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ACCOUNTING FOR THE SLOWDOWN IN OUTPUT GROWTH AFTER THE GREAT RECESSION: A WEALTH PREFERENCE APPROACH

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Accounting for the slowdown in output growth after the Great Recession: A wealth preference approach*

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Abstract

Previous studies have argued that output growth in advanced economies declined during the Great Recession and remained low afterward. This paper proposes a model to explain this slowdown in output growth. We incorporate wealth preferences and downward nominal wage rigidity into a standard monetary growth model. Our model demonstrates that output initially grows at the same rate as productivity and slows endogenously in the transition path to the stagnation steady state. This stagnation is persistent even if productivity continues to grow at a steady rate. Applying our model to US data, we show that it successfully explains the declines observed in the real interest rate, inflation, and the velocity of money, along with the slowdown in output growth.

JEL Classification: E41, E47, O10

Keywords: Secular stagnation, Wealth preferences, Liquidity preferences, the Great Recession, Downward nominal wage rigidity

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1 Introduction

The persistent decline in output growth in advanced economies after the Great Recession has formed one of the most well-known debates in macroeconomics. In his "secular stagnation hypothesis," Summers (2014) argues that while long-run US output was expected to grow steadily prior to the Great Recession, the actual output after the Great Recession failed to catch up with the expected output trend.[1] Figure 1 plots two estimated linear trends for log real GDP per capita using the data from 1990:Q1 to 2019:Q4. The dashed line is the linear output trend under the assumption that there was a structural break starting in 2007:Q1 in the growth rate of real GDP per capita. The dot-dashed line is the counterfactual linear output trend that removes the structural break.[2] The two estimated output trends grow at the same rate of 1.93 percent up until 2007:Q1. However, after 2007:Q1, the growth rate of the output trend shown in the dashed line declines to 0.77 percent. That is, there was a slowdown in output growth.[3]

[Figure 1 about here.]

This paper accounts for this slowdown in output growth using wealth preferences. What motivates our approach is the data on the velocity of money. Figure 2 shows that while the velocity of M2 increased in the 1990s, it quickly decreased after the 2000s. In the standard monetary growth model (e.g., Sidrauski (1967)), velocity is often constant over time and thus difficult to reconcile with the data. If we introduce wealth preferences into the standard monetary growth model, the strong demand for money as part of wealth may slow the velocity of money. Furthermore, the literature on wealth preferences has found that strong wealth preferences could lead to an inherently stagnant economy in the steady state.[4] As such, we consider that the dynamics of velocity are not irrelevant to the observed slowdown in output growth.

[Figure 2 about here.]

[1] Similar observations have been made for the Eurozone countries after the Great Recession and Japan after the 1997 financial crisis.

[2] We regress the log of real GDP per capita ($\ln y_t$) on a constant, time (t), and $d_t \times (t - t^*)$, where d_t is a dummy variable that takes a value of one for t after 2007:Q1 and t^* is the 2007:Q1 period. The estimation equation is $\ln y_t = a_0 + a_1 t + b_1 d_t(t - t^*) + \varepsilon_t$, where ε_t is the error term. In plotting the dot-dashed line, we remove the structural break by setting b_1 at zero. The regressor of $d_t \times (t - t^*)$ ensures that the two output trends are connected at $t =$2007:Q1 with the adjustment of a constant term. The coefficient on this regressor b_1 is estimated to be -0.0029 with a standard error of 0.0003 and statistically different from zero.

[3] We confirmed a very similar pattern using quadratic, cubic, and quartic detrending. We also confirmed a similar pattern when using the potential GDP series estimated by the Congressional Budget Office.

[4] The earliest examples of these studies are Ono (1994) and Ono (2001), among others.

We incorporate two components into the standard monetary growth model. As discussed, the first component is wealth preferences. Following the literature (e.g., Michau (2018), Michaillat and Saez (2021), Hashimoto, Ono, and Schlegl (2021)), we introduce wealth preferences with a strictly positive marginal utility in equilibrium. This preference assumption leads to a strong desire for saving compared with the case without wealth preferences. The second component is downward nominal wage rigidity (DNWR), which is widely discussed in recent studies.[5] Together with wealth prefrences, this DNWR plays an important role for generating secular stagnation in the monetary growth model.

We demonstrate that our model endogenously generates a slowdown in output growth in the transition path to the stagnation steady state. In particular, we theoretically show that output initially follows productivity that represents potential output in our model, but output later falls below productivity. In our model, the household with wealth preferences accumulates wealth rather than consuming enough to reach the potential output. The aggregate demand weakened by wealth preferences leads to disinflation while the DNWR is not binding. However, once the DNWR binds, inflation no longer decreases, the aggregate demand determines output, and output growth is endogenously determined. Then, the weakened aggregate demand falls short of the potential output, the aggregate demand shortage occurs, and the growth rate of output is lower than that of productivity.

We conduct numerical simulations assuming that the slowdown in output growth takes place in 2007:Q1. These numerical simulations show that the simulated output tracks the estimated output trend with a structural break. More specifically, our model explains 88.8 percent of the decline in output in 2015 and almost 100 percent of the decline in output in 2017. While our model is extremely simplified to derive our main results theoretically, it predicts the slowdown in US output growth after the Great Recession remarkably well.

Our model can also predict the permanent decline in the real interest rate, which is closely linked to the secular stagnation hypothesis.[6] In the standard consumption Euler equation, the real interest rate is determined by the household's subjective discount rate and the growth rate of

[5] See Barattieri, Basu, and Gottschalk (2014), Sigurdsson and Sigurdardottir (2016), Schmitt-Grohé and Uribe (2016), Fallick, Lettau, and Wascher (2016), Hazell and Taska (2020), and Grigsby, Hurst, and Yildirmaz (2021), among others.

[6] Characterizations of secular stagnation appear in Baldwin and Teulings (2014) and Krugman (2014).

consumption. However, as pointed out by Michaillat and Saez (2021), wealth preferences in the consumption Euler equation create discounting in the real interest rate. This discounting leads to a persistently low real interest rate, consistent with the data under secular stagnation.

In our model, inflation also declines until the output trend deviates from the productivity trend. This observation is roughly consistent with inflation in the US before and after the Great Recession. Hall (2011) points out that US inflation declined during the 1990s, but became stable even in the presence of long-lasting slack in the economy from the Great Recession. Our model interprets the missing deflation as being a consequence of the binding DNWR, where inflation stops declining even if aggregate demand falls short of aggregate supply.

Our model can also replicate the decline in the velocity of money in the 2000s shown in Figure 2. In our model, households have strong wealth preference so that they accumulate money rather than increasing consumption. These strong preferences for money generate nominal money holdings that grow faster than does nominal aggregate demand. Consequently, velocity declines over time. This prediction sharply contrasts with the standard monetary growth model, which predicts the constant velocity of money along the balanced growth path.

Previous studies have fallen into one of four groups in explaining secular stagnation. The first focuses on the productivity slowdown (e.g., Fernald (2015), Gordon (2015), Takahashi and Takayama (2022)). While this group considers the decline in productivity as the main driver of secular stagnation, our paper assumes a constant productivity growth rate to highlight the degree to which wealth preferences alone can explain the observed slowdown in output growth. The second group emphasizes the impact of demographic changes on saving in explaining the declining real interest rate. (e.g., Carvalho, Ferrero, and Nechio (2016), Gagnon, Johannsen, and Lopez-Salido (2021), Jones (2022)). We exclude this potentially important factor from our model because we focus on the mechanism behind the impact of wealth preferences on saving. The third group relies on debt deleveraging (e.g., Hall (2011), Eggertsson and Krugman (2012), Mian and Sufi (2014), Guerrieri and Lorenzoni (2017), Eggertsson, Mehrotra, and Robbins (2019)). Among these studies, Eggertsson, Mehrotra, and Robbins (2019) integrate these three factors to replicate secular stagnation and discuss policy evaluation in their model.[7]

[7] Ikeda and Kurozumi (2019) discuss monetary policy rules to prevent secular stagnation in a model with financial frictions and endogenous total factor productivity growth.

Our paper is categorized into the fourth group, which introduces wealth preferences into a standard macroeconomic model. This group assumes a strong desire for liquidity or wealth (e.g., Michau (2018), Illing, Ono, and Schlegl (2018)).[8] The study closest to ours is Michau (2018). He incorporates wealth preferences and the DNWR into the standard neoclassical growth model and shows the existence of both the neoclassical and stagnation steady states in his model. By contrast, our model has a unique steady state, and endogenously generates a regime change from efficient allocation in the neoclassical economy to inefficient allocation in the stagnant economy.[9]

The paper is organized as follows. Section 2 presents our simple growth model. Section 3 studies the model dynamics and presents the main analytical results. In Section 4, we simulate the model and show that its predictions are consistent with the data. Section 5 concludes.

2 The model

2.1 Setup

The representative household solves the following maximization problem:

$$\max \quad \int_0^\infty e^{-\rho t} \left[\ln c_t + v(m_t) + \beta(a_t) \right] dt, \tag{1}$$

$$\text{s.t.} \quad \dot{a}_t = r_t(a_t - m_t) - \pi_t m_t + w_t n_t - c_t + \tau_t, \tag{2}$$

$$n_t \leq 1, \tag{3}$$

where a_0 is given. The notation is standard: c_t is consumption, m_t is real money balances ($m_t = M_t/P_t$, where M_t is nominal money balances and P_t is the price level), and a_t is total real asset holdings. In the budget constraint, r_t is the real interest rate ($a_t - m_t = b_t$ represents the illiquid asset holdings of the household), π_t is inflation or the opportunity cost of holding money, w_t is real wages, n_t is labor supply, and τ_t are lump-sum transfers from government. The parameter $\rho > 0$ is the household's subjective discount rate. The budget constraint (2) suggests that the

[8] In recent studies, models with wealth preferences are analyzed using the New Keynesian framework (Michaillat and Saez (2021)) and search models (Michaillat and Saez (2022)).

[9] In this sense, our analysis also differs from Benigno and Fornaro (2018), who develop an endogenous growth model with downward nominal wage rigidities. They show that weak growth depresses aggregate demand and that the resulting aggregate demand shortage may lead to the stagnation steady state. In contrast to our study, stagnation arises as a self-fulfilling equilibrium.

sources of consumption and saving ($\dot{a}_t$) equal income from asset holdings ($r_t(a_t - m_t) - \pi_t m_t$), labor income ($w_t n_t$), and the lump-sum transfers from government (τ_t). In this maximization problem, we simplify the labor supply and assume that $n_t \leq 1$. We also assume no capital in the economy, so $a_t = m_t$ holds for all t (i.e., $b_t = 0$ in equilibrium).

