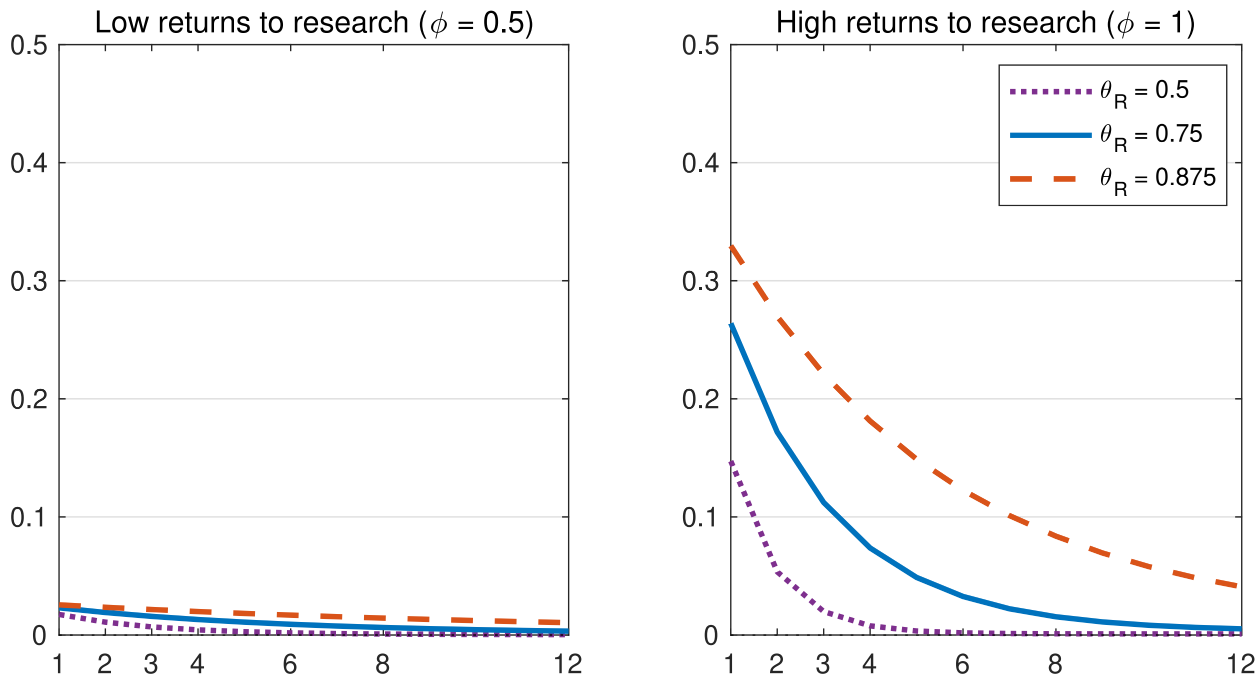

Figure 7: Role of stickiness in researcher wages

Notes: The left panel depicts the responses of real GDP to a one-percent increase in the money supply when the returns to research are low in idea production ($\phi = 0.5$). The right panel is when the returns to research are high ($\phi = 1$). For the configuration of nominal wage rigidity, worker wages are flexible, but researcher wages are sticky (W-flexible/R-sticky). In each panel, dotted lines represent the responses of real GDP when $\theta_R = 0.5$; dashed lines correspond to when $\theta_R = 0.75$; solid lines are when $\theta_R = 0.875$. Vertical axes measure percentage deviations from the steady state. Time is given in quarters on the horizontal axes.

4.4 Depreciation rate of knowledge capital

In the calibration, we have chosen the depreciation rate of knowledge capital δ, together with the returns to knowledge in the goods production function α, to match the data on the R&D expenditure share to GDP and the labor cost ratio. However, the evidence on δ is not necessarily ample and the value of δ may vary according to the interpretation of knowledge capital. We thus check the robustness of our results to different values of δ. In this sensitivity analysis to δ, we keep α unchanged at 0.58, which is the baseline parameter value when the returns to research are high ($\phi = 1$).

