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COST OF COMPLEXITY IN IMPLEMENTING THE SHAPLEY VALUE BY CHOOSING A PROPOSER THROUGH A BIDDING PROCEDURE

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Cost of complexity in implementing the Shapley value by choosing a proposer through a bidding procedure*

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Abstract

We experimentally compare a simplified version of two mechanisms that implement the Shapley value as an (ex ante) equilibrium outcome of a noncooperative bargaining procedure: one proposed by Hart and Mas-Colell (1996, H-MC) and the other by Pérez-Castrillo and Wettstein (2001, PC-W). While H-MC induces the Shapley value only on average, PC-W does so as a unique equilibrium outcome by introducing an additional bidding stage on top of H-MC. We investigate the effect of this additional complexity that PC-W introduces on the resulting outcomes such as the frequency of grand coalition formation, efficiency, and the distance between the realized allocation and the Shapley value. Our experiment shows that H-MC not only results in significantly greater efficiency than PC-W, but also the average allocation is closer to the Shapley value for those groups that formed the grand coalition.

JEL codes: C70, C71, C92
Keywords: Nash program, Bargaining procedures, Shapley value, Experiments

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1 Introduction

The Nash program (Nash, 1953) provides a noncooperative foundation for cooperative solution concepts. In this seminal work, Nash showed that the solution to the cooperative bargaining problem (Nash, 1950) can be obtained as an equilibrium outcome of a noncooperative game, and noted that the fact that a different approach yielding the same solution "indicates that the solution is appropriate for a wider variety of situations" (Nash, 1953, p. 136). Indeed, the main idea of the Nash program "is both simple and important: the relevance of a concept [...] is enhanced if one arrives at it from different points of view" (Serrano, 2005, p. 220).

Many authors have contributed to the development of the Nash program (see, Serrano, 2005, 2008, 2014, 2021, for surveys). Since the inception of the Nash program, the cooperative solution that has attracted most attention is the Shapley value (Shapley, 1953). Indeed, various papers (for example, Gul, 1989; Harsanyi, 1981; Hart and Moore, 1990; Krishna and Serrano, 1995; Winter, 1994; Hart and Mas-Colell, 1996; Pérez-Castrillo and Wettstein, 2001) have proposed its implementation.

Chessa et al. (2022, 2021) contributed to this literature by providing experimental comparisons between Winter (1994) and Hart and Mas-Colell (1996) in Chessa et al. (2022) and among three versions of Winter (1994) in Chessa et al. (2021). Chessa et al. (2022) found that the demand-based mechanism proposed by Winter (1994) resulted in an allocation that better reflects players' effective bargaining power, while the efficiency and the frequency of the grand coalition formation are lower than those from an offer-based mechanism proposed by Hart and Mas-Colell (1996). However, because this result was obtained from a simplified version of the approach of Winter (1994), Chessa et al. (2021) investigated the robustness of the outcome by comparing the outcome of the simplified one-period version of Winter (1994) considered in Chessa et al. (2022) with a more complex—but closer to the original theoretical analysis—two-period version of Winter (1994). Chessa et al. (2021) found that the results were very similar between the one-period and two-period versions.

In this paper, we extend the experimental analyses of Chessa et al. (2022, 2021) by comparing the mechanisms proposed by Hart and Mas-Colell (1996) and Pérez-Castrillo and Wettstein (2001). Both mechanisms are offer-based. Namely, in both mechanisms, there will be a proposer who proposes an allocation, which is voted on by the remaining players sequentially. The key difference between the two is the way the proposer is chosen. On the one hand, a proposer is chosen randomly among the players in Hart and Mas-Colell (1996). On the other hand, in Pérez-Castrillo and Wettstein (2001), the proposer is determined through a bidding procedure. While Hart and Mas-Colell (1996) implements the Shapley value as an *ex ante* equilibrium payoff (i.e., it is only achieved as an expected outcome), Pérez-Castrillo and Wettstein (2001) implements it as a unique equilibrium outcome of the game.

While it is a nice theoretical investigation, the complexity of the bidding procedure introduced by Pérez-Castrillo and Wettstein (2001) makes one wonders whether it would indeed result in the Shapley value if the game was played by participants in a laboratory experiment. Various experimental studies on extensive form games (see, for example, Kagel and Roth, 1995, Ch.4) show that participants have difficulty behaving in accordance with the prediction of the subgame perfect Nash equilibrium. Furthermore, as demonstrated by the idea of "obviously strategy proofness" (Li, 2017), a mechanism needs to be extremely simple for it to result in the outcome intended by its designer.

Indeed, our experiment shows that the simpler mechanism *à la* Hart and Mas-Colell (1996) not only results in greater efficiency than that by Pérez-Castrillo and Wettstein

(2001), but also that the average allocation is closer to the Shapley value for those groups that formed a grand coalition. Thus, the complexity of the mechanism of Pérez-Castrillo and Wettstein (2001) imposes substantial costs to implementing the Shapley value.

The rest of the paper is organized as follows. We present the two mechanisms considered in our experiment in the next section. Section 3 describes the experiment procedure, while the results are presented in Section 4. Section 5 concludes.

2 Two mechanisms

In this section, we present the H-MC mechanism (Section 2.1) and the PC-W mechanism (Section 2.2) in more detail.

2.1 The Hart and Mas-Colell mechanism

The bargaining procedure proposed by Hart and Mas-Colell (1996) implements the Shapley value in monotonic cooperative games.[1] In this mechanism, the bargaining starts with a randomly chosen proposer making an offer to the other players (responders). The responders, sequentially, may either accept or reject the offer. If the offer is accepted by all responders, it is implemented. If one of the responders rejects the offer, the game moves to the next stage. If the offer is rejected, the proposer leaves the game with the worth of her stand-alone coalition with probability p and the bargaining continues with the remaining players, with a new player randomly chosen as the new proposer. In our laboratory implementation, we set $p = 1$ to make it comparable with the PC-W mechanism.

[1] See Peleg and Sudhölter (2007) as well as Chessa et al. (2022) for the general definition and the properties of a cooperative transferable utility (TU) game (a cooperative game), as well as the Shapley value.

We present here a more formal description of the H-MC mechanism. A decision point position at time t is given simply by the vector (S^t, j), where:

$S^t \subseteq N$ is the set of players still in the game, and

$j \in S^t$ is the player making an offer to the remaining players $(t_i)_{i \in S^t \setminus \{j\}}$ such that $\sum_{i \in S^t \setminus \{j\}} t_i \leq v(S^t)$.

With j's offer, the game proceeds now in the following way:

1) if all $i \in S^t \setminus \{j\}$ accept the offer one after the other, then players in $S^t \setminus \{j\}$ are paid $(t_i)_{i \in S^t \setminus \{j\}}$, player j is paid $v(S^t) - \sum_{i \in S^t \setminus \{j\}} t_i$, and the game ends;

2) if at least one player $i \in S^t \setminus \{j\}$ refuses the offer, then there are two possibilities:

$2a$) if $|S^t| = 2$ (only one more player is left, together with j), then they both get the worth of their own stand-alone coalition $v(\{i\})$ for each $i \in S^t$, and the game ends;

$2b$) if $|S^t| > 2$, then player i is removed from the game, she gets her individual payoff $v(\{i\})$, a new proposer $k \in S^{t+1} = S^t \setminus \{j\}$ is randomly selected and the new position is (S^{t+1}, k).

