Discussion Paper No. 1192

ISSN (Print) 0473-453X
ISSN (Online) 2435-0982

PERSONALIZED PRICING WHEN CONSUMERS CAN PURCHASE MULTIPLE ITEMS

Qiuyu Lu
Noriaki Matsushima

October 2022

The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

Personalized pricing when consumers can purchase multiple items*

Qiuyu Lu[†]
Graduate School of Economics, Osaka University

Noriaki Matsushima[‡]
Institute of Social and Economic Research, Osaka University

October 12, 2022

Abstract

We discuss the effect of personalized pricing on profits and welfare in a Hotelling model in which consumers can simultaneously purchase from both firms. As the additional gain from the second purchase increases, personalized pricing is more likely to harm (resp., benefit) consumers (resp., firms). If the additional gain is intermediate, personalized pricing improves consumer welfare and firms' profits, contrasting with the standard result: personalized pricing benefits consumers but harms firms. When firms can choose one of the pricing policies: uniform or personalized, both choose uniform (resp., personalized) pricing under some parameters (resp., in any case); multiple equilibria can co-exist.

Keywords: Personalized pricing, Multi-unit purchase, Hotelling model

JEL Codes: L13, D43.

---

*We thank Andrew Rhodes, Shiva Shekhar, and Mark J. Tremblay for their valuable discussions and comments. We gratefully acknowledge the financial support from the Japan Society for the Promotion of Science (JSPS), KAKENHI Grant Numbers JP19H01483, JP20H05631, JP21H00702, JP21K01452, and JP21K18430. Although Matsushima serves as a member of the Competition Policy Research Center (CPRC) at the Japan Fair Trade Commission (JFTC), the views expressed in this paper are solely ours and should not be attributed to the JFTC. The usual disclaimer applies.

†Quiyu Lu, Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka, 560-0043, Japan. E-mail: ncwssdukeloo@gmail.com

‡Corresponding author: Noriaki Matsushima, Institute of Social and Economic Research, Osaka University, 6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan. E-mail: nmatsush@iser.osaka-u.ac.jp

1 Introduction

We theoretically investigate the effect of personalized pricing on profits and welfare in a Hotelling duopoly model in which consumers can simultaneously purchase from both firms, given the current competitive environment explained below. Advances in information technology, particularly the rapid adoption of smartphones, have made personalized pricing a reality (Esteves and Resende, 2016), as exemplified by route-based pricing (Uber, a taxi platform) and the "JustforU" program (Safeway, a traditional retailer).[1] In addition to the anecdotal evidence, several academic articles have detected personalized offers and search discrimination (steering customers to particular product categories) even on regular e-commerce sites (Mikians et al., 2012, Hannak et al., 2014). Related to the impact of personalized pricing on profits, Shiller (2020) simulates counterfactual situations in which Netflix hypothetically engages in personalized pricing based on Web-browsing histories, using data about Web site visits and transactions during 2006. He shows that history-based personalized pricing will increase Netflix's profits by about 13%.[2]

Our assumption regarding consumers' multi-unit purchases from firms coincides with real-world purchasing behavior. The low cost of visiting online retailers and online services helps consumers purchase items from multiple online retailers and join multiple online services, including online music stores and games (e.g., Landsman and Stremersch (2011) for game consoles and Li and Zhu (2021) for a daily deals market). Even when consumers purchase from offline retailers, more than half of consumers visit multiple offline retailers

[1] The following articles explain the details of the cases: Uber Testing New Policy: Charge What It Thinks You're Able to Pay (May 22, 2017) and Worth The Deal? Groceries Get a Personalized Price (August 20, 2012). The URLs of the articles are as follows:
http://www.thedrive.com/tech/10487/uber-testing-new-policy-charge-what-it-thinks-youre-able-to-pay
http://knkx.org/post/worth-deal-groceries-get-personalized-price

[2] Smith et al. (2022) and Shiller (2022) are further empirical investigations on targeted pricing.

and consider whether to purchase from those offline stores (Gijsbrechts et al., 2008). Below, we explain two typical examples of consumers' multi-unit purchases.

A typical example of consumers' multi-unit purchases is the market for subscription video on demand (SVOD), in which more than one-third of consumers subscribe to multiple SVOD services (Ishihara and Oki, 2021, p.15). One of the leading firms in the SVOD market, Netflix, provides personalized recommendations to customers (Kim et al., 2017), enabling it to potentially use personalized pricing based on its recommendation system as discussed in Shiller (2020). The same would apply to Amazon Prime Video because of its ability to provide personalized recommendations.[3]

Furthermore, playing multiple online games in the same genre (e.g., shooter video games) is common. For example, consider three famous shooter video game series: Call of Duty, Battlefield, and Halo. In a survey with 8,024 respondents in the US, UK, Germany, and France, almost half of the respondents have played at least two of the three (Melcher, 2021). Those game series sell some functionalities to users within the game applications. The nature of these products means that the games' producers can potentially use personalized pricing to sell personalized functionalities.

Following the recent market environments, we discuss the effect of personalized pricing on profits and welfare in a Hotelling duopoly model in which consumers can simultaneously purchase from both firms. We borrow the framework in Jeitschko et al. (2017) who investigate a Hotelling model in which consumers can purchase multiple units. The additional intrinsic utility from the second product is smaller than the intrinsic utility from the first product. Consumers observe prices proposed by firms and choose one of the options: purchasing from (i) one of the firms or (ii) both firms. We compare the results when firms

[3] Zhou and Zou (2022) theoretically investigate competitive personalized recommendations in online markets.

use uniform prices and when they use personalized prices.

