ISSN (Print) 0473-453X
Discussion Paper No. 1175
ISSN (Online) 2435-0982

AN EXPERIMENT ON THE NASH PROGRAM: COMPARING TWO STRATEGIC MECHANISMS IMPLEMENTING THE SHAPLEY VALUE

Michela Chessa
Nobuyuki Hanaki
Aymeric Lardon
Takashi Yamada

May 2022

The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

An experiment on the Nash program: Comparing two strategic mechanisms implementing the Shapley value*

Michela Chessa† Nobuyuki Hanaki‡ Aymeric Lardon§ Takashi Yamada¶

May 30, 2022

Abstract

We experimentally compare two well-known mechanisms inducing the Shapley value as an *ex ante* equilibrium outcome of a noncooperative bargaining procedure: the demand-based Winter's demand commitment bargaining mechanism and the offer-based Hart and Mas-Colell procedure. Our results suggest that the offer-based Hart and Mas-Colell mechanism better induces players to cooperate and to agree on an efficient outcome, whereas the demand-based Winter mechanism better implements allocations that reflect players' effective power.

JEL code: C70, C71, C92
Keywords: Nash program, Bargaining procedures, Shapley value, Experiments

*This is a revised version, with an additional experiment, of the Chessa, M., N. Hanaki, A. Lardon, and T. Yamada (2021): "An experiment on the Nash program: Comparing two bargaining implementations of the Shapley value," GREDEG Working Paper 2021-07, GREDEG.

†Université Côte d'Azur, CNRS, GREDEG, France. E-mail: michela.chessa@univ-cotedazur.fr

‡Institute of Social and Economic Research, Osaka University, Japan. E-mail: nobuyuki.hanaki@iser.osaka-u.ac.jp

§GATE Lyon Saint-Etienne, UMR 5824 CNRS, Université de Lyon. France. E-mail: aymeric.lardon@univ-st-etienne.fr

¶Faculty of Global and Science Studies, Yamaguchi University, Japan. E-mail: tyamada@yamaguchi-u.ac.jp

1 Introduction

Whenever a facility is shared by different customers, departments, or other units of an organization, the problem of how to allocate the costs or the payoffs among players arises. Relevant examples of this situation include airports, transit systems, water distribution networks, inventory models, and scheduling. These contexts are well known as *cost* or *payoff allocation (or sharing) problems*.[1] Similar sharing problems arise in the context of "co-opetition" (Brandenburger and Nalebuff, 1996) where competitors cooperate to achieve a common goal. Usually, in such contexts, two approaches based on game theoretical concepts may be adopted.

One approach is for the players to bargain among themselves to determine how costs or payoffs should be shared. However, this implies a strategic interaction, which may result in unnecessary additional costs if it is conducted in an unrestricted fashion (see, e.g., the arguments by Roth and Verrecchia, 1979). Instead, many bargaining procedures follow the tradition of setting up sequential, perfect information games based on offers, that is, games in which, at each stage, one of the players becomes a proposer of a cost (payoff) allocation, with a requirement for reaching unanimous agreement. Such bargaining procedures implement negotiations in the style of the well-known two-player bargaining over a pie in Rubinstein's problem (Rubinstein, 1982), which is then extended to the n-player case.

Alternatively, one can view the problem as a normative one, in which an external player, a so-called *regulator*, designs a pricing (rewarding) scheme that maximizes some measures of social welfare or that imposes axioms of equity or stability. Shubik (1962) was among the original proponents of the Shapley value (Shapley, 1953) as a method

[1]Many specific concrete examples of analogous situations come from one of the most typical economic phenomena: consumption and the contribution to public goods.

of joint-cost allocation. At present, this value continues to attract the greatest interest among the allocation schemes predicated on notions of cooperative game theory (see, e.g., Littlechild and Owen, 1973; Schulz and Uhan, 2010; Timmer et al., 2013).

Bridging the gap between the strategic and cooperative approaches is recognized as a fundamental issue of game theory. Attempted resolutions of this issue, well known as the *Nash program* (Nash, 1953), have provided many different strategic bargaining mechanisms that sustain the Shapley value at equilibrium (for example, among others, Hart and Mas-Colell, 1996; Pérez-Castrillo and Wettstein, 2001). Such mechanisms fit and unify the two approaches, allowing the players facing an allocation problem (in our specific context) to bargain in a restricted way, and to converge to a stable solution without the need for an intermediary.

Both the original normative implementation of the Shapley value by a regulator and the implementation of classical bargaining mechanisms based on offers that lead to the Shapley value require complete information, either on the part of the regulator, or the players. In many contexts, complete, transparent, and accountable information is often desirable and encouraged. However, in some specific domains, this may represent an unrealistic assumption, for example, when players are customers of a facility and do not necessarily know about other customers (Young, 1998), or when computing such allocations requires data from each player, some of which may be private (McSherry and Talwar, 2007). We argue that even when such information is publicly available, it is difficult to guess prices (awards) to charge (give) that are likely to be accepted by every single player in the bargaining because this requires managing a wide and complex information set about the structure of the underlying cooperative game and the marginal contributions of each player.

An alternative but less common approach is to describe a bargaining mechanism based on demands rather than offers. A demand-based mechanism was the basis of the implementation by Nash (1953) of the cooperative bargaining solution by Nash (1950). Other examples of mechanisms based on demands, though not common, include Young (1998), who, sharing our concern that complete information may be a difficult assumption in practice, describes a demand revelation mechanism in which potential customers of a public facility simply bid to be served. Bargaining mechanisms based on demands resemble oral auctions, in which each player, standing alone, reveals the charges he or she is ready to pay to be served, or the payoff he or she is ready to accept for offering collaboration, and waits for such a request to be met. By allowing the players to bargain, such mechanisms allow them to focus only on their specific role in the organization and on their expectation of how much they should contribute to or obtain from the facility. This approach drastically reduces the information a player is required to possess or process to make a proposal. Moreover, in a demand-based mechanism, acceptance of a proposal by the organization typically depends on objective feasibility conditions rather than on subjective approval by its members.

In this paper, we aim to contribute to this dispute concerning demand- vs. offer-based bargaining mechanisms. We experimentally compare two well-known mechanisms inducing the Shapley value as an *ex ante* equilibrium outcome of a noncooperative bargaining procedure. We choose two mechanisms that are based on these opposing approaches (demand vs. offer) but that remain, in our opinion, similar in terms of their implementation and the ease with which they can be understood by the participants in a laboratory experiment.[2] The first mechanism is *Winter's demand commitment bargaining mechanism* (Winter, 1994, referred to as the *Winter mechanism* below). The second

[2]A comparison between offer-based and demand-based mechanisms has been conducted experimentally for voting games by Fréchette et al. (2005a), as well as empirically by, for example, Warwick and Druckman (2001) and Ansolabehere et al. (2005), employing field data.

is the *Hart and Mas-Colell procedure* (Hart and Mas-Colell, 1996, referred to as the *H–MC mechanism* below).

Both procedures are described as sequential, perfect information games, where, at each stage, a player becomes a proposer. In accordance with the theoretical presentation of the two mechanisms, we illustrate and implement the bargaining procedures to define a sharing of payoffs rather than an allocation of costs.[3] In the first mechanism, which is defined for cooperative games with increasing returns to scale for cooperation (strictly convex games), the proposer makes a demand for him- or herself concerning the payoff that he or she is willing to receive from a possible collaboration. In the second mechanism, which is defined for monotonic games (a much weaker assumption), the proposer makes a proposal to each of the other players concerning the payoff he or she is willing to offer them.

Two main issues arise with most strategic bargaining models, as observed by Fréchette et al. (2005a) in an experimental work implementing some well-known legislative bargaining processes. First, the theoretical predictions that they propose are very sensitive to variations in the rules of the game, for example, in our case, whether a demand-based or an offer-based mechanism is considered. Even if experiments show that actual bargaining behavior is not always as sensitive to the different bargaining rules as the theory suggests, we expect our analysis to confirm such a statement and the two mechanisms to perform very differently, despite the similar theoretical predictions. Second, the equilibrium solution may require an unrealistic degree of rationality on the part of the players, such that the experimental evidence is very far from the theoretical prediction. We claim that the degree of rationality required in a demand-based mechanism is much lower than

[3] It is straightforward to establish the theoretical implementation of a cost allocation bargaining procedure.

that for an offer-based mechanism. This is because, as argued above, compared with the offer-based mechanism, the demand-based mechanism requires a player to know and process a smaller amount of information to make a proposal, as he or she can focus only on his or her own specific role in the organization and his or her marginal contributions, ignoring the role of all other players. Then, we aim at investigating the consequences of such an issue in the results of our experiment.

It has been argued that the difference between a demand-based vs. an offer-based mechanism is less relevant when considering two-player games, such as in Rubinstein (1982)'s bargaining-over-a-pie game (see, Fréchette et al., 2005a). However, it becomes crucial when considering groups with at least three members. In particular, offer-based mechanisms are comparable with a voting procedure in which all the other players either accept or reject the proposed utility share put forward by the proposer. As such, they are theoretically expected to show a high degree of asymmetry between the proposer and all the other players. In our case, both mechanisms are expected to show some form of proposer advantage. In fact, for both mechanisms the *ex post* predicted solution strongly depends on the selected proposer. In the case of the Winter mechanism in particular, it even depends on the complete ordering.

Our analysis mainly focuses on (i) analyzing whether these mechanisms lead to formation of the grand coalition and (ii) testing the convergence in expected value and, as predicted by the theory, to the Shapley value.

Our results show that the H–MC mechanism results in a higher frequency of grand coalition formation and a higher efficiency than does the Winter mechanism. Conversely, the Winter mechanism better implements the Shapley value as the average payoff provided that the grand coalition is formed. Therefore, our results suggest that an offer-based H–MC mechanism better induces players to cooperate and to agree on an efficient outcome, whereas a demand-based Winter mechanism better implements allocations that reflect players' effective power.

The remainder of the paper is organized as follows. Section 2 reviews existing studies that are most relevant to our work. Section 3 presents the general definition and the properties of a cooperative transferable utility (TU) game, as well as the Shapley value. Section 4 presents the two mechanisms that we investigate, namely the Winter and the H–MC mechanisms. Section 5 describes the setting of our experiment. The results are presented in Section 6, and Section 7 concludes. Additional analyses aimed at reinforcing our results and at providing new points for reflection are contained in the Appendix II to V.

2 Related work

Bridging the gap between the noncooperative models, in which the primitives are the sets of possible actions of individual players, and the cooperative models, in which they are the sets of possible joint actions of groups of players, has been recognized as a fundamental issue of game theory. The very first attempt at this so-called Nash program dates back almost 70 years to Nash himself (Nash, 1953). His idea was to provide a noncooperative foundation for cooperative solution concepts, and he began implementing it by designing a noncooperative game that sustained the Nash solution of his two-player *bargaining problem* (Nash, 1950) as its equilibrium. Following this first attempt by Nash, many alternative procedures for implementing solutions of two-player bargaining problems or n-player pure bargaining problems[4] have been implemented. Some mechanisms intended to obtain the Nash solution, exactly or approximately, at equilibrium

[4] A pure bargaining problem is a cooperative game in which only the grand coalition N creates a positive surplus with respect to what each player can achieve if he or she does not cooperate with anyone.

(see, among others, Binmore et al., 1986; Trockel, 2002). Others aimed instead to obtain the Kalai–Smorodinsky solution (Kalai and Smorodinsky, 1975), that is, the main alternative solution to such problems (Moulin, 1984b; Trockel, 1999; Haake, 2000).

Many different theoretical mechanisms have been designed with the aim of implementing other cooperative solution concepts via a strategic interaction of the players for more generic cases, that is, when there are more than two players or when the bargaining problem is not pure. This is the case, for example, in the seminal work of Harsanyi (1974), who reinterpreted the von Neumann–Morgenstern solution as an equilibrium of a noncooperative bargaining mechanism, and of the many works sustaining the most famous axiomatic solution concept by Shapley (1953), the Shapley value. For a relevant and extensive review of the theoretical literature on the Nash program, we refer readers to the surveys by Serrano (2005, 2008, 2014, 2021).

In this section, we focus on the literature devoted to testing cooperative game theory through experiments. To date, this literature has focused mainly on three different directions. The first direction provides a normative interpretation, as in De Clippel and Rozen (2021), in which subjects designated as decision-makers express their view on what is fair for others by recommending a payoff allocation. De Clippel and Rozen (2021) show that the decision-maker's choices can be described as a convex combination of the Shapley value and the equal division solution.

The second direction investigates how an unstructured interaction affects the final agreement. One example is the paper by Kalisch et al. (1954), in which groups of players are asked to freely discuss the formation of coalitions and to reach an agreement on how to split the related values. The authors identify many different factors influencing the final outcome of such a procedure, including personality differences or the geometrical arrangement of players around the table. Similarly, but with a greater focus on voting games, Montero et al. (2008) propose an unstructured bargaining protocol in which participants propose and vote on how to distribute a fixed budget among themselves. The paper provides experimental evidence of the so-called *paradox of new members*, according to which enlargement of a voting body (i.e., the addition of a new voter) can increase the voting power of an existing member. Guerci et al. (2014) study the impact of variations in the experimental protocol of Montero et al. (2008) on the formation of the so-called minimal winning coalitions, that is, coalitions for which each player is crucial.

Most experimental works in the literature follow a third direction, studying the outcome when a more formal (or structured) bargaining protocol is imposed. Our paper broadens this last direction of research.

Formal bargaining protocols have been implemented to tackle different aspects of the cooperative inclination of the players under different settings. For example, Murnighan and Roth (1977) investigate the effect of various communication/information conditions on the final outcome in a specific game played by a monopolist and two weaker players. They show how the results over the entire set of conditions closely approximate the Shapley value, although they often report a clear tendency for an equal split of the pie. Similarly, Murnighan and Roth (1982) introduce bargaining models to investigate the influence of information shared by subjects about the games (e.g., payoffs) on the final outcome. They show that the quality of the information has an impact on the final outcome and that the Nash bargaining solution has a good predictive performance in many cases. Bolton et al. (2003) investigate how the communication configuration affects coalition negotiation and show how players with weaker alternatives would benefit from a more constrained structure, especially if they can be the conduit of communication, whereas those endowed with stronger alternatives benefit from working within a more public communication structure that promotes competitive bidding. Other works focus more specifically on the coalition formation process, including Nash et al. (2012); Shinoda and Funaki (2019); Abe et al. (2021). In the first paper, the authors implement finitely repeated three-person coalition formation games, showing how efficiency requires people's willingness to accept the agency of others, such as political leaders. The second paper is then presented as a follow-up, in which the authors maintain the same value of the coalitions as in Nash et al. (2012), but implement a different bargaining protocol. They report a rare formation of a grand coalition, which can be induced by some external factors, such as the presence of a chat window. The third paper presents a comparison between two mechanisms that invite players to join a meeting simultaneously or sequentially. The authors report that the sequential mechanism induces a higher social surplus than the simultaneous mechanism. Moreover, players make choices consistent with the subgame perfect Nash equilibrium (SPNE) in the sequential setting and choose the dominant strategy in the simultaneous setting, when a dominant strategy exists.

Formal bargaining protocols are mostly based on the implementation of theoretical mechanisms, which are shown to converge to some specific well-known solutions. This is the case, for example, in Nash (1953) and Harsanyi (1974), which we have referred to above, or in the case of the bargaining mechanism proposed by Raiffa (1953) to implement the Raiffa solution (as opposed to the Nash solution) to the Nash cooperative bargaining problem. Several experimental implementations have been proposed, with the final goal of testing Nash axioms, or of comparing Nash and Raiffa solutions (see, e.g., Nydegger and Owen, 1975; Rapoport et al., 1977). In addition, there is a large literature devoted to studying the class of bidding mechanisms. Bidding mechanisms are introduced by Demange (1984) and Moulin (1984a), and Moulin and Jackson (1992) study them in economic environments. They are developed by Pérez-Castrillo and Wettstein (2001) and Ju and Wettstein (2009) to implement solution concepts in the framework of cooperative TU games.

In particular, many different theoretical mechanisms have been designed specifically with the aim of implementing the best-known cooperative solution, the Shapley value (see Shapley, 1953). Because this solution is applied in many economic problems, supporting it through strategic explanation is considered to be particularly important. See among others, Harsanyi (1981), Gul (1989), Hart and Moore (1990), Winter (1994), and Hart and Mas-Colell (1996).[5]

Despite the large body of existing literature, the Nash program *"is not ready for retirement yet"*, but is, on the contrary, *"still full of energy"* and *"waiting for good papers to be written"* (Serrano, 2021). In this paper, we aim to contribute to this research agenda by providing new insights gained from a controlled laboratory experiment. In particular, we propose an experimental comparison of two mechanisms. The first mechanism is the one-period version developed by Winter (1994) (this simplified version was also previously used by Bennett and van Damme (1991) to treat Apex games, a type of weighted majority games). The second mechanism is by Hart and Mas-Colell (1996), in the specific case in which a proposer whose proposal is rejected leaves the game with a probability 1. Our work is similar to Fréchette et al. (2005a), who experimentally compare an offer-based model of Baron and Ferejohn (1989) with a demand-based model of Morelli (1999) in weighted majority voting games. Earlier experimental studies of the Baron–Ferejohn model include Fréchette et al. (2003, 2005b), and Fréchette et al. (2005a) provide an experimental study of demand bargaining.[6] However, Fréchette

[5]Krishna and Serrano (1995) deepen the study of the set of subgame perfect equilibria associated with the bargaining mechanism proposed by Hart and Mas-Colell (1996).

[6]Fiorina and Plott (1978) propose multiple experiments on committee decision-making under majority rules to test a wide range of solution concepts of noncooperative games.

et al. (2005a) present the first work to directly compare the two within an experimental framework. Their results show that proposers have some first-mover advantage in both the demand and offer games, but their power does not differ nearly as much between the two mechanisms as theory predicts.

