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# THE EFFECT OF CHOOSING A PROPOSER THROUGH A BIDDING PROCEDURE IN IMPLEMENTING THE SHAPLEY VALUE

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# The effect of choosing a proposer through a bidding procedure in implementing the Shapley value*

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**Abstract**

We experimentally compare a simplified version of two mechanisms that implement the Shapley value as an (ex ante) equilibrium outcome of a noncooperative bargaining procedure: one proposed by Hart and Mas-Colell (1996, H-MC) and the other by Perez-Castrillo and Wettstein (2001, PC-W). While H-MC induces the Shapley value only on average, PC-W does so as a unique equilibrium outcome by introducing an additional bidding stage on top of H-MC. We investigate the effect of this additional bidding stage on the resulting outcomes such as the frequency of grand coalition formation, efficiency, and the distance between the realized allocation and the Shapley value. Our experiment shows that H-MC not only results in significantly greater efficiency than PC-W, but also that the average allocation is closer to the Shapley value for those groups that formed the grand coalition. This difference is because those proposers who won the bidding stage in PC-W tend to offer an allocation that favors themselves more than the randomly chosen proposers in H-MC, and such offers are more likely to be rejected.

**JEL codes:** C70, C71, C92
**Keywords:** Nash program, Bargaining procedures, Shapley value, Experiments

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# 1 Introduction

The Nash program (Nash, 1953) provides a noncooperative foundation for cooperative solution concepts. In this seminal work, Nash showed that the solution to the cooperative bargaining problem (Nash, 1950) can be obtained as an equilibrium outcome of a noncooperative game. The main idea of the Nash program "is both simple and important: the relevance of a concept [...] is enhanced if one arrives at it from different points of view" (Serrano, 2005, p. 220). Then, noticing that a different noncooperative approach yields the same solution, which "indicates that the solution is appropriate for a wider variety of situations" (Nash, 1953, p. 136), and then that the cooperative solution is widely applicable (even through a normative approach and without relying on precise assumptions about the bargaining protocol).

Many authors have contributed to the development of the Nash program (see, Serrano, 2005, 2008, 2014, 2021, for surveys). Since the inception of the Nash program, the cooperative solution that has attracted most attention is the Shapley value (Shapley, 1953). The popularity of this solution concept comes from its intuitive definition (an average of marginal contributions), its desirable properties (such as efficiency, monotonicity, or symmetry), and consequently, from the many theoretical and real-world applications (cost and payoff sharing, voting power, fair division, etc.). Indeed, various papers (for example, Gul, 1989; Harsanyi, 1981; Hart and Moore, 1990; Krishna and Serrano, 1995; Winter, 1994; Hart and Mas-Colell, 1996; Perez-Castrillo and Wettstein, 2001) have proposed its implementation.

Chessa et al. (2022a,b) contributed to this literature by providing experimental comparisons between Winter (1994) and Hart and Mas-Colell (1996) in Chessa et al. (2022b) and among three versions of Winter (1994) in Chessa et al. (2022a). Chessa et al.

(2022b) found that the demand-based mechanism proposed by Winter (1994) resulted in an allocation that better reflects players' effective bargaining power, while the efficiency and the frequency of the grand coalition formation are lower than those from an offer-based mechanism proposed by Hart and Mas-Colell (1996). However, because this result was obtained from a simplified version of the approach of Winter (1994), Chessa et al. (2022a) investigated the robustness of the outcome by comparing the outcome of the simplified one-period version of Winter (1994) considered in Chessa et al. (2022b) with a more complex—but closer to the original theoretical analysis—two-period version of Winter (1994). Chessa et al. (2022a) found that the results of the one-period and two-period versions were very similar.

In this paper, we extend the experimental analyses of Chessa et al. (2022a,b) by comparing the mechanisms proposed by Hart and Mas-Colell (1996) and Perez-Castrillo and Wettstein (2001). Both mechanisms are offer-based. Namely, in both mechanisms, there will be a proposer who proposes an allocation, which is voted on by the remaining players sequentially. The key difference between the two is the way the proposer is chosen. On the one hand, a proposer is chosen randomly among the players in Hart and Mas-Colell (1996). On the other hand, in Perez-Castrillo and Wettstein (2001), the proposer is determined through a bidding procedure. Both mechanisms predict at equilibrium that the grand coalition will form. Moreover, while Hart and Mas-Colell (1996) implement the Shapley value as an *ex ante* equilibrium payoff (i.e., it is only achieved as an expected outcome), Perez-Castrillo and Wettstein (2001) implement it as a unique equilibrium outcome of the game.

The bidding procedure introduced by Perez-Castrillo and Wettstein (2001) not only constitutes an interesting theoretical investigation, but it is also a good representation of some real-world scenarios. One main characteristic is that regardless of the random selection of the proposer that many theoretical bidding mechanisms rely on, participants in a negotiation may often be willing to take costly measures to be selected as the proposer because of the well-known *proposer advantage* (an individual who makes a proposal often obtains a greater share than others). Thus, a preliminary bidding stage may be, for example, a good representation of pre-negotiations in government coalition formation: in Italy, a preliminary round of consultation with the possible prime ministers allows the head of state to select the "best" prime minister according to her/his bid-like preliminary proposal.[1]

The literature on competition to win the proposal for the subsequent bargaining procedure is somewhat vast at present (see, e.g., Yildirim, 2007; Kim and Kim, 2022), and relies on different approaches such as preliminary lottery contests, ultimatum bargaining games, or bidding procedures. However, these additional preliminary levels in the bargaining procedures make one wonder whether they would indeed result in the predicted theoretical solution if the game was played by participants in a laboratory experiment. For example, Navarro and Veszteg (2011) experimentally investigated the impact of introducing a bidding stage to determine the proposer in a two-player ultimatum bargaining game. Although they found that participants' behavior stabilized after many trials (after 20 out of 30 repetitions) and the payoff gap between the proposer and the responder becomes smaller as predicted by the theory, the observed behavior, even in these final rounds of the experiment, deviate substantially from the theoretical prediction.

Furthermore, as demonstrated by the idea of "obviously strategy proofness" (Li, 2017), a mechanism needs to be extremely simple for it to result in the outcome in-

[1] Similarly, in France, a similar preliminary round of consultation was recently implemented because of the relative majority of prime minister Macron's parliamentarian group.

tended by its designer, and such additional stages certainly make the procedure more complex. In this paper, we investigate whether the bidding procedure introduced by Perez-Castrillo and Wettstein (2001) improves (as theoretically predicted) or in fact lowers the performance of the underlying mechanism of Hart and Mas-Colell (1996) in implementing the Shapley in a laboratory experiment.

Indeed, our experiment shows that the simpler mechanism *à la* Hart and Mas-Colell (1996) not only results in greater efficiency than that of Perez-Castrillo and Wettstein (2001), but also that the average allocation is closer to the Shapley value for those groups that formed a grand coalition. Our analyses suggest that this happens because those proposers who won the bidding stage in Perez-Castrillo and Wettstein (2001) tend to make offers that favor themselves (thus, they are less equal and deviate more from the Shapley value) and are more likely to be rejected than those randomly selected proposers in Hart and Mas-Colell (1996). We believe that this may be a consequence of the fact that winners at the bidding stage consider themselves to be entitled to a larger share (due to winning at the bidding phase, or because the winners of a bidding phase are those players who are the most motivated in exploiting the proposer advantage), whereas randomly selected proposers feel less legitimate in exploiting such an advantage.

The remainder of the paper is organized as follows. We present several theoretical preliminaries in cooperative game theory and the two mechanisms considered in our experiment in Section 2. Section 3 describes the experiment procedure, while the results are presented in Section 4. Section 5 concludes.

# 2 Theoretical preliminaries

## 2.1 Cooperative games and values

Let $N = \{1, \ldots, n\}$ be a finite set of players. Each subset $S$ of $N$ is called a *coalition* while $N$ is called the *grand coalition*. A *cooperative game with transferable utility* (hereafter, a *cooperative game*) on a fixed player set $N$ is a function $v : 2^N \to \mathbb{R}$ such that $v(\emptyset) = 0$. For each coalition $S \subseteq N$, $v(S)$ describes the worth that members of $S$ can achieve by agreeing to cooperate. We use $\mathcal{G}^N$ to denote the set of all games with player set $N$. A game $v \in \mathcal{G}^N$ is said to be *monotonic* if $v(S) \leq v(T)$ for each $S \subseteq T \subseteq N$. In monotonic games, the bigger the coalition is, the higher its worth becomes. A game $v \in \mathcal{G}^N$ is said to be *zero-monotonic* if $v(S) + v(\{i\}) \leq v(S \cup \{i\})$ for any subset $S \subseteq N$ with $i \notin S$. In zero-monotonic games, there are no negative externalities when a single player joins a coalition.[2]

A *value* is a mapping $\psi : \mathcal{G}^N \to \mathbb{R}^N$ which uniquely determines, for each $v \in \mathcal{G}^N$ and each player $i \in N$, a payoff $\psi_i(v) \in \mathbb{R}$ for participating to $v \in \mathcal{G}^N$. The *Shapley value* is the best-known solution concept, which is widely applied in economic and political models, and is defined as:

$$\phi_i(v) = \sum_{S \subseteq N, i \in S} \frac{(|S|-1)!(|N|-|S|)!}{|N|!}(v(S) - v(S \setminus \{i\})) \; \forall i \in N.$$

The Shapley value assigns to every player his or her expected marginal contribution to the coalition of players that enter the game before this player, given that every order of entrance has equal probability. Various characterizations of the Shapley value have been

[2]Note that the classes of zero-monotonic and monotonic games are not equivalent. For example, let $N = \{1, 2\}$, $v$ be such that $v(\{1\}) = 4$, $v(\{2\}) = -2$ and $v(\{1, 2\}) = 3$, and $w$ be such that $w(\{1\}) = 3$, $w(\{2\}) = 2$ and $w(\{1, 2\}) = 4$. We can see that $v$ is zero-monotonic but not monotonic while $w$ is monotonic but not zero-monotonic. It is worth noting that all the four games we consider in experiments respect these two assumptions.

provided in the literature. One of the most famous characterizations uses efficiency, symmetry, the null player, and additivity axioms and can be easily deduced from the seminal article by Shapley (1953).

## 2.2 Two mechanisms

In this subsection, we present the Hart and Mas-Colell (1996, referred to as H-MC below) mechanism (Section 2.2.1) and the Perez-Castrillo and Wettstein (2001, referred to as PC-W below) mechanism (Section 2.2.2) in more detail.

### 2.2.1 H-MC mechanism

The bargaining procedure proposed by Hart and Mas-Colell (1996) implements the Shapley value in monotonic cooperative games. In this mechanism, the bargaining starts with a randomly chosen proposer making an offer to the other players (responders). The responders, sequentially, may either accept or reject the offer. If the offer is accepted by all responders, it is implemented. If one of the responders rejects the offer, the game moves to the next stage. If the offer is rejected, the proposer leaves the game with the worth of her stand-alone coalition with probability $p$ and the bargaining continues with the remaining players, with a new player randomly chosen as the new proposer. In our laboratory implementation, we set $p = 1$ to make it comparable with the PC-W mechanism. We present here a more formal description of the H-MC mechanism. A decision point position at time $t$ is given simply by the vector $(S^t, j)$, where:

$S^t \subseteq N$ is the set of players still in the game, and

$j \in S^t$ is the player making an offer to the remaining players $(t_i)_{i \in S^t \setminus \{j\}}$ such that $\sum_{i \in S^t \setminus \{j\}} t_i \leq v(S^t)$.

