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BEHAVIOR-BASED PRICE DISCRIMINATION IN THE DOMESTIC AND INTERNATIONAL MIXED DUOPOLY

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Behavior-based Price Discrimination in the Domestic and International Mixed duopoly *

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Abstract

This study investigates mixed markets in which a social welfare-maximizing public firm and a private firm engage in behavior-based price discrimination (BBPD). Total of two cases are considered: one where domestic shareholders completely own the private firm and one where foreign shareholders completely own it. In the domestic mixed duopoly, BBPD is irrelevant from the viewpoint of social welfare. This is because poaching does not occur. In the international mixed duopoly, BBPD improves domestic social welfare, as it allows the public firm to lower its poaching price. In both cases, privatization is more undesirable under BBPD than uniform pricing.

JEL classification: D43, H42,L13
Keywords : Behavior-based price discrimination, Mixed oligopoly, Foreign firms, Privatization

1 Introduction

In the telecommunication industry, a firm that is fully or partially owned by the governments provides mobile services, and offers different prices depending on consumers' past purchases; this is called behavior-based price discrimination (BBPD).[1] Orange and Docomo offer discounts on cell phone bills to new consumers. Telkomsel offers discounts to

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[1]In mobile phone markets, consumers can choose between pre- or post-paid plans. When firms contract post-paid plans with rivals' consumers, not only the private firms but also those that the governments fully or partially own offer discounts.

consumers with new numbers. Telenor offers new consumers a two-month trial period. On the other hand, BSNL does not offer new consumers discounts.

Recently, government ownership in these markets has become issue. In Japan, NTT, a partially privatized company, made Docomo its subsidiary in 2021. In India, the world's second-largest cell phone market, BSNL, which the Indian government fully owns, is expected to be privatized. The impacts of nationalization and privatization on welfare in these markets have not been fully discussed. Therefore, this study investigates whether the presence of a social welfare-maximizing public firm improves welfare in the market where firms engage in price discrimination.

This study provides analyses of BBPD in mixed duopoly markets. We follow the brand preference approach based on Hotelling models (Fudenberg and Tirole, 2000). We assume that the public firm maximizes domestic social welfare and the private firm maximizes its profit. In a domestic (resp. international) mixed duopoly, the public firm competes with a private firm completely owned by domestic (resp. foreign) shareholders.[2] Consumers' preferences are private information. In the first period, firms offer uniform prices. In the second, firms offer different prices based on the consumers' purchase histories. This study investigates the impacts of BBPD on domestic social welfare and discusses privatization.

BBPD is found to be irrelevant from the viewpoint of social welfare in the domestic mixed duopoly, in contrast to the result in Fudenberg and Tirole (2000), which show that BBPD is detrimental to social welfare in a domestic pure duopoly. In Fudenberg and Tirole (2000), two profit-maximizing private firms poach their rival's consumers by offering discounts, and poaching induces socially inefficient switching in the second period. However, in our model, the public firm does not offer a discount to its rival's consumers, and poaching does not occur in the second period since the public firm minimizes the sum of disutilities of taste mismatch in each turf.

BBPD is beneficial to domestic social welfare in the international mixed duopoly. The public firm has an incentive to lower its prices to reduce the outflow to foreign shareholders in the international mixed duopoly. By lowering its prices, it faces an increase in the sum of disutilities of taste mismatch if firms do not engae in BBPD. However, BBPD allows the public firm to lower its poaching price without increasing the sum of disutilities of taste mismatch in the private firm's turf in the second period. Therefore, BBPD lowers equilibrium prices and reduces the outflow to foreign shareholders, although BBPD increases the total discounted sum of disutilities of taste mismatch in the international mixed duopoly.

Privatization is more undesirable under BBPD than uniform pricing.[3] Welfare loss

[2]Some compete with domestic private firms (e.g., Docomo) and others compete with foreign private firms (Orange, Telenor, Telia, Telkomsel, BSNL).

[3]Recently, the literature on privatization policies has become richer. Matsumura and Shimizu (2010) examine sequential privatization. Matsumura and Okamura (2015) and Haraguchi et al. (2018) discuss the relationship between competition and the optimal privatization policy. Lin and Matsumura

from privatization is found to be larger under BBPD than uniform pricing in the domestic and international mixed duopoly. In the domestic mixed duopoly, privatization reduces domestic social welfare under BBPD, although it is irrelevant from the viewpoint of domestic social welfare under uniform pricing. In the international mixed duopoly, privatization reduces domestic social welfare under both uniform pricing and BBPD. However, the welfare loss from privatization is larger under BBPD than under uniform pricing. These results indicate that, from the viewpoint of domestic social welfare, the subsidiary acquisition of Docomo improves social welfare and privatization of BSNL is more detrimental to the market of the post-paid plan than that of the pre-paid plan.

The remainder of this paper is organized as follows. Section 2 provides a review of the literature. Section 3 describes the model setup. The effects of BBPD on firms' profit, consumer surplus, and social welfare in the domestic mixed duopoly are examined in Section 4. Section 5 examines the effects of the foreign mixed duopoly. Section 6 discusses the welfare implications of BBPD. Finally, concluding remarks arepresented in Section 7.

2 Related literature

Our study relates to two strands of literature. Firstly, it is related to a strand that deals with BBPD. Since the seminal works by Chen (1997), Villas-Boas (1999), and Fudenberg and Tirole (2000), numerous studies have investigated BBPD.[4] These three studies and Choe and Matsushima (2021) demonstrate that BBPD is detrimental to firms unless they are sufficiently asymmetric since it intensifies competition. Carroni (2016), Jing (2017), and Rhee and Thomadsen (2017) introduce asymmetric competitive advantage and indicate that BBPD can be beneficial to firms. We assume that firms maximize different objective functions and show that BBPD is detrimental to firms. The reason is that the pricing of the public firm limits the private firm's price in the second period.

This paper is also related to the strand of literature on mixed oligopoly. In particular, this paper has strong connections to (i) spatial competition and (ii) international competition.

(2018a) discuss optimal privatization policy in the Stackelberg competition. Sato and Matsumura (2019a, 2019b) investigate the impact of the shadow cost of public funds on the optimal privatization policy. Kim (2018, 2019), Haraguchi and Matsumura (2020b, 2020c), and Liu et al. (2021) discuss optimal privatization policies by introducing pollution, corporate social responsibility, asymmetric private firms, commitment, and corporate taxation policies, respectively. See Matsumura and Okumura (2013), Matsumura and Okumura (2017) for privatization neutrality theorem.

[4] Chen and Pearcy (2010) discuss the intertemporal dependence of brand preference. Liu and Shuai (2013, 2016) investigate price discrimination in a multi-dimensional model. Esteves and Reggiani (2014) and Zhang et al. (2019) introduce elastic demand.

The literature includes two types of spatial competition: price competition under uniform pricing and spatial price discrimination.

In the first type model, Cremer et al. (1991) examine a mixed duopoly with horizontal differentiation in a Hotelling model. They show that the equilibrium location is optimal, and first-best is achieved in the domestic mixed duopoly, whereas maximal differentiation occurs in a domestic pure duopoly. This difference occurs because the public firm minimizes the transport cost. Matsumura and Matsushima (2004) introduce cost-reducing activities into the model in Cremer et al. (1991). Kitahara and Matsumura (2013) and Matsumura and Tomaru (2015) extend this model by incorporating elastic demand and shadow cost of public finding, respectively.