3 Theoretical model

3.1 Cooperative TU games and solutions

Let $N = \{1, \dots, n\}$ be a finite set of *players*. Each subset $S \subseteq N$ is called a *coalition*, and N is called the *grand coalition*. A *cooperative TU game* (from now on, *cooperative game*) consists of a couple (N, v), where N is the set of players and $v : 2^N \to \mathbb{R}$ is the *characteristic function*, which assigns to each coalition $S \subseteq N$ the *worth* $v(S)$, with the normalization condition $v(\emptyset) = 0$. The worth of a coalition represents the value that members of S can achieve by agreeing to cooperate. To simplify the notation if no ambiguity appears, we consider the set of players N as fixed and we write v instead of (N, v). We use $\mathcal{G}^N$ to denote the set of all games with player set N.

A game $v \in \mathcal{G}^N$ is said to be

- *monotonic* if $v(S) \leq v(T)$ for each $S \subseteq T \subseteq N$,
- *superadditive* if $v(S) + v(T) \leq v(S \cup T)$ whenever $S \cap T = \emptyset$, with $S, T \subseteq N$,
- *convex* if $v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$, for each $S, T \subseteq N$, and *strictly convex* if the inequality holds strictly.

We observe that convexity $\Rightarrow$ superadditivity $\Rightarrow$ monotonicity. In (strictly) convex games, cooperation becomes increasingly appealing, and a so-called "snowball effect" is expected, leading to the formation of the grand coalition. Another equivalent definition for convexity can be stated as $v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T)$, for each $S \subseteq T \subseteq N \setminus \{i\}$.

Given a game $v \in \mathcal{G}^N$, an *allocation* is an n-dimensional vector $(x_1, \dots, x_n) \in \mathbb{R}^N$, assigning to player i the amount $x_i \in \mathbb{R}$. For each $S \subseteq N$, we assume that $x(S) = \sum_{i \in S} x_i$. The *imputation set* is defined by:

$$I(v) = \{x \in \mathbb{R}^n | x(N) = v(N) \text{ and } x_i \geq v(\{i\}) \ \forall i \in N\},$$

that is, it contains all the allocations that are *efficient* ($x(N) = v(N)$) and *individually rational* ($x_i \geq v(\{i\}) \forall i \in N$).

The core is the set of imputations that are also *coalitionally rational*, that is,

$$C(v) = \{x \in I(v) | x(S) \geq v(S) \ \forall S \subseteq N\}.$$

An element of the core is stable in the sense that if such a vector is proposed as an allocation for the grand coalition, no coalition will have an incentive to split off and cooperate on its own. Intuitively, the idea behind the core is analogous to that behind a (strong) Nash equilibrium of a noncooperative game, namely an outcome is stable if no deviation is profitable. For the Nash equilibrium, the possible deviation concerns a single player, whereas in the core, deviations of groups of players are relevant.

A *solution* is a function $\psi : \mathcal{G}^N \to \mathbb{R}^N$ that assigns an allocation $\psi(v)$ to every game $v \in \mathcal{G}^N$. The *Shapley value* is the best-known solution concept, which is widely applied in economic models, and is defined as:

$$\phi_i(v) = \sum_{S \subseteq N, i \in S} \frac{(|S|-1)!(|N|-|S|)!}{|N|!}(v(S) - v(S \setminus \{i\})) \; \forall i \in N.$$

The Shapley value assigns to every player his or her expected marginal contribution to the coalition of players that enter the game before this player, given that every order of entrance has equal probability. This solution concept has been defined as respecting some notion of fairness (see Appendix III for more discussion about its properties), but it is not necessarily stable. However, if the game is superadditive, the Shapley value is an imputation, and if the game is convex, it belongs to the core (in particular, it is its barycenter).

4 Two mechanisms

In this section, we present the demand-based Winter mechanism (Section 4.1) and the offer-based H–MC mechanism (Section 4.2) in more detail. Section 4.3 compares the equilibrium predictions of the two mechanisms with a simple example.

4.1 The Winter mechanism

Winter (1994) presented a bargaining model based on sequential demands for strictly convex cooperative games. As noted, in such games, cooperation becomes increasingly appealing and a "snowball effect" is expected, leading to the formation of the grand coalition. Moreover, in convex games, the Shapley value is a central point in the core, which is always nonempty.

In this model, players announce their demands publicly in turns. That is, the players effectively state "I am willing to join any coalition that yields me..." and wait for these demands to be met by other players. The bargaining starts with a randomly chosen player from N, say player i. This player publicly announces his or her demand d_i and then points to a second player, who has to state his or her demand. Then, the game proceeds by having each player introduce a demand then point at a new player to take a turn. If or when, at some point, a compatible demand is introduced, which means that there exists a coalition S for which the total demand for players in S does not exceed $v(S)$, then the first player with such a demand selects a compatible coalition S. The players in S receive their demands and leave the game, and the bargaining continues with the rest of the players using the same rule on v restricted on $N \setminus S$.

Here, we present the one-period Winter mechanism more formally. This is a simplified version of the more general mechanism in Winter (1994), which allows for more periods and includes a discount factor. A decision point position at time t of the one-period demand commitment game is given by the vector $(S_1^t, S_2^t, d_{S_2^t}, j)$, where:

$S_1^t \subseteq N$ is the set of players remaining in the game,

$S_2^t \subset S_1^t$ is the set of players who have submitted demands that are not yet met,

$d_{S_2^t} = (d_i)_{i \in S_2^t}$ is the vector of demands submitted by players in S_2^t, ($0 \leq d_i \leq \max_{S \subseteq N} v(S)$), and

$j \in S_1^t \setminus S_2^t$ is the player taking the decision by introducing a demand d_j. His or her demand d_j is said to be *compatible* if there exists some $S \subseteq S_2^t$ with $v(S \cup \{j\}) - \sum_{i \in S} d_i \geq d_j$. Otherwise, d_j is not compatible.

With j's decision, the game proceeds in the following way:

1) If d_j is compatible, then j specifies a compatible coalition S, each player $i \in S \cup \{j\}$ is paid d_i, and a player $k \neq j$ is randomly chosen from $S_1^t \setminus S_2^t$. The new position is now given by $(S_1^{t+1}, S_2^{t+1}, d_{S_2^{t+1}}, k)$, with $S_1^{t+1} = S_1^t \setminus (S \cup \{j\})$ and $S_2^{t+1} = S_2^t \setminus (S \cup \{j\})$.

2) If d_j is noncompatible, then two cases are distinguished:

2_a) if $S_2^t = S_1^t \setminus \{j\}$ (j is the last player to make a demand), then each player $i \in S_1^t$ (j included) gets his or her individual payoff $v(\{i\})$, and the game ends;

2_b) if $S_2^t \subset S_1^t \setminus \{j\}$, then j specifies a new player $k \neq j$ in $S_1^t \setminus S_2^t$ and the new position is $(S_1^{t+1}, S_2^{t+1}, d_{S_2^{t+1}}, k)$, with $S_1^{t+1} = S_1^t$ and $S_2^{t+1} = S_2^t \cup \{j\}$.

The game starts with a randomly chosen player $j \in N$. Then, the initial position is set to be $(N, \emptyset, d_\emptyset, j)$. It terminates either when there are no more players in the game (see point 1 above), or when $S_1^t \cup \{j\} = S_2^t$ (see point 2_a above).

As shown by Winter for the more generic case, this mechanism has a unique subgame perfect equilibrium, which assigns equal probabilities according to the principle of indifference. At this equilibrium, the grand coalition forms and the *a priori* expected equilibrium payoff coincides with the Shapley value. Moreover, given a specific ordering of the players, the *a posteriori* equilibrium payoff of each player depends on the order of players only through the set of the player's successors but it is not influenced by the way that these players are ordered, as each player demands a marginal contribution to the set of successors.

4.2 The Hart and Mas-Colell mechanism

Hart and Mas-Colell (1996) proposed a bargaining procedure for monotonic cooperative games. This is a much weaker assumption compared with the strict convexity required by the Winter mechanism. Thus, the H–MC procedure is applicable for a larger set of cooperative games.

In this mechanism, the bargaining starts with a randomly chosen proposer making an offer to the other players, with the meaning “If you wish to form a coalition with me, I will give you...”. Then the other players, who act sequentially, may either accept or reject the proposal. The requirement for agreement is unanimity. The key modeling issue is the specification of what happens if there is no agreement and, as a consequence, the game moves to the next stage. In our implementation, if the proposal is rejected, the proposer leaves the game with his or her individual value and the bargaining continues among the rest of the players, with a new player randomly chosen as a new proposer.

We present a more formal description of the H–MC mechanism. A decision point position at time t is simply given by the vector (S^t, j), where:

$S^t \subseteq N$ is the set of players remaining in the game,

$j \in S^t$ is the player making an offer to the remaining players $(t_i)_{i \in S^t \setminus \{j\}}$ such that $\sum_{i \in S^t \setminus \{j\}} t_i \leq v(S^t)$.

With j's proposal, the game proceeds now in the following way:

1) If all $i \in S^t \setminus \{j\}$, who decide sequentially, accept the proposal one after the other, then players in $S^t \setminus \{j\}$ are paid $(t_i)_{i \in S^t \setminus \{j\}}$, player j is paid $v(S^t) - \sum_{i \in S^t \setminus \{j\}} t_i$, and the game ends;

2) If at least one player $i \in S^t \setminus \{j\}$ refuses the offer, then two cases are distinguished:

$2a$) if $|S^t| = 2$ (only one more player is left, together with j), then they both receive their individual value $v(\{i\})$ for each $i \in S^t$, and the game ends;

$2b$) if $|S^t| > 2$, then player i is removed from the game, he or she receives his or her individual payoff $v(\{i\})$, a new proposer $k \in S^{t+1} = S^t \setminus \{j\}$ is randomly selected, and the new position is (S^{t+1}, k).

The game starts with a randomly chosen player $j \in N$. Then, the initial position is set to be (N, j). It terminates either when there are no more players in the game (see point $2a$ above), or when the proposal is unanimously accepted (see point 1 above).

Hart and Mas-Colell (1996) show that this game has a unique subgame perfect equilibrium. At this equilibrium, the grand coalition forms and the *a priori* expected equilibrium payoff coincides with the Shapley value. In contrast to the Winter mechanism, given a specific initial proposer $j \in N$ (in the previous mechanism, it was necessary to specify the order of all the players, whereas in this case only one player, the proposer, needs to be specified at equilibrium), the *a posteriori* equilibrium payoff assigns to each other player his or her Shapley value in the cooperative game, reduced to the set of players $N \setminus \{j\}$, and the proposer is assigned his or her marginal contribution to the grand coalition $v(N) - v(N \setminus \{j\})$.

4.3 A comparison between the Winter and the H–MC mechanisms

We illustrate the two mechanisms using the strictly convex three-player game shown in Table 1. Although our experiment is based on four-player games, a three-player game example is of particular interest because it allows us to graphically represent the imputation set, the core, and the different solutions, as illustrated in Figure 1.

Table 1: A three-player game

S	1	2	3	1,2	1,3	2,3	N
$v(S)$	20	20	30	45	55	60	100

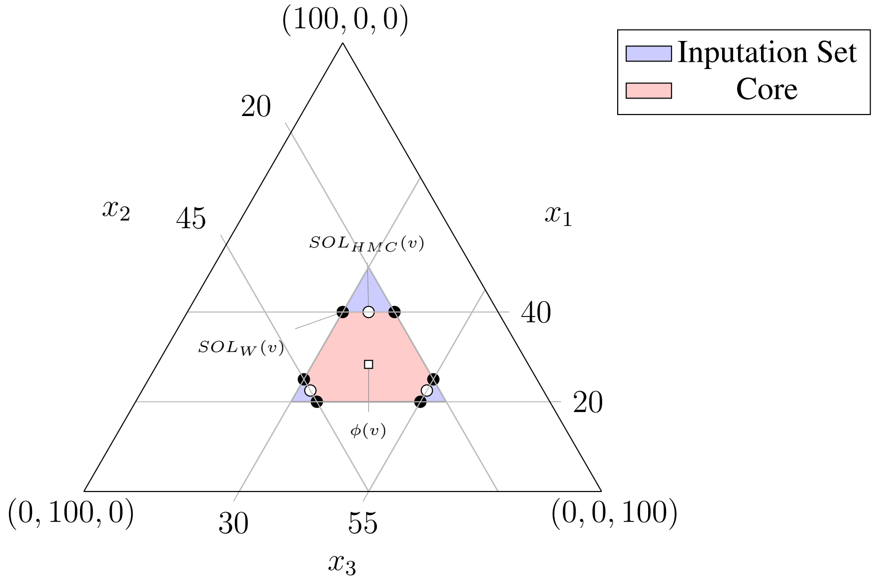
As we have already observed, the convexity assumption implies the monotonicity. Thus, the game satisfies the assumptions of both the Winter and H–MC mechanisms. The Shapley value of this game is given by the vector $\phi(v) = \left(\frac{170}{6}, \frac{185}{6}, \frac{245}{6}\right) = (28.33, 30.83, 40.83)$, which corresponds to the *a priori* equilibrium payoff for both the Winter and H–MC mechanisms.

We suppose now that player 1 is chosen randomly as the first proposer in both mechanisms. Independently of the order of the following players in the Winter mechanism, the proposer will receive an *a posteriori* equilibrium payoff equal to 40 in both mechanisms, which corresponds to his or her marginal contribution to the grand coalition $v(N) - v(N \setminus \{1\})$. We can see that both mechanisms lead to a proposer advantage, as $40 > \frac{170}{6}$, meaning that, as the first proposer, player 1 can obtain more than his or her Shapley value.

Suppose now that the total ordering of the players in the Winter mechanism is given by 1, 2, and 3. The *a posteriori* equilibrium payoff of the Winter mechanism is given by the vector $SOL_W(v) = (40, 30, 30)$, in which player 2 demands his or her marginal contribution $v(\{2, 3\}) - v(\{3\})$, and player 3 demands his or her individual value $v(\{3\})$.

Conversely, in the case of the H–MC mechanism, the proposer offers the Shapley value of the reduced game to players 2 and 3. Thus, the *a posteriori* equilibrium payoff is given by the vector $SOL_{HMC}(v) = (40, 25, 35)$. Even with the disadvantage of not being the first mover, player 2, as the second mover, manages to obtain more under the

Figure 1: The core of the three-player game



Winter mechanism than under the H–MC mechanism even if, in both cases, he or she obtains less than his or her Shapley value.

Figure 1 shows the imputation set $I(v) = co\,\langle(20, 50, 30), (50, 20, 30), (20, 20, 60)\rangle$, the core $C(v) = co\,\langle(40, 30, 30), (40, 20, 40), (25, 20, 55), (20, 25, 55), (20, 45, 35), (25, 45, 30)\rangle$, the Shapley value $\phi(v)$, and possible *a posteriori* solutions $SOL_W(v)$ (6 black dots) and $SOL_{HMC}(v)$ (3 white dots). A point in the simplex corresponds to an allocation (x_1, x_2, x_3). For example, the height of a point from the edge that is opposite to the apex labeled $(100, 0, 0)$ represents the payoff allocated to player 1. Thus, a point on the bottom edge represents an observed allocation that gives a zero payoff to player 1. Similarly, the height of a point from the edge that is opposite to the apex labeled $(0, 0, 100)$ represents the payoff allocated to player 3.

We make the following observation to conclude this example and the comparison between the two mechanisms.

Table 2: The games

S	1	2	3	4	1,2	1,3	1,4	2,3	2,4	3,4	1,2,3	1,2,4	1,3,4	2,3,4	N
$v_1(S)$	0	5	5	10	20	20	25	20	25	25	50	60	60	60	100
$v_2(S)$	0	20	20	30	20	20	30	45	55	60	45	55	60	100	100
$v_3(S)$	$= v_1(S) + v_2(S)$														
$v_4(S)$	$= 2v_1(S)$														

Observation 1. *The core is always a convex polyhedron. The a posteriori equilibrium of the Winter mechanism always coincides with a vertex of this polyhedron. The a posteriori equilibrium of the H–MC mechanism always provides a vector on a face of this polyhedron.*

5 The experimental setting

5.1 The games

For our analysis, we implement the four four-player games shown in Table 2. These games are chosen to test the properties of the Shapley value that are presented in Appendix III. Note that:

- games 1, 3, and 4 are strictly convex, whereas game 2 is only convex. All four games are, by consequence, monotonic. Therefore, all four games respect the assumptions for the implementation of the H–MC mechanism, whereas all except game 2 respect the assumption for the implementation of the Winter mechanism. However, with game 2 being only convex, we consider that "strict convexity" could be relaxed and the mechanism could still be implemented in such a case;
- in games 1 and 4, players 2 and 3 are symmetric;

Table 3: The Shapley values of games 1, 2, 3, and 4

	$\phi_1(v)$	$\phi_2(v)$	$\phi_3(v)$	$\phi_4(v)$
Game 1	22.08	23.75	23.75	30,42
Game 2	0	28.33	30.83	40.83
Game 3	22.08	52.08	54.58	71.25
Game 4	44.16	47.5	47.5	60.83

- in game 2, player 1 is a null player. This is the reason why the game is only convex, but not strictly convex, as the presence of a null player does not allow, by definition, the possibility of having a strictly increasing marginal contribution for such a player;
- game 3 is defined as the sum of games 1 and 2;
- game 4 is defined as twice game 1 and it preserves the symmetry of players 2 and 3;
- the marginal contributions of player 1 are always higher in game 1 than in game 2, and higher in game 4 than in game 3.

The Shapley values of the four games are presented in Table 3. The equal division payoff vector is simply equal to $ED(v_k) = (25, 25, 25, 25)$ when $k = 1, 2$, and to $ED(v_k) = (50, 50, 50, 50)$ when $k = 3, 4$.