Our preference assumptions on the utility from wealth are critical. We assume that the household has an insatiable desire for wealth. The utility from wealth satisfies $\beta'(a_t) > 0$, $\beta''(a_t) \leq 0$, and $\beta'(a_t)$ is strictly positive and constant in equilibrium. Except for $\beta(a_t)$, the utility functions are standard and take a constant-relative-risk-aversion form. The utility from consumption is given by $\ln c_t$, and the utility from real money balances satisfies $v'(m_t) > 0$, $v''(m_t) < 0$, and $\lim_{m_t \to \infty} v'(m_t) = 0$.

The simplest specification that satisfies the above conditions for $\beta(a_t)$ is a linear function $\beta(a_t) = \beta \times a_t$, where β is a positive constant. The linearity assumption follows Michau (2018) and extends Ono (1994) and Ono (2001), in which the household has an insatiable desire for liquidity. We assume the linearity of $\beta(a_t)$ for simplicity, not for the necessity of our main results.

A necessary condition for our main results is that the marginal utility from wealth is strictly positive and constant in the stagnation steady state. As argued by Michau (2018) and Michaillat and Saez (2021), there are a variety of alternative specifications for the utility from wealth that generates positive constant marginal utility. In these studies, while the concavity of the utility function is ensured, the marginal utility from the wealth is constant in equilibrium.[10] Because these specifications lead to the same results, we employ linear utility for simplicity.[11]

[10] Michau (2018) and Hashimoto, Ono, and Schlegl (2021) consider the preferences for wealth excluding money, $\beta(a_t - m_t^s)$ where m_t^s is the real money supply and the household takes it as given. They assume that $\beta'(a_t - m_t^s) > 0$, $\beta''(a_t - m_t^s) < 0$, and $\lim_{a_t \to \infty} \beta'(a_t - m_t^s) = 0$, but $\beta'(0) > 0$. Thus, the marginal utility from the wealth is constant in equilibrium, where $b_t = a_t - m_t^s = 0$. Michaillat and Saez (2021) allow for the utility from relative wealth $a_t(i) - \tilde{a}_t$. Here, $a_t(i)$ denotes the wealth at the individual household level and $\tilde{a}_t$ is the average wealth in the economy, and the household takes $\tilde{a}_t$ as given. They assume that $\beta'(a_t(i) - \tilde{a}_t) > 0$, $\beta''(a_t(i) - \tilde{a}_t) < 0$ and $\lim_{a_t(i) \to \infty} \beta'(a_t(i) - \tilde{a}_t) = 0$, but again $\beta'(0) > 0$ where $a_t(i) = \tilde{a}_t$.

[11] The constant marginal utility results in equilibrium money holdings beyond the amount the consumers use for their transactions. Nevertheless, constant marginal utility is not necessarily inconsistent with the neuroscientific evidence. Based on lab experiments, Camerer, Loewenstein, and Prelec (2005) argue that "people value money without carefully computing what they plan to buy with it." (p.35)

The first-order conditions are

$$\begin{aligned} c_t^{-1} &= \lambda_t, \\ v'(m_t) &= (r_t + \pi_t)\lambda_t, \\ \dot{\lambda}_t &= (\rho - r_t)\lambda_t - \beta'(a_t), \end{aligned}$$

where λ is the Lagrange multiplier for the budget constraint. The transversality condition is given by $\lim_{t\to\infty} e^{-\rho t}\lambda_t a_t = 0$.

We rewrite the above equations as

$$v'(m_t)c_t = r_t + \pi_t, \tag{4}$$

$$\frac{\dot{c}_t}{c_t} = r_t - \rho + \beta'(a_t)c_t. \tag{5}$$

In (4), the household pays the opportunity cost of holding money, $r_t + \pi_t$, to receive the marginal benefits $v'(m_t)$ (or $v'(m_t)c_t$ when measured by the unit of consumption goods). In (5), the household pays the marginal cost of saving, $\rho + \dot{c}_t/c_t$. This is the household's consumption discount rate that allows for the household's risk aversion.[12] Regarding the marginal benefits of saving, the household receives market returns on illiquid assets r_t and the marginal utility from wealth $\beta'(a_t)$ (or $\beta'(a_t)c_t$ when measured by the unit of consumption goods). When the wealth preferences are absent, (5) reduces to the standard Euler equation $\dot{c}_t/c_t = r_t - \rho$ and saving only yields market returns r_t. When the wealth preferences are present, however, saving generates additional benefits of $\beta'(a_t)c_t$. Thus, the household would give up more consumption and accept a lower interest to enjoy holding more wealth.

Eliminating r_t from (4) and (5) and allowing for $b_t = 0$ in equilibrium yield the condition for the substitution between consumption and liquidity.

$$\begin{aligned} \Omega(m_t, c_t) &= \rho + \frac{\dot{c}_t}{c_t} + \pi_t, \\ \text{where} \qquad & \Omega(m_t, c_t) \equiv [v'(m_t) + \beta'(m_t)]c_t. \end{aligned} \tag{6}$$

[12] The consumption discount rate is, in general, represented by the sum of the steady-state discount rate and the growth rate of marginal utility. In equation, it is given by $\rho - [du'(c_t)/dt][1/u'(c_t)]$, where $u(c_t)$ is the (instantaneous) utility from consumption. Under our assumption that $u(c_t) = \ln c_t$, we obtain $\rho - [du'(c_t)/dt][1/u'(c_t)] = \rho + \dot{c}_t/c_t$.

Here $\Omega(m_t, c_t)$ denotes the marginal benefits of holding real money balances (measured by the unit of consumption goods). In $\Omega(m_t, c_t)$, $v'(m_t)$ is benefits from increasing money and $\beta'(m_t)$ is an additional benefit of increasing wealth as a whole. The right-hand side of (6) represents the opportunity cost of holding money. To hold an additional unit of the liquid asset, the household has to give up consumption goods by an amount equal to the household's consumption discount rate $(\rho + \dot{c}/c_t)$ and the inflation rate (π_t).

There is a representative firm in the competitive market in the economy. In our model, the firm's technology is linear:

$$y_t = \theta_t n_t, \tag{7}$$

where y_t is output and θ_t is productivity. With this production function, the firm's labor demand condition is

$$w_t = \theta_t. \tag{8}$$

Critical assumptions in our model are that nominal wage inflation $\dot{W}_t/W_t$ cannot be lower than the lower bound γ (i.e., the DNWR), and that θ_t increases at $g > 0$.[13] Real wage inflation is g from (8) and thus nominal wage inflation is $\pi_t + g$. As a result, the DNWR is expressed as $\dot{W}_t/W_t = \pi_t + g \geq \gamma$, or $\pi_t \geq \gamma - g$, where $\gamma - g$ is the lowest level of price inflation.

The government has a budget constraint $\tau_t = \mu m_t^s$, where $m_t^s = M_t^s/P_t$ and M_t^s is the nominal money supply. Throughout the paper, we assume that the money growth rate is strictly positive $(\mu > 0)$ and sufficiently high:

$$\mu > \gamma - g, \tag{9}$$

which means that the money growth rate always exceeds the lowest level of inflation.

We rewrite the assumption for the DNWR as the complementarity slackness condition in the labor market:

$$(\pi_t + g - \gamma)(1 - n_t) = 0. \tag{10}$$

[13] We assume that θ_t grows at an exogenous rate of g, but with a sufficiently large natural limit $\bar{\theta}$. We require this assumption for a technical reason. Later, we show that aggregate demand converges to a constant level. Unless we impose a natural limit on θ_t, labor demand decreases with productivity and will eventually disappear. To circumvent this, we assume that $\bar{\theta}$ is at least as large as $(\rho + \gamma)/\beta$. We further assume that the household does not know when productivity hits $\bar{\theta}$ in the future. In our model, these assumptions ensure that the representative household makes all decisions under the prediction that θ_t eternally grows at g.

The economy has two regimes depending on the complementary slackness condition. If $n_t = 1$, the economy is in the state of full employment and $\pi_t > \gamma - g$. We refer to this regime as the high-inflation regime. Alternatively, if $n_t < 1$, there is unemployment in the labor market with the binding DNWR and $\pi_t = \gamma - g$. We refer to this regime as the low-inflation regime.

The market-clearing conditions are

1. Goods market $c_t = y_t = \theta_t n_t$,

2. Labor market $(\pi_t + g - \gamma)(1 - n_t) \geq 0$,

3. Money market $m_t = m_t^s$,

4. Bond market $b_t = 0$.

A competitive equilibrium of the model is the set of allocations $\{c_t, y_t, n_t, m_t, a_t\}$ and prices $\{w_t, r_t, \pi_t\}$ that satisfy the following: i) The representative household maximizes (1) subject to (2) and (3); ii) The representative firm maximizes profits; iii) The government's transfers and money supply are specified as above; and iv) All markets clear except for labor market. The labor market-clearing condition depends on the complementary slackness condition (10).

2.2 High-inflation regime

The high-inflation regime is characterized by full employment $n_t = 1$ and high inflation $\pi_t > \gamma - g$ where the DNWR is not binding. We summarize the equilibrium conditions as follows:

$$\frac{\dot{y}_t}{y_t} = g, \tag{11}$$

$$\frac{\dot{m}_t}{m_t} = \mu - \Omega(m_t, y_t) + \rho + g, \tag{12}$$

$$\pi_t = \Omega(m_t, y_t) - (\rho + g), \tag{13}$$

$$r_t = \rho + g - \beta'(m_t) y_t. \tag{14}$$

To derive (11), recall that the production function (7) and goods market-clearing condition imply $c_t = y_t = \theta_t$. Here, $y_t = \theta_t$ means that the growth rate of output is g. Next, (5) leads to (14), where we use $\dot{y}_t / y_t = g$ and $a_t = m_t$ in equilibrium. It is straightforward to show (13) from (6). By assumption, inflation is higher than $\gamma - g$ in this regime. Finally, (12) is obtained from the definition of $m_t = M_t/P_t$, $\dot{m}_t/m_t = \mu - \pi_t$. Given the initial value θ_0, we obtain the output level $y_t = \theta_t$. Moreover, we can numerically solve (12) for m_t given m_0.

It is convenient to define the threshold value of $\Omega(m_t, y_t)$ in which output grows at the rate of g but inflation is as low as $\gamma - g$. Substituting $\dot{y}_t/y_t = g$ and $\pi_t = \gamma - g$ into (6) gives the threshold value of $\Omega(m_t, y_t)$:

$$\Omega^* = \rho + \gamma. \tag{15}$$

We use the threshold Ω^* to evaluate the allocation in the low-inflation regime.

