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HORIZONTAL MERGER ANALYSIS WITH ENDOGENOUS PRODUCT RANGE CHOICE

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Horizontal Merger Analysis with Endogenous Product Range Choice*

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Abstract

We consider mergers between multi-product firms in a market with monopolistically competitive fringe of single-product firms. Aggregate product variety is determined by product variety choices of multi-product firms and entry/exit decisions of single-product firms. Mergers can generate marginal cost synergies (affecting marginal cost of quantity) or fixed cost synergies (affecting marginal cost of variety). We show that with marginal cost synergies, consumer welfare decreases whenever aggregate variety increases following a merger. However, with fixed cost synergies, an increase in aggregate variety can indicate that the merger is beneficial. Our results also show high synergies do not necessarily improve consumer welfare.

Keywords: Antitrust policy; merger analysis; multiproduct firms; product range choice; entry; cost synergies

JEL classification: L11; L41; D43

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1 Introduction

Antitrust authorities regard post-merger product diversity as an important consideration in determining the social desirability of mergers.[1] Indeed, many mergers involve multi-product firms and may result in a readjustment of the merged entity's product range. For example, after the merger in 2007 between Whole Foods Market and Wild Oats, the two leading retailers of organic and natural food in the US, Whole Foods Market sold 35, closed 12, and converted one-third of the remaining Wild Oats stores into Whole Foods outlets.[2] Another example is the consolidation of the American radio broadcasting industry prompted by the Telecommunications Act of 1996, which raised both the per-station and overall variety in terms of programming formats (Berry and Waldfogel, 2001). As a result, it is important to understand under what circumstances mergers resulting in changes in the merged entity's product range can harm consumer welfare.

We study this issue in a model which allows for merger-generated efficiencies and post-merger entry. Merger-generated efficiencies and post-merger entry are two features evaluated positively by antitrust authorities while assessing the welfare impact of a merger. The general thinking is that a merger is likely to be procompetitive if merger efficiencies are substantial or the prospect of entry in the relevant market is high.[3]

Combining merger-generated efficiencies, product range change and entry in a

[1] For example, the 2018 OECD document *Considering non-price effects in merger control – Background note by the Secretariat* (paragraph 86) states that "a merger that results in changes to the available variety of differentiated products can have implications for consumer welfare." The 2010 U.S. Horizontal Merger Guidelines (Section 6.4) states that "reductions in variety following a merger may or may not be anticompetitive."

[2] See Harvard Business Review Case Study called "Whole Foods and Wild Oats Merger," available at https://store.hbr.org/product/whole-foods-market-and-wild-oats-merger/UV1019.

[3] For example, it is stated in the US Horizontal Merger Guidelines that "a primary benefit of mergers to the economy is their potential to generate significant efficiencies and thus enhance the merger firm's ability and incentive to compete, which may result in lower prices, improved quality, enhanced service, or new products." Similarly, the European Commission's Horizontal Merger Guidelines state the following: "The relevant benchmark in assessing efficiency claims is that consumers will not be worse off as a result of the merger. For that purpose, efficiencies should be substantial ...." As far as potential entrants are concerned, the US Horizontal Merger Guidelines consider entry into the relevant market as part of the full assessment of competitive effect: "The prospect of entry into the relevant market will alleviate concerns about adverse competitive effects only if such entry will deter or counteract any competitive effects of concern so the merger will not substantially harm customers."

coherent theoretical framework is necessary to determine how they interact with each other. The interaction raises a number of questions. How do multi-product firms adjust their product range following a merger in the face of efficiency gains? What are the reactions of non-merging firms, including both incumbents and potential entrants? What are the impacts of such mergers on aggregate product variety in the marketplace and welfare?

To address these questions, we consider a horizontally differentiated goods market where a few multi-product firms and a host of single-product firms compete in quantity. We assume that the multi-product firms act strategically while the single-product firms are monopolistically competitive.[4] Such a mixed market structure consisting of multi-product and single-product firms has been shown to be prevalent in many industries.[5]

The aggregate product range in the market is determined endogenously in our model through the multi-product firms' product range adjustments and single-product firms' entry/exit decisions. We assume that each multi-product firm strategically supplies a continuum of varieties, and the variety range it offers is a choice variable. A single-product firm is infinitesimal and can freely enter or exit the market.

In our analysis, we allow for two kinds of merger-generated synergies. We assume that a merger may result in marginal cost savings or fixed cost savings. The fixed cost in our model is a per-variety fixed cost. Since we allow the multi-product firms to determine their product range endogenously, the fixed cost represents the marginal cost of producing one more variety. Hence, it is different from costs related to back-office functions. For example, in his analysis of fixed cost efficiencies from mergers in the US radio industry, Jeziorski (2014) identifies "large within-format cost synergies."[6] Specifically, he finds that "operating an extra station in

[4] The theoretical framework with large oligopolists and small monopolistic competitors is first considered by Shimomura and Thisse (2012), who assume that all the firms are single-product firms and examine the impact of a large firm's entry. Pan and Hanazono (2018) and Parenti (2018) consider the coexistence of multi-product large firms and single-product small firms, examining the impact of large firms' entry and trade liberalization, respectively. We draw on this framework with particular attention to merger analysis.

[5] For instance, in the US manufacturing industry, 91% of total output is supplied by 41% of firms, 89% of which adjust product range every five years (Bernard et al., 2010).

[6] Each radio station is characterized by a format which includes information about type of music, the number of news and talk shows, as well as information about being inactive.

the already-owned format costs more than 60% less."

For mergers with marginal cost synergies, we show that if such synergies are sufficiently high, the merging multi-product firms expand their variety range and the mass of single-product firms shrinks. The net effect of these two changes on aggregate product variety in the market depends on whether the multi-product firms have a fixed cost advantage over the single-product firms. If the multi-product firms have a fixed cost advantage, then high synergies result in an increase in aggregate product variety. Interestingly, the merger, despite generating high synergies and increasing the aggregate product variety, harms consumer welfare. This is because of a reallocation effect: the varieties produced by the single-product firms are replaced by the varieties produced by the multi-product firms. Since the latter charges a higher price for its products (both before and after the merger), the reallocation hurts consumers.

Hence, our analysis reveals that if the multi-product firms have a fixed cost advantage over the single-product firms, then high marginal cost synergies harm consumers, but a moderate level of marginal cost synergies increases consumer welfare despite causing aggregate product variety to decrease. If the multi-product firms do not have a fixed cost advantage, the results are reversed. In this case, high (low) marginal cost synergies cause aggregate product variety to decrease (increase) and consumer welfare to increase (decrease). These results imply that in mergers with marginal cost synergies, any consumer welfare gain does not originate from an increase in aggregate variety. Rather, it is the lower prices that consumers face as a result of the reallocation of production between the multi-product and single-product firms that benefit the consumers. Moreover, contrary to the view taken in merger guidelines, entry is not always a necessary condition for a merger to increase consumer welfare. When single-product firms have fixed cost advantage, consumer welfare increases despite exit by single-product firms which is caused by sufficiently high merger-generated synergies.

We next consider the impact of mergers which generate fixed cost synergies. Fixed cost synergies reduce the cost of introducing new varieties. An important difference between mergers with marginal-cost synergies and mergers with fixed cost synergies is that mergers with fixed cost synergies also have a price effect. This is because as the fixed cost level decreases and the multi-product firm introduces more varieties, it charges higher prices in order to mitigate the cannibalization between their own varieties.

Due to this price effect which increases with the level of synergies, mergers with sufficiently high fixed cost synergies are harmful to consumers. We further show that if multi-product firms are more efficient to start with, both in terms of their fixed and marginal cost of production, then all mergers are harmful for consumers. Otherwise, some mergers with moderate levels of synergies may increase consumer welfare. Importantly, in contrast with the case of marginal cost synergies, an increase in aggregate variety can be an indication that the merger is beneficial for consumers in this case if the multi-product firms have a fixed cost advantage over the single-product firms before the merger.

These results imply that in terms of antitrust analysis, an increase in aggregate product variety or high synergies should not be seen as reasons for favorable treatment of mergers. Even mergers which result in both increased aggregate product variety and high synergies can still reduce consumer welfare. Mergers which result in an increase in aggregate product variety can still harm consumers if producers of lower-priced varieties are forced out of the market and the varieties they offer are replaced by the varieties offered by the merged firms. Mergers with high synergies can still harm consumers if they result in higher prices for the merged firms' varieties.

Although there exists a large literature on mergers, most of the theoretical literature rely on models with single-product firms. Two exceptions are Nocke and Schutz (2018) and Johnson and Rhodes (2021).[7] Nocke and Schutz (2018) analyze horizontal mergers in a model of multi-product firms and price competition with nested CES or nested logit demands. They show that the Herfindahl index provides an adequate measure of the oligopoly distortions to consumer surplus and aggregate surplus. However, they do not explicitly consider endogenous product range choice by multi-product firms. Johnson and Rhodes (2021) investigate mergers where firms may reposition their product lines by adding or removing products of different qualities following a merger. They identify a product-mix effect by which

[7] See also Lommerud and Sogard (1997) who examine merger profitability in a model with three firms where brand withdrawal by the merged entity and new brand introduction by the non-merging firm are allowed. The firms are assumed to be single-product firms before the merger.

mergers without synergies may raise consumer surplus whereas mergers with synergies may lower consumer surplus. We complement their work by considering a market with horizontally differentiated products, where multi-product and single-product firms coexist. Our paper also differs from both of these two papers by allowing for entry by the single-product firms.

Post-merger entry constitutes an important factor in merger evaluation and has been investigated in several studies, such as Werden and Froeb (1998), Cabral (2003), Spector (2003), Davidson and Mukherjee (2007), Erkal and Piccinin (2010), Anderson et al. (2020) and Caradonna et al. (2021).[8] Nevertheless, all of these papers assume an exogenous product range for each firm. Endogenous product range choice by multi-product firms is central to our paper.

Our paper is also related to the literature which use merger simulation or structural methods to investigate the impact of product choice and product repositioning on merger outcomes. This literature has considered mergers in different varieties of markets, such as radio stations (Berry and Waldfogel, 2001; Sweeting, 2010), airlines (Li et al., 2019), smartphones (Fan and Yang, 2020a), and craft beer (Fan and Yang, 2020b). One key message from this literature is that price effects and welfare consequences of mergers emerging from models with a fixed set of products may be quite different from those emerging from models with endogenous product choice or product repositioning.[9]

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 conducts the equilibrium analysis without a merger. Section 4 analyzes the equilibrium with a merger and examines the impact of marginal cost synergies on market performance and welfare. Section 5 considers the case of fixed cost synergies. Section 6 concludes.

[8] Gowrisankaran (1999) develops a dynamic Cournot game with endogenous investment, merger, entry and exit decisions. His computational analysis suggests that mergers' anticompetitive effects are unlikely to be reversed by entry.

[9] For example, while Gandhi et al. (2008) and Sweeting (2010) show that allowing for product repositioning in merger analysis makes mergers appear less anticompetitive, Mazzeo et al. (2019) find the opposite.

2 Model

We draw on Parenti (2018) and Pan and Hanazono (2018) to describe the market structure with multi-product (MP) and single-product (SP) firms.[10] Consider a closed economy with two goods, a homogeneous good and a horizontally differentiated good. In the differentiated good market, there are a discrete number $N \geq 2$ of MP firms and a continuum $[0, S]$ of symmetric SP firms. MP firm $n = 1, ..., N$ supplies a continuum of varieties, with its product range represented by ω_n.

2.1 Preferences and Demand

Specifically, the utility of the representative consumer is described by

$$\begin{aligned} U \;=\; & \alpha \left[\int_0^S q_{sp}(i)di + \sum_{n=1}^{N} \int_0^{\omega_n} q_{mp}^n(x)dx \right] - \frac{\beta}{2} \left[\int_0^S (q_{sp}(i))^2 di + \sum_{n=1}^{N} \int_0^{\omega_n} (q_{mp}^n(x))^2 dx \right] \\ & -\frac{\gamma}{2} \left[\int_0^S q_{sp}(i)di + \sum_{n=1}^{N} \int_0^{\omega_n} q_{mp}^n(x)dx \right]^2 + q_0, \end{aligned} \tag{1}$$

where $q_{sp}(i)$ is the quantity produced by SP firm $i \in [0, S]$, and $q_{mp}^n(x)$ denotes the quantity of variety $x \in [0, \omega_n]$ for MP firm $n = 1, ..., N$. We treat ω_n as a continuous variable and let q_0 stand for the consumption of the homogeneous good which is the numeraire.