With $j$'s offer, the game proceeds now in the following way:

1) if all $i \in S^t \setminus \{j\}$ accept the offer one after the other, then players in $S^t \setminus \{j\}$ are paid $(t_i)_{i \in S^t \setminus \{j\}}$, player $j$ is paid $v(S^t) - \sum_{i \in S^t \setminus \{j\}} t_i$, and the game ends;

2) if at least one player $i \in S^t \setminus \{j\}$ refuses the offer, then there are two possibilities:

2$a$) if $|S^t| = 2$ (only one more player is left, together with $j$), then they both obtain the worth of their own stand-alone coalition $v(\{i\})$ for each $i \in S^t$, and the game ends;

2$b$) if $|S^t| > 2$, then player $i$ is removed from the game, she obtains her individual payoff $v(\{i\})$, a new proposer $k \in S^{t+1} = S^t \setminus \{j\}$ is randomly selected, and the new position is $(S^{t+1}, k)$.

The game starts with a randomly chosen proposer $j \in N$, so that the initial position is $(N, j)$. It terminates either when there are no more players in the game (see point $2a$ above), or when the offer is unanimously accepted (see point $1$ above).

Hart and Mas-Colell (1996) showed that this game has a unique subgame perfect equilibrium that supports the grand coalition and yields the Shapley value payoff vector in expectation.

Given a specific initial proposer $j \in N$, the *a posteriori* equilibrium payoff assigns to each other player her Shapley value in the cooperative game reduced to the set of players $N \setminus \{j\}$, and to the proposer, the marginal contribution to the grand coalition $v(N) - v(N \setminus \{j\})$.

### 2.2.2 PC-W mechanism

Perez-Castrillo and Wettstein (2001) proposed a bidding mechanism to implement the Shapley value for zero-monotonic cooperative games. In this mechanism, unlike Hart and Mas-Colell (1996), the bargaining starts with a bidding stage where each of the players makes a bid to each of the other players. The proposer is chosen as the player making the *highest net bid*, calculated as the difference between the sum of the bids a player makes to the others minus the sum of the bids the others make to her. If several players make the highest net bid, the proposer is then selected randomly from them. At the end of the bidding stage, the proposer pays the promised bids to the other players. In the second stage, the proposer makes an offer to the other players (responders). This offer is sequentially accepted or rejected by the responders. If all the responders accept, the game ends and the proposer pays the other players according to their offers and receives what remains of the worth of the coalition. If one of the responders rejects the offer, the proposer leaves the game and obtains the worth of her stand-alone coalition minus the bids she has already paid in the first stage. The remaining players keep what they have received, and they begin a new bargaining round.

We present here a more formal description of the PC-W mechanism. A decision point position at time $t$ is given simply by the set of players $S^t \subseteq N$ remaining in the game, (when $t = 1$, $S^t = N$). The game is described as follows:

(1) Each player $i \in S^t$ makes bids $b^i_j$ to the other players $j \in S^t \backslash \{i\}$. Player $i$ with the highest net bid $\sum_{j \in S \backslash \{i\}} (b^i_j - b^j_i)$ is chosen to be the proposer and pays $b^i_j$ to every $j \in S^t \backslash \{i\}$; if several players make the highest net bid, the proposer is chosen randomly among them.

(2) If player $i$ is the proposer, she makes an offer $y^i_j$ to the other players $j \in S^t \backslash \{i\}$.

(3) With $i$'s offer, the game proceeds in the following way:

(3a) If the offer is sequentially accepted by the other players, each player $j \in S^t \backslash \{i\}$ receives $b_j^i + y_j^i$, the proposer obtains $v(S^t) - \sum_{j \in S^t \backslash \{i\}} (b_j^i + y_j^i)$, and the game ends.

(3b) If at least one player $j \in S^t \backslash \{i\}$ refuses the offer, then proposer $i$ is removed and obtains $v(\{i\}) - \sum_{j \in S^t \backslash \{i\}} b_j^i$; the new set of players becomes $S^{t+1} = S^t \backslash \{i\}$ in which each member $j$ receives $b_j^i$ and a new bidding mechanism starts among them (if only one player remains in the game, i.e., $|S^{t+1}| = 1$, then that player obtains the worth of her stand-alone coalition and the game ends).

Perez-Castrillo and Wettstein (2001) show that any subgame perfect equilibrium of this game implements the Shapley value as the *a posteriori* equilibrium payoff. At these equilibria, the bid of player $i$ to player $j$ corresponds to $j$'s Shapley value of the original game minus $j$'s Shapley value in the cooperative game reduced to the set of players $N \setminus \{i\}$. The balanced contribution property (Myerson, 1980) then ensures that all net bids are equal to zero, leading to a random selection of the proposer among all the players. Moreover, player $i$, if she is randomly selected to be the proposer, offers to player $j$ her Shapley value in the reduced cooperative game with player set $N \setminus \{i\}$. It is worth noting that when the game is strictly zero-monotonic,[3] the grand coalition always forms. Otherwise, rejection of some offers could also constitute a subgame perfect equilibrium (this is the case, for example, in game 2 in Table 1 below when Player 1 is chosen as the proposer).

[3] A game is strictly zero-monotonic if $v(S) + v(\{i\}) < v(S \cup \{i\})$ for any nonempty subset $S \subseteq N$ with $i \notin S$. Note that only games 1, 3, and 4 that we consider in our experiments are strictly zero-monotonic.

# 3 The experimental setting

We consider the four games that are studied in (Chessa et al., 2022a,b). These games and their corresponding Shapley values are reported in Table 1.

Table 1: Four games and corresponding Shapley values

| $S$ | Game 1 $v_1(S)$ | Game 2 $v_2(S)$ | Game 3 $v_3(S)$ $= v_1(S) + v_2(S)$ | Game 4 $v_4(S)$ $= 2v_1(S)$ |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 2 | 5 | 20 | 25 | 10 |
| 3 | 5 | 20 | 25 | 10 |
| 4 | 10 | 30 | 40 | 20 |
| 1,2 | 20 | 20 | 40 | 40 |
| 1,3 | 20 | 20 | 40 | 40 |
| 1,4 | 25 | 30 | 55 | 50 |
| 2,3 | 20 | 45 | 65 | 40 |
| 2,4 | 25 | 55 | 80 | 50 |
| 3,4 | 25 | 60 | 85 | 50 |
| 1,2,3 | 50 | 45 | 95 | 100 |
| 1,2,4 | 60 | 55 | 115 | 120 |
| 1,3,4 | 60 | 60 | 120 | 120 |
| 2,3,4 | 60 | 100 | 160 | 120 |
| N | 100 | 100 | 200 | 200 |
| Shapley values for each player | | | | |
| $\phi_1(v)$ | 22.08 | 0 | 22.08 | 44.16 |
| $\phi_2(v)$ | 23.75 | 28.33 | 52.08 | 47.5 |
| $\phi_3(v)$ | 23.75 | 30.83 | 54.58 | 47.5 |
| $\phi_4(v)$ | 30.42 | 40.83 | 71.25 | 60.84 |

## 3.1 Procedure

Upon arrival, participants received a copy of the instruction slides. The instructions were divided into two parts: an explanation of the rules of the game and an explanation of the computer interface. First, a prerecorded video of the first part of the instructions (explanation of the rules of the game) was played. Then, a comprehension quiz (computerized) was administered to make sure participants understood the rules of the game. They needed to answer each question correctly before proceeding to the next one. Once all the participants finished answering the quiz questions, two practice rounds of a game in which the Shapley value for all the players was 25 was run to familiarize the participants with the software before the real experiment started. During these practice rounds, participants were asked to look at the second part of the instructions about the computer interface. In the first round of the practice, the experimenter explained each screen following the instructions. See Appendix D for English translations of the instruction slides and the comprehension quiz.

In the actual experiment, just as in Chessa et al. (2022a,b), each participant played all four games twice in one of the following four orderings: 1234, 2143, 3412, and 4321. Between each play of a game (called a round), players were randomly rematched into groups of four players, and participants were randomly assigned a new role within the newly created group.[4] At the end of the experiment, two rounds (one from the first four rounds and another from the last four rounds) were randomly selected for payments. Participants received cash rewards based on the points they earned in these two selected rounds with an exchange rate of 20 JPY = 1 point in addition to 1500 JPY and 1900 JPY participation fees for H-MC and PC-W. The participation fee for PC-W was set larger

[4]As Chessa et al. (2022b) note, this design choice may have slowed down participants' learning how to play the game, and therefore makes the data noisier compared with a design in which participants play only one game. Despite this concern, this design is chosen in Chessa et al. (2022b) to have within-session variations of games because the tests of the axioms involve comparing outcomes across different games. Furthermore, as suggested by Guerci et al. (2014), this random reassignment of the roles across rounds instead of fixing the roles may have slowed down participants' learning how to play the game, and thus may have made our data noisier. Chessa et al. (2022b) opt for this design, however, to avoid participants feeling the experiment is unfair because of the existence of the null player in game 2. Because the data of H-MC used in this paper are the same as those reported in Appendix V of Chessa et al. (2022b) as H-$\text{MC}_{sim}$, we also follow these design choices in PC-W. See Appendix C for analyses as well as discussions related to subject learning.

to compensate for the longer experiment, as well as to cover possible losses participants may have made.

## 3.2 Simplification of, and an additional difference between, the mechanisms

In our experiment, we simplified both the H-MC and PC-W mechanisms. Namely, after a proposer made an offer, instead of the remaining players (responders) approving or rejecting the offer one by one sequentially, all responders decided simultaneously.

Coalition formation processes may perform differently in the two settings (sequential versus simultaneous). For example, recently, Abe et al. (2021) compared experimentally the efficiency of (a) a mechanism in which participants decide to join the coalition sequentially, as in the original theoretical analysis of Hart and Mas-Colell (1996) and Perez-Castrillo and Wettstein (2001) and (b) an analogous mechanism in which participants decide to join the coalition simultaneously, as in our experimental implementation. The authors found that the former resulted in higher efficiency than the latter. However, Chessa et al. (2022b) investigated the effect of this simplification in H-MC in the same four games we consider in this paper (see their Appendix V). This simplification gives rise to other Nash equilibria, in which two or more players refuse the proposal. However, they reported that the simplified H-MC indeed resulted in a significantly lower frequency of grand coalition formation and efficiency in game 2, but not in the remaining three games, such that the performances of the two implementations were comparable. Moreover, in our experiment this simplification is done both for H-MC and PC-W. Therefore, for the purpose of investigating the effect of adding a bidding stage to determine the proposer, the impact of this simplification, if any, should be analogous on two mechanisms and should not affect their comparison.

