ISSN (Print) 0473-453X
Discussion Paper No. 1175
ISSN (Online) 2435-0982

AN EXPERIMENT ON THE NASH PROGRAM: A COMPARISON OF TWO STRATEGIC MECHANISMS IMPLEMENTING THE SHAPLEY VALUE

Michela Chessa
Nobuyuki Hanaki
Aymeric Lardon
Takashi Yamada

Revised February 2023
May 2022

The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

An experiment on the Nash program: A comparison of two strategic mechanisms implementing the Shapley value

Michela Chessa* Nobuyuki Hanaki† Aymeric Lardon‡ Takashi Yamada§

February 21, 2023

Abstract

We experimentally compare two well-known mechanisms inducing the Shapley value as an *ex ante* equilibrium outcome of a noncooperative bargaining procedure: the demand-based Winter's demand commitment bargaining mechanism and the offer-based Hart and Mas-Colell procedure. Our results suggest that the offer-based Hart and Mas-Colell mechanism better induces players to cooperate and to agree on an efficient outcome, whereas the demand-based Winter mechanism better implements allocations that reflect players' effective power, provided that the grand coalition is formed.

JEL code: C70, C71, C92
Keywords: Nash program, Bargaining procedures, Shapley value, Experiments

*Université Côte d'Azur, CNRS, GREDEG, France. E-mail: michela.chessa@univ-cotedazur.fr
†Institute of Social and Economic Research, Osaka University, Japan. E-mail: nobuyuki.hanaki@iser.osaka-u.ac.jp
‡GATE Lyon Saint-Etienne, UMR 5824 CNRS, Université de Lyon. France. E-mail: aymeric.lardon@univ-st-etienne.fr
§Faculty of Global and Science Studies, Yamaguchi University, Japan. E-mail: tyamada@yamaguchi-u.ac.jp

1 Introduction

Whenever a facility is shared by different customers, departments, or other units of an organization, the problem of how to allocate the costs or the payoffs among players arises. Relevant examples of this situation include airports, transit systems, water distribution networks, inventory models, and scheduling. These contexts are well known as *cost* or *payoff allocation (or sharing) problems*. Similar sharing problems arise in the context of "co-opetition" (Brandenburger and Nalebuff, 1996) where competitors cooperate to achieve a common goal. Usually, in such contexts, two approaches based on game theoretical concepts may be adopted.

One approach is for the players to bargain among themselves to determine how costs or payoffs should be shared. However, this implies a strategic interaction, which may result in unnecessary additional costs if it is conducted in an unrestricted fashion (see, e.g., the arguments by Roth and Verrecchia, 1979). Instead, many bargaining procedures follow the tradition of setting up sequential, perfect information games based on offers, that is, games in which, at each stage, one of the players becomes a proposer of a cost (payoff) allocation, with a requirement for reaching unanimous agreement. Such bargaining procedures represent negotiations in the style of the well-known two-player bargaining over a pie in Rubinstein's problem (Rubinstein, 1982), which is then extended to the n-player case.

Alternatively, one can view the problem as a normative one, in which an external player, a so-called *regulator*, designs a pricing (rewarding) scheme that maximizes some measures of social welfare or that imposes axioms of equity or stability. Shubik (1962) was among the original proponents of the Shapley value (Shapley, 1953) as a method of joint-cost allocation. At present, this value continues to attract the greatest interest among the allocation schemes predicated on notions of cooperative game theory (see, e.g., Littlechild and Owen, 1973; Schulz and Uhan, 2010; Timmer et al., 2013).

Bridging the gap between the strategic and cooperative approaches is recognized as a fundamental issue of game theory. Attempted resolutions of this issue, well known as the *Nash program* (Nash, 1953), have provided many different strategic bargaining mechanisms that sustain the Shapley value at equilibrium (for example, among others, Hart and Mas-Colell, 1996; Pérez-Castrillo and Wettstein, 2001). Such mechanisms fit and unify the two approaches, allowing the players who face an allocation problem (in our specific context) to bargain in a restricted way, and to converge to a stable solution without the need for an intermediary.

Both the original normative implementation of the Shapley value by a regulator and the playing of classical bargaining mechanisms based on offers that lead to the Shapley value require one single agent, either the regulator or one of the players, to propose a complete allocation. In many contexts, the centralization in the hands of a single player is often desirable and encouraged. However, in some specific domains, this may represent an unrealistic assumption, for example, when players are customers of a facility and do not necessarily know about other customers (Young, 1998), or when computing such allocations requires data from each player, some of which may be private (McSherry and Talwar, 2007).

An alternative but less common approach is to describe a bargaining mechanism based on demands rather than offers. A demand-based mechanism was the basis of the implementation by Nash (1953) of the cooperative bargaining solution by Nash (1950). Other examples of mechanisms based on demands, though not common, include Young (1998), who describes a demand revelation mechanism in which potential customers of a public facility simply bid to be served. Bargaining mechanisms based on demands resemble oral auctions, in which each player, standing alone, reveals the charges he or she is ready to pay to be served, or the payoff he or she is ready to accept for offering collaboration, and waits for such a request to be met. In a demand-based mechanism, acceptance of a proposal by the organization typically depends on objective feasibility conditions rather than on subjective approval by its members.

In this paper, we aim to investigate the differences between demand- and offer-based bargaining mechanisms by experimentally comparing the two well-known mechanisms inducing the Shapley value as an *ex ante* equilibrium outcome of a noncooperative bargaining procedure. We choose two mechanisms that are based on these opposing approaches (demand vs. offer) but that remain, in our opinion, similar in terms of the ease with which they can be understood by the participants in a laboratory experiment.[1] The first mechanism is *Winter's demand commitment bargaining mechanism* (Winter, 1994, referred to as the *Winter mechanism* below). The second is the *Hart and Mas-Colell procedure* (Hart and Mas-Colell, 1996, referred to as the *H–MC mechanism* below).

Both procedures are described as sequential, perfect information games, where, at each stage, a player becomes a proposer. In accordance with the theoretical presentation of the two mechanisms, we illustrate the bargaining procedures to define a sharing of payoffs rather than an allocation of costs.[2] In the first mechanism, which is defined for cooperative games with increasing returns to scale for cooperation (strictly convex games), the proposer makes a demand for him- or herself concerning the payoff that he or she is willing to receive from a possible collaboration. In the second mechanism, which is defined for monotonic games (a much weaker assumption), the proposer makes

[1] A comparison between offer-based and demand-based mechanisms has been conducted experimentally for voting games by Fréchette et al. (2005a), as well as empirically by, for example, Warwick and Druckman (2001) and Ansolabehere et al. (2005), employing field data.

[2] It is straightforward to theoretically establish the equivalence between these two.

a proposal to each of the other players concerning the payoff he or she is willing to offer them.

Two main issues arise with most strategic bargaining models, as observed by Fréchette et al. (2005a), in experimental analyses of some well-known legislative bargaining processes. First, the equilibrium solution may require an unrealistic degree of rationality on the part of the players, such that the experimental evidence is very far from the theoretical prediction. Second, partly related to the first point, while the theoretical predictions are very sensitive to variations in the rules of the game, the observed bargaining behaviors in the experiment are not always so. In our case, although two mechanisms have the same ex ante prediction (the Shapley value as expected payoffs), it is possible that the degree to which the observed behavior deviates from the theoretical prediction, and the reason for doing so, may differ greatly between the two.

Note that, it has been argued that the difference between a demand-based vs. an offer-based mechanism is less relevant when considering two-player games, such as in Rubinstein (1982)'s bargaining-over-a-pie game (see, Fréchette et al., 2005a). However, it may become crucial when considering groups with at least three members. Notice that, on one hand, in an offer-based mechanism, because proposers propose an allocation by dividing the worth of the coalition, it is not difficult for a proposal to satisfy both the feasibility and efficiency conditions. In demand-based mechanisms, on the other hand, because each player separately makes his/her demand, coordination among them to make the submitted set of demands as a whole to satisfy these two conditions becomes more difficult as the group becomes larger.

Our analysis mainly focuses on (i) analyzing whether these mechanisms lead to formation of the grand coalition and (ii) testing the convergence in expected value and, as predicted by the theory, to the Shapley value.

Our results show that the H–MC mechanism results in a higher frequency of grand coalition formation and a higher efficiency than does the Winter mechanism. Conversely, the Winter mechanism better implements the Shapley value as the average payoff provided that the grand coalition is formed. Therefore, our results suggest that an offer-based H–MC mechanism better induces players to cooperate and to agree on an efficient outcome, whereas a demand-based Winter mechanism better implements allocations that reflect players' effective power provided the grand coalition is formed.

The remainder of the paper is organized as follows. Section 2 reviews existing studies that are most relevant to our work. Section 3 presents the general definition and the properties of a cooperative transferable utility (TU) game, as well as the Shapley value. Section 4 presents the two mechanisms that we investigate, namely the Winter and the H–MC mechanisms. Section 5 describes the setting of our experiment. The results are presented in Section 6, and Section 7 concludes. Additional analyses that supplement our results and provide new points for reflection are contained in the Online Appendices I to VI.

2 Related work

Bridging the gap between the noncooperative models, in which the primitives are the sets of possible actions of individual players, and the cooperative models, in which they are the sets of possible joint actions of groups of players, has been recognized as a fundamental issue of game theory. The very first attempt at this so-called Nash program dates back almost 70 years to Nash himself (Nash, 1953). His idea was to provide a noncooperative foundation for cooperative solution concepts, and he began doing so by proposing a noncooperative game that sustained the Nash solution of his two-player *bargaining problem* (Nash, 1950) as its equilibrium. Following this first attempt by Nash, many alternative procedures for implementing solutions of two-player bargaining problems or n-player pure bargaining problems[3] have been proposed. Some mechanisms intended to obtain the Nash solution, exactly or approximately, at equilibrium (see, among others, Binmore et al., 1986; Trockel, 2002). Others aimed instead to obtain the Kalai–Smorodinsky solution (Kalai and Smorodinsky, 1975), that is, the main alternative solution to such problems (Moulin, 1984b; Trockel, 1999; Haake, 2000).

Many different theoretical mechanisms have been designed with the aim of implementing other cooperative solution concepts via a strategic interaction of the players for more generic cases, that is, when there are more than two players or when the bargaining problem is not pure. This is the case, for example, in the seminal work of Harsanyi (1974), who reinterpreted the von Neumann–Morgenstern solution as an equilibrium of a noncooperative bargaining mechanism, and of the many works sustaining the most famous axiomatic solution concept by Shapley (1953), the Shapley value. For a relevant and extensive review of the theoretical literature on the Nash program, we refer readers to the surveys by Serrano (2005, 2008, 2014, 2021).

In this section, we focus on the literature devoted to testing cooperative game theory through experiments. To date, this literature has focused mainly on three different directions. The first direction provides a normative interpretation, as in De Clippel and Rozen (2022), in which subjects designated as decision-makers express their view on what is fair for others by recommending a payoff allocation. De Clippel and Rozen (2022) show that the decision-maker's choices can be described as a convex combination of the Shapley value and the equal division solution.

[3] A pure bargaining problem is a cooperative game in which only the grand coalition N creates a positive surplus with respect to what each player can achieve if he or she does not cooperate with anyone.

The second direction investigates how an unstructured interaction affects the final agreement. One example is the paper by Kalisch et al. (1954), in which groups of players are asked to freely discuss the formation of coalitions and to reach an agreement on how to split the related values. The authors identify many different factors that influence the final outcome of such a procedure, including personality differences or the geometrical arrangement of players around the table. Similarly, but with a greater focus on voting games, Montero et al. (2008) propose an unstructured bargaining protocol in which participants propose and vote on how to distribute a fixed budget among themselves. The paper provides experimental evidence of the so-called *paradox of new members*, according to which enlargement of a voting body (i.e., the addition of a new voter) can increase the voting power of an existing member. Guerci et al. (2014) study the impact of variations in the experimental protocol of Montero et al. (2008) on the formation of the so-called minimal winning coalitions, that is, coalitions for which each player is crucial.

Most experimental works in the literature follow a third direction, studying the outcome when a more formal (or structured) bargaining protocol is imposed. Our paper broadens this last direction of research.

Formal bargaining protocols have been designed to tackle different aspects of the cooperative inclination of the players under different settings. For example, Murnighan and Roth (1977) investigate the effect of various communication/information conditions on the final outcome in a specific game played by a monopolist and two weaker players. They show how the results over the entire set of conditions closely approximate the Shapley value, although they often report a clear tendency for an equal split of the pie. Similarly, Murnighan and Roth (1982) introduce bargaining models to investigate the influence of information shared by subjects about the games (e.g., payoffs) on the final outcome. They show that the quality of the information has an impact on the final outcome and that the Nash bargaining solution has a good predictive performance in many cases. Bolton et al. (2003) investigate how the communication configuration affects coalition negotiation and show how players with weaker alternatives would benefit from a more constrained structure, especially if they can be the conduit of communication, whereas those endowed with stronger alternatives benefit from working within a more public communication structure that promotes competitive bidding. Other works focus more specifically on the coalition formation process, including Nash et al. (2012); Shinoda and Funaki (2019); Abe et al. (2021). In the first paper, the authors propose finitely repeated three-person coalition formation games, showing how efficiency requires people's willingness to accept the agency of others, such as political leaders. The second paper is then presented as a follow-up, in which the authors maintain the same value of the coalitions as in Nash et al. (2012), but design a different bargaining protocol. They report a rare formation of a grand coalition, which can be induced by some external factors, such as the presence of a chat window. The third paper presents a comparison between two mechanisms that invite players to join a meeting simultaneously or sequentially. The authors report that the sequential mechanism induces a higher social surplus than the simultaneous mechanism. Moreover, players make choices consistent with the subgame perfect Nash equilibrium (SPNE) in the sequential setting and choose the dominant strategy in the simultaneous setting, when a dominant strategy exists.

Formal bargaining protocols are mostly based on the implementation of allocations, which are shown to converge to some specific well-known solutions. This is the case, for example, in Nash (1953) and Harsanyi (1974), which we have referred to above, or in the case of the bargaining mechanism proposed by Raiffa (1953) to implement the Raiffa solution (as opposed to the Nash solution) to the Nash cooperative bargaining problem. Several experiments have been conducted, with the final goal of testing Nash axioms, or of comparing Nash and Raiffa solutions (see, e.g., Nydegger and Owen, 1975; Rapoport et al., 1977). In addition, there is a large literature devoted to studying the class of bidding mechanisms. Bidding mechanisms are introduced by Demange (1984) and Moulin (1984a), and Moulin and Jackson (1992) study them in economic environments. They are developed by Pérez-Castrillo and Wettstein (2001) and Ju and Wettstein (2009) to implement solution concepts in the framework of cooperative TU games.

In particular, many different theoretical mechanisms have been designed specifically with the aim of implementing the best-known cooperative solution, the Shapley value (see Shapley, 1953). Because this solution is applied in many economic problems, supporting it through strategic explanation is considered to be particularly important. See among others, Harsanyi (1981), Gul (1989), Hart and Moore (1990), Winter (1994), and Hart and Mas-Colell (1996).[4]

Despite the large body of existing literature, the Nash program *"is not ready for retirement yet"*, but is, on the contrary, *"still full of energy"* and *"waiting for good papers to be written"* (Serrano, 2021). In this paper, we aim to contribute to this research agenda by providing new insights gained from a controlled laboratory experiment. In particular, we propose an experimental comparison of two mechanisms. The first mechanism is the one-period version developed by Winter (1994) (this simplified version was also previously used by Bennett and van Damme (1991) to treat Apex games, a type of weighted majority games). The second mechanism is by Hart and Mas-Colell (1996), in the specific case in which a proposer whose proposal is rejected leaves the game with a

[4] Krishna and Serrano (1995) deepen the study of the set of subgame perfect equilibria associated with the bargaining mechanism proposed by Hart and Mas-Colell (1996).

probability 1. Our work is similar to Fréchette et al. (2005a), who experimentally compare an offer-based model of Baron and Ferejohn (1989) with a demand-based model of Morelli (1999) in weighted majority voting games. Earlier experimental studies of the Baron–Ferejohn model include Fréchette et al. (2003, 2005c), and Fréchette et al. (2005b) provide an experimental study of demand bargaining.[5] However, Fréchette et al. (2005a) present the first work to directly compare the two within an experimental framework. Their results show that proposers have some first-mover advantage in both the demand and offer games, but their power does not differ nearly as much between the two mechanisms as theory predicts.

3 Theoretical model

3.1 Cooperative TU games and solutions

Let $N = \{1, \dots, n\}$ be a finite set of *players*. Each subset $S \subseteq N$ is called a *coalition*, and N is called the *grand coalition*. A *cooperative TU game* (from now on, *cooperative game*) consists of a couple (N, v), where N is the set of players and $v : 2^N \to \mathbb{R}$ is the *characteristic function*, which assigns to each coalition $S \subseteq N$ the *worth* $v(S)$, with the normalization condition $v(\emptyset) = 0$. The worth of a coalition represents the value that members of S can achieve by agreeing to cooperate. To simplify the notation if no ambiguity appears, we consider the set of players N as fixed and we write v instead of (N, v). We use $\mathcal{G}^N$ to denote the set of all games with player set N.

A game $v \in \mathcal{G}^N$ is said to be

- *monotonic* if $v(S) \leq v(T)$ for each $S \subseteq T \subseteq N$,

[5] Fiorina and Plott (1978) propose multiple experiments on committee decision-making under majority rules to test a wide range of solution concepts of noncooperative games.

- *superadditive* if $v(S) + v(T) \leq v(S \cup T)$ whenever $S \cap T = \emptyset$, with $S, T \subseteq N$,
- *convex* if $v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$, for each $S, T \subseteq N$, and *strictly convex* if the inequality holds strictly.

We observe that convexity $\Rightarrow$ superadditivity $\Rightarrow$ monotonicity. In (strictly) convex games, cooperation becomes increasingly appealing, and a so-called "snowball effect" is expected, leading to the formation of the grand coalition. Another equivalent definition for convexity can be stated as $v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T)$, for each $S \subseteq T \subseteq N \setminus \{i\}$.

Given a game $v \in \mathcal{G}^N$, an *allocation* is an n-dimensional vector $(x_1, \ldots, x_n) \in \mathbb{R}^N$, assigning to player i the amount $x_i \in \mathbb{R}$. For each $S \subseteq N$, we assume that $x(S) = \sum_{i \in S} x_i$. The *imputation set* is defined by:

$$I(v) = \{x \in \mathbb{R}^n | x(N) = v(N) \text{ and } x_i \geq v(\{i\}) \; \forall i \in N\},$$

that is, it contains all the allocations that are *efficient* ($x(N) = v(N)$) and *individually rational* ($x_i \geq v(\{i\}) \forall i \in N$).