We compare two cases: one to reduce the value δ to one-third of the baseline parameter value (i.e., $\delta = 1.05 \times 10^{-4}$), and the other to triple the value of δ $(= 9.45 \times 10^{-4})$. We confirm that the R&D expenditure share to GDP and the labor cost ratio with these values of δ remain within reasonable ranges. Table 2 shows the R&D expenditure shares to GDP and the labor cost ratios implied from δ where α is

Table 2: Steady-state R&D expenditure shares and labor cost ratios for different values of δ

High returns to research ($\phi = 1$)		
δ	R&D exp. share	Labor cost ratio
1.05×10^{-4}	0.01	0.03
3.15×10^{-4}	0.03	0.08
9.45×10^{-4}	0.08	0.22
Low returns to research ($\phi = 0.5$)		
δ	R&D exp. share	Labor cost ratio
1.05×10^{-4}	0.01	0.01
3.15×10^{-4}	0.03	0.04
9.45×10^{-4}	0.08	0.11

Notes: The table reports steady-state R&D expenditure shares (denoted by "R&D exp. Share") and labor cost ratios with different values of δ. We fix α at $\alpha = 0.58$, which is the baseline parameter value under $\phi = 1$. Labor cost ratios depend on the choice of ϕ. The steady-state R&D expenditure share is given by (24) and the labor cost ratio is given by (25).

fixed at $\alpha = 0.58$. The top panel of Table 2 shows the case when the returns to research are high ($\phi = 1$). As shown in the second column of the table, if we reduce δ from 3.15×10^{-4} to 1.05×10^{-4}, the R&D expenditure share reduces to 1 percent. If we triple δ ($= 9.45 \times 10^{-4}$), the R&D expenditure share amounts to 8 percent. We observe similar patterns regarding the labor cost ratio. As shown in the third column of the table, the resulting labor cost ratio ranges from 0.03 to 0.22. The bottom panel of Table 2 presents the case when the returns to research are low ($\phi = 0.5$). While the steady-state R&D expenditure shares do not depend on the value of ϕ, the labor cost ratios decrease by half.

Figure 8 plots the impulse responses. The dotted line represents the case of $\delta = 1.05 \times 10^{-4}$ and the dashed line corresponds to the case of $\delta = 9.45 \times 10^{-4}$. The solid line uses the baseline parameterization under $\phi = 1$ (i.e., $\delta = 3.15 \times 10^{-4}$) for comparisons. Overall, impulse responses are amplified through the increased δ, especially when the returns to research are high. As shown in the right panel of the figure, output increases at impact by 0.20 percent for $\delta = 1.05 \times 10^{-4}$ and by 0.33 percent for $\delta = 9.45 \times 10^{-4}$.

The stronger real effect of money under a larger δ results from the increased steady-state expenditure share of R&D to GDP (i.e., $(Q/P)X/[C + (Q/P)X]$). As the steady-state expenditure share of R&D to GDP increases, the effect of X_t on total expenditure $C_t + (Q_t/P_t)X_t$ is amplified. For this reason, the output response is somewhat large when $\delta = 9.45 \times 10^{-4}$ even under $\phi = 0.5$. Nevertheless, the real effects