We assume that the demand parameters α, β and γ are all positive. The parameters α and γ capture the degree of substitutability between the varieties of the differentiated product and the numeraire: an increase in α or a decrease in γ shifts out the demand for the differentiated good relative to the numeraire. The parameter β represents the degree of differentiation between the differentiated varieties. The degree of differentiation increases with β since a higher β corresponds to a stronger preference for diversified consumption.

[10] The seminal work by Shimomura and Thisse (2012) is the first paper that characterizes the coexistence of large oligopolists and and small monopolistic competitors, but they assume that all the firms are single-product. Also assuming single-product firms, Anderson et al. (2020) validate the results in Shimomura and Thisse (2012) in a general framework via the aggregative games approach.

The representative consumer's budget constraint is given by

$$\int_0^S p_{sp}(i)q_{sp}(i)di + \sum_{n=1}^{N}\int_0^{\omega_n} p^n_{mp}(x)q^n_{mp}(x)dx + q_0 = I, \tag{2}$$

where $p_{sp}(i)$ and $p^n_{mp}(x)$ are the prices of SP firm i's product and MP firm n's variety x, respectively. The price of the numeraire is normalized to 1. The representative consumer's income is I, which is exogenously given.

The inverse demand function facing each SP and MP firm is determined by the maximization of the consumer's utility function subject to the budget constraint. The inverse demand functions are given by

$$p^n_{mp}(x) = \alpha - \beta q^n_{mp}(x) - \gamma Q \qquad n = 1, \cdots, N,\ x \in [0, \omega_n] \tag{3}$$

and

$$p_{sp}(i) = \alpha - \beta q_{sp}(i) - \gamma Q \qquad i \in [0, S] \tag{4}$$

where

$$Q = \int_0^S q_{sp}(i)di + \sum_{n=1}^{N}\int_0^{\omega_n} q^n_{mp}(x)dx \tag{5}$$

is the aggregate output of the differentiated good.

2.2 Firms

In the absence of a merger, we assume that each MP firm has a constant marginal cost C and each SP firm has a constant marginal cost c. In addition, both MP and SP firms incur fixed costs. SP firms incur a fixed cost f, which consists of a fixed cost of production and an entry cost. As incumbent firms, MP firms only incur a fixed cost of production per variety that they produce, denoted by F. In the following analysis, our results will critically depend on whether the per-variety fixed cost of the MP firms is smaller or larger than that of SP firms. For example, if the entry cost of SP firms is sufficiently high or MP firms enjoy economies of scope, then we may have $F < f$. On the other hand, if producing multiple varieties means increased coordination costs, then we may have $F > f$.

The firms play a non-cooperative game in which they choose their actions simultaneously. As incumbent firms, each MP firm produces a range of varieties and strategically chooses both its product range and the quantity of each variety. SP firms freely enter or exit the market, and once entering the market, each SP firm decides on its quantity. Hence, the aggregate product range in the model is determined by both the product range choices of the MP firms and the entry decisions of the SP firms. Note that there is an essential difference in the strategic behavior between MP and SP firms. Supplying a continuum of varieties and occupying a substantial market share, each MP firm generates a non-negligible market impact and competes in an oligopolistic manner. In contrast, each SP firm is negligible in the market and hence behaves non-strategically as a monopolistic competitor. We next explain the profit functions of the two type of firms.

2.2.1 MP Firms

Each MP firm n optimally chooses the range ω_n of varieties it will provide and the quantity $q_{mp}^n(x)$ for variety $x \in [0, \omega_n]$. The profit of MP firm n is expressed by

$$\Pi_{mp}^n(\omega_n, q_{mp}^n(\cdot)) = \int_0^{\omega_n} (p_{mp}^n(x) - C) q_{mp}^n(x) dx - \omega_n F. \tag{6}$$

Substituting $p_{mp}^n(x)$ from equation (3) into (6) gives us

$$\begin{aligned} \Pi_{mp}^n(\omega_n, q_{mp}^n(\cdot)) &= (\alpha - C) \int_0^{\omega_n} q_{mp}^n(x) dx - \beta \int_0^{\omega_n} (q_{mp}^n(x))^2 dx \\ &\quad -\gamma \left(\int_0^{\omega_n} q_{mp}^n(x) dx \right)^2 - \gamma Q^{-n} \int_0^{\omega_n} q_{mp}^n(x) dx - \omega_n F. \end{aligned} \tag{7}$$

Each MP firm takes as given the total output of other firms, $Q^{-n} = \int_0^S q_{sp}(i) di + \sum_{k \neq n} \int_0^{\omega_k} q_{mp}^k(x) dx$, and maximizes its profit with respect to ω_n and $q_{mp}^n(x)$. Hence, it takes its impact on the aggregate output into account while solving its maximization problem.

2.2.2 SP Small Firms

The profit of a SP firm is expressed by

$$\pi_{sp}(i) = (p_{sp}(i) - c)q_{sp}(i) - f. \tag{8}$$

Plugging $p_{sp}(i)$ from equation (4) into (8) yields

$$\pi_{sp}(i) = (\alpha - c)q_{sp}(i) - \beta[q_{sp}(i)]^2 - \gamma Q q_{sp}(i) - f. \tag{9}$$

As different from a MP firm, a SP firm is negligible in the market and understands that its decision does not affect the aggregate output Q. Therefore, each SP firm maximizes its profit with respect to its quantity $q_{sp}(i)$, treating the aggregate output Q parametrically.

2.2.3 Equilibrium

In the following analysis, we compare equilibria with and without a merger, by taking into account the impact of the merger on the MP firms' product choice and the mass of SP firms. We focus on the equilibrium where both $\omega > 0$ and $S > 0$. The equilibrium size S of the monopolistically competitive fringe is endogenously determined by the zero-profit condition:

$$\pi_{sp}(i) = (\alpha - c)q_{sp}(i) - \beta[q_{sp}(i)]^2 - \gamma Q q_{sp}(i) - f = 0. \tag{10}$$

Hence, in equilibrium, (i) each firm chooses its own action to maximize its own profits, (ii) the MP firms earn positive profits with a positive product range, and (iii) the mass of SP firms is positive and such that the zero-profit condition holds.

A market structure with MP and SP firms exists if and only if Assumption 1 holds.

Assumption 1

$$\text{(i) } \alpha > \max \left\{ \begin{array}{c} c(N+1) - CN + 2\sqrt{\beta}\left[(N+1)\sqrt{f} - N\sqrt{F}\right], \\ cN - C(N-2) + 2\sqrt{\beta}\left[N\sqrt{f} - (N-1)\sqrt{F}\right], \\ cN - C(N-1) + 2\sqrt{\beta}\left[(N-1)\sqrt{f} - (N-3)\sqrt{F}\right] \end{array} \right\}.$$

(ii) $c + 2\sqrt{\beta f} > C + 2\sqrt{\beta F}$.

We show the derivation of the conditions stated in Assumption 1 in Appendix A. As will become clear later on, Assumption 1(i) is a lower-bound condition on the demand intercept α such that the mass of SP firms $S > 0$ in equilibrium. The assumption ensures that $S > 0$ in equilibrium in all the market structures we consider in this paper: when there is no merger, when there is a merger with marginal cost synergies, and when there is a merger with fixed cost synergies.

Assumption 1(ii) is a condition on the product range of MP firms and ensures that MP firms earn a positive profit with a positive product range in equilibrium. This condition is sufficient both in the case without a merger and in the case with a merger. A closer look at Assumption 1(ii) shows that if the SP and MP firms have the same costs (i.e., $c = C$ and $f = F$), there will be no MP firms in the market. If $f \leq F$, then Assumption 1(ii) requires that $c > C$. On the other hand, if $c \leq C$, then Assumption 1(ii) requires that $f > F$. Hence, the MP firms must have either a marginal or a fixed cost advantage in comparison to the SP firms. This is because a MP firm maximizing the profits from all its varieties ends up internalizing externalities. This implies that at any given total output level, they earn less than they would if they were acting as a single-product firm. Hence, it would not be profitable for MP firms to exist if they had the same cost structure as or higher costs than the SP firms.

3 Market Outcome without a Merger

3.1 SP Firms' Entry and Profit Maximization

Since each SP firm's impact on the market is negligible, it does not internalize its impact on the market output. Maximizing the profit given in (9) with respect to $q_{sp}(i)$ yields

$$q_{sp}(i) = q_{sp} = \frac{\alpha - c - \gamma Q}{2\beta}. \tag{11}$$

Due to symmetry, all SP firms choose the same level of output. We can solve for the price of the SP firms by using (4). This yields:

$$p_{sp} = \frac{\alpha + c - \gamma Q}{2}. \tag{12}$$

After substituting (10) in (11) and using that $Q = Q_{mp} + Sq_{sp}$, where Q_{mp} is the aggregate quantity of MP firms, we can find the equilibrium mass of SP firms as a function of Q_{mp}:

$$S(Q_{mp}) = \frac{1}{\gamma}\left[\sqrt{\frac{\beta}{f}}(\alpha - c - \gamma Q_{mp}) - 2\beta\right]. \tag{13}$$

The equilibrium mass of SP firms decreases with Q_{mp}.

Expressing $Q = Q_{mp} + Sq_{sp}$, we can now substitute (13) into (11). Simplifying yields the optimal quantity of each SP firm as

$$q^*_{sp} = \sqrt{\frac{f}{\beta}}. \tag{14}$$

Finally, substituting for (11) and (13) in (12) yields the equilibrium price of SP firms as

$$p^*_{sp} = c + \sqrt{\beta f}. \tag{15}$$

Note that p^*_{sp}, q^*_{sp} and the revenue $p^*_{sp}q^*_{sp}$ are positively correlated with f. At a higher fixed cost, the equilibrium mass of SP firms will be lower. Each SP firm, expecting less competition, charges a higher price.

Furthermore, substituting (14) back into (10), the aggregate quantity in equilibrium is pinned down by the zero-profit condition of the SP firms:

$$Q^* = \frac{\alpha - c - 2\sqrt{\beta f}}{\gamma}. \tag{16}$$

Owing to the free entry and exit of SP firms, the aggregate quantity of the differentiated good depends on the technology of SP firms, but is independent of the technology of MP firms. Furthermore, the aggregate output is negatively correlated with the marginal and fixed costs of SP firms. In other words, higher efficiencies of SP firms would expand the market at the aggregate level.

3.2 MP Firms' Profit Maximization

Unlike SP firms, MP firms impose a non-negligible impact on the market. MP firm n maximizes its profit given in (7) with respect to its per-variety quantity and product range. The first-order condition of MP firm n with respect to the quantity $q^n_{mp}(x)$ of its xth variety is

$$\alpha - C - 2\beta q^n_{mp}(x) - 2\gamma \int_0^{\omega_n} q^n_{mp}(v)dv - \gamma Q^{-n} = 0, \tag{17}$$

which implies symmetric production behavior across the varieties within firm n. That is, $q^n_{mp}(x) = q^n_{mp}$ for $x \in [0, \omega_n]$. Hence, we have

$$\int_0^{\omega_n} q^n_{mp}(v)dv = \omega_n q^n_{mp}.$$

Substituting the above symmetry property into equation (17), we can solve for the optimal quantity q^n_{mp} of each variety as a function of MP firm n's product range ω_n and the expected aggregate output Q^{-n} of other firms:

$$q^n_{mp} = \frac{\alpha - C - \gamma Q^{-n}}{2(\beta + \gamma\omega_n)}. \tag{18}$$

Everything else being equal, an increase in firm n's product range (larger ω_n) results in a decrease in the quantity of each variety (smaller q^n_{mp}) produced due to a cannibalization effect.

Large firm n also maximizes its profit with respect to its product range, ω_n. The first-order condition is

$$\left(\alpha - C - (\beta + 2\gamma\omega_n)\, q^n_{mp} - \gamma Q^{-n}\right) q^n_{mp} = F. \tag{19}$$

Since $Q^{-n} = Sq_{sp}^* + \sum\limits_{k \neq n} \omega_k q_{mp}^*$, (19) can be expressed as

$$2\left(\beta + \gamma\omega_n\right) = \sqrt{\frac{\beta}{F}}\left[\alpha - C - \gamma\left(Sq_{sp}^* + \sum_{k \neq n} \omega_k q_{mp}^*\right)\right]. \tag{20}$$

From equations (18) and (19), we obtain the optimal per-variety output for the MP firm:

$$q_{mp}^{n*} = q_{mp}^* = \sqrt{\frac{F}{\beta}}. \tag{21}$$

Substituting for $q_{mp}^* = \sqrt{F/\beta}$ and $q_{sp}^* = \sqrt{f/\beta}$ in (20) yields

$$\gamma\omega_n = \sqrt{\frac{\beta}{F}}\left[\alpha - C - \gamma\left(S\sqrt{\frac{f}{\beta}} + \sum_{k=1}^{N} \omega_k \sqrt{\frac{F}{\beta}}\right)\right] - 2\beta, \tag{22}$$

which implies that $\omega_n = \omega$ for each $n = 1, .., N$. Simplifying gives an expression for ω which indicates that as the mass of SP firms increases, MP firms react by decreasing their variety range:

$$\omega(S) = \frac{\sqrt{\beta/F}\left(\alpha - C - \gamma S\sqrt{f/\beta}\right) - 2\beta}{\gamma(N+1)}. \tag{23}$$

3.3 Mixed Market Equilibrium

A mixed market equilibrium exists if and only if $S^* > 0$ and $\omega^* > 0$, which are ensured by the conditions given in Assumption 1. In such an equilibrium, the variety range is endogenously determined by the total mass of SP firms and the product range of each MP firm. In this section, we characterize the mixed market equilibrium.