The game starts with a randomly chosen proposer $j \in N$, so that the initial position is (N, j). It terminates either when there are no more players in the game (see point $2a$ above), or when the offer is unanimously accepted (see point 1 above).

Hart and Mas-Colell (1996) showed that this game has a unique subgame perfect equilibrium. At this equilibrium, the grand coalition forms and the *a priori* expected equilibrium payoff coincides with the Shapley value.

Given a specific initial proposer $j \in N$, the *a posteriori* equilibrium payoff assigns to each other player her Shapley value in the cooperative game reduced to the set of players $N \setminus \{j\}$, and to the proposer, the marginal contribution to the grand coalition $v(N) - v(N \setminus \{j\})$.

2.2 PC-W mechanism

Pérez-Castrillo and Wettstein (2001) proposed a bidding mechanism to implement the Shapley value for zero-monotonic cooperative games.[2] In this mechanism, unlike Hart and Mas-Colell (1996), the bargaining starts with a bidding stage where each of the players makes a bid to each of the other players. The proposer is chosen as the player making the *highest net bid*, calculated as the difference between the sum of the bids a player makes to the others minus the sum of the bids the others make to her. If several players make the highest net bid, the proposer is then selected randomly among them. At the end of the bidding stage, the proposer pays the promised bids to the other players. In the second stage, the proposer makes an offer to the other players (responders). This offer is sequentially accepted or rejected by the responders. If all the responders accept, the game ends and the proposer pays the other players according to her offers, and receives what remains of the worth of the coalition. If one of the responders rejects the offer, the proposer leaves the game and obtains the worth of her stand-alone coalition minus the bids she has already paid in the first stage. The rest of the players keep what they have received and a new bargaining starts among them.

We present here a more formal description of the PC-W mechanism. A decision point position at time t is given simply by the set of players $S^t \subseteq N$ remaining in the

[2]In zero-monotonic games, there are no negative externalities when a single player joins a coalition. Formally, we have $v(S) + v(\{i\}) \leq v(S \cup \{i\})$ for any subset $S \subseteq N$ with $i \notin S$. Note that all the four games we consider in experiments respect this assumption.

game, (when $t = 1$, $S^t = N$). The game is described as follows:

(1) Each player $i \in S^t$ makes bids b^i_j to the other players $j \in S^t \backslash \{i\}$. Player i with the highest net bid $\sum_{j \in S \backslash \{i\}} (b^i_j - b^j_i)$ is chosen to be the proposer and pays b^i_j to every $j \in S^t \backslash \{i\}$; if several players make the highest net bid, the proposer is chosen randomly among them.

(2) If player i is the proposer, she makes an offer y^i_j to the other players $j \in S^t \backslash \{i\}$.

(3) With i's offer, the game proceeds now in the following way:

(3a) If the offer is sequentially accepted by the other players, each player $j \in S^t \backslash \{i\}$ receives $b^i_j + y^i_j$, the proposer obtains $v(S^t) - \sum_{j \in S^t \backslash \{i\}} (b^i_j + y^i_j)$, and the game ends.

(3b) If at least one player $j \in S^t \backslash \{i\}$ refuses the offer, then proposer i is removed and obtains $v(\{i\}) - \sum_{j \in S^t \backslash \{i\}} b^i_j$; the new set of players becomes $S^{t+1} = S^t \backslash \{i\}$ in which each member j receives b^i_j and a new bidding mechanism starts among them (if only one player remains in the game, i.e. $|S^{t+1}| = 1$, then that player just obtains the worth of her stand-alone coalition and the game ends).

Pérez-Castrillo and Wettstein (2001) showed that any subgame perfect equilibrium of this game implements the Shapley value as the *a posteriori* equilibrium payoff. At these equilibria, the bid of player i to player j corresponds to her Shapley value of the original game minus her Shapley value in the cooperative game reduced to the set of players $N \setminus \{i\}$. The balanced contribution property (Myerson, 1980) then ensures that all net bids are equal to zero, leading to a random selection of the proposer among all the players. Moreover, player i, if she is randomly selected to be the proposer, offers to player j her Shapley value in the reduced cooperative game with player set $N \setminus \{i\}$. It is worth noting that when the game is strictly zero-monotonic,[3] the grand coalition always forms. Otherwise, rejection of some offers could also constitute a subgame perfect equilibrium (this is the case, for example, in game 2 when player 1 is chosen as the proposer).

3 The experimental setting

We consider the four games that are studied in (Chessa et al., 2022, 2021). These games and their corresponding Shapley values are reported in Table 1.

3.1 Procedure

Upon arrival, participants received a copy of the instruction slides. The instructions were divided into two parts: an explanation of the rules of the game and an explanation of the computer interface. First, a prerecorded video of the first part of the instructions (explanation of the rules of the game) was played. Then, a comprehension quiz (computerized) was administered to make sure participants understood the rules of the game. They needed to answer each question correctly before proceeding to the next one. Once all the participants finished answering the quiz questions, two practice rounds of a game in which the Shapley value for all the players was 25 was run to familiarize the participants with the software before the real experiment started. During these practice rounds, participants were asked to look at the second part of the instructions about

[3] A game is strictly zero-monotonic if $v(S) + v(\{i\}) < v(S \cup \{i\})$ for any nonempty subset $S \subseteq N$ with $i \notin S$. Note that only games 1, 3, and 4 that we consider in our experiments are strictly zero-monotonic.

Table 1: Four games and corresponding Shapley values

S	Game 1 $v_1(S)$	Game 2 $v_2(S)$	Game 3 $v_3(S)$ $= v_1(S) + v_2(S)$	Game 4 $v_4(S)$ $= 2v_1(S)$
1	0	0	0	0
2	5	20	25	10
3	5	20	25	10
4	10	30	40	20
1,2	20	20	40	40
1,3	20	20	40	40
1,4	25	30	55	50
2,3	20	45	65	40
2,4	25	55	80	50
3,4	25	60	85	50
1,2,3	50	45	95	100
1,2,4	60	55	115	120
1,3,4	60	60	120	120
2,3,4	60	100	160	120
N	100	100	200	200
Shapley values for each player				
$\phi_1(v)$	22.08	0	22.08	44.16
$\phi_2(v)$	23.75	28.33	52.08	47.5
$\phi_3(v)$	23.75	30.83	54.58	47.5
$\phi_4(v)$	30.42	40.83	71.25	60.84

the computer interface. In the first round of the practice, the experimenter explained each screen following the instructions. See Appendix A for English translations of the instruction slides and the comprehension quiz.

In the actual experiment, just as in Chessa et al. (2022, 2021), each participant played all four games twice in one of the following four orderings: 1234, 2143, 3412, and 4321. Between each play of a game (called a round), players were randomly rematched into groups of four players, and participants were randomly assigned a new role within the newly created group. At the end of the experiment, two rounds (one from the first four rounds and another from the last four rounds) were randomly selected for payments.