In our experiment, in addition to the way the proposer is chosen, there is a difference in the way the simplified version of the H-MC and PC-W is implemented. Namely, on the one hand, under our H-MC implementation, the responders observed the proposed allocation to all the players when deciding to accept or reject the proposal. On the other hand, under our PC-W implementation, each responder observed only their own proposed allocation and not those to other players (see the screenshots entitled "respondent's input" for H-MC and "STEP 2 (Respondent)" for PC-W included in the English translation of the instruction material available in Appendix D). The reason for introducing this difference is that in PC-W, responders are presented not only with the offer in the second stage, but also the allocated amount in the first bidding stage and the sum of the two. Presenting the proposed allocation for all the responders in this manner for PC-W substantially increases the amount of information that participants must process when deciding to approve or reject the offer. While this difference in the presentation of the information does not influence the theoretical analyses (based on the standard set of assumptions), it may influence the results we report below. We investigate whether and how this difference in the procedure may have impacted the results in Section 4.2.2.

# 4 Results

The computerized experiment was conducted at the Institute of Social and Economic Research (ISER), Osaka University, between June and December 2019. The experiment was computerized and used z-Tree (Fischbacher, 2007). A total of 164 students among those registered in the participant database managed by ORSEE (Greiner, 2015), who had not participated in similar experiments previously, were recruited as subjects of the experiment. Among these 164 participants, 84 played H-MC and 80 played PC-W.[5] As noted in footnote 4 above, the data of H-MC used in this paper are the same as those reported in Appendix V of Chessa et al. (2022b) as H-MC$_{sim}$.

The experiment lasted on average 1h30m for H-MC and 2h05m for PC-W including the instructions, comprehension quiz, and payment. The average earning was 2780 JPY for H-MC and 2990 JPY for PC-W (approximately 25.7 and 27.7 USD using an average exchange rate of 1 USD = 108 JPY for the period over which the experiment was conducted.)

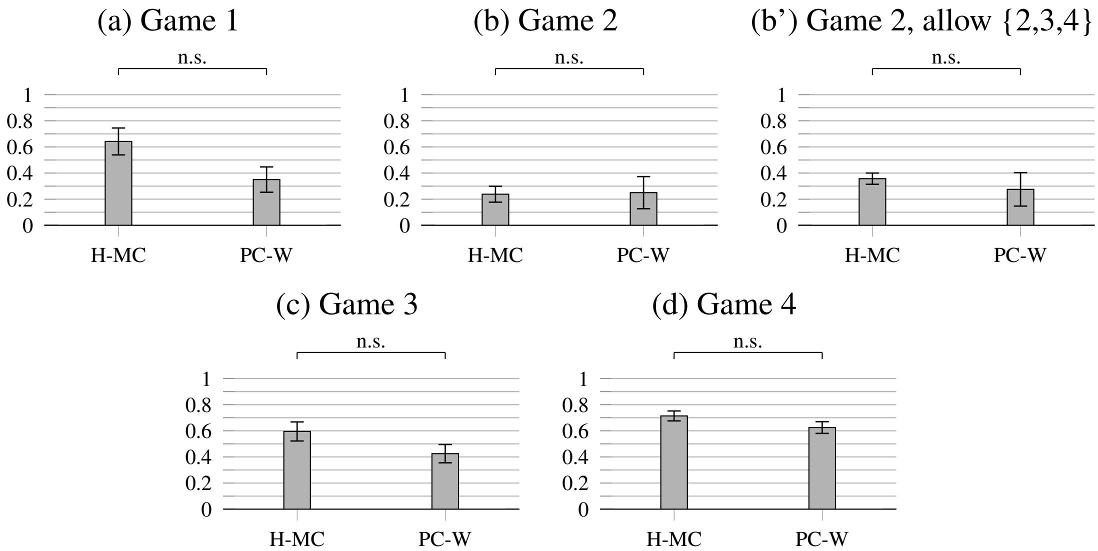
## 4.1 Grand coalition formation and efficiency

Figure 1 presents the results relating to the grand coalition formation in the H-MC mechanism and in the PC-W mechanism, for the four games. The figure is created based on the estimated coefficients of the following linear regression: $gc_i = \beta_1 HMC_i + \beta_2 PCW_i + \mu_i$ where $gc_i$ is a dummy variable that takes the value 1 if the grand coalition is formed, and zero otherwise, in group $i$, and $HMC_i$ ($PCW_i$) is a dummy variable that takes the value 1 if the H-MC (PC-W) mechanism is used, and zero otherwise. Standard errors are corrected for within-session clustering effects (there are 8 clusters, 4 clusters within each treatment, in total). The statistical tests are based on the Wald test for equality of the estimated coefficients of two treatment dummies.

For game 2, we also consider the case in which we allow a coalition without the null player as the grand coalition (panel b'). We observe from Figure 1 that the frequency of grand coalition formation is higher under H-MC than under PC-W; however, the difference is not statistically significant.

[5]There were four sessions, corresponding to four orderings of the games, for each mechanism. For H-MC, we had 24 participants in one session (with the order of games being 4321) and 20 participants in three other sessions. For PC-W, we had 20 participants in all four sessions.

Figure 1: H-MC and PC-W mechanisms: proportion of times the grand coalition formed

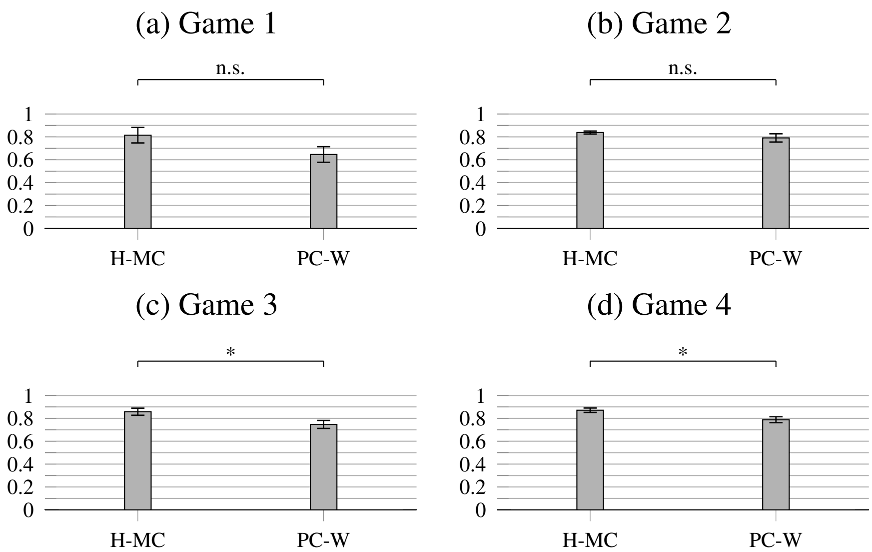



Note: Error bars show one standard error range. ***, **, and * indicate the proportion of times that grand coalition formation is significantly different between the H-MC and the PC-W at the 0.1, 1, and 5% significance levels, respectively (Wald test).

As a direct consequence of the grand coalition not always being formed, both mechanisms fail to achieve full efficiency. Figure 2 compares the efficiency of the two mechanisms for the four games. Efficiency is computed as the fraction of the sum of the payoffs obtained by the four players compared with the value of the grand coalition of the considered game ($100$ for games 1 and 2 and $200$ for games 3 and 4). The figure is created based on the estimated coefficients of the following linear regressions: $Eff_i = \beta_1 HMC_i + \beta_2 PCW_i + \mu_i$ where $Eff_i \equiv \frac{\sum_i \pi_i}{v(N)}$ is the efficiency measure for group $i$, $HMC_i$ ($PCW_i$) is a dummy variable that takes the value 1 if the H-MC (PC-W) mechanism is used, and zero otherwise. Standard errors are corrected for within-session clustering effects. The statistical tests are based on the Wald test for the equality of the estimated coefficients of two treatment dummies.

Figure 2 shows that efficiency is higher under the H-MC mechanism than under the

Figure 2: Efficiency




Note: Error bars show the one standard error range. ***, **, and * indicate that efficiency is significantly different between H-MC and PC-W at the 0.1, 1, and 5% significance levels, respectively (Wald test).

PC-W mechanism, and that the difference is statistically significant at the 5% level for games 3 and 4.

The significantly lower efficiency under PC-W compared with H-MC (for game 3, the mean efficiency is 0.747 and 0.858 for PC-W and H-MC, respectively, and for game 4, it is 0.788 and 0.871) demonstrates that the cost introduced by the bidding procedure of the former is not negligible.

## 4.2 The first proposals and responses against them

To better understand the reason for the lower efficiency of PC-W compared to H-MC, we analyze the first proposals and responses against them. We focus on the first proposer because for the grand coalition to be formed, we require the first proposal to be accepted. We start by looking at who became the first proposer.

Table 2: Frequency of being the first proposer

(a) Frequencies in PC-W and H-MC (all games combined)

| | Player 1 | Player 2 | Player 3 | Player 4 |
|---|---|---|---|---|
| PC-W | 21 | 38 | 39 | 62 |
| H-MC | 41 | 40 | 45 | 42 |

(b) Result of multi-nominal logistic regression

| Proposer | 1 | 2 | 3 |
|---|---|---|---|
| H-MC | 1.059** | 0.441 | 0.533** |
| | (0.349) | (0.318) | (0.168) |
| Constant | -1.083*** | -0.490 | -0.464*** |
| | (0.228) | (0.264) | (0.131) |

Note: Proposer 4 is the baseline outcome. Standard errors are corrected for session-level clustering effect. *** and ** are statistically significant at 0.1 and 1% level respectively.

Panel (a) of Table 2 shows the frequency of each player being chosen as the first proposer in PC-W and H-MC. We have pooled all the games from all the sessions. In H-MC, by construction, all four players have been selected with similar frequencies to be a proposer. In PC-W, however, Player 4 is almost three times more likely to become the proposer than Player 1, and Players 2 and 3 are almost twice more likely to become the proposer than Player 1.

To test whether there is a significant difference between PC-W and H-MC regarding the frequency of chosen proposer, we have conducted multi-nominal logistic regression where the dependent variable is the chosen proposer (Players 1 to 4) and independent variables are constant and the H-MC dummy that takes value 1 if the treatment is H-MC and zero in case of PC-W. Standard errors are corrected for within-session clustering effect. The result, with Player 4 used as the base outcome, is shown in panel (b) of Table 2. As we have observed in panel (a), for PC-W, Players 1 and 3 are significantly (at 1% significance level) less likely to be selected as the proposer than Player 4 (the null hypothesis, constant=0, is rejected (p-value $< 0.001$)). For H-MC, however, the null hypothesis, constant + H-MC=0, is not rejected (p-value = 0.928, Wald test) suggesting that all the players are equally likely to become the proposer.

#### 4.2.1 Offer made by the first proposer

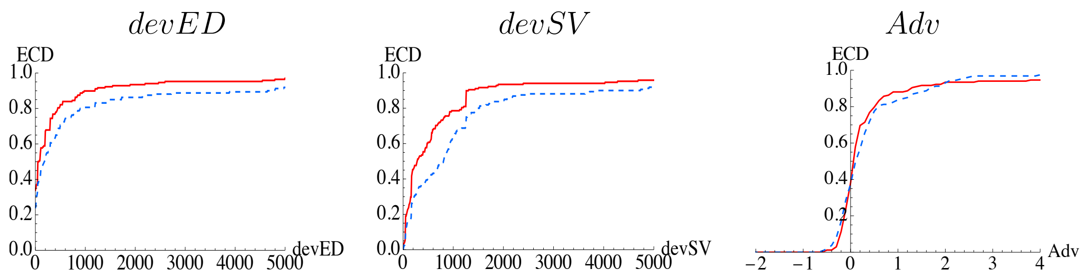
We now investigate the offers made by the first proposer in H-MC and PC-W. We have pooled all the games from all the sessions. We consider the following three characteristics regarding the offer made by the first proposers:

- Deviation from the equal division: $devED = \sum_j (o_j - v(N)/4)^2$ where $o_j$ is the offered amount to player $j$
- Deviation from the Shapley value: $devSV = \sum_j (o_j - \phi_j)^2$ where $o_j$ is the offered amount to player $j$ and $\phi_j$ is the Shapley value of player $j$ in the game
- Relative proposer advantage: $Adv = (o_p - \phi_p)/\phi_p$ where $o_p$ and $\phi_p$ are the amount the proposer allocated to him/herself in the proposal and $\phi_p$ is her/his Shapley value in the game

In all the measures, for PC-W we include the amount allocated as a result of the first bidding stage in the first proposal to be able to better compare H-MC and PC-W. Appendix A provides analyses of the bidding stage in PC-W.