6 Results

The experiment was conducted at the Institute of Social and Economic Research (ISER), Osaka University, in January and February 2019 (Winter mechanism) and January and

February 2022 (H–MC mechanism).[7] A total of 176 students, who had never participated in similar experiments before, were recruited as subjects of the experiment, 96 playing the Winter mechanism and 80 playing the H–MC mechanism.[8] The experiment was computerized with z-Tree (Fischbacher, 2007) and participants were recruited using ORSEE (Greiner, 2015).

To control for potential ordering effects, each participant played all four games twice in one of the following four orderings: 1234, 2143, 3412, and 4321.[9] Between each play of a game (called a round), players were randomly rematched into groups of four players, and participants were randomly assigned a new role within the newly created group.[10] At the end of the experiment, two rounds (one from the first four rounds and another from the last four rounds) were randomly selected for payments. Participants received cash rewards based on the points that they earned in these two selected rounds, with an exchange rate of 20 JPY = 1 point, as well as a 1,500 JPY participation fee. On average, the experiments lasted for 1 hour 40 minutes for Winter and 1 hour 45 minutes for H–MC, including the instructions (15 minutes for Winter and 11 minutes for H–MC), a comprehension quiz (5 minutes), and payment.[11] The average earnings

[7]The experiments were conducted in 2019 and 2022 because the original H–MC experiment conducted in December 2019 (which we refer to as the pseudo-H–MC or H–MC$_{sim}$ in Appendix V) did not reflect the H–MC model precisely, and we have redone the H–MC experiment to correct this. Appendix V compares the outcomes of the pseudo-H–MC conducted in December 2019 and the (corrected) H–MC conducted in January–February 2022.

[8]The difference in the number of participants between the two mechanisms is a result of variations in the show-up rate among experimental sessions.

[9]We let participants play all four games, instead of just one, in each session. Although this design choice may have meant participants were slower in learning how to play the game, we consider that having within-session variations is desirable because the tests of the axioms involve comparing outcomes across different games.

[10]We implemented random reassignment of the roles across rounds instead of fixing the role. Again, this may make learning the game slower for players given that their roles change, as Guerci et al. (2014) suggest. However, given the existence of the null player in one of the four games considered, we chose reassignment of the role to avoid participants feeling the experiment was unfair.

[11]Participants received a copy of instruction slides, and a prerecorded instruction video was played. The quiz was given on the screen after the explanation of the game. The user interface was explained

were 2,650 JPY for Winter and 2,850 JPY for H–MC.

We first compare the Winter and H–MC mechanisms in terms of the frequency of grand coalition formation and efficiency. Then, we analyze whether the resulting allocations from the two mechanisms match the Shapley values. We contrast the experimental results with the allocation predicted under the SPNE as well as under an equal division. Additional analyses of our experimental results are presented in the Appendix II to IV.

6.1 Grand coalition formation and efficiency

Figure 2 presents the results concerning the grand coalition formation under the H–MC and Winter mechanisms for the four games.[12]

We observe that for game 2 and the Winter mechanism, the grand coalition never forms (because player 1 is a null player and, consequently, the game is only convex and not strictly convex[13]). Therefore, for game 2, we consider the partition $\{\{1\},\{2,3,4\}\}$ as a realization of the grand coalition for both the H–MC and Winter mechanisms.

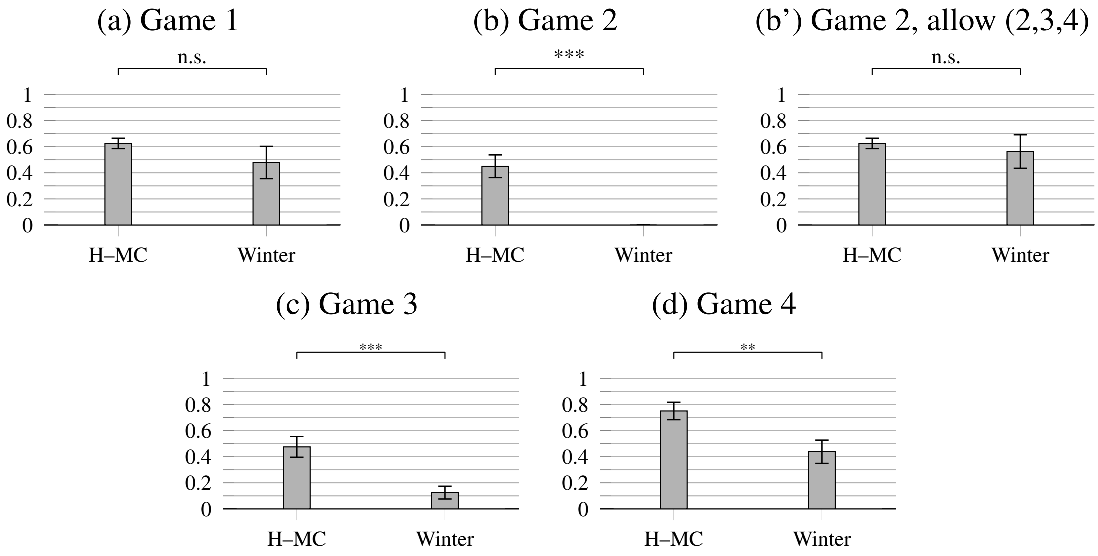
Considering the four games together, the grand coalition (in the case of game 2, either the grand coalition or the $\{2,3,4\}$ coalition) is formed in 61.9% of the cases under the H–MC mechanism, but only in 40.1% of the cases under the Winter mechanism. In particular, we observe that the grand coalition is formed more frequently under the H–MC mechanism than under the Winter mechanism in games 3 and 4 at the 1% and 5%

during the practice rounds, referring to the handout about the computer screen. See Appendix VI for English translations of the instruction materials and the comprehension quiz.

[12]The figure is created based on the estimated coefficients of the following linear regressions: $gc_i = \beta_1 HMC_i + \beta_2 Winter_i + \mu_i$ where gc_i is a dummy variable that takes a value of 1 if the grand coalition is formed, and zero otherwise, in group i, HMC_i ($Winter_i$) is a dummy variable that takes a value of 1 if the H–MC (Winter) mechanism is used, and zero otherwise. The standard errors are corrected for within-session clustering effects. The statistical tests are based on the Wald test for the equality of the estimated coefficients of two treatment dummies.

[13]Recall that the Winter mechanism is theoretically defined for strictly convex games. In this game, Player 1 always has a zero marginal contribution and, as such, can be left out of any coalition at no cost for either him/her or the other players.

Figure 2: Proportion of times the grand coalition is formed



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate the proportion of times that the formation of the grand coalition is significantly higher for the H–MC mechanism compared with the Winter mechanism at the 1%, 5%, and 10% significance levels (Wald test), respectively.

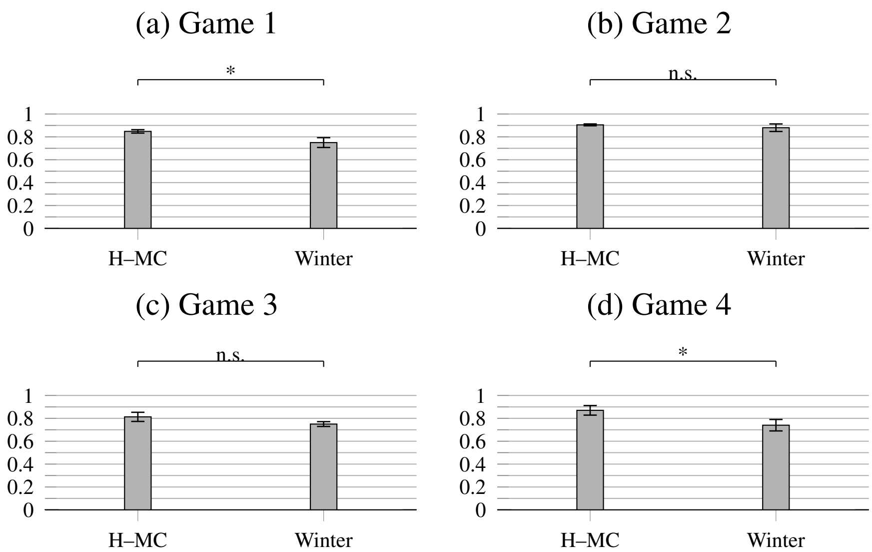
significance levels, respectively.

As a direct consequence of the grand coalition being formed in less than 100% of the cases, both mechanisms fail to achieve full efficiency. Efficiency is computed as the sum of the payoffs obtained by the four players as a proportion of the value of the grand coalition of the considered game (100 for games 1 and 2, and 200 for games 3 and 4). As Figure 3 shows, efficiency is significantly higher under the H–MC mechanism than under the Winter mechanism in games 1 and 4 (both at the 10% level).[14]

Therefore, we conclude as follows.

[14] The figure is created based on the estimated coefficients of the following linear regressions: $Eff_i = \beta_1 HMC_i + \beta_2 Winter_i + \mu_i$, where $Eff_i \equiv \frac{\sum_i \pi_i}{v(N)}$ is the efficiency measure for group i, HMC_i ($Winter_i$) is a dummy variable that takes a value of 1 if the H–MC (Winter) mechanism is used, and zero otherwise. The standard errors are corrected for within-session clustering effects. The statistical tests are based on the Wald test for the equality of the estimated coefficients of the two treatment dummies.

Figure 3: Efficiency



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate that the efficiency for the H–MC mechanism is significantly higher than for the Winter mechanism, at the 1%, 5%, and 10% significance levels (Wald test), respectively.

Result 1. *Although the grand coalition is not always formed under the two mechanisms, it is more frequently formed under the H–MC mechanism than under the Winter mechanism. Consequently, efficiency is higher under the H–MC mechanism than under the Winter mechanism.*

Note that under the H–MC mechanism, the proposer is forced to offer feasible demands, that is, if S is the set of players remaining in the game, the proposer has to propose a total distribution of payoffs no larger than $v(S)$. Conversely, under the Winter mechanism, the players, speaking one after the other, may make unfeasible demands. As a result, the formation of a coalition under the H–MC mechanism is simply determined by whether the players choose to accept the proposal or reject it, whereas under the Winter mechanism, the formation of the coalition can be blocked by unfeasibility conditions. Such a difference between the two mechanisms can cause the significantly higher frequency of the grand coalition formation under the H–MC mechanism compared with the Winter mechanism.[15]

6.2 Allocations

We use $\pi^{HMC}(v_k)$ to denote a vector of payoffs obtained by the players in the H–MC mechanism in game k, with $k = 1, 2, 3, 4$. Analogously, let $\pi^{W}(v_k)$ denote a vector of payoffs obtained by the players under the Winter mechanism. The *ex ante* theoretical prediction for both mechanisms states that the mean of such vectors (based on many realizations with different orderings of the players) should converge to the Shapley value.

When players fail to form the grand coalition, the total payoff obtained by the players is smaller than the value under the grand coalition. As a result, the average realized payoff vectors are significantly different from the Shapley value, as shown in Figure I.1 of Appendix I. Therefore, we focus our analyses on those groups that formed the grand coalition.[16]

Our main analyses are based on a set of ordinary least squares (OLS) regressions (using only the data from groups that formed the grand coalition) for the following

[15] In Appendix II, we report the frequency of the grand coalition formation and efficiency by separating the data for the first half (rounds 1–4) and the second half (rounds 5–8) of the experiment. We observe an increase in both the frequency of the grand coalition formation and efficiency, at least in some of the games, for both mechanisms. A significantly higher frequency of grand coalition formation and efficiency is observed under the H–MC mechanism than under the Winter mechanism even in the second half of the experiment.

[16] In Appendix IV, we report the results based on all groups using payoff shares, instead of restricting our attention to groups that formed the grand coalition.

Table 4: Results of linear regression based only on the groups that formed the grand coalition

	H–MC					Winter			
	π_1	π_2	π_3	π_4		π_1	π_2	π_3	π_4
g1	23.44	25.48	25.00	26.08	g1	23.09	24.57	22.43	26.65
	(1.23)	(0.50)	(0.59)	(1.29)		(2.28)	(1.11)	(1.01)	(3.11)
g2	13.00	25.8	28.08	33.12	g2	0.0	29.15	31.56	38.81
	(1.98)	(0.63)	(0.91)	(0.80)		-	(1.03)	(0.54)	(0.73)
g3	45.53	48.68	50.53	55.26	g3	21.00	52.67	57.33	68.00
	(3.82)	(0.65)	(2.28)	(3.19)		(3.57)	(5.42)	(4.11)	(5.19)
g4	47.10	51.53	50.03	51.33	g4	44.24	48.14	45.57	56.86
	(2.46)	(1.02)	(1.15)	(1.27)		(3.22)	(4.12)	(8.58)	(4.68)
R^2	0.95	0.98	0.98	0.98	R^2	0.90	0.93	0.91	0.95
Obs.	99	99	99	99	Obs.	77	77	77	77

Note: The standard errors are corrected for within-group clustering effects.

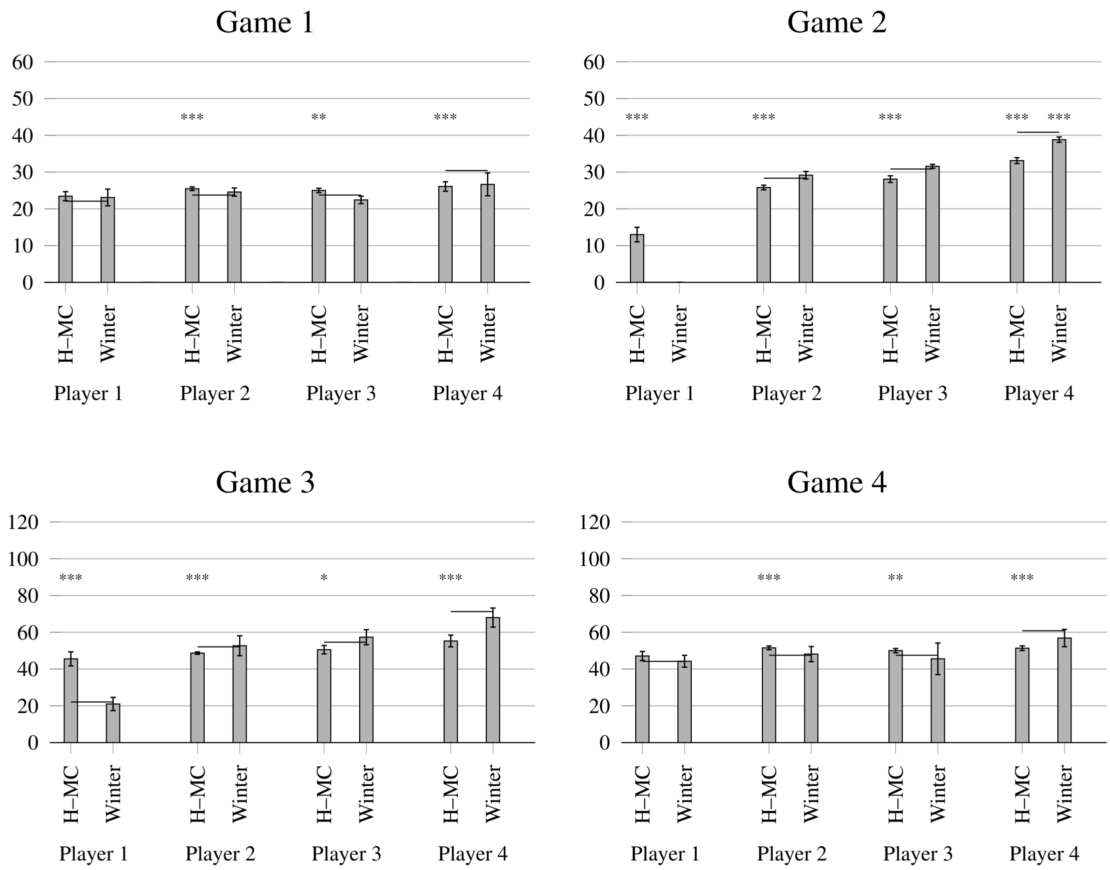
system of equations:

$$\begin{aligned} \pi_1 &= a_1 g_1 + a_2 g_2 + a_3 g_3 + a_4 g_4 + u_1 \\ \pi_2 &= b_1 g_1 + b_2 g_2 + b_3 g_3 + b_4 g_4 + u_2 \\ \pi_3 &= c_1 g_1 + c_2 g_2 + c_3 g_3 + c_4 g_4 + u_3 \\ \pi_4 &= d_1 g_1 + d_2 g_2 + d_3 g_3 + d_4 g_4 + u_4 \end{aligned} \tag{1}$$

where π_i is the payoff of player i, g_j is a dummy variable that takes a value of 1 if the game $j \in \{1, 2, 3, 4\}$ is played, and zero otherwise. Because participants play all four games twice, we correct the standard errors for within-group clustering effects. Note that the estimated coefficients a_j, b_j, c_j, and d_j are the average payoffs in game j for players 1, 2, 3, and 4, respectively. Table 4 reports the results of these regressions, showing the H–MC (Winter) mechanism in the left (right) panel.

Figure 4 shows the average payoffs obtained by each player in the four games con-

Figure 4: Mean payoffs based only on the groups that formed the grand coalition



Note: The horizontal lines indicate the Shapley values. The error bars show the one standard error range. The symbols ***, **, and * indicate the frequency with which the average normalized payoff is significantly different from the Shapley values at the 1%, 5%, and 10% significance levels (Wald test), respectively.

ditional on the grand coalition being formed. The horizontal lines indicate the Shapley values for each game. It can be observed that for the Winter mechanism, the average payoffs are not significantly different from the Shapley values for all four players in games 1, 3, and 4. Conversely, for the H–MC mechanism, they are significantly different from the Shapley values for almost all players in all four games. This indicates that provided that the grand coalition is formed, the average payoffs under the Winter mechanism are closer to the Shapley values than those under the H–MC mechanism.