In the second type model, Matsushima and Matsumura (2003) investigate a mixed oligopoly by using the spatial price discrimination model with Cournot competition.[5] Heywood and Ye (2009b), Ebina et al. (2009), Ye and Wu (2015), and Heywood et al. (2021) extend Matsushima and Matsumura (2003) by incorporating, price competition, additional transportation costs, two-firm mergers, and an upstream mixed oligopoly, respectively. Matsushima and Matsumura (2006) and Heywood and Ye (2009a) analyze spatial price discrimination models of mixed markets with foreign private firms.

In our model, it is assumed that firms offer different prices based on the consumers' purchase histories. Poaching does not occur and first-best is achieved in the domestic mixed duopoly, whereas poaching induces socially inefficient switching in the domestic pure duopoly as shown in Fudenberg and Tirole (2000). The reason for this difference is that the public firm minimizes the sum of disutilities of taste mismatch.

Since Fjell and Pal (1996) and Pal and White (1998), many studies have discussed the international mixed oligopoly.[6] Fjell and Pal (1996) suggest that the public firm's price is below its marginal cost as the domestic social welfare does not include foreign firms' profits in the international mixed oligopoly. We demonstrate that BBPD allows the public firm to further reduce outflows to foreign shareholders.

[5]In spatial price discrimination models, firms offer different prices based on consumers' location.

[6]Matsumura (2003) and Matsumura and Tomaru (2012) investigate Stackelberg competition in the international mixed oligopoly. Matsumura et al. (2009), and Haraguchi and Matsumura (2014) discuss the international mixed oligopoly with differentiated products. Lin and Matsumura (2012), Cato and Matsumura (2012), Cato and Matsumura (2015), and Lee et al. (2018) examine the effect of partial privatization, developed by Matsumura (1998) on social welfare. Lyu and Shuai (2017) discuss subcontracting in the international mixed duopoly by using the Hotelling model. Lin and Matsumura (2018b) discuss the privatization neutrality theorem when domestic and foreign investors own the private firm. Matsumura and Matsushima (2012) indicate that the privatization of international airports can improve welfare. Haraguchi and Matsumura (2020a) discuss technology transfer from foreign enterprises.

3 Preliminary

We extend Fudenberg and Tirole (2000) by considering mixed duopoly markets. Two firms, A and B, compete in two periods. Let Firm A be a public firm, which maximizes domestic social welfare, and Firm B be a private firm, which maximizes its profit. The firms produce goods at a constant marginal cost, c, and simultaneously set their prices in each period.

Consumers uniformly distribute over $[\underline{\theta},\ \overline{\theta}]$ ($\underline{\theta} = -\overline{\theta},\ \underline{\theta} < 0$). The parameter θ represents how much consumers prefer goods B over goods A. Consumers' preferences are constant over time. Consumers buy one unit of product from Firm A or Firm B or do not buy at all. The per-period utilities for a consumer indexed by θ purchasing from A and B at price p are $u_A = v - \frac{\theta}{2} - p$ and $u_B = v + \frac{\theta}{2} - p$, respectively. We assume that v is sufficiently large so that all consumers purchase in equilibrium. Firms and consumers discount their future by the common factor $\delta \in [0, 1)$.

4 Domestic mixed duopoly

4.1 Uniform pricing

We consider the benchmark case where BBPD is not feasible. The firms cannot identify the first-period decisions of consumers, or BBPD is illegal. The two-period model reduces to two replications of the static equilibrium. We solve the static model.

Let θ^U denote the consumer who is indifferent between choosing Firm A and Firm B. Consumers in $[\underline{\theta},\ \theta^U]$ choose Firm A and consumers in $[\theta^U,\ \overline{\theta}]$ choose Firm B. Firm A and Firm B offer p_A and p_B to consumers, respectively. Then, θ^U satisfies $v - \frac{\theta^U}{2} - p_A = v + \frac{\theta^U}{2} - p_B$. Solving it for θ^U yields

$$\theta^U = p_B - p_A. \tag{1}$$

Per-period domestic social welfare under uniform pricing is the sum of per-period consumer surplus and per-period profits of firms and is given by

$$sw^U = (v - c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{1}{2}(\theta^U)^2. \tag{2}$$

Firm A chooses p_A to maximize (2), and Firm B chooses p_B to maximize its profit. The profit function of Firm B is

$$\pi_B = (p_B - c)(\overline{\theta} - \theta^U). \tag{3}$$

The first-order conditions are

$$p_B - p_A = 0, \tag{4}$$

$$c + \overline{\theta} + p_A - 2p_B = 0. \tag{5}$$

The first-order conditions yield $p_A^* = p_B^* = c + \overline{\theta}$ and $\theta^{U*} = 0$.

Hence, the total discounted consumer surplus, total discounted profits, and total discounted social welfare in equilibrium under uniform pricing in the domestic mixed duopoly are

$$\begin{aligned} CS^U &= (1+\delta)[(v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - 2\overline{\theta}^2], &(6)\\ \Pi_A^U &= \Pi_B^U = (1+\delta)\overline{\theta}^2, &(7)\\ SW^U &= (1+\delta)[(v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2]. &(8) \end{aligned}$$

4.2 BBPD

We now assume that BBPD is feasible. In the first period, each firm sets a uniform price. In the second, they offer different prices to consumers who have different purchase histories after they observe consumers' first-period behaviors. We derive the subgame-perfect Nash equilibrium by backward induction.

The second period

Let θ^* denote the consumer who is indifferent between choosing Firm A and Firm B in the first period. Consumers in $[\underline{\theta}, \theta^*]$ chose Firm A and consumers in $[\theta^*, \overline{\theta}]$ chose Firm B in the first period. In the second period, Firm A and Firm B offer α and β to their past consumers and offer $\hat{\alpha}$ and $\hat{\beta}$ to the rival's past consumers, respectively. Consumer θ on $[\underline{\theta}, \theta^*]$ continues to purchase from Firm A if $v - \frac{\theta}{2} - \alpha \geq v + \frac{\theta}{2} - \hat{\beta}$, otherwise, they switche to Firm B. Consumer θ on $[\theta^*, \overline{\theta}]$ continues to purchase from Firm B if $v - \frac{\theta}{2} - \hat{\alpha} \leq v + \frac{\theta}{2} - \beta$, otherwise, they switche to Firm A.