Figure 3 shows the empirical cumulative distribution of $devED$ (left), $devSV$ (center), and $Adv$ (right) for H-MC (solid red) and PC-W (dashed blue). In each panel, the distribution of H-MC lies on the left of that of PC-W, suggesting that first proposals under H-MC tend to be more equal, closer to SV, and not to give a large amount to the proposer relative to his/her Shapley value, compared to PC-W.

Figure 3: Empirical cumulative distribution of $devED$, $devSV$, and $Adv$




Note: Solid Red: H-MC. Dashed Blue: PC-W.

To test whether there are significant differences in terms of these three measures of the first proposals between H-MC and PC-W, we conduct an ordinary least square (OLS) regression that takes either $devED$, $devSV$, or $Adv$ as a dependent variable and constant and the H-MC dummy as the independent variables (regressions (1), (3), and (5) in Table 3). We also run a regression in which we add the proposer's ID (1, 2, 3, or 4) as an independent variable in order to control for the difference in the frequency of proposer between PC-W and H-MC (regressions (2), (4), and (6) in Table 3). Standard errors are corrected for session-level clustering effect.

The regression results shown in Table 3 confirm our observation from Figure 3. Namely, the first proposal results in H-MC are significantly more equal, closer to SV, and the amount allocated to the proposer is significantly closer to his/her Shapley value than in PC-W.

#### 4.2.2 Responses to the first proposal

We have seen that the first proposals under PC-W tend to allocate a much larger share to the proposer (relative to her/his Shapley value). This may result in responders rejecting the first proposal with a higher frequency (which results in a lower frequency of the

Table 3: Results of OLS regression

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | $devED$ | | $devSV$ | | $Adv$ | |
| H-MC | -888.84* | -765.14 | -1120.79** | -1096.84* | -0.195*** | -0.25*** |
| | (337.28) | (330.31) | (307.51) | (342.34) | (0.0383) | (0.0214) |
| Proposer | | 340.12 | | 65.86 | | -0.18** |
| (1,2,3,4) | | (195.74) | | (233.75) | | (0.0489) |
| Constant | 1694.15*** | 712.04 | 2089.07*** | 1898.91* | 0.38*** | 0.91*** |
| | (316.06) | (627.46) | (287.99) | (803.57) | (0.033) | (0.012) |
| No. obs | 328 | 328 | 328 | 328 | 317 | 317 |
| $R^2$ | 0.012 | 0.020 | 0.020 | 0.020 | 0.015 | 0.076 |

Note: Standard errors are corrected for session-level clustering effect. ***, **, * indicate statistically significant at 0.1%, 1%, and 5% level.

grand coalition formation and also a lower efficiency, as we have observed).

We investigate whether, controlling for the characteristics of the first proposals (using the three measures we have considered above), there are significant differences in terms of frequency of the first proposal being accepted or not between H-MC and PC-W. In particular, we consider the logit regression[6] with the dependent variable being a rejection dummy that takes the value 1 if the proposal is rejected and zero otherwise, and independent variables being constant, the H-MC dummy, one of the three measures of the proposal (either $devED$, $devSV$, and $Adv$) and its interaction with the H-MC dummy.

The results reported in Table 4 show that for each of the three measures of the first proposal ($devED$, $devSV$, and $Adv$), the larger the measure is, the higher is the likelihood of the proposal being rejected. However, except for the regression where we use $Adv$ as the measure of the proposal, there is no statistically significant difference between the H-MC and PC-W regarding the likelihood of the proposal being rejected.

[6]We obtain qualitatively the same results if we use probit specification

Table 4: Results of logit regression

| | Dependent variable: Rejection dummy | | |
|---|---|---|---|
| | $X = devED$ | $X = devSV$ | $X = Adv$ |
| X | 0.002** | 0.0005** | 1.8445*** |
| | (0.0008) | (0.0002) | (0.2681) |
| X × H-MC | -0.0007 | 0.00001 | -0.2661 |
| | (0.0011) | (0.00026) | (0.4057) |
| H-MC | -0.214 | -0.3690 | -0.481** |
| | (0.281) | (0.2479) | (0.1556) |
| Constant | -0.425* | -0.1770 | -0.034 |
| | (0.185) | (0.2071) | (0.1249) |
| No. Obs | 328 | 328 | 317 |
| pseudo $R^2$ | 0.1563 | 0.084 | 0.1250 |

Note: Standard errors are corrected for session-level clustering effect. ***, **, * indicate statistically significant at 0.1%, 1%, and 5% level.

Therefore, a lower efficiency observed in PC-W compared to H-MC is a result of the differences in the proposals. The results suggest that those participants who won the bidding stage and have actively become the proposer in PC-W, instead of being randomly chosen in H-MC, seem to claim a large amount to themselves, and as a result, their proposals are more likely to be rejected.

The regression results also suggest that the differences in the way the information regarding the proposal is presented between the two mechanisms when responders decide to approve or reject it (c.f. Section 3.2) had some, but not too strong, impacts on the result once the characteristics of the proposal are controlled for.

Let us now analyze the realized allocations.

## 4.3 Allocations

We denote by $\pi^{HMC}(v_k)$ a vector of payoffs obtained by the players in the H-MC mechanism in game $k$, with $k = 1, 2, 3, 4$. Analogously, we denote by $\pi^{PCW}(v_k)$ a vector of payoffs obtained by the players in the PC-W mechanism. As noted above, H-MC implements the Shapley value only on average, whereas PC-W, thanks to the bidding stage, does so as a unique equilibrium outcome. Therefore, we first compare the average realized payoff vectors between the two mechanisms. However, when the players fail to form the grand coalition, the total payoffs obtained by the players are smaller than the value of the grand coalition. This results in the average realized payoff vectors being significantly different from the Shapley value. We therefore focus our analyses on those groups that formed the grand coalition.[7]

Our main analyses are based on a set of OLS regressions (using only the data from groups that formed the grand coalition) for the following system of equations.

$$\begin{aligned}
\pi_1 &= a_1 g_1 + a_2 g_2 + a_3 g_3 + a_4 g_4 + u_1 \\
\pi_2 &= b_1 g_1 + b_2 g_2 + b_3 g_3 + b_4 g_4 + u_2 \\
\pi_3 &= c_1 g_1 + c_2 g_2 + c_3 g_3 + c_4 g_4 + u_3 \\
\pi_4 &= d_1 g_1 + d_2 g_2 + d_3 g_3 + d_4 g_4 + u_4,
\end{aligned} \tag{1}$$

where $\pi_i$ is the payoff of player $i$, $g_j$ is a dummy variable that takes the value 1 if the game $j \in \{1, 2, 3, 4\}$ is played, and zero otherwise. Because participants play all four games twice, we correct the standard errors for any within-group clustering effect. Note that the estimated coefficients $a_j$, $b_j$, $c_j$, and $d_j$ are the average payoffs in game $j$ for players 1, 2, 3, and 4, respectively.

[7] In Appendix B we report the distribution of realized allocation in four games under two mechanisms for those groups that formed the grand coalition.

Table 5: Results of linear regression based only on the groups that formed the grand coalition

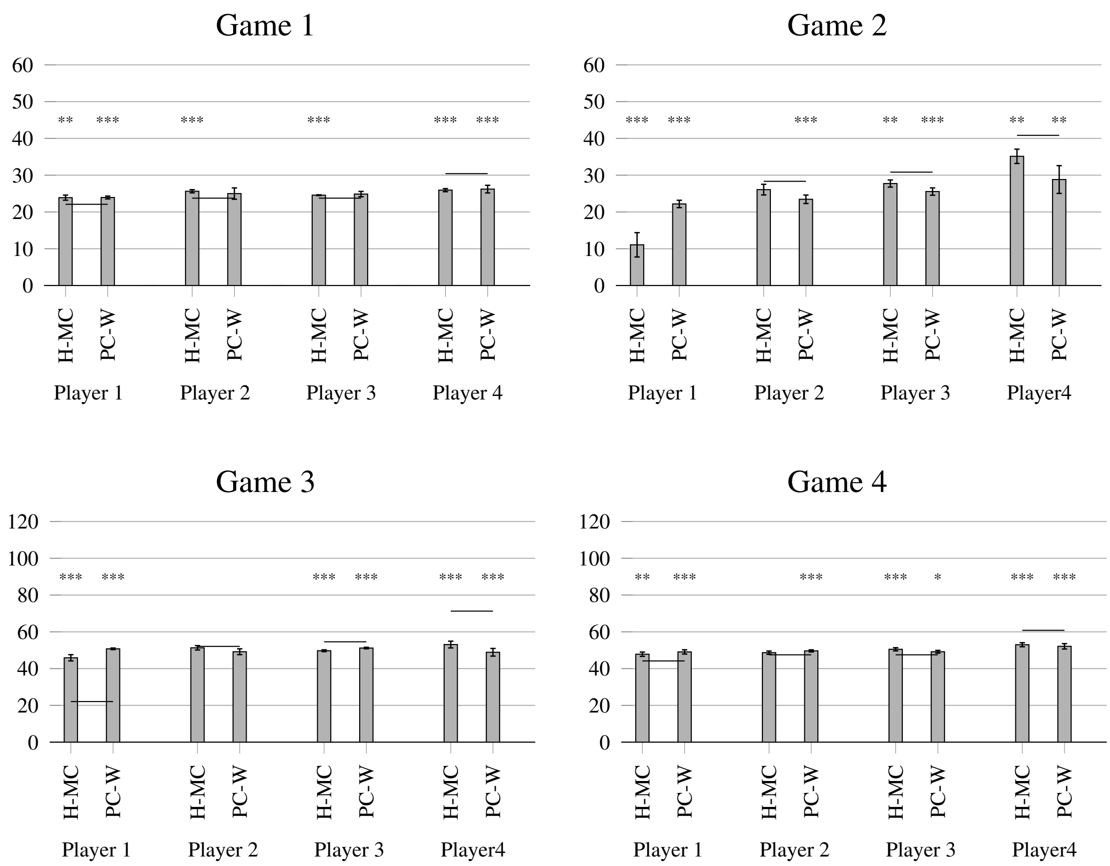
| H-MC | $\pi_1$ | $\pi_2$ | $\pi_3$ | $\pi_4$ | PC-W | $\pi_1$ | $\pi_2$ | $\pi_3$ | $\pi_4$ |
|---|---|---|---|---|---|---|---|---|---|
| g1 | 23.88 | 25.63 | 24.56 | 25.93 | g1 | 23.93 | 25.00 | 24.86 | 26.21 |
| | (0.70) | (0.43) | (0.11) | (0.42) | | (0.39) | (1.54) | (0.72) | (1.04) |
| g2 | 11.07 | 26.07 | 27.73 | 35.13 | g2 | 22.18 | 23.45 | 25.55 | 28.82 |
| | (3.31) | (1.45) | (0.97) | (1.95) | | (4.01) | (1.15) | (0.99) | (3.78) |
| g3 | 45.88 | 51.32 | 49.72 | 53.08 | g3 | 50.76 | 49.18 | 51.18 | 48.88 |
| | (1.70) | (1.19) | (0.50) | (1.84) | | (0.44) | (1.59) | (0.41) | (2.08) |
| g4 | 47.83 | 48.67 | 50.5 | 53.00 | g4 | 49.08 | 49.64 | 49.16 | 52.12 |
| | (1.17) | (0.89) | (0.87) | (1.10) | | (1.16) | (0.57) | (0.75) | (1.46) |
| $R^2$ | 0.96 | 0.99 | 0.99 | 0.97 | $R^2$ | 0.98 | 0.98 | 0.99 | 0.97 |
| Obs. | 97 | 97 | 97 | 97 | Obs. | 67 | 67 | 67 | 67 |

Note: Standard errors are corrected for any within-group clustering effect.