To compare the two mechanisms in terms of how close their average payoffs are to the Shapley values, we employ the following measure:

$$Dis2_{\phi} = \sqrt{\sum_{i}(\overline{\pi_i} - \phi_i)^2} \tag{2}$$

where $\overline{\pi_i}$ and ϕ_i are the average payoff and Shapley value for player i, respectively, in the given game.

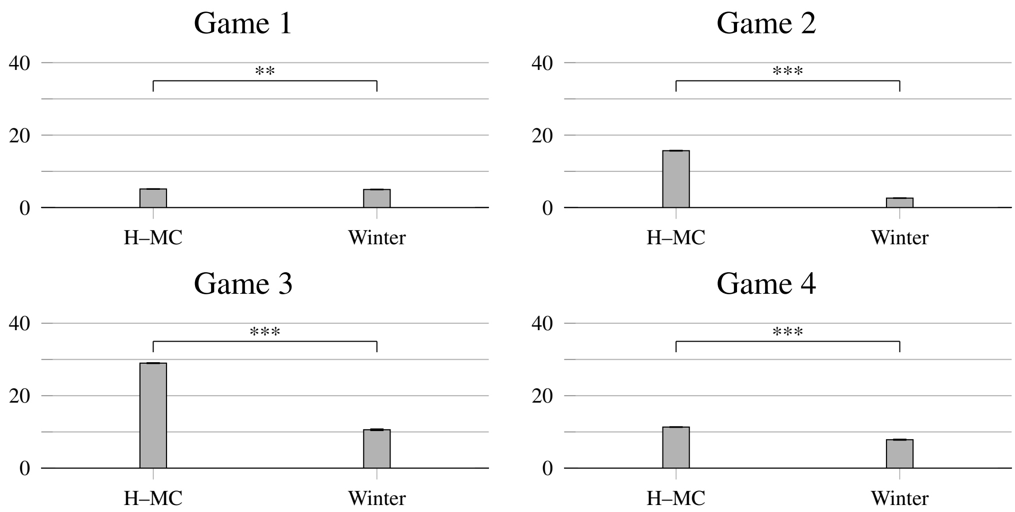
To conduct a statistical test, we employ a bootstrapping technique. For each iteration, we use a sub-sample (with replacement) of our data, and run the system of regressions (Eq. 1). Based on the obtained estimated coefficients (i.e., the average payoffs for the sub-sample), we compute $Dis2_{\phi}$.

Figure 5 shows the result based on the outcomes of 1,000 repetitions. For all four games, $Dis2_{\phi}$ is statistically significantly smaller under the Winter mechanism than under the H–MC mechanism.[17]

Result 2. *Provided the grand coalition is formed, the average payoffs follow the Shapley*

[17]Based on two-sample t-test with unequal variance using the sample generated by the bootstrap. The means $Dis2_{\phi}$ (standard errors) for the H–MC mechanism are 5.13 (0.035) in game 1, 15.70 (0.064) in game 2, 28.99 (0.074) in game 3, and 11.34 (0.047) in game 4. For the Winter mechanism, the corresponding values are 4.99 (0.047) for game 1, 2.61 (0.030) for game 2, 10.60 (0.178) for game 3, and 7.85 (0.094) for game 4.

Figure 5: Distance from Shapley



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate the significant difference between the H–MC and the Winter mechanisms at the 1%, 5%, and 10% significance levels (two-sample t-test), respectively.

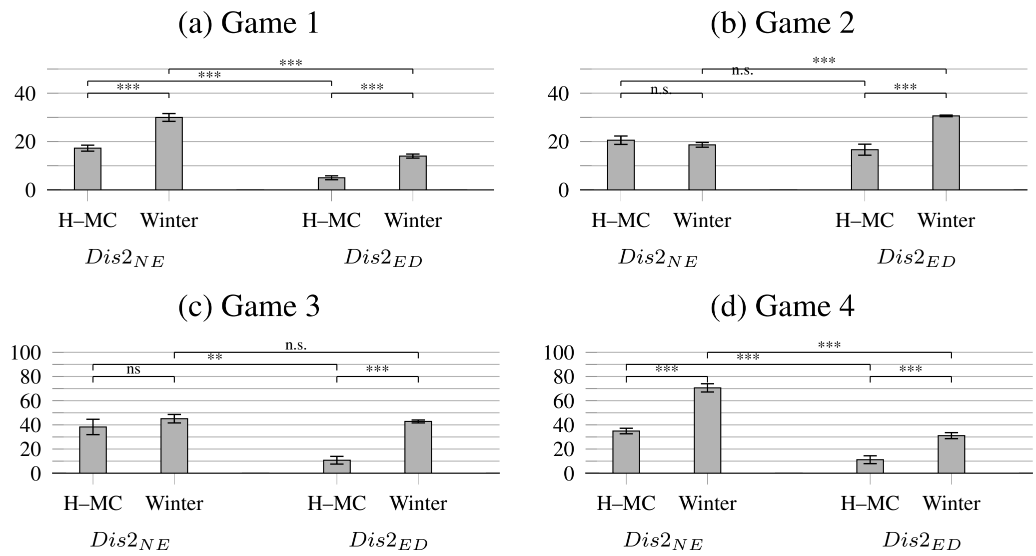
values more closely under the Winter mechanism than under the H–MC mechanism.

6.3 Realized allocations and *a posteriori* equilibria

Now, let us analyze the realized payoffs in the light of the *a posteriori* equilibrium payoff vectors. We continue to focus only on the groups that formed the grand coalition. We measure the distance between the realized payoff vectors and the allocation under the SPNE for the four games by their Euclidean distance. Let eq_i be the equilibrium payoff for player i for the given game, the realized order of the players (making a proposal or demand), and the mechanism. The distance of the realized payoff from the equilibrium is computed as $Dis2_{NE} = \sqrt{\sum_i (\pi_i - eq_i)^2}$.[18] We also consider the distance between the realized payoff vectors and equal division payoffs, defined by $Dis2_{ED} = \sqrt{\sum_i (\pi_i - ED_i)^2}$ where ED_i is the equal division payoff for player i for

[18] For the sake of simplicity, we omit the specifications about the considered mechanism and the game.

Figure 6: Mean of the distances of the realized payoff vectors from the SPNE and the equal division



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate that the distance of the normalized payoff vectors from the equilibrium allocations or from the equal division was significantly different between the H–MC and the Winter implementation, at the 1%, 5%, and 10% significance levels (Wald test), respectively.

the given game.

Figure 6 shows the mean $Dis2_{NE}$ and the mean $Dis2_{ED}$ for the two mechanisms in the four games.[19] We observe that the distance to the equal division is significantly smaller (at the 1% level) for the H–MC mechanism than for the Winter mechanism in all four games. This may not be surprising because, as Observation 1 states, the *a posteriori* equilibrium payoff vectors tend to be less unequal under the H–MC mechanism than under the Winter mechanism. In fact, as we can observe, the distance to the equilibrium allocation is significantly smaller for the H–MC mechanism than for the Winter mecha-

[19]The figure is created based on the estimated coefficients of the following linear regressions: $Dis_i = \beta_1 HMC_i + \beta_2 Winter_i + \mu_i$, where Dis_i is the relevant distance measure for group i, HMC_i ($Winter_i$) is a dummy variable that takes a value of 1 if the H–MC (Winter) mechanism is used, and zero otherwise. The standard errors are corrected for within-session clustering effects. The statistical tests are based on the Wald test for the equality of the estimated coefficients of the two treatment dummies.

nism in games 1 and 4 (in which the equilibrium payoffs are less unequal than in games 2 and 3) at the 1% level. For games 2 and 3, however, the distance to the equilibrium allocations is not significantly different between the two mechanisms.

Figure 6 shows that, on the one hand, the payoff vectors realized under the H–MC mechanism are significantly closer to the equal division than to the equilibrium ones in all but game 2 (in which $Dis2_{NE}$ and $Dis2_{ED}$ are not significantly different). On the other hand, under the Winter mechanism, the realized payoff vectors are significantly closer to the equal division than to the equilibrium ones only in games 1 and 4, but the opposite is the case for game 2. In game 3, $Dis2_{NE}$ and $Dis2_{ED}$ are not significantly different under the Winter mechanism.

Result 3. *The H–MC mechanism more often results in payoffs that are closer to the equal division than to the equilibrium payoffs compared with the Winter mechanism.*

This indicates that, albeit imperfectly, the Winter mechanism achieves the allocation that better reflects the power of the players than does the H–MC mechanism.

7 Conclusions

We have experimentally compared two of the best-known bargaining procedures in the Nash program, the H–MC and the Winter mechanisms. Our main rationale for this choice is simplicity, which is a key desideratum when considering possible applicability to the real world. These two mechanisms are simple and similar in their implementation, making them suitable for a direct comparison. They differ mainly in the way that they implement bargaining, as the H–MC mechanism is based on offers, and the Winter on demands.

Previous studies have found a certain closeness of the experimental results when making a similar comparison (see Fréchette et al., 2005a), despite the sharply different theoretical predictions. Our findings partially contradict these results, showing how two very similar mechanisms can behave differently, despite the close theoretical predictions. In particular, the H–MC mechanism results in higher frequencies of the grand coalition formation and, consequently, higher efficiency than the Winter mechanism. We suggest that the H–MC mechanism is better suited to bargaining over cost or payoff allocation problems when the main target is efficiency, or when full cooperation represents a crucial goal for society (e.g., full cooperation in the airport problem (Littlechild and Owen, 1973) results in one single airport being built instead of many, and this is certainly desirable for environmental reasons). Conversely, provided that the grand coalition is formed, the Winter mechanism results in average payoffs that are closer to the Shapley values and better satisfy various axioms. We suggest that the Winter mechanism is best suited to allocation problems in which it is important to value players' effective power (e.g., production games (Owen, 1975), or in which arguments such as social welfare and symmetry are inescapable (e.g., allocation of resources in health or social care (Kluge, 2007)).

Our findings suggest that when facing a cost or payoff allocation problem, the choice of which bargaining procedure to implement, one based on offers or on demands, may have some unexpected effects, regardless of the theoretical prediction. This should be taken into account when making such a choice in various applications. In fact, different bargaining mechanisms, even when equivalent from the theoretical point of view, favor different properties that are reflected in the resulting allocations. An example of such effects may be found in the verification of the null player property of the Shapley value in Appendix III. Theoretically, a player who always has a zero marginal contribution should receive a zero payoff, according to Shapley. According to the theoretical prediction, in a demand-based mechanism, nonnull players should systematically refuse a strictly positive demand by a null player. However, we find that nonnull players seem to be uncomfortable with making a zero offer to a null player in an offer-based mechanism, and this contributes to a final payoff share that is closer to the equal division solution. A deeper analysis of how different mechanisms can lead players toward respecting or violating some properties would be a fruitful direction for future research.

Many potentially important complementary questions can be addressed in future research. Among others, an analysis of the more complex versions of our proposed mechanisms (e.g., the Winter mechanism with more periods and a discount factor, or the H–MC mechanism where the proposer whose offer is refused then leaves the game with a probability strictly smaller than one) can be compared with our actual results. Comparing the outcomes of the experiments based on noncooperative mechanisms with those of unstructured bargaining experiments would be an interesting topic for future research.

Acknowledgments

The experiments reported in this paper were approved by the Institutional Review Board (IRB) of Yamaguchi University (No. 5, for the experiments conducted in 2019) as well as by the IRB of ISER, Osaka University (for the experiment conducted in 2022). We gratefully acknowledge financial support from the Joint Usage/Research Center at ISER, Osaka University, and Grants-in-Aid for Scientific Research, Japan Society for the Promotion of Science (15K01180, 15H05728, 18K19954, 20H05631), the Fund for the Promotion of Joint International Research (Fostering Joint International Research)

(15KK0123), and the French government-managed l'Agence Nationale de la Recherche (les Programmes d'Investissements d'Avenir UCA^{JEDI}) (ANR-15-IDEX-01). In particular, we thank the UCAinACTION project. We thank Guillaume Fréchette and participants of the seminar at the University of Technology Sydney, Monash University, Osaka University, Tokyo University, and Keio University, as well as participants of ASFEE 2019 (Toulouse, France), Game Theory Workshop 2020 (Tokyo, Japan), SEET 2020 (Naples, Italy), SAET 2021 (Seoul, Korea), and ESA Global 2021 (online) for comments and suggestions.

References

ABE, T., Y. FUNAKI, AND T. SHINODA (2021): "Invitation Games: An Experimental Approach to Coalition Formation," Games, 3, 64.

ANSOLABEHERE, S., J. SNYDER, A. STRAUSS, AND M. TING (2005): "Voting Weights and Formateur Advantages in the Formation of Coalition Governments," American Journal of Political Science, 49, 550–563.

BARON, D. P. AND J. A. FEREJOHN (1989): "Bargaining in Legislatures," American Political Science Review, 83, 1181–1206.

BENNETT, E. AND E. VAN DAMME (1991): "Demand Commitment Bargaining: - The Case Of Apex Games," in Game Equilibrium Models III, ed. by S. R., Springer, Berlin, Heidelberg.

BINMORE, K., A. RUBINSTEIN, AND A. WOLINSKY (1986): "The Nash Bargaining Solution in Economic Modelling," RAND Journal of Economics, 17, 176–188.

BOLTON, G. E., K. CHATTERJEE, AND K. L. MCGINN (2003): "How Communication Links Influence Coalition Bargaining: A Laboratory Investigation," Management Science, 49, 583–598.

BRANDENBURGER, A. AND B. NALEBUFF (1996): Co-Opetition, New York, NY: Doubleday Business.

DE CLIPPEL, G. AND K. ROZEN (2021): "Fairness through the lens of cooperative game theory: An experimental approach," American Economic Journal: Microeconomics, forthcoming (https://www.aeaweb.org/articles?id=10.1257/mic.20200015).

DEMANGE, G. (1984): "Implementing efficient egalitarian outcomes," Econometrica, 52, 1167–1177.

FIORINA, M. P. AND C. R. PLOTT (1978): "Committee Decisions under Majority Rule: An Experimental Study," The American Political Science Review, 72, 575–598.

FISCHBACHER, U. (2007): "z-Tree: Zurich toolbox for ready-made economic experiments," Experimental Economics, 10, 171–178.

FRÉCHETTE, G., J. H. KAGEL, AND S. F. LEHRER (2003): "Bargaining in Legislatures: An Experimental Investigation of Open versus Closed Amendment Rules," The American Political Science Review, 97, 221–232.

FRÉCHETTE, G., J. H. KAGEL, AND M. MORELLI (2005a): "Behavioral Identification in Coalition Bargaining: An Experimental Analysis of Demand Bargaining and Alternating Offers," Econometrica, 73, 1893–1937.

——— (2005b): "Behavioral identification in coalitional bargaining: An experimental analysis of demand bargaining and alternating offers," Econometrica, 73, 1893–1937.

——— (2005a): "Gamson's Law versus Non-Cooperative Bargaining Theory," Games and Economic Behavior, 51, 365–390.

——— (2005b): "Nominal Bargaining Power, Selection Protocol and Discounting in Legislative Bargaining," Journal of Public Economics, 89, 1497–1518.

GREINER, B. (2015): "An Online Recruitment System for Economic Experiments," Journal of the Economic Science Association, 1, 114–125.

GUERCI, E., N. HANAKI, N. WATANABE, G. ESPOSITO, AND X. LU (2014): "A methodological note on a weighted voting experiment," Social Choice and Welfare, 43, 827–850.

GUL, F. (1989): "Bargaining Foundations of Shapley Value," Econometrica, 57, 81–95.

HAAKE, C.-J. (2000): "Support and Implementation of the Kalai-Smorodinsky Bargaining Solution," in Operations Research Proceedings 1999 (Selected Papers of the Symposium on Operations Research (SOR '99) Magdeburg, September 1-3, 1999), vol 1999, ed. by K. Inderfurth, G. Schwödiauer, W. Domschke, F. Juhnke, P. Kleinschmidt, and W. G., Mannheim: Bibliographisches Institut Mannheim.

HARSANYI, J. C. (1974): "An Equilibrium-Point Interpretation of Stable Sets and a Proposed Alternative Definition," Management Science, 20, 1472–1495.

——— (1981): "The Shapley Value and the Risk Dominance Solutions of Two Bargaining Models for Characteristic-Function Games," in Essays in Game Theory and

Mathematical Economics, ed. by R. J. A. et al., Mannheim: Bibliographisches Institut Mannheim, 43–68.

HART, O. AND J. MOORE (1990): "Property Rights and the Nature of the Firm," Journal of Political Economy, 98, 1119–1157.

HART, S. AND A. MAS-COLELL (1996): "Bargaining and Value," Econometrica, 64, 357–380.

JU, Y. AND D. WETTSTEIN (2009): "Implementing cooperative solution concepts: a generalized bidding approach," Economic Theory, 39, 307–330.

KALAI, E. AND M. SMORODINSKY (1975): "Other Solutions to Nash's Bargaining Problem," Econometrica, 43, 513–518.

KALISCH, G. K., J. W. MILNOR, J. F. NASH, AND E. D. NERING (1954): "Some experimental n-person games," in Decision Processes, ed. by R. Thrall, C. Coombs, and R. Davis, John Wiley & Sons, Inc, 301–327.

KLUGE, E.-H. W. (2007): "Resource allocation in healthcare: implications of models of medicine as a profession," MedGenMed, 9(1):57.

KRISHNA, V. AND R. SERRANO (1995): "Perfect Equilibria of a Model of N-Person Noncooperative Bargaining," International Journal of Game Theory, 24, 259–272.

LITTLECHILD, S. C. AND G. OWEN (1973): "A Simple Expression for the Shapely Value in a Special Case," Management Science, 20, 370–372.