Considering the possibility of corner solutions, we obtain the locations of the indifferent consumers on $[\underline{\theta}, \theta^*]$ and $[\theta^*, \overline{\theta}]$, θ_A and θ_B, respectively.

$$\theta_A = \begin{cases} \hat{\beta} - \alpha, & \text{if } \hat{\beta} - \alpha \leq \theta^*, \\ \theta^*, & \text{if } \hat{\beta} - \alpha \geq \theta^*, \end{cases} \tag{9}$$

$$\theta_B = \begin{cases} \theta^*, & \text{if } \beta - \hat{\alpha} \leq \theta^*, \\ \beta - \hat{\alpha}, & \text{if } \beta - \hat{\alpha} \geq \theta^*. \end{cases} \tag{10}$$

Domestic social welfare in the second period is given by

$$sw_2^D = (v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 + \frac{1}{2}\theta^{*2} - \frac{1}{2}\theta_A^2 - \frac{1}{2}\theta_B^2, \tag{11}$$

where the superscript "D" stands for price discrimination. The second-period profit of firm B is given by

$$\pi_{B2}^{D} = (\beta - c)(\overline{\theta} - \theta_B) + (\hat{\beta} - c)(\theta^* - \theta_A). \tag{12}$$

Firm A chooses α and $\hat{\alpha}$ to maximize (11), and Firm B chooses β and $\hat{\beta}$ to maximize (12). By taking the parametric conditions in (9) and (10) into account, we solve the maximization problems, leading to

$$\alpha = \begin{cases} \hat{\beta} - \theta^*, & \text{if} \quad \underline{\theta} \le \theta^* \le 0, \quad (13) \\ \hat{\beta}, & \text{if} \quad 0 \le \theta^* \le \overline{\theta}, \quad (14) \end{cases}$$

$$\hat{\alpha} = \begin{cases} \beta, & \text{if} \quad \underline{\theta} \le \theta^* \le 0, \quad (15) \\ \beta - \theta^*, & \text{if} \quad 0 \le \theta^* \le \overline{\theta}, \quad (16) \end{cases}$$

$$\hat{\beta} = \begin{cases} \alpha + \theta^*, & \text{if} \quad \alpha \le c - \theta^*, \quad (17) \\ \dfrac{1}{2}\alpha + \dfrac{c + \theta^*}{2}, & \text{if} \quad \alpha \ge c - \theta^*, \quad (18) \end{cases}$$

$$\beta = \begin{cases} \dfrac{1}{2}\hat{\alpha} + \dfrac{c + \overline{\theta}}{2}, & \text{if} \quad \hat{\alpha} \le c + \overline{\theta} - 2\theta^*, \quad (19) \\ \hat{\alpha} + \theta^*, & \text{if} \quad \hat{\alpha} \ge c + \overline{\theta} - 2\theta^*. \quad (20) \end{cases}$$

These best-response functions are depicted in Figure 1.[7] The best-response function for Firm A in its turf, which is given by (13) when $\underline{\theta} \le \theta^* \le 0$ and (14) when $0 \le \theta^* \le \overline{\theta}$, is depicted in Figures 1 (a) and (b), respectively. The best-response function for Firm A in Firm B's turf, which is given by (15) when $\underline{\theta} \le \theta^* \le 0$ and (16) when $0 \le \theta^* \le \overline{\theta}$, is depicted in Figures 1 (c) and (d), respectively. It is clear that corner solutions are achieved in Figures 1 (a) and (d) and interior solutions are achieved in Figures 1 (b) and (c).

The best-response function for Firm B in Firm A's turf, which are given by (17) and (18), is depicted in Figures 1 (a) and (b). That for Firm B's turf, which are given by (19) and (20), is depicted in Figures 1 (c) and (d). Since the private firm does not set its price lower than its marginal cost, Firm B's best-response functions have horizontal parts in Figure 1.[8]

[7]Figure 1 (d) depicts equilibrium under the condition $c+\overline{\theta}-2\theta^* > 0$. The figure under the condition $c + \overline{\theta} - 2\theta^* \le 0$ is omitted since equilibrium prices under the condition $c + \overline{\theta} - 2\theta^* \le 0$ is the same as under the condition $c + \overline{\theta} - 2\theta^* > 0$.

[8]Following Fudenberg and Tirole (2000), we assume that a profit-maximizing firm does not charge prices below the marginal cost.

(Figure 1 about here)

The best-response functions of the public firm can be interpreted as follows. Firm A's best-response functions have slopes of 1 and vertical intercepts of 0 in Figures 1 (b) and (c). The public firm chooses its price equal to that of the private firm when $\theta^* \geq 0$ in the public firm's turf and when $\theta^* \leq 0$ in the private firm's turf, respectively. Firm A's best-response functions have intercepts of θ^* in Figures 1 (a) and (d). The public firm lowers its price, α, so that θ_A is equal to θ^* when $\theta^* \leq 0$ in the public firm's turf, and increases its price, $\hat{\alpha}$, so that θ_B is equal to θ^* when $\theta^* \geq 0$ in the private firm's turf given the private firm's prices. This is because the sum of disutilities of taste mismatch is minimized if the public firm supplies consumers in $[\underline{\theta}, 0]$ and the private firm supplies consumers in $[0, \overline{\theta}]$.

The best-response functions of the private firm can be interpreted as follows. Firm B's best-response function in Firm A's turf has a slope of one-half if $\alpha \geq c - \theta^*$ and a horizontal part if $\alpha \leq c - \theta^*$ as shown in Figures 1 (a) and (b). The private firm attacks the public firm's turf by lowering its price, $\hat{\beta}$, but not by lowering prices below marginal cost, c. That in the Firm B's turf has a slope of one if $\hat{\alpha} \geq c + \overline{\theta} - 2\theta^*$ and a slope of one-half and a horizontal part if $\hat{\alpha} \leq c + \overline{\theta} - 2\theta^*$ as shown in Figures 1 (c) and (d). The private firm captures the entire market if the public firm's price, $\hat{\alpha}$ is sufficiently high.

Figure 1 indicatess that there is a unique equilibrium represented by E_A in (a) and (b), and E_B in (c), respectively. However, there are multiple equilibria in (d). We assume that a social welfare-maximizing firm puts a little more value on the consumer surplus than on the producer surplus. Firm A lowers its price as much as possible and the equilibrium is then determined at E_B in Figure 1 (d).

Thus, equilibrium prices in the second period are

$$\alpha = c - \theta^*,\ \hat{\beta} = c,\ \hat{\alpha} = \beta = c + \overline{\theta}, \qquad \text{if} \quad \underline{\theta} \leq \theta^* \leq 0, \tag{21}$$

$$\alpha = \hat{\beta} = c + \theta^*,\ \hat{\alpha} = c + \overline{\theta} - 2\theta^*,\ \beta = c + \overline{\theta} - \theta^*, \qquad \text{if} \quad 0 \leq \theta^* \leq \overline{\theta}. \tag{22}$$

Since $\theta_A = \theta^*$ and $\theta_B = 0$ when $\underline{\theta} \leq \theta^* \leq 0$ and $\theta_A = 0$, $\theta_B = \theta^*$ when $0 \leq \theta^* \leq \overline{\theta}$, Firm A supplies consumers $[\underline{\theta}, 0]$ and Firm B supplies $[0, \overline{\theta}]$ for all θ^*. We have

$$sw_2^D = (v - c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2, \quad \text{for all} \quad \underline{\theta} \leq \theta^* \leq \overline{\theta}, \tag{23}$$

$$\pi_{B2}^D = \begin{cases} \overline{\theta}^2, & \text{if} \quad \underline{\theta} \leq \theta^* \leq 0, \quad (24) \\ \theta^{*2} + (\overline{\theta} - \theta^*)^2, & \text{if} \quad 0 \leq \theta^* \leq \overline{\theta}. \quad (25) \end{cases}$$

Equation (23) shows that the sum of disutilities of taste the mismatch is minimized for all θ^*.