Table 5 reports the results of these regressions, with H-MC in the left panel and PC-W in the right panel. These estimated coefficients and standard errors are also visualized in Figure 4. In the figure, the horizontal lines indicate the Shapley values for each game. The stars above each bar indicate that the average payoff is significantly different from the corresponding Shapley value at the 0.1, 1, or 5% levels. As one can observe, there are only a few cases (players 2 and 3 in game 1 as well as Player 2 in game 3 for PC-W, and Player 2 in games 2, 3, and 4 for H-MC) where the average payoffs are not significantly different from the Shapley value at the 5% level. Thus, neither H-MC nor PC-W implement the Shapley values.

To better compare the two mechanisms in terms of how close their average payoffs

Figure 4: Mean payoffs based only on the groups that formed the grand coalition



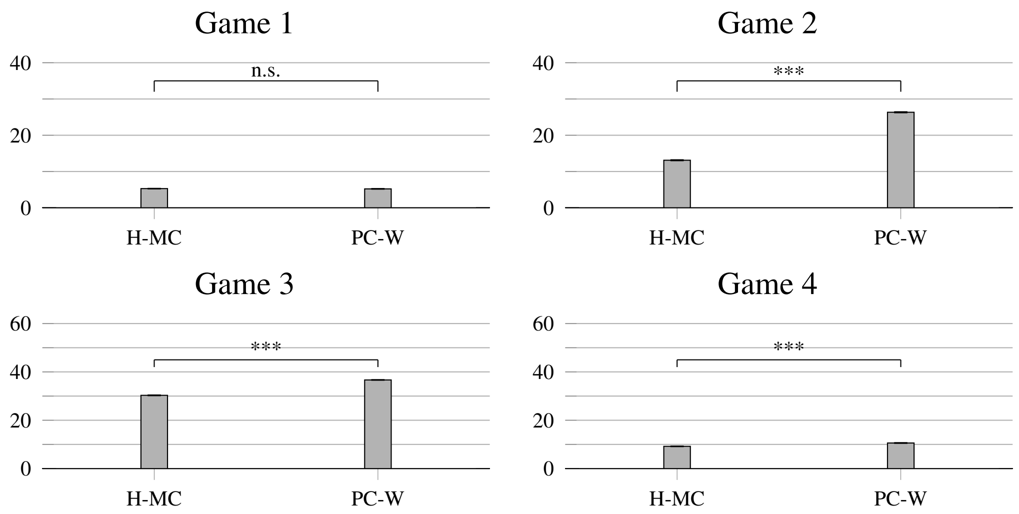

Note: The horizontal lines indicate the Shapley values. Error bars show one standard error range. ***, **, and * indicate the average payoff being significantly different from the Shapley value at the 0.1, 1, and 5% significance levels ($\chi^2$ test).

are to the Shapley value, we compute the following measure:

$$Dis_\phi = \sqrt{\sum_i (\overline{\pi_i} - \phi_i)^2}, \tag{2}$$

where $\overline{\pi_i}$ and $\phi_i$ are the average payoff and Shapley value for player $i$, respectively, in the given game. As above, we focus on only those groups that formed a grand coalition.

Figure 5: H-MC and PC-W mechanisms: distance from Shapley




Note: Error bars show the one standard error range. ***, **, and * indicate significant difference between H-MC and PC-W at the 0.1, 1, and 5% significance levels (two-sample t-test).

Furthermore, we employ a bootstrapping technique to conduct statistical tests.[8] Namely, in each iteration, we use a subsample (with replacement) of our data, run the system of regressions presented above (Eq. 1), and compute $Dis_{\phi}$ based on the obtained estimated coefficients (i.e., the average payoffs for the subsample).

Figure 5 shows the results based on the outcomes of 1000 repetitions. As one can observe, $Dis_{\phi}$ is significantly higher for games 2, 3, and 4 under PC-W than H-MC.[9] Thus, the bidding stage of PC-W not only results in a lower efficiency, but also a larger deviation from the Shapley value even among those groups that successfully formed the grand coalition.

[8]Note that we have one observation of $Dis_{\phi}$ for each game because $Dis_{\phi}$ is based on average payoffs. By using a bootstrapping technique, we generate multiple observations of $Dis_{\phi}$ using sub-samples.

[9]Based on a two-sample t-test with unequal variance using the sample generated by the bootstrap. The means $Dis_{\phi}$ (standard errors) for the H–MC mechanism are 5.21 (0.041) in game 1, 13.11 (0.088) in game 2, 30.31 (0.089) in game 3, and 9.21 (0.058) in game 4. For the PC-W mechanism, the corresponding values are 5.21 (0.041) for game 1, 26.34 (0.098) for game 2, 36.67 (0.053) for game 3, and 10.59 (0.077) for game 4.

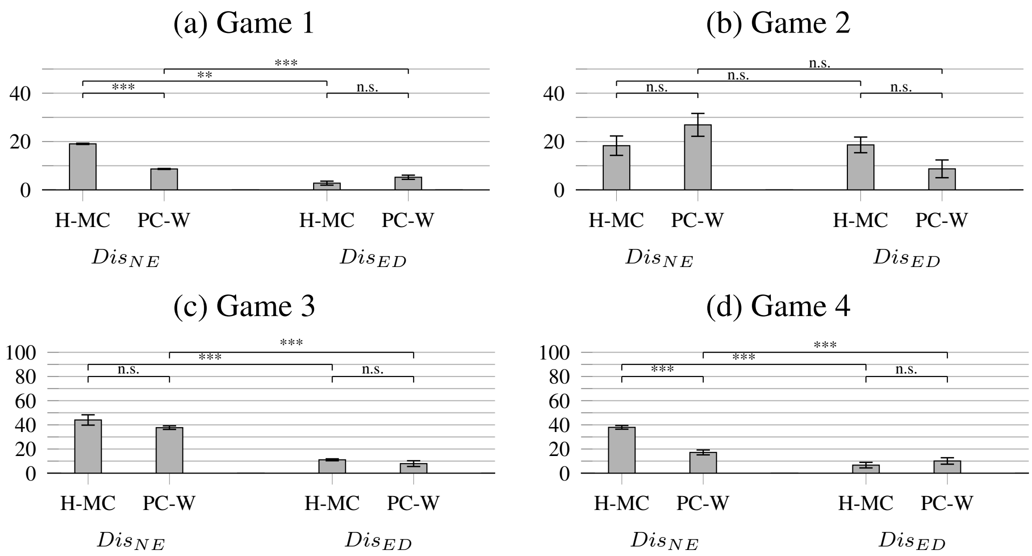
## 4.4 Realized allocations and the a posteriori equilibria

Let us now analyze the realized payoffs in light of the a posteriori equilibrium payoff vectors. We continue to focus only on the groups that formed the grand coalition. We measure the distance between the realized payoff vectors and the equilibrium allocation for the four games by the Euclidean distances between the two. Let $eq_i$ be the equilibrium payoff for player $i$ for the given game and the mechanism. Note that $eq_i$ depends also on the realized proposer in the case of H-MC. The distance of the realized payoff from the equilibrium is computed as $Dis_{NE} = \sqrt{\sum_i (\pi_i - eq_i)^2}$. We also consider the distance between the realized payoff vectors and equal division payoffs, defined by $Dis_{ED} = \sqrt{\sum_i (\pi_i - ED_i)^2}$, where $ED_i$ is the equal division payoff for player $i$ for the given game.

Figure 6 shows the mean $Dis_{NE}$ and the mean $Dis_{ED}$ for the two mechanisms in the four games.[10] Figure 6 shows that, except for game 2, the realized allocations are closer to the equal division than for the equilibrium under both H-MC and PC-W. The differences between $Dis_{NE}$ and $Dis_{ED}$ are significant at the 1% significance level, except for game 2. Because the allocation is closer to the equal division than that for equilibrium, and the equilibrium allocation of PC-W is less extreme than those of H-MC, we observe that PC-W results in an allocation closer to that of equilibrium than H-MC for games 1 and 4 at the 0.1% significance level. For games 2 and 3, $Dis_{NE}$ is not significantly different between the two mechanisms.

[10]The figure is created based on the estimated coefficients of the following linear regressions: $Dis_i = \beta_1 HMC_i + \beta_2 PCW_i + \mu_i$ where $Dis_i$ is the relevant distance measure for group $i$, $HMC_i$ ($PCW_i$) is a dummy variable that takes the value 1 if the H-MC (P-WC) mechanism is used, and zero otherwise. Standard errors are corrected for any within-session clustering effect. The statistical tests are based on the Wald test for the equality of the estimated coefficients of the two treatment dummies.

Figure 6: Mean of the distances of the realized payoff vectors from the subgame perfect Nash equilbrium and the equal division




Note: Error bars show the one standard error range. ***, **, and * indicate that the distance of the payoff vectors from the equilibrium allocations or from the equal division was significantly different between H-MC and PC-W at the 0.1, 1, and 5% significance levels (Wald test).

## 4.5 Shapley distance

Finally, following Aguiar et al. (2018), we decompose *Shapley distance* (the distance of payoff vectors from the corresponding Shapley value) into the failure of axioms characterizing the Shapley value. While Aguiar et al. (2018) proposes the decomposition into the failure of the symmetry, the efficiency, and the marginality axioms, we follow Chessa et al. (2022a) and decompose it into the failure of the axioms considered by Shapley (1953), namely, the efficiency, the symmetry, the additivity, and the null player. For this exercise, we consider all the groups, i.e., including those groups that did not form the grand coalition.

Let $\pi = (\pi_1, \pi_2, \pi_3, \pi_4)$ be a vector of payoffs obtained by the four players. Below, we summarize the detailed procedure of this decomposition.

(1) First, find a vector of payoffs, $\pi^{sym}$, that satisfies symmetry and is closest to $\pi$. That is, for symmetric players ($j = \{2, 3\}$ in games 1 and 4), $\pi_j^{sym} = \sum_{j \in N^s} \pi_j / |N^s|$, where $N^s$ is the set of symmetric players in the game. For other players, $\pi_k^{sym} = \pi_k$.