Participants received cash rewards based on the points they earned in these two selected rounds with an exchange rate of 20 JPY = 1 point in addition to 1500 JPY and 1900 JPY participation fees for H-MC and PC-W. The participation fee for PC-W was set larger to compensate for the longer experiment, as well as to cover possible losses participants may have made.

3.2 Simplification of, and an additional difference between, the mechanisms

In our experiment, we simplified both the H-MC and PC-W mechanisms. Namely, after a proposer makes an offer, instead of the remaining players (responders) approving or rejecting the offer one by one sequentially, all responders decided simultaneously.

Coalition formation processes may perform differently in the two settings, sequential versus simultaneous. For example, recently, Abe et al. (2021) compared experimentally the efficiency of (a) a mechanism in which participants decide to join the coalition sequentially as in the original theoretical analysis of Hart and Mas-Colell (1996) and Pérez-Castrillo and Wettstein (2001) and (b) an analogous mechanism in which participants decide to join the coalition simultaneously as in our experimental implementation, and found that the former resulted in higher efficiency than the latter. However, Chessa et al. (2022) investigated the effect of this simplification in H-MC in the same four games we consider in this paper (see their Appendix V). This simplification gives rise to other Nash equilibria, in which two or more players refuse the proposal. However, they reported that the simplified H-MC indeed resulted in a significantly lower frequency of grand coalition formation and efficiency in game 2, but not in the remaining three games, such that the performances of the two implementation resulted as comparable. Moreover, in our experiment this simplification is done both for H-MC and PC-W. Therefore, for the purpose of investigating the effect of the complexity introduced by adding the bidding stage to determine the proposer, the impact of this simplification, if any, should be analogous on two mechanisms and should not affect their comparison.

In our experiment, in addition to the way the proposer is chosen, there is a difference in the way the simplified version of the H-MC and PC-W is implemented. Namely, on the one hand, under our H-MC implementation, the responders observed the proposed allocation to all the players when deciding to accept or reject the proposal. On the other hand, under our PC-W implementation, each responder observed only their own proposed allocation and not those to other players (see the screenshots entitled "respondent's input" for H-MC and "STEP 2 (Respondent)" for PC-W included in the English translation of the instruction material available in Appendix A). The reason for introducing this difference is that in PC-W, responders are presented not only with the offer in the second stage, but also the allocated amount in the first bidding stage and the sum of the two. Presenting the proposed allocation for all the responders in this manner for PC-W substantially increases the amount of information participants have to process when deciding to approve or reject the offer. While this difference in the presentation of the information does not influence the theoretical analyses (based on the standard set of assumptions), it may influence the results we report below, but it is not clear in what way.

4 Results

The computerized experiment was conducted at the Institute of Social and Economic Research (ISER), Osaka University, between June and December 2019. The experiment was computerized and used z-Tree (Fischbacher, 2007). A total of 164 students among those registered in the participants database managed by ORSEE (Greiner, 2015), who had not participated in similar experiments previously, were recruited as subjects of the experiment. Among these 164 participants, 84 played H-MC and 80 played PC-W. Note that the data of H-MC used in this paper is the same as those reported in Appendix V of Chessa et al. (2022) as H-MC$_{sim}$.

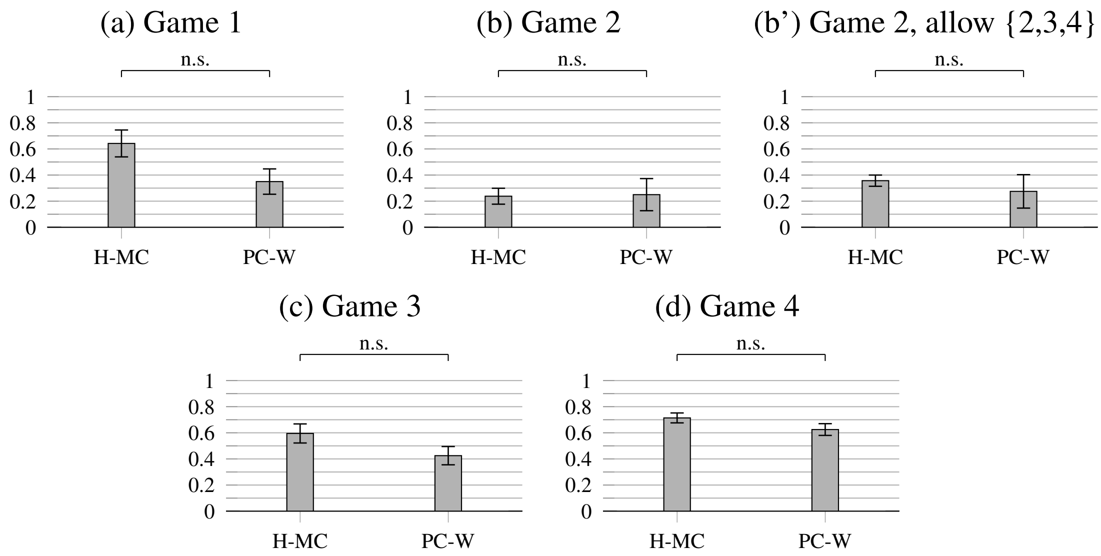
The experiment lasted on average 1h30m for H-MC and 2h05m for PC-W including the instructions, comprehension quiz, and payment. The average earning was 2780 JPY for H-MC and 2990 JPY for PC-W (approximately 25.7 and 27.7 USD using an average exchange rate of 1 USD = 108 JPY for the period over which the experiment was conducted.)

4.1 Grand coalition formation and efficiency

Figure 1 presents the results about the grand coalition formation, in the H-MC mechanism and in the PC-W mechanism, for the four games. The figure is created based on the estimated coefficients of the following linear regression: $gc_i = \beta_1 HMC_i + \beta_2 PCW_i + \mu_i$ where gc_i is a dummy variable that takes the value 1 if the grand coalition is formed, and zero otherwise, in group i, and HMC_i (PCW_i) is a dummy variable that takes the value 1 if the H-MC (PC-W) mechanism is used, and zero otherwise. The standard errors are corrected for any within-session clustering effect. The statistical tests are based on the Wald test for the equality of the estimated coefficients of two treatment dummies.