The first period

We obtain the indifferent consumers in the first period. In the case of $\underline{\theta} \leq \theta^* \leq 0$, the consumer, θ^*, is indifferent between choosing Firm A in the first period at price a and then choosing Firm A in the second period at price α, or choosing Firm B in the first period at price b and then choosing Firm A at price $\hat{\alpha}$. Thus, θ^* satisfies

$$v - \frac{\theta^*}{2} - a + \delta(v - \frac{\theta^*}{2} - \alpha) = v + \frac{\theta^*}{2} - b + \delta(v - \frac{\theta^*}{2} - \hat{\alpha}). \tag{26}$$

Substituting the second-period equilibrium prices and solving it for θ^* yields

$$\theta^* = \frac{b - a + \delta\overline{\theta}}{1-\delta}, \quad \text{if} \quad \underline{\theta} \leq \theta^* \leq 0. \tag{27}$$

In the same way, in the case of $0 \leq \theta^* \leq \overline{\theta}$, the consumer, θ^*, is indifferent between choosing Firm A in the first period at a, and then choosing Firm B in the second period at $\hat{\beta}$, or choosing Firm B in the first period at b, and then choosing Firm B at β. Thus, θ^* satisfies

$$v - \frac{\theta^*}{2} - a + \delta(v + \frac{\theta^*}{2} - \hat{\beta}) = v + \frac{\theta^*}{2} - b + \delta(v + \frac{\theta^*}{2} - \beta). \tag{28}$$

Substituting the second-period equilibrium prices and solving it for θ^* yields

$$\theta^* = \frac{b - a + \delta\overline{\theta}}{1+2\delta}, \quad \text{if} \quad 0 \leq \theta^* \leq \overline{\theta}. \tag{29}$$

Domestic social welfare in the first period is given by

$$sw_1^D = \left[(v-c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{1}{2}\theta^{*2}\right], \tag{30}$$

and the first-period profit of Firm B is given by

$$\pi_{B1}^D = (b-c)(\overline{\theta} - \theta^*). \tag{31}$$

Firm A chooses a to maximize $SW^D = sw_1^D + \delta sw_2^D$ and firm B chooses b to maximize $\Pi_B^D = \pi_{B1}^D + \delta\pi_{B2}^D$.

The differentiation of the above objective functions yields

$$a = b + \delta\overline{\theta}, \qquad \text{for all} \quad \underline{\theta} \leq \theta^* \leq \overline{\theta}, \tag{32}$$

$$b = \begin{cases} \frac{1}{2}a + \frac{c + (1-2\delta)\overline{\theta}}{2}, & \text{if} \quad \underline{\theta} \leq \theta^* \leq 0, \quad (33) \\ \frac{1-2\delta}{2}a + \frac{(1+2\delta)c + (1+\delta+2\delta^2)\overline{\theta}}{2}, & \text{if} \quad 0 \leq \theta^* \leq \overline{\theta}. \quad (34) \end{cases}$$

Solving these best-response functions, we have the following proposition.

Proposition 1 *Equilibrium prices in the domestic mixed duopoly are given by*

$$a = \hat{\alpha} = \beta = c + \overline{\theta},\ \alpha = \hat{\beta} = c,\ b = c + (1-\delta)\overline{\theta}.$$

Thus, we have

$$p_A^* = a = \hat{\alpha} > \alpha,\ p_B^* = \beta \geq b > \hat{\beta},\ \theta^* = \theta_A = \theta_B = 0.$$

Proof. See Appendix A.

Proposition 1 reveals the public firm does not offer a discount to its rival's consumers, and both firms do not poach their rival's consumers in the domestic mixed duopoly.[9] Equations (27) and (32) indicates that it is optimal that Firm A offers its price so that θ^* is 0 in the first period. Given θ^*, Firm A offers its prices to minimize the sum of disutilities in the second period. Hence, poaching, which induces socially inefficient switching, does not occur.

Total discounted consumer surplus, total discounted profits, and total discounted domestic social welfare in equilibrium under BBPD in the domestic mixed duopoly are

$$CS^D = (1+\delta)[(v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2] - 2\overline{\theta}^2, \tag{35}$$

$$\Pi_A^D = \Pi_B^D = \overline{\theta}^2, \tag{36}$$

$$SW^D = (1+\delta)[(v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2]. \tag{37}$$

Proposition 2 *Comparing the outcome under uniform pricing,* (6)*,* (7)*, and* (8)*, we have*

$$CS^U \leq CS^D, \Pi_i^U \geq \Pi_i^D, SW^U = SW^D,\ (i = A, B).$$

Thus, BBPD does not affect domestic social welfare since poaching does not occur in the domestic mixed duopoly.

In the domestic mixed duopoly, BBPD is beneficial to consumers, detrimental to firms, and neutral to domestic social welfare. Fudenberg and Tirole (2000) consider two symmetric profit-maximizing firms and demonstrate that BBPD is beneficial to consumers and detrimental to firms and social welfare. Our result indicates that BBPD does not harm domestic social welfare, unlike Fudenberg and Tirole (2000).

Domestic social welfare in the first period equals in both cases since the first-period differential consumers are located at 0. However, the second-period domestic social welfare in Fudenberg and Tirole (2000) is smaller than our model. This is because two firms poach their rival's consumers in Fudenberg and Tirole (2000), whereas poaching does not occur, and no consumers switch in our model.

[9] Poaching occurs if the public firm engages in uniform pricing and the private firm engages in BBPD, or the former maximizes the weighted sum of social welfare and its profit.

5 International mixed duopoly

We next discuss a market where a public firm competes with a foreign private firm owned by foreign shareholders.

5.1 Uniform pricing

We consider the case where BBPD is not feasible. Since the foreign private firm's profit flows out to foreign owners, per-period domestic social welfare under uniform pricing in the international mixed duopoly is the sum of per-period consumer surplus and the per-period profit of Firm A and is given by

$$sw^{Uf} = (v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{1}{2}(\tilde{\theta}^U)^2 - (p_B^f - c)(\overline{\theta} - \tilde{\theta}^U), \tag{38}$$

where the superscript "f" represents for the international mixed duopoly and $\tilde{\theta}^U$ is given by $p_B^f - p_A^f$. Solving as in the previous section, we obtain $p_A^{f*} = c$, $p_B^{f*} = c + \frac{\overline{\theta}}{2}$, and $\tilde{\theta}^{U*} = \frac{\overline{\theta}}{2}$. Thus, we have

$$\begin{aligned} CS^{Uf} &= SW^{Uf} = (1+\delta)[(v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2] - (1+\delta)\frac{3}{8}\overline{\theta}^2, && (39)\\ \Pi_A^{Uf} &= 0, && (40)\\ \Pi_B^{Uf} &= (1+\delta)\frac{1}{4}\overline{\theta}^2. && (41) \end{aligned}$$

The public firm's price in the international mixed duopoly is equal to its marginal cost and lower than that in the domestic mixed duopoly when firms engage in uniform pricing. This is because that the public firm's lower price increases the sum of disutilities of taste mismatch but decreases the outflow to foreign shareholders. The former effect is smaller than the latter effect when $p_A^{f*} \geq c$ and vice versa.