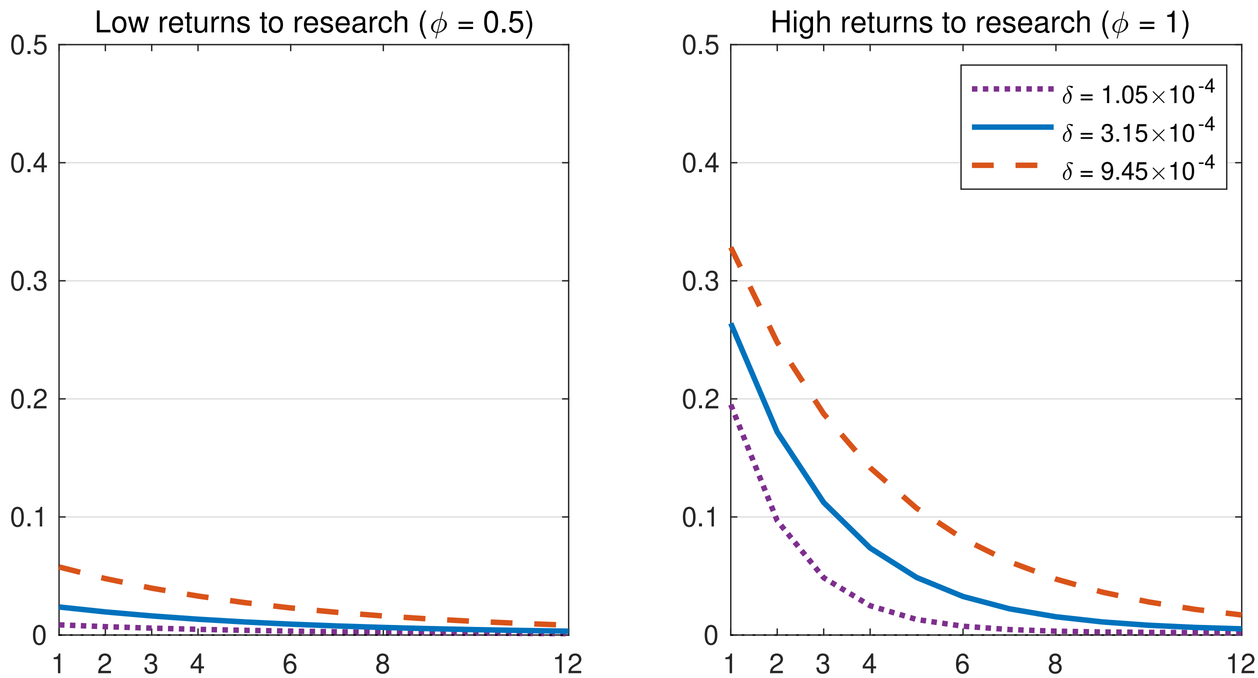

Figure 8: Responses of real GDP with different values of δ

Notes: The left panel depicts the responses of real GDP to a one-percent increase in the money supply when the returns to research are low in idea production ($\phi = 0.5$). The right panel is when the returns to research are high ($\phi = 1$). For the configuration of nominal wage rigidity, worker wages are flexible, but researcher wages are sticky (W-flexible/R-sticky). In each panel, dotted lines represent the responses of real GDP when $\delta = 1.05 \times 10^{-4}$; solid lines are when $\delta = 3.15 \times 10^{-4}$; dashed lines correspond to when $\delta = 9.45 \times 10^{-4}$. Vertical axes measure percentage deviations from the steady state. Time is given in quarters on the horizontal axes.

of monetary shocks substantially differ between $\phi = 0.5$ and $\phi = 1$ for each value of δ. Therefore, our key message that the shape of the idea production function matters for the real effect of monetary shocks remains unchanged.

4.5 Returns to knowledge in the goods production function

In the calibration, we chose the returns to knowledge in the goods production function α under the assumption of constant returns to scale. However, the evidence on knowledge as a factor of production is limited, and the value of α may differ, depending on the interpretation of knowledge capital. Thus, it would be sensible to check the robustness of our results, using a wide range of the value of α.[21]

To implement sensitivity analysis to the returns to knowledge in the goods production function, we

[21] Jones (2019a) also considers constant and increasing returns to scale in the goods production function.