For game 2, we also consider the case in which we allow a coalition without the null player as the grand coalition (panel b'). We observe from Figure 1 that the frequency of grand coalition formation is higher under H-MC than under PC-W; however, the

Figure 1: H-MC and PC-W mechanisms: proportion of times the grand coalition formed

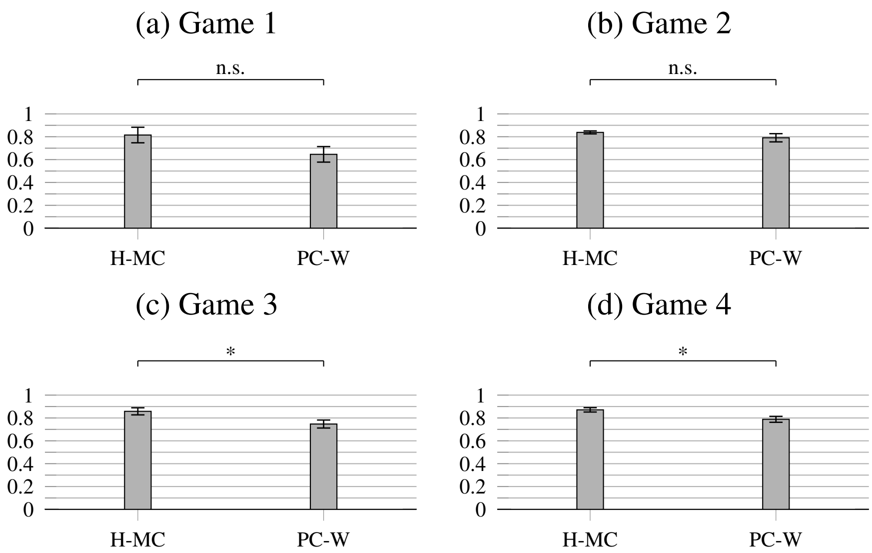


Note: Error bars show one standard error range. ***, **, and * indicate the proportion of times that grand coalition formation is significantly different between the H-MC and the PC-W at the 0.1, 1, and 5% significance levels, respectively (Wald test).

difference is not statistically significant.

As a direct consequence of the grand coalition not always being formed, both mechanisms fail to achieve full efficiency. Figure 2 compares the efficiency between the two mechanisms for four games. Efficiency is computed as the fraction of the sum of the payoffs obtained by the four players compared with the value of the grand coalition of the considered game (100 for games 1 and 2 and 200 for games 3 and 4). The figure is created based on the estimated coefficients of the following linear regressions: $Eff_i = \beta_1 HMC_i + \beta_2 PCW_i + \mu_i$ where $Eff_i \equiv \frac{\sum_i \pi_i}{v(N)}$ is the efficiency measure for group i, HMC_i (PCW_i) is a dummy variable that takes the value 1 if the H-MC (PC-W) mechanism is used, and zero otherwise. The standard errors are corrected for any within-session clustering effect. The statistical tests are based on the Wald test for the equality of the estimated coefficients of two treatment dummies.

Figure 2: Efficiency



Note: Error bars show the one standard error range. ***, **, and * indicate the efficiency is significantly different between H-MC and PC-W at the 0.1, 1, and 5% significance levels, respectively (Wald test).

Figure 2 shows that the efficiency is higher under the H-MC mechanism than under the PC-W mechanism, and that the difference is statistically significant at the 5% level for games 3 and 4.

The significantly lower efficiency under PC-W compared with H-MC (for game 3, the mean efficiency is 0.747 and 0.858 for PC-W and H-MC, respectively, and for game 4, they are 0.788 and 0.871) demonstrates that the cost of complexity introduced by the bidding procedure of the former is not negligible.

4.2 Allocations

We denote by $\pi^{HMC}(v_k)$ a vector of payoffs obtained by the players in the H-MC mechanism in game k, with $k = 1, 2, 3, 4$. Analogously, we denote by $\pi^{PCW}(v_k)$ a vector of payoffs obtained by the players in the PC-W mechanism. As noted above, H-MC implements the Shapley value only on average, whereas PC-W, thanks to the bidding stage, does so as a unique equilibrium outcome. Therefore, we first compare the average realized payoff vectors between the two mechanisms. However, when the players fail to form the grand coalition, the total payoffs obtained by the players is smaller than the value of the grand coalition. This results in the average realized payoff vectors being significantly different from the Shapley value. We therefore focus our analyses on those groups that formed the grand coalition.

Our main analyses are based on a set of OLS regressions (using only the data from groups that formed the grand coalition) for the following system of equations.

$$\begin{aligned}
\pi_1 &= a_1 g_1 + a_2 g_2 + a_3 g_3 + a_4 g_4 + u_1 \\
\pi_2 &= b_1 g_1 + b_2 g_2 + b_3 g_3 + b_4 g_4 + u_2 \\
\pi_3 &= c_1 g_1 + c_2 g_2 + c_3 g_3 + c_4 g_4 + u_3 \\
\pi_4 &= d_1 g_1 + d_2 g_2 + d_3 g_3 + d_4 g_4 + u_4
\end{aligned} \tag{1}$$

where π_i is the payoff of player i, g_j is a dummy variable that takes the value 1 if the game $j \in \{1, 2, 3, 4\}$ is played, and zero otherwise. Because participants play all four games twice, we correct the standard errors for any within-group clustering effect. Note that the estimated coefficients a_j, b_j, c_j, and d_j are the average payoffs in game j for players 1, 2, 3, and 4, respectively.

Table 2 reports the results of these regressions, with H-MC in the left panel and PC-W in the right panel. These estimated coefficients and standard errors are also visualized in Figure 3. In the figure, the horizontal lines indicate the Shapley values for each game. The stars above each bar indicate that the average payoff is significantly different from the corresponding Shapley value at the 0.1, 1, or 5% levels. As one can observe, there

Table 2: Results of linear regression based only on the groups that formed the grand coalition

H-MC

	π_1	π_2	π_3	π_4
g1	23.88	25.63	24.56	25.93
	(0.70)	(0.43)	(0.11)	(0.42)
g2	11.07	26.07	27.73	35.13
	(3.31)	(1.45)	(0.97)	(1.95)
g3	45.88	51.32	49.72	53.08
	(1.70)	(1.19)	(0.50)	(1.84)
g4	47.83	48.67	50.5	53.00
	(1.17)	(0.89)	(0.87)	(1.10)
R^2	0.96	0.99	0.99	0.97
Obs.	97	97	97	97

PC-W

	π_1	π_2	π_3	π_4
g1	23.93	25.00	24.86	26.21
	(0.39)	(1.54)	(0.72)	(1.04)
g2	22.18	23.45	25.55	28.82
	(4.01)	(1.15)	(0.99)	(3.78)
g3	50.76	49.18	51.18	48.88
	(0.44)	(1.59)	(0.41)	(2.08)
g4	49.08	49.64	49.16	52.12
	(1.16)	(0.57)	(0.75)	(1.46)
R^2	0.98	0.98	0.99	0.97
Obs.	67	67	67	67

Note: The standard errors are corrected for any within-group clustering effect.