5.2 BBPD

We now assume that BBPD is feasible.

The second period

Let θ^{*f} denote the first-period indifferent consumer in the international mixed duopoly. Firm A and Firm B offer α^f and β^f to their past consumers and offer $\hat{\alpha}^f$ and $\hat{\beta}^f$ to the rival's past consumers, respectively, in the second period. As in the previous section, we obtain the locations of the indifferent consumers on $[\underline{\theta}, \theta^{*f}]$ and $[\theta^{*f}, \overline{\theta}]$, θ_A^f and θ_B^f, respectively.

$$\theta_A^f = \begin{cases} \hat{\beta}^f - \alpha^f, & \text{if} \quad \hat{\beta}^f - \alpha^f \leq \theta^{*f}, \\ \theta^{*f}, & \text{if} \quad \hat{\beta}^f - \alpha^f \geq \theta^{*f}, \end{cases} \tag{42}$$

$$\theta_B^f = \begin{cases} \theta^{*f}, & \text{if} \quad \beta^f - \hat{\alpha}^f \leq \theta^{*f}, \\ \beta^f - \hat{\alpha}^f, & \text{if} \quad \beta^f - \hat{\alpha}^f \geq \theta^{*f}. \end{cases} \tag{43}$$

In the case of the international mixed duopoly, domestic social welfare in the second period under BBPD is given by

$$\begin{aligned} sw_2^{Df} &= (v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 + \frac{1}{2}(\theta^{*f})^2 - \frac{1}{2}(\theta_A^f)^2 - \frac{1}{2}(\theta_B^f)^2 \\ &\quad -(\beta^f - c)(\overline{\theta} - \theta_B^f) - (\hat{\beta}^f - c)(\theta^{*f} - \theta_A^f). \end{aligned} \tag{44}$$

Firm A chooses α^f and $\hat{\alpha}^f$ to maximize (44) and firm B chooses β^f and $\hat{\beta}^f$ to maximize

$$\pi_{B2}^{Df} = (\beta^f - c)(\overline{\theta} - \theta_B^f) + (\hat{\beta}^f - c)(\theta^{*f} - \theta_A^f). \tag{45}$$

By taking the parametric conditions in (42) and (43) into account, we solve the public firm's maximization problems, leading to

$$\alpha^f = \begin{cases} \hat{\beta}^f - \theta^{*f}, & \text{if } \hat{\beta}^f \geq c + \theta^{*f}, \quad (46) \\ c, & \text{if } \hat{\beta}^f \leq c + \theta^{*f}, \quad (47) \end{cases}$$

and

$$\hat{\alpha}^f = \begin{cases} c, & \text{if } \beta^f \geq c + \theta^{*f}, \quad (48) \\ \beta^f - \theta^{*f}, & \text{if } \beta^f \leq c + \theta^{*f}. \quad (49) \end{cases}$$

The private firm's best-response function is the same as in (17), (18), (19), and (20). By comparing (13)-(16) and (46)-(49), it can be seen that the public firm offers a lower price in the international mixed duopoly than the domestic mixed duopoly for a given the private firm's prices. The outflow to foreign shareholders makes the public firm more aggressive in pricing in the international mixed duopoly.

Figure 2 depicts these best-response functions. Figures 2(a) and (b) show equilibrium in the Firm A's turf. Firm A's best-response function is given by (46) and (47), and that of Firm B by (17) and (18) in Figures 2(a) and (b). Firm A supplies all consumers in its turf if $\theta^* \leq 0$ and supplies consumers on $[\underline{\theta}, \frac{\theta^*}{2}]$ if $\theta^* \geq 0$. The public firm supplies the consumers who prefer the private firm to the public firm to reduce the outflow when θ^* is larger than 0. Figures 2(c) and (d) show equilibrium in Firm B's turf. Firm A's best-response function is given by (48) and (49), and that of Firm B is given by (19) and (20) in Figures 2(c) and (d). Firm B supplies all consumers in its turf if $\theta^* \geq \frac{\overline{\theta}}{2}$ and supplies consumers on $[\frac{\overline{\theta}}{2}, \overline{\theta}]$ if $\theta^* \leq \frac{\overline{\theta}}{2}$. The public firm does not supply consumers in the private firm's turf to prevent an increase in the sum of disutilities of taste mismatch when θ^* is sufficiently large. The results are different from the second period's equilibrium in the domestic mixed duopoly in that the public firm can supply consumers who prefer the private firm to the public firm.

(Figure 2 about here)

Proceeding in the same way as in Section 4, equilibrium prices in the second period are

$$\alpha^f = c - \theta^{*f},\ \hat{\beta}^f = c,\ \hat{\alpha}^f = c,\ \beta^f = c + \frac{\overline{\theta}}{2}, \quad \text{if } \underline{\theta} \leq \theta^{*f} \leq 0, \tag{50}$$

$$\alpha^f = c,\ \hat{\beta}^f = c + \frac{\theta^{*f}}{2},\ \hat{\alpha}^f = c,\ \beta^f = c + \frac{\overline{\theta}}{2}, \quad \text{if } 0 \leq \theta^{*f} \leq \frac{\overline{\theta}}{2}, \tag{51}$$

$$\alpha^f = c,\ \hat{\beta}^f = c + \frac{\theta^{*f}}{2},\ \hat{\alpha}^f = c + \overline{\theta} - 2\theta^{*f},\ \beta^f = c + \overline{\theta} - \theta^{*f}, \quad \text{if } \frac{\overline{\theta}}{2} \leq \theta^{*f} \leq \overline{\theta}. \tag{52}$$

Therefore, we have

$$sw_2^{Df} = (v - c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{3}{8}(\theta^{*f})^2 - (\overline{\theta} - \theta^{*f})^2, \tag{53}$$

$$\pi_{B2}^{Df} = (\overline{\theta} - \theta^{*f})^2 + \frac{1}{4}(\theta^{*f})^2, \tag{54}$$

if $\frac{\overline{\theta}}{2} \leq \theta^{*f} \leq \overline{\theta}$. In this case, Firm A does not poach Firm B's old consumer since $\theta_B^f = \theta^{*f}$ but Firm B poaches that of Firm A since $\theta_A^f < \theta^{*f}$.
In Appendix B, we show that only the case of $\frac{\overline{\theta}}{2} \leq \theta^{*f} \leq \overline{\theta}$ is possible in equilibrium.