The core is the set of imputations that are also *coalitionally rational*, that is,

$$C(v) = \{x \in I(v) | x(S) \geq v(S) \; \forall S \subseteq N\}.$$

An element of the core is stable in the sense that if such a vector is proposed as an allocation for the grand coalition, no coalition will have an incentive to split off and cooperate on its own. Intuitively, the idea behind the core is analogous to that behind a (strong) Nash equilibrium of a noncooperative game, namely an outcome is stable if no deviation is profitable. For the Nash equilibrium, the possible deviation concerns a single player, whereas in the core, deviations of groups of players are relevant.

A *solution* is a function $\psi : \mathcal{G}^N \to \mathbb{R}^N$ that assigns an allocation $\psi(v)$ to every game $v \in \mathcal{G}^N$. The *Shapley value* is the best-known solution concept, which is widely applied in economic models, and is defined as:

$$\phi_i(v) = \sum_{S \subseteq N, i \in S} \frac{(|S|-1)!(|N|-|S|)!}{|N|!}(v(S) - v(S \setminus \{i\}))\ \forall i \in N.$$

The Shapley value assigns to every player his or her expected marginal contribution to the coalition of players that enter the game before this player, given that every order of entrance has equal probability. This solution concept has been defined as respecting some notion of fairness, but it is not necessarily stable. However, if the game is super-additive, the Shapley value is an imputation, and if the game is convex, it belongs to the core (in particular, it is its barycenter).

Players i and j are *symmetric* in $v \in \mathcal{G}^N$, if $v(S \cup \{i\}) = v(S \cup \{j\})$ for all $S \subseteq N \setminus \{i, j\}$. Player i is a *null player* in $v \in \mathcal{G}^N$ if $v(S) = v(S \setminus \{i\})$ for all $S \subseteq N$.

In the literature, we find various axiomatic characterizations of cooperative solutions and, in particular, of the Shapley value (Shapley, 1953; Young, 1985; van den Brink, 2002). Given a solution $\psi : \mathcal{G}^N \to \mathbb{R}^N$, we list here the four axioms that are used in the characterization by Shapley (1953), and later be used in analyzing our data.[6]

Axiom 1 (Efficiency): for every v in $\mathcal{G}^N$, $\sum_{i \in N} \psi_i(v) = v(N)$.

Axiom 2 (Symmetry): if i and j are symmetric players in game $v \in \mathcal{G}^N$, then $\psi_i(v) = \psi_j(v)$.

Axiom 3 (Additivity): for all $v, w \in \mathcal{G}^N$, $\psi(v + w) = \psi(v) + \psi(w)$.

[6]See Online Appendix IV for other axioms that have been proposed to characterize the Shapley value.

Axiom 4 (Null player property): if i is a null player in game $v \in \mathcal{G}^N$, then $\psi_i(v) = 0$.

4 Two mechanisms

In this section, we present the demand-based Winter mechanism (Section 4.1) and the offer-based H–MC mechanism (Section 4.2) in more detail. Section 4.3 compares the equilibrium predictions of the two mechanisms with a simple example.

4.1 The Winter mechanism

Winter (1994) presented a bargaining model based on sequential demands for strictly convex cooperative games. As noted, in such games, cooperation becomes increasingly appealing and a "snowball effect" is expected, leading to the formation of the grand coalition. Moreover, in convex games, the Shapley value is a central point in the core, which is always nonempty.

In this model, players announce their demands publicly in turns. That is, the players effectively state "I am willing to join any coalition that yields me..." and wait for these demands to be met by other players. The bargaining starts with a randomly chosen player from N, say player i. This player publicly announces his or her demand d_i and then points to a second player, who has to state his or her demand. Then, the game proceeds by having each player introduce a demand then point at a new player to take a turn. If or when, at some point, a compatible demand is introduced, which means that there exists a coalition S for which the total demand for players in S does not exceed $v(S)$, then the first player with such a demand selects a compatible coalition S. The players in S receive their demands and leave the game, and the bargaining continues with the rest of the players using the same rule on v restricted on $N \setminus S$.

Here, we present the one-period Winter mechanism we consider in our experiment more formally. This is a simplified version of the more general mechanism in Winter (1994), which allows for more periods and includes a discount factor. A decision point position at time t of the one-period demand commitment game is given by the vector $(S_1^t, S_2^t, d_{S_2^t}, j)$, where:

$S_1^t \subseteq N$ is the set of players remaining in the game,

$S_2^t \subset S_1^t$ is the set of players who have submitted demands that are not yet met,

$d_{S_2^t} = (d_i)_{i \in S_2^t}$ is the vector of demands submitted by players in S_2^t, $(0 \leq d_i \leq \max_{S \subseteq N} v(S))$, and

$j \in S_1^t \setminus S_2^t$ is the player taking the decision by introducing a demand d_j. His or her demand d_j is said to be *compatible* if there exists some $S \subseteq S_2^t$ with $v(S \cup \{j\}) - \sum_{i \in S} d_i \geq d_j$. Otherwise, d_j is not compatible.

With j's decision, the game proceeds in the following way:

1) If d_j is compatible, then j specifies a compatible coalition S, each player $i \in S \cup \{j\}$ is paid d_i, and a player $k \neq j$ is randomly chosen from $S_1^t \setminus S_2^t$. The new position is now given by $(S_1^{t+1}, S_2^{t+1}, d_{S_2^{t+1}}, k)$, with $S_1^{t+1} = S_1^t \setminus (S \cup \{j\})$ and $S_2^{t+1} = S_2^t \setminus (S \cup \{j\})$.

2) If d_j is noncompatible, then two cases are distinguished:

2_a) if $S_2^t = S_1^t \setminus \{j\}$ (j is the last player to make a demand), then each player $i \in S_1^t$ (j included) gets his or her individual payoff $v(\{i\})$, and the game ends;

2_b) if $S_2^t \subset S_1^t \setminus \{j\}$, then j specifies a new player $k \neq j$ in $S_1^t \setminus S_2^t$ and the new position is $(S_1^{t+1}, S_2^{t+1}, d_{S_2^{t+1}}, k)$, with $S_1^{t+1} = S_1^t$ and $S_2^{t+1} = S_2^t \cup \{j\}$.

The game starts with a randomly chosen player $j \in N$. Then, the initial position is set to be $(N, \emptyset, d_\emptyset, j)$. It terminates either when there are no more players in the game (see point 1 above), or when $S_1^t \cup \{j\} = S_2^t$ (see point 2_a above).

As shown by Winter for the more generic case, this mechanism has a unique subgame perfect equilibrium, which assigns equal probabilities according to the principle of indifference. At this equilibrium, the grand coalition forms and the *a priori* expected equilibrium payoff coincides with the Shapley value. Moreover, given a specific ordering of the players, the *a posteriori* equilibrium payoff of each player depends on the order of players only through the set of the player's successors but it is not influenced by the way that these players are ordered, as each player demands a marginal contribution to the set of successors.

4.2 The Hart and Mas-Colell mechanism

Hart and Mas-Colell (1996) proposed a bargaining procedure for monotonic cooperative games. This is a much weaker assumption compared with the strict convexity required by the Winter mechanism. Thus, the H–MC procedure is applicable for a larger set of cooperative games.

In this mechanism, the bargaining starts with a randomly chosen proposer making an offer to the other players, with the meaning "If you agree to form a coalition with me, I will give you...". Then the other players, who act sequentially, may either accept or reject the proposal. The requirement for agreement is unanimity. The key modeling issue is the specification of what happens if there is no agreement and, as a consequence, the game moves to the next stage. The more general mechanism by Hart and Mas-Colell (1996) allows for a proposer, even after a rejection, to remain in the game and join the other to next stage with a given probability. In our experiment, we consider the special case in which such a probability is zero, and then, if the proposal is rejected, the proposer leaves the game with his or her individual value and the bargaining continues among the rest of the players, with a new player randomly chosen as a new proposer[7].

We present a more formal description of the H–MC mechanism. A decision point position at time t is simply given by the vector (S^t, j), where:

$S^t \subseteq N$ is the set of players remaining in the game,

$j \in S^t$ is the player making an offer to the remaining players $(t_i)_{i \in S^t \setminus \{j\}}$ such that $\sum_{i \in S^t \setminus \{j\}} t_i \leq v(S^t)$.

With j's proposal, the game proceeds now in the following way:

1) If all $i \in S^t \setminus \{j\}$, who decide sequentially, accept the proposal one after the other, then players in $S^t \setminus \{j\}$ are paid $(t_i)_{i \in S^t \setminus \{j\}}$, player j is paid $v(S^t) - \sum_{i \in S^t \setminus \{j\}} t_i$, and the game ends;

2) If at least one player $i \in S^t \setminus \{j\}$ refuses the offer, then two cases are distinguished:

$2a$) if $|S^t| = 2$ (only one more player is left, together with j), then they both receive their individual value $v(\{i\})$ for each $i \in S^t$, and the game ends;

$2b$) if $|S^t| > 2$, then player i is removed from the game, he or she receives his or her individual payoff $v(\{i\})$, a new proposer $k \in S^{t+1} = S^t \setminus \{j\}$ is randomly

[7] A first simplified version of the mechanism by Hart and Mas-Colell had already appeared in Mas-Colell (1988).

selected, and the new position is (S^{t+1}, k).

The game starts with a randomly chosen player $j \in N$. Then, the initial position is set to be (N, j). It terminates either when there are no more players in the game (see point $2a$ above), or when the proposal is unanimously accepted (see point 1 above).

Hart and Mas-Colell (1996) show that this game has a unique subgame perfect equilibrium. At this equilibrium, the grand coalition forms and the *a priori* expected equilibrium payoff coincides with the Shapley value. In contrast to the Winter mechanism, given a specific initial proposer $j \in N$ (in the previous mechanism, it was necessary to specify the order of all the players, whereas in this case only one player, the proposer, needs to be specified at equilibrium), the *a posteriori* equilibrium payoff assigns to each other player his or her Shapley value in the cooperative game, reduced to the set of players $N \setminus \{j\}$, and the proposer is assigned his or her marginal contribution to the grand coalition $v(N) - v(N \setminus \{j\})$.

4.3 A comparison between the Winter and the H–MC mechanisms

We illustrate the two mechanisms using the strictly convex three-player game shown in Table 1. Although our experiment is based on four-player games, a three-player game example is of particular interest because it allows us to graphically represent the imputation set, the core, and the different solutions, as illustrated in Figure 1.

Table 1: A three-player game

S	1	2	3	1,2	1,3	2,3	N
$v(S)$	20	20	30	45	55	60	100

As we have already observed, the convexity assumption implies the monotonicity. Thus, the game satisfies the assumptions of both the Winter and H–MC mechanisms. The Shapley value of this game is given by the vector $\phi(v) = \left(\frac{170}{6}, \frac{185}{6}, \frac{245}{6}\right) = (28.33, 30.83, 40.83)$, which corresponds to the *a priori* equilibrium payoff for both the Winter and H–MC mechanisms.

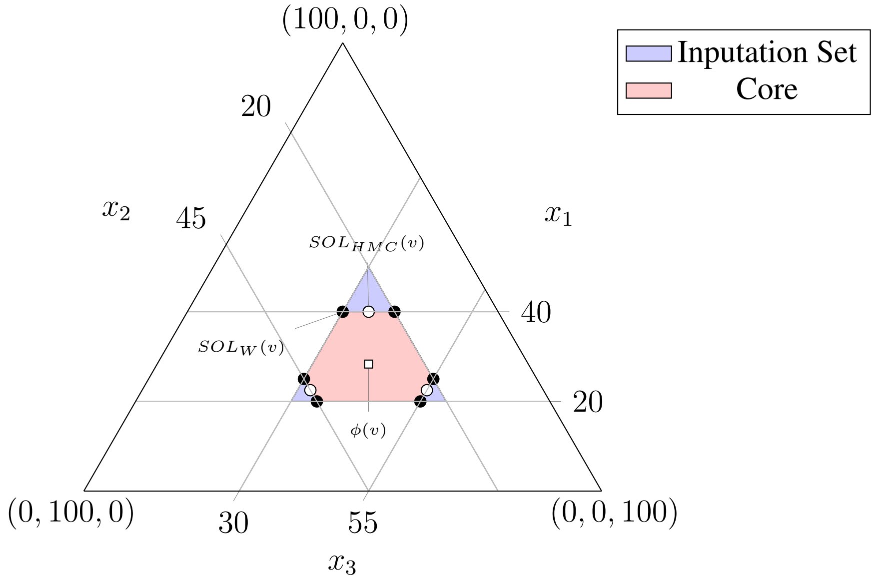
We suppose now that player 1 is chosen randomly as the first proposer in both mechanisms. Independently of the order of the following players in the Winter mechanism, the proposer will receive an *a posteriori* equilibrium payoff equal to 40 in both mechanisms, which corresponds to his or her marginal contribution to the grand coalition $v(N) - v(N \setminus \{1\})$. We can see that both mechanisms lead to a proposer advantage, as $40 > \frac{170}{6}$, meaning that, as the first proposer, player 1 can obtain more than his or her Shapley value.

Suppose now that the total ordering of the players in the Winter mechanism is given by 1, 2, and 3. The *a posteriori* equilibrium payoff of the Winter mechanism is given by the vector $SOL_W(v) = (40, 30, 30)$, in which player 2 demands his or her marginal contribution $v(\{2,3\}) - v(\{3\})$, and player 3 demands his or her individual value $v(\{3\})$.

Conversely, in the case of the H–MC mechanism, the proposer offers the Shapley value of the reduced game to players 2 and 3. Thus, the *a posteriori* equilibrium payoff is given by the vector $SOL_{HMC}(v) = (40, 25, 35)$. Even with the disadvantage of not being the first mover, player 2, as the second mover, manages to obtain more under the Winter mechanism than under the H–MC mechanism even if, in both cases, he or she obtains less than his or her Shapley value.

Figure 1 shows the imputation set $I(v) = co\,\langle(20, 50, 30), (50, 20, 30), (20, 20, 60)\rangle$, the core $C(v) = co\,\langle(40, 30, 30), (40, 20, 40), (25, 20, 55), (20, 25, 55), (20, 45, 35), (25, 45, 30)\rangle$, the Shapley value $\phi(v)$, and possible *a posteriori* solutions $SOL_W(v)$ (6 black dots)

Figure 1: The core of the three-player game



and $SOL_{HMC}(v)$ (3 white dots). A point in the simplex corresponds to an allocation (x_1, x_2, x_3). For example, the height of a point from the edge that is opposite to the apex labeled $(100, 0, 0)$ represents the payoff allocated to player 1. Thus, a point on the bottom edge represents an observed allocation that gives a zero payoff to player 1. Similarly, the height of a point from the edge that is opposite to the apex labeled $(0, 0, 100)$ represents the payoff allocated to player 3.

We make the following observation to conclude this example and the comparison between the two mechanisms.

Observation 1. *The core is always a convex polyhedron. The a posteriori equilibrium of the Winter mechanism always coincides with a vertex of this polyhedron. The a posteriori equilibrium of the H–MC mechanism always provides a vector on a face of this polyhedron.*

Table 2: The games

S	1	2	3	4	1,2	1,3	1,4	2,3	2,4	3,4	1,2,3	1,2,4	1,3,4	2,3,4	N
$v_1(S)$	0	5	5	10	20	20	25	20	25	25	50	60	60	60	100
$v_2(S)$	0	20	20	30	20	20	30	45	55	60	45	55	60	100	100
$v_3(S)$								$= v_1(S) + v_2(S)$							
$v_4(S)$								$= 2v_1(S)$							

5 The experimental setting

5.1 The games

We consider the four four-player games shown in Table 2 in our experiment. These games are chosen to test the properties of the Shapley value that are discussed in Section 3.1. Note that:

- games 1, 3, and 4 are strictly convex, whereas game 2 is only convex. All four games are, by consequence, monotonic. Therefore, all four games respect the assumptions of the H–MC mechanism, whereas all except game 2 respect the assumptions of the Winter mechanism. However, with game 2 being only convex, we consider that “strict convexity” could be relaxed and the game could still be played in such a case;

- in games 1 and 4, players 2 and 3 are symmetric;

- in game 2, player 1 is a null player. This is the reason why the game is only convex, but not strictly convex, as the presence of a null player does not allow, by definition, the possibility of having a strictly increasing marginal contribution for such a player;

Table 3: The Shapley values of games 1, 2, 3, and 4

	$\phi_1(v)$	$\phi_2(v)$	$\phi_3(v)$	$\phi_4(v)$
Game 1	22.08	23.75	23.75	30,42
Game 2	0	28.33	30.83	40.83
Game 3	22.08	52.08	54.58	71.25
Game 4	44.16	47.5	47.5	60.83

- game 3 is defined as the sum of games 1 and 2;
- game 4 is defined as twice game 1 and it preserves the symmetry of players 2 and 3;

The Shapley values of the four games are presented in Table 3. The equal division payoff vector is simply equal to $ED(v_k) = (25, 25, 25, 25)$ when $k = 1, 2$, and to $ED(v_k) = (50, 50, 50, 50)$ when $k = 3, 4$.

6 Results

The experiment was conducted at the Institute of Social and Economic Research (ISER), Osaka University, in January and February 2019 (Winter mechanism) and January and February 2022 (H–MC mechanism).[8] A total of 176 students, who had never participated in similar experiments before, were recruited as subjects of the experiment, 96 playing the Winter mechanism and 80 playing the H–MC mechanism.[9] The experiment

[8]The experiments were conducted in 2019 and 2022 because the original H–MC experiment conducted in December 2019 (which we refer to as the pseudo-H–MC or H–MC$_{sim}$ in Online Appendix VI) did not reflect the H–MC model precisely (we thank an anonymous reviewer for pointing this out), and we have redone the H–MC experiment to correct this. Online Appendix VI compares the outcomes of the pseudo-H–MC conducted in December 2019 and the (corrected) H–MC conducted in January–February 2022.

[9]The difference in the number of participants between the two mechanisms is a result of variations in the show-up rate among experimental sessions.

was computerized with z-Tree (Fischbacher, 2007) and participants were recruited using ORSEE (Greiner, 2015).