Table 3: Steady-state R&D expenditure shares and labor cost ratios for different values of α_K

High returns to research ($\phi = 1$)		
α_K	R&D exp. share	Labor cost ratio
0.28	0.02	0.04
0.58	0.03	0.08
0.88	0.05	0.12
1.18	0.06	0.16
Low returns to research ($\phi = 0.5$)		
α_K	R&D exp. share	Labor cost ratio
0.28	0.02	0.02
0.58	0.03	0.04
0.88	0.05	0.06
1.18	0.06	0.08

Notes: The table reports the steady-state expenditure shares of R&D to GDP (denoted by "R&D exp. share") and labor cost ratios with different values of α_K. We fix δ at $\delta = 3.15 \times 10^{-4}$, which is the baseline parameter value. Labor cost ratios depend on the choice of ϕ. The R&D expenditure share is given by $\alpha_K \delta[1-\beta(1-\delta)]^{-1}/\{\mu_P + \alpha_K \delta[1-\beta(1-\delta)]^{-1}\}$ and the labor cost ratio is given by $(\alpha_K/\alpha_N)(\phi\delta)[1-\beta(1-\delta)]^{-1}$.

replace the goods production function (21) by

$$F(N_{W,t}(j), K_t(j)) = N_{W,t}(j)^{\alpha_N} K_t(j)^{\alpha_K}, \tag{32}$$

where $\alpha_N > 0$ and $\alpha_K > 0$ but not necessarily $\alpha_N + \alpha_K = 1$. In our sensitivity analysis, we consider $\alpha_K = \{0.28, 0.88, 1.18\}$ in comparison to the baseline parameter value $\alpha = 0.58$. We fix α_N at 0.42, which is the baseline parameter value of the returns to workers under $\phi = 1$ (i.e., $1-\alpha = 0.42$), so that we can consider the decreasing returns to scale ($\alpha_N + \alpha_K = 0.7$), as well as the increasing returns to scale ($\alpha_N + \alpha_K = 1.3$ and 1.6). In this robustness analysis, we fix the remaining parameters at the baseline parameter value including δ ($= 3.15 \times 10^{-4}$).

We confirm that the R&D expenditure share to GDP and the labor cost ratio resulting from the newly parameterized α_K do not deviate substantially from the data. Table 3 shows how the R&D expenditure share and the labor cost ratio vary as we change α_K. The top panel of Table 3 shows the case when the returns to research are high ($\phi = 1$). Recall that our calibration achieves a steady-state R&D share to GDP of 3 percent and the labor cost ratio of 0.08 when $\alpha_K = \alpha = 0.58$ and $\alpha_N = 1 - \alpha = 0.42$. The second