The first period

In the case of $\frac{\overline{\theta}}{2} \leq \theta^{*f} \leq \overline{\theta}$, the consumer, θ^{*f}, is indifferent between choosing Firm A in the first period at price, a^f, and then choosing Firm B in the second period at price, $\hat{\beta}^f$, or choosing Firm B in the first period at price, b^f, and then choosing Firm B at price, β^f. Thus, θ^{*f} satisfies

$$v - \frac{\theta^{*f}}{2} - a^f + \delta(v + \frac{\theta^{*f}}{2} - \hat{\beta}^f) = v + \frac{\theta^{*f}}{2} - b^f + \delta(v + \frac{\theta^{*f}}{2} - \beta^f). \tag{55}$$

Substituting the second-period equilibrium prices and solving it for θ^* yields

$$\theta^{*f} = \frac{2(b^f - a^f + \delta\overline{\theta})}{2 + 3\delta}, \quad \text{if} \quad \frac{\overline{\theta}}{2} \leq \theta^{*f} \leq \overline{\theta}. \tag{56}$$

Domestic social welfare in the first period is given by

$$sw_1^{Df} = \left[(v - c)(\overline{\theta} - \underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{1}{2}(\theta^{*f})^2 - (b^f - c)(\overline{\theta} - \theta^{*f})\right], \tag{57}$$

and the first-period profit of Firm B is given by

$$\pi_{B1}^{Df} = (b^f - c)(\overline{\theta} - \theta^{*f}). \tag{58}$$

Firm A chooses a^f to maximize $SW^{Df} = sw_1^{Df} + \delta sw_2^{Df}$ and Firm B chooses b^f to maximize $\Pi_B^{Df} = \pi_{B1}^{Df} + \delta\pi_{B2}^{Df}$.

The differentiation of the above objective functions yields

$$2(2 + 3\delta)c - (4 + \delta)\delta\overline{\theta} + 5\delta b^f - (4 + 11\delta)a^f = 0, \tag{59}$$
$$2(2 + 3\delta)c + (4 + \delta^2)\overline{\theta} + 4(1 - \delta)a^f - 2(4 + \delta)b^f = 0. \tag{60}$$

Solving (59) and (60) gives equilibrium prices in the first period and substituting these into (52), we can obtain equilibrium prices.

Proposition 3 *Equilibrium prices and the first-period indifferent consumer in the international mixed duopoly are given by*

$$a^f = c - \frac{\delta(6 - \delta)}{2(7\delta + 8)}\overline{\theta}, \quad b^f = c + \frac{5\delta^2 + 2\delta + 8}{2(7\delta + 8)}\overline{\theta}, \quad \theta^{*f} = \frac{2(3\delta + 2)}{7\delta + 8}\overline{\theta},$$

$$\alpha^f = c, \quad \hat{\beta}^f = c + \frac{3\delta + 2}{7\delta + 8}\overline{\theta}, \quad \hat{\alpha}^f = c - \frac{5\delta}{7\delta + 8}\overline{\theta}, \quad \beta^f = c + \frac{\delta + 4}{7\delta + 8}\overline{\theta}.$$

Thus, we have

$$p_A^f = \alpha^f \geq a^f \geq \hat{\alpha}^f, \;\; p_B^f \geq b^f \geq \beta^f > \hat{\beta}^f, \;\; \theta_A^f < \theta^{*f}, \;\; \theta_B^f = \theta^{*f}.$$

We can check that θ^{*f} satisfies $\frac{\overline{\theta}}{2} \leq \theta^{*f} \leq \overline{\theta}$.

Proposition 3 implies that BBPD decreases equilibrium prices of the public firm, a^f and $\hat{\alpha}^f$, to below its marginal cost. BBPD allows the public firm to determine its poaching price, $\hat{\alpha}^f$, by considering only the reduction of the outflow without considering disutilities of taste mismatch in the private firm's turf. Consumers in the private firm's turf have lower relative preferences for the public firm's goods as θ^{*f} is larger. As shown in (52), no consumers in the private firm's turf switch to the public firm's goods even if the latter offers $\hat{\alpha}^f$ lower than c when $\frac{\overline{\theta}}{2} \leq \theta^{*f} \leq \overline{\theta}$. Therefore, the public firm can reduce the private firm's profit by reducing $\hat{\alpha}^f$ without increasing taste mismatch in this situation. If $\theta^{*f} \leq \frac{\overline{\theta}}{2}$, the public firm does not make $\hat{\alpha}^f$ lower than c to prevent an increase in disutilities of taste mismatch.

This increases an incentive for the public firm to lower its first-period price. The equation (54) demonstrates that the second-period's profit of the private firm is decreasing in θ^* in the range of $\frac{\overline{\theta}}{2} \leq \theta^* \leq \frac{4}{5}\overline{\theta}$. The public firm can reduce the second period's profit for the private firm by capturing a larger market share in the first period. Therefore, the public firm's first-period price, a^f, is lower than c and is decreasing in δ.

Proposition 3 implies that only the private firm poaches its rival's consumers because $\theta_A^f = \frac{1}{2}\theta^{*f}$ and $\theta_B^f = \theta^{*f}$. The public firm supplies consumers in $[\underline{\theta}, \frac{1}{2}\theta^{*f}]$ and the private firm supplies consumers in $[\frac{1}{2}\theta^{*f}, \overline{\theta}]$ in the second period as illustrated in Figure 3.

(Figure 3 about here)

BBPD decreases the second period's sum of disutilities of taste mismatch since $\theta_A^f < \frac{\overline{\theta}}{2}$ but increases that of the first period since $\theta^{*f} > \frac{\overline{\theta}}{2}$. As a result, BBPD increases the total discounted sum of disutilities of taste mismatch, although BBPD decreases equilibrium prices in the international mixed duopoly.

Total discounted consumer surplus, total discounted profits, and total discounted domestic social welfare in equilibrium under BBPD in the international mixed duopoly are

$$CS^{Df} = (1+\delta)[(v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2] - \frac{47\delta^3 + 44\delta^2 + 36\delta + 48}{2(7\delta+8)^2}\overline{\theta}^2, \quad (61)$$

$$\Pi_A^{Df} = -\frac{\delta(6-\delta)(13\delta+12)}{2(7\delta+8)^2}\overline{\theta}^2, \quad (62)$$

$$\Pi_B^{Df} = \frac{25\delta^3 + 62\delta^2 + 56\delta + 32}{2(7\delta+8)^2}\overline{\theta}^2, \quad (63)$$

$$SW^{Df} = (1+\delta)[(v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2] - \frac{17\delta^3 + 55\delta^2 + 54\delta + 24}{(7\delta+8)^2}\overline{\theta}^2. \quad (64)$$

Proposition 4 *Comparing the outcome under uniform pricing,* (39)*,* (40)*, and* (41) *yields*

$$CS^{Uf} \leq CS^{Df},\ \Pi_i^{Uf} \geq \Pi_i^{Df},\ SW^{Uf} \leq SW^{Df},\ \ (i = A, B).$$

Thus, BBPD improves domestic social welfare, although it increases the total discounted sum of disutilities of taste mismatch in the international mixed duopoly.

BBPD is beneficial to consumers in the international mixed duopoly since it increases the sum of disutilities of taste mismatch but decreases equilibrium prices. BBPD is detrimental to the firms in the international mixed duopoly since it intensifies competition. The public firm's profit falls into the red. The private firm's profit decreases, although BBPD increases its market share. BBPD is beneficial to domestic social welfare in the international mixed duopoly. BBPD reduces the outflow to foreign shareholders, although it is detrimental to international social welfare since it increases the sum of disutilities of taste mismatch. Fudenberg and Tirole (2000) indicate that private firms offer discounts to poach their rival's consumers in the second period. The same is true for the private firm in our model. However, the public firm offers its poaching price to regulate one of the rival's prices in the second period.