We have the following results by analyzing the duopoly model. The consumer surplus under personalized pricing is higher than under uniform pricing if the additional intrinsic utility from the second product is not large. Firms benefit from personalized pricing if the additional intrinsic utility from the second product is large. The consumer surplus and firms' profits under personalized pricing are higher than under uniform pricing if the additional intrinsic utility from the second product is intermediate. In this case, personalized pricing expands the market demand because of the standard mechanism of first-degree price discrimination. The latter two results sharply contrast with those under the single-unit purchase assumption in the personalized pricing literature on Hotelling models: personalized pricing benefits consumers but harms firms.

We extend the model by endogenizing firms' pricing policies. At the beginning of the game, each firm chooses one of the pricing policies: uniform and personalized pricing. After the decisions, they compete in price. As a result, we obtain two types of equilibrium outcomes. First, both firms choose personalized pricing in any case; Second, both firms choose uniform pricing *only if* the following two hold: all consumers purchase from only one of the firms in the case where the firms employ uniform pricing; some consumers purchase from both firms in the case where one of the firms employs personalized pricing. The second type of equilibrium outcome is uncommon and novel in the context of personalized pricing, although several papers show that only one of the firms employs personalized pricing in *asymmetric* duopoly models (Ghose and Huang, 2009, Matsumura and Matsushima, 2015).

Our results contrast with the insights suggested by the earlier works. Some previous works show that the feasibility of personalized pricing intensifies competition, leading to a worse outcome for competing firms (e.g., Thisse and Vives, 1988, Choe et al., 2018).[4]

[4] The negative effect of personalized pricing on profitability appears even in monopoly models (e.g.,

Contrasting to the earlier finding, we show that the feasibility of personalized pricing does not always lead to worse outcomes when consumers are more likely to purchase multiple items. Our result implies that when we discuss the impact of personalized pricing on profits and welfare, we need to consider the propensity of consumers' multistore shopping (Gijsbrechts et al., 2008, Bell et al., 2011, Landsman and Stremersch, 2011).

Our paper contributes to two strands of literature on (i) multi-unit purchases in single-side product differentiation models and (ii) the effect of personalized pricing on profits and welfare.

In the first strand of literature, several papers discuss multi-unit purchases in standard vertical differentiation models (Mussa and Rosen, 1978, Shaked and Sutton, 1982) and horizontal differentiation models (d'Aspremont et al., 1979) since the early 21st century.[5] Gabszewicz et al. (2001) and Gabszewicz and Wauthy (2003) consider vertical differentiation models in which consumers can purchase multiple units and characterize price equilibria. Guo (2006) and Kim and Serfes (2006) independently investigate firms' location choices in Hotelling duopoly models in which consumers can purchase multiple units and show agglomeration of firms. Anderson et al. (2017) embed multiple product functionalities to multi-purchasing models based on the Hotelling duopoly framework. Those papers do not discuss personalized pricing.

Recently, Jeitschko et al. (2017) investigate a Hotelling model in which consumers can purchase multiple units and derive the condition that consumers purchase a single item or multiple items (Proposition 1 and Figure 3).[6] They also discuss discount offers to

Hajihashemi et al., 2022).

[5] de Palma et al. (1999) consider multi-unit purchases in a Cournot model with network externality.

[6] Jeitschko and Tremblay (2020) provide a general discussion on consumers' multi-unit purchases. Several papers extend Jeitschko et al. (2017) to discuss a monopolistic content provider's optimal licensing to downstream firms (e.g., Jiang et al., 2019, Ishihara and Oki, 2021). Wu and Chiu (2022) discuss content

consumers who purchase from both firms, although they do not discuss personalized pricing. Therefore, we complement their discussion by investigating the effect of personalized pricing on profits and welfare.

In the second strand of literature, following the growth in personalized pricing, researchers in the personalized pricing literature investigate the impact of personalized pricing on profitability and welfare by using the standard Hotelling model in which all consumers purchase from only one of the firms in equilibrium, the so-called full coverage assumption (e.g., Thisse and Vives, 1988, Shaffer and Zhang, 2002, Choe et al., 2018).[7] They show that personalized pricing tends to increase competition and improve consumer welfare.[8] Those papers also assume that consumers do not choose both firms' products. We relax this assumption and allow consumers to purchase from both firms.

In the literature of personalized pricing, several papers show that personalized pricing can be a profitable pricing strategy in contrast to the standard impact of personalized pricing on profits and welfare. Liu and Serfes (2013) consider two-sided markets to investigate the profitability of personalized pricing.[9] They show that personalized pricing is profitable but harms consumers if they can purchase from both firms.[10]

creation by a downstream firm. Carroni et al. (2020) also discuss a related issue in the context of two-sided markets.

[7] Many papers also discuss the impacts of personalized pricing on monopolists and consumers (e.g., Acquisti and Varian, 2005, Xu and Dukes, 2021, Hajihashemi et al., 2022).

[8] We refer to papers that show contrasting results later.