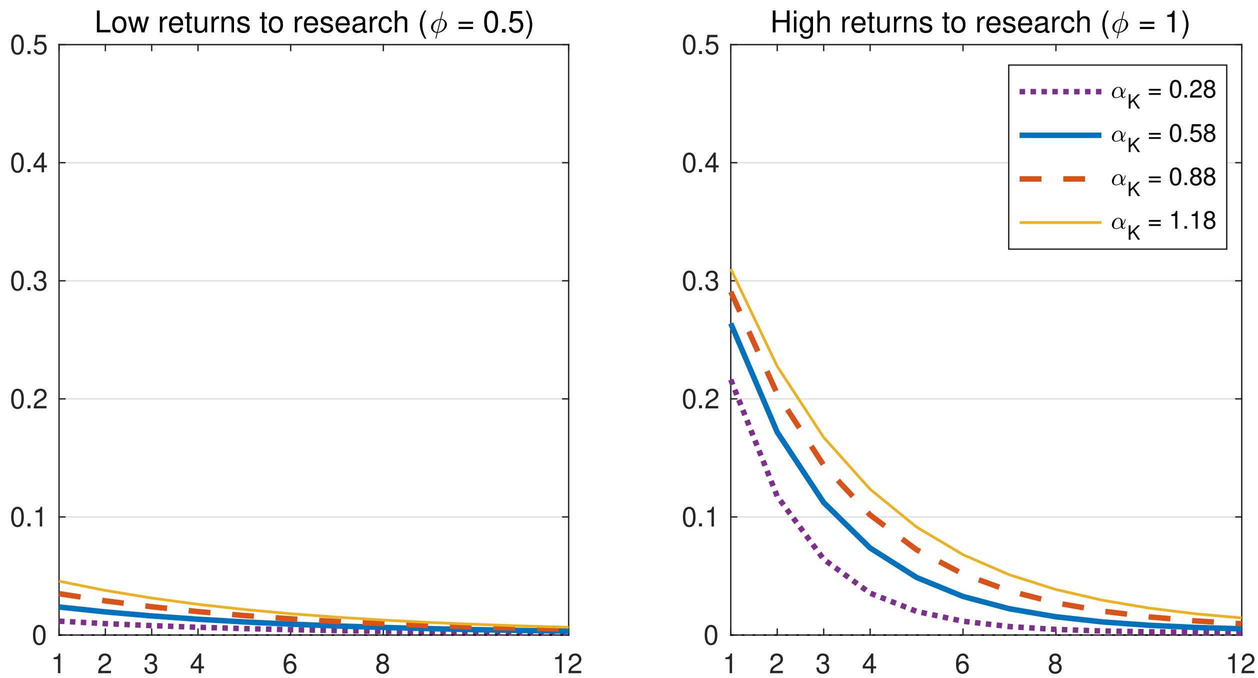

Figure 9: Responses of real GDP for different values of α

Notes: The left panel depicts the responses of real GDP to a one-percent increase in the money supply when the returns to research are low in idea production ($\phi = 0.5$). The right panel is when the returns to research are high ($\phi = 1$). For the configuration of nominal wage rigidity, worker wages are flexible, but researcher wages are sticky (W-flexible/R-sticky). In each panel, the dashed line is the case of $\alpha_K = 0.28$, the bold solid line is the case of $\alpha_K = 0.58$, the dashed line is the case of $\alpha_K = 0.88$ and the thin solid line is the case of $\alpha_K = 1.18$. Vertical axes measure percentage deviations from the steady state. Time is given in quarters on the horizontal axes.

column indicates that the R&D expenditure share to GDP increases from 2 to 6 percent when we increase α_K from 0.28 to 1.18. The third column represents the labor cost ratios that range from 0.04 to 0.16. The bottom panel of Table 3 displays the steady-state R&D expenditure share and the labor cost ratio when the returns to research are low ($\phi = 0.5$).

Figure 9 depicts the impulse response functions for four values of α_K. The results are robust to the value of returns to knowledge in goods production. When $\phi = 0.5$, the real effect of a monetary shock under W-flexible/R-sticky is weak, regardless of α_K. In contrast, when $\phi = 1$, the output responses are large. The size of the output responses also increases with α_K. In particular, following a one-percent increase in the money supply, output increases by 0.22 percent when $\alpha_K = 0.28$ (shown by the dotted line in the right panel of Figure 9) whereas output increases by 0.31 percent when $\alpha_K = 1.18$ (shown by the thin solid line in the same panel).

As in Section 4.4, the stronger real effect of money under a larger α_K results from the increased steady-state expenditure share of R&D to GDP (i.e., $(Q/P)X/[C+(Q/P)X]$). The effect of X_t on total expenditure $C_t+(Q_t/P_t)X_t$ becomes larger when the steady-state expenditure share of R&D to GDP increases. The real effects of monetary shocks substantially differ between $\phi = 0.5$ and $\phi = 1$ for each value of α_K. Therefore, our key message that the shape of the idea production function matters for the real effects of monetary shock remains unchanged.

4.6 Functional forms of production functions

Before closing this section, we discuss the robustness of our results to alternative functional forms of the idea and goods production functions.