MCSHERRY, F. AND K. TALWAR (2007): "Mechanism Design via Differential Privacy," in 48th Annual IEEE Symposium on Foundations of Computer Science (FOCS'07), 94–103.

MONTERO, M., M. SEFTON, AND P. ZHANG (2008): "Enlargement and the balance of power: an experimental study," Social Choice and Welfare, 30, 69–87.

MORELLI, M. (1999): "Demand Competition and Policy Compromise in Legislative Bargaining," American Political Science Review, 93, 809–820.

MOULIN, H. (1984a): "The conditional auction mechanism for sharing a surplus," Review of Economic Studies, 51, 157–170.

——— (1984b): "Implementing the Kalai-Smorodinsky Bargaining Solution," Journal of Economic Theory, 33, 32–45.

MOULIN, H. AND M. O. JACKSON (1992): "Implementing a public project and distributing its costs," Journal of Economic Theory, 57, 125–140.

MURNIGHAN, J. K. AND A. E. ROTH (1977): "The Effects of Communication and Information Availability in an Experimental Study of a Three-Person Game," Management Science, 23, 1336–1348.

——— (1982): "The Role of Information in Bargaining: An Experimental Study," Econometrica, 50, 1123–1142.

NASH, J. F. (1950): "The Bargaining Problem," Econometrica, 18, 155–162.

——— (1953): "Two person coooperative games," Econometrica, 21, 128–140.

NASH, J. F., R. NAGEL, A. OCKENFELS, AND R. SELTEN (2012): "The agencies method for coalition formation in experimental games," Proceedings of the National Academy of Sciences, 109, 20358–20363.

NYDEGGER, R. V. AND G. OWEN (1975): "Two-person bargaining: An experimental test of the Nash axioms," International Journal of Game Theory, 3, 239–249.

OWEN, G. (1975): "On the Core of Linear Production Games," Mathematical Programming, 9, 358–370.

PÉREZ-CASTRILLO, D. AND D. WETTSTEIN (2001): "Bidding for the Surplus : A Non-cooperative Approach to the Shapley Value," Journal of Economic Theory, 100, 274–294.

RAIFFA, H. (1953): "Arbitration Schemes for Generalized Two Person Games," in Contributions to the Theory of Games II, ed. by H. Kuhn and A. Tucker, Princeton: Princeton University Press, 361–387.

RAPOPORT, A., O. FRENKEL, AND J. PERNER (1977): "Experiments with cooperative 2 X 2 games," Theory and Decision, 8, 67–92.

ROTH, A. E. AND R. E. VERRECCHIA (1979): "The Shapley Value As Applied to Cost Allocation: A Reinterpretation," Journal of Accounting Research, 17, 295–303.

RUBINSTEIN, A. (1982): "Perfect Equilibrium in a Bargaining Model," Econometrica, 50, 97–110.

SCHULZ, A. S. AND N. A. UHAN (2010): "Sharing Supermodular Costs," Operations Research, 58, 1051–1056.

SERRANO, R. (2005): "Fifty Years of the Nash Program, 1953-2003," Investigaciones Economicas, 29, 219–258.

——— (2008): "Nash Program," in The New Palgrave Dictionary of Economics, 2nd edition, ed. by S. Durlauf and L. Blume, McMillan, London.

——— (2014): "The Nash Program: A Broader Interpretation," Ensayos, 33, 105–106.

——— (2021): "Sixty-Seven Years of the Nash Program: Time for Retirement?" SERIEs, 12, 35–48.

SHAPLEY, L. S. (1953): "A Value for n-Person Games," in Contribution to the Theory of Games, ed. by H. Kuhn and A. Tucker, Princeton, vol. II, 303–317.

SHINODA, T. AND Y. FUNAKI (2019): "Unstructured Bargaining Experiment on Three-person Cooperative Games," Tech. Rep. E1915, September 2019, Waseda Institute of Political EConomy.

SHUBIK, M. (1962): "Incentives, Decentralized Control, the Assignment of Joint Costs and Internal Pricing," Management Science, 8, 325–343.

TIMMER, J., M. CHESSA, AND R. J. BOUCHERIE (2013): "Cooperation and game-theoretic cost allocation in stochastic inventory models with continuous review," European Journal of Operational Research, 231, 567–576.

TROCKEL, W. (1999): "Unique Nash Implementation for a Class of Bargaining Solutions," International Game Theory Review, 1, 267–272.

——— (2002): "A Universal Meta Bargaining Realization of the Nash Solution," Social Choice and Welfare, 19, 581–586.

VAN DEN BRINK, R. (2002): "An Axiomatization of the Shapley Value using a Fairness Property," International Journal of Game Theory, 30, 309–319.

——— (2006): "Null or nullifying players: The difference between the Shapley value and equal division solutions," Journal of Economic Theory, 136, 767–775.

WARWICK, P. V. AND J. N. DRUCKMAN (2001): "Portfolio Salience and the Proportionality of Payoffs in Coalition Government," British Journal of Political Science, 31, 627–649.

WINTER, E. (1994): "The Demand Commitment Bargaining and Snowballing Cooperation," Economic Theory, 4, 255–273.

YOUNG, H. P. (1985): "Monotonic Solutions of Cooperative Games," International Journal of Game Theory, 14, 65–72.

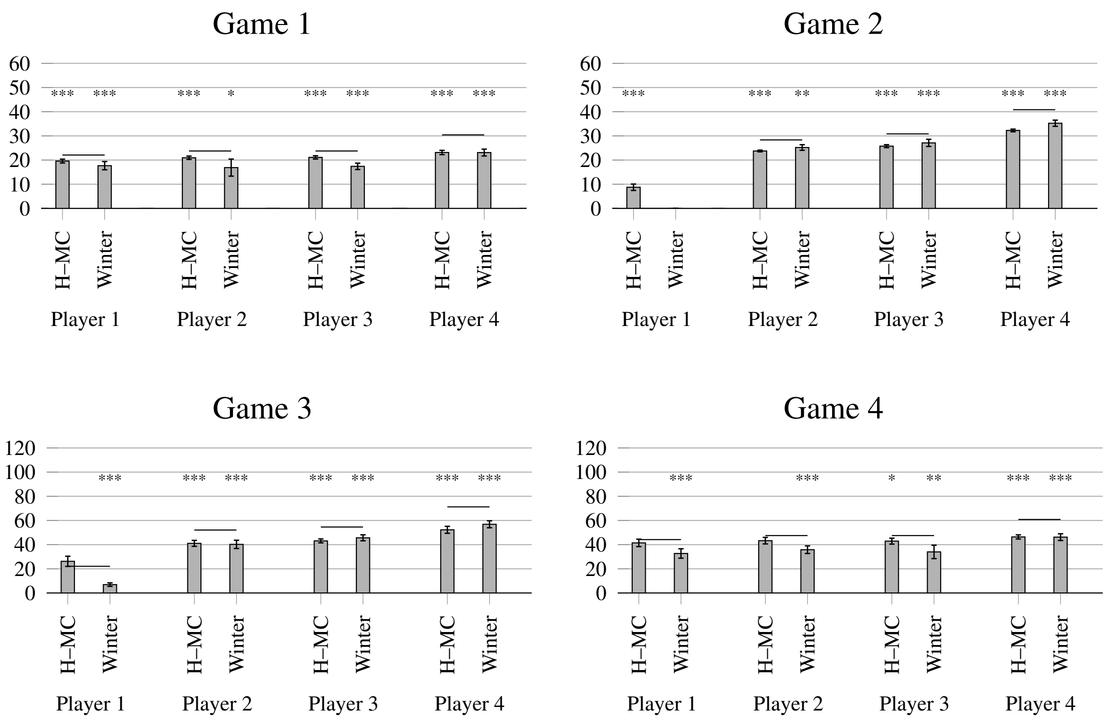
——— (1998): "Cost allocation, demand revelation, and core implementation," Mathematical Social Sciences, 36, 213–228.

Appendix

I The average payoffs

Figure I.1 shows the mean realized payoffs based on all groups in each of the four games, and the horizontal lines indicate the Shapley values for each game.[20]

Figure I.1: Mean payoffs, all groups



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate the average payoff is significantly different from the Shapley values at the 1%, 5%, and 10% significance levels, respectively (Wald test).

[20] As in Figure 4, the mean and the standard errors are obtained by running the system of linear regressions that take each player's payoff as the dependent variables and four game dummy variables without the constant. The standard errors are corrected for session-level clustering effects. The statistical tests are based on these regressions.

II Effect of learning and bargaining dynamics

II.1 Grand coalition formation and efficiency

We have already shown in Section 3 that both mechanisms fail to achive an efficient outcome. However, H–MC mechanism performs significantly better in this matter. A possible explanation is because, as we have already observed in Section 3 and with Result 1, H–MC mechanism forces feasible offers, while Winter mechanism allows for unfeasible demands which, as a result, lead to inefficiencies. This also naturally leads to the fact that the grand coalition is formed more often under the H–MC mechanism, than under the Winter mechanism.

One may hypothesize that this generalized failure (more for Winter, but partially also for H-MC) in reaching an efficient outcome can explained by some limited rationality arguments: even if we chose two mechansims that are in our opinion simple, the games' optimal dynamics is hard to understand for participants to the experiment especially in the beginning.

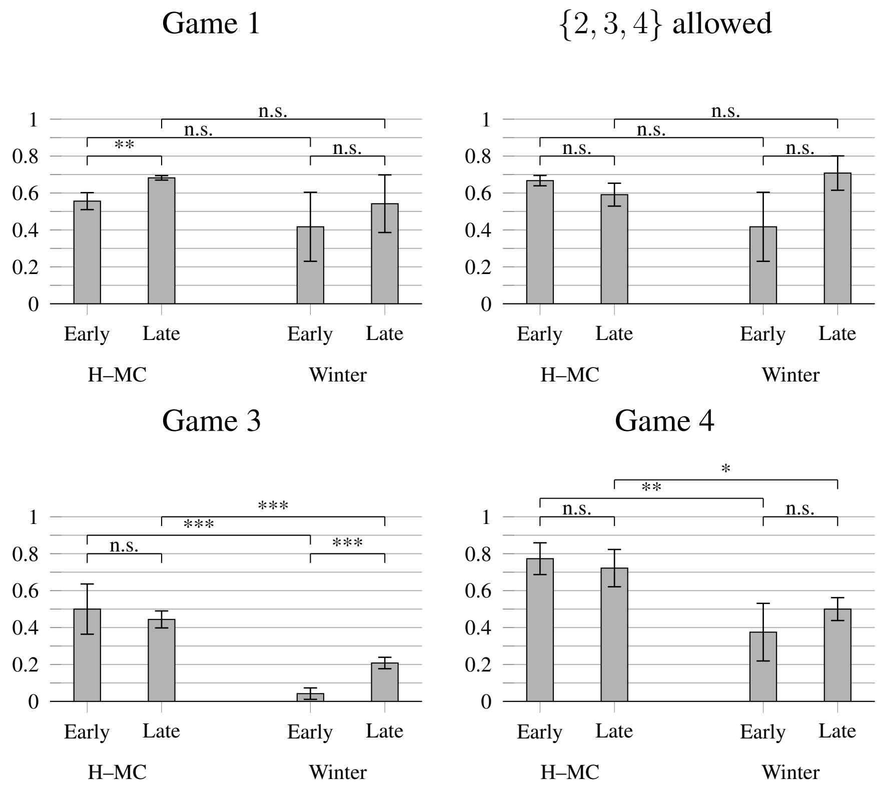
We check this hypothesis by investigating the presence of a learning effect by comparing the outcomes in the first half of four rounds (1-4) and the second half of four rounds (5-8). Because the number of groups that formed a grand coalition becomes

Table II.1: Number of groups with Grand Coalition

	game 1	game 2	game 3	game 4
Winter early	10	10	1	9
Winter late	13	17	5	12
H–MC early	10	12	11	17
H–MC late	15	13	8	13

Note: game 2 allows $\{2, 3, 4\}$ to be the grand coalition.

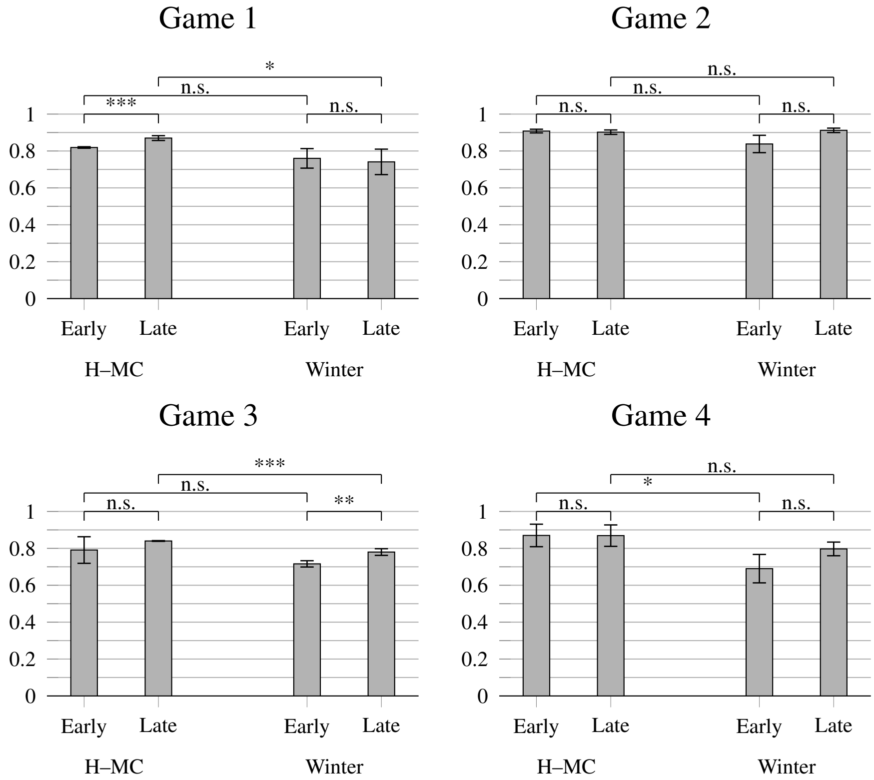
Figure II.2: Proportion of times the grand coalition formed in early and late rounds



small if we separate the data into the first half and second half (see Table II.1), we do not report the results of the analyses comparing the distance between the average realized payoff vectors and the Shapley values, or whether realized payoff vectors satisfy various axioms characterizing the Shapley value. Instead, we investigate only the frequency of grand coalition formation and efficiency.

Figures II.2 and II.3 show the frequency of the grand coalition formation and the average efficiency (i.e., the average total payoff / value of the grand coalition) for the first half and the second half (i.e., the first four rounds vs. the second four rounds) of each game. For H–MC, the frequency of the grand coalition formation and the average

Figure II.3: Efficiency in early and late rounds



efficiency are significantly higher in the later rounds only in game 1; for the remaining three games, there are no significant differences between the early and late rounds. For the Winter mechanism, both the frequency of the grand coalition formation and the average efficiency are significantly higher in the later rounds only in game 3, with no significant differences in other games.

As conclusion, we report no statistically significant learning effect, when implementing either the H–MC or the the Winter mechanism. This does not rule our the possibility that by implementing a higher number of repetitions, a significant learning effect could be observed.

II.2 Departure from the Shapley payoff share

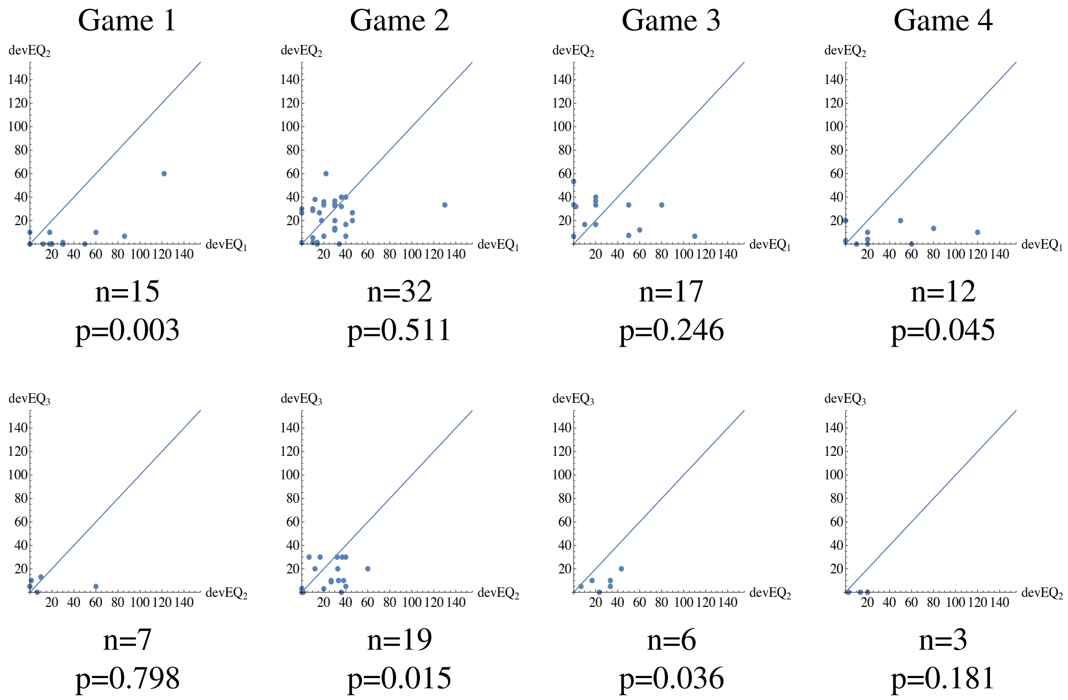
Here, we investigate some possible dynamics that could lead the experimental results of H–MC mechanism to be closer to the equal division solution (Result 3), when compared to the experimental results of the Winter mechanism that are instead closer to the Shapley value (Result 2).