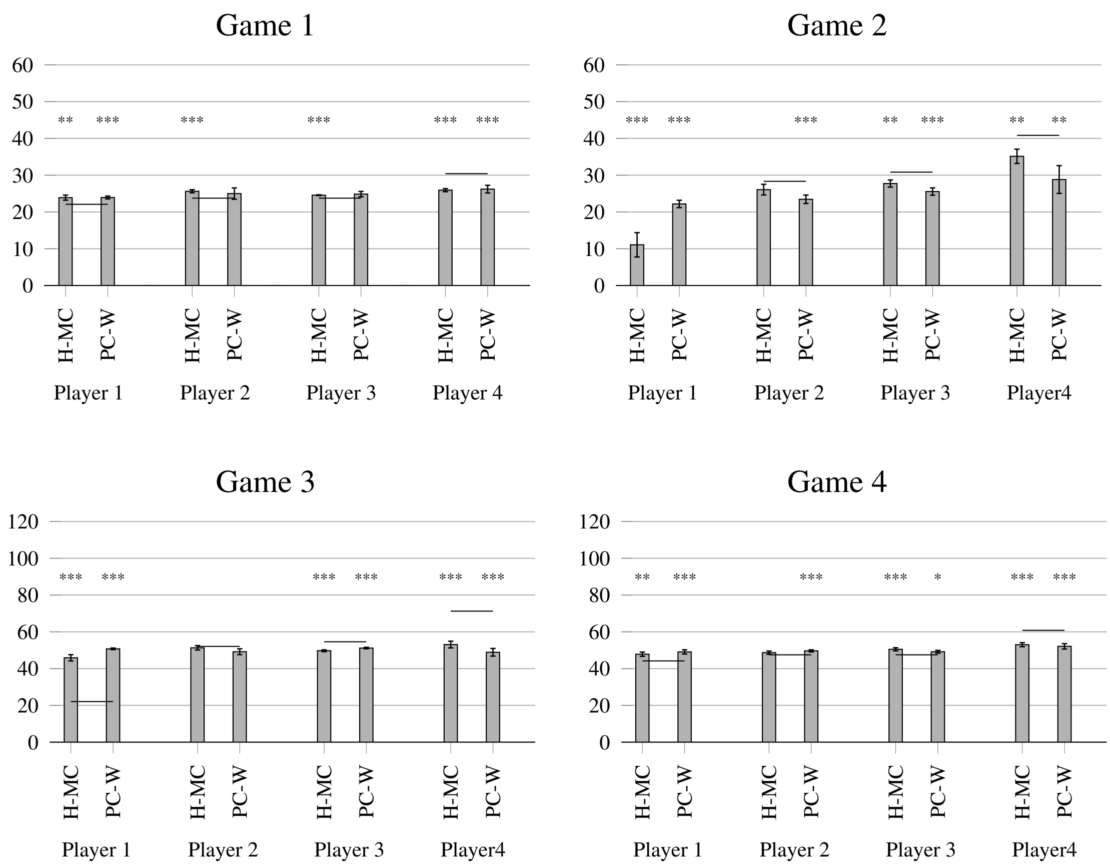
are only a few cases (players 2 and 3 in game 1 as well as player 2 in game 3 for PC-W, and player 2 in games 2, 3, and 4 for H-MC) where the average payoffs are not significantly different from the Shapley value at the 5% level. Thus, neither H-MC nor PC-W implements the Shapley values.

To better compare the two mechanisms in terms of how close their average payoffs are to the Shapley value, we compute the following measure:

$$Dis_{\phi} = \sqrt{\sum_{i} (\overline{\pi_i} - \phi_i)^2} \tag{2}$$

where $\overline{\pi_i}$ and ϕ_i are the average payoff and Shapley value for player i, respectively, in the given game. As above, we focus on only those groups that formed a grand coalition. Furthermore, we employ a bootstrapping technique to conduct statistical tests. Namely, in each interaction, we use a subsample (with replacement) of our data, run the system of

Figure 3: Mean payoffs based only on the groups that formed the grand coalition



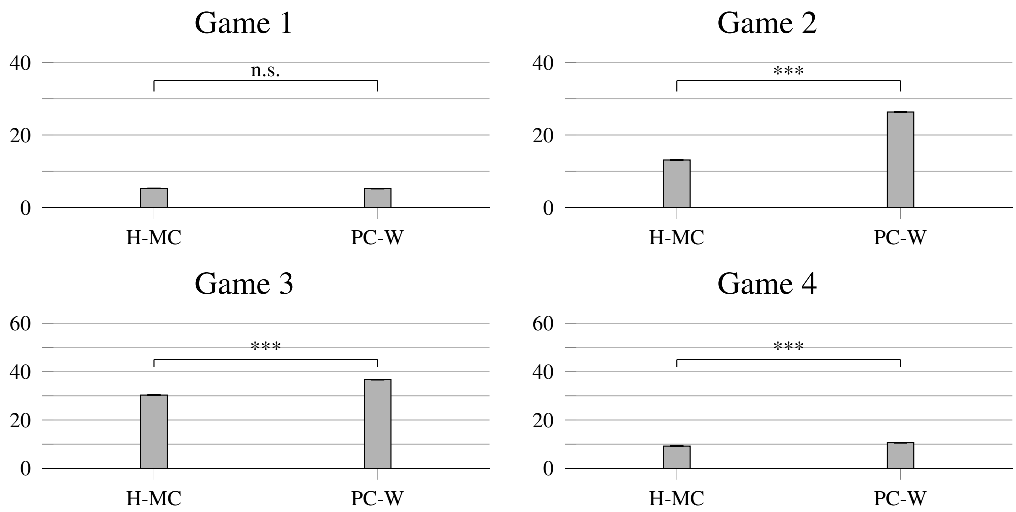
Note: The horizontal lines indicate the Shapley values. Error bars show one standard error range. ***, **, and * indicate the average payoff being significantly different from the Shapley value at the 0.1, 1, and 5% significance levels (χ^2 test).

regressions presented above (Eq. 1), and compute Dis_ϕ based on the obtained estimated coefficients (i.e., the average payoffs for the subsample).

Figure 4 shows the results based on the outcomes of 1000 repetitions. As one can observe, Dis_ϕ is significantly higher for games 2, 3 and 4 under PC-W than H-MC.[4]

[4] Based on two-sample t-test with unequal variance using the sample generated by the bootstrap. The means Dis_ϕ (standard errors) for the H–MC mechanism are 5.21 (0.041) in game 1, 13.11 (0.088) in game 2, 30.31 (0.089) in game 3, and 9.21 (0.058) in game 4. For the PC-W mechanism, the corresponding values are 5.21 (0.041) for game 1, 26.34 (0.098) for game 2, 36.67 (0.053) for game 3, and 10.59 (0.077) for game 4.