(2) Second, find a vector of payoffs, $\pi^{sym,eff}$, that satisfies efficiency and is closest to $\pi^{sym}$. In particular, for each player $i = 1, 2, 3, 4$, $\pi^{sym,eff} = \pi^{sym} + [v(N) - \sum_{j \in N} \pi_j]/|N|$.

(3) Third, find a vector of payoffs, $\pi^{sym,eff,null}$, that satisfies the null player property and is closest to $\pi^{sym,eff}$. Namely, if player $i$ is a null player (Player 2 in game 2), $\pi_i^{sym,eff,null} = 0$. For the other players $j$, $\pi_j^{sym,eff,null} = \pi_j^{sym,eff} + \sum_{i \in N^n} \pi_i^{sym,eff} / (|N| - |N^n|)$, where $N^n$ is the set of null players.

(4) Fourth, using these payoff vectors, $\pi$, $\pi^{sym}$, $\pi^{sym,eff}$, $\pi^{sym,eff,null}$, and the corresponding vector of the Shapley values, $\phi(v)$, we can define the following "errors" for each Player $i$

- $e_i^{\phi} = \pi_i - \phi_i(v)$
- $e_i^{sym} = \pi_i - \pi_i^{sym}$,
- $e_i^{eff} = \pi_i^{sym} - \pi_i^{sym,eff}$,
- $e_i^{null} = \pi_i^{sym,eff} - \pi_i^{sym,eff,null}$,
- $e_i^{add} = \pi_i^{sym,eff,null} - \phi_i(v)$.

Given these "errors", the Shapley distance $||e^{\phi}||^2$ can be decomposed into the failure of the symmetry, the efficiency, the null player, and the additivity axioms as follows:

$$||e^{\phi}||^2 = ||e^{sym}||^2 + ||e^{eff}||^2 + ||e^{null}||^2 + ||e^{add}||^2 + 2 < e^{add}, e^{null} >,$$

where $< \cdot, \cdot >$ is the scalar product.[11] As one can observe from the presence of $< e^{add}, e^{null} >$, unlike the decomposition of Aguiar et al. (2018) that ensures decomposition into three orthogonal components, in the decomposition of Chessa et al. (2022a), vectors $e^{null}$ and $e^{add}$ are not orthogonal, and $< e^{add}, e^{null} >$ is not equal to zero. However, $< e^{add}, e^{null} >$ in our data are several orders of magnitude smaller than other components and can be safely ignored (on average, they are 0.014 and 0.007 in PC-W and H-MC, respectively).

To test for differences between PC-W and H-MC, we run the following OLS regression by pooling the data from all four games:

$$||e^k||_g^2 = \beta_1 HMC + \beta_2 PCW + U. \tag{3}$$

The dependent variable is the components of the Shapley distance corresponding to the four axioms as well as the Shapley distance itself ($||e^k||_g^2$ with $k \in \{sym, eff, null, add, \phi\}$) for group $g$ and the independent variables are $HMC$ and $PCW$, which take a value of 1 if the corresponding mechanism is used, and zero otherwise. Standard errors are corrected for within-session clustering effects.

Table 6 shows the results of the regressions. The Shapley distance is significantly larger for PC-W (1604.27) than for H-MC (769.12, p-value = 0.008, Wald test). In both PC-W and H-MC, the main components of the Shapley distance are due to the failure of the efficiency and the additivity axioms. They account for 49.2% and 40.5% in PC-W and 50.3% and 41.2% in H-MC. Furthermore, as we have observed in Figure 4, the failure of the null player axiom is significantly larger for PC-W than H-MC (p-value = 0.028, Wald test) which accounts for, respectively, 7.3% in PC-W and 5.1% in H-MC

[11] For any vector $y \in \mathbb{R}^n$, $||y||^2 =< y, y >= \sum_{i \in N} y_i^2$.

| | $\lvert\lvert e^{sym}\rvert\rvert^2$ | $\lvert\lvert e^{eff}\rvert\rvert^2$ | $\lvert\lvert e^{null}\rvert\rvert^2$ | $\lvert\lvert e^{add}\rvert\rvert^2$ | $\lvert\lvert e^{\phi}\rvert\rvert^2$ |
|---|---|---|---|---|---|
| PC-W | 48.58 | 789.34 | 117.32 | 649.15 | 1604.27 |
| | (7.67) | (96.88) | (26.04) | (136.96) | (220.94) |
| H-MC | 25.90 | 386.64 | 39.55 | 317.10 | 769.12 |
| | (2.28) | (40.36) | (10.47) | (21.81) | (55.17) |
| No. Obs | 328 | 328 | 328 | 328 | 328 |
| $R^2$ | 0.103 | 0.248 | 0.125 | 0.265 | 0.427 |
| p-value* | 0.025 | 0.006 | 0.028 | 0.048 | 0.008 |

Standard errors are corrected for session-level clustering effects and shown in parentheses.
* p-values for testing $H_0$: PC-W = H-MC (based on the Wald test)

Table 6: Result of Shapley distance decomposition. Based on pooling the data of all groups and all games

of the Shapley distance.

# 5 Summary and conclusion

In this paper, we aim to experimentally contribute to the Nash program (Nash, 1953) by extending the experimental analyses of Chessa et al. (2022a,b). We do so by comparing the simplified version of the mechanisms proposed by Hart and Mas-Colell (1996) and Perez-Castrillo and Wettstein (2001). In the original theoretical investigation, in both mechanisms, there is a proposer who proposes an allocation, which is voted on sequentially by the remaining players (in our experiment, the voting is done simultaneously). The proposal is accepted if all the remaining players accept the proposal; if not, with some probability in H-MC and with probability one in PC-W (in our experiment, both are set to be with probability 1 to be comparable), the proposer leaves the game with his/her individual value (minus the sum allocated in the bidding stage in PC-W), and the game continues with the remaining players. The key difference between the two mechanisms is the way the proposer is chosen. On the one hand, a proposer is chosen randomly from among the players in Hart and Mas-Colell (1996). On the other hand, in Perez-Castrillo and Wettstein (2001), the proposer is determined through a bidding procedure. While Hart and Mas-Colell (1996) implements the Shapley value as an *ex ante* equilibrium payoff (i.e., it is only achieved as an expected outcome), Perez-Castrillo and Wettstein (2001) implements it as a unique equilibrium of the game.

Our experiment shows that the simpler mechanism a la Hart and Mas-Colell (1996) not only results in higher efficiency than the one by Perez-Castrillo and Wettstein (2001), but also that the average allocation is closer to the Shapley value for those groups that formed a grand coalition. Thus, the complexity of the mechanism of Perez-Castrillo and Wettstein (2001) imposes substantial costs for implementing the Shapley value. In particular, we find that those proposers who have been selected via the bidding in PC-W tend to propose an allocation that favors themselves than those selected randomly in H-MC. Namely, the allocations proposed in PC-W are significantly less equal and deviate more from Shapley value than those proposed in H-MC. However, such proposals are more likely to be rejected by other players, and as a result, the grand coalition are less likely to be formed and efficiency is lower under PC-W than under H-MC. This suggests a possibility that while those who win the bidding stage in PC-W consider themselves to be entitled to receive a larger share, others do not think in the same way.

Chessa et al. (2022a,b), as well as the present paper, showed that participants have difficulty forming a grand coalition and achieving full efficiency under various noncooperative mechanisms (namely, Winter, 1994; Hart and Mas-Colell, 1996; Perez-Castrillo and Wettstein, 2001). A natural question is whether participants can better form a grand coalition and achieve a higher efficiency if they can negotiate freely in an unstructured bargaining environment. Establishing such a benchmark in more "cooperative" environments and comparing them with the results obtained under "noncooperative" environments would be fruitful future research.

Other possible future work could investigate the performance of modifications of the mechanism by Perez-Castrillo and Wettstein (2001) when implementing some variations of the Shapley value, such as the egalitarian Shapley value (van den Brink et al., 2013) or when we introduce discounting in the bidding mechanism (van den Brink and Funaki, 20015).

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# A Bidding stage in PC-W

In this section, we analyze the result of the bidding stage in PC-W. In particular, we investigate (1) whether a small number of participants have won the bidding stage most of the time, and (2) the distribution of the submitted bids.

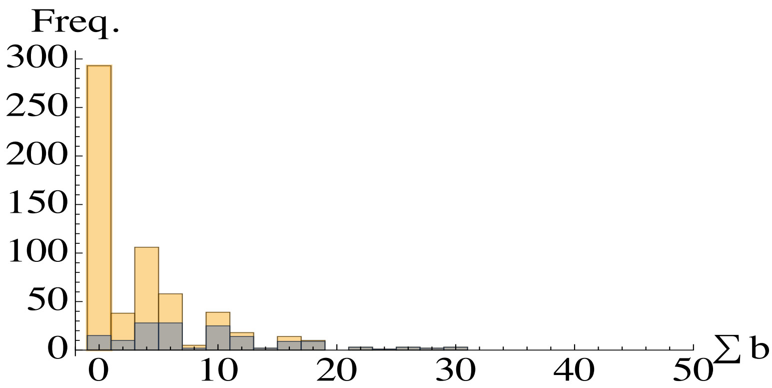
## A.1 Frequency of winning the first bidding stage

Table 7: Frequency of becoming the first proposer

| | Session | | | |
|---|---|---|---|---|
| Fr. of winning | 1 | 2 | 3 | 4 |
| 0 | 6 | 3 | 3 | 2 |
| 1 | 3 | 5 | 6 | 7 |
| 2 | 4 | 7 | 4 | 5 |
| 3 | 2 | 2 | 3 | 5 |
| 4 | 2 | 1 | 3 | 0 |
| 5 | 3 | 1 | 1 | 0 |
| 6 | 0 | 1 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 1 |
| total | 20 | 20 | 20 | 20 |

Table 7 shows the distribution of the frequency of becoming the first proposer in each of four sessions. There is one participant in session 4 who won the first bidding

Figure 7: Histogram of non-negative total bids and winning total bids




Light color: Total bids. Dark color: Total winning bids.

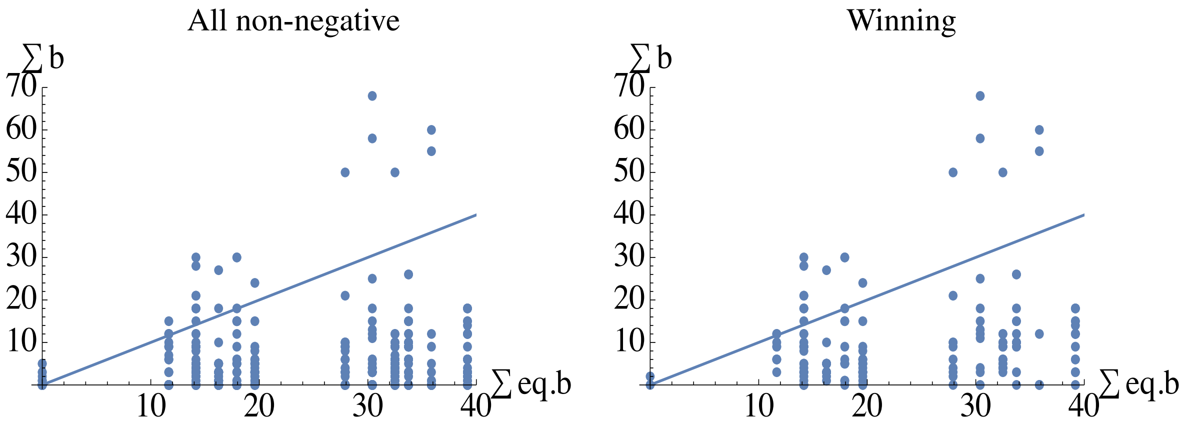
stage and became the first proposer in all the 8 games in which s/he participated. There are also a few participants who became the first proposer more than half (4) of the times. Thus, there are some participants who bid aggressively to become a proposer.