[9] Kodera (2015) considers discriminatory pricing for only one of the sides (advertisers' side) using the model in Liu and Serfes (2013). He shows that discriminatory pricing benefits the competing platforms if consumers' advertising aversion is strong. Furthermore, if discriminatory pricing harms the competing platforms, discriminatory pricing always increases advertisers' total profits. Adachi and Tremblay (2020) incorporate bilateral negotiations between a platform and firms in a two-sided market. They show that such personalized negotiations are not always exploitative if the bargaining power of the platform over firms is strong. Shekhar (2022) extends Liu and Serfes (2013) and shows the conditions under which firms choose personalized pricing ("exclusive deal" in his paper) and/or uniform pricing.

[10] We can calculate the consumer surplus on each side using the result in Liu and Serfes (2013), and

Chen et al. (2020) consider a static Hotelling duopoly model in which each firm has information about the location of consumers on the range from the firm's location to a particular point.[11] They assume, in contrast to the standard assumption in the literature on personalized pricing (e.g., Choe et al., 2018), that consumers can actively avoid personalized prices if those are higher than uniform prices. They show the possibility that personalized pricing can be an exploitative device. The key point in their result is the asymmetric distribution of customer information.

Jullien et al. (2022) investigate the optimal distribution strategy of a monopolistic manufacturer that initially distributes its product through an independent retailer. When the manufacturer opens its direct channel, the independent retailer and the direct channel compete in the downstream market. Jullien et al. (2022) show that personalized pricing can be an exploitative device if the manufacturer designs a proper wholesale tariff. The interaction between vertical contracts and personalized pricing is the key element of this result.

Laussel and Resende (2022) extend the two-period model in Choe et al. (2018) to investigate the interaction between product customization and personalized pricing based on purchase histories in the first period.[12] They show that product-price personalization

we mention that personalized pricing worsens consumer surplus in the main text of our paper.

[11] Esteves (2022) and Matsushima et al. (2022) discuss the conditions under which personalized pricing is more profitable for firms than uniform pricing in static Hotelling models by incorporating consumer heterogeneity (purchasing quantities, Esteves (2022); mismatch costs, Matsushima et al. (2022)). Furthermore, several studies show that personalized pricing does not necessarily result in a prisoner's dilemma in the case of firm asymmetry (e.g., quality difference (Shaffer and Zhang, 2002), quality choice (Choudhary et al., 2005, Ghose and Huang, 2009), and initial cost difference with R&D (Matsumura and Matsushima, 2015)).

[12] Chen et al. (2022) consider a two-market model in which one market deals with electric devices to gather consumer data and the other deals with data-applicable services (e.g., health care). A pair of firms in the former and the latter markets merge and use customer data gathered in the device market. They show the condition that the merger leads to the monopolization of the two markets.

can be a profitable pricing strategy, contrasting with the finding in Choe et al. (2018).[13]

Rhodes and Zhou (2022) discuss generalized oligopoly models based on Perloff and Salop (1985) to investigate the effects of personalized pricing on profits and welfare.[14] They point out that the degree of market coverage and the number of firms are key factors in the effects. They show that consumers are more likely to benefit from personalized pricing as the degree of market coverage increases, and the converse holds for firms (Figure 2 in Rhodes and Zhou (2022)). Moreover, they show that consumers and firms benefit from personalized pricing when the degree of market coverage is intermediate.

The relationship between the degree of market coverage and the gains of consumers and firms from personalized pricing in Rhodes and Zhou (2022) differs from ours in that we show that consumers are more likely to benefit from personalized pricing as the degree of market coverage becomes lower (each consumer purchases from *only one of the two firms*), and the converse holds for firms.[15] The difference comes from the number of items each consumer can purchase (unit demand in Rhodes and Zhou (2022) and up to two items in our paper). Therefore, we complement their discussion by considering the standard Hotelling framework with multi-unit purchases.

2 Model

We use the model in Jeitschko et al. (2017). Consumers are on the line segment of length one, $[0, 1]$ (Hotelling line). The mass of consumers is 1, and the distribution of consumers

[13] Choe et al. (2022) also extend the two-period model in Choe et al. (2018) to investigate firms' incentive to precommit to sharing customer information gathered at the end of the first period. They show that at the beginning of the game, firms agree to share customer information to mitigate competition in the first period.

[14] Zhou (2021) also use Perloff and Salop (1985) to investigate mixed bundling.

[15] As in the standard Hotelling model, we focus on situations in which each consumer purchases at least one item. In this sense, the market is fully covered in our model.

is uniform along the Hotelling line. Two firms (firms 1 and 2) are at the edges of the Hotelling line, 0 and 1, respectively. The utility from purchasing firm 1's product, firm 2's product, or both products is:

$$\begin{cases} w_1 - tx - p_1, & \text{purchasing from only firm 1,} \\ w_2 - t(1-x) - p_2, & \text{purchasing from only firm 2,} \\ w_1 + w_2 - V - t - p_1 - p_2, & \text{purchasing from firms 1 and 2,} \end{cases} \tag{1}$$

where w_i is the intrinsic utility of firm i's product, t is the per-length transportation cost, $x(\in [0,1])$ is the location of consumers at x on the Hotelling line, p_i is firm i's price, and V is the cross-effects of joint consumption. For simplicity, we assume that $w_i = w \geq 3t/2$ to ensure that each consumer purchases at least one of the products, $V \in (0, \min\{w_1, w_2\})$, and $v \equiv w - V$.

We consider two cases: (i) firms use personalized pricing, and (ii) they use uniform pricing. In the former case, the firms recognize the locations of all consumers and can offer personalized prices to them. That is, prices become a function of x, $p_i(x)$. In the latter case, the firms offer uniform prices to all consumers. We consider one-shot games in the two cases.