From symmetry, we have $Q_{mp} = N\omega q_{mp}^* = N\omega\sqrt{F/\beta}$. Therefore, (13) can be re-expressed as

$$S(\omega) = \frac{\sqrt{\beta/f}(\alpha - c - \gamma N\omega\sqrt{F/\beta}) - 2\beta}{\gamma}. \tag{24}$$

Equations (23) and (24) jointly determine the equilibrium mass of SP firms S^* and the equilibrium product range of a MP firm ω^*. Solving these two equations simultaneously for S and ω yields

$$S^* = \sqrt{\frac{\beta}{f}} \frac{\alpha - [c(N+1) - CN] - 2\sqrt{\beta}\left[(N+1)\sqrt{f} - N\sqrt{F}\right]}{\gamma} \tag{25}$$

$$\omega^* = \sqrt{\frac{\beta}{F}} \frac{(c - C) + 2\sqrt{\beta}\left(\sqrt{f} - \sqrt{F}\right)}{\gamma}. \tag{26}$$

It is straightforward to check that both the mass of SP firms and the per-firm variety of MP firms are decreasing in own fixed cost and increasing in rival's fixed cost.[11]

We can now solve for the equilibrium price and profits of the MP firms using (3) and (6). We obtain

$$\begin{aligned} p^*_{mp} &= c + 2\sqrt{\beta f} - \sqrt{\beta F} \\ &= p^*_{sp} + \sqrt{\beta}\left(\sqrt{f} - \sqrt{F}\right) \end{aligned} \tag{27}$$

and

$$\Pi^*_{mp} = \frac{\left[2\sqrt{\beta}\left(\sqrt{f} - \sqrt{F}\right) + (c - C)\right]^2}{\gamma}. \tag{28}$$

The MP firms' equilibrium price is increasing in the SP firms' fixed cost and decreasing in MP firms' per-variety fixed cost. As f decreases, the MP firms face more competition from the SP firms. This increase in competitive pressure causes them to decrease their prices. As F decreases, each MP firm produces more varieties because it pays less for each variety it produces. This causes the firm to charge a higher price per unit of each variety in order to soften the competition between its varieties.

Lemma 1 compares the equilibrium per-variety quantity and price of SP firms and MP firms. It plays a key role in the welfare analysis of mergers with marginal cost synergies that we conduct in the next section.

Lemma 1 *$q^*_{sp} > q^*_{mp}$ and $p^*_{sp} < p^*_{mp}$ if and only if $f > F$.*

MP firms produce less (more) per variety and charge a higher (lower) price if

[11] In Appendix A, we show that q^{n*}_L and ω^*_n derived from (21) and (26) are locally optimal.

their fixed cost of production is lower (higher) than the per-variety fixed cost of SP firms. If $f > F$, then the cost for an MP firm to add a new variety is lower than the cost for a SP firm to enter the market and supply a variety. Since consumers prefer a diversified consumption, each MP firm takes advantage of a lower fixed cost by expanding its variety range. With an increased product range, each MP firm finds it profitable to reduce the per-variety quantity and increase the mark-up on each variety to ease the competition between its products.

4 Market Outcome with a Merger: Marginal Cost Synergies

If two MP firms merge, then there are $(N-1)$ MP firms in the market, including the merged firm (insider) and $(N-2)$ MP outsiders. We start by examining the role of marginal cost synergies. We use the superscript MC to denote the variables after a merger with marginal cost synergies.

We assume the merged entity enjoys a marginal cost synergy of $(1-\lambda)C$, so its marginal cost is reduced to λC, with $0 < \lambda < 1$. Following the merger, the SP firms simultaneously decide whether or not to enter the market and what quantity to produce. The MP incumbent firms make their product range and per-variety quantity choices. This determines their total fixed cost of production.[12] The merging firms maximize their joint profits while the rest maximize individual profits.

4.1 Outsiders

We first examine the behavior of non-merging firms. In a market structure with a merger, each SP outsider still maximizes (9) and its optimal choice still satisfies (11). Hence, the equilibrium total output level with and without the merger is pinned down by the same zero-profit condition given in (10) and is equal to (16).[13]

[12] Hence, we assume that after merging, the merged entity experiences a change in its marginal cost of production due to synergies and a change in its fixed cost of production due to the change in its product range.

[13] More generally, this result follows from the fact that under the twin assumptions of quadratic utility and Cournot competition, the firms play an aggregative game, where their payoff functions

It follows from (11) and (12) that the SP outsiders' equilibrium quantity and price are the same with and without the merger. Moreover, it is straightforward to see from (3), (17) and (19) that since $Q^{MC*} = Q^*$, the merger has no impact on the behavior of the MP outsiders either. We summarize these observations in the next lemma:

Lemma 2 *A bilateral merger between two MP firms has no impact on (i) the equilibrium total output level (i.e., $Q^{MC*} = Q^*$); (ii) the SP firms' behavior (i.e., $p_{sp}^{MC*} = p_{sp}^*$ and $q_{sp}^{MC*} = q_{sp}^*$); and (iii) the MP outsiders' behavior and profits (i.e., $p_{mp}^{MC*} = p_{mp}^*$, $\omega^{MC*} = \omega^*$, $q_{mp}^{MC*} = q_{mp}^*$ and $\Pi_{mp}^{MC*} = \Pi_{mp}^*$).*

4.2 MP Insider

The merging MP firms choose their product range and quantity for each variety to jointly maximize their profits. Let $\Pi_I^{MC} = \Pi_{I1}^{MC} + \Pi_{I2}^{MC}$ stand for the joint profits of the merging parties, where I stands for insider, and Π_{I1}^{MC} and Π_{I2}^{MC} represent the profits of the two parties of the merged firm. We have

$$\begin{aligned}
\Pi_I^{MC} = & \int_0^{\omega_{I1}^{MC}} \left(\alpha - \lambda C - \beta q_{I1}^{MC}(v) - \gamma \int_0^{\omega_{I1}^{MC}} q_{I1}^{MC}(x)dx - \gamma \int_0^{\omega_{I2}^{MC}} q_{I2}^{MC}(y)dy - \gamma Q_{-I}\right) q_{I1}^{MC}(v)dv \\
& + \int_0^{\omega_{I2}^{MC}} \left(\alpha - \lambda C - \beta q_{I2}^{MC}(w) - \gamma \int_0^{\omega_{I2}^{MC}} q_{I2}^{MC}(y)dy - \gamma \int_0^{\omega_{I1}^{MC}} q_{I1}^{MC}(x)dx - \gamma Q_{-I}\right) q_{I2}^{MC}(w)dw \\
& - \left(\omega_{I1}^{MC} F + \omega_{I2}^{MC} F\right),
\end{aligned}$$

where ω_{I1}^{MC} and ω_{I2}^{MC} represent the product range of the two parties of the merged firm, $q_{I1}^{MC}(v)$ represents the quantity of variety $v \in [0, \omega_{I1}^{MC}]$, and $q_{I2}^{MC}(w)$ represents the quantity of variety $w \in [0, \omega_{I2}^{MC}]$.

The first-order conditions with respect to $q_{I1}^{MC}(v)$ and ω_{I1}^{MC} are

$$\alpha - \lambda C - 2\beta q_{I1}^{MC}(v) - 2\gamma \int_0^{\omega_{I1}^{MC}} q_{I1}^{MC}(x)dx - 2\gamma \int_0^{\omega_{I2}^{MC}} q_{I2}^{MC}(y)dy - \gamma Q_{-I} = 0 \quad (29)$$

can be written as a function of their own action and an aggregate of all players' actions (Q in this case). See Anderson et al. (2020) on the long-run equilibrium properties of aggregative games.

and

$$\left(\alpha - \lambda C - \beta q_{I1}^{MC}(v) - 2\gamma \int_0^{\omega_{I1}^{MC}} q_{I1}^{MC}(x)dx - 2\gamma \int_0^{\omega_{I2}^{MC}} q_{I2}^{MC}(y)dy - \gamma Q_{-I}\right) q_{I1}^{MC}(v) - F = 0 \tag{30}$$

where $Q_{-I} = \int_0^{S^{MC}} q_{sp}^{MC}(i)di + \sum_{k=1}^{N-2} \int_0^{\omega^{MC}} q_{mpk}^{MC}(v)dv$ denotes the total output of all outsider firms. The first-order conditions with respect to $q_{I2}^{MC}(v)$ and ω_{I2}^{MC} can be written similarly.

The first-order conditions with respect to $q_{I1}^{MC}(v)$ and $q_{I2}^{MC}(w)$ imply symmetry across the quantities produced of the different varieties: $q_{I1}^{MC} = q_{I2}^{MC} = q_I^{MC}$. Define $\omega_I^{MC} = \omega_{I1}^{MC} + \omega_{I2}^{MC}$ as the product range of the merged firm. The first-order conditions could be rewritten as

$$\alpha - \lambda C - 2\beta q_I^{MC} - 2\gamma \omega_I^{MC} q_I^{MC} - \gamma Q_{-I} = 0 \tag{31}$$

and

$$\left[\alpha - \lambda C - \beta q_I^{MC} - 2\gamma \omega_I^{MC} q_I^{MC} - \gamma Q_{-I}\right] q_I^{MC} - F = 0. \tag{32}$$

It is immediate from these two equations that

$$q_I^{MC*} = \sqrt{\frac{F}{\beta}}. \tag{33}$$

Comparing with (21), we note that the merger has no impact on the per-variety quantity choice of the merged entity.

Since $q_I^{MC*} = q_{mp}^*$ and $Q^{MC*} = Q^*$, we obtain from (3) that $p_I^{MC*} = p_{mp}^*$. That is, the merger has no impact on the price of the merged entity. However, as we show below, the merger causes the merged entity to change the product range it offers. This finding stands in contrast to what would happen in a set-up where product variety was not endogenously determined. In such a set-up, the full impact of the merger would be felt through a price change.[14]

To find the insider's equilibrium product range from (32), we first note that

[14]The result we obtain here is specific to marginal cost synergies. As we show in the next section, with fixed cost synergies, mergers affect consumer welfare through the twin channels of price and product variety changes.

$Q_{-I} = Q^{MC} - \omega_I^{MC} q_I^{MC}$. Then, after substituting for Q^{MC*} and q_I^{MC*} in (33), we obtain

$$\omega_I^{MC*} = \sqrt{\frac{\beta}{F}} \frac{2\sqrt{\beta}\left(\sqrt{f} - \sqrt{F}\right) + (c - \lambda C)}{\gamma}. \tag{34}$$

Consequently, the insider's profit is

$$\Pi_I^{MC*} = \frac{\left[2\sqrt{\beta}\left(\sqrt{f} - \sqrt{F}\right) + (c - \lambda C)\right]^2}{\gamma}. \tag{35}$$

A merger is profitable if $\Pi_I^{MC*} > 2\Pi_{mp}^*$. Simplifying gives

$$\lambda < 1 - \frac{(\sqrt{2} - 1)\left[2\sqrt{\beta}\left(\sqrt{f} - \sqrt{F}\right) + (c - C)\right]}{C} = \overline{\lambda}.$$

Hence, while the merger has no impact on the profits of the outsider firms, it has a positive impact on the insider firms if and only if $\lambda < \overline{\lambda}$.

The merged firm expands its product range after the merger (i.e., $\omega_I^{MC*} > 2\omega^*$) if

$$\lambda < 1 - \frac{2\sqrt{\beta}\left(\sqrt{f} - \sqrt{F}\right) + (c - C)}{C} = \underline{\lambda}.$$

We make the following assumption which ensures that $0 < \underline{\lambda}$.

Assumption 2 $2\sqrt{\beta}\left(\sqrt{f} - \sqrt{F}\right) + (c - 2C) < 0.$

Since $0 < \underline{\lambda} < \overline{\lambda}$, this assumption allows us to consider profitable mergers with and without a product range expansion. Any merger which results in product expansion must be profitable. However, profitable mergers may be accompanied by a decrease in the range of products offered by the merged firms if the level of synergies is such that $\underline{\lambda} < \lambda < \overline{\lambda}$.