To control for potential ordering effects, each participant played all four games twice in one of the following four orderings: 1234, 2143, 3412, and 4321.[10] Between each play of a game (called a round), players were randomly rematched into groups of four players, and participants were randomly assigned a new role within the newly created group.[11] At the end of the experiment, two rounds (one from the first four rounds and another from the last four rounds) were randomly selected for payments. Participants received cash rewards based on the points that they earned in these two selected rounds, with an exchange rate of 20 JPY = 1 point, as well as a 1,500 JPY participation fee. On average, the experiments lasted for 1 hour 40 minutes for Winter and 1 hour 45 minutes for H–MC, including the instructions (15 minutes for Winter and 11 minutes for H–MC), a comprehension quiz (5 minutes), and payment.[12] The average earnings

[10]We let participants play all four games, instead of just one, in each session. Although this design choice may have meant participants were slower in learning how to play the game, we consider that having within-session variations is desirable because the tests of the axioms involve comparing outcomes across different games.

[11]We implemented random reassignment of the roles across rounds instead of fixing the role. Again, this may make learning the game slower for players given that their roles change, as Guerci et al. (2014) suggest. However, given the existence of the null player in one of the four games considered, we chose reassignment of the role to avoid participants feeling the experiment was unfair.

[12]Participants received a copy of instruction slides, and a pre-recorded instruction video was played. Quiz was given on the screen after the explanation of the game. The user interface was explained during the practice rounds referring to the handout about the computer screen. The quiz was given on the screen after the explanation of the game. The user interface was explained during the practice rounds, referring to the handout about the computer screen. See Online Appendix VII for English translations of the instruction materials and the comprehension quiz. At each decision screen, there was a non-binding time limit. The time limit was set to 60 seconds to make a demand (Winter) or a proposal (H-MC) and 30 seconds to choose a coalition (Winter) or decide to approve or reject the proposal (H-MC). When the time limit was reached, the message "please make a decision" appears on the top of the screen to encourage participants to make their decisions. On average (the standard deviation, the max, and the min), participants took 48.62 (25.28, 304, and 2) second for making a demand (n=768) and 20.36 (13.25, 164, and less than 1) seconds to choose a coalition (n=550) in the Winter. For H-MC, they took, on average (the standard deviation, the max, and the min), 40.83 (23.23, 160, and 7) seconds for making a proposal (n=257) and 16.18 (11.54, 87, and 2) seconds to approve or reject a proposal (n=531).

were 2,650 JPY for Winter and 2,850 JPY for H–MC.

We first compare the Winter and H–MC mechanisms in terms of the frequency of grand coalition formation and efficiency. Then, we analyze whether the resulting allocations from the two mechanisms match the Shapley values, and also try understanding the reasons for the discrepancies between the realized allocations and the Shapley values from the failure of four axioms that characterize the Shapley values presented in Section 3.1. We contrast the experimental results with the allocation predicted under the SPNE as well as under an equal division. Additional analyses of our experimental results are presented in the Online Appendix I to VI.

6.1 Grand coalition formation and efficiency

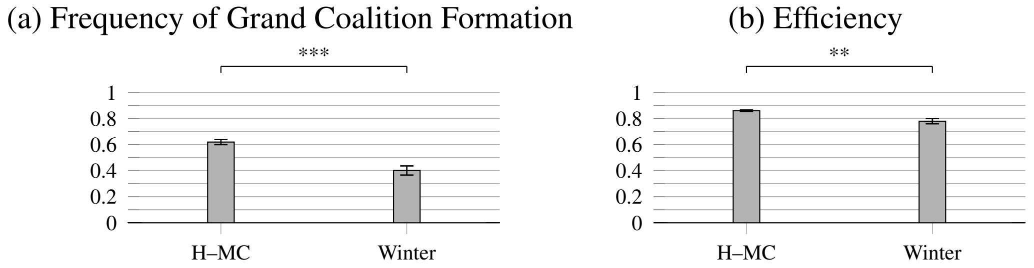
Panel (a) of Figure 2 presents the results concerning the grand coalition formation under the H–MC and Winter mechanisms pooling the data of all the four games.[13] For game 2, we consider the partition $\{\{1\}, \{2, 3, 4\}\}$ as a realization of the grand coalition for both the H–MC and Winter mechanisms because player 1 is a null player and, consequently, the game is only convex and not strictly convex.[14]

Considering the four games together, the grand coalition (in the case of game 2, either the grand coalition or the $\{2, 3, 4\}$ coalition) is formed in 61.9% of the cases under the H–MC mechanism, but only in 40.1% of the cases under the Winter mechanism.

[13]The figure is created based on the estimated coefficients of the following linear regressions: $gc_i = \beta_1 HMC_i + \beta_2 Winter_i + \mu_i$ where gc_i is a dummy variable that takes a value of 1 if the grand coalition is formed, and zero otherwise, in group i, HMC_i ($Winter_i$) is a dummy variable that takes a value of 1 if the H–MC (Winter) mechanism is used, and zero otherwise. The standard errors are corrected for within-session clustering effects. The statistical tests are based on the Wald test for the equality of the estimated coefficients of two treatment dummies. See Online Appendix I for results for four games separately.

[14]Recall that the Winter mechanism is theoretically defined for strictly convex games. In this game, Player 1 always has a zero marginal contribution and, as such, can be left out of any coalition at no cost for either him/her or the other players.

Figure 2: Proportion of times the grand coalition is formed and efficiency



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate the outcomes of H–MC mechanism and the Winter mechanism are significantly different at the 0.1%, 1%, and 5% significance levels (Wald test), respectively.

As a direct consequence of the grand coalition being formed in less than 100% of the cases, both mechanisms fail to achieve full efficiency. Efficiency is computed as the sum of the payoffs obtained by the four players as a proportion of the value of the grand coalition of the considered game (100 for games 1 and 2, and 200 for games 3 and 4). As Panel (b) of Figure 2 shows, considering all the four games together, efficiency is significantly higher under the H–MC mechanism than under the Winter mechanism.[15]

Therefore, we conclude as follows.

Result 1. *Although the grand coalition is not always formed under the two mechanisms, it is more frequently formed under the H–MC mechanism than under the Winter mechanism. Consequently, efficiency is higher under the H–MC mechanism than under the Winter mechanism.*

Note that under the H–MC mechanism, the proposer proposes an allocation to all

[15]The figure is created based on the estimated coefficients of the following linear regressions: $Eff_i = \beta_1 HMC_i + \beta_2 Winter_i + \mu_i$, where $Eff_i \equiv \frac{\sum_i \pi_i}{v(N)}$ is the efficiency measure for group i, HMC_i ($Winter_i$) is a dummy variable that takes a value of 1 if the H–MC (Winter) mechanism is used, and zero otherwise. The standard errors are corrected for within-session clustering effects. The statistical tests are based on the Wald test for the equality of the estimated coefficients of the two treatment dummies. If we consider four games separately, however, the efficiency is not statistically significantly different at 5% significance level in any of the game. See Online Appendix I.

the remaining members given the feasibility condition. Conversely, under the Winter mechanism, the players, speaking one after the other, may make unfeasible demands or form a small coalition early without waiting for others making their demands. As a result, the formation of the grand coalition under the H–MC mechanism is simply determined by whether the remaining players choose to accept the proposal or reject it, whereas under the Winter mechanism, it can be blocked by either by players forming a smaller coalition prematurely or by unfeasibility conditions. Such a difference between the two mechanisms can cause the significantly higher frequency of the grand coalition formation under the H–MC mechanism compared with the Winter mechanism. Indeed, in Online Appendix III.1, we report that the main reason for the failure of grand coalition formation in Winter in our experiment is a coalition being formed before reaching the fourth player.[16]

6.2 Allocations

We use $\pi^{HMC}(v_k)$ to denote a vector of payoffs obtained by the players in the H–MC mechanism in game k, with $k = 1, 2, 3, 4$. Analogously, let $\pi^{W}(v_k)$ denote a vector of payoffs obtained by the players under the Winter mechanism. The *ex ante* theoretical prediction for both mechanisms states that the mean of such vectors (based on many realizations with different orderings of the players) should converge to the Shapley value.

Figure 3 shows the mean realized payoffs based on all groups in each of the four games, and the horizontal lines indicate the Shapley values for each game. The mean

[16] In Online Appendix II, we report the frequency of the grand coalition formation and efficiency by separating the data for the first half (rounds 1–4) and the second half (rounds 5–8) of the experiment. We observe an increase in both the frequency of the grand coalition formation and efficiency, at least in some of the games, for both mechanisms. A significantly higher frequency of grand coalition formation and efficiency is observed under the H–MC mechanism than under the Winter mechanism even in the second half of the experiment.

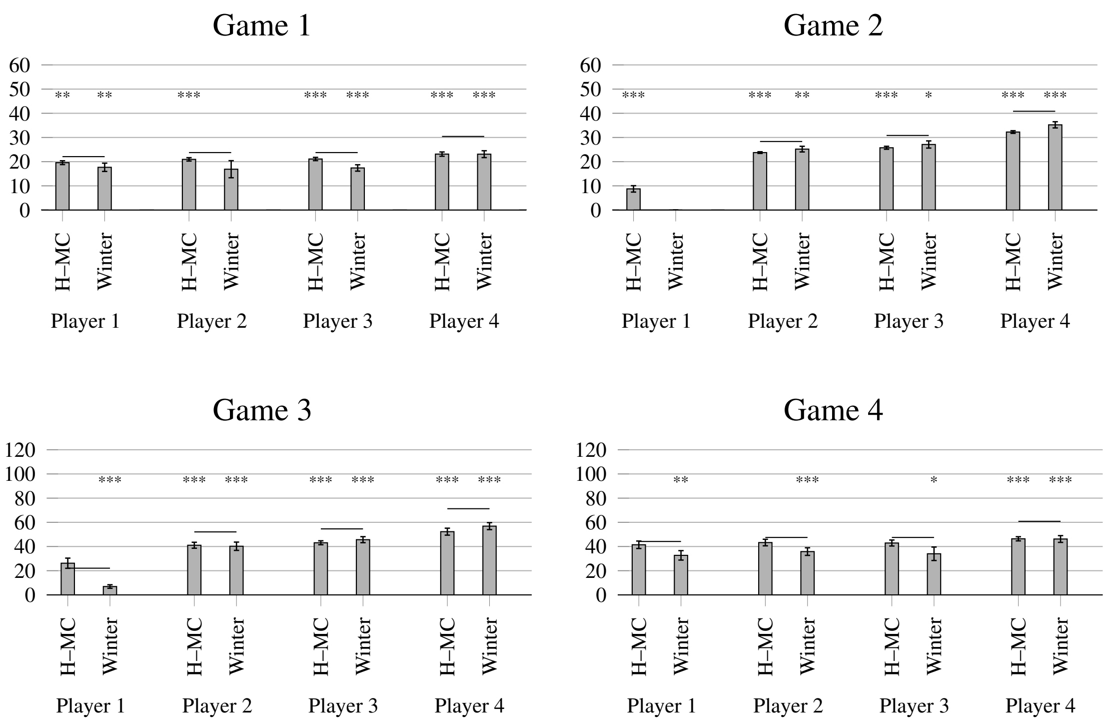
and the standard errors are obtained by running a set of ordinary least squares (OLS) regressions for the following system of equations:

$$\begin{aligned}
\pi_1 &= a_1 g_1 + a_2 g_2 + a_3 g_3 + a_4 g_4 + u_1 \\
\pi_2 &= b_1 g_1 + b_2 g_2 + b_3 g_3 + b_4 g_4 + u_2 \\
\pi_3 &= c_1 g_1 + c_2 g_2 + c_3 g_3 + c_4 g_4 + u_3 \\
\pi_4 &= d_1 g_1 + d_2 g_2 + d_3 g_3 + d_4 g_4 + u_4
\end{aligned} \tag{1}$$

where π_i is the payoff of player i, g_j is a dummy variable that takes a value of 1 if the game $j \in \{1, 2, 3, 4\}$ is played, and zero otherwise. Because participants play all four games twice, we correct the standard errors for within-group clustering effects. Note that the estimated coefficients a_j, b_j, c_j, and d_j are the average payoffs in game j for players 1, 2, 3, and 4, respectively.

When players fail to form the grand coalition, the total payoff obtained by the players is smaller than the value under the grand coalition. As a result, the average realized payoff vectors are significantly different from the Shapley value, as shown in Figure 3. We do observe, however, that in game 2, the average payoff of player 1 under Winter is zero (just as Shapley value) while it is positive under H–MC. Below, we first follow the approach proposed by Aguiar et al. (2018) and compute the *Shapley distance* to decompose the reasons behind the deviations of the realized payoff vectors from the Shapley values into the failure of its four main properties. We then further investigate the realized payoff vectors by focusing on those groups that formed the grand coalition.

Figure 3: Mean payoffs, all groups



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate the average payoff is significantly different from the Shapley values at the 0.1%, 1%, and 5% significance levels, respectively (Wald test).

6.2.1 Shapley distance

The approach of Aguiar et al. (2018) we apply is based on the decomposition of the distance of the payoff vectors from the Shapley value into the failure of efficiency, symmetry, additivity and null player property. The same decomposition has been used in Chessa et al. (2022). We present the procedure below.

Let $\pi = (\pi_1, \pi_2, \pi_3, \pi_4)$ be the realized vector of payoffs in a game. We first find a vector of payoffs closest to π that satisfies the symmetry. Call such a vector $\pi^{sym} = (\pi_1^{sym}, \pi_2^{sym}, \pi_3^{sym}, \pi_4^{sym})$. Namely, we take the sum of payoffs obtained by symmetric players s (players 2 and 3 in games 1 and 4) and divide the sum equally among them. Thus, in games 1 and 4, $\pi_s^{sym} = \sum_{s\in\{2,3\}} \pi_s/2$, and for non-symmetric players k (including all the players in games 2 and 3), $\pi_k^{sym} = \pi_k$.

Next, we find a vector of payoffs satisfying efficiency that is closest to π^{sym}. Call the new payoff vector $\pi^{sym,eff} = (\pi_1^{sym,eff}, \pi_2^{sym,eff}, \pi_3^{sym,eff}, \pi_4^{sym,eff})$. Namely, for each player $i = 1, 2, 3, 4$, $\pi_i^{sym,eff} = \pi_i^{sym} + [v(N) - \sum_{j\in N} \pi_j]/4$.

We then find a vector of payoffs satisfying null player property that is closest to $\pi^{sym,eff}$. Let $\pi^{sym,eff,null} = (\pi_1^{sym,eff,null}, \pi_2^{sym,eff,null}, \pi_3^{sym,eff,null}, \pi_4^{sym,eff,null})$ be the resulting vector of payoffs. Specifically, if player n is a null player (player 1 in game 2), then her new payoff must be equal to zero, that is, $\pi_n^{sym,eff,null} = 0$. Three other players j in the game (players 2, 3, 4 in game 2) equally share $\pi_n^{sym,eff}$ of the null player. That is, $\pi_j^{sym,eff,null} = \pi_j^{sym,eff} + \pi_n^{sym,eff}/3$. When there is no null player in the game, $\pi_i^{sym,eff,null} = \pi_i^{sym,eff}$ for all i.

Theorem 3 in Aguiar et al. (2018) shows that a vector of payoffs $\pi = (\pi_1, \pi_2, \pi_3, \pi_4)$ obtained when playing game v can be decomposed as follows: $\pi = \phi(v) + e^{sym} + e^{eff} + e^{null} + e^{add}$. Thus, the Shapley error, $e^{\phi} = \pi - \phi(v)$, is $e^{\phi} = e^{sym} + e^{eff} + e^{null} + e^{add}$ where

$$e_i^{sym} = \pi_i - \pi_i^{sym} \text{ for all } i,$$

$$e_i^{eff} = \pi_i^{sym} - \pi_i^{sym,eff} \text{ for all } i,$$

$$e_i^{null} = \pi_i^{sym,eff} - \pi_i^{sym,eff,null} = \text{for all } i,$$

$$e_i^{add} = \pi_i^{sym,eff,null} - \phi_i(v) \text{ for all } i.$$

Given this decomposition, the Shapley distance is given by:

$$||e^{\phi}||^2 = ||e^{sym}||^2 + ||e^{eff}||^2 + ||e^{null}||^2 + ||e^{add}||^2 + 2 < e^{add}, e^{null} >$$

Table 4: Result of Shapley distance decomposition. Based on pooling the data of all groups and all games

	$\|\|e^{sym}\|\|^2$	$\|\|e^{eff}\|\|^2$	$\|\|e^{null}\|\|^2$	$\|\|e^{add}\|\|^2$	$\|\|e^{\phi}\|\|^2$
H-MC	38.19	429.96	63.97	270.84	802.88
	(12.73)	(53.41)	(8.27)	(20.70)	(62.70)
Winter	85.18	606.81	7.28	321.49	1020.72
	(18.97)	(101.35)	(1.87)	(17.33)	(72.22)
No. Obs	352	352	352	352	352
R^2	0.132	0.220	0.111	0.369	0.421
p-value*	0.079	0.167	0.0003	0.103	0.057

Note: Standard errors are corrected for session-level clustering effects and shown in parentheses. $< e^{add}, e^{null} >$ are not reported in the table as they are negligible (the mean values are 0.0093 for H–MC and 0.0026 for Winter).
* p-values for testing H_0: H–MC = Winter (based on the Wald test)

where $< \cdot, \cdot >$ is the scalar product and for any vector $y \in \mathbb{R}^n$, $||y||^2 =< y, y >= \sum_{i \in N} y_i^2$.[17]

We perform the Shapley distance decomposition of each payoff vector and the corresponding Shapley value, and compute the average distance, pooling data of all groups and all games, to compare between H–MC and Winter. Results are presented in Table 4.

One can observe from the last column of Table 4, indeed, the Shapley distance is (marginally significantly) larger under Winter than under H–MC. And this is because the distance due to the violations of symmetry, efficiency, and additivity axioms tend to be lager under Winter than under H–MC (and marginally significantly so for the symmetry). However, as we have observed in Figure 3, the distance due to the violation of null player property is significantly smaller under Winter than under H–MC. Note that when decomposed in this way, the distance due to the violation of efficiency axiom

[17] Differently from the original decomposition by Aguiar et al. (2018), that ensures orthogonal components, with our decomposition, in general, vectors e^{null} and e^{add} are not orthogonal so that $< e^{add}, e^{null} >$ is not equal to zero.

is not significantly different between the two mechanisms unlike what we have seen above. However, this is due to the difference in the definition. The efficiency measure used above ($\sum_i \pi_i/v(N)$) and $||e^{eff}||^2$ are negatively correlated (the Pearson correlation coefficient is -0.78) but not perfectly so.