6 Privatization

This section discusses the privatization of the public firm. We assume that a privatized public firm maximizes its profit. Domestic social welfare under uniform pricing in the domestic mixed duopoly is given by (8) and that in a domestic pure duopoly, where there are two profit-maximizing private firms completely owned by domestic shareholders, can be obtained as follows:

$$SW^{Up} = (1+\delta)[(v-c) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2], \tag{65}$$

where "p" represents for the pure duopoly. Domestic social welfare under BBPD in the domestic mixed duopoly is given by (37) and that in the domestic pure duopoly can be obtained as follows:

$$SW^{Dp} = (1+\delta)[(v-c) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2] - \frac{1}{9}\delta\overline{\theta}^2. \tag{66}$$

We can see that (8) is equal to (65) and (37) is larger than (66).

Proposition 5 *In the domestic mixed duopoly, privatization reduces domestic social welfare under BBPD, whereas it is neutral to it under uniform pricing.*

Domestic social welfare under uniform pricing in the international mixed duopoly is given by (39) and that in an international pure duopoly can be obtained as follows:

$$SW^{Upf} = (1+\delta)[(v-c) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2] - (1+\delta)\overline{\theta}^2. \tag{67}$$

In the international pure duopoly, a profit-maximizing firm completely owned by domestic shareholders competes with a profit-maximizing firm completely owned by foreign shareholders. Domestic social welfare under BBPD in the international mixed duopoly is given by (64) and that in the international pure duopoly is equal to (67). We can see that (39) and (64) are larger than (67) and Proposition 4 indicates that (64) is larger than (39).

Proposition 6 *In the international mixed duopoly, privatization reduces domestic social welfare under both uniform pricing and BBPD. The welfare loss from privatization is larger under BBPD than under uniform pricing.*

From Proposition 5 and 6, we can conclude that the existence of the public firm increases domestic social welfare more when firms engage in BBPD than when they engage in uniform pricing. Therefore, privatization is more undesirable under BBPD than uniform pricing.

7 Conclusion

We extend Fudenberg and Tirole (2000) by considering a mixed duopoly to investigate mixed markets in which the social welfare-maximizing public firm and the private firm engage in behavior-based price discrimination.

We find that BBPD is irrelevant from the viewpoint of domestic social welfare when the public firm competes with the domestic private firm. Since the public firm chooses its prices to minimize the sum of disutilities of taste mismatch in each turf, poaching does not occur in the second period.

BBPD is also found to improves domestic social welfare when the public firm competes with the foreign private firm. The former chooses its prices by considering minimization of the sum of disutilities of taste mismatch and the reduction of the outflow to foreign investors in the international mixed duopoly. However, it lowers its poaching price by considering only the reduction of the outflow to foreign investors in the second period when firms engage in BBPD. Although BBPD increases the total discounted sum of disutilities of taste mismatch, it reduces the outflow to foreign investors by lowering equilibrium prices.

Privatization worsens domestic social welfare in both domestic and international mixed duopoly when firms engage in BBPD. Compared to the case of uniform pricing, BBPD is more desirable from the viewpoint of domestic social welfare.

Introducing long-term contracts, in which firms promise to supply their goods at the same prices in the first and second periods, remains for future research.

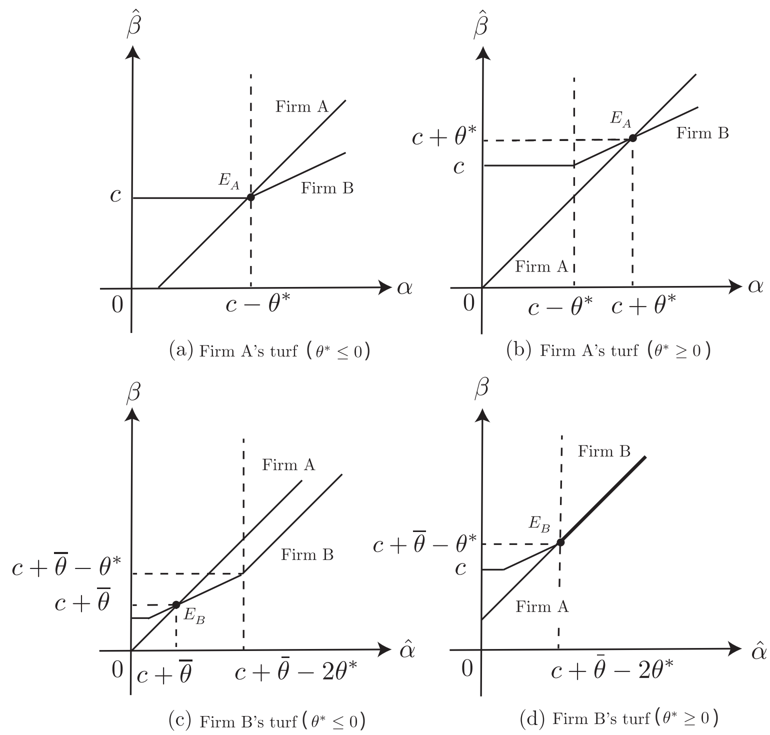
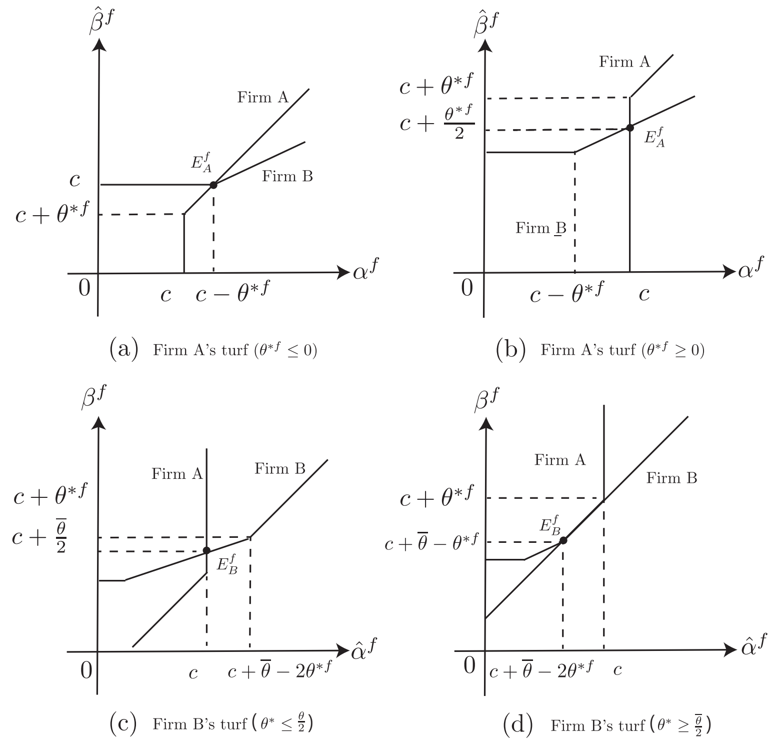


Figure 1: The second period's equilibrium in the domestic mixed duopoly



(a) Firm A's turf ($\theta^{*f} \leq 0$)

(b) Firm A's turf ($\theta^{*f} \geq 0$)

(c) Firm B's turf ($\theta^{*} \leq \frac{\overline{\theta}}{2}$)

(d) Firm B's turf ($\theta^{*} \geq \frac{\overline{\theta}}{2}$)