2.3 Low-inflation regime

The low-inflation regime is characterized by unemployment $n_t < 1$ and the lower bound of inflation $\pi_t = \gamma - g$ where the DNWR is binding. We summarize the equilibrium conditions as follows:

$$\frac{\dot{y}_t}{y_t} = g - [(\rho + \gamma) - \Omega(m_t, y_t)], \tag{16}$$

$$\frac{\dot{m}_t}{m_t} = \mu - (\gamma - g), \tag{17}$$

$$\pi_t = \gamma - g, \tag{18}$$

$$r_t = v'(m_t)y_t - (\gamma - g). \tag{19}$$

Our assumption corresponds to (18). Equation (17) immediately follows from (18) because $\dot{m}_t/m_t = \mu - \pi_t$. To obtain the expression for output growth (16), we use (6) and the equilibrium condition $c_t = y_t$. The real interest rate r_t can be obtained from (4) together with $\pi_t = \gamma - g$.

To compare the system of equations between the high- and low-inflation regimes, we rewrite (16) – (19) as

$$\frac{\dot{y}_t}{y_t} = g - [\Omega^* - \Omega(m_t, y_t)], \tag{20}$$

$$\frac{\dot{m}_t}{m_t} = \mu - \Omega^* + \rho + g, \tag{21}$$

$$\pi_t = \Omega^* - (\rho + g), \tag{22}$$

$$r_t = \rho + g - \beta'(m_t)y_t - [\Omega^* - \Omega(m_t, y_t)], \tag{23}$$

where we use the definitions of $\Omega(m_t, y_t)$ and Ω^* shown in (6) and (15), respectively.

Comparisons between the two regimes reveal that there is additional downward pressure on output growth and the real interest rate when the DNWR binds. Equation (20) indicates that output growth can be lower than g depending on the sign of $\Omega^* - \Omega(m_t, y_t)$. In particular, it indicates that output growth is lower than or equal to g if $\Omega(m_t, y_t) \leq \Omega^*$. Similar interpretations can apply to the real interest rate from a comparison between (14) and (23).

3 The model dynamics

In this section, we specify the functional forms of $v(m_t)$ and $\beta(a_t)$ and investigate the model dynamics. We first show that the stagnation steady state exists in the presence of the wealth preferences. We then demonstrate that output initially grows at the same rate as productivity but slows in the transition path to this stagnation steady state. This slowdown in output growth endogenously occurs even if productivity continues to grow at a steady rate. Finally, we also show that the model without wealth preferences fails to generate the slowdown in output growth.

In what follows, we assume that the functions $v(m_t)$ and $\beta(a_t)$ are given by

$$v(m_t) = v\frac{m_t^{1-\eta}}{1-\eta}, \qquad v > 0, \quad \eta > 0, \tag{24}$$

$$\beta(a_t) = \beta a_t, \qquad \beta \geq 0, \tag{25}$$

where, with a slight abuse of notations, $v > 0$ and $\beta \geq 0$ represent a parameter for the functions $v(m_t)$ and $\beta(a_t)$.

3.1 The stagnation steady state

We characterize the steady state as a preparatory step for analyzing the model dynamics. The steady state in our model is the "stagnation" steady state in which the DNWR binds and output converges to a constant value. We show that the stagnation steady state can arise under our preference assumptions (24) and (25).

Suppose that nominal wage inflation hits the lower bound γ at $t = t^*$. Equation (17) implies that m_t goes to ∞ as $t \to \infty$ because of (9). In particular,

$$m_t = m_{t^*} \exp[(\mu - \gamma + g)(t - t^*)] \Rightarrow \lim_{t\to\infty} m_t = \infty, \tag{26}$$

where m_{t^*} is the real money balances in the period from which the DNWR starts binding. Then, in the stagnation steady state where y_t converges to a constant value, we have

$$\lim_{t\to\infty} \Omega(m_t, y_t) = \lim_{t\to\infty} [\beta'(m_t) + v'(m_t)]y_t = \beta y^{ss},$$

where a superscript ss on a variable denotes the stagnation steady-state value.

To prove the existence of the stagnation steady state, we use the transversality condition: $\lim_{t\to\infty} \lambda_t a_t \exp(-\rho t) = \lim_{t\to\infty} \lambda_t m_t \exp(-\rho t) = 0$. Taking the time derivative of this condition translates the condition into $\dot{\lambda}_t/\lambda_t + \dot{m}_t/m_t - \rho < 0$. In the stagnation steady state, $\dot{\lambda}_t/\lambda_t = -\dot{c}_t/c_t = 0$ and $\dot{m}_t/m_t = \mu - (\gamma - g)$. Thus, $\mu - (\gamma - g) < \rho$ ensures the transversality condition. Note that as long as the assumption of $\mu > 0$ holds, the transversality condition implies that $g < \rho + \gamma$.

Now, the allocations in the steady state are characterized as follows. Regarding output, (6) becomes $\beta y^{ss} = \rho + \gamma - g$ because $\lim_{t\to\infty} \Omega(m_t, y_t) = \beta y^{ss}$, $\dot{y}_t/y_t = 0$, and $\pi_t = \gamma - g$. The steady-state output is given by

$$y^{ss} = \frac{\rho + \gamma - g}{\beta}, \tag{27}$$

which is strictly positive because $g < \rho + \gamma$. Because of the DNWR, we have $\pi^{ss} = \gamma - g$. The nominal interest rate in this steady state is zero because of (4): $\lim_{t\to\infty}(r_t + \pi_t) = \lim_{t\to\infty}[v'(m_t)y_t] = 0$. From this equation, the real interest rate is given by $r^{ss} = -\pi^{ss} = g - \gamma$. Clearly, $\dot{y}_t/y_t = 0$ and $\dot{m}_t/m_t = \mu - (\gamma - g) > 0$ from (9).

3.2 Transition dynamics to the stagnation steady state

We are now ready to discuss the transition path to the stagnation steady state. The following proposition summarizes our main results.

Proposition 1. *Suppose that productivity grows at a strictly positive growth rate ($g > 0$) and that the money growth rate exceeds the lower bound of inflation ($\mu > \gamma - g$) and is strictly positive*

($\mu > 0$). Then, under the preference assumptions specified by (24) and (25) with a strictly positive β, we have a unique dynamic path of output toward the stagnation steady state. Output growth is endogenously determined. In particular, let t^* be the period of time from which the DNWR binds. Then,

- *For $0 < t < t^*$, the growth rate of output is equal to that of productivity.*
- *For $t = t^*$, the growth rate of output starts declining.*
- *For $t > t^*$, the growth rate of output is lower than that of productivity.*
- *As $t \to \infty$, the growth rate of output converges to zero.*

Here, we assume that the initial productivity θ_0 is such that $\pi_0 = \Omega(m_0, \theta_0) - (\rho + g) > \Omega^* - (\rho + g)$. To ensure the transversality condition, we assume that $\mu - (\gamma - g) < \rho$.

Proof. See Appendix A.1. □

The model endogenously generates the slowdown in output growth. The economy experiences a regime change from the high- to the low-inflation regime. Given that $\pi_0 > \Omega^* - (\rho + g)$, output initially equals productivity, or equivalently, the potential level (i.e., $y_t = \theta_t$). Thus, output grows at the same rate as productivity. A steady growth of output persists when the DNWR is not binding (i.e., $\pi_t > \gamma - g$). However, once the DNWR binds at $t \geq t^*$, the growth rate becomes lower than g. Output growth then declines over time and converges to zero in the stagnation steady state.

To understand the model dynamics, we focus on $\Omega(m_t, y_t)$ given by (6). Using the market-clearing conditions $c_t = y_t$ in equilibrium, we have

$$\Omega(m_t, y_t) = \rho + \frac{\dot{y}_t}{y_t} + \pi_t. \tag{28}$$

Equation (28) is the condition for substitution between consumption and saving, where the saving is in the form of money. Under our specification, $\Omega(m_t, y_t) = (\beta + v m_t^{-\eta}) y_t$ is decreasing in m_t and increasing in y_t. That is, there are two offsetting effects on $\Omega(m_t, y_t)$. In the growing economy, both m_t and y_t grow over time so that the dynamics of $\Omega(m_t, y_t)$ are not clear analytically. Numerically, however, the marginal benefits of holding real money balances $\Omega(m_t, y_t)$ decreases over time because the effect of m_t on $\Omega(m_t, y_t)$ overwhelms the effect of y_t on $\Omega(m_t, y_t)$. Intuitively, this is because strong preferences for wealth make real money balances grow faster than consumption.

Now, given the declining marginal benefits of holding money, the marginal cost represented by the right-hand side of (28) must decline in equilibrium. As (28) shows, given a constant ρ, either output growth ($\dot{y}_t/y_t$) or inflation (π_t) must decline. If the economy is in the high-inflation regime, output growth can be kept at $\dot{y}_t/y_t = g$. This is because decreased inflation can lower the marginal cost of holding money. By contrast, inflation can no longer decrease if the economy is in a low-inflation regime. Only through the slowdown in output growth can the marginal cost of holding money decrease.

We emphasize that the aggregate demand shortage drives this slowdown in output growth. In our model, the household's wealth preferences weaken the aggregate demand growth by substituting wealth for consumption goods. Initially, the weakened aggregate demand leads to disinflation since the DNWR is not binding. Because prices are flexible, the economy can achieve the efficient level of output. However, when the DNWR makes price adjustment rigid, the weakened aggregate demand determines equilibrium output. Moreover, the equilibrium output is inefficient in terms of welfare. Therefore, the growth rate of output is g for $t < t^*$ and becomes lower than g for $t \geq t^*$.

Some remarks on the transition path are in order. First, all variables including $\dot{y}_t/y_t$ and $\dot{m}_t/m_t$ smoothly switch from a path described from (11)–(14) to a path specified by (20)–(23) without a jump. We can confirm this by evaluating $\Omega(m_t, y_t)$ at Ω^*. When the marginal benefit of holding money is at the threshold level, (20)–(23) at $t = t^*$ all reduce to (11)–(14), suggesting smooth transitions from the high- to the low-inflation regime. Second, output growth also exhibits a smooth transition to the stagnation steady state. Since $\Omega(m_t, y_t)$ in (16) smoothly converges to its limit of βy^{ss}, output growth smoothly converges to zero as $t \to \infty$:

$$\lim_{t\to\infty} \frac{\dot{y}_t}{y_t} = \lim_{t\to\infty} [g + \Omega(m_t, y_t) - (\rho + \gamma)] = g + \beta y^{ss} - \rho - \gamma = 0 \tag{29}$$

where the last equality comes from (27).