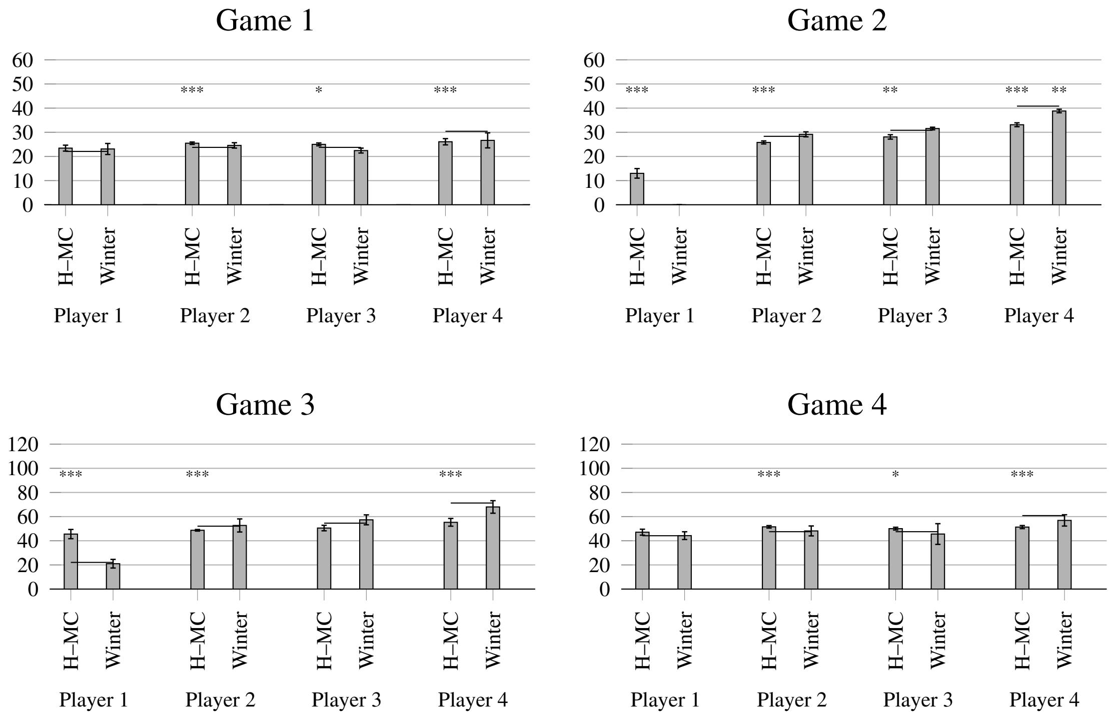
To further compare the realized allocations between the two mechanisms in light of theoretical predictions, let us now focus on those groups that formed the grand coalition. The reason for focusing on groups that formed the grand coalition is that different coalition structures provide a different value to be shared, and therefore a comparison between the payoff vectors in relation to some theoretical benchmark would not be clear cut. We complement our analyses in Online Appendix V and report the corresponding results based on all groups using payoff shares, instead of restricting our attention to groups that formed the grand coalition. The idea behind these additional analyses is to check whether payoff shares respect the hierarchy among the players as predicted by the theory.

6.2.2 Allocations when the grand coalition is formed

Figure 4 shows the average payoffs obtained by each player in the four games conditional on the grand coalition being formed.[18] The horizontal lines indicate the Shapley values for each game. It can be observed that for the Winter mechanism, the average payoffs are not significantly different from the Shapley values for all four players in games 1, 3, and 4. Conversely, for the H–MC mechanism, they are significantly different from the Shapley values for at least three out of four players in all four games. This indicates that provided that the grand coalition is formed, the average payoffs under the

[18]The mean and the standard errors are obtained by running a system of linear regressions as we have done to generate Figure 3 but restricting to those cases where the grand coalition is formed.

Figure 4: Mean payoffs based only on the groups that formed the grand coalition



Note: The horizontal lines indicate the Shapley values. The error bars show the one standard error range. The symbols ***, **, and * indicate that the average payoff is significantly different from the Shapley values at the 0.1%, 1%, and 5% significance levels (Wald test), respectively.

Winter mechanism are closer to the Shapley values than those under the H–MC mechanism. Furthermore, we report in Online Appendix IV that, provided that the grand coalition if formed, the realized allocations under the Winter mechanism better satisfy the axioms characterizing the Shapley values than ones realized under the H–MC mechanism.

Result 2. *Provided the grand coalition is formed, the average payoffs follow the Shapley values more closely under the Winter mechanism than under the H–MC mechanism.*

6.2.3 Realized allocations in the grand coalition and *a posteriori* equilibria

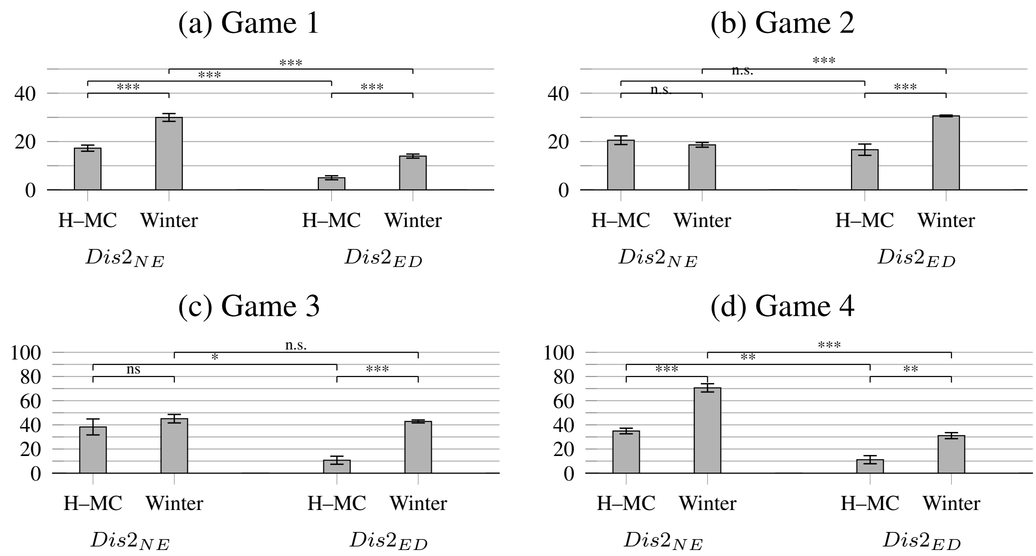
Now, let us analyze the realized payoffs in the light of the *a posteriori* equilibrium payoff vectors. We continue to focus only on the groups that formed the grand coalition. We measure the distance between the realized payoff vectors and the allocation under the SPNE for the four games by their Euclidean distance. Let eq_i be the equilibrium payoff for player i for the given game, the realized order of the players (making a proposal or demand), and the mechanism. The distance of the realized payoff from the equilibrium is computed as $Dis2_{NE} = \sqrt{\sum_i (\pi_i - eq_i)^2}$.[19] We also consider the distance between the realized payoff vectors and equal division payoffs, defined by $Dis2_{ED} = \sqrt{\sum_i (\pi_i - ED_i)^2}$ where ED_i is the equal division payoff for player i for the given game.

Figure 5 shows the mean $Dis2_{NE}$ and the mean $Dis2_{ED}$ for the two mechanisms in the four games.[20] We observe that the distance to the equal division is significantly smaller (at the 1% level) for the H–MC mechanism than for the Winter mechanism in all four games. This may not be surprising because, as Observation 1 states, the *a posteriori* equilibrium payoff vectors tend to be less unequal under the H–MC mechanism than under the Winter mechanism. In fact, as we can observe, the distance to the equilibrium allocation is significantly smaller for the H–MC mechanism than for the Winter mechanism in games 1 and 4 (in which the equilibrium payoffs are less unequal than

[19] For the sake of simplicity, we omit the specifications about the considered mechanism and the game.

[20] The figure is created based on the estimated coefficients of the following linear regressions: $Dis_i = \beta_1 HMC_i + \beta_2 Winter_i + \mu_i$, where Dis_i is the relevant distance measure for group i, HMC_i ($Winter_i$) is a dummy variable that takes a value of 1 if the H–MC (Winter) mechanism is used, and zero otherwise. The standard errors are corrected for within-session clustering effects. The statistical tests are based on the Wald test for the equality of the estimated coefficients of the two treatment dummies. For the difference between $Dis2_{NE}$ and $Dis2_{ED}$ we compute $\Delta = Dis2_{NE} - Dis2_{ED}$ and run the same regression as above and test whether the estimated coefficients of treatment dummies are significantly different from zero.

Figure 5: Mean of the distances of the realized payoff vectors from the SPNE and the equal division among those groups that formed the grand coalition.



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate that the differences are statistically significant at the 0.1%, 1%, and 5% significance levels (Wald test for across treatment differences and t-test for within treatment difference between $Dis2_{NE}$ and $Dis2_{ED}$), respectively.

in games 2 and 3) at the 0.1% level. For games 2 and 3, however, the distance to the equilibrium allocations is not significantly different between the two mechanisms.

Figure 5 shows that, on the one hand, the payoff vectors realized under the H–MC mechanism are significantly closer to the equal division than to the equilibrium ones in all but game 2 (in which $Dis2_{NE}$ and $Dis2_{ED}$ are not significantly different). On the other hand, under the Winter mechanism, the realized payoff vectors are significantly closer to the equal division than to the equilibrium ones only in games 1 and 4, but the opposite is the case for game 2. In game 3, $Dis2_{NE}$ and $Dis2_{ED}$ are not significantly different under the Winter mechanism.

Result 3. *When the grand coalition is formed, the H–MC mechanism more often results in payoffs that are closer to the equal division than to the equilibrium payoffs compared* *with the Winter mechanism.*

This indicates that, albeit imperfectly, the Winter mechanism achieves the allocation that better reflects the power of the players than does the H–MC mechanism. In Online Appendix III.2, we show that in H-MC, the proposals become more equal after players observing rejection of earlier, less equal, proposals. Such dynamics lead to this result.

6.3 Exploitation of the first mover advantage

We investigate now for the strategic behavior of the players in our games. At first, we observe that a direct comparison between H-MC and Winter mechanisms on this point is rather challenging because the strategic behavior to optimally participate in an offered based vs. a demand based mechanism is much different off the equilibrium path. After a first proposal has been made, on the one side, and as already noticed, an offer based mechanism resemble a voting situation, in which each player simply accepts or reject the proposal. We may observe that, in this case, the best response simply depends on the set of players who are left in the game and on the offer he or she received, independently on what has been offered to the other players. Moreover, at a given period, the only possibility for a player is to accept forming the coalition containing all the remaining players, or refusing to do that. On the other side, in a demand based mechanism, the announcement of a player who is not playing first in the period, i.e., after the first demand of the remaining players has been declared, not only depends on the set of players who are left in the game, but also on all precedent demands in the period which are not yet satisfied. In this case, the choice is not only on whether forming a coalition containing all the remaining players, but also on whether eventually forming other feasible sub-coalitions. Thus, a direct comparison of the strategic behavior along the entire strategic interaction under the two different mechanisms is not possible.

Instead, the two mechanisms are fully comparable when investigating the strategic behavior of the first mover. In fact, once the first player has been randomly chosen, he or she is theoretically expected to offer him- or herself (under the H-MC mechanism), or to demand (under the Winter mechanism) exactly the same payoff, i.e., his or her marginal contribution to the coalition formed by the remaining players (which is equal to the his or her a posteriori equilibrium payoff). For this reason, we focus our analysis on the behavior of the first mover to analyze the degree to which participants play the game as predicted by the theory.

When asking his-or her marginal contribution to the coalition formed by the remaining players, the first mover experiences what we call the *first mover advantage*, i.e., he or she can strategically ask more than what given by the *a priori* expected equilibrium payoff, i.e., the Shapley value. We therefore investigate whether the first mover successfully exploit this advantage.

In the following, let FA_i be the degree to which the first mover i exploits his or her first mover advantage. Namely, $FA_i = (a_i - \phi_i) \,/\, (eq_i - \phi_i)$, where i is the first mover and a_i is either the proposed allocation to i in H-MC or the demand by i in Winter, ϕ_i is i's Shapley value, and eq_i is i's *a posteriori* equilibrium payoff. Note that since FA_i is not defined for the null player (player 1 in Game 2), we exclude the null player from the following analysis.[21] But we consider all the groups, otherwise.

Table 5 show the average FA_i in two mechanisms for four games. The standard errors are corrected for session clustering effect. The table is created based on the

[21] When the null player was the first mover, the average a_i was 35 in H–MC and 16.5 in Winter. The standard errors adjusted for the session clustering effect were 6.830 and 2.284 in H–MC and in Winter, respectively. The difference is significant at 5% level ($p = 0.037$) according to the Wald test for the equality of coefficients of the two treatment dummies in a linear regression where a_i is the dependent variable and two treatment dummies, without the constant, are the only independent variables.

Table 5: Exploitation of the first move advantage

	Game 1	Game 2	Game 3	Game 4
H-MC	0.354***	-0.055	0.674*	0.406**
	(0.022)	(0.156)	(0.264)	(0.089)
Winter	-0.096	-0.024	-0.123	-0.218
	(0.152)	(0.166)	(0.104)	(0.095)
No. Obs	88	62	88	88
R^2	0.245	0.002	0.232	0.238
p-value†	0.022	0.897	0.026	0.002

Note: Standard errors are corrected for session-level clustering effects and shown in parentheses.
***, **, *: estimated coefficient is significantly different from zero at 0.1, 1, and 5% significance level.
† p-values for testing H_0: H–MC = Winter (based on the Wald test)

estimated coefficients of the linear regression that takes FA_i as the independent variable and two dummy variables representing the two mechanisms as only dependent variables without the constant. We observe that, on average, while the first mover advantage is exploited, although not fully, in H–MC except for Game 2, it is not in Winter. Thus, the first movers are acting significantly closer to the *a posteriori* equilibrium prediction in H–MC than in Winter. This finding is in line with the result we have seen in Figure 6.2.3, where we have shown that the H-MC mechanism provides result that are closer to the SPNE when the grand coalition is formed.

7 Conclusion

We have experimentally compared two of the best-known bargaining procedures in the Nash program, the H–MC and the Winter mechanisms. Our main rationale for this choice is that the two mechanisms stand out in the literature for their distinctive features, and they are a fundamental contribution on the Nash program. Moreover, they are recognized for their simplicity, which is a key desideratum when considering possible applicability of a theoretical mechanism to the real world. These two mechanisms have the same ex ante equilibrium prediction, but differ mainly in their processes: H–MC mechanism is based on offers, and the Winter on demands.

Previous studies have found a certain closeness of the experimental results when making a similar comparison (see Fréchette et al., 2005a), despite the sharply different theoretical predictions. Instead, we show that our two mechanisms behave very differently, despite the close theoretical predictions. In particular, the H–MC mechanism results in higher frequencies of the grand coalition formation and, consequently, higher efficiency than the Winter mechanism. We suggest that the H–MC mechanism is better suited to bargaining over cost or payoff allocation problems when the main target is efficiency, or when full cooperation represents a crucial goal for society (e.g., full cooperation in the airport problem (Littlechild and Owen, 1973) results in one single airport being built instead of many, and this is certainly desirable for environmental reasons). Conversely, provided that the grand coalition is formed, the Winter mechanism results in average payoffs that are closer to the Shapley values and better satisfy various axioms. We suggest that the Winter mechanism, when it leads to collaboration, is best suited to allocation problems in which it is important to value players' effective power (e.g., production games (Owen, 1975), or in which arguments such as social welfare and symmetry are inescapable (e.g., allocation of resources in health or social care (Kluge, 2007)). Of course, the major drawback of the Winter mechanism is its failure of reaching the full collaboration.

Our findings suggest that when facing a cost or payoff allocation problem, the choice of which bargaining procedure to use, one based on offers or on demands, may have some unexpected effects, regardless of the theoretical prediction. This should be taken into account when making such a choice in various applications. In fact, different bargaining mechanisms, even when equivalent from the theoretical point of view, favor different properties that are reflected in the resulting allocations. An example of such effects may be found in the verification of the null player property of the Shapley value. Theoretically, a player who always has a zero marginal contribution should receive a zero payoff, according to Shapley. In accordance with the theoretical prediction, in a demand-based mechanism, non-null players have refused a strictly positive demand by a null player in our experiment. However, we find that non-null players in our experiment seem to be uncomfortable with making a zero offer to a null player in an offer-based mechanism, and this contributes to a final payoff share that is closer to the equal division solution. A deeper analysis of how different mechanisms can lead players toward respecting or violating some properties would be a fruitful direction for future research.

Many potentially important complementary questions can be addressed in future research. Among others, an analysis of the more complex versions of our proposed mechanisms (e.g., the Winter mechanism with more periods and a discount factor (see, Chessa et al., 2022, for two periods version), or the H–MC mechanism where the proposer whose offer is refused then leaves the game with a probability strictly smaller than one) can be compared with our actual results. Comparing the outcomes of the experiments based on noncooperative mechanisms with those of unstructured bargaining experiments would be an interesting topic for future research.

Acknowledgments

The experiments reported in this paper were approved by the Institutional Review Board (IRB) of Yamaguchi University (No. 5, for the experiments conducted in 2019) as well as by the IRB of ISER, Osaka University (for the experiment conducted in 2022). We gratefully acknowledge financial support from the Joint Usage/Research Center at ISER, Osaka University, and Grants-in-Aid for Scientific Research, Japan Society for the Promotion of Science (15K01180, 15H05728, 18K19954, 20H05631), the Fund for the Promotion of Joint International Research (Fostering Joint International Research) (15KK0123), and the French government-managed l'Agence Nationale de la Recherche (les Programmes d'Investissements d'Avenir UCA^{JEDI}) (ANR-15-IDEX-01). In particular, we thank the UCAinACTION project. We thank Guillaume Fréchette and participants of the seminar at the University of Technology Sydney, Monash University, Osaka University, Tokyo University, and Keio University, as well as participants of ASFEE 2019 (Toulouse, France), Game Theory Workshop 2020 (Tokyo, Japan), SEET 2020 (Naples, Italy), SAET 2021 (Seoul, Korea), and ESA Global 2021 (online) for comments and suggestions.

References

ABE, T., Y. FUNAKI, AND T. SHINODA (2021): "Invitation Games: An Experimental Approach to Coalition Formation," Games, 3, 64.

AGUIAR, V.H., R. PONGOU AND J-B. TONDJI (2018): "A non-parametric approach to testing the axioms of the Shapley value with limited data," Games and Economic Behavior, 111, 41–63.