In what follows, we refer to the case of $\overline{\lambda} < \lambda$ as low synergies, $\underline{\lambda} < \lambda < \overline{\lambda}$ as moderate synergies, and $\lambda < \underline{\lambda}$ as high synergies. The following lemma summarizes the impact of different levels of marginal cost synergies on the merged entity.

Lemma 3 *A merger with marginal cost synergies is profitable if $\lambda < \overline{\lambda}$. Although the merger has no impact on the per-variety quantity or price of the merged firms,* *it has a positive impact on the product range of the merged firms if* $\lambda < \underline{\lambda} < \overline{\lambda}$ *(high synergies), and has a negative impact on the product range of the merged firms if* $\underline{\lambda} < \lambda < \overline{\lambda}$ *(moderate synergies).*

Lemma 2 and 3 show that although a merger between two MP firms has no impact on the SP firms' equilibrium quantity, the MP firms' equilibrium quantity and the MP outsiders' product range, it changes the product range of the merged firms if the level of synergies is sufficiently high. Intuitively, a marginal increase in product variety has two kinds of effects on the merged entity's profits. The first effect is negative due to cannibalization of its own products and the second one is positive due to business stealing from outsiders. In the absence of marginal cost synergies, the first effect dominates and the merged firms end up reducing their product range. When the cost synergies are sufficiently high, the second effect dominates and the net result is an increase in product variety.

4.3 Market Performance

We are now in a position to evaluate the welfare implications of the merger. We start by analyzing the impact of the merger on the mass of SP firms. The pre-merger aggregate output Q^* and post-merger aggregate output Q^{MC*} can be expressed as

$$\begin{aligned} Q^* &= S^* q_{sp}^* + N\omega^* q_{mp}^*, \\ Q^{MC*} &= S^{MC*} q_{sp}^{MC*} + (N-2)\omega^{MC*} q_{mp}^{MC*} + \omega_I^{MC*} q_I^{MC*}. \end{aligned}$$

As shown in Lemma 2, the aggregate output Q^{MC*}, the quantity of each SP firm q_{sp}^{MC*}, and the quantity of each MP outsider $\omega^{MC*} q_{mp}^{MC*}$ do not change with the merger. In addition, Lemma 3 states that for the MP insider, $q_I^{MC*} = q_{mp}^{MC*}$. Hence, rearranging these two expressions gives us

$$\Delta S^{MC} q_{sp}^* + \Delta V_{mp}^{MC} q_{mp}^* = 0, \tag{36}$$

where $\Delta S^{MC} = S^{MC*} - S^*$ is the change in the mass of SP firms, and $\Delta V_{mp}^{MC} = \omega_I^{MC*} - 2\omega^*$ is the change in the variety range of MP firms. Equation (36) implies that $\Delta S^{MC} > 0$ if and only if $\Delta V_{mp}^{MC} < 0$, i.e., any product variety expansion by the merged entity displaces SP firms. Alternatively, when the insider chooses to shrink its product range (with moderate merger efficiencies from Lemma 3), it produces less than the sum of the two merging firms' outputs without the merger. This leaves room in the market for more SP firms to enter.

We can solve for the equilibrium number of SP firms with a merger as a function of the merger-generated marginal cost synergies:

$$S^{MC*}(\lambda) = \sqrt{\frac{\beta}{f}} \frac{\alpha - [cN - C(N - 2 + \lambda)] - 2\sqrt{\beta}\left[\sqrt{f}N - \sqrt{F}(N-1)\right]}{\gamma}. \quad (37)$$

It is straightforward to note that this expression is increasing in λ. Hence, a larger level of synergies (i.e., lower λ) implies a smaller mass of SP firms.

Lemma 4 summarizes the impact of merger-generated marginal cost synergies on the mass of SP firms.

Lemma 4 *Moderate merger efficiencies (that is, $\underline{\lambda} < \lambda < \overline{\lambda}$), which induce a shrinkage of the insider's product range, increase the mass of SP firms. By contrast, high merger efficiencies, (that is, $0 < \lambda \leq \underline{\lambda}$), which induce an expansion of the insider's product range, reduce the mass of SP firms.*

The change in consumer welfare from a merger can be expressed as:

$$\Delta U^{MC} = \underbrace{\frac{\beta}{2}\Delta S^{MC} q_{sp}^*(q_{sp}^* - q_{mp}^*)}_{\text{Reallocation Effect}}.$$

The reallocation effect represents the reallocation of production between the MP and SP firms. A change in the MP insider's product range after the merger results in the reallocation of production across MP and SP firms. The sign of the reallocation effect depends on the change in the mass of SP firms, ΔS^{FC}, and the comparison of per-variety quantity between SP and MP firms, q_{sp}^* and q_{mp}^*.

Consider first the case when a SP firm's fixed cost is larger than that of a MP firm, i.e., $f > F$. As shown in Lemma 1, if $f > F$, then a SP firm produces more at a lower price than a MP firm for each variety, i.e., $q_{sp}^* > q_{mp}^*$ and $p_{sp}^* < p_{mp}^*$.

Furthermore, by Lemmas 3 and 4, with a moderate marginal cost synergy, the MP insider shrinks its production and the mass of SP firms increases ($\Delta S^{MC} > 0$). Thus, a portion of MP firms' production is reallocated to the SP firms. Since SP firms supply a larger per-variety quantity at a lower price than MP firms, the entry of SP firms generates a positive reallocation effect on consumer welfare. Hence, taking the endogenous post-merger product choice of the MP insider into account, our welfare results imply that a moderate level of merger-generated marginal cost synergies may improve consumer welfare (and high synergies may not) in a market where the SP firms are more aggressive.

In contrast, if a SP firm's fixed cost is smaller than a MP firm's, i.e., $f < F$,[15] then a SP firm behaves less aggressively than a MP firm, i.e., $q^*_{sp} < q^*_{mp}$ and $p^*_{sp} > p^*_{mp}$. In this case, moderate marginal cost synergies, which increase the mass of SP firms, generate a negative reallocation effect on consumer welfare. Therefore, when $f < F$, moderate marginal cost synergies that invite more SP firms into the market are detrimental to consumer welfare, but high marginal cost synergies that cause SP firms to exit are beneficial to consumer welfare.

Proposition 1 summarizes the impact of mergers with marginal cost synergies on consumer welfare.

Proposition 1 *When the fixed cost of a SP firm is higher than the per-variety fixed cost of a MP firm, i.e., $f > F$, all profitable mergers with moderate marginal cost synergies ($\underline{\lambda} < \lambda < \overline{\lambda}$) are beneficial to consumer welfare, but all profitable mergers with high marginal cost synergies ($0 < \lambda \leq \underline{\lambda}$) are detrimental to consumer welfare. In contrast, when the fixed cost of a SP firm is lower than the per-variety fixed cost of a MP firm, i.e., $f < F$, all profitable mergers with moderate marginal cost synergies ($\underline{\lambda} < \lambda < \overline{\lambda}$) are detrimental to consumer welfare, but all profitable mergers with high marginal cost synergies ($0 < \lambda \leq \underline{\lambda}$) are beneficial to consumer welfare.*

Proposition 1 underlines that while analyzing the potential impact of a merger with marginal cost synergies, it is important to consider firms' per-variety fixed

[15] By the second condition in Assumption 1, this is satisfied only if $c > C$, i.e. a SP small firm's marginal cost is strictly higher than a MP large firm's.

costs of production. The existence of high marginal cost synergies is not sufficient to reach a conclusion on the welfare impact of a merger. Mergers with high marginal cost synergies are detrimental to consumer welfare if $f > F$.[16]

As a corollary to Proposition 1, we emphasize the link between a merger's impact on the entry/exit of SP firms and its impact on consumer welfare.

Corollary 1 *When $f > F$, all profitable mergers which induce entry (exit) of SP firms are beneficial (harmful) to consumer welfare. In contrast, when $f < F$, all profitable mergers which induce entry (exit) of SP firms are harmful (beneficial) to consumer welfare.*

On the policy front, Corollary 1 has significant implications. We know from Lemma 4 that mergers with sufficiently high marginal cost synergies cause SP firms to exit. Corollary 1 states that such exit may sometimes (i.e., when $f < F$) be an indication that the merger is beneficial to consumer welfare. Hence, using Corollary 1, policy makers can adopt a "fringe test" to evaluate the impact of a merger on consumer welfare. The impact of a merger on the fringe firms (SP firms) may help them understand whether the merger is likely to harm or hurt consumers. Importantly, Corollary 1 shows that, contrary to the view taken in merger guidelines, entry is not always a necessary condition for a merger to increase consumer welfare.

We next investigate how a change in aggregate variety affects consumer welfare. As a first step, the following lemma establishes the link between the level of synergies and aggregate variety. Rearranging (36) gives

$$\frac{q_{sp}^{*} - q_{mp}^{*}}{q_{mp}^{*}} \Delta S^{MC} = -\Delta V^{MC},$$

where $\Delta V^{MC} = \Delta S^{MC} + \Delta V_L^{MC}$ is the change in aggregate variety. By Lemma 1, if $f > F$, then $q_{sp}^{*} > q_{mp}^{*}$. That is, the per-variety quantity of a SP firm is larger than that of a MP firm. Since aggregate quantity does not change with the merger, $q_{sp}^{*} > q_{mp}^{*}$ implies that $Sign(\Delta V^{MC}) = -Sign(\Delta S^{MC})$. As a result, an increase in the mass of SP firms is accompanied with a decrease in aggregate

[16] Covarrubias et al. (2020) present evidence in line with this condition.

variety, and vice versa. On the contrary, if $f < F$, which implies that $q^*_{sp} < q^*_{mp}$, then $Sign(\Delta V^{MC}) = Sign(\Delta S^{MC})$. Lemma 5 summarizes the impact of a merger on aggregate variety.

Lemma 5 *When the fixed cost of a SP firm is higher than the per-variety fixed cost of a MP firm, i.e., $f > F$, a merger with moderate marginal cost synergies ($\underline{\lambda} < \lambda < \overline{\lambda}$) decreases aggregate variety while a merger with high marginal cost synergies ($0 < \lambda \leq \underline{\lambda}$) increases aggregate variety. In contrast, when the fixed cost of a SP firm is lower than the per-variety fixed cost of a MP firm, i.e., $f < F$, a merger with moderate marginal cost synergies ($\underline{\lambda} < \lambda < \overline{\lambda}$) increases aggregate variety while a merger with high marginal cost synergies ($0 < \lambda \leq \underline{\lambda}$) decreases aggregate variety.*

Finally, combining Proposition 1 with Lemma 5 yields that in mergers with marginal cost synergies, there exists an inverse relationship between aggregate variety and consumer welfare. Whenever aggregate variety increases, consumer welfare decreases.

Proposition 2 *A merger with marginal cost synergies increases consumer welfare if and only if it decreases aggregate variety.*

This result implies that in mergers with marginal cost synergies, any consumer welfare gain does not originate from an increase in aggregate variety. Rather, it is the lower prices that consumers face as a result of the reallocation of production between MP and SP firms that benefit the consumers.

5 Market Outcome with a Merger: Fixed Cost Synergies

In this section, we analyze the impact of a merger between two MP firms that results in fixed cost synergies. While marginal cost synergies decrease the per-unit production cost, fixed cost synergies reduce the per-variety production cost. Specifically, suppose that with the merger, the merged firm's per-variety fixed cost is reduced to $\delta^2 F$ where $\delta \in (0, 1)$.

As in the case of marginal cost synergies, a merger with fixed cost synergies has no impact on the aggregate quantity or the individual behavior of non-merging firms. To see this, note first that each SP outsider still maximizes (9) and the free entry condition is still given by (10). This implies that the equilibrium behavior of SP outsiders is given by the same expression with and without a merger, and $p_{sp}^{FC*} = p_{sp}^*$ and $q_{sp}^{FC*} = q_{sp}^*$ where the superscript FC denotes fixed cost synergies. As the behavior of SP outsiders does not change with a merger, by the free entry condition (10), the equilibrium total output does not change either: $Q^{FC*} = Q^*$. Finally, since $Q^{FC*} = Q^*$ and the first-order conditions of MP outsiders are still given by (17) and (19), the merger has no impact on the behavior of the MP outsiders. That is, $p_{mp}^{FC*} = p_{mp}^*$, $q_{mp}^{FC*} = q_{mp}^*$ and $\Pi_{mp}^{FC*} = \Pi_{mp}^*$. Therefore, Lemma 2 continues to hold.