3 Results

We consider two cases: (i) firms use personalized pricing, and (ii) they use uniform pricing. Then, we compare the outcomes.

3.1 Personalized pricing

Given that consumers at x purchase from firm j at a positive personalized price, they also buy from firm i if and only if

$$2w - V - t - p_i(x) - p_j(x) \geq w - td_j(x) - p_j(x)$$

$$\Rightarrow \quad p_i(x) \leq w - V - t(1 - d_j(x)) = v - t(1 - d_j(x)),$$

where $d_j(x)$ is the distance between firm j and the consumer at x. If firm j cannot attract consumers at x at a nonnegative personalized price, it does not supply to them and sets $p_j(x) = 0$. Then, firm i's personalized price is acceptable for consumers at x if and only if

$$w - td_i(x) - p_i(x) \geq w - td_j(x) - 0 \quad \Rightarrow \quad p_i(x) \leq t(d_j(x) - d_i(x)).$$

We obtain the following lemma:

Lemma 1. *The schedules of personalized prices depend on v and t. Concretely,*

1. *both firms can offer personalized prices that induce all consumers to purchase from both firms if and only if $v > t$;*

2. *firms 1 and 2 offer positive personalized prices for consumers on $[0, v/t)$ and $(1 - v/t, 1]$, respectively, if and only if $t/2 < v \leq t$. In this case, consumers on $(1 - v/t, v/t)$ purchase from both firms;*

3. *no firm can offer personalized prices that induce consumers to buy from both firms if and only if $v \leq t/2$.*

The personalized prices of firms 1 and 2 are:

$$p_1(x) = \begin{cases} v - tx & \text{if } t < v, \\ \begin{array}{ll} \max\{t(1-2x), 0\} & \text{for } x \in [0, 1 - v/t], \\ \max\{v - tx, 0\} & \text{for } x \in [1 - v/t, 1], \end{array} & \text{if } t/2 < v \leq t, \\ \max\{t(1-2x), 0\} & \text{if } v \leq t/2, \end{cases}$$

$$p_2(x) = \begin{cases} v - t(1-x) & \text{if } t < v, \\ \begin{array}{ll} \max\{t(2x-1), 0\} & \text{for } x \in [v/t, 1], \\ \max\{v - t(1-x), 0\} & \text{for } x \in [0, v/t], \end{array} & \text{if } t/2 < v \leq t, \\ \max\{t(2x-1), 0\} & \text{if } v \leq t/2. \end{cases}$$

When $t < v$, all consumers purchase from both firms under $p_1(x) = v - tx$ and $p_2(x) = v - t(1-x)$. The total payments of the consumer at x are:

$$p_1(x) + p_2(x) = 2v - t.$$

The net utility of each consumer is

$$2w - V - t - (2v - t) = 2w - 2v - V = V (> 0).$$

We summarize the outcome as a lemma:

Lemma 2. *Suppose that $v > t$. When firms use personalized pricing, firm i completely extracts the additional gross consumer surplus, $p_i(x) = v - td_i(x)$, from each consumer at x. Each consumer obtains the remaining consumer surplus, V.*

That is, as the cross-effects of joint consumption become larger, the gains of consumers from personalized pricing are larger.

We derive the consumer surplus. When $t/2 < v \leq t$, consumers on $[0, 1 - v/t]$ purchase from firm 1 under $p_1(x) = t(1-2x)$, consumers on $(1 - v/t, v/t)$ purchase from both firms under $p_1(x) = v - tx$ and $p_2(x) = v - t(1-x)$, and consumers on $[v/t, 1]$ purchase from firm 2 under $p_2(x) = t(2x-1)$. Consumer surplus when $t/2 < v \leq t$ is

$$(w - v)(2v/t - 1) + \int_0^{1-v/t} (w - tx - t(1-2x))dx + \int_{v/t}^1 (w - t(1-x) - t(2x-1))dx.$$

When $v \leq t/2$, consumers on $[0, 1/2]$ purchase from firm 1 under $p_1(x) = t(1-2x)$ and consumers on $(1/2, 1]$ purchase from firm 2 under $p_2(x) = t(2x-1)$. Consumer surplus when $v \leq t/2$ is

$$\int_0^{1/2} (w - tx - t(1-2x))dx + \int_{1/2}^1 (w - t(1-x) - t(2x-1))dx.$$

In sum, consumer surplus is

$$CS^P = \begin{cases} V & \text{if } t < v, \\ w + v - t - \dfrac{v^2}{t} & \text{if } t/2 < v \leq t, \\ w - \dfrac{3t}{4} & \text{if } v \leq t/2. \end{cases}$$

We derive the profit of each firm. When $t < v$, the profit of each firm is

$$\pi_1^P = \pi_2^P = \int_0^1 (v - tx)dx = v - \frac{t}{2}.$$

When $t/2 < v \leq t$, the profit of each firm is

$$\pi_1^P = \pi_2^P = \int_0^{1-v/t} t(1-2x)dx + \int_{1-v/t}^{v/t} (v - tx)dx = \frac{t}{2} - \frac{v(t-v)}{t}.$$

When $v \leq t/2$, the profit of each firm is

$$\pi_1^P = \pi_2^P = \int_0^{1/2} t(1-2x)dx = \frac{t}{4}.$$

In sum, the profit of each firm is

$$\pi_1^P = \pi_2^P = \begin{cases} v - \dfrac{t}{2} & \text{if } t < v, \\ \dfrac{t}{2} - \dfrac{v(t-v)}{t} & \text{if } t/2 < v \leq t, \\ \dfrac{t}{4} & \text{if } v \leq t/2. \end{cases}$$

Total surplus is

$$TS^P = CS^P + \pi_1^P + \pi_2^P = \begin{cases} w + v - t & \text{if } t < v, \\ w - v + \dfrac{v^2}{t} & \text{if } t/2 < v \leq t, \\ w - \dfrac{t}{4} & \text{if } v \leq t/2. \end{cases}$$

3.2 Uniform pricing and comparisons

Using Proposition 1 in Jeitschko et al. (2017), we describe the results under uniform pricing. After that, we compare the results with those under personalized pricing.