Ansolabehere, S., J. Snyder, A. Strauss, and M. Ting (2005): "Voting Weights and Formateur Advantages in the Formation of Coalition Governments," American Journal of Political Science, 49, 550–563.

Baron, D. P. and J. A. Ferejohn (1989): "Bargaining in Legislatures," American Political Science Review, 83, 1181–1206.

Bennett, E. and E. van Damme (1991): "Demand Commitment Bargaining: - The Case Of Apex Games," in Game Equilibrium Models III, ed. by S. R., Springer, Berlin, Heidelberg.

Binmore, K., A. Rubinstein, and A. Wolinsky (1986): "The Nash Bargaining Solution in Economic Modelling," RAND Journal of Economics, 17, 176–188.

Bolton, G. E., K. Chatterjee, and K. L. McGinn (2003): "How Communication Links Influence Coalition Bargaining: A Laboratory Investigation," Management Science, 49, 583–598.

Brandenburger, A. and B. Nalebuff (1996): Co-Opetition, New York, NY: Doubleday Business.

Chessa, M., N. Hanaki, A. Lardon, and T. Yamada (2022): "An experiment on demand commitment bargaining," Dynamic Games and Applications, https://doi.org/10.1007/s13235-022-00463-x.

De Clippel, G. and K. Rozen (2022): "Fairness through the lens of cooperative game theory: An experimental approach," American Economic Journal: Microeconomics, 14, 810-836 (https://www.aeaweb.org/articles?id=10.1257/mic.20200015).

DEMANGE, G. (1984): "Implementing efficient egalitarian outcomes," Econometrica, 52, 1167–1177.

FIORINA, M. P. AND C. R. PLOTT (1978): "Committee Decisions under Majority Rule: An Experimental Study," The American Political Science Review, 72, 575–598.

FISCHBACHER, U. (2007): "z-Tree: Zurich toolbox for ready-made economic experiments," Experimental Economics, 10, 171–178.

FRÉCHETTE, G., J. H. KAGEL, AND S. F. LEHRER (2003): "Bargaining in Legislatures: An Experimental Investigation of Open versus Closed Amendment Rules," The American Political Science Review, 97, 221–232.

FRÉCHETTE, G., J. H. KAGEL, AND M. MORELLI (2005a): "Behavioral Identification in Coalition Bargaining: An Experimental Analysis of Demand Bargaining and Alternating Offers," Econometrica, 73, 1893–1937.

——— (2005b): "Gamson's Law versus Non-Cooperative Bargaining Theory," Games and Economic Behavior, 51, 365–390.

——— (2005c): "Nominal Bargaining Power, Selection Protocol and Discounting in Legislative Bargaining," Journal of Public Economics, 89, 1497–1518.

GREINER, B. (2015): "An Online Recruitment System for Economic Experiments," Journal of the Economic Science Association, 1, 114–125.

GUERCI, E., N. HANAKI, N. WATANABE, G. ESPOSITO, AND X. LU (2014): "A methodological note on a weighted voting experiment," Social Choice and Welfare, 43, 827–850.

GUL, F. (1989): "Bargaining Foundations of Shapley Value," Econometrica, 57, 81–95.

HAAKE, C.-J. (2000): "Support and Implementation of the Kalai-Smorodinsky Bargaining Solution," in Operations Research Proceedings 1999 (Selected Papers of the Symposium on Operations Research (SOR '99) Magdeburg, September 1-3, 1999), vol 1999, ed. by K. Inderfurth, G. Schwödiauer, W. Domschke, F. Juhnke, P. Kleinschmidt, and W. G., Mannheim: Bibliographisches Institut Mannheim.

HARSANYI, J. C. (1974): "An Equilibrium-Point Interpretation of Stable Sets and a Proposed Alternative Definition," Management Science, 20, 1472–1495.

——— (1981): "The Shapley Value and the Risk Dominance Solutions of Two Bargaining Models for Characteristic-Function Games," in Essays in Game Theory and Mathematical Economics, ed. by R. J. A. et al., Mannheim: Bibliographisches Institut Mannheim, 43–68.

HART, O. AND J. MOORE (1990): "Property Rights and the Nature of the Firm," Journal of Political Economy, 98, 1119–1157.

HART, S. AND A. MAS-COLELL (1996): "Bargaining and Value," Econometrica, 64, 357–380.

JU, Y. AND D. WETTSTEIN (2009): "Implementing cooperative solution concepts: a generalized bidding approach," Economic Theory, 39, 307–330.

KALAI, E. AND M. SMORODINSKY (1975): "Other Solutions to Nash's Bargaining Problem," Econometrica, 43, 513–518.

KALISCH, G. K., J. W. MILNOR, J. F. NASH, AND E. D. NERING (1954): "Some experimental n-person games," in Decision Processes, ed. by R. Thrall, C. Coombs, and R. Davis, John Wiley & Sons, Inc, 301–327.

KLUGE, E.-H. W. (2007): "Resource allocation in healthcare: implications of models of medicine as a profession," MedGenMed, 9(1):57.

KRISHNA, V. AND R. SERRANO (1995): "Perfect Equilibria of a Model of N-Person Noncooperative Bargaining," International Journal of Game Theory, 24, 259–272.

LITTLECHILD, S. C. AND G. OWEN (1973): "A Simple Expression for the Shapely Value in a Special Case," Management Science, 20, 370–372.

MAS-COLELL, A. (1988): "Algunos comentarios sobre la teoría cooperativa de los juegos," Cuadernos Económicos, 40, 143–162.

MCSHERRY, F. AND K. TALWAR (2007): "Mechanism Design via Differential Privacy," in 48th Annual IEEE Symposium on Foundations of Computer Science (FOCS'07), 94–103.

MONTERO, M., M. SEFTON, AND P. ZHANG (2008): "Enlargement and the balance of power: an experimental study," Social Choice and Welfare, 30, 69–87.

MORELLI, M. (1999): "Demand Competition and Policy Compromise in Legislative Bargaining," American Political Science Review, 93, 809–820.

MOULIN, H. (1984a): "The conditional auction mechanism for sharing a surplus," Review of Economic Studies, 51, 157–170.

——— (1984b): "Implementing the Kalai-Smorodinsky Bargaining Solution," Journal of Economic Theory, 33, 32–45.

MOULIN, H. AND M. O. JACKSON (1992): "Implementing a public project and distributing its costs," Journal of Economic Theory, 57, 125–140.

MURNIGHAN, J. K. AND A. E. ROTH (1977): "The Effects of Communication and Information Availability in an Experimental Study of a Three-Person Game," Management Science, 23, 1336–1348.

——— (1982): "The Role of Information in Bargaining: An Experimental Study," Econometrica, 50, 1123–1142.

NASH, J. F. (1950): "The Bargaining Problem," Econometrica, 18, 155–162.

——— (1953): "Two person coooperative games," Econometrica, 21, 128–140.

NASH, J. F., R. NAGEL, A. OCKENFELS, AND R. SELTEN (2012): "The agencies method for coalition formation in experimental games," Proceedings of the National Academy of Sciences, 109, 20358–20363.

NYDEGGER, R. V. AND G. OWEN (1975): "Two-person bargaining: An experimental test of the Nash axioms," International Journal of Game Theory, 3, 239–249.

OWEN, G. (1975): "On the Core of Linear Production Games," Mathematical Programming, 9, 358–370.

PÉREZ-CASTRILLO, D. AND D. WETTSTEIN (2001): "Bidding for the Surplus : A Non-cooperative Approach to the Shapley Value," Journal of Economic Theory, 100, 274–294.

RAIFFA, H. (1953): "Arbitration Schemes for Generalized Two Person Games," in Contributions to the Theory of Games II, ed. by H. Kuhn and A. Tucker, Princeton: Princeton University Press, 361–387.

RAPOPORT, A., O. FRENKEL, AND J. PERNER (1977): "Experiments with cooperative 2 X 2 games," Theory and Decision, 8, 67–92.

ROTH, A. E. AND R. E. VERRECCHIA (1979): "The Shapley Value As Applied to Cost Allocation: A Reinterpretation," Journal of Accounting Research, 17, 295–303.

RUBINSTEIN, A. (1982): "Perfect Equilibrium in a Bargaining Model," Econometrica, 50, 97–110.

SCHULZ, A. S. AND N. A. UHAN (2010): "Sharing Supermodular Costs," Operations Research, 58, 1051–1056.

SERRANO, R. (2005): "Fifty Years of the Nash Program, 1953-2003," Investigaciones Economicas, 29, 219–258.

——— (2008): "Nash Program," in The New Palgrave Dictionary of Economics, 2nd edition, ed. by S. Durlauf and L. Blume, McMillan, London.

——— (2014): "The Nash Program: A Broader Interpretation," Ensayos, 33, 105–106.

——— (2021): "Sixty-Seven Years of the Nash Program: Time for Retirement?" SERIEs, 12, 35–48.

SHAPLEY, L. S. (1953): "A Value for n-Person Games," in Contribution to the Theory of Games, ed. by H. Kuhn and A. Tucker, Princeton, vol. II, 303–317.

SHINODA, T. AND Y. FUNAKI (2019): "Unstructured Bargaining Experiment on Three-person Cooperative Games," Tech. Rep. E1915, September 2019, Waseda Institute of Political EConomy.

SHUBIK, M. (1962): "Incentives, Decentralized Control, the Assignment of Joint Costs and Internal Pricing," Management Science, 8, 325–343.

TIMMER, J., M. CHESSA, AND R. J. BOUCHERIE (2013): "Cooperation and game-theoretic cost allocation in stochastic inventory models with continuous review," European Journal of Operational Research, 231, 567–576.

TROCKEL, W. (1999): "Unique Nash Implementation for a Class of Bargaining Solutions," International Game Theory Review, 1, 267–272.

——— (2002): "A Universal Meta Bargaining Realization of the Nash Solution," Social Choice and Welfare, 19, 581–586.

VAN DEN BRINK, R. (2002): "An Axiomatization of the Shapley Value using a Fairness Property," International Journal of Game Theory, 30, 309–319.

——— (2006): "Null or nullifying players: The difference between the Shapley value and equal division solutions," Journal of Economic Theory, 136, 767–775.

WARWICK, P. V. AND J. N. DRUCKMAN (2001): "Portfolio Salience and the Proportionality of Payoffs in Coalition Government," British Journal of Political Science, 31, 627–649.

WINTER, E. (1994): "The Demand Commitment Bargaining and Snowballing Cooperation," Economic Theory, 4, 255–273.

YOUNG, H. P. (1985): "Monotonic Solutions of Cooperative Games," International Journal of Game Theory, 14, 65–72.

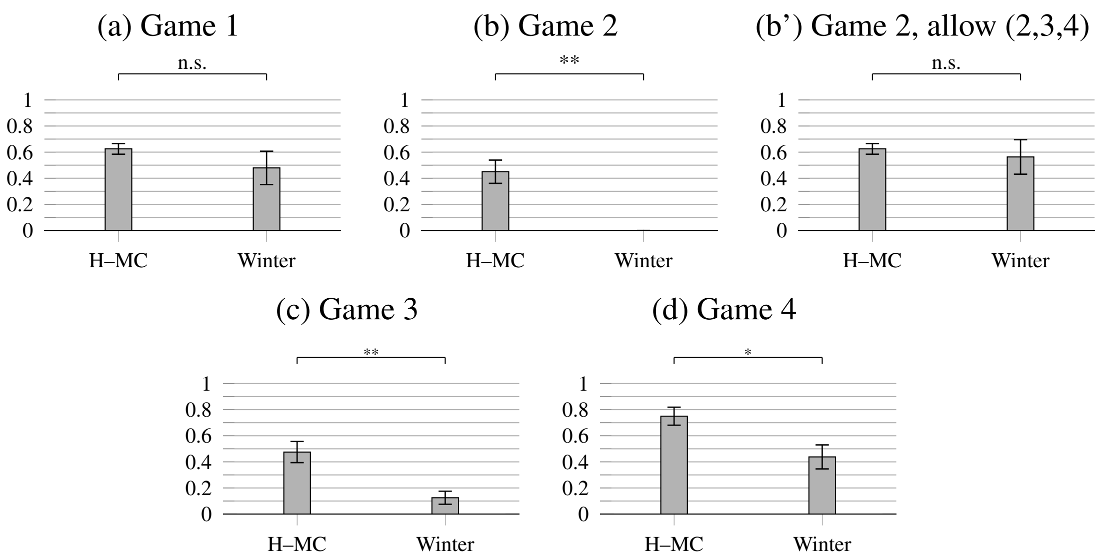
——— (1998): "Cost allocation, demand revelation, and core implementation," Mathematical Social Sciences, 36, 213–228.

Online Appendix

I Grand coalition formation and efficiency for each game

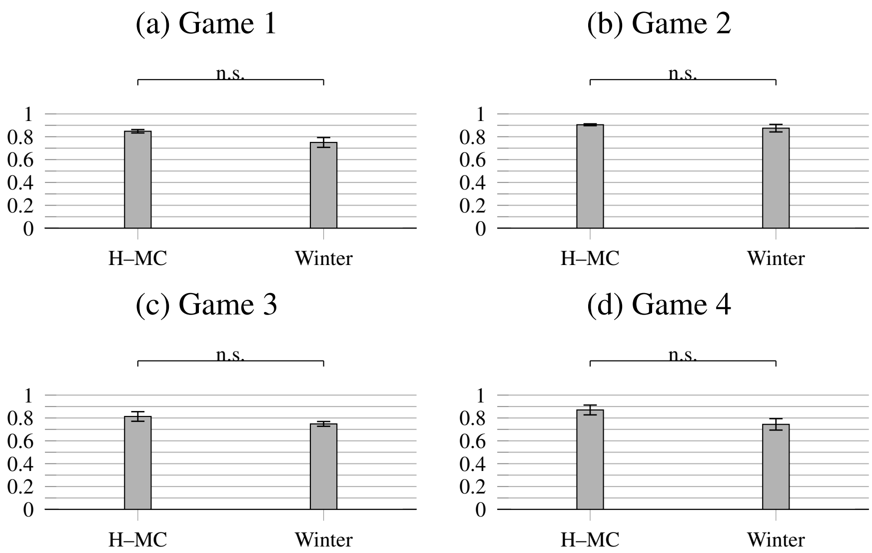
Figure I.1 shows the frequency of grand coalition formation under H-MC and Winter for four games separately. We observe that for game 2 and the Winter mechanism, the grand coalition never forms (because player 1 is a null player and, consequently, the game is only convex and not strictly convex. Therefore, for game 2, we consider the partition $\{\{1\},\{2,3,4\}\}$ as a realization of the grand coalition for both the H–MC and Winter mechanisms. Grand coalition is significantly more frequently formed under H-MC than under Winter in games 3 and 4.

Figure I.1: Proportion of times the grand coalition is formed



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate the proportion of times that the formation of the grand coalition is significantly different between the two mechanisms at the 0.1%, 1%, and 5% significance levels (Wald test), respectively.

Figure I.2: Efficiency



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate that the efficiency is significantly different between the two mechanisms, at the 0.1%, 1%, and 5% significance levels (Wald test), respectively.

Figure I.2 shows the average efficiency under H-MC and Winter for four games separately. Unlike the case where we pool all the data, the efficieny is not significantly different under two mechanisms in any of the games at 5% significance level.

II Effect of learning

II.1 Grand coalition formation and efficiency

We have already shown in Section 6.1 that both mechanisms fail to achieve an efficient outcome. However, H–MC mechanism performs significantly better in this matter. A possible explanation is because, as we have already observed in Section 6.1 and with Result 1, H–MC mechanism forces feasible offers, while Winter mechanism allows for unfeasible demands or players forming smaller coalitions prematually which, as a result, lead to inefficiencies. This also naturally leads to the fact that the grand coalition is formed more often under the H–MC mechanism, than under the Winter mechanism.

One may hypothesize that this generalized failure (more for Winter, but partially also for H-MC) in reaching an efficient outcome can explained by some limited rationality arguments: even if we chose two mechanisms that are in our opinion simple, the games' optimal dynamics is hard to understand for participants to the experiment especially in the beginning.

We check this hypothesis by investigating the presence of a learning effect by comparing the outcomes in the first half of four rounds (1-4) and the second half of four rounds (5-8). Because the number of groups that formed a grand coalition becomes small if we separate the data into the first half and second half (see Table II.1), we investigate only the frequency of grand coalition formation and efficiency.

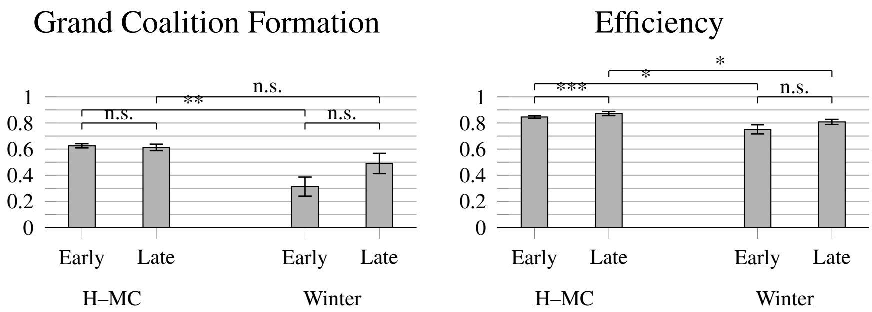
Figure II.1 shows the frequency of the grand coalition formation (left) and the average efficiency (i.e., the average total payoff / value of the grand coalition, right) for the first half and the second half (i.e., the first four rounds vs. the second four rounds). We pool four games. For H–MC, while the frequency of the grand coalition formation is not significantly different in the early and later rounds, the efficiency is significantly higher in the later rounds than in the early rounds. For the Winter mechanism, both the fre-

Table II.1: Number of groups with Grand Coalition

	game 1	game 2	game 3	game 4
Winter early	10	10	1	9
Winter late	13	17	5	12
H–MC early	10	12	11	17
H–MC late	15	13	8	13

Note: game 2 allows $\{2, 3, 4\}$ to be the grand coalition.