We next consider the impact of fixed cost synergies on the merged entity. The merged firm's profit is

$$\begin{aligned} \Pi_I^{FC} &= \Pi_{I1}^{FC} + \Pi_{I2}^{FC} \\ &= \int_0^{\omega_{I1}^{FC}} \left(\alpha - C - \beta q_{I1}^{FC}(v) - \gamma \int_0^{\omega_{I1}^{FC}} q_{I1}^{FC}(x)dx - \gamma \int_0^{\omega_{I2}^{FC}} q_{I2}^{FC}(y)dy - \gamma Q_{-I}^{FC} \right) q_{I1}^{FC}(v)dv \\ &\quad + \int_0^{\omega_{I2}^{FC}} \left(\alpha - C - \beta q_{I2}^{FC}(w) - \gamma \int_0^{\omega_{I2}^{FC}} q_{I2}^{FC}(y)dy - \gamma \int_0^{\omega_{I1}^{FC}} q_{I1}^{FC}(x)dx - \gamma Q_{-I}^{FC} \right) q_{I2}^{FC}(w)dw \\ &\quad -\delta^2 \left(\omega_{I1}^{FC} F + \omega_{I2}^{FC} F \right). \end{aligned} \tag{38}$$

The first-order conditions with respect to $q_{I1}^{FC}(v)$ and ω_{I1}^{FC} are

$$\alpha - C - 2\beta q_{I1}^{FC}(v) - 2\gamma \int_0^{\omega_{I1}^{FC}} q_{I1}^{FC}(x)dx - 2\gamma \int_0^{\omega_{I2}^{FC}} q_{I2}^{FC}(y)dy - \gamma Q_{-I}^{FC} = 0 \tag{39}$$

and

$$\left(\alpha - C - \beta q_{I1}^{FC}(v) - 2\gamma \int_0^{\omega_{I1}^{FC}} q_{I1}^{FC}(x)dx - 2\gamma \int_0^{\omega_{I2}^{FC}} q_{I2}^{FC}(y)dy - \gamma Q_{-I}^{FC} \right) q_{I1}^{FC}(v) - \delta^2 F = 0 \tag{40}$$

where $Q_{-I}^{FC} = \int_0^{S^{FC}} q_{sp}^{FC}(i)di + \sum_{k=1}^{N-2} \int_0^{\omega_{mpk}^{FC}} q_{mpk}^{FC}(v)dv$ denotes the total output of all outsider firms. The first-order conditions with respect to $q_{I2}^{FC}(v)$ and ω_{I2}^{FC} can be written similarly. Imposing symmetry across the quantities produced of the different varieties and setting $q_{I2}^{FC} = q_{I1}^{FC} = q_I^{FC}$ gives us

$$\alpha - C - 2\beta q_I^{FC} - 2\gamma\omega_{I1}^{FC} q_I^{FC} - 2\gamma\omega_{I2}^{FC} q_I^{FC} - \gamma Q_{-I}^{FC} = 0 \tag{41}$$

and

$$(\alpha - C - \beta q_I^{FC} - 2\gamma\omega_{I1}^{FC} q_I^{FC} - 2\gamma\omega_{I2}^{FC} q_I^{FC} - \gamma Q_{-I}^{FC}) q_I^{FC} - \delta^2 F = 0. \tag{42}$$

It is immediate from these two equations that

$$q_I^{FC*} = \delta\sqrt{\frac{F}{\beta}}. \tag{43}$$

Substituting q_I^{FC*} into the inverse demand, the insider's price is

$$p_I^{FC*} = c + 2\sqrt{\beta f} - \delta\sqrt{\beta F} > p_{mp}^* = c + 2\sqrt{\beta f} - \sqrt{\beta F}. \tag{44}$$

In contrast with the case of marginal cost synergies, we immediately observe that fixed cost synergies affect both the per-variety quantity and price of the merged entity. Specifically, per-variety quantity decreases and price increases as fixed cost synergies increase. An increase in fixed cost synergies gives the merged entity an advantage in the cost of introducing new varieties. As the merged entity introduces more varieties, it finds it optimal to increase the price of each variety in order to reduce the competitive pressure between them.

Moreover, since $p_{sp}^* = c + \sqrt{\beta f}$, we can also write (44) as $p_I^{FC*} = p_{sp}^* + \sqrt{\beta}\left(\sqrt{f} - \delta\sqrt{F}\right)$. Hence, the price difference between the merged entity and the SP firms depends on the difference between their (post-merger) fixed costs. This implies that the ranking between the prices of the MP and SP firms may be reversed if it is the case that $f < F$ before the merger and the magnitude of fixed cost synergies are such that $f > \delta^2 F$ after the merger.

Note that $Q_{-I}^{FC} = Q^{FC} - \omega_{I1}^{FC} q_I^{FC} - \omega_{I2}^{FC} q_I^{FC}$ and that $\omega_{I1}^{FC} = \omega_{I2}^{FC}$ under symmetry. Define $\omega_I^{FC} = \omega_{I1}^{FC} + \omega_{I2}^{FC}$ as the variety range of the merged firm. Substituting (43) into (42) and using Lemma 2, the equilibrium variety range of the merged entity can be expressed as

$$\omega_I^{FC*} = \frac{1}{\delta}\sqrt{\frac{\beta}{F}}\frac{2\sqrt{\beta}\left(\sqrt{f}-\delta\sqrt{F}\right)+(c-C)}{\gamma}. \tag{45}$$

Its profit is

$$\Pi_I^{FC*} = \frac{\left[2\sqrt{\beta}\left(\sqrt{f}-\delta\sqrt{F}\right)+(c-C)\right]^2}{\gamma}. \tag{46}$$

We next consider how the merged entity's profits, product range and total production depend on the level of synergies. From (45) and (46), we can readily verify that both ω_I^{FC*} and Π_I^{FC*} decrease with δ. That is, the merged entity's product range and profit increase as it enjoys larger fixed cost synergies. A merger is profitable if and only if $\Pi_I^{FC*} > 2\Pi_{mp}^*$. Simplifying yields:

$$\delta < 1 - \frac{\left(\sqrt{2}-1\right)}{2\sqrt{\beta F}}\left[2\sqrt{\beta}\left(\sqrt{f}-\sqrt{F}\right)+(c-C)\right] = \delta_1. \tag{47}$$

A merger increases its total variety range (i.e., $\omega_I^{FC*} > 2\omega^*$) if and only if

$$\delta < 1 - \frac{2\sqrt{\beta}\left(\sqrt{f}-\sqrt{F}\right)+(c-C)}{2\left[\sqrt{\beta}\left(2\sqrt{f}-\sqrt{F}\right)+(c-C)\right]} = \delta_2. \tag{48}$$

Finally, a merged entity increases its total quantity (i.e., $\omega_I^{FC*}q_I^{FC*} > 2\omega^* q_{mp}^*$) if and only if

$$\delta < 1 - \frac{2\sqrt{\beta}\left(\sqrt{f}-\sqrt{F}\right)+(c-C)}{2\sqrt{\beta F}} = \delta_3. \tag{49}$$

It is readily verifiable that $\delta_3 < \delta_2 < \delta_1 < 1$, where the last inequality follows from Assumption 1(ii). We summarize the impact of different levels of fixed cost synergies on the behavior of the merged entity in the next lemma.

Lemma 6 *A merger with fixed cost synergies causes the per-variety quantity of the merged firms to decrease and the per-variety price of the merged firms to increase. Moreover, there exist threshold values* $0 < \delta_3 < \delta_2 < \delta_1 < 1$ *such that (i) all mergers with* $0 < \delta < \delta_1$ *are profitable, (ii) all mergers with* $0 < \delta < \delta_2$ *result* *in an expansion of variety range of the merged entity, and (iii) all mergers with* $0 < \delta < \delta_3$ *result in an increase in the total production of the merged entity.*

Similar to Lemma 4, the next lemma establishes the impact of a merger with fixed cost synergies on the mass of SP firms. As discussed above, the merger has no impact on the aggregate quantity, the SP firms' behavior, and the MP outsiders' behavior. Using

$$Q^* = Q^{FC*}$$

and expanding both sides, we get

$$S^* q_{sp}^* + 2\omega^* q_{mp}^* + (N-2)\omega^* q_{mp}^* = S^{FC*} q_{sp}^* + \omega_I^{FC*} q_I^{FC*} + (N-2)\omega^* q_{mp}^*,$$

which implies

$$\Delta S^{FC} q_{sp}^* = 2\omega^* q_{mp}^* - \omega_I^{FC*} q_I^{FC*},$$

where $\Delta S^{FC} = S^{FC*} - S^*$ denotes the change in the mass of SP firms. As stated in the next lemma, this expression implies that the mass of SP firms decreases whenever the MP insider expands its total production.

Lemma 7 *A profitable merger with merger-generated fixed cost synergies* $0 < \delta < \delta_3$ *causes the mass of SP firms to decrease. A profitable merger with merger-generated fixed cost synergies* $\delta_3 < \delta < \delta_1$ *causes the mass of SP firms to increase.*

This result is consistent with the one we have for marginal cost synergies (Lemma 4). Since a merger with marginal cost synergies has no impact on the merged entity's per-variety quantity, an increase in total variety is equivalent to an increase in total production with marginal cost synergies. In the case with fixed cost synergies, an increase in total variety is not equivalent to an increase in total production because a merger reduces the per-variety quantity of the MP insider.

5.1 Market performance

Since the price of the merged entity changes with the level of fixed cost synergies, as shown in (44), the impact of a merger on consumer welfare can be decomposed into the following two effects:[17]

$$\Delta U^{FC} = \underbrace{\frac{\beta}{2}\Delta S^{FC} q_{sp}^{*}\left(q_{sp}^{*} - q_{mp}^{*}\right)}_{\text{Reallocation Effect}} + \underbrace{\omega_I^{*} q_I^{FC*}\left(p_{mp}^{*} - p_I^{FC*}\right)}_{\text{Price Effect}}. \tag{50}$$

As before, the reallocation effect represents the reallocation of production between the MP and SP firms. As in the case of marginal cost synergies, the sign of the reallocation effect depends on the change in the mass of SP firms, ΔS^{FC}, and the comparison of per-variety quantity between SP and MP firms, q_{sp}^{*} and q_{mp}^{*}. The following lemma establishes when the reallocation effect is positive. The result follows from Lemma 7 and the expressions for q_{sp}^{*} and q_{mp}^{*} given in (11) and (21).

Lemma 8 *For $f > F$, the reallocation effect is positive if and only if the mass of SP firms increases, i.e., $\Delta S^{FC} > 0$, which occurs for $\delta_3 < \delta$. For $f < F$, the reallocation effect is positive if and only if the mass of SP firms decreases, i.e., $\Delta S^{FC} < 0$, which occurs for $0 < \delta < \delta_3$.*

Lemma 8 implies that when $f < F$ ($f > F$), a necessary condition for mergers to benefit consumer welfare is to have exit (entry) by SP firms. When $f < F$, even though society pays a lower fixed cost per variety with each SP firm, the SP firms charge a higher price per unit (Lemma 1). Due to the higher price charged by the SP firms, it is beneficial to have less of them.

The second effect in (50), the price effect, is about the impact of the merger on the price of the merging firms. From (44), we observe that the price effect is negative whenever the merged entity incurs fixed cost synergies since fixed cost synergies increase the price of the MP insider, i.e., $p_{mp}^{*} < p_I^{FC*}$. Furthermore, the price effect strengthens as the merged entity enjoys higher fixed cost synergies, as both $\omega_I^{*} q_I^{FC*}$ and $(p_{mp}^{*} - p_I^{FC*})$ increase with δ. Therefore, the negative price effect may dominate whenever the merged entity enjoys sufficiently high fixed cost synergies. The existence of the negative price effect suggests that mergers with fixed cost synergies may be more likely to decrease consumer welfare in comparison to marginal cost synergies.

[17] Specifically, using that $Q^{FC*} = Q^{*}$, $\Delta U = \frac{\beta}{2}[\Delta S^{FC}(q_{sp}^{*})^2 + \omega_I^{FC*}(q_I^{FC*})^2 - 2\omega^{*}(q_{mp}^{*})^2] = \frac{\beta}{2}[\Delta S^{FC}(q_{sp}^{*})^2 + \omega_I^{FC*}(q_I^{FC*})^2 - q_{mp}^{*}(\Delta S^{FC} q_{sp}^{*} + \omega_I^{FC*} q_I^{FC*})] = \frac{\beta}{2}[\Delta S^{FC} q_{sp}^{*}(q_{sp}^{*} - q_{mp}^{*}) + \omega_I^{FC*} q_I^{FC*}(q_I^{FC*} - q_{mp}^{*})] = \frac{\beta}{2}[\Delta S^{FC} q_{sp}^{*}(q_{sp}^{*} - q_{mp}^{*}) + \omega_I^{FC*} q_I^{FC*}(p_{mp}^{*} - p_I^{FC*})]$

We next examine more closely whether a profitable merger with fixed cost synergies improves or harms consumer welfare. To do this, we first establish how different levels of fixed cost synergies impact consumer welfare without taking into account merger profitability.