## A.2 Submitted bids

Figure 7 shows the histogram of the total bids (sum of the bids to three other players in the group) that are non-negative as well as the winning total bids. Note that there are 42 (out of 640) cases in which participants submitted a negative total bid to avoid becoming the proposer (in some cases, the total bid was -300).

Figure 8 shows the distribution of the total bids (that are non-negative). The left panel shows all the bids, and the right panel shows the only the winning bids. In both panels, the sum of the equilibrium bids (equilibrium total bids) is indicated in the x-axis as well as the straight line in the figure. While there are cases of participants bidding more than the equilibrium amount, in most of the cases, total bids were less than the equilibrium amounts. The same is true among those who won the bidding stage shown

Figure 8: Distribution of total bids vis-a-vis the equilibrium total bids




on the right panel.

# B Realized allocations in the grand coalitions

Table 8 shows the distribution of realized allocations in four games for those groups that formed a grand coalition. For games 1, 3, and 4, there are many groups that opted for the equal division. There are also groups that decided to allocate points among four players in the reverse ordering of the Shapley value.

# C Comparison of the first two games and the last two games

So far, we have considered all the rounds and compared H-MC and PC-W. However, given that PC-W is more complex than H-MC, it is possible that it takes longer for

Table 8: Distribution of realized allocation in the grand coalitions.

### H-MC, Game 1

| $\pi_1$ | $\pi_2$ | $\pi_3$ | $\pi_4$ | Freq. |
|---|---|---|---|---|
| 25 | 25 | 25 | 25 | 16 |
| 20 | 25 | 25 | 30 | 6 |
| 21 | 37 | 21 | 21 | 1 |
| 23 | 31 | 23 | 23 | 1 |
| 25 | 25 | 23 | 27 | 1 |
| 26 | 26 | 22 | 26 | 1 |
| 30 | 23 | 24 | 23 | 1 |

### PC-W, Game 1

| $\pi_1$ | $\pi_2$ | $\pi_3$ | $\pi_4$ | Freq. |
|---|---|---|---|---|
| 25 | 25 | 25 | 25 | 8 |
| 15 | 25 | 25 | 35 | 1 |
| 16 | 25 | 27 | 32 | 1 |
| 20 | 30 | 20 | 30 | 1 |
| 25 | 35 | 21 | 21 | 1 |
| 30 | 10 | 30 | 30 | 1 |
| 31 | 25 | 25 | 19 | 1 |

### H-MC, Game 2

| $\pi_1$ | $\pi_2$ | $\pi_3$ | $\pi_4$ | Freq. |
|---|---|---|---|---|
| 0 | 30 | 30 | 40 | 2 |
| 0 | 33 | 33 | 34 | 2 |
| 20 | 20 | 30 | 30 | 2 |
| 22 | 22 | 22 | 34 | 2 |
| 0 | 25 | 25 | 50 | 1 |
| 5 | 30 | 30 | 35 | 1 |
| 9 | 27 | 27 | 37 | 1 |
| 10 | 25 | 30 | 35 | 1 |
| 15 | 26 | 26 | 33 | 1 |
| 20 | 25 | 25 | 30 | 1 |
| 23 | 23 | 23 | 31 | 1 |

### PC-W, Game 2

| $\pi_1$ | $\pi_2$ | $\pi_3$ | $\pi_4$ | Freq. |
|---|---|---|---|---|
| 25 | 25 | 25 | 25 | 2 |
| 0 | 30 | 30 | 40 | 1 |
| 16 | 23 | 23 | 38 | 1 |
| 21 | 25 | 21 | 33 | 1 |
| 23 | 22 | 24 | 31 | 1 |
| 23 | 24 | 30 | 23 | 1 |
| 25 | 24 | 27 | 24 | 1 |
| 25 | 22 | 25 | 28 | 1 |
| 26 | 26 | 22 | 26 | 1 |
| 35 | 12 | 29 | 24 | 1 |

### H-MC, Game 3

| $\pi_1$ | $\pi_2$ | $\pi_3$ | $\pi_4$ | Freq. |
|---|---|---|---|---|
| 50 | 50 | 50 | 50 | 12 |
| 40 | 50 | 50 | 60 | 3 |
| 35 | 55 | 55 | 55 | 2 |
| 20 | 65 | 45 | 70 | 1 |
| 30 | 50 | 50 | 70 | 1 |
| 35 | 65 | 45 | 55 | 1 |
| 38 | 54 | 54 | 54 | 1 |
| 40 | 40 | 40 | 80 | 1 |
| 51 | 51 | 51 | 47 | 1 |
| 70 | 50 | 50 | 30 | 1 |
| 73 | 48 | 48 | 31 | 1 |

### PC-W, Game 3

| $\pi_1$ | $\pi_2$ | $\pi_3$ | $\pi_4$ | Freq. |
|---|---|---|---|---|
| 50 | 50 | 50 | 50 | 6 |
| 45 | 50 | 50 | 55 | 2 |
| 40 | 45 | 65 | 50 | 1 |
| 47 | 63 | 45 | 45 | 1 |
| 49 | 55 | 49 | 47 | 1 |
| 51 | 51 | 51 | 47 | 1 |
| 52 | 52 | 52 | 44 | 1 |
| 54 | 46 | 46 | 54 | 1 |
| 56 | 40 | 53 | 51 | 1 |
| 60 | 30 | 55 | 55 | 1 |
| 64 | 54 | 54 | 28 | 1 |

### H-MC, Game 4

| $\pi_1$ | $\pi_2$ | $\pi_3$ | $\pi_4$ | Freq. |
|---|---|---|---|---|
| 50 | 50 | 50 | 50 | 17 |
| 45 | 50 | 50 | 55 | 3 |
| 45 | 45 | 65 | 45 | 2 |
| 25 | 45 | 45 | 85 | 1 |
| 40 | 45 | 45 | 70 | 1 |
| 40 | 50 | 50 | 60 | 1 |
| 44 | 44 | 44 | 68 | 1 |
| 45 | 50 | 55 | 50 | 1 |
| 51 | 51 | 51 | 47 | 1 |
| 55 | 35 | 55 | 55 | 1 |
| 60 | 50 | 40 | 50 | 1 |

### PC-W, Game 4

| $\pi_1$ | $\pi_2$ | $\pi_3$ | $\pi_4$ | Freq. |
|---|---|---|---|---|
| 50 | 50 | 50 | 50 | 11 |
| 44 | 68 | 44 | 44 | 2 |
| 30 | 41 | 39 | 90 | 1 |
| 39 | 44 | 44 | 73 | 1 |
| 40 | 50 | 50 | 60 | 1 |
| 42 | 45 | 61 | 52 | 1 |
| 42 | 56 | 51 | 51 | 1 |
| 51 | 51 | 51 | 47 | 1 |
| 51 | 51 | 47 | 51 | 1 |
| 55 | 30 | 55 | 60 | 1 |
| 55 | 35 | 55 | 55 | 1 |
| 57 | 61 | 47 | 35 | 1 |
| 62 | 46 | 46 | 46 | 1 |
| 65 | 45 | 45 | 45 | 1 |

participants to learn to play better. In this section, therefore, we separately investigate the outcomes in the early rounds (i.e., the first two games that participants played) and in the late rounds (i.e., the last two games that participants played). The figures below are generated in a similar way to the corresponding figures presented above.[12]

Figure 9 shows the frequency of grand coalition formation in four games for the early and the late rounds. For H-MC, the grand coalition is significantly more frequently formed in the late rounds than in the early rounds for game 1 (at the 1% significance level). For PC-W, the frequency of grand coalition formation is significantly higher in the late rounds than in the early rounds for games 3 and 4 (at the 0.1 and 1% significance levels, respectively). A similar tendency can be observed for the efficiency shown in Figure 10.

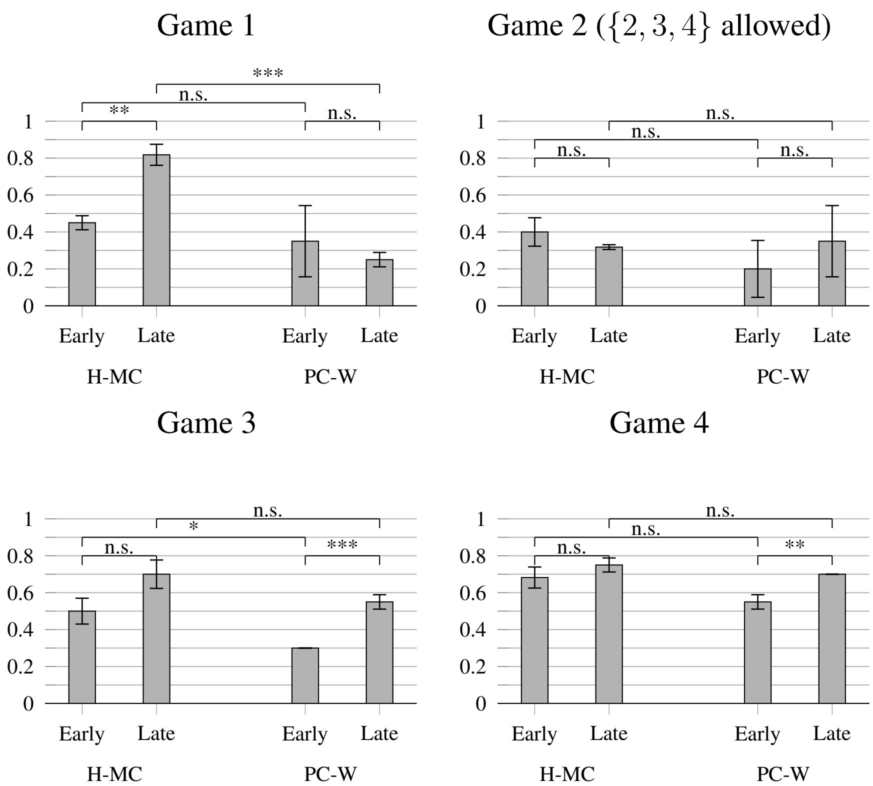
Figure 11 shows $Dis_\phi$ for the early and late rounds in H-MC and PC-W for the four games. While for H-MC, $Dis_\phi$ becomes smaller in the late rounds than in the early rounds for all four games, this is not the case for PC-W.[13] For PC-W, $Dis_\phi$ becomes smaller in the late rounds than in the early rounds only for games 3 and 4, whereas for games 1 and 2, it becomes larger in the late rounds.[14] Thus, gaining experience playing the game does not necessarily lead to allocation according to the Shapley value under PC-W.