Figure II.1: Grand coalition formation and efficiency in early and late rounds (all games pooled)



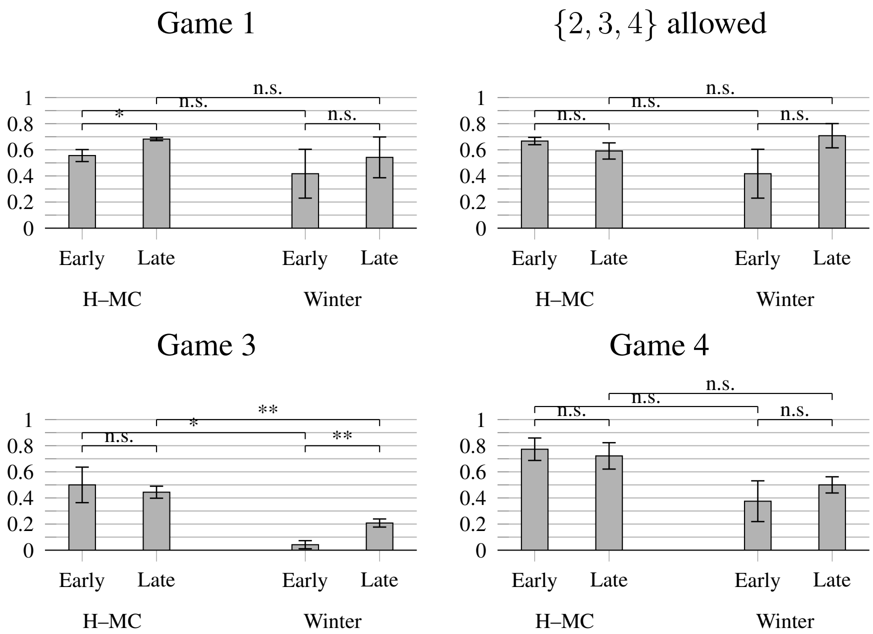
Note: The error bars show the one standard error range. The symbols ***, **, and * indicate the outcomes shown in two bars are significantly different at 0.1%, 1%, and 5% significance levels, respectively (Wald test).

quency of the grand coalition formation and the average efficiency are not significantly different between early and late rounds.

Figures II.2 and II.3 show the frequency of the grand coalition formation and the average efficiency for the first half and the second half of each game. For H–MC, the frequency of the grand coalition formation and the average efficiency are significantly higher in the later rounds only in game 1; for the remaining three games, there are no significant differences between the early and late rounds. For the Winter mechanism, both the frequency of the grand coalition formation and the average efficiency are significantly higher in the later rounds only in game 3, with no significant differences in other games.

As conclusion, we report no statistically significant learning effect, when testing either the H–MC or the the Winter mechanism. This does not rule out the possibility that by implementing a higher number of repetitions, a significant learning effect could be observed.

Figure II.2: Proportion of times the grand coalition formed in early and late rounds



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate the outcomes shown in two bars are significantly different at 0.1%, 1%, and 5% significance levels, respectively (Wald test).

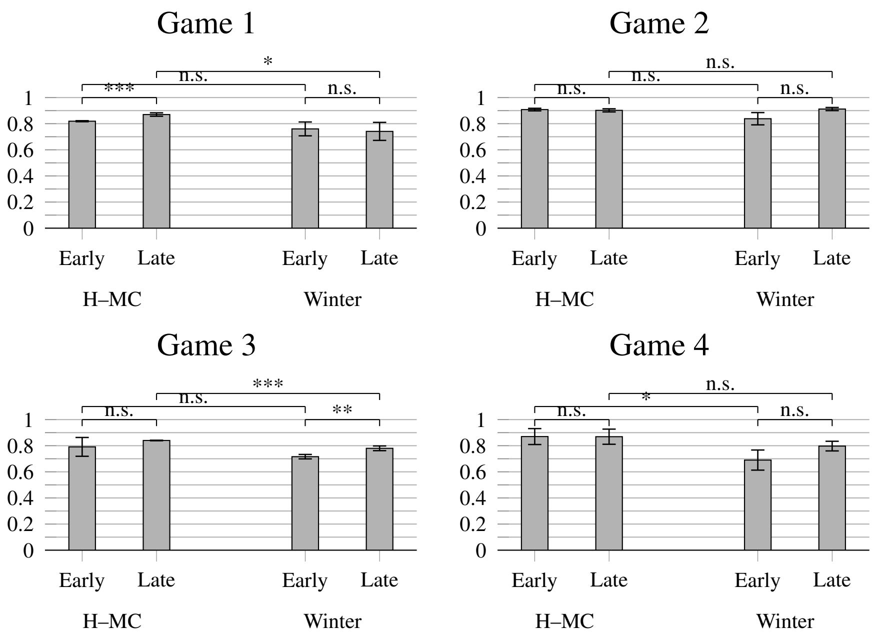
III Observed dynamics in Winter and H-MC

In this section, we analyze the coalition formation dynamics of Winter as well as the way proposals evolved in H–MC to better understand the results presented in the main text. Namely, the reason for failure of the grand coalition formation in Winter and the allocation becoming closer to the equal division solution in H–MC.

III.1 Dynamics of coalition formation in Winter

We first show the low frequency of the grand coalition formation under Winter is due to participants forming smaller coalitions before reaching the 4th player making the demand.

Figure II.3: Efficiency in early and late rounds



As one can observe from Table III.1, out of 176 play of the games where the first mover was not the null player, the first mover exited the game by belonging to a coalition without waiting for the fourth player making the demand 77 times. Thus, there is a clear tendency for participants forming a coalition and exiting the game prematurely.

Among 70 cases in games other than game 2 where the first move belonged to the coalition formed by the fourth mover, only 50 formed the grand coalition. For game 2, in 2 out of 2 cases in which the first move belonged to the coalition formed by the fourth mover and 17 out of 18 in which the first move belonged to the coalition formed by the third mover, the resulting coalition were $\{2, 3, 4\}$ which, in our analyses, considered as the grand coalition.

Furthermore, among these 21 cases where the grand coalition (including $\{2, 3, 4\}$ in game 2) was not formed even when the coalition to which the first player belonged to

Table III.1: Frequency of the timing of the formation of the coalition to which the first mover belongs to.

Formed by	All	All*	Game 1	Game 2	Game 2*	Game 3	Game 4
Self	4	4	0	3	3	0	1
Second mover	8	8	0	2	2	3	3
Third mover	65	65	15	18	18	19	13
Fourth mover	72	72	27	2	2	17	26
None	43	27	6	23	7	9	5
Total	192	176	48	48	32	48	48

* Excluding the cases where the null player was the first mover.

was formed by the 4th (or the 3rd in case of game 2) player, the total amount demanded by four players exceeded $v(N)$, and thus it was not possible to form the grand coalition, in 14 cases. In remaining 7 cases, the grand coalition was not formed although doing so was possible. An interesting observation is that in 2 cases in game 2 where the first mover belonged to the coalition formed by the fourth mover and the coalition $\{2, 3, 4\}$ instead of $\{1, 2, 3, 4\}$ was formed, the sum of the demand including the one by the null player did not exceed $v(N)$.

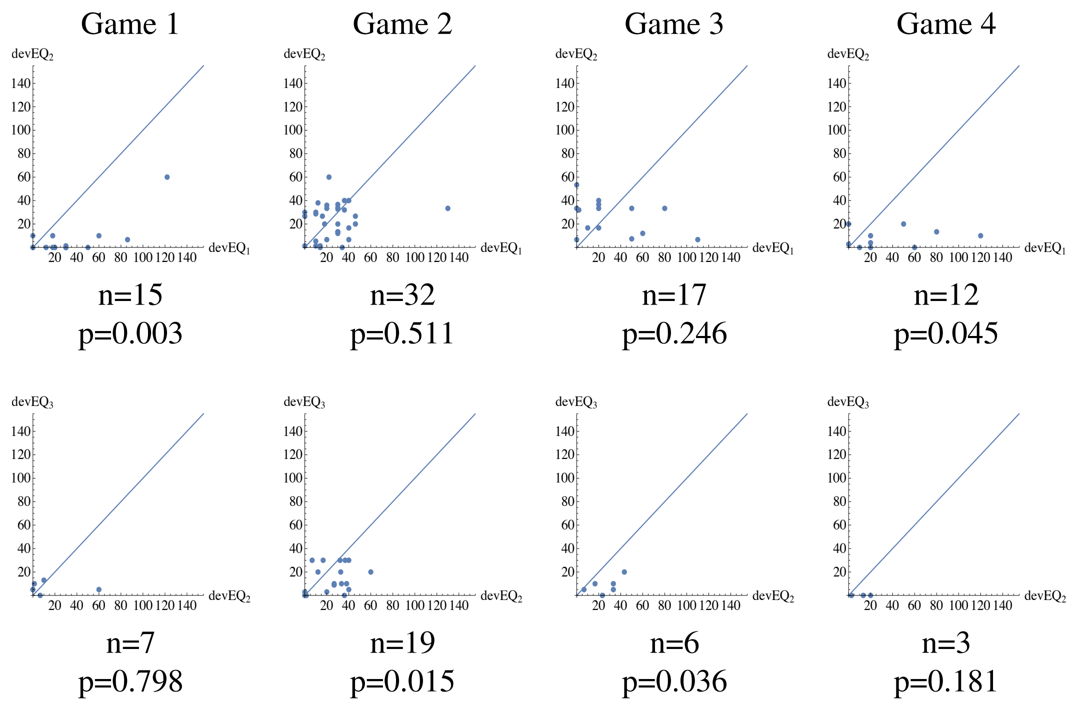
III.2 Dynamics of proposals in H–MC

In Section 6, we have already shown that accepted proposals in the H–MC mechanism go in the sense of equal division. This result is not surprising, as experimental results of offer-based mechanisms (such as the well-known two-player bargaining over a pie of Rubinstein (1982)) often show that, contrary to the theoretical prediction, players tend to go for an equal split of the pie. We show that, in our experiment, this behavior becomes more evident after a first rejection of a proposal, as second proposals are closer to the equal share than the first ones.

Let the distance between the proposal and equal devision in Lth proposal (for a group), $devEQ_L$, be $devEQ_L = \sum_i |a_{L,i} - ED|$ where $a_{L,i}$ is the proposed allocation for player i in Lth proposal (for the group) and ED is the equal division payoff for the game.

First, we observe the first proposal is less likely to be accepted if its distance from the equal division is larger. The estimated coefficient for the $devEQ_1$ is negative and significant in a linear regression in which the dependent variable is the dummy variable that takes value of 1 if the proposal is accepted and 0 otherwise, and the independent variables are the constant and $devEQ_1$ (-0.005 with the standard error (corrected for the session clustering effect) being 0.0009 and p-value $= 0.008$. $N = 160$. $R^2 = 0.1352$)).

Figure III.1: H–MC mechanism: Distance from equal division for the first and the second proposals (top) and the second and the third proposals (bottom)



Note: In each panel, only those groups in which the first proposal (top) or the second proposal (bottom) is rejected are plotted. p-values are based on the Signed-Rank test (two-tailed) with the null hypothesis $devEQ_L = devEQ_{L+1}$.

In Figure III.2, $devEQ_L = \sum_i |a_{L,i} - ED|$ for the first ($L = 1$, horizontal axis) and the second (vertical axis) proposals (top) and the second ($L = 2$, horizontal axis) and the third (vertical axis) proposals (bottom) for each game. Each dot corresponds to a pair of the proposals of a group.

We observe a clear tendency for either the second proposal to be more equal than the first one ($devEQ_1 > devEQ_2$) or the second proposal to be more equal than the third one ($devEQ_2 > devEQ_3$) depending on the game.

IV Testing for the axioms of the Shapley value

We test the axioms that are historically the most relevant to characterizing the Shapley value. In doing so, we aim to provide greater insight into whether a demand-based bargaining mechanism is more appropriate than an offer-based bargaining mechanism for cost or payoff allocation problems when the allocation scheme is constructed on the main axiomatic solution notion of cooperative game theory, that is, the Shapley value.

In the literature, we find various axiomatic characterizations of cooperative solutions and, in particular, of the Shapley value. Given a solution $\psi : \mathcal{G}^N \to \mathbb{R}^N$, we have already listed in Section 3 four axioms that are used in the characterization by Shapley (1953): efficiency, symmetry, additivity and null player property. Here, we list three additional commonly used axioms to provide a characterization.

Axiom 5 (Homogeneity): for all $v \in \mathcal{G}^N$ and $a \in \mathbb{R}$, $\psi(av) = a\psi(v)$.

Axiom 6 (Strong monotonicity): if $i \in N$ is such that $v(S \cup \{i\}) - v(S) \leq w(S \cup \{i\}) - w(S)$ for each $S \subseteq N$, then $\psi_i(v) \leq \psi_i(w)$.

Axiom 7 (Fairness): if i, j are symmetric in $w \in \mathcal{G}^N$, then $\psi_i(v + w) - \psi_i(v) =$

$\psi_j(v+w) - \psi_j(v)$ for all $v \in \mathcal{G}^N$.

Fairness states that if we add a game $w \in \mathcal{G}^N$, in which players i and j are symmetric, to a game $v \in \mathcal{G}^N$, then the payoffs of players i and j change by the same amount.

The axiomatization of Young (1985) involves axioms 1, 2, and 6, whereas that of van den Brink (2002) involves axioms 1, 4, and 7. Note that axiom 5, even if not directly involved in any of these axiomatizations, is crucial because, together with axiom 3, it guarantees the linearity of the solution.[1]

We noted in Section 6 that both mechanisms fail to satisfy efficiency (axiom 1) if we examine overall data. Here, we examine the remaining six axioms focusing on the groups that formed grand coalition. These axioms are tested based on the estimated coefficients obtained from running the regression of Eq. 1 as follows.

- **Symmetry (axiom 2)** requires $b_1 = c_1$ and $b_4 = c_4$.
- **Additivity (axiom 3)** and **homogeneity (axiom 5)** require that $x_3 = x_1 + x_2$ and $x_4 = 2x_1$ for $x \in \{a, b, c, d\}$, respectively.
- **Null player** property **(axiom 4)** requires that $a_2 = 0$.
- **Strong monotonicity (axiom 6)** requires that $a_1 > a_2$ and $a_4 > a_3$.
- **Fairness (axiom 7)** requires that $b_3 - b_2 = c_3 - c_2$.

[1]The equal division solution satisfies 1, 2, and 3, but does not satisfy the null player property in 4. However, it satisfies a similar property when null players are replaced with nullifying players. Player i is a *nullifying player* if $v(S) = 0$ for each $S \subseteq N$ such that $i \in S$. Then, we can state the following additional axiom that can be called the nullifying player property: if i is a nullifying player in game $v \in \mathcal{G}^N$, then $\psi_i(v) = 0$. Replacement of the null player property in the axiomatization of the Shapley value in Shapley (1953) with the nullifying player property characterizes the equal division solution (see van den Brink, 2006).

Table IV.1: Results of Wald tests for the verification of the symmetry, additivity, homogeneity, strong monotonicity, and fairness axioms (based only on the groups that formed a grand coalition)

		H–MC		Winter	
Axiom	H_0	χ^2	p-value	χ^2	p-value
Symmetry	$a_2 = a_3$	0.35	0.552	1.85	0.174
	$d_2 = d_3$	1.60	0.206	0.06	0.811
Additivity	$c_1 = a_1 + b_1$	6.69	0.001	0.13	0.721
	$c_2 = a_2 + b_2$	3.23	0.072	0.02	0.878
	$c_3 = a_2 + b_3$	2.16	0.142	0.47	0.492
	$c_4 = a_4 + b_4$	1.52	0.218	0.78	0.376
Homogeneity	$d_1 = 2a_1$	0.00	0.946	0.10	0.749
	$d_2 = 2a_2$	0.08	0.772	0.11	0.745
	$d_3 = 2a_3$	0.00	0.983	0.00	0.947
	$d_4 = 2a_4$	0.06	0.813	0.82	0.365
Null player	$a_2 = 0$	42.91	0.000	.	.
Strong monotonicity	$a_1 = b_1$	10.76	0.001	102.24	0.000
(H_0 should be rejected)	$c_1 = d_1$	0.16	0.692	26,84	0.000
Fairness	$b_3 - b_2 = c_3 - c_2$	0.62	0.433	0.74	0.391

In Table IV.1, we present the results of the Wald tests for the verification of these axioms, together with the null hypothesis (H_0).

Note that the symmetry (according to which H_0 should not be rejected) is confirmed for the two cases under both the Winter and the H–MC mechanisms. The additivity (according to which H_0 should not be rejected) is always confirmed under the Winter mechanism, but is not confirmed in one of four cases under the H–MC mechanism. The homogeneity (according to which H_0 should not be rejected) is always confirmed for both mechanisms. The null player property (according to which H_0 should not be rejected) is not confirmed in the H–MC mechanism, but it is confirmed (respected 100% of the time) for the Winter mechanism. The strong monotonicity (according to which

Table IV.2: Tests of axioms (based only on the groups that formed a grand coalition)

Axiom	H–MC	Winter
Symmetry	+	+
Additivity	+	+
Homogeneity	+	+
Null player property	-	+
Strong monotonicity	-	+
Fairness	+	+

$+$ indicates that the axiom is considered to be satisfied on average. $-$ indicates the opposite.

H_0 should be rejected) is confirmed for the Winter mechanism but only for half of the time for the H–MC mechanism. The fairness (according to which H_0 should not be rejected) is confirmed for both mechanisms.

Let us consider that the axiom is satisfied on average if it is confirmed for strictly more than half of the cases being tested. Table IV.2 summarizes whether each axiom is satisfied on average (+) or not (-) for two mechanisms. We can state the following.