The change in consumer welfare with fixed cost synergies can be expressed as

$$\Delta U^{FC} = \frac{\sqrt{\beta}}{2\gamma} \left\{ \begin{array}{c} (c - C)\left[\sqrt{f} - \sqrt{F}(2-\delta)\right] \\ +2\sqrt{\beta}\left[f - 2\sqrt{fF}\,(2-\delta) + F(2-\delta^2)\right] \end{array} \right\}, \tag{51}$$

which has a negative sign with respect to the quadratic term δ^2. The two solutions to $\Delta U^{FC} = 0$ are given by

$$\delta_4 = \frac{4\sqrt{\beta f} + (c-C) - \sqrt{32\beta(\sqrt{f}-\sqrt{F})^2 + 16\sqrt{\beta}(\sqrt{f}-\sqrt{F})(c-C) + (c-C)^2}}{4\sqrt{\beta F}}, \tag{52}$$

$$\delta_5 = \frac{4\sqrt{\beta f} + (c-C) + \sqrt{32\beta(\sqrt{f}-\sqrt{F})^2 + 16\sqrt{\beta}(\sqrt{f}-\sqrt{F})(c-C) + (c-C)^2}}{4\sqrt{\beta F}}, \tag{53}$$

where $\delta_4 < \delta_5$. Therefore, $\Delta U^{FC} > 0$ if and only if $\delta_4 < \delta < \delta_5$. It is readily verified that $\delta_5 > 0$.[18] $\delta_4 > 0$ if and only if $\left(\sqrt{f} - 2\sqrt{F}\right)\left(2\sqrt{\beta}(\sqrt{f}-\sqrt{F}) + (c-C)\right) < 2\sqrt{\beta f F}$.

We summarize the results in Lemma 9.

Lemma 9 *(i) If* $\left(\sqrt{f} - 2\sqrt{F}\right)\left(2\sqrt{\beta}(\sqrt{f}-\sqrt{F}) + (c-C)\right) \geq 2\sqrt{\beta f F}$*, then* $\delta_4 \leq 0$*. In this case, consumer welfare increases if* $\delta \in (0, \delta_5)$*, and decreases if* $\delta \in (\delta_5, 1)$*.*

(ii) If $\left(\sqrt{f} - 2\sqrt{F}\right)\left(2\sqrt{\beta}(\sqrt{f}-\sqrt{F}) + (c-C)\right) < 2\sqrt{\beta f F}$*, then* $\delta_4 > 0$*. In this case, consumer welfare increases if* $\delta \in (\delta_4, \delta_5)$*, and decreases if* $\delta \in (0, \delta_4) \cup (\delta_5, 1)$*.*

Assuming $\delta_4 > 0$, we define $\delta \in (0, \delta_4)$ as high fixed cost synergies, $\delta \in (\delta_4, \delta_5)$ as moderate fixed cost synergies, and $\delta \in (\delta_5, 1)$ as low fixed cost synergies. Then,

[18] By Assumption 1(ii), $2\sqrt{\beta}\left(\sqrt{f} - \sqrt{F}\right) + (c-C) > 0$, which is equivalent to $2\sqrt{\beta f} + (c-C) > 2\sqrt{\beta F} > 0$. This is sufficient for $4\sqrt{\beta f} + (c-C) > 0$. Furthermore, we assume that the roots δ_4 and δ_5 are real, which is guaranteed by (55). Therefore, $\sqrt{32\beta\left(\sqrt{f}-\sqrt{F}\right)^2 + 16\sqrt{\beta}\left(\sqrt{f}-\sqrt{F}\right)(c-C) + (c-C)^2} > 0$, and $\delta_5 > 0$ holds.

Lemma 9 implies that mergers with moderate fixed cost synergies increase consumer welfare, whereas mergers with high or low fixed cost synergies decrease consumer welfare. Because the price effect is increasing in the level of synergies, its effect makes mergers with high fixed cost synergies consumer welfare decreasing.

We are interested in exploring how profitable mergers affect consumer welfare. Since Lemma 6 demonstrates that a merger is profitable if $\delta \in (0, \delta_1)$, we make the following assumption to ensure that $\delta_1 > 0$ and there exist some profitable mergers.

Assumption 3 $(c - C) < 2(\sqrt{2} + 2)\sqrt{\beta F} - 2\sqrt{\beta f}$.

In order to examine the impact of profitable mergers on consumer welfare, we need to compare δ_1 with δ_4 and δ_5. Suppose that the conditions in Lemma 9-(ii) are satisfied such that δ_4 and δ_5 exist and are both positive. The next proposition establishes the impact of mergers with fixed cost synergies on consumer welfare.

Proposition 3 *Consider a merger with fixed cost synergies and suppose that Assumptions 1 and 3 hold. Then:*

(i) If $f > F$ and $c \geq C$, then all profitable mergers decrease consumer welfare.

(ii) If $f > F$ and $c < C$, then there exist some profitable mergers with moderate fixed cost synergies which increase consumer welfare.

(iii) If $f < F$ and $c > C$, then there are two subcases depending on how efficient the MP firms are:

a. Suppose $(c - C) > (-8 + 4\sqrt{2})\sqrt{\beta}(\sqrt{f} - \sqrt{F})$. Then all profitable mergers decrease consumer welfare.

b. Suppose $(c - C) < (-8 + 4\sqrt{2})\sqrt{\beta}(\sqrt{f} - \sqrt{F})$. Then there exist some profitable mergers with moderate fixed cost synergies which increase consumer welfare.

Proof. See Appendix C. ∎

In Proposition 3(i), MP firms are more efficient than SP firms in terms of both their fixed cost and their marginal cost. As shown in Appendix C, we have $\delta_1 \leq \delta_4$. Hence, mergers are profitable under high synergies only. We know from Lemma 9 that high synergies are not consumer welfare increasing because they result in a strong price effect which is always negative. Proposition 3(i) implies that a necessary condition for mergers with fixed cost synergies to increase consumer welfare is that SP firms are more efficient in terms of either their fixed cost or their marginal cost. Recall that if SP firms are more efficient in terms of both types of costs, we get a contradiction with Assumption 1 and a mixed market structure would not exist.

In Proposition 3(ii), MP firms are more efficient than SP firms in terms of their fixed cost but not in terms of their marginal cost. Since $\delta_4 < \delta_1 < \delta_5$, some mergers with moderate level of synergies can be profitable and consumer welfare increasing at the same time. In Proposition 3(iii), MP firms are less efficient in terms of their fixed cost but more efficient in terms of their marginal cost than SP firms. Subcase (a) in Proposition 3(iii) shows that even if the MP firms do not have a fixed cost advantage, if their marginal cost advantage is sufficiently large, then all mergers harm consumer welfare. Otherwise, there exist profitable mergers with moderate synergies which increase consumer welfare.

Figure 1 provides a graphical representation of Proposition 3. On the horizontal axis is $\sqrt{f}$ and on the vertical axis is the marginal cost difference between SP and MP firms, $c - C$. We divide the parameter space which satisfy Assumptions 1 and 3 (represented by the orange and blue lines) into four subregions. The green line represents the cut-off condition for case (iii) in Proposition 3. In subregions (i) and (iiia), all profitable mergers decrease consumer welfare. In subregions (ii) and (iiib), mergers may increase consumer welfare only when the synergies are moderate.

[Figure 1 around here]

In summary, Proposition 3 and Figure 1 imply that antitrust agencies should be cautious of mergers between MP firms which generate fixed cost synergies if the MP firms are more efficient than the SP firms. Moreover, they should be cautious of the efficiency defense when merger related efficiencies are substantial. This is because the benefit from fixed cost efficiencies is not necessarily passed on to consumers. Mergers with fixed cost synergies (unlike mergers with marginal cost synergies) result in higher prices. That is, with a low per-variety fixed cost, the merging parties find it optimal to supply a wide range of varieties and charge a higher price per variety. This negative price effect of a merger is increasing in the level of synergies and may dominate whenever the merged entity enjoys sufficiently high fixed cost synergies. At the same time, the potential expansion of production by the merging parties may lead to the exit of some SP firms who supply differentiated goods at lower prices. Therefore, mergers with high fixed cost synergies may generate anticompetitive effects, raising the average market price and decreasing consumer welfare.

We next investigate the link between a merger's impact on aggregate variety and its impact on consumer welfare. We know from Proposition 2 that with marginal cost synergies, consumer welfare decreases whenever aggregate variety increases. The next proposition states that this is not necessarily the case with fixed cost synergies.

Proposition 4 *Consider a merger with fixed cost synergies and suppose that Assumptions 1 and 3 hold.*

(i) Suppose $f < F$. If there is an increase in aggregate variety, then consumer welfare always decreases. If there is a decrease in aggregate variety, then consumer welfare may increase or decrease.

(ii) Suppose $f > F$. If there is a decrease in aggregate variety, then consumer welfare always increases. If there is an increase in aggregate variety, then consumer welfare may increase or decrease.

Proof. See Appendix D. ■

Proposition 4 implies that if the SP firms have a fixed cost advantage over the MP firms before the merger, then an increase in aggregate variety always implies that the merger is detrimental to consumer welfare. In this case, any consumer welfare increasing merger is accompanied with a decrease in aggregate variety. This result is consistent with the result for mergers with marginal cost synergies. However, if the MP firms have a fixed cost advantage over the SP firms before the merger, then an increase in aggregate variety can be an indication that the merger is beneficial for consumers. In this case, it is possible that a merger with fixed cost synergies increases both the aggregate variety and consumer welfare. This result differs from the result for mergers with marginal cost synergies, in which case consumer welfare decreases whenever aggregate variety increases.

6 Conclusion

We consider bilateral mergers between multi-product firms in a market structure with a monopolistically competitive fringe of single-product firms. Aggregate product variety in the market is determined by the endogenous product variety choices of the multi-product firms and entry/exit decisions of single-product firms. We consider and compare the impact of mergers when there are merger-generated marginal cost synergies or fixed cost synergies. Fixed cost synergies in our model correspond to savings on the cost of producing one more variety .

The consumer welfare impact of mergers with marginal cost synergies originates from a reallocation effect. Due to this reallocation effect, when the multi-product firms have a fixed cost advantage over the single-product firms, mergers with sufficiently high marginal cost synergies harm consumers, but mergers with a moderate level of cost synergies increase consumer welfare. However, if single-product firms have a fixed cost advantage, then mergers with sufficiently high marginal cost synergies increase consumer welfare while mergers with moderate synergies decrease consumer welfare.

The consumer welfare impact of mergers with fixed cost synergies originates from a reallocation effect as well as a price effect. Because the price effect is negative and increasing in the level of synergies, mergers with sufficiently high synergies always hurt consumer welfare. Similarly, mergers which take place in industries where the multi-product firms are more efficient than the single-product firms also always hurt consumer welfare.

These results indicate that in general, we should be cautious of the efficiency defense when merger-generated efficiencies are substantial. Moreover, it is of policy interest whether a merger's impact on aggregate variety can be used as an indication of its impact on consumer surplus. For example, are those mergers which result in an increase in aggregate variety more likely to increase consumer welfare? Our findings reveal that in mergers with marginal cost synergies, consumer welfare decreases whenever aggregate variety increases. Hence, any consumer welfare gain does not originate from an increase in aggregate variety. In contrast, an increase in aggregate variety can be an indication that the merger is beneficial for consumers in the case of mergers with fixed cost synergies if the multi-product firms have a fixed cost advantage over the single-product firms before the merger.

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A Derivation of Assumption 1

In this appendix, we show the derivation of the expressions in Assumption 1.

Assumption 1(i) ensures that the mass of SP firms is positive in equilibrium. S^*, S^{MC*} and S^{FC*} denote the equilibrium mass of SP firms when no merger takes place, when there is a merger with marginal cost synergies, and when there is a merger with fixed cost synergies, respectively. Hence, we need to find the condition such that $min\left\{S^*, S^{MC*}, S^{FC*}\right\} > 0$.

From (25), the condition for $S^* > 0$ is $\alpha > c(N+1)-CN+2\sqrt{\beta}\left[(N+1)\sqrt{f}-N\sqrt{F}\right]$. From (37), we see that S^{MC*} is increasing in λ. Therefore, a sufficient condition for $S^{MC*}(\lambda) > 0$ is $S^{MC*}(0) > 0$, which requires $\alpha > cN - C\left(N-2\right) - 2\sqrt{\beta}\left[N\sqrt{f}-(N-1)\sqrt{F}\right]$. Finally, when there is merger with fixed cost synergies, the equilibrium mass of small firms is given by

$$S^{FC*} = \sqrt{\frac{\beta}{f}}\frac{\alpha - [cN - C(N-1)] - 2\sqrt{\beta}\left[\sqrt{f}(N-1) - \sqrt{F}(N-3+\delta)\right]}{\gamma},$$

which is increasing in δ. Therefore, a sufficient condition for $S^{FC*} > 0$ is $\alpha > cN - C(N-1) + 2\sqrt{\beta}\left[\sqrt{f}(N-1) - \sqrt{F}(N-3)\right]$. Combining all three conditions gives us the expression in Assumption 1(i).