In Section 6, we have already shown that accepted proposals in the H–MC mechanism go in the sense of equal division. This result is not surprising, as experimental results of offer-based mechanisms (such as the well-known two-player bargaining over a pie of Rubinstein (1982)) often show that, contrary to the theoretical prediction, players tend to go for an equal split of the pie. We show that, in our experiment, this behavior becomes more evident after a first rejection of a proposal, as second proposals are closer to the equal share than the first ones.

In Figure II.2, we show the distance from equal division, $devEQ_L = \sum_i |a_{L,i} - ED|$ where $a_{L,i}$ is the proposed allocation for player i in Lth proposal (for a group) and ED is the equal division payoff for the game, for the first ($L = 1$, horizontal axis) and the second (vertical axis) proposals (top) and the second ($L = 2$, horizontal axis) and the third (vertical axis) proposals (bottom) for each game. Each dot corresponds to a pair of the proposals of a group.

We observe a clear tendency for either the second proposal to be more equal than the first one ($devEQ_1 > devEQ_2$) or the second proposal to be more equal than the third one ($devEQ_2 > devEQ_3$) depending on the game.

Figure II.4: H–MC mechanism: Distance from equal division for the first and the second proposals (top) and the second and the third proposals (bottom)



Note: In each panel, only those groups in which the first proposal (top) or the second proposal (bottom) is rejected are plotted. p-values are based on the Signed-Rank test (two-tailed) with the null hypothesis $devEQ_L = devEQ_{L+1}$.

III Testing for the axioms of the Shapley value

We test the axioms that are historically the most relevant to characterizing the Shapley value. In doing so, we aim to provide greater insight into whether a demand-based bargaining mechanism is more appropriate than an offer-based bargaining mechanism for cost or payoff allocation problems when the allocation scheme is constructed on the main axiomatic solution notion of cooperative game theory, that is, the Shapley value.

First, we provide two definitions, which are used in the following.

Players i and j are *symmetric* in $v \in \mathcal{G}^N$, if $v(S \cup \{i\}) = v(S \cup \{j\})$ for all $S \subseteq N \setminus \{i, j\}$. Player i is a *null player* in $v \in \mathcal{G}^N$ if $v(S) = v(S \setminus \{i\})$ for all $S \subseteq N$.

In the literature, we find various axiomatic characterizations of cooperative solutions and, in particular, of the Shapley value. Given a solution $\psi : \mathcal{G}^N \to \mathbb{R}^N$, we list some of the most commonly used axioms to provide a characterization.

Axiom 1 (Efficiency): for every v in $\mathcal{G}^N$, $\sum_{i \in N} \psi_i(v) = v(N)$.

Axiom 2 (Symmetry): if i and j are symmetric players in game $v \in \mathcal{G}^N$, then $\psi_i(v) = \psi_j(v)$.

Axiom 3 (Additivity): for all $v, w \in \mathcal{G}^N$, $\psi(v + w) = \psi(v) + \psi(w)$.

Axiom 4 (Homogeneity): for all $v \in \mathcal{G}^N$ and $a \in \mathbb{R}$, $\psi(av) = a\psi(v)$.

Axiom 5 (Null player property): if i is a null player in game $v \in \mathcal{G}^N$, then $\psi_i(v) = 0$.

Axiom 6 (Strong monotonicity): if $i \in N$ is such that $v(S \cup \{i\}) - v(S) \leq w(S \cup \{i\}) - w(S)$ for each $S \subseteq N$, then $\psi_i(v) \leq \psi_i(w)$.

Axiom 7 (Fairness): if i, j are symmetric in $w \in \mathcal{G}^N$, then $\psi_i(v+w) - \psi_i(v) = \psi_j(v+w) - \psi_j(v)$ for all $v \in \mathcal{G}^N$.

Fairness states that if we add a game $w \in \mathcal{G}^N$, in which players i and j are symmetric, to a game $v \in \mathcal{G}^N$, then the payoffs of players i and j change by the same amount.

In particular, among many others, the axiomatization of Shapley (1953), which is the most classical one, involves axioms 1, 2, 3, and 5. The axiomatization of Young (1985) involves axioms 1, 2, and 6, whereas that of van den Brink (2002) involves axioms 1, 5, and 7. Note that axiom 4, even if not directly involved in any of these axiomatizations, is crucial because, together with axiom 3, it guarantees the linearity of the solution.[21]

We noted in Section 6 that both mechanisms fail to satisfy efficiency (axiom 1) if we examine overall data. Here, we examine the remaining six axioms. These axioms are tested based on the estimated coefficients obtained from running the regression of Eq. 1 as follows.

- **Symmetry (axiom 2)** requires $b_1 = c_1$ and $b_4 = c_4$.
- **Additivity (axiom 3)** and **homogeneity (axiom 4)** require that $x_3 = x_1 + x_2$ and $x_4 = 2x_1$ for $x \in \{a, b, c, d\}$, respectively.
- **Null player** property **(axiom 5)** requires that $a_2 = 0$.
- **Strong monotonicity (axiom 6)** requires that $a_1 > a_2$ and $a_4 > a_3$.

[21]The equal division solution satisfies 1, 2, and 3, but does not satisfy the null player property in 5. However, it satisfies a similar property when null players are replaced with nullifying players. Player i is a *nullifying player* if $v(S) = 0$ for each $S \subseteq N$ such that $i \in S$. Then, we can state the following additional axiom that can be called the nullifying player property: if i is a nullifying player in game $v \in \mathcal{G}^N$, then $\psi_i(v) = 0$. Replacement of the null player property in the axiomatization of the Shapley value in Shapley (1953) with the nullifying player property characterizes the equal division solution (see van den Brink, 2006).

Table III.2: Results of Wald tests for the verification of the symmetry, additivity, homogeneity, strong monotonicity, and fairness axioms (based only on the groups that formed a grand coalition)

		H–MC		Winter	
Axiom	H_0	χ^2	p-value	χ^2	p-value
Symmetry	$a_2 = a_3$	0.35	0.552	1.85	0.174
	$d_2 = d_3$	1.60	0.206	0.06	0.811
Additivity	$c_1 = a_1 + b_1$	6.69	0.001	0.13	0.721
	$c_2 = a_2 + b_2$	3.23	0.072	0.02	0.878
	$c_3 = a_2 + b_3$	2.16	0.142	0.47	0.492
	$c_4 = a_4 + b_4$	1.52	0.218	0.78	0.376
Homogeneity	$d_1 = 2a_1$	0.00	0.946	0.10	0.749
	$d_2 = 2a_2$	0.08	0.772	0.11	0.745
	$d_3 = 2a_3$	0.00	0.983	0.00	0.947
	$d_4 = 2a_4$	0.06	0.813	0.82	0.365
Null player	$a_2 = 0$	42.91	0.000	.	.
Strong monotonicity	$a_1 = b_1$	10.76	0.001	102.24	0.000
(H_0 should be rejected)	$c_1 = d_1$	0.16	0.692	26,84	0.000
Fairness	$b_3 - b_2 = c_3 - c_2$	0.62	0.433	0.74	0.391

- **Fairness (axiom 7)** requires that $b_3 - b_2 = c_3 - c_2$.

In Table III.2, we present the results of the Wald tests for the verification of these axioms, together with the null hypothesis (H_0).

Note that the symmetry (according to which H_0 should not be rejected) is confirmed for the two cases under both the Winter and the H–MC mechanisms. The additivity (according to which H_0 should not be rejected) is always confirmed under the Winter mechanism, but is confirmed in only two of four cases under the H–MC mechanism. The homogeneity (according to which H_0 should not be rejected) is always confirmed for both mechanisms. The null player property (according to which H_0 should not be

Table III.3: Tests of axioms (based only on the groups that formed a grand coalition)

Axiom	H–MC	Winter
Symmetry	+	+
Additivity	-	+
Homogeneity	+	+
Null player property	-	+
Strong monotonicity	-	+
Fairness	+	+

$+$ indicates that the axiom is considered to be satisfied on average. $-$ indicates the opposite.

rejected) is not confirmed in the H–MC mechanism, but it is confirmed (respected 100% of the time) for the Winter mechanism. The strong monotonicity (according to which H_0 should be rejected) is confirmed for the Winter mechanism but only for half of the time for the H–MC mechanism. The fairness (according to which H_0 should not be rejected) is confirmed for both mechanisms.

Let us consider that the axiom is satisfied on average if it is confirmed for strictly more than half of the cases being tested. Table III.3 summarizes whether each axiom is satisfied on average (+) or not (-) for two mechanisms. We can state the following.

Result 4. *Provided the grand coalition is formed, the Winter mechanism better satisfies axioms that characterize the Shapley value than the H–MC mechanism.*

IV Additional results based on payoff shares

In this section, we report the results based on all the groups using the payoff share instead of restricting our attention to those that formed the grand coalition. Payoff shares are defined as $\widetilde{\pi}_i^W(v_k) = \frac{\pi_i^W(v_k)}{\sum_{j \in N} \pi_j^W(v_k)} \times v_k(N)$ and $\widetilde{\pi}_i^{H--MC}(v_k) = \frac{\pi_i^{H--MC}(v_k)}{\sum_{j \in N} \pi_j^{H--MC}(v_k)} \times v_k(N)$ for each $i = 1, 2, 3, 4$.

As in the main text, our analyses are based on running a set of OLS regressions shown by Eq. 1 but using payoff shares as dependent variables. Table IV.4 shows the results of the regression.

Table IV.4: Results of linear regression for normalized payoffs

H–MC

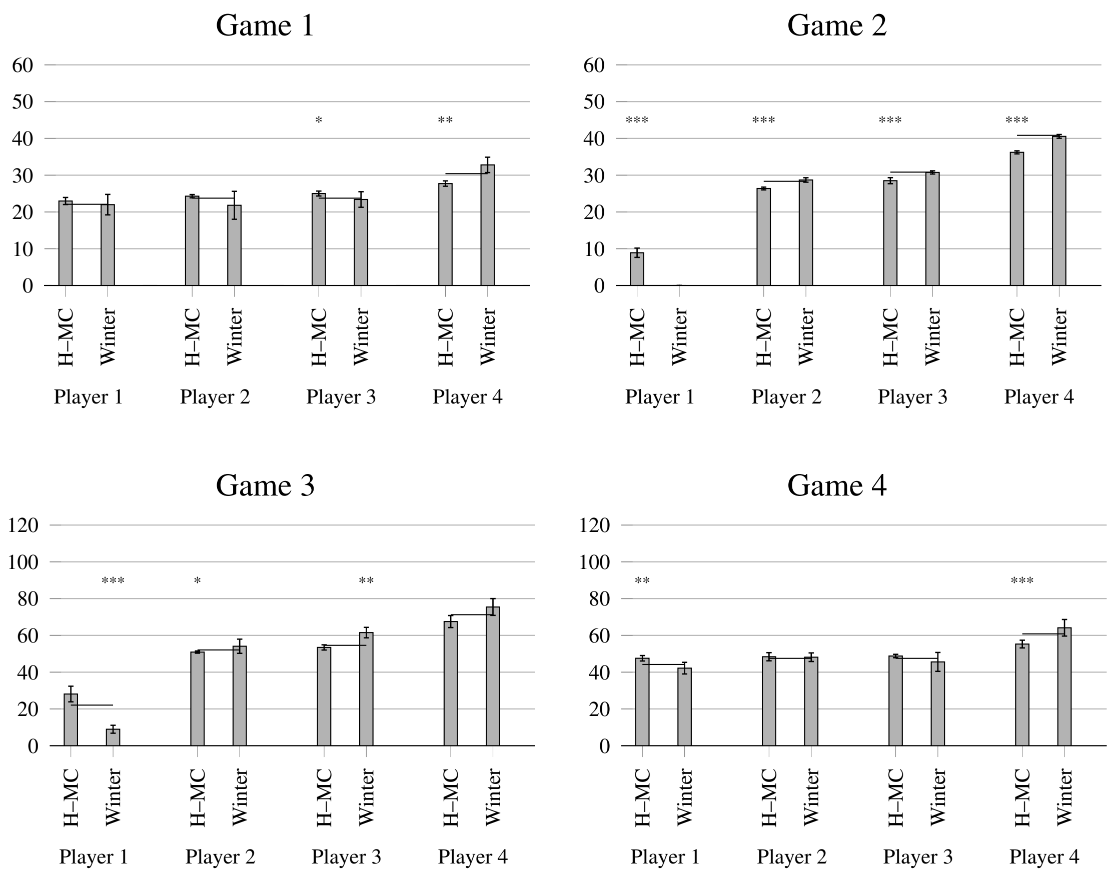
	$\widetilde{\pi}_1$	$\widetilde{\pi}_2$	$\widetilde{\pi}_3$	$\widetilde{\pi}_4$
g1	22.98	24.29	25.01	27.72
	(0.96)	(0.44)	(0.67)	(0.73)
g2	8.91	26.38	28.51	36.21
	(1.27)	(0.38)	(0.82)	(0.42)
g3	28.11	50.92	53.45	67.52
	(4.25)	(0.60)	(1.42)	(3.27)
g4	47.55	48.39	48.78	55.28
	(1.48)	(2.22)	(0.92)	(2.10)
R^2	0.81	0.95	0.95	0.95
Obs.	160	160	160	160

Winter

	$\widetilde{\pi}_1$	$\widetilde{\pi}_2$	$\widetilde{\pi}_3$	$\widetilde{\pi}_4$
g1	21.99	21.91	23.39	32.80
	(2.78)	(3.80)	(2.12)	(2.09)
g2	0.0	28.70	30.75	40.55
	-	(0.61)	(0.44)	(0.51)
g3	8.95	54.09	61.52	75.55
	(2.16)	(3.86)	(2.84)	(4.57)
g4	42.18	48.13	45.57	64.13
	(3.15)	(2.34)	(5.14)	(4.51)
R^2	0.73	0.90	0.92	0.93
Obs.	192	192	192	192

Figure IV.5 shows the mean of the normalized payoffs in the four games, where the horizontal lines indicate the Shapley values for each game. It can be observed that for the Winter mechanism, the average normalized payoffs are not significantly different from the Shapley values for all four players in games 1, 2, and 4. However, for the H–MC mechanism, the average normalized payoffs for all four players do not respect the Shapley values in any of the games at the 10% significance level.

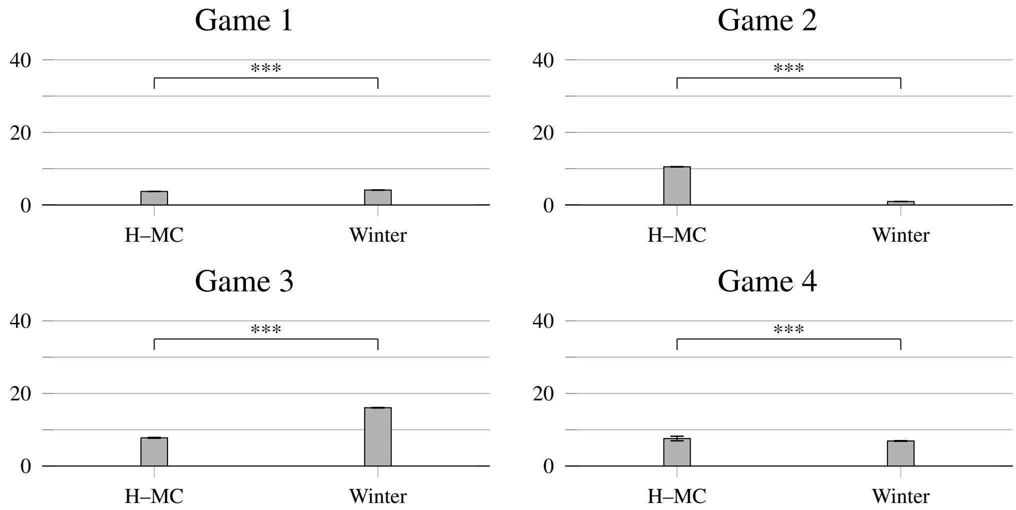
Figure IV.5: Mean of the normalized payoffs



Note: The horizontal lines indicate the Shapley values. The error bars show the one standard error range. The symbols ***, **, and * indicate the frequency with which the average normalized payoff is significantly different from the Shapley values at the 1%, 5%, and 10% significance levels, respectively (Wald test).

Figure IV.6 shows the mean $Dis2_\phi$ (based on the normalized payoff). As in the main text, we use a bootstrapping technique with 1,000 repetitions to create the figure and conduct the statistical tests. In contrast to the analyses restricted to the groups that formed a grand coalition, we now observe that $Dis2_\phi$ is significantly smaller for the H–MC mechanism than for the Winter mechanism in games 1 and 3. For the other two games, as before, $Dis2_\phi$ is significantly smaller for the Winter mechanism than for the

Figure IV.6: Distance of the normalized payoffs from Shapley



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate the significant difference between the H–MC and Winter mechanisms at the 1%, 5%, and 10% significance levels, respectively (two-sample t-test).

H–MC mechanism.[22]

Tables IV.5 and IV.6 summarize the results of testing the six axioms. Based on the normalized payoff, on average, the symmetry, strong monotonicity, and fairness axioms are now satisfied under the H–MC mechanism. For the Winter mechanism, with normalized payoffs, the fairness axiom is no longer satisfied.

Thus, if we consider all the groups and normalized payoffs, the Winter and H–MC mechanisms are comparable in terms of their distance to the Shapley value and satisfaction of its properties.