All consumers purchase from only one of the firms (S: single unit) When all consumers purchase from one of the firms, the equilibrium prices, profit of each firm, and resulting consumer and total surpluses are

$$p_i^{US} = t, \quad \pi_i^{US} = \frac{t}{2}, \quad CS^{US} = w - \frac{5t}{4}, \quad TS^{US} = w - \frac{t}{4}.$$

The outcome is effective if and only if $v \leq \sqrt{2}t \simeq 1.414t$.

We compare the outcomes in the cases of personalized pricing and uniform pricing. The differences between the values under personalized pricing and those under uniform pricing are as follows:

$$\Delta CS^S = \begin{cases} \dfrac{5t}{4} - v & \text{if } t < v, \\ \dfrac{t}{4} + \dfrac{v(t-v)}{t} & \text{if } t/2 < v \leq t, \\ \dfrac{t}{2} & \text{if } v \leq t/2, \end{cases} \quad \Delta \pi_i^S = \begin{cases} v - t & \text{if } t < v, \\ -\dfrac{v(t-v)}{t} & \text{if } t/2 < v \leq t, \\ -\dfrac{t}{4} & \text{if } v \leq t/2, \end{cases}$$

$$\Delta TS^S = \begin{cases} v - \dfrac{3t}{4} & \text{if } t < v, \\ \dfrac{(t-2v)^2}{4t} & \text{if } t/2 < v \leq t, \\ 0 & \text{if } v \leq t/2. \end{cases}$$

We summarize the comparison as Proposition 1.

Proposition 1. *Suppose that all consumers purchase from only one of the firms under uniform pricing, that is, $v \leq \sqrt{2}t \simeq 1.414t$. Compared with uniform pricing, personalized pricing increases the*

1. *profit of each firm if and only if $v > t$;*
2. *consumer surplus if and only if $v < 5t/4$;*
3. *total surplus if and only if $v > t/2$.*

In particular, firm profits and consumer surplus improve if and only if $t < v < 5t/4$.

We explain the reason that personalized pricing can improve profits and consumer surplus. Personalized pricing allows firms to expand their quantities supplied to consumers as in standard monopolistic personalized pricing. The gains of firms from personalized pricing are larger as the value of v increases (see Lemma 2). Furthermore, even when all consumers purchase from both firms ($v > t$) and firms fully extract the *additional* gross consumer surplus from the second product, consumers obtain the residual surplus V (see Lemma 2). Therefore, when v satisfies $t < v < 5t/4$, that is, when $t < w - V < 5t/4$, personalized pricing improves firms' profits and consumer surplus. Note that V must be larger than $t/4$ to obtain the result because we assume that $w \geq 3t/2$. Note also that the net surpluses of consumers around the edges decrease from $w - p_i^{US} - td_i(x) \simeq w - t$ to V if $d_i(x)$ is sufficiently small ($t < w - V = v$ holds).

At least some consumers purchase from both firms (M: multiple units) When at least some consumers purchase from both firms, the equilibrium prices, profit of each firm, and resulting consumer and total surpluses are:

$$p_i^{UM} = \begin{cases} v - t & \text{if } 2t \leq v, \\ \dfrac{v}{2} & \text{if } v < 2t, \end{cases} \quad \pi_i^{UM} = \begin{cases} v - t & \text{if } 2t \leq v, \\ \dfrac{v^2}{4t} & \text{if } v < 2t, \end{cases}$$

$$CS^{UM} = \begin{cases} w - v + t & \text{if } 2t \leq v, \\ w + \dfrac{v(v-4t)}{4t} & \text{if } v < 2t, \end{cases} \quad TS^{UM} = \begin{cases} w + v - t & \text{if } 2t \leq v, \\ w + \dfrac{v(3v-4t)}{4t} & \text{if } v < 2t, \end{cases}$$

The outcome is effective if and only if $v > 2(2\sqrt{2}+1)t/7 \simeq 1.094t$.

We compare the outcomes in the cases of personalized pricing and uniform pricing when some of the consumers purchase from both firms. The differences between the values under personalized pricing and those under uniform pricing are as follows:

$$\Delta CS^M = \begin{cases} -t & \text{if } 2t \leq v, \\ -\dfrac{v^2}{4t} & \text{if } 1.094t \leq v < 2t, \end{cases} \quad \Delta \pi_i^M = \begin{cases} \dfrac{t}{2} & \text{if } 2t \leq v, \\ \dfrac{t}{2} - \dfrac{(2t-v)^2}{4t} & \text{if } 1.094t \leq v < 2t, \end{cases}$$

$$\Delta TS^M = \begin{cases} 0 & \text{if } 2t \leq v, \\ \dfrac{(2t-v)(3v-2t)}{4t} & \text{if } 1.094t \leq v < 2t. \end{cases}$$

We summarize the comparison.