In the idea production function, we assume that knowledge becomes nonexcludable with only a one-period delay. This assumption may be too strong. We thus replace the assumption with a new assumption that the transformation of newly created ideas into nonexcludable knowledge requires multiple periods. In particular, we can consider the following generalized idea production function with partial transformation:

$$G(N_{R,t}(j), \bar{K}_{t-1}) = N_{R,t}(j)^{\phi} \bar{K}_{t-1}^{\lambda}, \tag{33}$$

instead of (22). Here $\bar{K}_{t-1}$ is a function of lagged nonexcludable knowledge

$$\bar{K}_{t-1} = \prod_{\tau=1}^{T} (K_{t-\tau})^{\nu_\tau}, \tag{34}$$

where $\sum_{\tau=1}^{T} \nu_\tau = 1$ and T can be infinity. In (33), the knowledge becomes nonexcludable more slowly because $\bar{K}_{t-1}$ is the weighted geometric mean of lagged nonexcludable knowledge of $\{K_{t-\tau}\}_{\tau=1}^{T}$, as indicated by (34). This specification reduces to the previous idea production function when $\nu_1 = 1$ and $T = 1$.

Our results concerning the real effects of money do not change, irrespective of how we specify $\bar{K}_t$ in the model.[22] The reason is straightforward, as suggested in Section 3.3. Under a sufficiently low δ, a monetary shock does not generate a sizable change in the stock of knowledge K_t. Because $\bar{K}_t$ is the geometric mean

[22] The simulation results are available upon request.

of lagged K_t, $\bar{K}_t$ is near constant. As long as $\bar{K}_t$ is near constant, the model's dynamics can be explained by the mechanism discussed in Section 3.3.[23]

Turning to the goods production function, the generalization (e.g., a constant elasticity of substitution production function to allow for the complementarity between workers and knowledge) does not affect our main results under W-flexible/R-sticky. To see this, recall that real GDP can be expressed as $F(N_{W,t}, K_t) + (Q_t/P_t)G(N_{R,t}, K_{t-1})$. As long as K_t is nearly constant and worker wages are flexible, neither $N_{W,t}$ nor Q_t/P_t fluctuates in response to a monetary shock. As suggested by (26) and (27), this result holds no matter how we generalize the goods production function $F(N_{W,t}, K_t)$. Output responses are quantitatively similar under any shape of the goods production function when the configuration of nominal wage rigidities is W-flexible/R-sticky.

5 Concluding remarks

In the economic growth literature, it is well-known that the production of new ideas and knowledge spillovers can be fundamental sources of endogenous growth. In contrast, in the business cycle literature, while the procyclicality of R&D activity suggests their importance for business cycles, there has been little attention to the role of idea production in understanding output and employment fluctuations. One may conjecture that the impact of R&D activity may be negligible in business cycles because both the shares of R&D expenditure to GDP and the labor cost ratio of researchers to workers are small.

We show that this conjecture is not always correct in the transmission mechanism for monetary shocks. Incorporating idea production and knowledge capital in a New Keynesian sticky-wage model provides new insights into monetary business cycles. Ideas and knowledge can play a crucial role in generating the real effects of monetary shock, depending on the degree of the returns to research in idea production. If the returns to research are high in idea production, nominal rigidities only in researcher wages generate a substantial real effect of money. Even though researchers are quantitatively small in the economy, the real effect of

[23] We can also consider the idea production function (33), where $\bar{K}_{t-1}(j) = K_{t-1}^{\nu_1}[K_{t-1}(j)]^{1-\nu_1}$. In this specification, only some knowledge capital becomes nonexcludable while the remainder remains firm specific. For the same reason discussed, the simulation results based on this idea production function are similar to the results described in the main text.

money is comparable to when there are nominal rigidities in the wages of workers, who are quantitatively dominant in the economy. By contrast, if the returns to research are low in idea production, nominal rigidities in researcher wages fail to produce a substantial real effect of money. Indeed, the model's dynamics are effectively the same as a model without researchers.

Our results imply that the shape of the idea production function is key to understanding monetary business cycle models with ideas and knowledge capital. In his survey of idea-based growth theory, Jones (2019a) concludes that "[t]he shape of the idea production function remains an intriguing subject of study." Our analysis suggests that this is indeed so, not only for economic growth but also monetary business cycles.