3.3 The role of wealth preferences

The results of Proposition 1 critically depend on the household's wealth preferences with a strictly positive β. In this subsection, we show that if there are no wealth preferences (i.e., $\beta = 0$), the model fails to generate the endogenous slowdown in output growth, in contrast to Proposition 1. The following proposition summarizes our results under $\beta = 0$.

Proposition 2. *Suppose that productivity grows at a strictly positive growth rate ($g > 0$) and that the money growth rate exceeds the lower bound of inflation ($\mu > \gamma - g$). Then, under the preference assumptions specified by (24) and (25) but with $\beta = 0$, we have a unique dynamic path of output growth such that*

- *For any $t > 0$, the growth rate of output is constant.*
- *The growth rate of output is equal to $\min\{g, \eta[\mu - (\gamma - g)]\}$.*

The growth rate of output is g when the DNWR does not bind or $\eta[\mu - (\gamma - g)]$ when the DNWR binds. The transversality condition is given by $(1 - \eta) \min\{g, \eta[\mu - (\gamma - g)]\} < \rho$.

Proof. See the Appendix A.2. □

The proposition suggests that the model without wealth preferences ($\beta = 0$) fails to generate an endogenous slowdown in output growth. Instead, parameters in the model predetermine the regime in equilibrium. For example, if the economy is initially in a high-inflation regime, the growth rate moves in tandem with productivity. Output always equals its potential level and the allocation is efficient. If the economy instead starts from a low-inflation regime, the economy experiences low economic growth given the rigid wage adjustment arising from the DNWR. In this case, output is always below its potential level and the allocation is inefficient. The aggregate demand shortage make economic growth slower than g. In either case, however, we do not observe a slowdown in output growth, in contrast to the case of $\beta > 0$. Moreover, the growth rate of output with the binding DNWR does not converge to zero. Even if the DNWR is binding, there is no stagnation steady state under $\beta = 0$. Therefore, the essential ingredient for generating slowdown in output growth and stagnation steady state is a strictly positive β in the wealth preferences.[14]

[14] A natural question is what happens if nominal wages are fully flexible, but β is strictly positive. As (28) suggests, π_t may continue to decline without the slowdown in output growth. However, it can be shown that there is no monetary equilibrium with a strictly positive β. For details, see Ono (2001).

Our analysis suggests that the model with wealth preferences is qualitatively consistent with the observations in advanced economies. However, it is unclear whether the model numerically explains the data. Moreover, it is also worth assessing macroeconomic variables other than output growth. The following section assesses the model with wealth preferences based on output growth, the real interest rate, inflation, and the velocity of money.

4 Simulating the model

4.1 Calibrations

This section simulates the US output trend and sees whether our model can replicate the slowdown in US output growth after the Great Recession. In the debate on secular stagnation, it was expected that output growth would remain high after the Great Recession. Given these expectations, we calibrate g at the growth rate of output before 2007:Q1, $g = 0.019$.[15] We set $\mu = 0.036$ based on the mean growth rate of M2 stock (per capita) over 1990:Q1–2007:Q1. Regarding the subjective discount rate ρ, we parameterize $\rho = 0.04$. As for the lower bound of nominal wage inflation, we set γ at a slightly larger value than that in previous studies (e.g., Schmitt-Grohé and Uribe (2016)). We set γ at 0.019.[16]

To pin down v, η, and β in the utility functions $v(m_t)$ and $\beta(a_t)$, we target the moments of the real interest rate and the velocity of money between 1990:Q1 and 2007:Q1. For the real interest rate, we use the 10-year real Treasury yields from the database of the Cleveland Federal Reserve Bank and estimated by Haubrich, Pennacchi, and Ritchken (2012).[17] We set our target for the real interest rate at 2.80 percent, which is the mean over the period 1990:Q1–2007:Q1. For the velocity of money, which is y_t/m_t in our model, we target the mean velocity of M2 (2.03) and the mean growth rate (0.45 percent) over 1990:Q1–2007:Q1. The resulting calibrated values are $v = 0.45$,

[15] We estimated g from a regression of log output discussed in footnote 2. Using the regression, we set g at $4 \times \hat{a}_1$ where $\hat{a}_1$ is the estimated coefficient on a linear time trend.

[16] The value of $\gamma = 0.019$ appears large because the actual nominal wage inflation temporarily falls short of 0.019. In the data, the index of compensation per hour in the nonfarm business sector declined by 10.5 percent in 2009:Q1. Nevertheless, we calibrate γ at 0.019 for the following reasons. First, we are calibrating the lower bound of the long-run nominal wage inflation that excludes the impact of temporary shocks. For the above compensation per hour, the mean wage inflation rate over 2007:Q2–2019:Q4 is higher than two percent, which exceeds our calibration value of the lower bound. Second, the parameter value of $\gamma = 0.019$ satisfies the parameter restriction, $0 < \mu - (\gamma - g) < \rho$, where the latter points to the transversality condition. If $\gamma = -0.105$, it violates the transversality condition.

[17] The most recent data are available at `https://www.clevelandfed.org/our-research/indicators-and-data/inflation-expectations/background-and-resources.aspx#research`.

$\eta = 5.84$, and $\beta = 0.01$, respectively.

While the data are discrete in time, our simulated data are continuous. We assume that the regime change takes place after 2007:Q1, that is, $t^* = 2007{:}Q1$. This timing is matched with our estimated output trend with a structural break because the deviation between the two estimated output trends starts at 2007:Q1.

We perform the numerical simulation as follows. First, we guess at the output level in the period of regime change (denoted by y_{t^*}). Second, given the guess of output at $t = t^*$, we compute the transition path backward from $t^* = 2007{:}Q1$ to $t = 1990{:}Q1$. In this transition path for $t < t^*$, output growth is equal to productivity growth. Third, we calculate the transition path forward from $t^* = 2007{:}Q1$ to $t = \bar{t}$, where $\bar{t}$ is a sufficiently large to approximate $t = \infty$. In this transition path, the economy moves toward the stagnation steady state. We compute the output at $t = \bar{t}$ (denoted by $y_{\bar{t}}$) and compare it with the steady state output y^{ss}. Finally, if $y_{\bar{t}} \simeq y^{ss}$, we conclude that the transition path to the stagnation steady state is obtained. If not, we update the guess of the output y_{t^*} and iterate computations until we have $y_{\bar{t}} \simeq y^{ss}$.

4.2 Simulation results

Figure 3 plots the simulated output trend predicted by the model (solid line) along with the two estimated output trends that replicate those in Figure 1. The initial value of the simulated output trend is the value in 1990:Q1 and is normalized by the value of the estimated output trend data in the same period. Because we directly calibrate g at 0.019 from the estimated output trend, this normalization makes the simulated output trend match perfectly with the estimated output trend until 2007:Q1. Thus, the figure particularly depicts the simulated and estimated output trends from 2000:Q1 to focus on the output trends after 2007:Q1.

The simulated output trend in Figure 3 indicates the slowdown in output growth after the Great Recession, as the theory predicts. Comparing the solid and dashed lines, we see that the model can account for a substantial fraction of the slowdown in output growth. For example, the model can explain 88.8 percent of the total decline in output in 2015.[18] The dashed and solid lines intersect

[18] Based on Figure 3, the estimated counterfactual output trend that removes a structural break is 48.3 percent higher in 2015:Q1 than in 1990:Q1. The estimated output trend with the structural break is 39.4 percent higher in 2015:Q1 than in 1990:Q1. Thus, the output declines are 8.9 percentage points between the trend lines with and without the structural break. Next, the model's simulation results suggest that the simulated output trend is 40.4 percent higher in 2015:Q1 than in 1990:Q1, so output declines 7.9 percentage points. Thus, the model can explain

in 2017. Thus, the model can almost entirely explain the total output decline in 2017.

[Figure 3 about here.]

The model can also successfully replicate the real interest rate that has decreased over time since the 1990s. Figure 4 shows the real interest rate (dotted line), the cubic trend component of the real interest rate (dashed line), and the simulated real interest rate predicted by the model (solid line). As suggested by the trend (dashed) line, the trend real interest rate declines over time. In particular, the decline in the trend real interest rate is 3.16 percentage points lower, from 3.82 percent in 1990:Q1 to 0.66 percent in 2019:Q1. The simulated real interest rate also decreases over time. As discussed in the previous section, there is additional downward pressure on the real interest rate represented by $\Omega^* - \Omega(m_t, y_t)$ when the DNWR binds (See (23)). This downward pressure arises from the household's insatiable strong desire for wealth. As a result, the simulated real interest rate was 3.31 percent in 1990:Q1 and 0.19 percent in 2019:Q1. The magnitude of the decline in the simulated real interest rate is 3.12 percentage points, which is close to the 3.16 percentage point decline in the trend real interest rate.

[Figure 4 about here.]

Figure 5 plots inflation. As before, the solid, dotted, and dashed lines are the simulated, actual, and cubic trend inflation, respectively. Here, we calculate inflation using the price index of Personal Consumption Expenditure, excluding foods and energy.[19] The figure indicates that estimated trend inflation decreased steadily by 2003:Q2 and did not exhibit large declines after 2003:Q2. In particular, the dashed line shows that the trend inflation after 2003:Q2 is stable at around 1.5–2 percent. The simulated inflation stops decreasing after 2007:Q1 and is equal to a constant value $\gamma - g$ (See (22)). Overall, the simulated trend inflation is lower than the estimated trend inflation, especially after 2007:Q1, and the timing in which the simulated trend inflation becomes stable is slightly delayed. Nevertheless, the model replicates the magnitude of disinflation well. In particular, while the estimated trend inflation decreases by 2.35 percent (from 3.84 percent in 1990:Q1 to 1.49 percent in 2019:Q1), the simulated inflation decreases by 3.20 percent (from 3.20 percent in 1990:Q1 to zero percent in 2019:Q1).[20] Clearly, the decline in the

88.8 percent of the total decline ($0.079/0.089 = 0.888$).

[19] We use year-on-year inflation to remove the noise in inflation.

[20] The simulated inflation after 2007:Q1 is $\pi_t = \gamma - g$ and becomes zero because we assume $\gamma = g$.

simulated inflation results from the household's wealth preferences and the resulting weak aggregate demand for consumption goods.

[Figure 5 about here.]

We finally assess the velocity of money. We have seen that the trend velocity y_t/m_t eventually decreases with time because wealth preferences make real money balances m_t grow faster than output y_t. Here, we evaluate how fast the velocity of money decreases. Figure 6 compares the simulated velocity (solid line) with the actual velocity (dotted line) and its trend (dashed line). Recall that our calibration target was the level and growth rate of trend velocity between 1990:Q1 and 2007:Q1. We thus assess the performance of the model using trend velocity after 2007:Q1. The estimated trend velocity decreases by 27.6 percent from 2007:Q1 to 2019:Q4 and the simulated trend velocity declines by 35.8 percent during the same period.[21] Therefore, our model explains the trend velocity quantitatively well.