Figure 4: H-MC and PC-W mechanisms: distance from Shapley



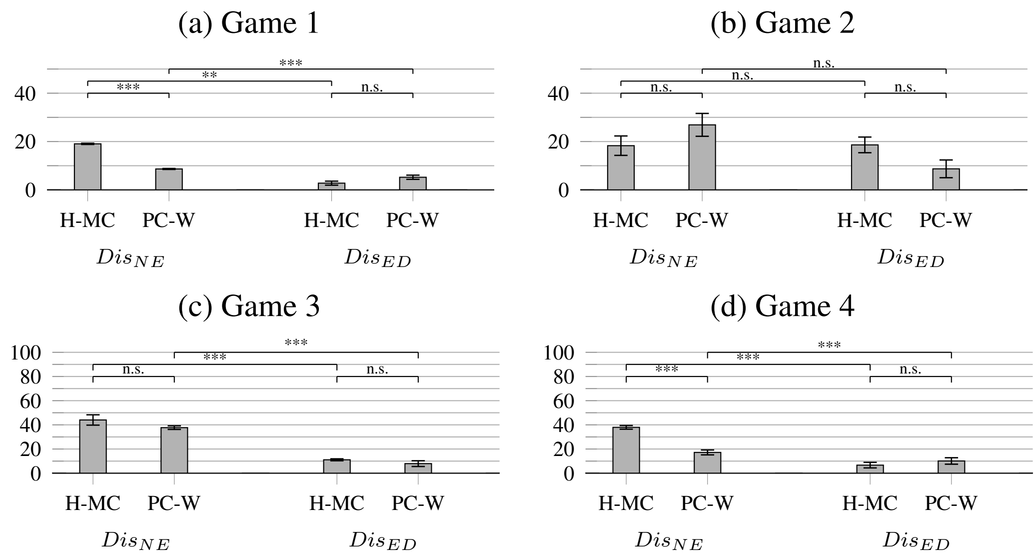
Note: Error bars show the one standard error range. ***, **, and * indicate significant difference between H-MC and PC-W at the 0.1, 1, and 5% significance levels (two-sample t-test).

Thus, the complexity introduced by the bidding stage of PC-W not only results in a lower efficiency, but also a larger deviation from the Shapley value even among those groups that successfully formed the grand coalition.

4.3 Realized allocations and the a posteriori equilibria

Let us now analyze the realized payoffs in light of the a posteriori equilibrium payoff vectors. We continue to focus only on the groups that formed the grand coalition. We measure the distance between the realized payoff vectors and the equilibrium allocation for the four games by the Euclidean distances between the two. Let eq_i be the equilibrium payoff for player i for the given game and the mechanism. Note that eq_i depends also on the realized proposer in the case of H-MC. The distance of the realized payoff from the equilibrium is computed as $Dis_{NE} = \sqrt{\sum_i (\pi_i - eq_i)^2}$. We also consider

Figure 5: Mean of the distances of the realized payoff vectors from the subgame perfect nash equilbrium and the equal division



Note: Error bars show the one standard error range. ***, **, and * indicate that the distance of the payoff vectors from the equilibrium allocations or from the equal division was significantly different between H-MC and PC-W at the 0.1, 1, and 5% significance levels (Wald test).

the distance between the realized payoff vectors and equal division payoffs, defined by $Dis_{ED} = \sqrt{\sum_i (\pi_i - ED_i)^2}$, where ED_i is the equal division payoff for player i for the given game.

Figure 5 shows the mean Dis_{NE} and the mean Dis_{ED} for the two mechanisms in the four games.[5] Figure 5 shows that, except for game 2, the realized allocations are closer to the equal division than the equilibrium one under both H-MC and PC-W. The differences between Dis_{NE} and Dis_{ED} are significant at the 1% significance level, except for game 2. Because the allocation is closer to the equal allocation than the

[5]The figure is created based on the estimated coefficients of the following linear regressions: $Dis_i = \beta_1 HMC_i + \beta_2 PCW_i + \mu_i$ where Dis_i is the relevant distance measure for group i, HMC_i (PCW_i) is a dummy variable that takes the value 1 if the H-MC (P-WC) mechanism is used, and zero otherwise. The standard errors are corrected for any within-session clustering effect. The statistical tests are based on the Wald test for the equality of the estimated coefficients of the two treatment dummies.

equilibrium one, and the equilibrium allocation of PC-W is less extreme than those of H-MC, we observe that PC-W results in an allocation closer to the equilibrium one than H-MC for games 1 and 4 at the 0.1% significance level. For games 2 and 3, Dis_{NE} is not significantly different between the two mechanisms.

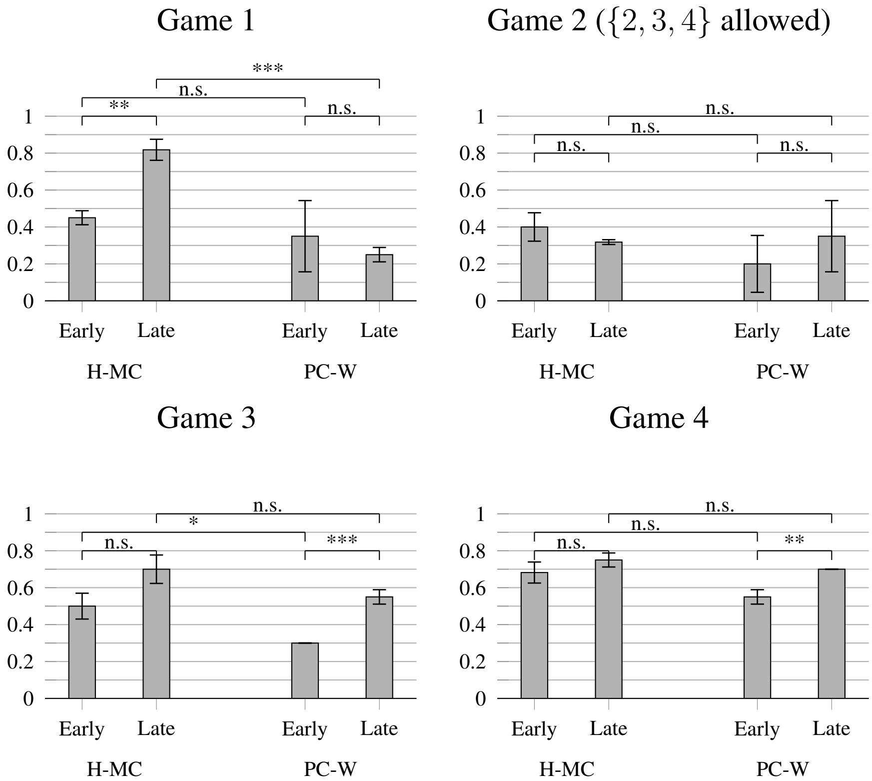
5 Comparison of the first two games and the last two games

So far, we have considered all the rounds and compared H-MC and PC-W. However, given that PC-W is more complex than H-MC, it is possible that it takes longer for participants to learn to play better. In this section, therefore, we separately investigate the outcomes in the early rounds (i.e., the first two games participants played) and in the late rounds (i.e., the last two games participants played). The figures below are generated in the similar way as the corresponding figures presented above.[6]

Figure 6 shows the frequency of grand coalition formation in four games for the early and the late rounds. For H-MC, the grand coalition is significantly more frequently formed in the late rounds than in the early rounds for game 1 (at the 1% significance level). For PC-W, the frequency of grand coalition formation is significantly higher in the late rounds than in the early rounds for games 3 and 4 (at the 0.1 and 1% significance levels, respectively). A similar tendency can be observed for the efficiency shown in Figure 7.