Figure 2: The second period's equilibrium in the international mixed duopoly

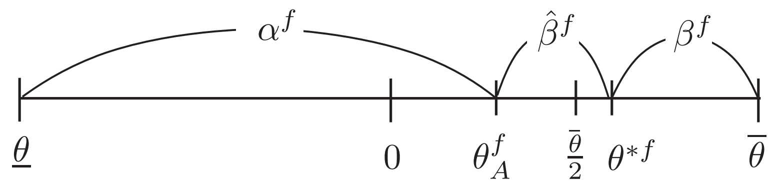


Figure 3: The second period's market in the international mixed duopoly

Appendix

Appendix A Equilibrium prices, a and b

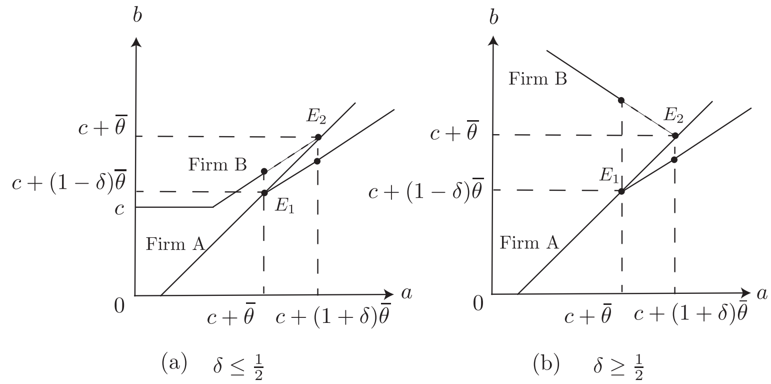


Figure 4: The first period's equilibrium in the domestic mixed duopoly

In this appendix, we obtains first-period equilibrium prices, a and b, in the domestic mixed duopoly. The best-response functions, (32), (33), and (34) can be depicted as in Figure 4. Figure 4(a) shows equilibrium when $\delta \leq \frac{1}{2}$ and Figure 4(b) illustrates it when $\delta \geq \frac{1}{2}$. Equation (32) shows that the Firm A's best-response function has a positive slope and its b-intercept is negative for all $\underline{\theta} \leq \theta^* \leq \overline{\theta}$. Equation (33) indicatess that Firm B's best-response function has a positive slope when $\underline{\theta} \leq \theta^* \leq 0$, which is located below Firm A's best-response function. Equation (34) shows that Firm B's best-response is located above Firm A's best-response function. It has a positive slope if $\delta \leq \frac{1}{2}$ as shown in Figure 4(a) and negative slope if $\delta \geq \frac{1}{2}$ as shown in Figure 4(b) when $0 \leq \theta^* \leq \overline{\theta}$.

There are two local optimal solutions, which are represented by E_1 and E_2 in each Figure 4(a) and Figure 4(b). Firm B earns the following profits, respectively.

$$\Pi_B^D = \frac{1}{4(1-\delta)}\left[a - c + (1-2\delta)\overline{\theta}\right]^2 + \delta\overline{\theta}^2, \qquad \text{if} \quad \underline{\theta} \leq \theta^* \leq 0, \tag{A1}$$

$$\Pi_B^D = \frac{1}{4}\left[a - c + (1-\delta)\overline{\theta}\right]^2 + \delta\overline{\theta}^2, \qquad \text{if} \quad 0 \leq \theta^* \leq \overline{\theta}. \tag{A2}$$

when $c + \overline{\theta} \leq a \leq c + (1+\delta)\overline{\theta}$. We find that (A1) is larger than (A2) in the range of $c + \overline{\theta} \leq a \leq c + (1+\delta)\overline{\theta}$. Hence, Firm B can earn more profit on the solid line than the dashd lines in the figures. Therefore, equilibrium is given by E_1 in Figure 4 and the first-period's equilibrium prices are given by $a = c + \overline{\theta}$ and $b = c + (1-\delta)\overline{\theta}$.

Appendix B Indifferent consumer, θ^{*f}

In the case of $\underline{\theta} \leq \theta^{*f} \leq 0$, the first-period indifferent consumer can be derived as

$$\theta^{*f} = \frac{b-a}{1-\delta}. \tag{A3}$$

The objective functions of Firm A and Firm B are

$$\begin{aligned} SW^{Df} = \big[(v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{1}{2}(\theta^{*f})^2 - (b^f-c)(\overline{\theta}-\theta^{*f})\big] \\ +\delta\big[(v-c)(\overline{\theta}-\underline{\theta}) - \frac{1}{8}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2\big], \end{aligned} \tag{A4}$$

$$\Pi_B^{Df} = (b^f-c)(\overline{\theta}-\theta^{*f}) + \delta\frac{1}{4}\overline{\theta}^2. \tag{A5}$$

First-order conditions are

$$(1-\delta)c + \delta b^f - a^f = 0, \tag{A4}$$

$$c + (1-\delta)\overline{\theta} + a^f - 2b^f = 0. \tag{A6}$$

Solving these equations yields $a^f = c + \frac{\delta(1-\delta)}{2-\delta}\overline{\theta}$ and $b^f = c + \frac{1-\delta}{2-\delta}\overline{\theta}$. Hence, we obtain $\theta^{*f} = \frac{1-\delta}{2-\delta}\overline{\theta}$, which violates $\underline{\theta} \leq \theta^{*f} \leq 0$.

In the case of $0 \leq \theta^{*f} \leq \frac{\overline{\theta}}{2}$, the first-period indifferent consumer can be derived as

$$\theta^{*f} = \frac{2(b-a)}{2-\delta}. \tag{A7}$$

The objective functions of Firm A and Firm B are

$$\begin{aligned} SW^{Df} = \big[(v-c)(\overline{\theta}-\underline{\theta}) + \frac{1}{4}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 - \frac{1}{2}(\theta^{*f})^2 - (b^f-c)(\overline{\theta}-\theta^{*f})\big] \\ +\delta\big[(v-c)(\overline{\theta}-\underline{\theta}) - \frac{1}{8}\overline{\theta}^2 + \frac{1}{4}\underline{\theta}^2 + \frac{1}{8}(\theta^{*f})^2\big], \end{aligned} \tag{A8}$$

$$\Pi_B^{Df} = (b^f-c)(\overline{\theta}-\theta^{*f}) + \delta\left(\frac{(\theta^{*f})^2}{4} + \frac{\overline{\theta}^2}{4}\right). \tag{A9}$$

First-order conditions are

$$2(2-\delta)c + \delta b^f - (4-\delta)a^f = 0, \tag{A10}$$

$$(2-\delta)^2\overline{\theta} + 2(2-\delta)c + 4(1-\delta)a^f - 2(4-3\delta)b^f = 0. \tag{A11}$$

Solving these equations yields $a^f = c + \frac{\delta(2-\delta)}{2(8-5\delta)}\overline{\theta}$ and $b^f = c + \frac{(2-\delta)(4-\delta)}{2(8-5\delta)}\overline{\theta}$. Hence, we obtain $\theta^{*f} = \frac{2(2-\delta)}{8-5\delta}\overline{\theta}$, which violates $0 \leq \theta^{*f} \leq \frac{\overline{\theta}}{2}$.

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