Assumption 1(ii) ensures that $N > 0$, or equivalently, $\omega > 0$ in equilibrium. The equilibrium ω values (with and without a merger) are given by (26), (34) and

(45). Comparing these three expressions reveals that (34) is decreasing in λ and (45) is decreasing in δ, so it is sufficient to show that (26) > 0, which corresponds to the condition assumed in Assumption 1(ii).

B Second-order conditions for a MP firm

The first-order derivatives of MP firm n's profit with respect to its per-variety quantity q_{mp}^n and product range ω_n are given by

$$\begin{aligned}
\frac{\partial \Pi_{mp}^n}{\partial q_{mp}^n} &= (\alpha - C - 2\beta q_{mp}^n - 2\gamma\omega_n q_{mp}^n - \gamma Q^{-n})\omega_n, \\
\frac{\partial \Pi_{mp}^n}{\partial \omega_n} &= (\alpha - C - (\beta + 2\gamma\omega_n) q_{mp}^n - \gamma Q^{-n}) q_{mp}^n - F.
\end{aligned}$$

Therefore, the Hessian matrix is given by

$$H = \begin{pmatrix} -2(\beta + \gamma\omega_n) & \alpha - C - 2(\beta + 2\gamma\omega_n) q_{mp}^n - \gamma Q^{-n} \\ \alpha - C - 2(\beta + 2\gamma\omega_n) q_{mp}^n - \gamma Q^{-n} & -2\gamma (q_{mp}^n)^2 \end{pmatrix},$$

with determinant

$$|H| = 4\beta\gamma(\beta + \gamma\omega_n)(q_{mp}^n)^2 - \left[\alpha - C - 2(\beta + 2\gamma\omega_n) q_{mp}^n - \gamma Q^{-n}\right]^2.$$

We can readily show that $-2(\beta + \gamma\omega_n) < 0$ and $-2\gamma(q_{mp}^n)^2 < 0$.

Substituting the first-order conditions in (17) and (22), we have

$$\alpha - C - 2\gamma\omega_n^* q_{mp}^{n*} - \gamma Q^{-n} = 2\beta q_{mp}^{n*}.$$

Therefore,

$$|H^*| = 4\beta\gamma\omega_n^* (q_{mp}^{n*})^2 > 0.$$

Therefore, q_{mp}^{n*} and ω_n^* are locally optimal.[19]

[19] The second order conditions for the MP large outsiders are also satisfied, following the same proof. As for the MP large insider, we can modify the proof by replacing C with λC.

C Proof of Proposition 3

Let $\phi = \sqrt{\beta}(\sqrt{f} - \sqrt{F})$ and $\chi = c - C$. ϕ and χ represent the relative efficiency of a SP and a MP firm. Recall that $2\phi + \chi > 0$ by Assumption 1(ii).

First, suppose $32\phi^2 + 16\phi\chi + \chi^2 < 0$. Joint with Assumption 1(ii), this condition is equivalent to

$$\phi < 0 \text{ and } \frac{\chi}{\phi} \in \left(-8 - 4\sqrt{2}, -8 + 4\sqrt{2}\right). \tag{54}$$

In this case, the roots δ_4 and δ_5, given in (52) and (53), are imaginary, and any merger decreases consumer welfare.

Now suppose $32\phi^2 + 16\phi\chi + \chi^2 > 0$. Joint with Assumption 1(ii), this condition is equivalent to

$$\begin{array}{c} \text{(a) } \phi \geq 0 \text{ and } \chi > -2\phi, \text{ and} \\ \text{(b) } \phi < 0 \text{ and } \frac{\chi}{\phi} \in \left(-\infty, -8 - 4\sqrt{2}\right] \cup \left[-8 + 4\sqrt{2}, -2\right). \end{array} \tag{55}$$

To examine the impact of a profitable merger with fixed cost synergies on consumer welfare, we compare δ_1 with δ_4 and δ_5.

Comparing δ_1 and δ_4, we have

$$\delta_1 - \delta_4 = \frac{\sqrt{32\phi^2 + 16\phi\chi + \chi^2} - \left[\left(2\sqrt{2} - 1\right)\chi + 4\sqrt{2}\phi\right]}{4\sqrt{\beta F}},$$

which is negative if and only if $(2\sqrt{2} - 1)\chi + 4\sqrt{2}\phi > 0$ and $\chi > 0$.[20] These conditions hold if (a) $\phi \geq 0$ and $\chi > 0$, or (b) $\phi < 0$ and $\chi/\phi < -\left(16 + 4\sqrt{2}\right)/7$. Combined with (55-b), we find that $\delta_1 < \delta_4$ if and only if

$$\text{(a) } \phi \geq 0 \text{ and } \chi > 0, \text{ or (b) } \phi < 0 \text{ and } \chi/\phi \leq -8 - 4\sqrt{2}. \tag{56}$$

It is straightforward to check that $\delta_4 > 1$ under condition (56-b).

[20]Specifically, $\delta_1 - \delta_4 < 0$ if and only if $(2\sqrt{2} - 1)\chi + 4\sqrt{2}\phi > \sqrt{32\phi^2 + 16\phi\chi + \chi^2} > 0$. Given that $2\phi + \chi > 0$ by Assumption 1(ii), $(32\phi^2 + 16\phi\chi + \chi^2) - [(2\sqrt{2} - 1)\chi + 4\sqrt{2}\phi]^2 = -4(2 - \sqrt{2})\chi(2\phi + \chi)$ is negative if and only if $\chi > 0$.

Comparing δ_1 and δ_5, we have

$$\delta_5 - \delta_1 = \frac{\sqrt{32\phi^2 + 16\phi\chi + \chi^2} + \left[(2\sqrt{2} - 1)\chi + 4\sqrt{2}\phi\right]}{4\sqrt{\beta F}},$$

which is negative if and only if $\phi < 0$ and $\chi/\phi \in \bigl(-(16 + 4\sqrt{2})/7, -2\bigr)$. Note that by condition (55-b), δ_5 does not exist if $\phi < 0$ and $\chi/\phi \in \bigl(-(16 + 4\sqrt{2})/7, -8 + 4\sqrt{2}\bigr)$. Therefore, $\delta_5 < \delta_1$ if and only if

$$\phi < 0 \text{ and } -8 + 4\sqrt{2} \leq \chi/\phi < -2. \tag{57}$$

In summary, we obtain the following three possible rankings of δ_1, δ_4 and δ_5:

(i) If $\phi \geq 0$ and $\chi > 0$, or $\phi < 0$ and $\chi/\phi \leq -8 - 4\sqrt{2}$, then $\delta_1 < \delta_4 < \delta_5$.

(ii) If $\phi > 0$ and $\chi/\phi \in (-2, 0]$, then $\delta_4 < \delta_1 < \delta_5$.

(iii) If $\phi < 0$ and $\chi/\phi \in \bigl(-8 + 4\sqrt{2}, -2\bigr)$, then $\delta_4 < \delta_5 < \delta_1 < 1$.

Combined with Lemma 9, we are able to establish the impact of mergers with different fixed cost synergies on consumer welfare in different parts of the parameter space:

(i) If $\phi \geq 0$ and $\chi > 0$, then $\delta_1 < \delta_4 < \delta_5$. In this case, all profitable mergers decrease consumer welfare. This gives Proposition 3(i).

(ii) If $\phi > 0$ and $\chi/\phi \in (-2, 0]$, then $\delta_4 < \delta_1 < \delta_5$. In this case, profitable mergers with synergies satisfying $\delta \in (\delta_4, \delta_1)$ increase consumer welfare, and profitable mergers with synergies satisfying $\delta \in (0, \delta_4)$ decrease consumer welfare. This gives Proposition 3(ii).

(iii) If $\phi < 0$ and $\chi/\phi \leq -8 - 4\sqrt{2}$, then $\delta_1 < 1 < \delta_4 < \delta_5$. If $\phi < 0$ and $\chi/\phi \in \bigl(-8 - 4\sqrt{2}, -8 + 4\sqrt{2}\bigr)$, then the roots of δ_4 and δ_5 are imaginary. In both cases, any profitable merger decreases consumer welfare. This gives Proposition 3(iii)a.

(iv) If $\phi < 0$ and $\chi/\phi \in \bigl(-8 + 4\sqrt{2}, -2\bigr)$, then $\delta_4 < \delta_5 < \delta_1$. In this case, profitable mergers with synergies satisfying $\delta \in (\delta_4, \delta_5)$ increase consumer welfare, and profitable mergers with synergies satisfying $\delta \in (0, \delta_4) \cup (\delta_5, \delta_1)$ decrease consumer welfare. This gives Proposition 3(iii)b.

D Proof of Proposition 4

Let $\phi = \sqrt{\beta}(\sqrt{f} - \sqrt{F})$ and $\chi = c - C$. ϕ and χ represent the relative efficiency of a SP and a MP firm.

We first note that, using $Q^* = Q^{FC*}$, we have

$$\Delta S^{FC} q^*_{sp} + \omega_I^{FC*} q_I^{FC*} - 2\omega^* q^*_{sp} = 0,$$
$$\Leftrightarrow \omega_I^{FC*} = \frac{2\omega^* q^*_{mp} - \Delta S^{FC} q^*_{sp}}{q_I^{FC*}}.$$

Therefore, we can express the change in aggregate variety as

$$\begin{aligned} \Delta V &= \Delta S^{FC} + \omega_I^{FC*} - 2\omega^* \\ &= \Delta S^{FC} + \frac{2\omega^* q^*_{mp} - \Delta S^{FC} q^*_{sp}}{q_I^{FC*}} - 2\omega^* \\ &= \frac{1}{q_I^{FC*}}[-\Delta S^{FC}(q^*_{sp} - q_I^{FC*}) + 2\omega^*(q^*_{mp} - q_I^{FC*})], \end{aligned} \tag{58}$$

and the change in consumer welfare as

$$\begin{aligned} \Delta U &= \frac{\beta}{2}[\Delta S^{FC}(q^*_{sp})^2 + \omega_I^{FC*}(q_I^{FC*})^2 - 2\omega^*(q^*_{mp})^2] \\ &= \frac{\beta}{2}[\Delta S^{FC}(q^*_{sp})^2 - 2\omega^*(q^*_{mp})^2 + q_I^{FC*}(2\omega^* q^*_{mp} - \Delta S^{FC} q^*_{sp})] \\ &= \frac{\beta q^*_{sp}}{2}[\Delta S^{FC}(q^*_{sp} - q_I^{FC*}) - 2\omega^* \frac{q^*_{mp}}{q^*_{sp}}(q^*_{mp} - q_I^{FC*})]. \end{aligned} \tag{59}$$

We next note the following two properties that will be useful later. Property 1 follows from (43) and Property 2 follows from Lemma 1.

Property 1: A merger with fixed cost synergies results in a reduction in the per-variety quantity of the insider, i.e., $q^*_{mp} - q_I^{FC*} = (1-\delta)\sqrt{F/\beta} > 0$.

Property 2: If $f < F$, then $q^*_{mp}/q^*_{sp} = \sqrt{F/f} > 1$. If $f > F$, then $q^*_{mp}/q^*_{sp} < 1$.

Proof of Proposition 4(i):

The first part of Proposition 4(i) states that when $f < F$, if there is an increase in aggregate variety, then consumer welfare always decreases. To see this, note that by (58), if $\Delta V > 0$, then $\Delta S^{FC}(q^*_{sp} - q_I^{FC*}) - 2\omega^*(q^*_{mp} - q_I^{FC*}) < 0$. Property 1 and 2 imply

$$-2\omega^*(q^*_{mp}/q^*_{sp})(q^*_{mp} - q_I^{FC*}) < -2\omega^*(q^*_{mp} - q_I^{FC*}).$$

Then, by (59) we have

$$\Delta U < (\beta q^*_{sp}/2)[\Delta S^{FC}(q^*_{sp} - q_I^{FC*}) - 2\omega^*(q^*_{mp} - q_I^{FC*})] < 0.$$

Therefore, when $f < F$, if $\Delta V > 0$, then $\Delta U < 0$.

The second part of Proposition 4(i) states that when $f < F$, if there is a decrease in aggregate variety, then consumer welfare may increase or decrease. We proceed in two steps.