The analysis in this paper could be extended in many important directions. First, it would be interesting to incorporate productivity and fiscal policy shocks into the model. While we focus on the real effects of monetary shock, the literature on business cycles emphasizes productivity shocks as an important driver of fluctuations. Likewise, introducing fiscal policies in this model would allow us to see how a fiscal policy shock influences R&D sectors. Second, although we did not introduce physical and human capital, incorporating these elements into our model would allow us to consider how our sticky-wage model is related to the growth models originally developed by Romer (1990) and Jones (1995). Finally, examining alternative specifications of monetary policy (e.g., the presence of an effective lower bound on the nominal interest rate) would also be an important direction for future research.

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Appendix

Equilibrium conditions

In what follows, we describe the log-linearized equilibrium conditions.

From the first-order conditions for households, together with (17) and (18),

$$\widehat{MRS}_{h,t} = \psi_h \widehat{N}_{h,t} + \sigma \widehat{C}_t, \tag{35}$$

$$\widehat{W}^*_{h,t} = \beta\theta_h \mathrm{E_t} \widehat{W}^*_{h,t+1} + (1-\beta\theta_h)\left[\frac{1}{1+\psi_h\varepsilon_h}\left(\widehat{MRS}_{h,t} - \widehat{W}_{h,t} + \widehat{P}_t\right) + \widehat{W}_{h,t}\right], \tag{36}$$

where $\widehat{W}^*_{h,t}$ represents the log deviation of the optimal nominal wage for $h = W, R$. Also, $\widehat{MRS}_{h,t}$ is the average marginal rate of substitution between consumption and labor. Here, the hatted lowercase letters are the log deviations from the initial steady state before a monetary shock materializes. Because of our assumption of sticky wages, we have log-linearized laws of motion for nominal wages:

$$\widehat{W}_{h,t} = \theta_h \widehat{W}_{h,t-1} + (1-\theta_h)\,\widehat{W}^*_{h,t}, \tag{37}$$

for $h = W, R$.

We next turn to the log-linearized equations for firms. Based on our assumption of the Cobb–Douglas production function, (12), (13), and (27) are linearized as

$$\widehat{W}_{W,t} - \widehat{P}_t - \widehat{C}_t = -\widehat{N}_{W,t}, \tag{38}$$

$$\widehat{Q}_t = \widehat{W}_{R,t} + (1-\phi)\,\widehat{N}_{R,t} - \lambda \widehat{K}_{t-1}, \tag{39}$$

$$\widehat{Q}_t - \widehat{P}_t - \sigma\widehat{C}_t = [1-\beta(1-\delta)]\left[(1-\sigma)\,\widehat{C}_t - \widehat{K}_t\right] + \beta(1-\delta)\,\mathrm{E_t}[\widehat{Q}_{t+1} - \widehat{P}_{t+1} - \sigma\widehat{C}_{t+1}]. \tag{40}$$

In terms of the firm's technology, we have the log-linearized law of motion for knowledge capital:

$$\widehat{K}_t = (1-\delta)\,\widehat{K}_{t-1} + \delta\widehat{X}_t. \tag{41}$$

Goods production and idea production functions are

$$\widehat{C}_t = (1-\alpha)\,\widehat{N}_{W,t} + \alpha\widehat{K}_t, \tag{42}$$

$$\widehat{X}_t = \phi\widehat{N}_{R,t} + \lambda\widehat{K}_{t-1}. \tag{43}$$

Using the definitions of nominal and real GDP, we derive the equations for GDP. The log deviation of real GDP is derived from (16):

$$\widehat{Y}_t^{GDP} = (1-\omega)\,\widehat{C}_t + \omega(\widehat{X}_t + \widehat{Q}_t - \widehat{P}_t), \tag{44}$$

where

$$\omega = \frac{(Q/P)X}{C + (Q/P)X} = \frac{\alpha\delta\left[1-\beta\left(1-\delta\right)\right]^{-1}}{\mu_P + \alpha\delta\left[1-\beta\left(1-\delta\right)\right]^{-1}}$$

is the steady-state expenditure share of R&D to GDP.