Figure IV.7 shows the mean $Dis2_{NE}$ and the mean $Dis2_{ED}$ for the two mechanisms in the four games computed based on the normalized payoffs using all the groups. The

[22]The mean $Dis2_{\phi}$ (standard error) values based on the normalized payoff for the H–MC mechanism are 3.73 (0.034) in game 1, 10.52 (0.056) in game 2, 7.78 (0.120) in game 3, and 7.58 (0.638) in game 4. For the Winter mechanism, the corresponding values are 4.11 (0.048) in game 1, 0.96 (0.016) in game 2, 16.07 (0.077) in game 3, and 6.91 (0.091) in game 4.

Table IV.5: Wald tests for the verification of the symmetry, additivity, homogeneity, strong monotonicity, and fairness axioms for normalized payoffs

		H–MC		Winter	
Axiom	H_0	χ^2	p-value	χ^2	p-value
Symmetry	$a_2 = a_3$	0.53	0.466	0.08	0.781
	$d_2 = d_3$	0.03	0.869	0.14	0.712
Additivity	$c_1 = a_1 + b_1$	0.99	0.319	7.25	0.007
	$c_2 = a_2 + b_2$	0.07	0.790	0.65	0.422
	$c_3 = a_2 + b_3$	0.00	0.952	2.54	0.111
	$c_4 = a_4 + b_4$	0.92	0.336	0.35	0.555
Homogeneity	$d_1 = 2a_1$	2.48	0.115	0.06	0.805
	$d_2 = 2a_2$	0.01	0.926	0.37	0.542
	$d_3 = 2a_3$	0.31	0.580	0.02	0.892
	$d_4 = 2a_4$	0.00	0.963	0.35	0.552
Null player	$a_2 = 0$	49.51	0.000	.	.
Strong monotonicity	$a_1 = b_1$	46.26	0.000	62.74	0.000
(H_0 should be rejected)	$c_1 = d_1$	14.57	0.001	147.12	0.000
Fairness	$b_3 - b_2 = c_3 - c_2$	0.58	0.447	7.53	0.006

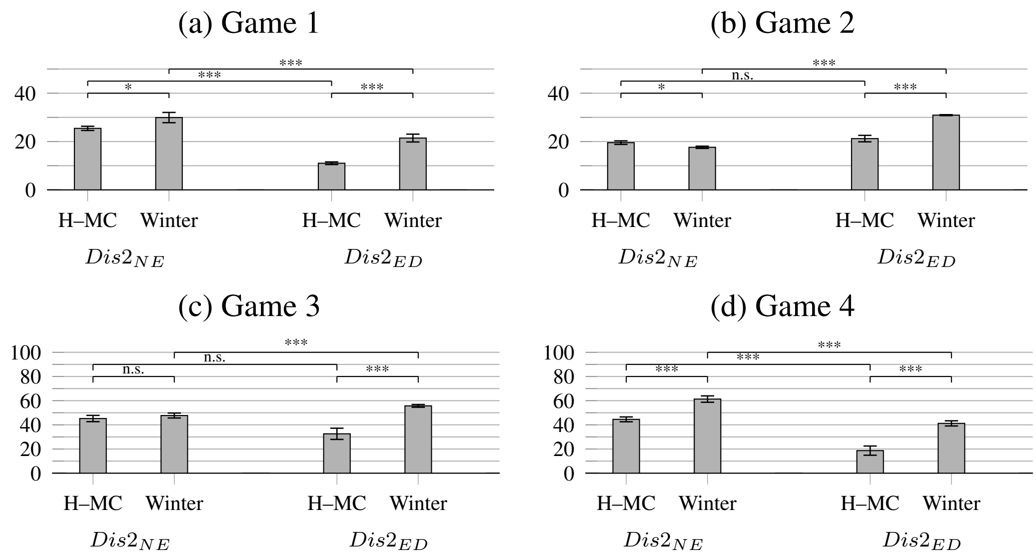
Table IV.6: Tests of axioms for normalized payoffs

Axiom	H–MC	Winter
Symmetry	+	+
Additivity	+	+
Homogeneity	+	+
Null player property	-	+
Strong monotonicity	+	+
Fairness	+	-

$+$ indicates that the axiom is considered to be satisfied on average. $-$ indicates the opposite.

normalized payoffs under the H–MC mechanism are significantly closer to the equal division than those under the Winter mechanism in all four games. Furthermore, those under H–MC are significantly closer to the equilibrium payoffs in games 1 and 4 than

Figure IV.7: Mean of the distances of the normalized payoff vectors from the SPNE and the equal division



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate that the distance of the normalized payoff vectors from the equilibrium allocations or from the equal division was significantly different between the H–MC and the Winter mechanism at the 1%, 5%, and 10% significance levels, respectively (Wald test).

those under the Winter mechanism. However, in these games, for both the Winter and H–MC mechanisms, normalized payoffs are significantly closer to the equal division than to the equilibrium payoffs. For games 2 and 3, the normalized payoffs under the Winter mechanism are significantly closer to the equilibrium than to the equal division. Under H–MC, $Dis2_{NE}$ and $Dis2_{ED}$ are not significantly different in games 2 and 3.

V Comparison of a classical H–MC sequential approval mechanism vs. a pseudo-H–MC simultaneous approval mechanism

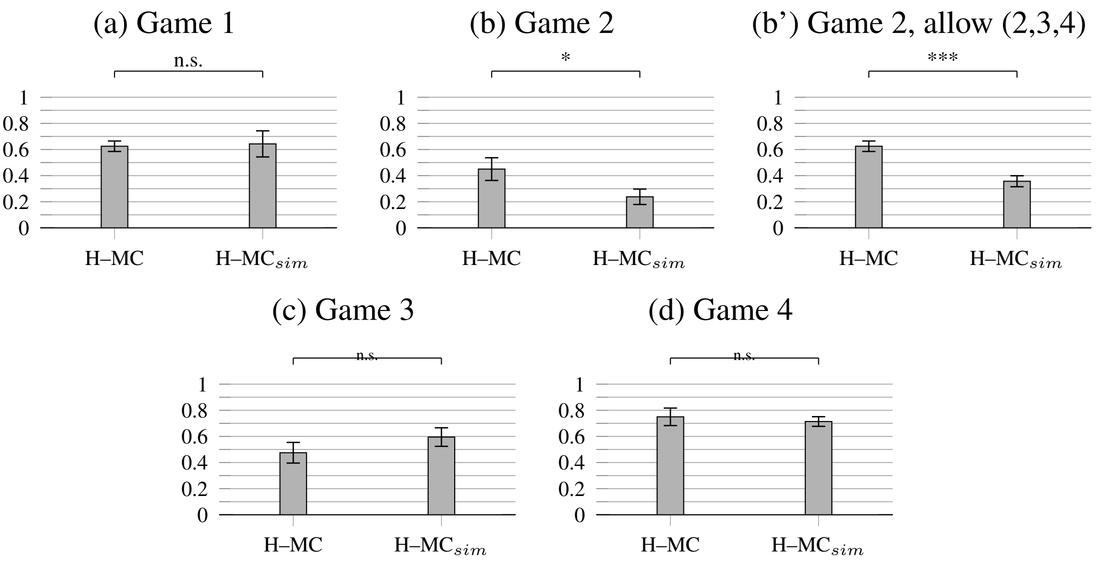
The comparison between sequential mechanisms and simultaneous ones in favoring the formation of efficient coalitions has been the object of recent experimental laboratory studies (Abe et al., 2021). Experimental evidence shows that subjects may perform very differently in these two proposed settings. Analogously, we propose a comparison between the performances of the H–MC mechanism and a pseudo-H–MC mechanism (in the following, denoted as H–MC$_{sim}$), whose structure is identical to that of the original mechanism except that after an offer is proposed, players are asked to either accept or refuse the proposal simultaneously. Theoretically, the H–MC$_{sim}$ mechanism allows for many more Nash equilibria in which two or more players refuse the proposal. We show that sometimes, as observed by Fréchette et al. (2005b), bargaining behavior is not as sensitive to the different bargaining rules as the theory suggests.

The H–MC$_{sim}$ experiment was conducted in December 2019 at ISER at Osaka University. In total, 84 participants, who had never participated in similar experiments before, were recruited. The experimental procedure was identical to the H–MC experiment reported in the main text. On average, the experiment lasted for 1 hour 30 minutes, including the instructions (11 minutes), a comprehension quiz (5 minutes), and payment. The average earnings were 2,780 JPY.

V.1 Grand coalition formation and efficiency

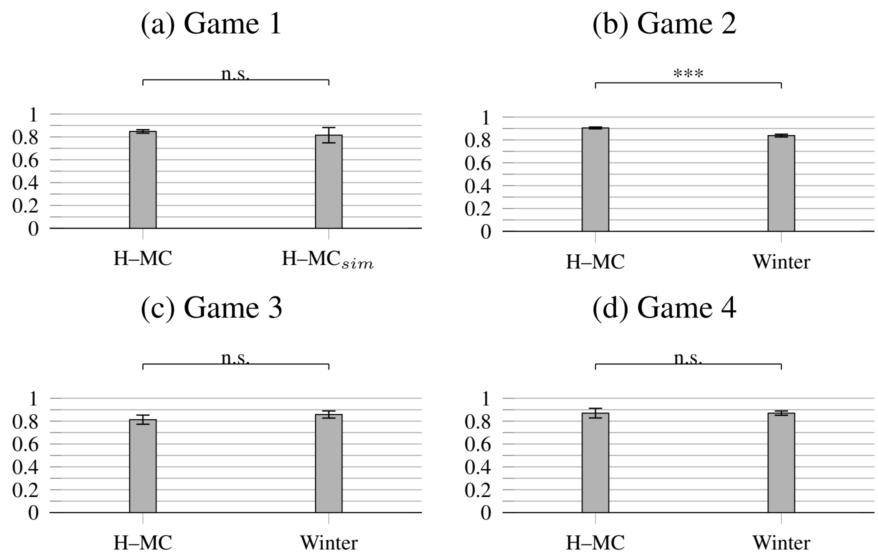
Figures V.8 and V.9 report the results concerning the grand coalition formation and efficiency. The only significant differences reported are for game 2.

Figure V.8: H–MC and H–MC$_{sim}$ mechanisms, proportion of times the grand coalition is formed



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate the proportion of times the grand coalition was formed, which was significantly higher for the H–MC implementation compared with the H–MC$_{sim}$ implementation at the 1%, 5%, and 10% significance levels (Wald test), respectively.

Figure V.9: H–MC and H–MC_{sim} mechanisms, efficiency



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate the efficiency for the H–MC mechanism is significantly higher than for the H–MC_{sim} implementation at the 1%, 5%, and 10% significance levels (Wald test), respectively.

V.2 Analyses based only on groups that formed the grand coalition

Table V.7 reports the results of running a set of OLS regressions as in Eq. 1 based on groups that formed the grand coalition.

Table V.7: H–MC$_{sim}$ mechanism, results of the linear regression based only on the groups that formed the grand coalition

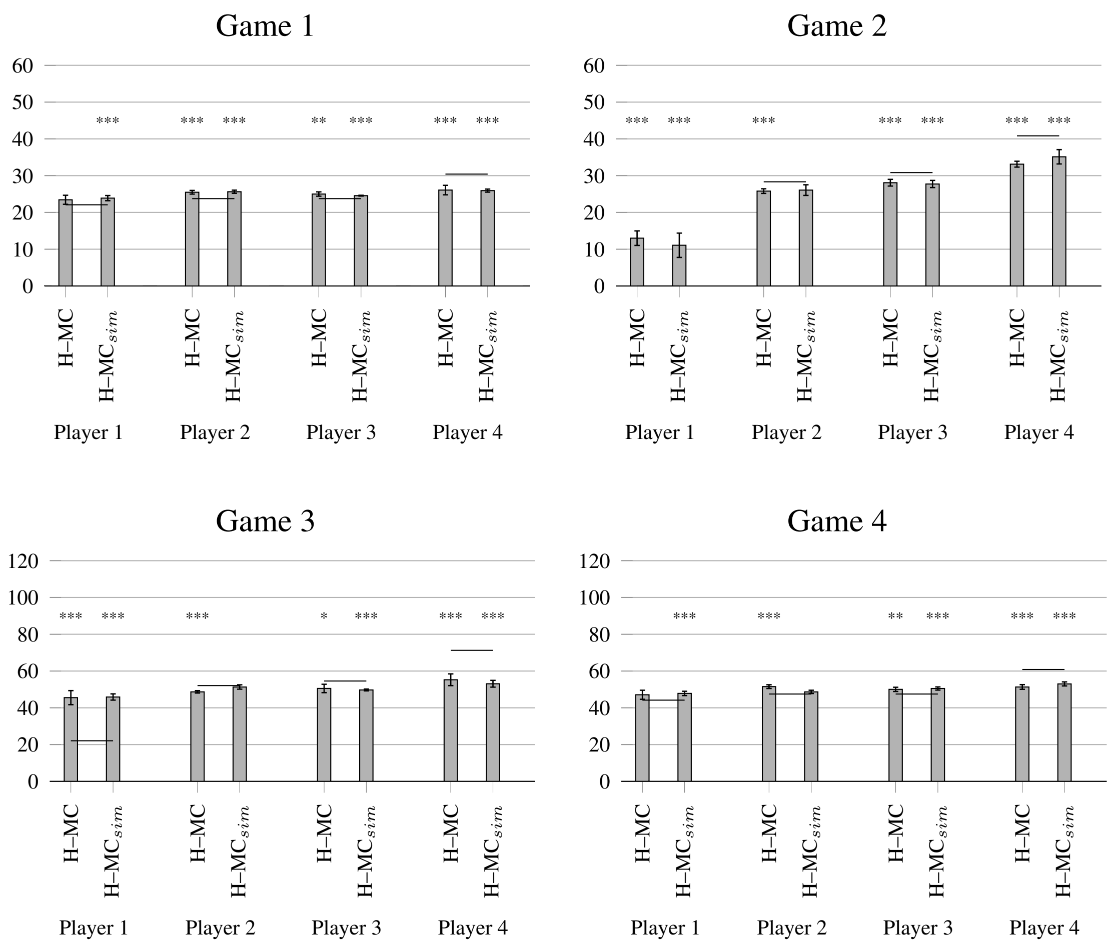
H–MC$_{sim}$

	π_1	π_2	π_3	π_4
g1	23.88	25.63	24.56	25.93
	(0.70)	(0.43)	(0.11)	(0.42)
g2	11.07	26.07	27.73	35.13
	(3.31)	(1.45)	(0.97)	(1.95)
g3	45.88	51.32	49.72	53.08
	(1.70)	(1.19)	(0.50)	(1.84)
g4	47.83	48.67	50.5	53.00
	(1.17)	(0.89)	(0.87)	(1.10)
R^2	0.96	0.99	0.99	0.97
Obs.	97	97	97	97

Note: The standard errors are corrected for within-group clustering effects.

Based on the estimated coefficients, Figure V.10 shows the average payoffs obtained by each player in the four games, conditional on the grand coalition being formed. The horizontal lines indicate the Shapley values for each game. We observe that the two mechanisms perform similarly in that there are players whose average payoff is significantly different from the Shapley value in all four games under both mechanisms.

Figure V.10: H–MC and H–MC$_{sim}$ mechanisms, mean payoffs based only on the groups that formed the grand coalition

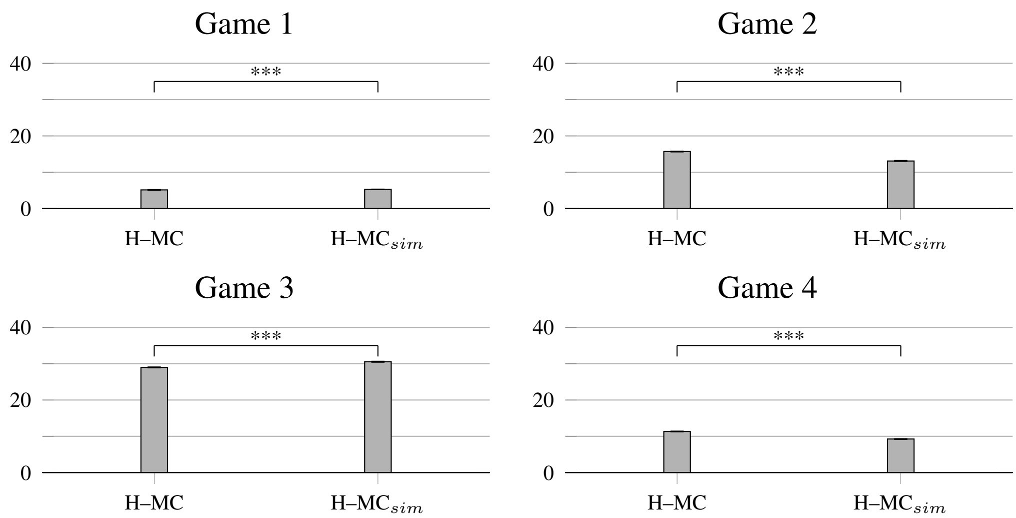


Note: The horizontal lines indicate the Shapley values. The error bars show the one standard error range. The symbols ***, **, and * indicate the average normalized payoff, which is significantly different from the Shapley value at the 1%, 5%, and 10% significance levels (Wald test), respectively.