Proposition 2. *Suppose that some consumers purchase from both firms under uniform pricing, that is,* $v \geq 2(2\sqrt{2}+1)t/7 \simeq 1.094t$*. Compared with uniform pricing, personalized pricing increases the*

1. *profit of each firm for any* v*;*

2. *total surplus if and only if* $v < 2t$*.*

However, personalized pricing worsens consumer surplus for any v *and is irrelevant to total surplus for* $v \geq 2t$*.*

The equilibrium uniform price in case M, p_i^{UM}, is smaller than that in case S, p_i^{US} if $v < 2t$ because firms need to induce some consumers around the center of the Hotelling line to consume both products by lowering their uniform prices. Actually, the total payment is $2p_i^{UM} = v$. Personalized pricing eliminates the downward pressure on uniform prices and increases prices. The total payment under personalized pricing is $2v - t$, which is larger than v if $v \geq 1.094t$ (see Lemma 2). The difference in the starting points under the cases M and S means that the impacts of personalized pricing on consumers in the two cases are quite different.

We compare our results with those in Rhodes and Zhou (2022). We show that consumers are more likely to benefit from personalized pricing as the degree of market coverage becomes lower (each consumer purchases from *only one of the two firms*), and the converse holds for firms. In a search theoretic model, Rhodes and Zhou (2022) show that consumers are more likely to benefit from personalized pricing as the degree of market coverage increases, and the converse holds for firms (Figure 2 in Rhodes and Zhou (2022)). Moreover, they also show that consumers and firms benefit from personalized pricing when the degree of market coverage is intermediate. The relationship between the degree of market coverage and the gains of consumers and firms from personalized pricing in Rhodes and Zhou (2022) differs from ours. The difference comes from the number of items each consumer can purchase (unit demand in Rhodes and Zhou (2022) and up to two items in our paper). Therefore, our results complement their findings by considering the standard Hotelling framework with multi-unit purchases.

3.3 Endogenous choices of pricing policies

We allow firms to choose one of the pricing policies endogenously: uniform and personalized pricing. We can discuss the endogenous choices by considering the asymmetric case in which one firm employs personalized pricing and the other employs uniform pricing. To discuss the asymmetric case, we assume that the latter firm (call it firm 2) sets its uniform price, and then observing the price, the former firm (call it firm 1) sets its personalized prices for consumers. The timing structure follows those in the related papers (e.g., Thisse and Vives, 1988, Shaffer and Zhang, 2002, Choe et al., 2018). The detail of the mathematical procedure is available in Appendix A.

We can classify the outcome of the asymmetric case into the following four: (i) all consumers purchase from both firms (if $v \geq 2t$); (ii) firm 1 serves all consumers but firm 2 serves the part of consumers (if $t \le v < 2t$); (iii) consumers around the center purchase from both firms but those around the edges purchase from the closest firm (if $\sqrt{2}t/2 < v < t$); (iv) all consumers purchase from only one of the firms, which is the same as the asymmetric case in Thisse and Vives (1988) (if $v \le \sqrt{2}t/2$). The following is the outcomes in the four cases:

1. When $v \ge 2t$, the prices of firms 1 and 2 are $p_1^*(x) = v - tx$ and $p_2^* = v - t$. The resulting demands for firms 1 and 2 are $N_1^* = N_2^* = 1$. The profits are $\pi_1^* = v - t/2$ and $\pi_2^* = v - t$. The adaptation of personalized pricing increases the profit of firm 1 from $v - t$ to $v - t/2$.

2. When $t \le v < 2t$, the prices of firms 1 and 2 are

$$p_1^* = \begin{cases} t(1-2x) + v/2 & \text{for } x \le 1 - v/(2t), \\ v - tx & \text{for } x \ge 1 - v/(2t), \end{cases} \quad \text{and} \quad p_2^* = v/2.$$

   The resulting demand for firms 1 and 2 are $N_1^* = 1$ and $N_2^* = v/(2t)$. The profits are $\pi_1^* = v(4t+v)/(8t)$ and $\pi_2^* = v^2/(4t)$. Firm 2's uniform price is higher than or equal to $t/2$, which is the uniform price under the asymmetric case in Thisse and Vives (1988). The higher uniform price of firm 2 implies that employing personalized pricing does not intensify competition, inducing firm 1 to choose personalized pricing.

3. When $\sqrt{2}t/2 \le v < t$, the prices of firms 1 and 2 are

$$p_1^* = \begin{cases} t(1-2x) + v/2 & \text{for } x < 1 - v/(2t), \\ v - tx & \text{for } 1 - v/(2t) \le x < v/t, \\ 0 & \text{for } x \ge v/t, \end{cases} \quad \text{and} \quad p_2^* = v/2.$$

   The resulting demands for firms 1 and 2 are $N_1^* = v/t$ and $N_2^* = v/(2t)$. The profits are $\pi_1^* = (4t^2 - 4tv + 5v^2)/(8t)$ and $\pi_2^* = v^2/(4t)$. Firm 2's uniform price is *lower* than $t/2$, contrasting with the previous case ($t \le v < 2t$). The lower uniform price of firm

2 implies that employing personalized pricing intensifies competition, diminishing firm 1's incentive to choose personalized pricing if v is small. In fact, the resulting profit of firm 1, $\pi_1^* = (4t^2 - 4tv + 5v^2)/(8t)$, is lower than that in case S in Section 3.2, $t/2$, if and only if $\sqrt{2}/2 \le v < 4t/5$ (note that the result in case S is the unique equilibrium outcome if $v < t$).