Result 4. *Provided the grand coalition is formed, the Winter mechanism better satisfies axioms that characterize the Shapley value than the H–MC mechanism.*

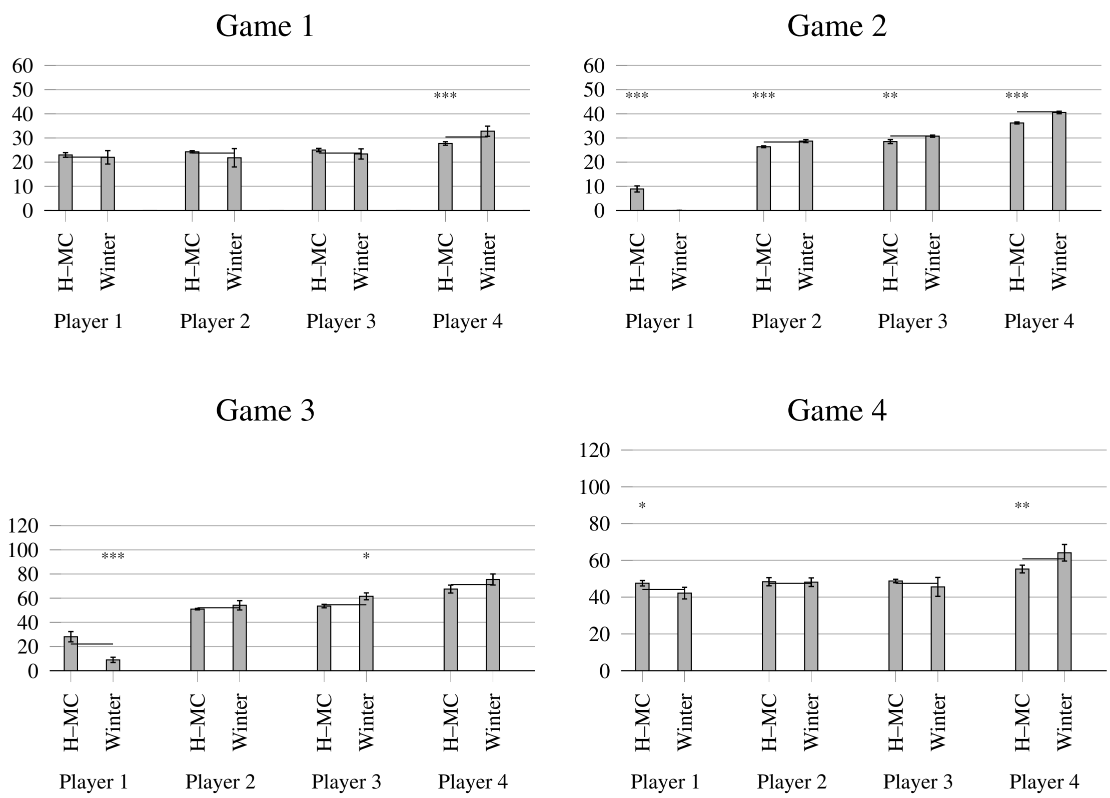
V Additional results based on payoff shares

In this section, we report the results based on all the groups and we use the payoff share instead of restricting our attention to those that formed the grand coalition. Payoff shares are defined as $\widetilde{\pi}_i^W(v_k) = \frac{\pi_i^W(v_k)}{\sum_{j\in N}\pi_j^W(v_k)} \times v_k(N)$ and $\widetilde{\pi}_i^{H--MC}(v_k) = \frac{\pi_i^{H--MC}(v_k)}{\sum_{j\in N}\pi_j^{H--MC}(v_k)} \times v_k(N)$ for each $i = 1, 2, 3, 4$.

As in the main text, our analyses are based on running a set of OLS regressions shown by Eq. 1 but using payoff shares as dependent variables.

Figure V.1 shows the mean of the normalized payoffs in the four games, where the horizontal lines indicate the Shapley values for each game. It can be observed that for the Winter mechanism, the average normalized payoffs are not significantly different from the Shapley values for all four players in games 1, 2, and 4. However, for the H–MC mechanism, the average normalized payoffs for all four players respect the Shapley values only in game 3 at the 5% significance level.

Figure V.1: Mean of the normalized payoffs



Note: The horizontal lines indicate the Shapley values. The error bars show the one standard error range. The symbols ***, **, and * indicate the frequency with which the average normalized payoff is significantly different from the Shapley values at the 0.1%, 1%, and 5% significance levels, respectively (Wald test).

Tables V.1 and V.2 summarize the results of testing the six axioms. Based on the normalized payoff, on average, the symmetry, strong monotonicity, and fairness axioms are now satisfied under the H–MC mechanism. For the Winter mechanism, with normalized payoffs, the fairness axiom is no longer satisfied.

Thus, if we consider all the groups and normalized payoffs, the Winter and H–MC mechanisms are comparable in terms of their distance to the Shapley value and satisfaction of its properties.

Table V.1: Wald tests for the verification of the symmetry, additivity, homogeneity, strong monotonicity, and fairness axioms for normalized payoffs

		H–MC		Winter	
Axiom	H_0	χ^2	p-value	χ^2	p-value
Symmetry	$a_2 = a_3$	0.53	0.466	0.08	0.781
	$d_2 = d_3$	0.03	0.869	0.14	0.712
Additivity	$c_1 = a_1 + b_1$	0.99	0.319	7.25	0.007
	$c_2 = a_2 + b_2$	0.07	0.790	0.65	0.422
	$c_3 = a_2 + b_3$	0.00	0.952	2.54	0.111
	$c_4 = a_4 + b_4$	0.92	0.336	0.35	0.555
Homogeneity	$d_1 = 2a_1$	2.48	0.115	0.06	0.805
	$d_2 = 2a_2$	0.01	0.926	0.37	0.542
	$d_3 = 2a_3$	0.31	0.580	0.02	0.892
	$d_4 = 2a_4$	0.00	0.963	0.35	0.552
Null player	$a_2 = 0$	49.51	0.000	.	.
Strong monotonicity	$a_1 = b_1$	46.26	0.000	62.74	0.000
(H_0 should be rejected)	$c_1 = d_1$	14.57	0.001	147.12	0.000
Fairness	$b_3 - b_2 = c_3 - c_2$	0.58	0.447	7.53	0.006

Figure V.2 shows the mean $Dis2_{NE}$ and the mean $Dis2_{ED}$ for the two mechanisms in the four games computed based on the normalized payoffs using all the groups. The normalized payoffs under the H–MC mechanism are significantly closer to the equal

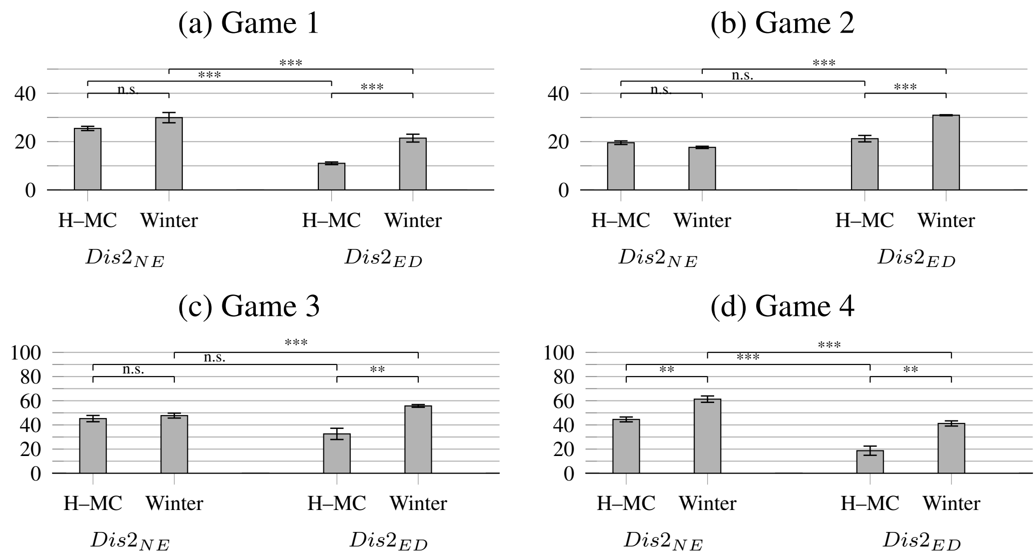
Table V.2: Tests of axioms for normalized payoffs

Axiom	H–MC	Winter
Symmetry	+	+
Additivity	+	+
Homogeneity	+	+
Null player property	-	+
Strong monotonicity	+	+
Fairness	+	-

$+$ indicates that the axiom is considered to be satisfied on average. $-$ indicates the opposite.

division than those under the Winter mechanism in all four games. Furthermore, those under H–MC are significantly closer to the equilibrium payoffs in game 4 than those under the Winter mechanism. However, for games 1 and 4, for both the Winter and H–MC mechanisms, normalized payoffs are significantly closer to the equal division than to the equilibrium payoffs. For games 2 and 3, the normalized payoffs under the Winter mechanism are significantly closer to the equilibrium than to the equal division. Under H–MC, $Dis2_{NE}$ and $Dis2_{ED}$ are not significantly different in games 2 and 3.

Figure V.2: Mean of the distances of the normalized payoff vectors from the SPNE and the equal division



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate statistically significant differences between the two bars at the 0.1%, 0.5%, and 5% significance levels (Wald test for across treatment differences and t-test for within treatment difference between $Dis2_{NE}$ and $Dis2_{ED}$), respectively.

VI Comparison of a classical H–MC sequential approval mechanism vs. a pseudo-H–MC simultaneous approval mechanism

The comparison between sequential mechanisms and simultaneous ones in favoring the formation of efficient coalitions has been the object of recent experimental laboratory studies (Abe et al., 2021). Experimental evidence shows that subjects may perform very differently in these two proposed settings. Analogously, we propose a comparison between the performances of the H–MC mechanism and a pseudo-H–MC mechanism (in the following, denoted as H–MC$_{sim}$), whose structure is identical to that of the original mechanism except that after an offer is proposed, players are asked to either accept or refuse the proposal simultaneously. Theoretically, the H–MC$_{sim}$ mechanism allows for many more Nash equilibria in which two or more players refuse the proposal.[1] We show that sometimes, as observed by Fréchette et al. (2005a), bargaining behavior is not as sensitive to the different bargaining rules as the theory suggests.

The H–MC$_{sim}$ experiment was conducted in December 2019 at ISER at Osaka University. In total, 84 participants, who had never participated in similar experiments before, were recruited. The experimental procedure was identical to the H–MC experiment reported in the main text. On average, the experiment lasted for 1 hour 30 minutes, including the instructions (11 minutes), a comprehension quiz (5 minutes), and payment.[2] The average earnings were 2,780 JPY.

[1] We thank an anonymous reviewer for pointing this out.

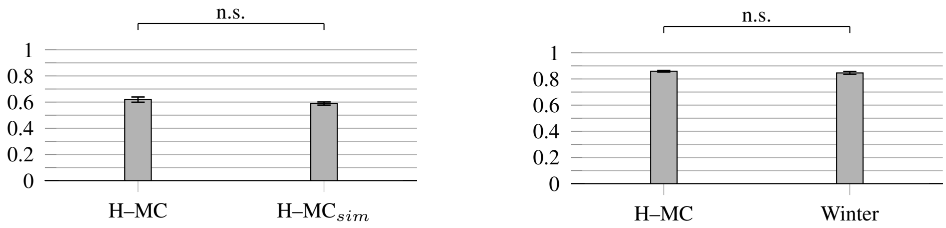
[2] Just as in the H-MC and the Winter, there was a non-binding time limit of 60-seconds to make a proposal and of 30-seconds to accept or reject the proposal. The average (the standard deviation, the maximum, and the minimum) time participants spent to make a proposal is 43.10 (22.31, 111, and 5) seconds (n=313), while those to accept or reject a proposal is 16.72 (10.83, 61, and 1) seconds (n-782).

VI.1 Grand coalition formation and efficiency

Figure VI.1 report the results concerning the grand coalition formation and efficiency by pooling four games. There are no significant difference between H–MC and H–MC$_{sim}$.

Figure VI.1: Proportion of times the grand coalition is formed and efficiency

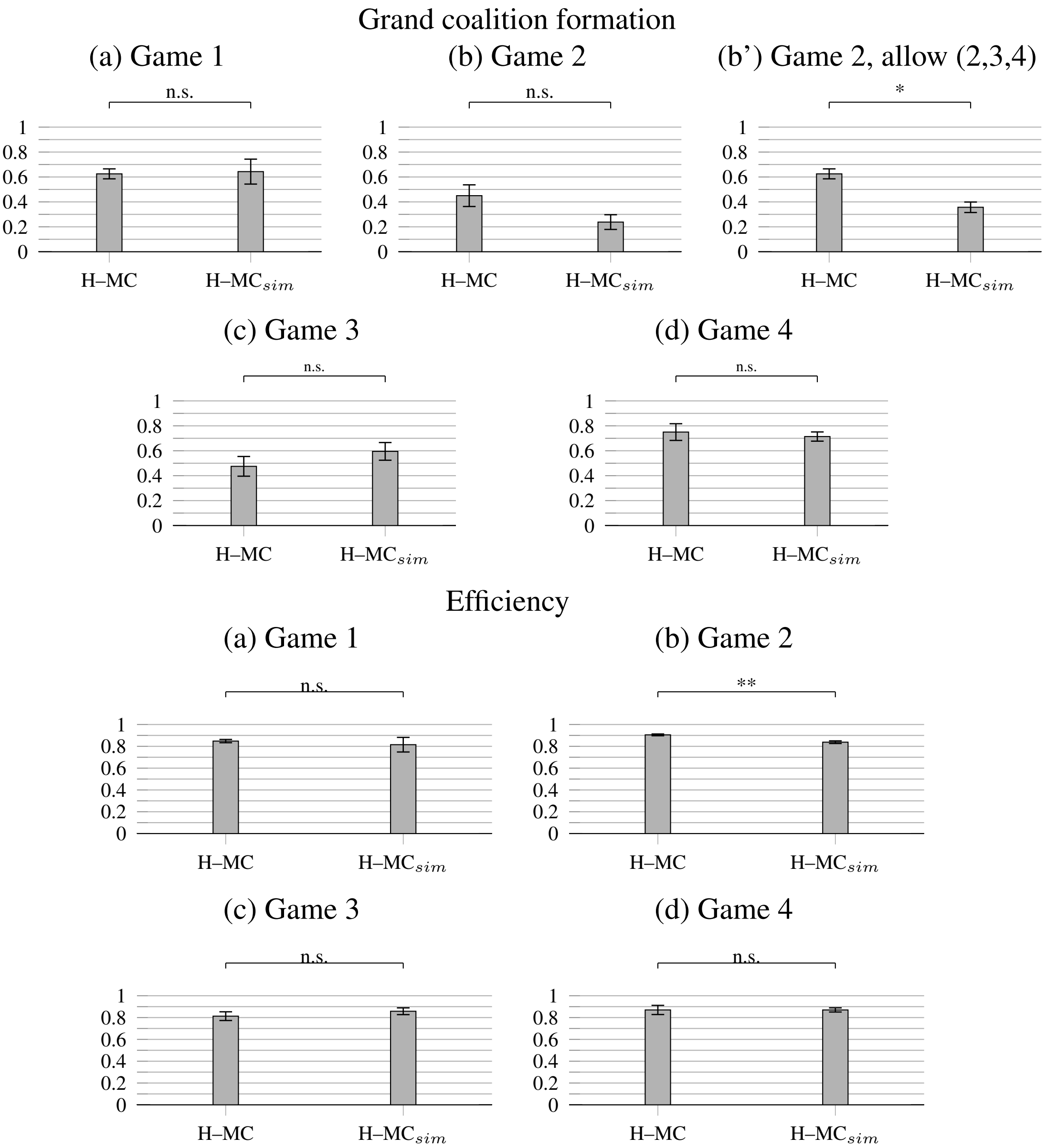
(a) Frequency of Grand Coalition Formation (b) Efficiency



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate the outcomes of H–MC mechanism and the H–MC$_{sim}$ mechanism are significantly different at the 0.1%, 1%, and 5% significance levels (Wald test), respectively.

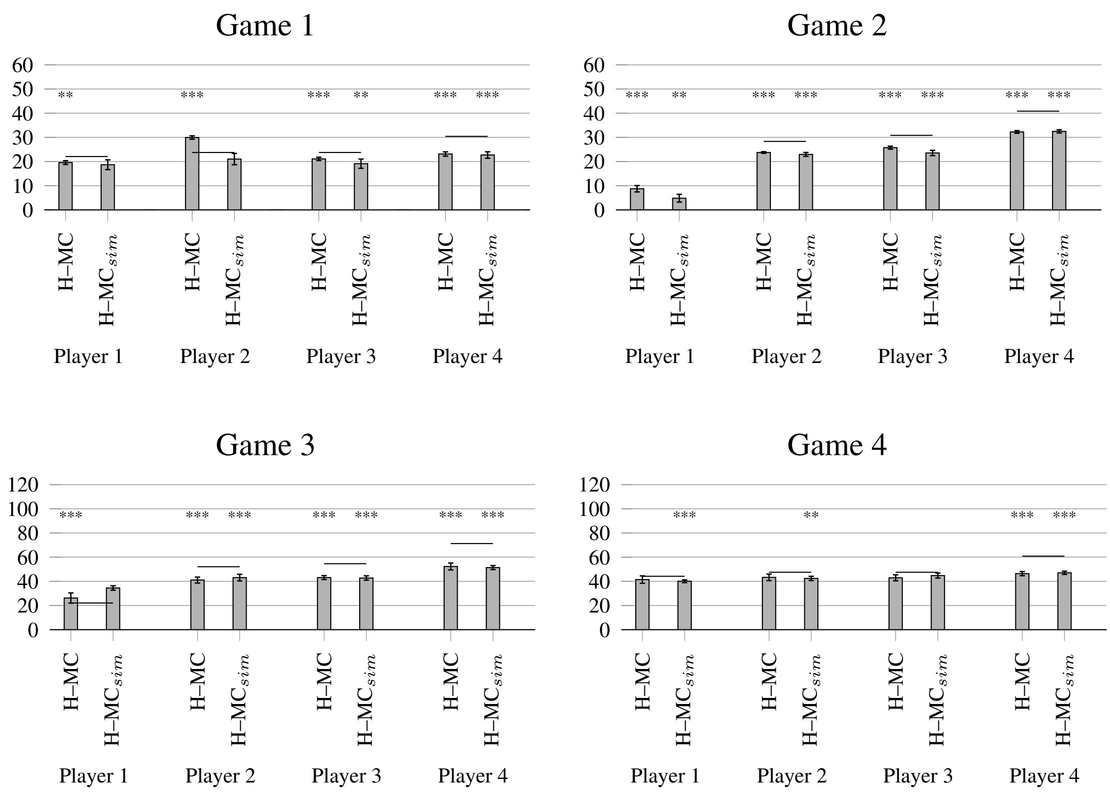
Figures VI.2 reports the results concerning the grand coalition formation and efficiency for each game separately. The only significant differences reported are for game 2.

Figure VI.2: H–MC and H–MC$_{sim}$ mechanisms, proportion of times the grand coalition is formed and efficiency



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate a statistically significant difference at the 0.1%, 1%, and 5% significance levels (Wald test), respectively.

Figure VI.3: H–MC and H–MC$_{sim}$ mechanisms, mean payoffs all the group



Note: The horizontal lines indicate the Shapley values. The error bars show the one standard error range. The symbols ***, **, and * indicate the average normalized payoff, which is significantly different from the Shapley value at the 0.1%, 1%, and 5% significance levels (Wald test), respectively.

VI.2 Allocations

We follow the analyses in the main text by first looking at the average payoffs of each player in each game based on the all the groups.