V.2.1 Distance between the ex ante payoffs and the Shapley value

We compute the distance of the average payoffs from the Shapley value, $Dis2_{\phi}$, using the bootstrapping technique with 1,000 repetitions as we have done in the main text comparing the H–MC and Winter mechanisms. The result is reported in Figure V.11. It can be observed that the ex ante payoffs of H–MC$_{sim}$ are closer to the Shapley values in games 2 and 4, whereas those of H–MC are closer to the Shapley values in games 1 and 3.[23]

Figure V.11: H–MC and H–MC$_{sim}$ mechanisms, distance from Shapley



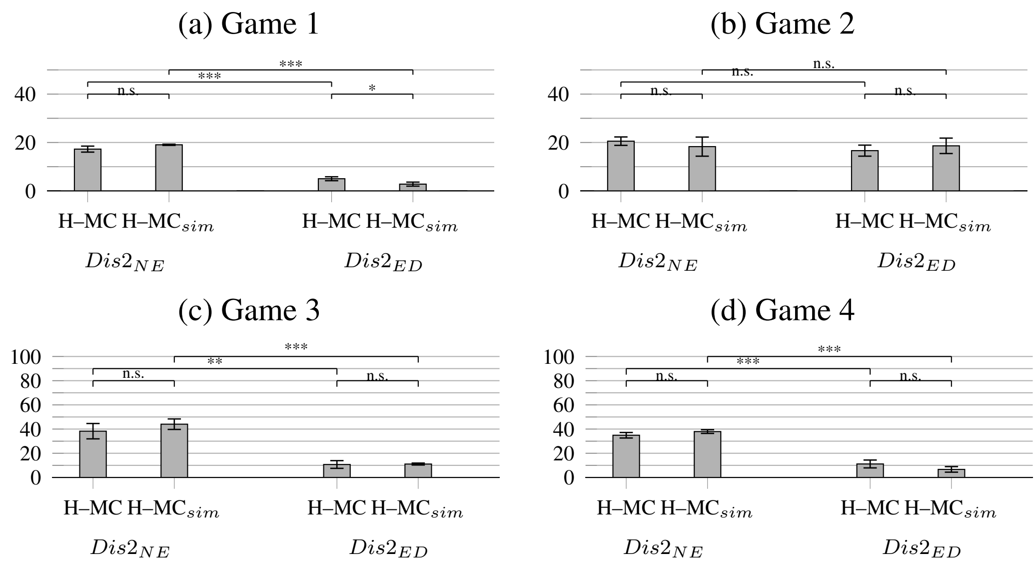
Note: The error bars show the one standard error range. The symbols ***, **, and * indicate significant differences between the H–MC and the H–MC$_{sim}$ mechanisms at the 1%, 5%, and 10% significance levels (two-sample t-test), respectively.

[23]The mean $Dis2_{\phi}$ (standard error) values for the H–MC mechanism are 5.14 (0.035) in game 1, 15.70 (0.064) for game 2, 28.99 (0.074) for game 3, and 11.34 (0.047) in game 4. The corresponding values for H–MC$_{sim}$ are 5.28 (0.018), in game 1, 13.09 (0.088) for game 2, 30.55 (0.091) for game 3, and 9.26 (0.057) in game 4.

V.2.2 Realized allocations and *a posteriori* equilibria

In terms of distance from SPNE or equal division, we observe from Figure V.12 that H–MC_{sim} results in outcomes significantly closer to equal division compared with H–MC only in game 1, whereas in the other games there is no significant difference.

Figure V.12: H–MC and H–MC_{sim} mechanisms, mean of the distances of the realized payoff vectors from the SPNE and the equal division



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate that the distance of the normalized payoff vectors from the equilibrium allocations or from the equal division was significantly different between the H–MC and the H–MC_{sim} implementations at the 1%, 5%, and 10% significance levels, respectively (Wald test).

V.2.3 Axioms

Finally, verification of the axioms (comparing Table V.8 and the left column of Tables III.2 and III.3) indicates that the differences in results between H–MC and H–MC$_{sim}$ are observed for symmetry and fairness (satisfied in H–MC but not in H–MC$_{sim}$).

Table V.8: Results of Wald tests for the verification of the symmetry, additivity, homogeneity, strong monotonicity, and fairness axioms (based only on the groups that formed the grand coalition)

Axiom	H_0	χ^2	p-value	Test
Symmetry	$a_2 = a_3$	5.07	0.024	-
	$d_2 = d_3$	1.11	0.293	
Additivity	$c_1 = a_1 + b_1$	4.84	0.028	-
	$c_2 = a_2 + b_2$	0.03	0.861	
	$c_3 = a_2 + b_3$	14.99	0.000	
	$c_4 = a_4 + b_4$	11.10	0.001	
Homogeneity	$d_1 = 2a_1$	0.00	0.983	+
	$d_2 = 2a_2$	13.12	0.000	
	$d_3 = 2a_3$	2.25	0.134	
	$d_4 = 2a_4$	0.43	0.513	
Null player	$a_2 = 0$	9.90	0.002	-
Strong monotonicity	$a_1 = b_1$	215.83	0.000	-
	$c_1 = d_1$	0.67	0.411	
Fairness	$b_3 - b_2 = c_3 - c_2$	3.02	0.082	-

$+$ indicates that the axiom is considered to be satisfied on average. $-$ indicates the opposite.

To summarize, there is no systematic difference between the H–MC and the H–MC$_{sim}$ mechanisms except that the H–MC better satisfies the symmetry and fairness axioms than does H–MC$_{sim}$ if we focus on the groups that formed grand coalitions.

V.3 Analyses based on all the groups but only on normalized payoffs

Below, we compare H–MC and H–MC$_{sim}$ based on the normalized payoffs but using the data for all groups. Table V.9 reports the results of running a set of OLS regressions as in Eq. 1 using the normalized payoffs as dependent variables.

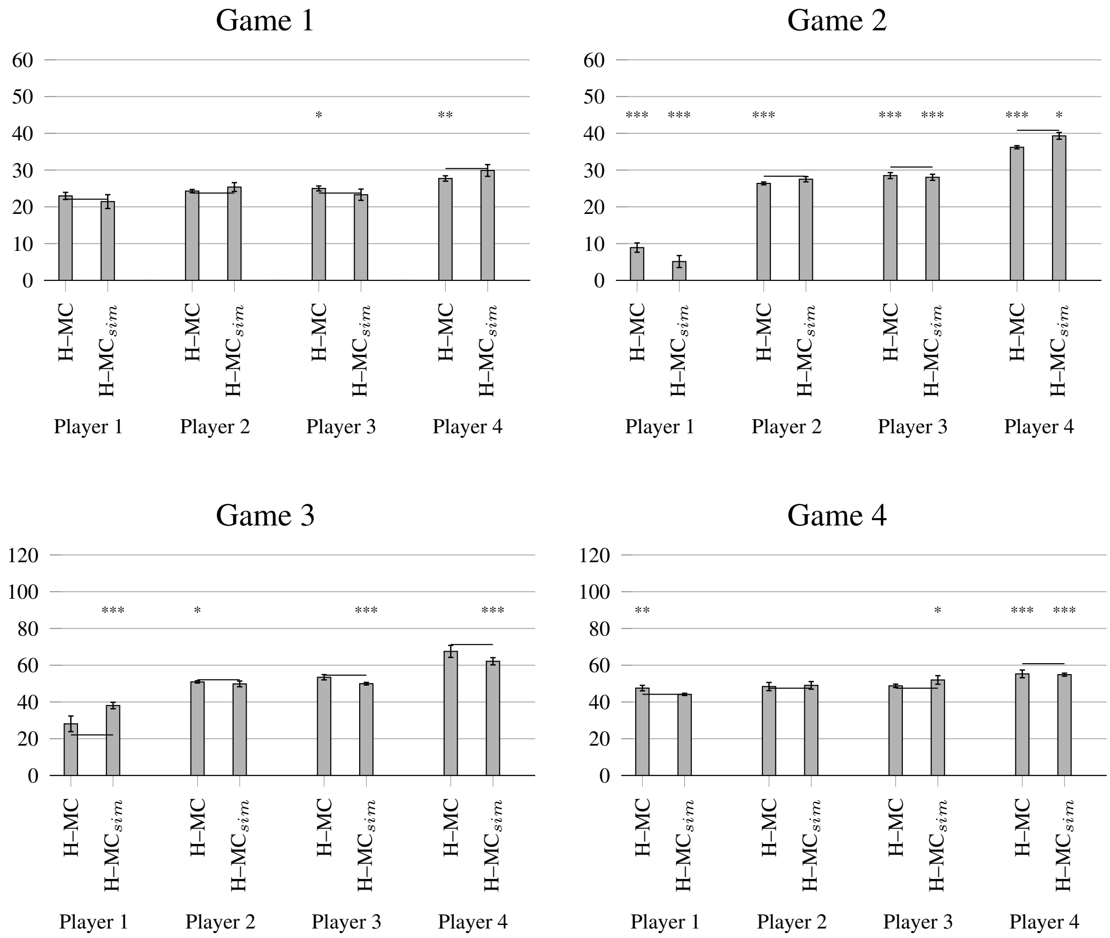
Table V.9: Results of linear regression for normalized payoffs

H–MC$_{sim}$

	$\widetilde{\pi}_1$	$\widetilde{\pi}_2$	$\widetilde{\pi}_3$	$\widetilde{\pi}_4$
g1	21.43	25.38	23.30	29.90
	(1.88)	(1.20)	(1.53)	(1.60)
g2	5.12	27.52	28.04	39.31
	(1.63)	(0.72)	(0.83)	(0.92)
g3	38.06	49.84	49.94	62.15
	(1.75)	(1.56)	(0.70)	(1.96)
g4	44.13	49.03	51.97	54.87
	(0.56)	(2.06)	(2.33)	(0.81)
R^2	0.83	0.96	0.96	0.96
Obs.	168	168	168	168

Based on the estimated coefficients reported in the left panel of Table IV.4 and Table V.9, Figure V.13 shows the average normalized payoffs obtained by each player in the four games under H–MC and H–MC$_{sim}$. The horizontal lines indicate the Shapley values for each game. We observe that for game 1 under H–MC$_{sim}$, the average normalized payoffs of each of the four players are not significantly different from the Shapley values.

Figure V.13: Mean of the normalized payoffs

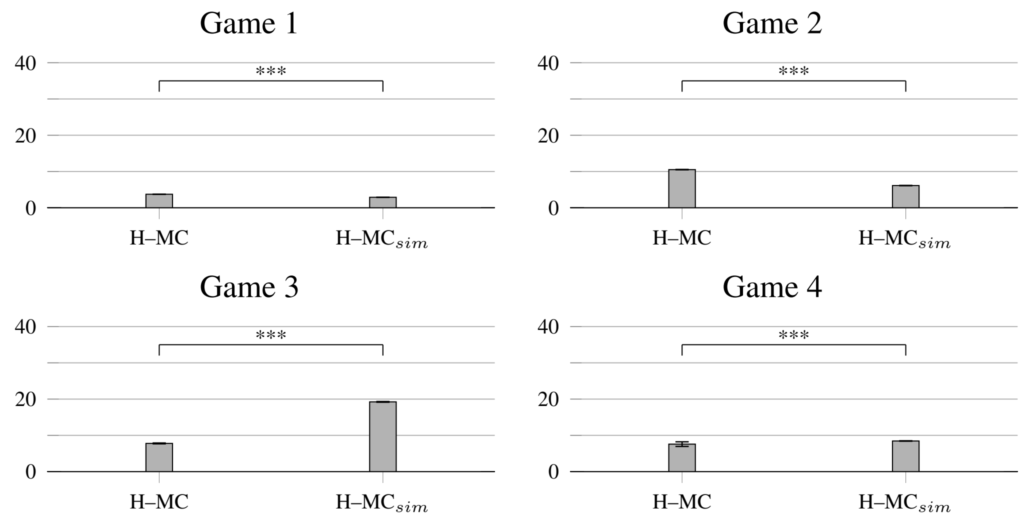


Note: The horizontal lines indicate the Shapley values. The error bars show the one standard error range. The symbols ***, **, and * indicate the average normalized payoff being significantly different from the Shapley values at the 1%, 5%, and 10% significance levels (Wald test), respectively.

V.3.1 Distance between the ex ante payoffs and the Shapley value

We compute the distance of the average normalized payoffs from the Shapley value, $Dis2_{\phi}$, using the bootstrapping technique with 1,000 repetitions, as we have done above. The result is reported in Figure V.14. It can be observed that the ex ante normalized payoffs of H–MC$_{sim}$ are closer to the Shapley values in games 1 and 2, whereas those

Figure V.14: Distance of the normalized payoffs from Shapley



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate the significant difference between the H–MC and the H–MC$_{sim}$ mechanisms at the 1%, 5%, and 10% significance levels (two-sample t-test), respectively.

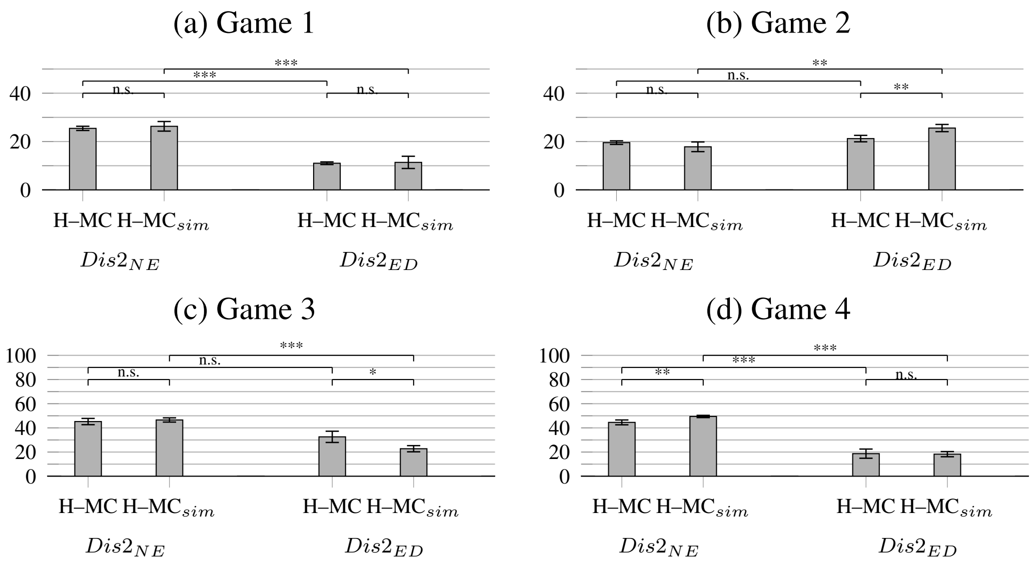
of H–MC are closer to Shapley values in games 3 and 4.[24]

V.3.2 Normalized payoffs and *a posteriori* equilibria

In terms of distance from SPNE or equal division, we observe from Figure V.15 that $Dis2_{NE}$ is significantly smaller under H–MC than under H–MC$_{sim}$ only in game 4. For other games, the values are not significantly different between the two mechanisms. In terms of $Dis2_{ED}$, although it is significantly smaller under H–MC in game 2, the opposite is the case for game 3. For games 1 and 4, there is no significant difference between the two mechanisms. We observe that normalized payoffs are significantly closer to the equal division than the SPNE for both mechanisms in games 1 and 4. For

[24]The mean $Dis2_{\phi}$ (standard error) values based on the normalized payoff for H–MC are 3.73 (0.034) in game 1, 10.52 (0.056) for game 2, 7.78 (0.120) for game 3, and 7.58 (0.638) in game 4. For H–MC$_{sim}$, the corresponding values are 2.89 (0.033) in game 1, 6.13 (0.042) for game 2, 19.24 (0.104) for game 3, and 8.45 (0.058) in game 4.

Figure V.15: Mean of the distances of the normalized payoff vectors from the SPNE and the equal division



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate that the distance of the normalized payoff vectors from the equilibrium allocations or from the equal division was significantly different between the H–MC and the H–MC$_{sim}$ implementations at the 1%, 5%, and 10% significance levels (Wald test), respectively.

H–MC$_{sim}$, $Dis2_{ED}$ is significantly larger than $Dis2_{NE}$ in game 2, whereas the opposite is the case for game 3. For H–MC, $Dis2_{NE}$ and $Dis2_{ED}$ are not significantly different in games 2 and 3.

V.3.3 Axioms

Finally, verification of the axioms (comparing Table V.10 and the left column of Tables IV.5 and IV.6) indicates that the differences in results between H–MC and H–MC$_{sim}$ are observed for additivity and homogeneity (satisfied in H–MC but not in H–MC$_{sim}$).

To summarize, even comparing the payoff shares using all the groups, there is no systematic difference between the H–MC and the H–MC$_{sim}$ mechanisms, except that

Table V.10: H–MC$_{sim}$ normalized payoffs, Wald tests for the verification of the symmetry, additivity, homogeneity, strong monotonicity and fairness axioms

Axiom	H_0	χ^2	p-value	Test
Symmetry	$a_2 = a_3$	1.01	0.314	+
	$d_2 = d_3$	0.47	0.492	
Additivity	$c_1 = a_1 + b_1$	36.91	0.000	-
	$c_2 = a_2 + b_2$	1.11	0.292	
	$c_3 = a_2 + b_3$	0.53	0.466	
	$c_4 = a_4 + b_4$	4.78	0.0288	
Homogeneity	$d_1 = 2a_1$	0.16	0.689	-
	$d_2 = 2a_2$	0.28	0.598	
	$d_3 = 2a_3$	5.90	0.015	
	$d_4 = 2a_4$	3.23	0.072	
Null player	$a_2 = 0$	9.90	0.002	-
Strong monotonicity	$a_1 = b_1$	23.87	0.000	+
(H_0 should be rejected)	$c_1 = d_1$	11.55	0.001	
Fairness	$b_3 - b_2 = c_3 - c_2$	0.15	0.694	+

$+$ indicates that the axiom is considered to be satisfied on average. $-$ indicates the opposite.

H–MC better satisfies the additivity and homogeneity axioms than H–MC$_{sim}$.

VI Translated instruction materials and screenshots of the comprehension quiz

- Winter mechanism: https://www.dropbox.com/s/galeo3todbah7iw/Winter_1_loop_handout.pdf?dl=0
- H–MC mechanism: https://www.dropbox.com/s/ctlw85momf96vmx/HMChandout_seq.pdf?dl=0
- Simultaneous voting version of the H–MC mechanism (H–MC$_{sim}$): https://www.dropbox.com/s/78lf5bn6qi3qfwp/HMChandout_sim.pdf?dl=0