4. When $v \le \sqrt{2}t/2$, the prices of firms 1 and 2 are

$$p_1^* = \begin{cases} t(1-2x) + t/2 & \text{for } x < 3/4, \\ 0 & \text{for } x \ge 3/4, \end{cases} \quad \text{and} \quad p_2^* = \frac{t}{2}.$$

The resulting demand for firms 1 and 2 are $N_1^* = 3/4$ and $N_2^* = 1/4$. The profits are $\pi_1^* = 9t/16$ and $\pi_2^* = t/8$. As in Thisse and Vives (1988), employing personalized pricing increases the profit of firm 1.

Using the outcomes in the three cases in Sections 3.1, 3.2, and 3.3, we can derive the following proposition.

Proposition 3. *Two types of pricing policy pairs can appear in equilibrium:*

1. *The following is always sustainable in equilibrium: both firms choose personalized pricing;*

2. *The following is also sustainable in equilibrium if and only if $\sqrt{2}t/2 \le v \le 4t/5$: both firms choose uniform pricing.*

That is, multiple equilibria co-exist if and only if $\sqrt{2}t/2 \le v \le 4t/5$.

In the context of personalized pricing, the second result is uncommon and novel. The key point of having the outcome is consumers' multi-unit purchases. Under the assumption of multi-unit purchases, each firm has an incentive to acquire consumers' second purchases if feasible. Because the market for consumers' second purchases is monopolistic, the demand for consumers' second purchases is more price elastic than when all consumers purchase from only one of the firms, which is a duopolistic case (see, Chen and Riordan, 2008, Cowan and Yin, 2008). Therefore, the uniform price of firm 2, $v/2$, is lower than that in Thisse and Vives (1988), $t/2$, in case 3 mentioned above ($v < t$).

We summarize the results and mention the implication. We show that personalized pricing is more likely to benefit firms if the additional gain from the second purchase, v, is larger than the threshold value, t (Propositions 1 and 2). We also show that firms can escape fierce price competition caused by personalized pricing if the additional gain from the second purchase is in the range where $\sqrt{2}t/2 \leq v \leq 4t/5$ because adopting personalized pricing induces its rival to set a sufficiently low uniform price (Proposition 3). Those results stem from incorporating the possibility of consumers' multi-unit purchases into the standard spatial competition model. Our result implies that when we discuss the impact of personalized pricing on profits and welfare, we need to consider the propensity of consumers' multistore shopping (Gijsbrechts et al., 2008, Bell et al., 2011, Landsman and Stremersch, 2011).

4 Heterogeneous firms

We relax the assumption on w_i and allow heterogeneous w_i ($v_i = w_i - V$). We assume that $w_1 + w_2 \geq 3t$ to ensure that all consumers purchase from at least one of the firms under uniform pricing. By a similar calculation procedure, we obtain the equilibrium prices and the players' benefits under personalized pricing and uniform pricing. As in the main model, when at least some consumers purchase from both firms, personalized pricing increases the profit of each firm but decreases the consumer surplus for any v_i, $i = 1, 2$. Combining these opposite effects, the total surplus improves if and only if $\exists i \in 1, 2,\ v_i < 2t$ (the detail is available in Appendix B).

Proposition 4. *Suppose that at least some consumers purchase from both firms under uniform pricing; equivalently, suppose that:*

$$\begin{cases} v_i > (1+\sqrt{2})(t - \frac{v_j}{2}) & \text{if } v_j > (6-4\sqrt{2})t, \\ v_i > 3t + \frac{1}{2}v_j - 2\sqrt{tv_j} & \text{if } v_j \leq (6-4\sqrt{2})t. \end{cases} \tag{2}$$

Compared with uniform pricing, personalized pricing increases the

1. *profit of each firm for any v_i;*

2. *total surplus if and only if $\exists i \in 1, 2,\ v_i < 2t$.*

However, personalized pricing worsens consumer surplus for any v_i and is irrelevant to total surplus when $\forall i \in 1, 2,\ v_i \geq 2t$.

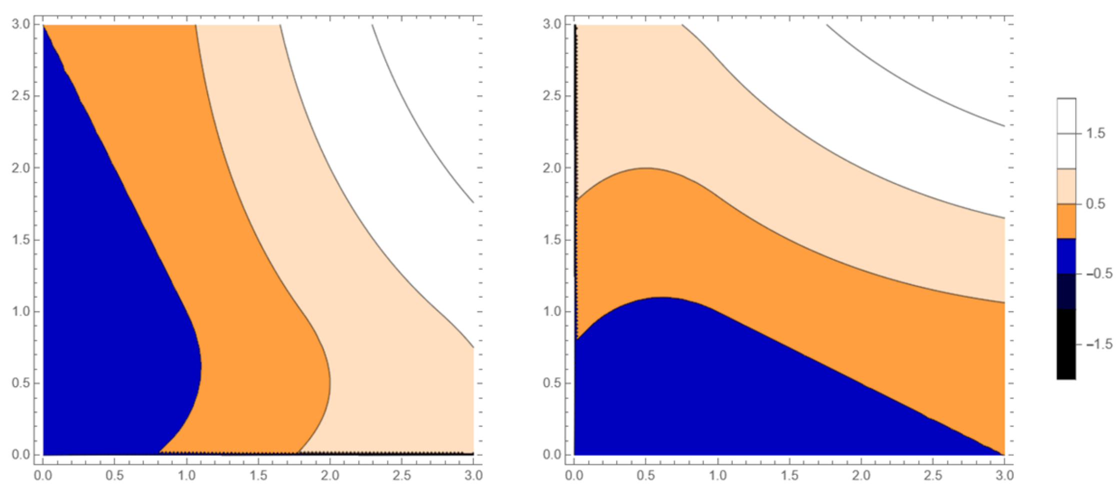

Figure 1: Difference in firm i's profits under personalized and uniform pricing ($\Delta\pi_i$)
Note: Lefthand-side: $\Delta\pi_1$; Righthand-side: $\Delta\pi_2$; Horizontal axis: v_1; Vertical axis: v_2.