[Figure 6 about here.]

Before closing this section, we provide some numerical examples of the model without wealth preferences. Proposition 2 implies that a high-inflation regime is chosen as long as $g \leq \eta[\mu-(\gamma-g)]$. Using the parameter values in the previous section, we translate the condition as $\eta \geq g/[\mu-(\gamma-g)] = 0.019/0.036 = 0.528$. We thus reassign two parameter values to η: $\eta = 1$ and $\eta = 1/2$. The former achieves the high-inflation regime, and the latter leads to the low-inflation regime.

If $\eta = 1$, the economy is in the high-inflation regime. It immediately follows from Proposition 2 and (14) that $\dot{y}_t/y_t = 0.019$ and $r_t = \rho + g = 0.059$. These results indicate that both output growth and the real interest rate are constant. Furthermore, $\Omega(m_t, y_t) = v'(m_t)y_t = vy_t/m_t$ under $\beta = 0$ and $\eta = 1$. Appendix A.2 proves that $\Omega(m_t, y_t)$ is constant in equilibrium when $\beta = 0$. Therefore, $\dot{m}_t/m_t = \dot{y}_t/y_t = 0.019$ and $\pi_t = \mu - g/\eta = 0.017$. The model fails to explain the disinflation observed in the 1990s and early 2000s.

If $\eta = 1/2$, the low-inflation regime is chosen. In this case, the DNWR binds, and output growth is lower than g. More specifically, Proposition 2 implies that $\dot{y}_t/y_t = \eta[\mu - (\gamma - g)] = 0.5 \times 0.036 =$

[21] The estimated trend velocity is 0.017 in 2007:Q1 and -0.259 in 2019:Q1 in logarithms, respectively. Thus, the percentage change in trend velocity is $-27.6(= -25.9 - 1.7)$ percent. Likewise, the simulated trend velocity in 2007:Q1 and 2019:Q1 is 0.092 and -0.266 in logarithms, respectively. Thus, the percentage change in the simulated trend velocity is $-35.8(= -26.6 - 9.2)$ percent.

0.018. From (17) and (18), we have $\dot{m}_t/m_t = \mu - (\gamma - g) = 0.036$ and $\pi_t = \gamma - g = 0$, respectively. Finally, the Euler equation (5) reduces to its standard form under $\beta = 0$: $r_t = \rho + \dot{y}_t/y_t$, so $r_t = \rho + \eta[\mu - (\gamma - g)] = 0.058$. While inflation is quite low, output growth, the real interest rate, and inflation do not decrease over time.

The velocity of money declines over time if $\eta < 1$. This is because of the constancy of $\Omega(m_t, y_t) = v y_t m_t^{-\eta}$ under $\beta = 0$ (See Appendix A.2). Given that $\Omega(m_t, y_t)$ is constant in equilibrium, we have $\dot{y}_t/y_t = \eta \dot{m}_t/m_t$. For the velocity of money to decrease, real money balances grow faster than does output. This is possible only if $\eta < 1$.

5 Conclusion

Output growth in advanced economies was persistently low after the Great Recession. In this paper, we explained this slowdown in output growth by introducing wealth preferences into a standard monetary growth model. We theoretically showed that our model generates a slowdown in output growth in the transition path to the stagnation steady state. Using numerical simulations, we also reported that our model explains 88.8 percent of the decline in output trend in 2015 and nearly 100 percent of the decline in output trend in 2017. In addition to output growth, the model successfully reproduces the magnitude of the decreases in the real interest rate. Our model also explains the declines in inflation and the velocity of money in the sample period.

It is quite surprising that a simple model can account for the long-run patterns in the data. Further work would enrich our understanding of secular stagnation and its policy prescriptions. Many questions are left open. What are the implications of growth-enhancing policy on the model dynamics? What happens to output growth and the real interest rate if we prompt nominal wage adjustment by removing institutional frictions in the labor market? What are the impacts of the forward guidance on output growth? Exploring these directions would be important for future research.

References

Baldwin, Richard and Coen Teulings. 2014. “Introduction.” In *Secular Stagnation: Facts, Causes, and Cures*, edited by Coen Teulings and Richard Baldwin, chap. 1. CEPR Press, 1–23.

Barattieri, Alessandro, Susanto Basu, and Peter Gottschalk. 2014. "Some evidence on the importance of sticky wages." *American Economic Journal: Macroeconomics* 6 (1):70–101.

Benigno, Gianluca and Luca Fornaro. 2018. "Stagnation traps." *The Review of Economic Studies* 85 (3):1425–1470.

Camerer, Colin, George Loewenstein, and Drazen Prelec. 2005. "Neuroeconomics: How neuroscience can inform economics." *Journal of Economic Literature* 43 (1):9–64.

Carvalho, Carlos, Andrea Ferrero, and Fernanda Nechio. 2016. "Demographics and real interest rates: Inspecting the mechanism." *European Economic Review* 88:208–226.

Eggertsson, Gauti B and Paul Krugman. 2012. "Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach." *The Quarterly Journal of Economics* 127 (3):1469–1513.

Eggertsson, Gauti B, Neil R Mehrotra, and Jacob A Robbins. 2019. "A model of secular stagnation: Theory and quantitative evaluation." *American Economic Journal: Macroeconomics* 11 (1):1–48.

Fallick, Bruce, Michael Lettau, and William Wascher. 2016. "Downward nominal wage rigidity in the United States during and after the Great Recession." Available at SSRN: https://ssrn.com/abstract=2725019.

Fernald, John G. 2015. "Productivity and potential output before, during, and after the Great Recession." *NBER Macroeconomics Annual* 29 (1):1–51.

Gagnon, Etienne, Benjamin K Johannsen, and David Lopez-Salido. 2021. "Understanding the new normal: The role of demographics." *IMF Economic Review* 69 (2):357–390.

Gordon, Robert J. 2015. "Secular stagnation: A supply-side view." *AEA Paper & Proceedings* 105 (5):54–59.

Grigsby, John, Erik Hurst, and Ahu Yildirmaz. 2021. "Aggregate nominal wage adjustments: New evidence from administrative payroll data." *American Economic Review* 111 (2):428–71.

Guerrieri, Veronica and Guido Lorenzoni. 2017. "Credit crises, precautionary savings, and the liquidity trap." *The Quarterly Journal of Economics* 132 (3):1427–1467.

Hall, Robert E. 2011. "The long slump." *American Economic Review* 101 (2):431–469.

Hashimoto, Ken-ichi, Yoshiyasu Ono, and Matthias Schlegl. 2021. "Structural unemployment, underemployment, and secular stagnation." ISER Discussion Papers No. 1088.

Haubrich, Joseph, George Pennacchi, and Peter Ritchken. 2012. "Inflation expectations, real rates, and risk premia: Evidence from inflation swaps." *The Review of Financial Studies* 25 (5):1588–1629.

Hazell, Jonathon and Bledi Taska. 2020. "Downward rigidity in the wage for new hires." Available at SSRN: https://ssrn.com/abstract=3728939.

Ikeda, Daisuke and Takushi Kurozumi. 2019. "Slow post-financial crisis recovery and monetary policy." *American Economic Journal: Macroeconomics* 11 (4):82–112.

Illing, Gerhard, Yoshiyasu Ono, and Matthias Schlegl. 2018. "Credit booms, debt overhang and secular stagnation." *European Economic Review* 108:78–104.

Jones, Callum. 2022. "Aging, secular stagnation and the business cycle." *The Review of Economics and Statistics* forthcoming.

Krugman, Paul. 2014. "Four observations on secular stagnation." In *Secular Stagnation: Facts, Causes, and Cures*, edited by Coen Teulings and Richard Baldwin, chap. 1. CEPR Press, 61–68.

Mian, Atif and Amir Sufi. 2014. "What explains the 2007–2009 drop in employment?" *Econometrica* 82 (6):2197–2223.

Michaillat, Pascal and Emmanuel Saez. 2021. "Resolving New Keynesian anomalies with wealth in the utility function." *The Review of Economics and Statistics* 103 (2):197–215.

———. 2022. "An economical business-cycle model." *Oxford Economic Papers* 74 (2):382–411.

Michau, Jean-Baptiste. 2018. "Secular stagnation: Theory and remedies." *Journal of Economic Theory* 176:552–618.

Ono, Yoshiyasu. 1994. *Money, Interest, and Stagnation: Dynamic Theory and Keynes's Economics.* Oxford University Press.

———. 2001. "A reinterpretation of chapter 17 of Keynes's general theory: Effective demand shortage under dynamic optimization." *International Economic Review* 42 (1):207–236.

Schmitt-Grohé, Stephanie and Martin Uribe. 2016. "Downward nominal wage rigidity, currency pegs, and involuntary unemployment." *Journal of Political Economy* 124 (5):1466–1514.

Sidrauski, Miguel. 1967. "Rational choice and patterns of growth in a monetary economy." *American Economic Review* 57 (2):534–544.

Sigurdsson, Jósef and Rannveig Sigurdardottir. 2016. "Time-dependent or state-dependent wage-setting? Evidence from periods of macroeconomic instability." *Journal of Monetary Economics* 78:50–66.

Summers, Lawrence H. 2014. "U.S. economic prospects: Secular stagnation, hysteresis, and the zero lower bound." *Business Economics* 49 (2):65–73.

Takahashi, Yuta and Naoki Takayama. 2022. "Tech-driven secular stagnation: Cross-country evidence." Unpublished manuscript.

A Proofs of Propositions

A.1 Proof of Proposition 1

To prove Proposition 1, the phase diagram in the (m_t, y_t) plane is convenient. We derive the loci in each inflation regime in Figure 7.

[Figure 7 about here.]

We first consider the high-inflation regime. Equation (11) implies that $\dot{y}_t > 0$ for any $t < t^*$. We thus focus on the $\dot{m}_t = 0$ locus. The $\dot{m}_t = 0$ locus is obtained from (12) and given by $\mu - \Omega(m_t, y_t) + \rho + g = 0$. Under our preference assumptions, $\Omega(m_t, y_t) = (\beta + v m_t^{-\eta}) y_t$. Therefore, the $\dot{m}_t = 0$ locus can be rewritten as

$$\begin{aligned} y_t &= \frac{\rho + g + \mu}{\beta + v m_t^{-\eta}} \\ &= \frac{\rho + \gamma + [\mu - (\gamma - g)]}{\beta + v m_t^{-\eta}} \\ &= f_H(m_t), \end{aligned} \tag{30}$$

where we define the $\dot{m}_t = 0$ locus by $y_t = f_H(m_t)$. In Figure 7, the locus is drawn as the red solid line. Here m_t increases with time whenever (m_t, y_t) lies to the right of the $\dot{m}_t = 0$ locus. Together with $\dot{y}_t > 0$ under the high-inflation regime, the directions of the changes are indicated by red arrows in the figure.