VI.2.1 Average payoffs

Figure VI.3 shows the average payoffs of each player in each game. The horizontal lines indicate the Shapley values for each game. As we have seen in the main text comparing

Table VI.1: Result of Shapley distance decomposition. Based on pooling the data of all groups and all games

	$\|\|e^{sym}\|\|^2$	$\|\|e^{eff}\|\|^2$	$\|\|e^{null}\|\|^2$	$\|\|e^{add}\|\|^2$	$\|\|e^{\phi}\|\|^2$
H-MC	38.19	429.96	63.97	270.84	802.88
	(12.73)	(53.41)	(8.27)	(20.70)	(62.70)
H-MC$_{sim}$	25.90	386.64	39.55	317.10	769.12
	(2.28)	(40.36)	(10.47)	(21.81)	(55.17)
No. Obs	328	328	328	328	352
R^2	0.085	0.170	0.107	0.261	0.378
p-value*	0.374	0.538	0.110	0.168	0.698

Note: Standard errors are corrected for session-level clustering effects and shown in parentheses. $< e^{add}, e^{null} >$ are not reported in the table as they are negligible (the mean values are 0.0093 for H–MC and 0.0074 for H–MC$_{sim}$).
* p-values for testing H_0: H–MC = H–MC$_{sim}$ (based on the Wald test)

the Winter and H–MC, because of the failure of forming the grand coalition, the average payoffs are significantly different from the Shapley values in both H-MC and H-MC$_{sim}$.

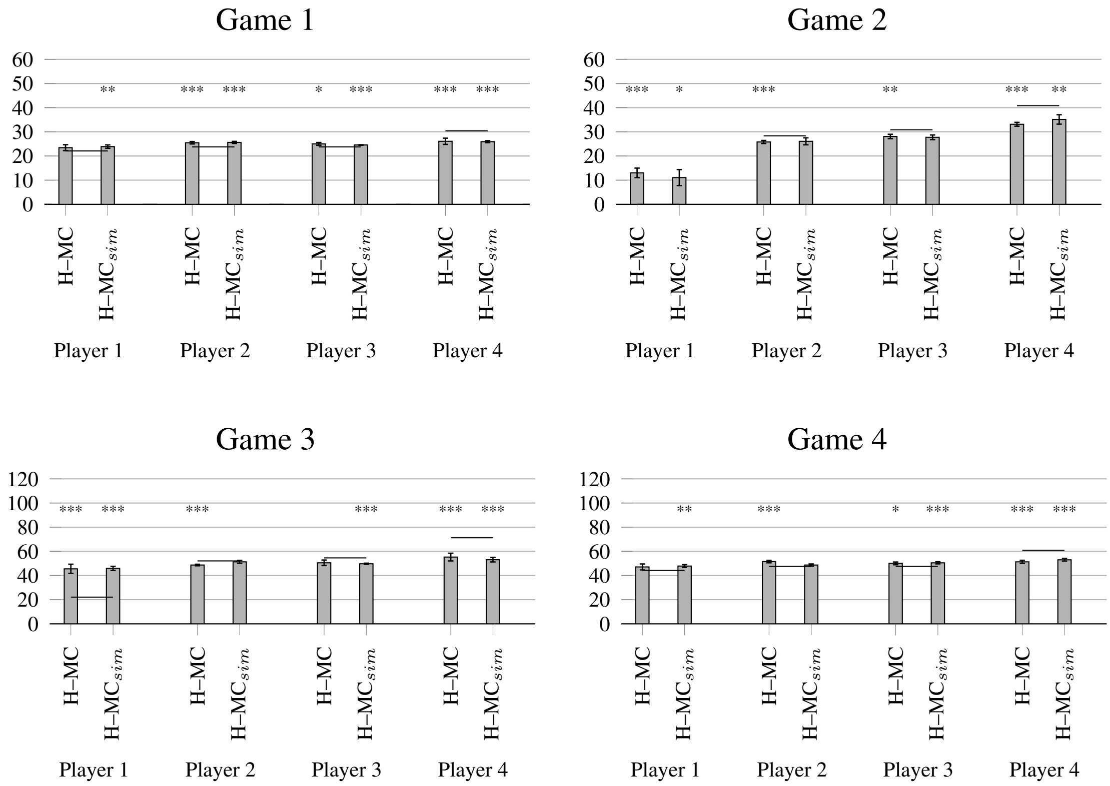
VI.2.2 Shapley distance decomposition

Are there significant differences in terms of the results of Shapley distance decomposition between H-MC and H-MC$_{sim}$? The results reported in Table VI.1 suggest that is not the case. The average Shapley distances and their four components are not significantly different between H-MC and H-MC$_{sim}$.

VI.3 Analyses based only on groups that formed the grand coalition

We now focus on those groups that formed grand coalition. Figure VI.4 shows the average payoffs obtained by each player in the four games, conditional on the grand coalition being formed. The horizontal lines indicate the Shapley values for each game.

Figure VI.4: H–MC and H–MC$_{sim}$ mechanisms, mean payoffs based only on the groups that formed the grand coalition



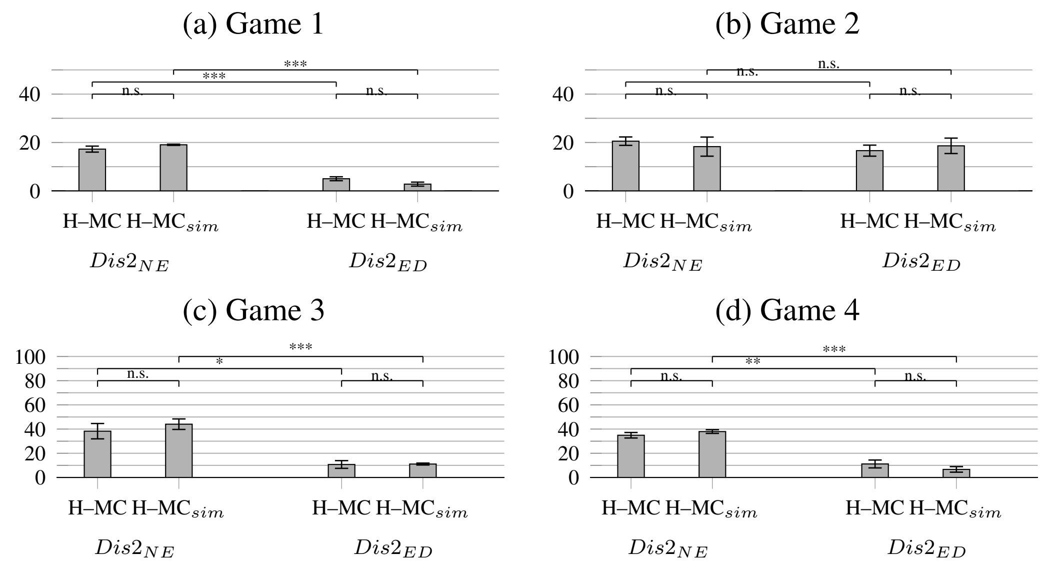
Note: The horizontal lines indicate the Shapley values. The error bars show the one standard error range. The symbols ***, **, and * indicate the average normalized payoff, which is significantly different from the Shapley value at the 0.1%, 1%, and 5% significance levels (Wald test), respectively.

We observe that the two mechanisms perform similarly in that there are players whose average payoff is significantly different from the Shapley value in all four games under both mechanisms even when we focus on only those groups that formed the grand coalition.

VI.3.1 Realized allocations and *a posteriori* equilibria

In terms of distance from SPNE or equal division, we observe from Figure VI.5 that H–MC$_{sim}$ results in outcomes significantly closer to equal division compared with H–MC only in game 1, whereas in the other games there is no significant difference.

Figure VI.5: H–MC and H–MC$_{sim}$ mechanisms, mean of the distances of the realized payoff vectors from the SPNE and the equal division



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate a statistically significant difference at the 0.1%, 0.5%, and 5% significance levels (Wald test for across treatment differences and t-test for within treatment difference between $Dis2_{NE}$ and $Dis2_{ED}$), respectively.

VI.3.2 Axioms

Finally, verification of the axioms (comparing Table VI.2 and the left column of Tables IV.1 and IV.2) indicates that the differences in results between H–MC and H–MC$_{sim}$ are observed for symmetry and fairness (satisfied in H–MC but not in H–MC$_{sim}$).

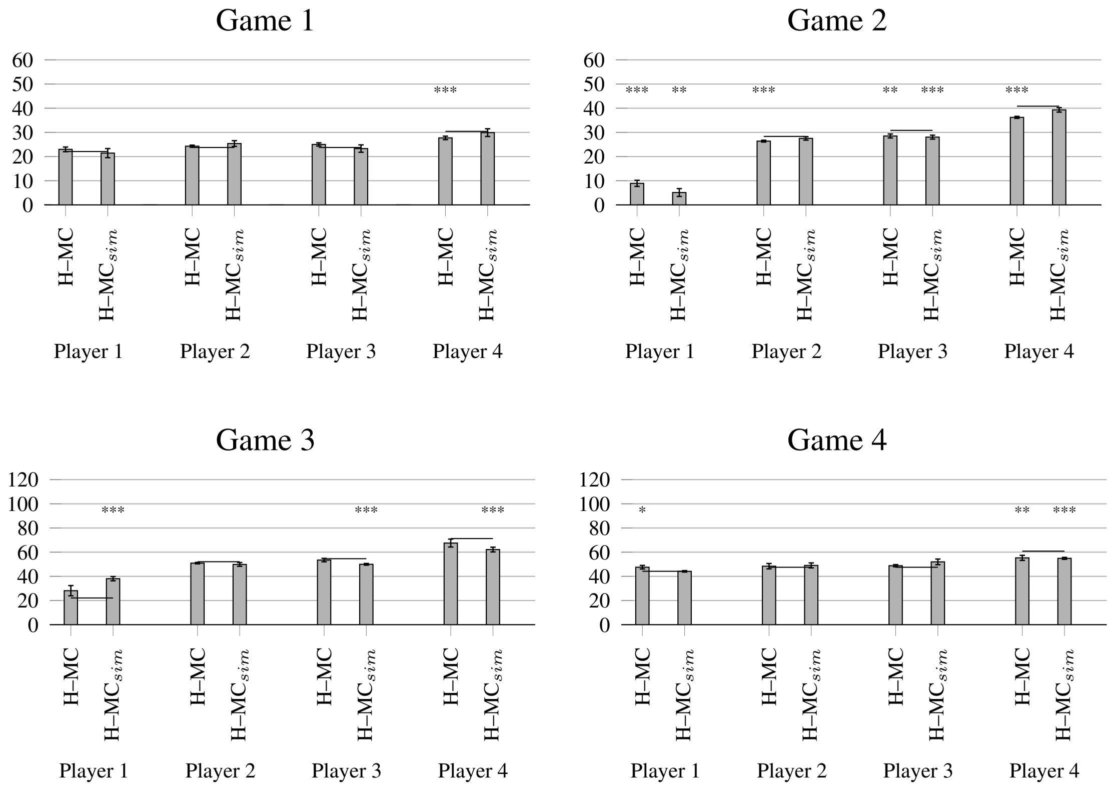
Table VI.2: Results of Wald tests for the verification of the symmetry, additivity, homogeneity, strong monotonicity, and fairness axioms (based only on the groups that formed the grand coalition)

Axiom	H_0	χ^2	p-value	Test
Symmetry	$a_2 = a_3$	5.07	0.024	-
	$d_2 = d_3$	1.11	0.293	
Additivity	$c_1 = a_1 + b_1$	4.84	0.028	-
	$c_2 = a_2 + b_2$	0.03	0.861	
	$c_3 = a_2 + b_3$	14.99	0.000	
	$c_4 = a_4 + b_4$	11.10	0.001	
Homogeneity	$d_1 = 2a_1$	0.00	0.983	+
	$d_2 = 2a_2$	13.12	0.000	
	$d_3 = 2a_3$	2.25	0.134	
	$d_4 = 2a_4$	0.43	0.513	
Null player	$a_2 = 0$	9.90	0.002	-
Strong monotonicity	$a_1 = b_1$	215.83	0.000	-
	$c_1 = d_1$	0.67	0.411	
Fairness	$b_3 - b_2 = c_3 - c_2$	3.02	0.082	+

$+$ indicates that the axiom is considered to be satisfied on average. $-$ indicates the opposite.

To summarize, there is no systematic difference between the H–MC and the H–MC$_{sim}$ mechanisms except that the H–MC better satisfies the symmetry axiom than does H–MC$_{sim}$ if we focus on the groups that formed grand coalitions.

Figure VI.6: Mean of the normalized payoffs



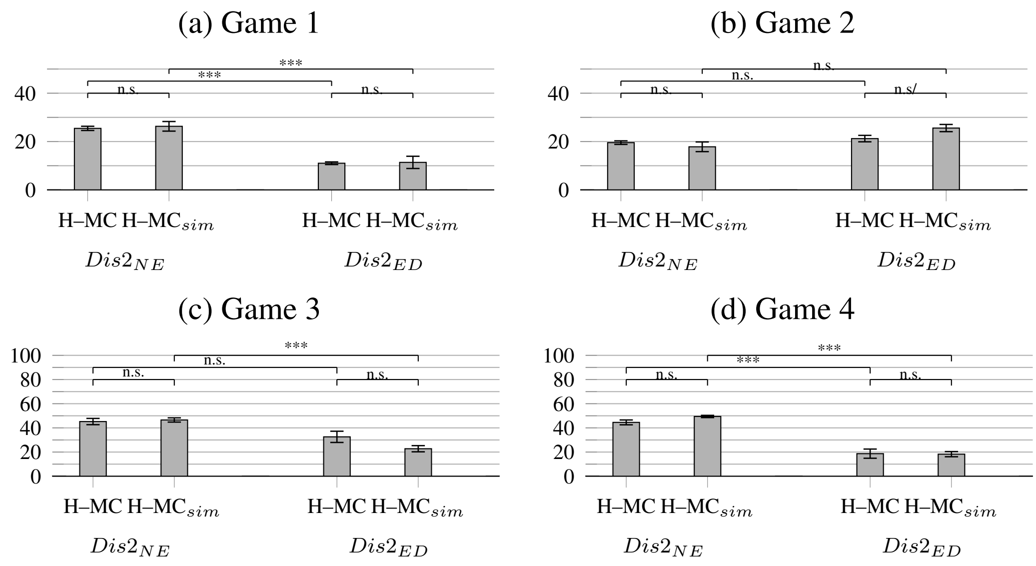
Note: The horizontal lines indicate the Shapley values. The error bars show the one standard error range. The symbols ***, **, and * indicate the average normalized payoff being significantly different from the Shapley values at the 0.1%, 1%, and 5% significance levels (Wald test), respectively.

VI.4 Analyses based on all the groups but only on normalized payoffs

Below, we compare H–MC and H–MC$_{sim}$ based on the normalized payoffs but using the data for all groups.

Figure VI.6 shows the average normalized payoffs obtained by each player in the four games under H–MC and H–MC$_{sim}$ based on results of running a set of OLS regressions as in Eq. 1. The horizontal lines indicate the Shapley values for each game.

Figure VI.7: Mean of the distances of the normalized payoff vectors from the SPNE and the equal division



Note: The error bars show the one standard error range. The symbols ***, **, and * indicate a statistically significant difference at the 0.1%, 1%, and 5% significance levels (Wald test for across treatment differences and t-test for within treatment difference between $Dis2_{NE}$ and $Dis2_{ED}$), respectively.

We observe that for games 1 and 4 under H–MC$_{sim}$, the average normalized payoffs of each of the four players are not significantly different from the Shapley values at 5% significance level, while those for H-MC is significantly different at 5% level for at least for one player in all the four games.

VI.4.1 Normalized payoffs and *a posteriori* equilibria

In terms of distance from SPNE or equal division, we observe from Figure VI.7 that $Dis2_{NE}$ is significantly smaller under H–MC than under H–MC$_{sim}$ only in game 4. For other games, the values are not significantly different between the two mechanisms. In terms of $Dis2_{ED}$, although it is significantly smaller under H–MC in game 2, the opposite is the case for game 3. For games 1 and 4, there is no significant difference between the two mechanisms. We observe that normalized payoffs are significantly closer to the equal division than the SPNE for both mechanisms in games 1 and 4. For H–MC$_{sim}$, $Dis2_{ED}$ is significantly larger than $Dis2_{NE}$ in game 2, whereas the opposite is the case for game 3. For H–MC, $Dis2_{NE}$ and $Dis2_{ED}$ are not significantly different in games 2 and 3.

VI.4.2 Axioms

Finally, verification of the axioms (comparing Table VI.3 and the left column of Tables V.1 and V.2) indicates that the differences in results between H–MC and H–MC$_{sim}$ are observed for additivity and homogeneity (satisfied in H–MC but not in H–MC$_{sim}$).

Table VI.3: H–MC$_{sim}$ normalized payoffs, Wald tests for the verification of the symmetry, additivity, homogeneity, strong monotonicity and fairness axioms

Axiom	H_0	χ^2	p-value	Test
Symmetry	$a_2 = a_3$	1.01	0.314	+
	$d_2 = d_3$	0.47	0.492	
Additivity	$c_1 = a_1 + b_1$	36.91	0.000	-
	$c_2 = a_2 + b_2$	1.11	0.292	
	$c_3 = a_2 + b_3$	0.53	0.466	
	$c_4 = a_4 + b_4$	4.78	0.0288	
Homogeneity	$d_1 = 2a_1$	0.16	0.689	+
	$d_2 = 2a_2$	0.28	0.598	
	$d_3 = 2a_3$	5.90	0.015	
	$d_4 = 2a_4$	3.23	0.072	
Null player	$a_2 = 0$	9.90	0.002	-
Strong monotonicity	$a_1 = b_1$	23.87	0.000	+
(H_0 should be rejected)	$c_1 = d_1$	11.55	0.001	
Fairness	$b_3 - b_2 = c_3 - c_2$	0.15	0.694	+

$+$ indicates that the axiom is considered to be satisfied on average. $-$ indicates the opposite.

To summarize, even comparing the payoff shares using all the groups, there is no systematic difference between the H–MC and the H–MC$_{sim}$ mechanisms, except that H–MC better satisfies the additivity axiom than H–MC$_{sim}$.

VII Translated instruction materials and screenshots of the comprehension quiz

- Winter mechanism: `https://www.dropbox.com/s/galeo3todbah7iw/Winter_1_loop_handout.pdf?dl=0`
- H–MC mechanism: `https://www.dropbox.com/s/ctlw85momf96vmx/HMChandout_seq.pdf?dl=0`
- Simultaneous voting version of the H–MC mechanism (H–MC$_{sim}$): `https://www.dropbox.com/s/78lf5bn6qi3qfwp/HMChandout_sim.pdf?dl=0`