Step 1**:** We first show that when $f < F$ and $(-8 + 4\sqrt{2})\phi < \chi$, a decrease in aggregate variety ($\Delta V < 0$) is always accompanied with a decrease in consumer welfare ($\Delta U < 0$). This follows from Proposition 3(iii)a, which states that all profitable mergers decrease consumer welfare if $f < F$ and $(-8 + 4\sqrt{2})\phi < \chi$.

Step 2**:** Next we prove that when $f < F$ and $-2\phi < \chi < (-8 + 4\sqrt{2})\phi$, if $\Delta V < 0$, then ΔU may be positive or negative. Note that these are the conditions given in Proposition 3(iii)b.

First, we can readily show that ΔV is U-shaped in δ.[21] Let δ_6 and δ_7, with $\delta_6 < \delta_7$, stand for the two roots of ΔV. Hence, $\Delta V < 0$ if and only if $\delta \in (\delta_6, \delta_7)$. In addition, we have shown in (51) that ΔU is an inverted U-shaped function of δ, with roots δ_4 and δ_5. $\Delta U > 0$ if and only if $\delta \in (\delta_4, \delta_5)$. Therefore, in order to establish the link between ΔV and ΔU, we need to rank δ_1, δ_4, δ_5, δ_6, and δ_7. We establish the ranking in the following two sub-steps.

Step 2.1**:** In this step, we show that $\delta_6 < \delta_4 < \delta_5 < \delta_7$.

At δ_4 and δ_5, $\Delta U = 0$, and by (59), $\Delta S^{FC}\left(q^*_{sp} - q_I^{FC*}\right) = 2\omega^*(q^*_{mp}/q^*_{sp})(q^*_{mp} - q_I^{FC*})$. Substituting this expression into (58), we get $\Delta V = (1/q_I^{FC*})2\omega^*(q^*_{mp} - q_I^{FC*})(1 - q^*_{mp}/q^*_{sp})$. By Property 1, $q^*_{mp} - q_I^{FC*} > 0$. By Property 2, if $f < F$, then $1 < q^*_{mp}/q^*_{sp}$. Therefore, $\Delta V < 0$.

Since ΔV is U-shaped in δ, and ΔU is inverted U-shaped in δ, $\Delta V(\delta_4) < 0$, $\Delta V(\delta_5) < 0$, and $\Delta U(\delta_4) = \Delta U(\delta_5) = 0$ imply that $\delta_6 < \delta_4 < \delta_5 < \delta_7$.

[21] Specifically, $\Delta V = \left(\frac{\delta[-\chi\phi - \chi\sqrt{\beta f} - 4\beta(f - \sqrt{fF} + F)] + \chi\sqrt{\beta f} + 2\beta F\delta^2 + 2\beta f}{\gamma\delta\sqrt{fF}}\right)$.

Step 2.2: We next show that $\delta_7 < \delta_1$. Note that for $\delta > \delta_7$, $\Delta V > 0$. Therefore, to show that $\delta_7 < \delta_1$, we show that $\Delta V(\delta_1) > 0$.

When $-2\phi < \chi < (-8 + 4\sqrt{2})\phi$, the denominator of $\Delta V(\delta_1)$ is always negative and the numerator of $\Delta V(\delta_1)$ is quadratic in χ.[22] Denote the numerator of $\Delta V(\delta_1)$ as NUV_{δ_1}. If $f < F$ and $\chi = -2\phi$, then $NUV_{\delta_1} = 0$ and $\partial(NUV_{\delta_1})/\partial\chi = 2\left(2 - \sqrt{2}\right)\beta\sqrt{F} < 0$. If $f < F$ and $\chi = \left(-8 + 4\sqrt{2}\right)\phi$, then $NUV_{\delta_1} \approx -8\phi^2\beta^{3/2}(0.01\sqrt{f} + 0.05\sqrt{F}) < 0$. Therefore, when $f < F$ and $-2\phi < \chi < \left(-8 + 4\sqrt{2}\right)\phi$, the numerator of $\Delta V(\delta_1)$ is always negative. Since the denominator of $\Delta V(\delta_1)$ is also negative, $\Delta(\delta_1) > 0$. Thus, we prove that $\delta_1 > \delta_7$.

By Steps 2.1 and 2.2, we conclude that when $f < F$ and $-2\phi < \chi < (-8 + 4\sqrt{2})\phi$, $\delta_6 < \delta_4 < \delta_5 < \delta_7 < \delta_1$. Hence, if aggregate variety decreases, which implies that fixed cost synergies satisfy $\delta \in (\delta_6, \delta_7)$, then consumer welfare may increase, which happens when $\delta \in (\delta_4, \delta_5)$, or decrease, which happens when $\delta \in (0, \delta_6) \cup (\delta_5, \delta_7)$.

By Steps 1 and 2, we conclude that when $f < F$ and $(-8 + 4\sqrt{2})\phi < \chi$, if aggregate variety decreases, then consumer welfare decreases. When $f < F$ and $-2\phi < \chi < (-8 + 4\sqrt{2})\phi$, if aggregate variety decreases, then consumer welfare may increase or decrease.

This completes the proof of Proposition 4(i).

Proof of Proposition 4(ii):

The proof technique is analogous to that of Proposition 4(i). The first part of Proposition 4(ii) states that when $f > F$, if there is a decrease in aggregate variety, then consumer welfare always increases. To see this, note that if $\Delta V < 0$, then $\Delta S^{FC}(q^*_{sp} - q_I^{FC*}) - 2\omega^*(q^*_{mp} - q_I^{FC*}) > 0$ by (58). Property 1 and 2 imply that

$$-2\omega^*(q^*_{mp}/q^*_{sp})(q^*_{mp} - q_I^{FC*}) > -2\omega^*(q^*_{mp} - q_I^{FC*}).$$

From (59) we have

$$\Delta U > (\beta q^*_{sp}/2)[\Delta S^{FC}(q^*_{sp} - q_I^{FC*}) - 2\omega^*(q^*_{mp} - q_I^{FC*})] > 0.$$

[22] Specifically, $\Delta V(\delta_1) = \dfrac{\left\{ \begin{array}{c} -\chi^2\sqrt{\beta}\left[2(\sqrt{2}-1)\sqrt{f} + (4-3\sqrt{2})\sqrt{F}\right] \\ -2\chi\phi\beta\left[4\left(\sqrt{2}-1\right)\sqrt{f} + \left(6-5\sqrt{2}\right)\sqrt{F}\right] - 8\beta\sqrt{\beta}\left(\sqrt{2}-1\right)\phi^3 \end{array} \right\}}{\left[\sqrt{fF}(\sqrt{2}-1)(\chi+2\sqrt{\beta f}) - 2\sqrt{2}\sqrt{\beta F}\right]\gamma}$

Therefore, when $f > F$, if $\Delta V < 0$, then $\Delta U > 0$.

The second part of Proposition 4(ii) states that when $f > F$, if there is an increase in aggregate variety, then consumer welfare may increase or decrease. As stated above, ΔV is U-shaped in δ with roots δ_6 and δ_7. Therefore, in order to establish the relationship between ΔV and ΔU, we need to rank δ_1, δ_4, δ_5, δ_6, and δ_7. We proceed in two steps.

Step 1**:** In this step, we show that when $f > F$, $\delta_4 < \delta_6 < \delta_7 < \delta_5$.

When $\delta = \delta_4$ or $\delta = \delta_5$, $\Delta U = 0$ and by (59), $\Delta S^{FC}\left(q^*_{sp} - q_I^{FC*}\right) = 2\omega^*(q^*_{mp}/q^*_{sp})(q^*_{mp} - q_I^{FC*})$. Substituting this expression in (58), we get $\Delta V = (1/q_I^{FC*})2\omega^*(q^*_{mp} - q_I^{FC*})(1 - q^*_{mp}/q^*_{sp})$. By Property 1, $q^*_{mp} - q_I^{FC*} > 0$. By Property 2, if $f > F$, then $q^*_{mp}/q^*_{sp} < 1$. Therefore, $\Delta V > 0$.

Since ΔV is U-shaped in δ, and ΔU is inverted U-shaped in δ, $\Delta V(\delta_4) > 0$, $\Delta V(\delta_5) > 0$, and $\Delta U(\delta_4) = \Delta U(\delta_5) = 0$ implying that $\delta_4 < \delta_6 < \delta_7 < \delta_5$.

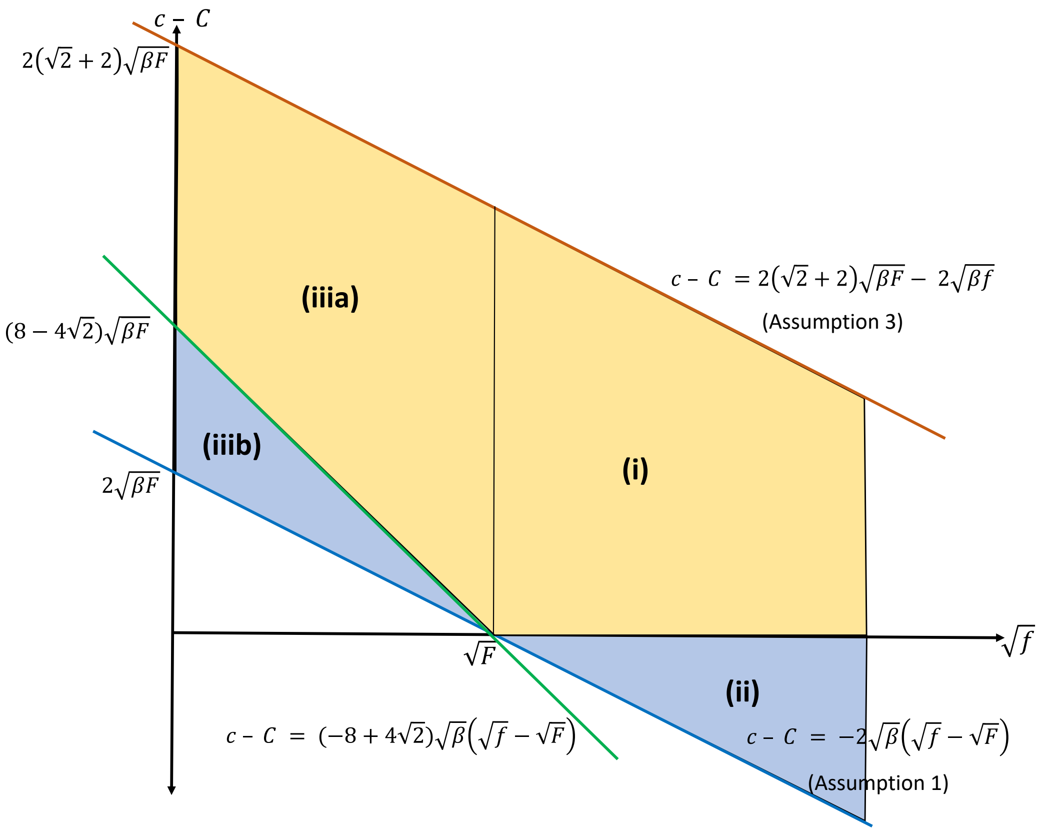
Step 2**:** In this step, we show that $\delta_6 < 1 < \delta_7$.

When $\delta = 1$, (43) implies that $q_I^{FC*} = q^*_{mp}$, and Lemma 7 implies that $\Delta S^{FC} > 0$. In addition, Property 2 implies that when $f > F$, $q^*_{sp} - q^*_{mp} > 0$. Thus, by (58), we have

$$\begin{aligned}\Delta V(1) &= (1/q_I^{FC*})[-\Delta S^{FC}(q^*_{sp} - q_I^{FC*})] \\ &= -\Delta S^{FC}(1/q_I^{FC*})(q^*_{sp} - q^*_{mp}) \\ &< 0.\end{aligned}$$

Since $\Delta V < 0$ if and only if $\delta_6 < \delta < \delta_7$, $\Delta V(1) < 0$ implies that $\delta_6 < 1 < \delta_7$.

By Steps 1 and 2, we conclude that when $f > F$, $\delta_4 < \delta_6 < 1 < \delta_7 < \delta_5$. Furthermore, from the proof of Proposition 3, we know that when $f > F$, $\delta_4 < \delta_1$. As a result, if aggregate variety increases, which implies that fixed cost synergies satisfy $\delta < \delta_6$, then consumer welfare may increase, which happens when $\delta \in (\delta_4, \min\{\delta_1, \delta_6\})$, or decrease, which happens when $\delta \in (0, \delta_4)$. This completes the proof for Proposition 4(ii).



All profitable mergers decrease consumer welfare.

Mergers with moderate fixed cost synergies increase consumer welfare.

Figure 1 The Impact of Fixed Cost Synergies on Consumer Welfare (Proposition 3)