Next, consider the low-inflation regime. Under the assumption of (9), (17) implies that $\dot{m}_t > 0$ for any $t > t^*$. We thus focus on the $\dot{y}_t = 0$ locus. The $\dot{y}_t = 0$ locus is obtained from (16) and given by $g - (\rho + \gamma) + \Omega(m_t, y_t) = 0$. Again, noting that $\Omega(m_t, y_t) = (\beta + v m_t^{-\eta}) y_t$, the $\dot{y}_t = 0$ locus can be rewritten as

$$\begin{aligned} y_t &= \frac{\rho + \gamma - g}{\beta + v m_t^{-\eta}} \\ &= f_L(m_t), \end{aligned} \tag{31}$$

where we define the $\dot{y}_t = 0$ locus by $y_t = f_L(m_t)$. Note that $f_L(m_t) < f_H(m_t)$ holds because $\mu - (\gamma - g) > 0$ from (9) and $g > 0$. In Figure 7, the locus is drawn as the blue solid line. Here y_t increases with time whenever (m_t, y_t) lies to the left of the $\dot{y}_t = 0$ locus. Together with $\dot{m}_t > 0$ under the low-inflation regime, the directions of the changes are indicated by blue arrows in the figure.

Let us introduce another locus that determines the regime change. This locus is defined as a set of (m_t, y_t) in which output growth is g and inflation is $\gamma - g$. Substitute $\dot{y}_t / y_t = g$ and $\pi_t = \gamma - g$ into (28) to get $\Omega(m_t, y_t) = \rho + \gamma$. Again, using $\Omega(m_t, y_t) = (\beta + v m_t^{-\eta}) y_t$, we rewrite this condition at the threshold as

$$\begin{aligned} y_t &= \frac{\rho + \gamma}{\beta + v m_t^{-\eta}} \\ &= f_T(m_t), \end{aligned} \tag{32}$$

where we define the locus as $y_t = f_T(m_t)$. As shown in Figure 7, we have $f_L(m_t) < f_T(m_t) < f_H(m_t)$ given m_t because of $\mu > \gamma - g$ and $g > 0$. It is easy to show that the economy is in the high-inflation regime when (m_t, y_t) lies above the locus.[22] When (m_t, y_t) lies below the locus, the DNWR is binding and the economy is in the low-inflation regime.

Figure 7 also draws the optimal time path (the curve with arrows) starting from the initial state of the economy. By assumption, at the initial state, $\pi_0 > \gamma - g$. Thus, the economy is in the high-inflation regime, and (m_0, y_0) is located above the locus of $f_T(m_t)$. Once (m_t, y_t) goes to the right of the $f_T(m_t)$ locus, the economy moves to the low-inflation regime. Recall that (20) and (21) indicate that (m_t, y_t) switches from the high-inflation regime to the low-inflation regime without a jump. Also, as $t \to \infty$, $m_t \to \infty$ from (26) and $\dot{y}_t/y_t \to 0$ from (29). Thus, (m_t, y_t) asymptotically converges to the dotted line located at the bottom in which $y_t = y^{ss}$. The optimal time path never exceeds the dotted line because $\dot{y}_t/y_t$ is always positive as long as (m_t, y_t) is located above the blue solid line (See Figure 7).

To complete the proof, we confirm that the time path shown in the figure is saddle-path stable. Let us redefine $z_t = 1/m_t$ to obtain the system of the equation under the low-inflation regime:

$$
\begin{aligned}
\frac{\dot{y}_t}{y_t} &= g - \left[(\rho + \gamma) - \Omega(1/z_t, y_t)\right], \\
\frac{\dot{z}_t}{z_t} &= -(\mu - \gamma + g)
\end{aligned}
$$

By linearizing the above two equations around the stagnation steady state $(z_t, y_t) = (0, y^{ss})$, we have the eigenvalues ζ such that

$$
\begin{vmatrix}
\rho + \gamma - g - \zeta & 0 \\
0 & -(\mu - \gamma + g) - \zeta
\end{vmatrix} = 0,
$$

where we use (27) to replace $\beta y^{ss} = \rho + \gamma - g$. As mentioned in the main text, combining the condition $\mu - (\gamma - g) < \rho$ derived from the transversality condition with the condition $\mu > 0$ implies that $g < \rho + \gamma$. The condition $g < \rho + \gamma$ then ensures that one eigenvalue is positive. In addition, the

[22] We can prove this claim by contradiction. Suppose that $\pi_t = \gamma - g$ and $\dot{y}_t/y_t < g$. We do not need to consider $\pi_t < \gamma - g$ and $\dot{y}_t/y_t > g$ because they are infeasible. Then, (28) implies that $\Omega(m_t, y_t) = \rho + \dot{y}_t/y_t + \pi_t < \rho + g + \gamma - g = \rho + \gamma = \Omega^*$. That is, $\Omega(m_t, y_t) < \Omega^*$. It immediately follows from $\Omega(m_t, y_t) = (\beta + v m_t^{-\eta}) y_t$ that $y_t < f_T(m_t)$, which contradicts the supposition.

assumption $\mu - (\gamma - g) > 0$ in (9) implies that the other eigenvalue is negative. Because the system of equations under the low-inflation regime includes one jump variable y_t and one predetermined variable z_t, the time path is saddle-path stable and unique.

Along any path located above the saddle path m_t eventually becomes negative so that it is infeasible. Along any path located below the saddle path c_t eventually becomes zero so that the transversality condition does not hold. Therefore, the saddle path is the unique equilibrium path.

A.2 Proof of Proposition 2

The proposition discusses the case of $\beta = 0$ to clarify the role of wealth preferences. To prove the proposition, we focus on the elasticity of $\Omega(m_t, y_t)$ with respect to m_t and study the dynamics of $\Omega(m_t, y_t) = [v'(m_t) + \beta'(m_t)]y_t$. We then discuss the equilibrium under the high- and low-inflation regimes.

Let ε_m be the elasticity of $\Omega(m_t, y_t)$ with respect to m_t.

$$\varepsilon_m(m_t) = \frac{\partial \Omega(m_t, y_t)}{\partial m_t}\frac{m_t}{\Omega_t} = -\eta \frac{v m_t^{-\eta}}{\beta + v m_t^{-\eta}}. \tag{33}$$

The growth rate of $\Omega_t = \Omega(m_t, y_t)$ is

$$\frac{\dot{\Omega}_t}{\Omega_t} = \frac{\dot{y}_t}{y_t} + \varepsilon_m(m_t)\frac{\dot{m}_t}{m_t}.$$

In general, the dynamics of Ω_t are described by a nonlinear differential equation because the elasticity of Ω_t depends on m_t. However, when $\beta = 0$, the above equation simplifies to

$$\frac{\dot{\Omega}_t}{\Omega_t} = \frac{\dot{y}_t}{y_t} - \eta\frac{\dot{m}_t}{m_t}. \tag{34}$$

Next, we study the dynamics of Ω_t under the high- and low-inflation regimes. We will show below that Ω_t is constant in equilibrium when $\beta = 0$. This constancy of $\Omega(m_t, y_t)$ along with (34) implies that $\dot{y}_t/y_t = \eta\dot{m}_t/m_t$ holds in equilibrium.

Output growth under the high-inflation regime Substituting (11) and (17) into (34) yields the differential equation for Ω_t under the high-inflation regime:

$$\begin{aligned}\frac{\dot{\Omega}_t}{\Omega_t} &= g - \eta(\mu - \Omega_t + \rho + g) \\ &= g - \eta[\mu - (\gamma - g)] + \eta(\Omega_t - \Omega^*),\end{aligned} \tag{35}$$

where we use (15) for the second equality. Equation (35) is the differential equation for Ω_t with a positive coefficient on Ω_t. Thus, if there is a deviation of Ω_t from its steady-state value, Ω_t would explode to ∞. Therefore, when $\beta = 0$, only $\dot{\Omega}_t = 0$ is feasible in equilibrium. Imposing $\dot{\Omega}_t = 0$ on (35), we have the steady-state value of Ω_t under the high-inflation regime with $\beta = 0$.

$$\Omega_H^{ss} = \Omega^* + [\mu - (\gamma - g)] - \frac{g}{\eta}. \tag{36}$$

The high-inflation regime under $\beta = 0$ is feasible only when $g < \eta[\mu - (\gamma - g)]$. To see this, suppose that $g \geq \eta[\mu - (\gamma - g)]$ under the high-inflation regime. In this case, (36) implies that $\Omega_H^{ss} \leq \Omega^*$. Given $\dot{y}_t/y_t = g$ in the high-inflation regime, (15) and (28) imply that $\Omega_H^{ss} \leq \Omega^*$ is rewritten as $\Omega_H^{ss} = \rho + \dot{y}_t/y_t + \pi_t = \rho + g + \pi_t < \Omega^* = \rho + \gamma$ and thus $\pi_t < \gamma - g$. However, it violates the assumption of the DNWR, $\pi > \gamma - g$.