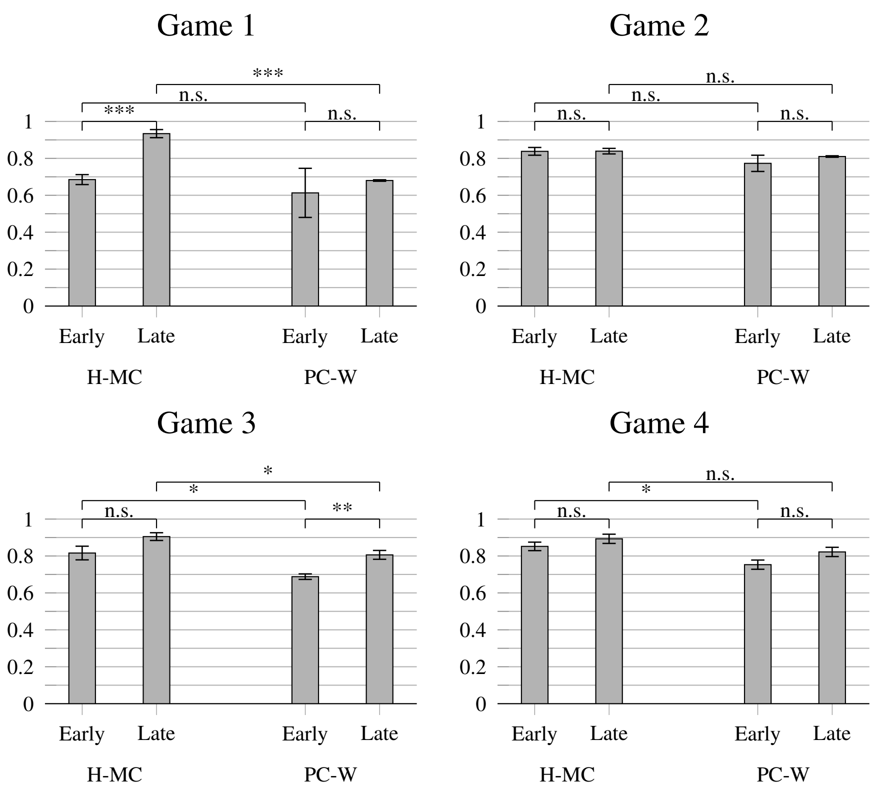
[6]Namely, the figure are created based on the estimated coefficients of the following linear regression: $y_i = \beta_1 HMC_i^e + \beta_2 HMC_i^l + \beta_3 PCW_i^e + \beta_4 PCW_i^l + \mu_i$ where y_i is the outcome variable of interest in group i, and HMC_i^τ (PCW_i^τ) is a dummy variable that takes the value 1 for $\tau \in \{e, l\}$ where e and l stand for early and late rounds of the H-MC (PC-W) mechanism, and zero otherwise. The standard errors are corrected for any within-session clustering effect. The statistical tests are based on the Wald test for the equality of the estimated coefficients of two treatment dummies.

Figure 6: H-MC and PC-W mechanisms: proportion of times the grand coalition formed in early and late rounds



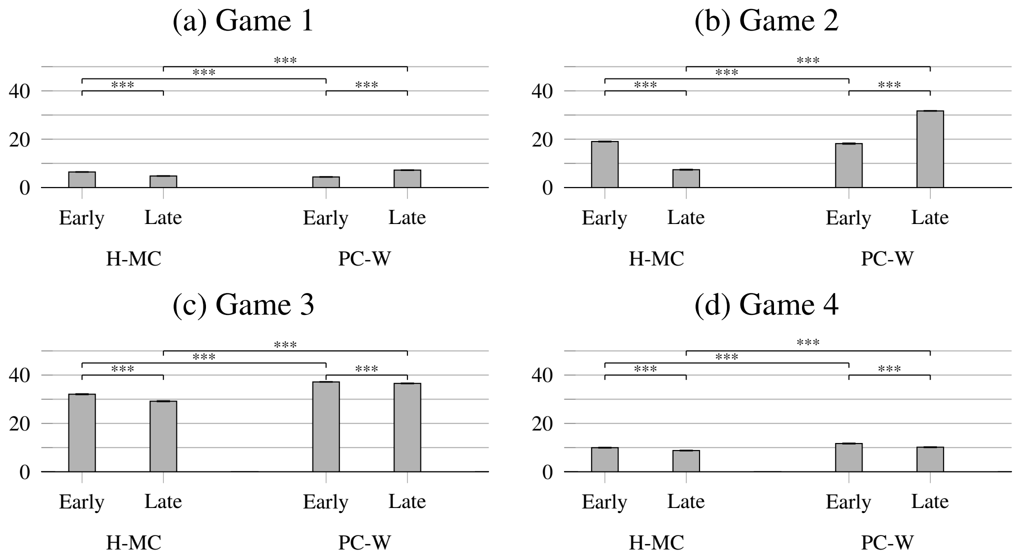
Note: Error bars show the one standard error range. ***, **, and * indicate that the two averages were significantly different at the 0.1, 1, and 5% significance levels (Wald test).

Figure 7: H-MC and PC-W mechanisms: efficiency in early and late rounds



Note: Error bars show the one standard error range. ***, **, and * indicate that the two averages were significantly different at the 0.1, 1, and 5% significance levels (Wald test).

Figure 8: Mean $Dis2_{\phi}$ in early and late rounds



Note: Error bars show the one standard error range. ***, **, and * indicate a statistically significant difference between the two means, at the 0.1, 1, and 5% significance levels (Wald test).