The remaining equations used for simulations are associated with the money market equilibrium. The money demand from firms can be approximated by

$$\widehat{M}_t = (1-\kappa)\left(\widehat{W}_{W,t} + \widehat{N}_{W,t}\right) + \kappa\left(\widehat{W}_{R,t} + \widehat{N}_{R,t}\right), \tag{45}$$

where

$$\kappa = \frac{W_R N_R}{W_W N_W + W_R N_R}$$

is the steady-state wage share of researcher labor as calculated from the labor cost ratio of researchers to workers. Finally, money supply follows a random walk

$$\widehat{M}_t = \widehat{M}_{t-1} + \xi_t, \tag{46}$$

where we use the money market equilibrium condition: $\widehat{M}_t = \widehat{M}_t^s$.

Equations (35) – (46) consist of the log-linearized system of equations characterizing the equilibrium conditions for 15 variables $\widehat{M}_t$, $\widehat{W}_{W,t}$, $\widehat{W}_{R,t}$, $\widehat{K}_t$, $\widehat{W}^*_{W,t}$, $\widehat{W}^*_{R,t}$, $\widehat{N}_{W,t}$, $\widehat{N}_{R,t}$, $\widehat{C}_t$, $\widehat{MRS}_{W,t}$, $\widehat{MRS}_{R,t}$, $\widehat{P}_t$, $\widehat{Q}_t$, $\widehat{X}_t$, and $\widehat{Y}^{GDP}_t$.

Data

For the labor cost ratio of researchers to workers, we take the data from the 2018 OES compiled by the Bureau of Labor Statistics.[24] The OES reports the number employed and annual mean wages of various occupations, among other things. We classify these occupations into our definition of "researchers" and "workers."

Our definition of "researchers" includes some research-related managers, computer and mathematical occupations, engineers, scientists and researchers, and doctors. The remaining occupations are considered "workers." All occupations included in our definition of "researchers" are presented in Table 4.

Using the mean annual wage and the number of employed, this classification results in a labor cost ratio of researchers to workers of $(W_R N_R)/(W_W N_W) = 0.08$.

[24]The data are available at https://www.bls.gov/oes/2018/may/oes_nat.htm.

Table 4: List of occupations classified as "researchers"

Medical and Health Services Managers	Misc. Life Scientists
Natural Sciences Managers	Astronomers and Physicists
Computer and Information Research Scientists	Atmospheric and Space Scientists
Mathematicians	Chemists and Materials Scientists
Operations Research Analysts	Environmental Scientists and Geoscientists
Statisticians	Misc. Physical Scientists
Misc. Mathematical Science Occupations	Economists
Architects, Except Naval	Survey Researchers
Surveyors, Cartographers, and Photogrammetrists	Psychologists
Aerospace Engineers	Sociologists
Agricultural Engineers	Urban and Regional Planners
Biomedical Engineers	Misc. Social Scientists and Related Workers
Chemical Engineers	Agricultural and Food Science Technicians
Civil Engineers	Biological Technicians
Computer Hardware Engineers	Chemical Technicians
Electrical and Electronics Engineers	Geological and Petroleum Technicians
Environmental Engineers	Nuclear Technicians
Industrial Engineers, Incl. Health and Safety	Social Science Research Assistants
Marine Engineers and Naval Architects	Misc. Life, Physical, and Social Science Technicians
Materials Engineers	Dentists
Mechanical Engineers	Optometrists
Mining and Geological Engineers, Incl. Mining Safety Engineers	Pharmacists
Nuclear Engineers	Physicians and Surgeons
Petroleum Engineers	Veterinarians
Misc. Engineers	Audiologists
Agricultural and Food Scientists	
Biological Scientists	
Conservation Scientists and Foresters	
Medical Scientists	