Output growth under the low-inflation regime Substituting (17) and (20) into (34) yields

$$\frac{\dot{\Omega}_t}{\Omega_t} = g - \Omega^* + \Omega_t - \eta[\mu - (\gamma - g)]. \tag{37}$$

As in (35), (37) is the differential equation for Ω_t with a positive coefficient on Ω_t. Once again, only $\dot{\Omega}_t = 0$ is feasible in equilibrium. Imposing $\dot{\Omega}_t = 0$ on (37), we have the steady-state value under the low-inflation regime with $\beta = 0$:

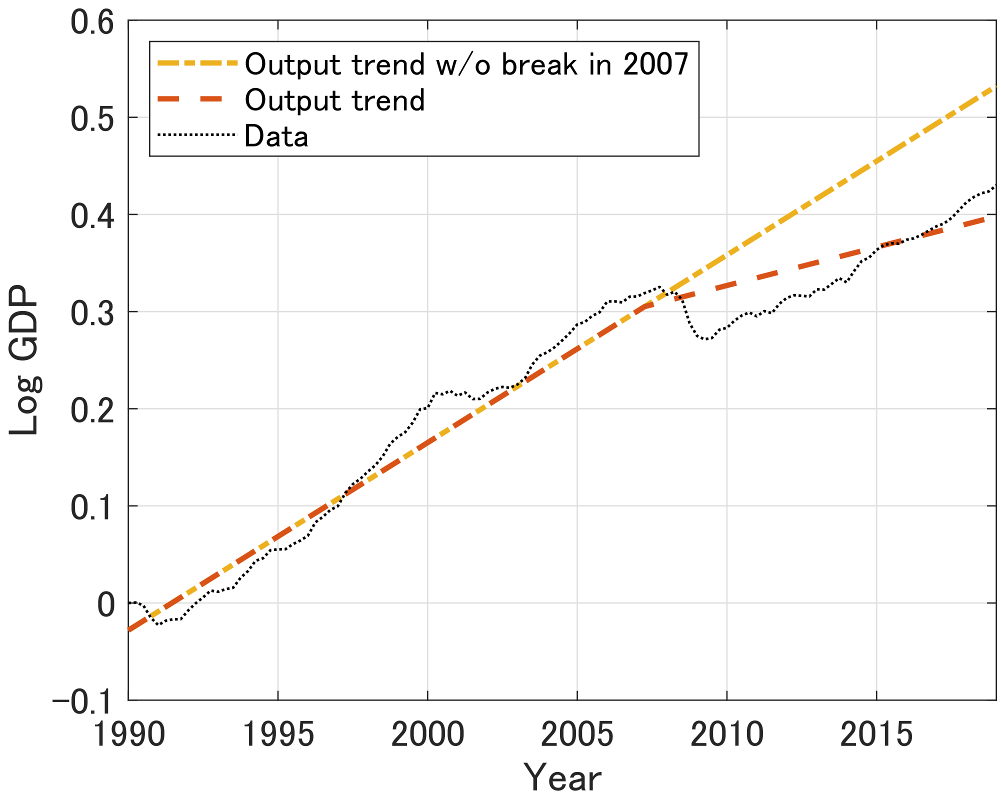
$$\Omega_L^{ss} = \Omega^* + \eta[\mu - (\gamma - g)] - g \tag{38}$$

The low-inflation regime under $\beta = 0$ is feasible only when $g \geq \eta[\mu - (\gamma - g)]$. To prove this, suppose that $g < \eta[\mu - (\gamma - g)]$ in the low-inflation regime. In this case, (38) implies that $\Omega_L^{ss} > \Omega^*$.

Given $\pi_t = \gamma - g$ in the low-inflation regime, (15) and (28) imply that the condition $\Omega_L^{ss} > \Omega^*$ is rewritten as $\Omega_L^{ss} = \rho + \dot{y}_t/y_t + \pi_t = \rho + \dot{y}_t/y_t + \gamma - g > \Omega^* = \rho + \gamma$ and thus $\dot{y}_t/y_t > g$. However, it violates the feasibility because $y_t = \theta_t n_t$ where $n_t \leq 1$.

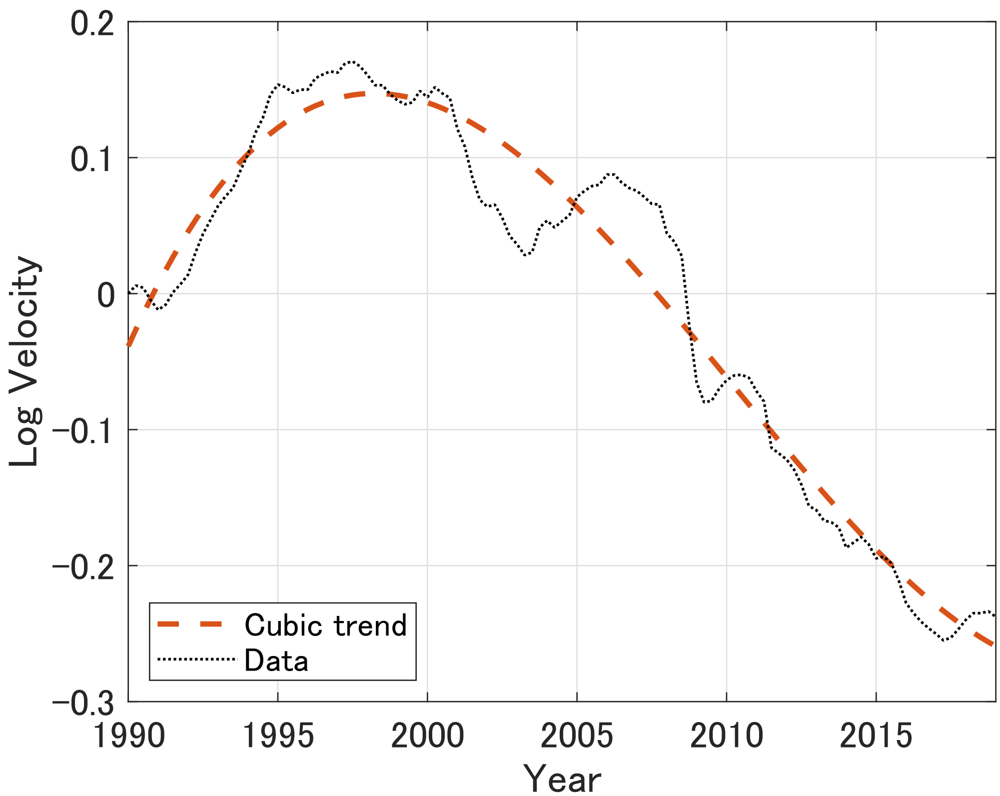
To summarize, there is no endogenous slowdown in output growth when $\beta = 0$. The parameters in the model fully determine the regime. If $g < \eta[\mu - (\gamma - g)]$, the growth rate of output is g and the economy remains in the high-inflation regime. Alternatively, if $g \geq \eta[\mu - (\gamma - g)]$, the growth rate of output is $\eta[\mu - (\gamma - g)]$ and the economy is always in the low-inflation regime from the initial period.

Figure 1: Real GDP per capita and trend



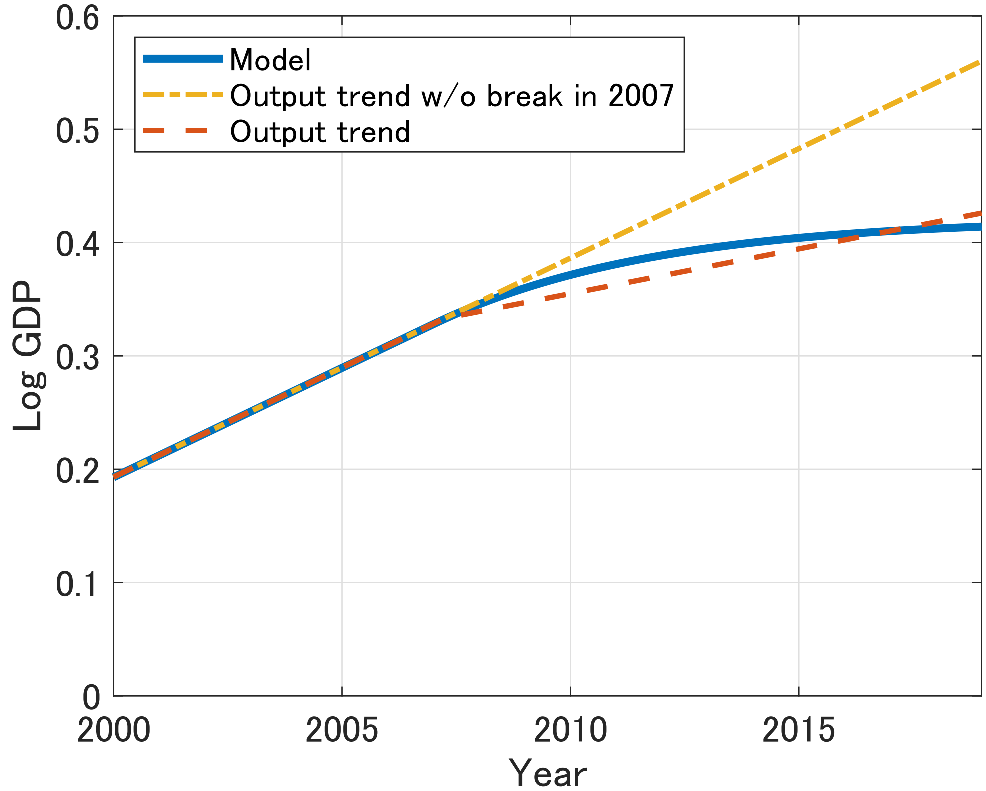
Notes: The figure plots log real GDP per capita from 1990:Q1 to 2019:Q4 and an estimated linear trend with and without a structural break in 2007:Q1. The dotted line represents actual GDP. The dashed (dotted-dashed) line is the linear output trend with (without) a structural break in trend in 2007:Q1. In the figure, actual GDP is normalized to its 1990:Q1 value and the linear trend lines are normalized accordingly.

Figure 2: Velocity of M2 and trend



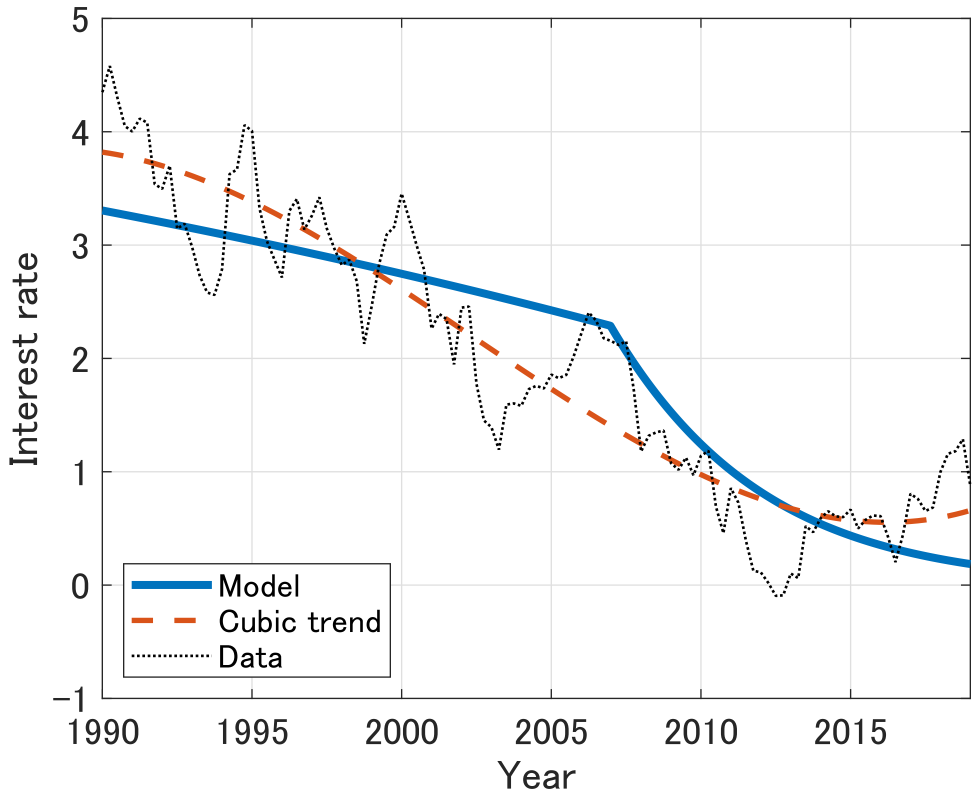
Notes: The figure plots the log velocity of M2 from 1990:Q1 to 2019:Q4. The dotted line represents the actual data for the log velocity of M2, and the dashed line corresponds to the estimated cubic trend. The actual velocity is normalized to its 1990:Q1 value, and the trend line is normalized accordingly.