[12]Namely, the figures are created based on the estimated coefficients of the following linear regression: $y_i = \beta_1 HMC_i^e + \beta_2 HMC_i^l + \beta_3 PCW_i^e + \beta_4 PCW_i^l + \mu_i$ where $y_i$ is the outcome variable of interest in group $i$, and $HMC_i^\tau$ ($PCW_i^\tau$) is a dummy variable that takes the value 1 for $\tau \in \{e, l\}$ where $e$ and $l$ stand for early and late rounds of the H-MC (PC-W) mechanism, and zero otherwise. The standard errors are corrected for any within-session clustering effect. The statistical tests are based on the Wald test for the equality of the estimated coefficients of two treatment dummies.

[13]The mean $Dis_\phi$ (the standard errors) for H-MC in early and late rounds are 6.44 (0.013) and 4.78 (0.027) for game 1, 19.03 (0.09) and 7.38 (0.11) for game 2, 32.08 (0.13) and 29.18 (0.13) for game 3, and 9.96 (0.09) and 8.78 (0.07) for game 4, respectively.

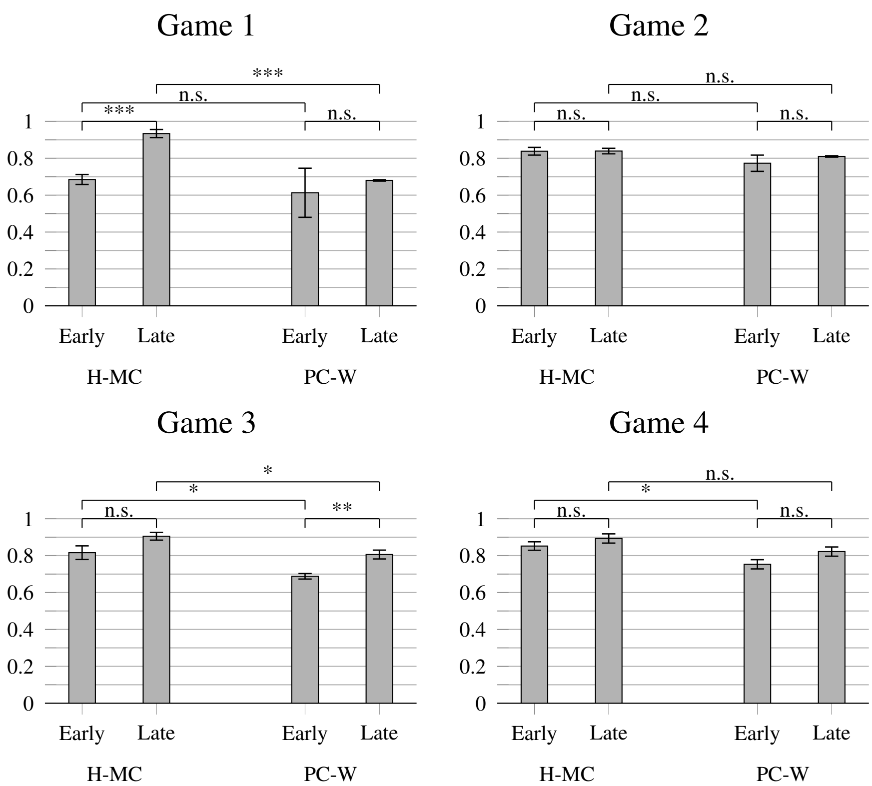
[14]For PC-W, the mean $Dis_\phi$ (the standard errors) in early and late rounds are 4.38 (0.056) and 7.21 (0.057) for game 1, 18.18 (0.17) and 31.70 (0.05) for game 2, 37.16 (0.03) and 36.55 (0.08) for game 3, and 11.68 (0.11) and 10.16 (0.10) for game 4, respectively.

Figure 9: H-MC and PC-W mechanisms: proportion of times the grand coalition was formed in early and late rounds



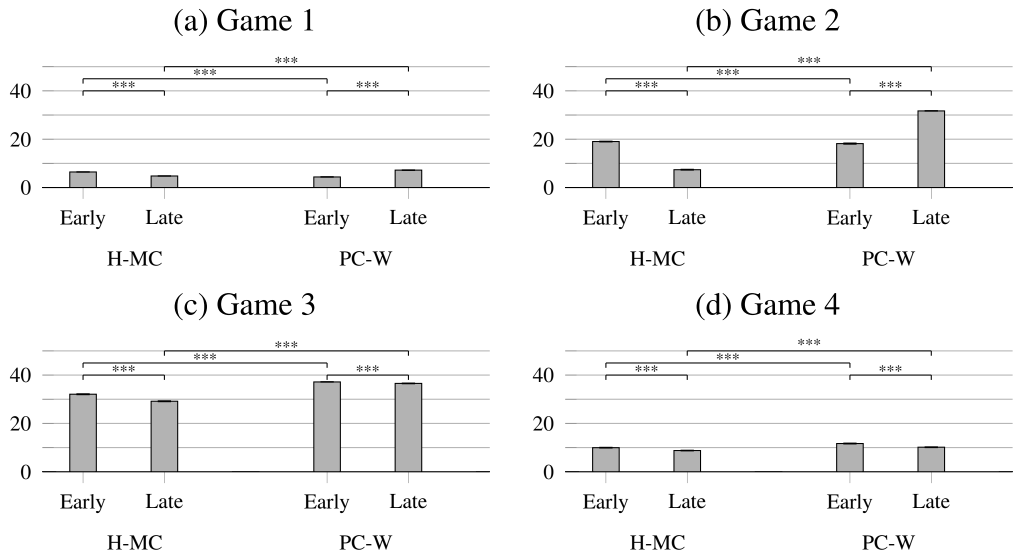

Note: Error bars show the one standard error range. ***, **, and * indicate that the two averages were significantly different at the 0.1, 1, and 5% significance levels (Wald test).

Figure 10: H-MC and PC-W mechanisms: efficiency in early and late rounds




Note: Error bars show the one standard error range. ***, **, and * indicate that the two averages were significantly different at the 0.1, 1, and 5% significance levels (Wald test).

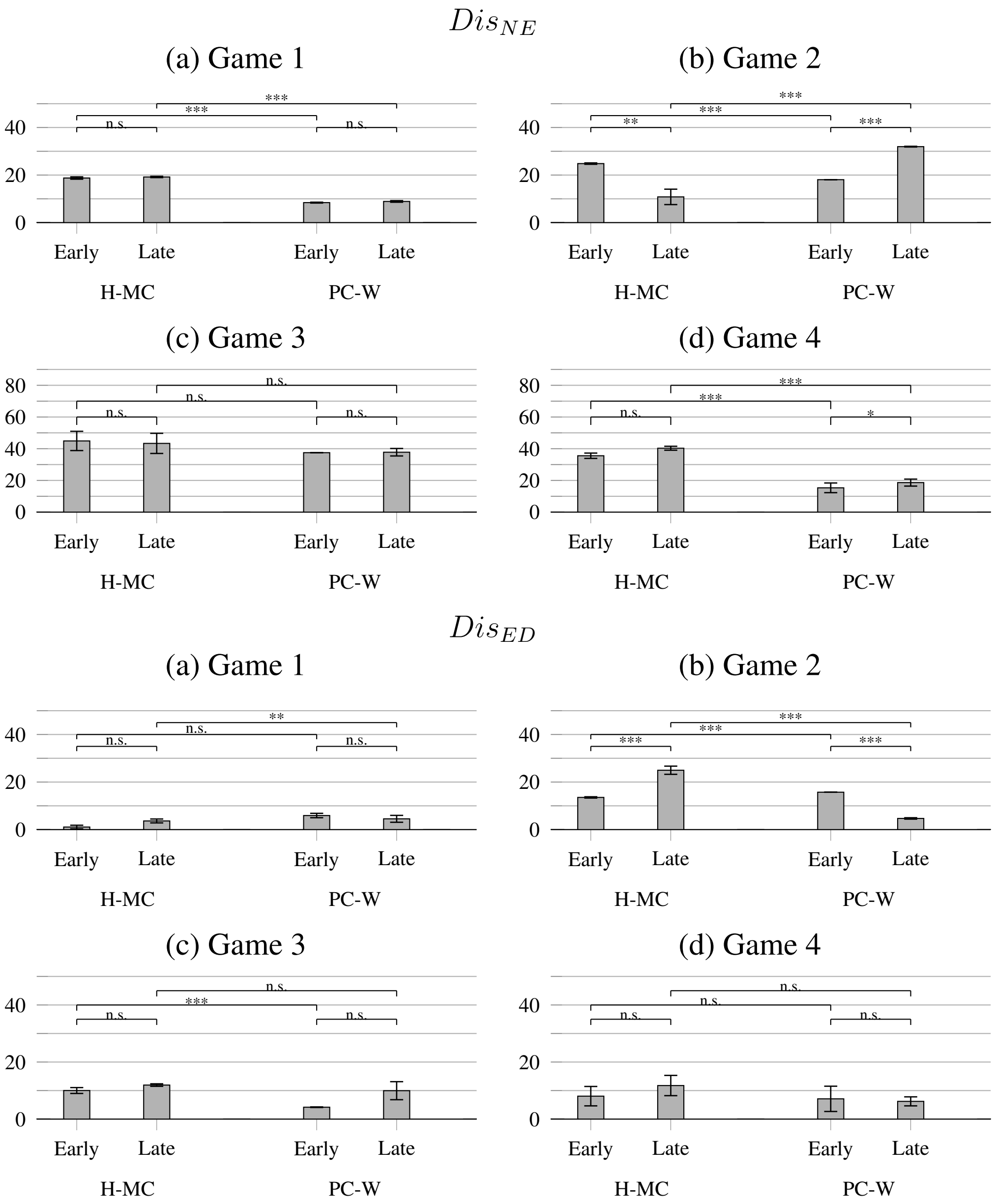
Figure 11: Mean $Dis2_{\phi}$ in early and late rounds




Note: Error bars show the one standard error range. ***, **, and * indicate a statistically significant difference between the two means, at the 0.1, 1, and 5% significance levels (Wald test).

This point can also be observed for $Dis_{NE}$. Figure 12 shows $Dis_{NE}$ (top four panels) and $Dis_{ED}$ (bottom four panels) in the early and the late rounds. For game 2, on the one hand, under H-MC, $Dis_{NE}$ becomes significantly smaller (and correspondingly, $Dis_{ED}$ becomes significantly larger) in the late rounds compared with the early rounds at the 1% significance level. On the other hand, under PC-W, the opposite is observed. Namely, $Dis_{NE}$ becomes significantly larger (and correspondingly, $Dis_{ED}$ becomes significantly smaller) in the late rounds compared with the early rounds at the 0.1% significance level. Furthermore, for PC-W in game 4, $Dis_{NE}$ in the late rounds is significantly larger than in the early rounds at the 5% significance level.

Figure 12: Mean $Dis_{NE}$ and $Dis_{ED}$ in early and late rounds




Note: Error bars show the one standard error range. ***, **, and * indicate a statistically significant difference between the two means at the 0.1, 1, and 5% significance levels (Wald test).

# D English translations of the instruction materials

The instruction materials and screenshots of the quiz for

- H-MC can be obtained from https://www.dropbox.com/s/78lf5bn6qi3qfwp/HMChandout_sim.pdf?dl=0
- PC-W can be obtained from https://www.dropbox.com/s/yc584hk4c58ceyr/PCW_handout.pdf?dl=0