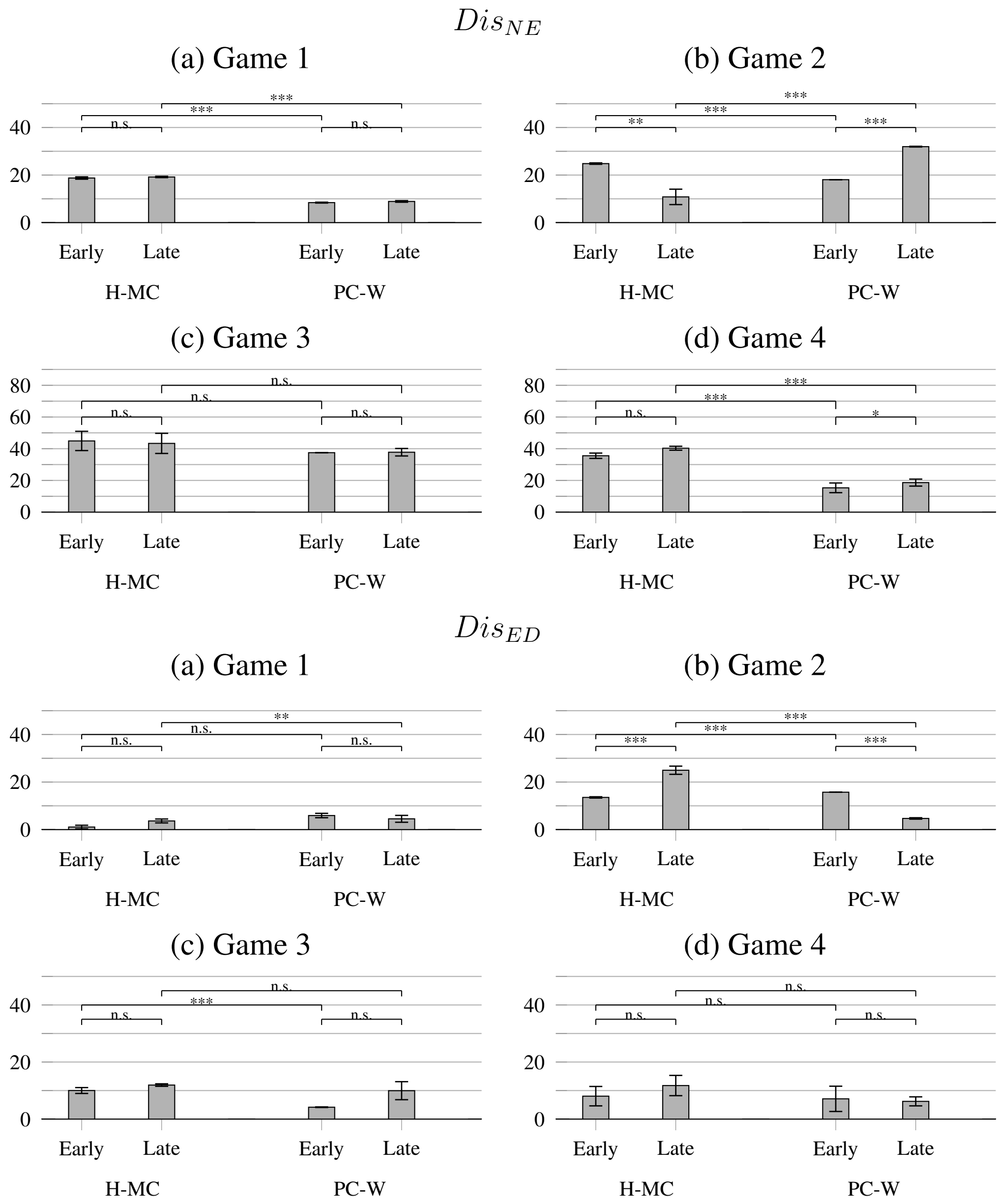
Figure 8 shows Dis_{ϕ} for the early and late rounds in H-MC and PC-W for the four games. While for H-MC, Dis_{ϕ} becomes smaller in the late rounds than in the early rounds for all four games, this is not the case for PC-W.[7] For PC-W, Dis_{ϕ} becomes smaller in the late rounds than in the early rounds only for games 3 and 4, whereas for games 1 and 2, it becomes larger in the late rounds.[8] Thus, gaining experience playing the game does not necessarily lead to allocation according to the Shapley value under PC-W.

This point can also be observed for Dis_{NE}. Figure 9 shows Dis_{NE} (top four panels) and Dis_{ED} (bottom four panels) in the early and the late rounds. For game 2, on the

[7]The mean Dis_{ϕ} (the standard errors) for H-MC in early and late rounds are 6.44 (0.013) and 4.78 (0.027) for game 1, 19.03 (0.09) and 7.38 (0.11) for game 2, 32.08 (0.13) and 29.18 (0.13) for game 3, and 9.96 (0.09) and 8.78 (0.07) for game 4, respectively.

[8]For PC-W, the mean Dis_{ϕ} (the standard errors) in early and late rounds are 4.38 (0.056) and 7.21 (0.057) for game 1, 18.18 (0.17) and 31.70 (0.05) for game 2, 37.16 (0.03) and 36.55 (0.08) for game 3, and 11.68 (0.11) and 10.16 (0.10) for game 4, respectively.

Figure 9: Mean Dis_{NE} and Dis_{ED} in early and late rounds



Note: Error bars show the one standard error range. ***, **, and * indicate a statistically significant difference between the two means at the 0.1, 1, and 5% significance levels (Wald test).

one hand, under H-MC, Dis_{NE} becomes significantly smaller (and correspondingly, Dis_{ED} becomes significantly larger) in the late rounds compared with the early rounds at the 1% significance level. On the other hand, under PC-W, the opposite is observed. Namely, Dis_{NE} becomes significantly larger (and correspondingly, Dis_{ED} becomes significantly smaller) in the late rounds compared with the early rounds at the 0.1% significance level. Furthermore, for PC-W in game 4, Dis_{NE} in the late rounds is significantly larger than in the early rounds at the 5% significance level.

6 Summary and conclusion

In this paper, we aim to experimentally contribute to the Nash program (Nash, 1953) by extending the experimental analyses of Chessa et al. (2022, 2021). We do so by comparing the simplified version of the mechanisms proposed by Hart and Mas-Colell (1996) and Pérez-Castrillo and Wettstein (2001). In the original theoretical investigation, in both mechanisms, there is a proposer who proposes an allocation, which is voted on sequentially by the remaining players (in our experiment, the voting is done simultaneously). The proposal is accepted if all the remaining players accept the proposal; if not, with some probability in H-MC and with probability one in PC-W (in our experiment, both are set to be with probability 1 to be comparable), the proposer leaves the game with his/her individual value (minus the sum allocated in the bidding stage in PC-W), and the game continues with the remaining players. The key difference between the two mechanisms is the way the proposer is chosen. On the one hand, a proposer is chosen randomly among the players in Hart and Mas-Colell (1996). On the other hand, in Pérez-Castrillo and Wettstein (2001), the proposer is determined through a bidding procedure. While Hart and Mas-Colell (1996) implements the Shapley value as an *ex ante* equilibrium payoff (i.e., it is only achieved as an expected outcome), Pérez-Castrillo and Wettstein (2001) implements it as a unique equilibrium of the game.

Our experiment shows that the simpler mechanism a la Hart and Mas-Colell (1996) not only results in higher efficiency than the one by Pérez-Castrillo and Wettstein (2001), but also the average allocation is closer to the Shapley value for those groups that formed a grand coalition. Thus, the complexity of the mechanism of Pérez-Castrillo and Wettstein (2001) imposes substantial costs for implementing the Shapley value. This finding is consistent with the recent work on the "obviously strategy proof" mechanism of (Li, 2017) which convincingly showed that, for a mechanism to implement the outcome intended by its designer, it must be extremely simple.

Chessa et al. (2022, 2021), as well as this paper, showed that participants have difficulty forming a grand coalition and achieving full efficiency under various noncooperative mechanisms (namely, Winter, 1994; Hart and Mas-Colell, 1996; Pérez-Castrillo and Wettstein, 2001). A natural question is whether participants can better form the grand coalition and achieve a higher efficiency if they can negotiate freely in an unstructured bargaining environment. Establishing such a benchmark in more "cooperative" environments and comparing them with the results obtained under "noncooperative" environments would be fruitful future research.

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A English translations of the instruction materials

The instruction materials as well as screenshots of the quiz for

- H-MC can be obtained from `https://www.dropbox.com/s/78lf5bn6qi3qfwp/HMChandout_sim.pdf?dl=0`
- PC-W can be obtained from `https://www.dropbox.com/s/yc584hk4c58ceyr/PCW_handout.pdf?dl=0`