Figure 3: Simulated output trend



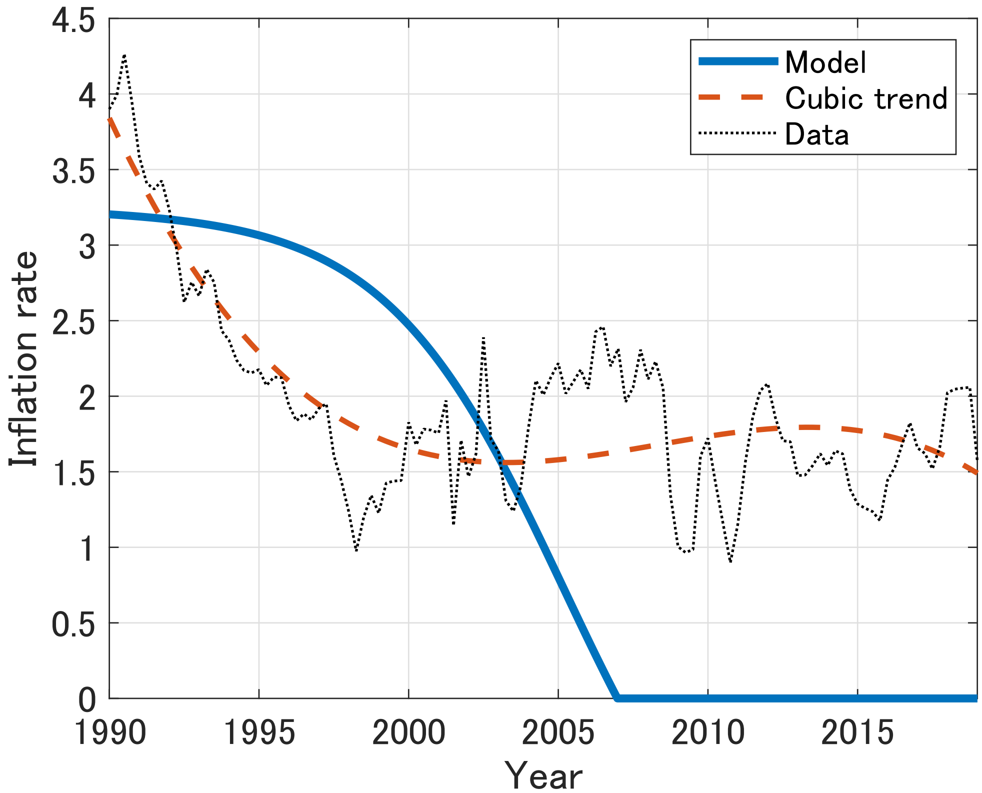
Notes: The figure compares the estimated output trend with the simulated output trend. The solid line represents the simulated output trend, where a regime change occurs in 2007:Q1. The estimation and simulation of trends cover the period from 1990:Q1 to 2019:Q4. For exposition, the figure depicts the trend lines from 2000. See Figure 1 for other details.

Figure 4: Simulated real interest rate



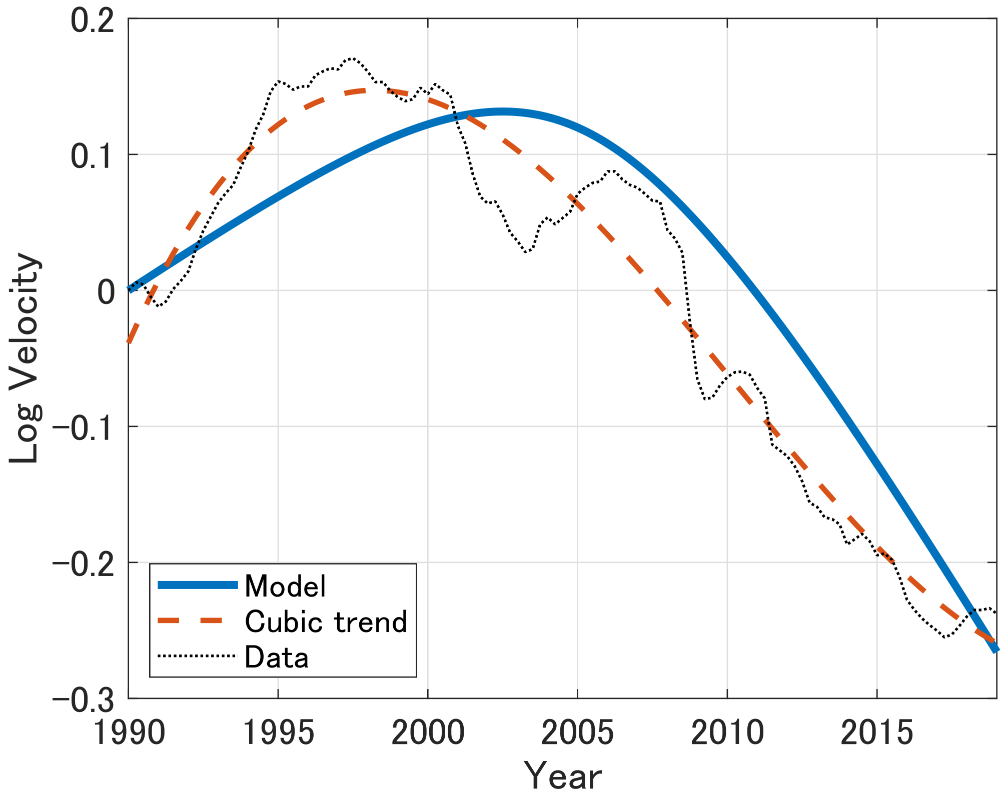
Notes: The solid line represents the simulated real interest rate. The dotted line is the real interest rate estimated using the approach in Haubrich, Pennacchi, and Ritchken (2012). The real interest rate corresponds to 10-year real Treasury yields. The dashed line is the estimated cubic trend, based on data from 1990:Q1 to 2019:Q4.

Figure 5: Simulated inflation



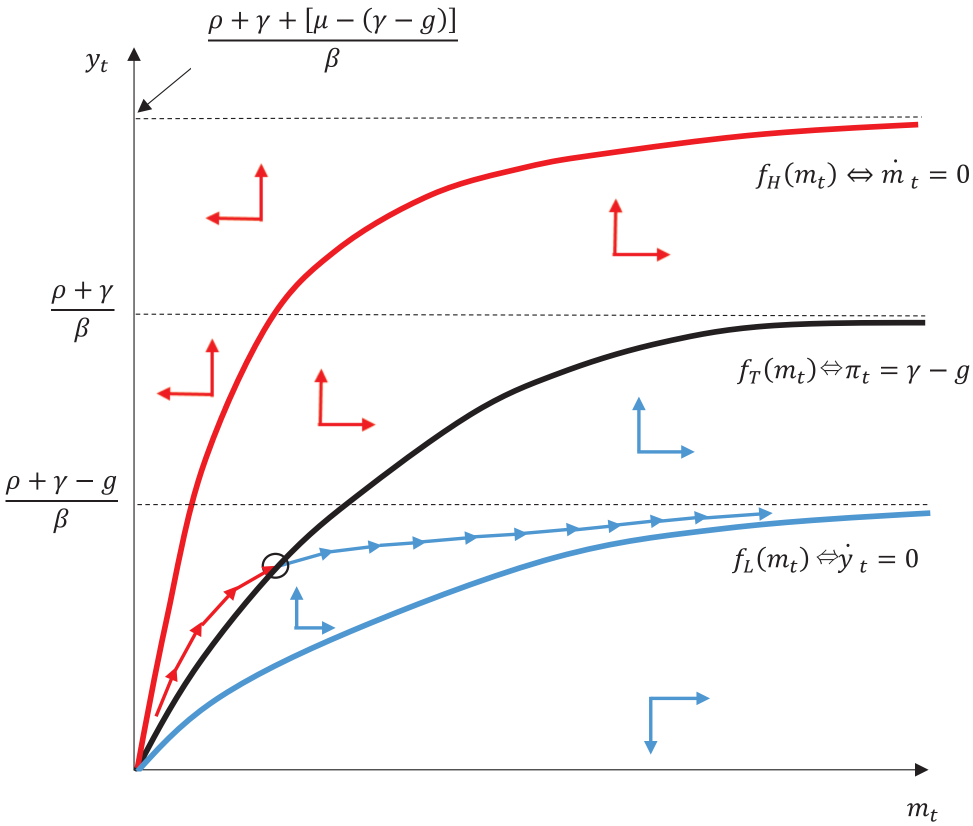
Notes: The solid line represents simulated inflation. The dotted line is the actual (year-on-year) core inflation rate. The dashed line represents the estimated cubic trend, based on data from 1990:Q1 to 2019:Q4.

Figure 6: Simulated trend of velocity



Notes: The solid line represents the simulated trend of velocity. The dotted line is the log of M2 velocity. The data are normalized to the value in 1990:Q1. The dashed line represents the cubic trend of velocity, based on data from 1990:Q1 to 2019:Q4.

Figure 7: Phase diagram



Notes: The red solid line denoted by $y_t = f_H(m_t)$ represents the locus that achieves $\dot{m}_t = 0$ when the economy is in the high-inflation regime. This $\dot{m}_t = 0$ locus determines the direction of change in m_t in the high-inflation regime. In this regime, $\dot{y}_t > 0$ always holds. The blue solid line denoted by $y_t = f_L(m_t)$ is the locus that achieves $\dot{y}_t = 0$ when the economy is in the low-inflation regime. This $\dot{y}_t = 0$ locus determines the direction of change in y_t in the low-inflation regime. In this regime, $\dot{m}_t > 0$ always holds. The black solid line denoted by $y_t = f_T(m_t)$ points to the locus that achieves $\dot{y}_t/y_t = g$ at the lowest level of inflation $\pi_t = \gamma - g$. If (m_t, y_t) is located above (below) the locus, the economy is in the high- (low-)inflation regime.