When at least one of v_1 and v_2 is large (say, v_1 is large), some consumers purchase from both firms even under uniform pricing (see equation (2)). In this situation, if the pricing regime changes to personalized pricing, firm 2 can offer personalized prices in the monopoly market in which consumers, who purchase from firm 1, consider additional purchases from firm 2. This pricing by firm 2 means that the interaction between the firms ceases, and the firms behave as if they are monopolists. Moreover, the elimination of the interaction vanishes firm 2's disadvantage over firm 1 when v_1 is larger than v_2. Because monopolistic personalized pricing expands the market supply, the regime of personalized pricing improves total surplus and profits. The converse holds for consumer surplus.

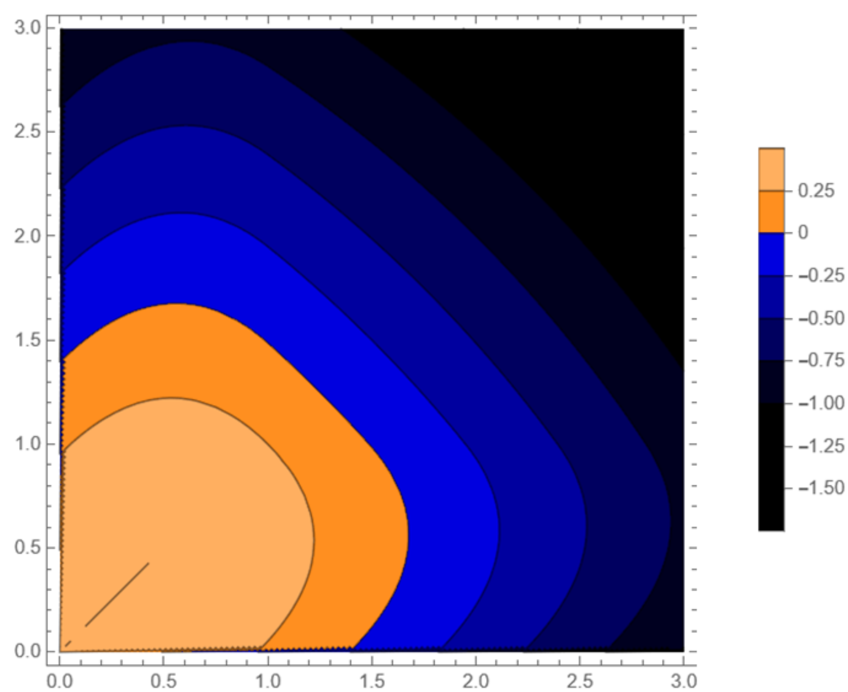

Figure 2: Difference in consumer surpluses under personalized and uniform pricing (ΔCS)
Note: Horizontal axis: v_1; Vertical axis: v_2.

Then we show a parameter range of v_1 and v_2 such that personalized pricing improves profits and consumer welfare. Combining Figures 1 and 2, we divide the parameter area into seven parts. As a result, we obtain the following proposition:

Proposition 5. *Suppose that all consumers purchase from only one of the firms under uniform pricing; equivalently, suppose that:*

$$\begin{cases} v_i \leq 3t - (\frac{3\sqrt{2}}{2} - 1)v_j & \text{if } v_j \leq 2t, \\ v_i \leq 3t + v_j - 3\sqrt{2}\sqrt{t(v_j - t)} & \text{if } 2t < v_j \leq 3t. \end{cases} \tag{3}$$

Compared with uniform pricing, personalized pricing increases the

1. *profit of each firm if and only if*

$$\begin{cases} 7(v_i - v_j) > (6\sqrt{2} - 3)t & \text{if } v_j \leq \frac{5 - 3\sqrt{2}}{7}t, \\ 8v_i + v_j - 3\sqrt{t^2 + 10tv_j - 7v_j^2} > 3t & \text{if } \frac{5 - 3\sqrt{2}}{7}t < v_j \leq \frac{1}{4}t, \\ v_i - v_j + 3\sqrt{3t^2 + v_j^2} > 6t & \text{if } \frac{1}{4}t < v_j \leq t, \\ 2v_i + v_j > 3t & \text{if } v_j > t; \end{cases}$$

2. *consumer surplus if and only if*

$$\begin{cases} v_i < v_j + 3\sqrt{2}\sqrt{6t^2 - v_j^2} - 9t & \text{if } v_j \leq t, \\ v_i < v_j + 3\sqrt{2}\sqrt{t(7t - v_j)} - 9t & \text{if } v_j > t; \end{cases}$$

3. *total surplus unless* $v_1 = v_2 \leq \frac{t}{2}$.

Figure 3 shows that when both v_1 and v_2 take intermediate values, personalized pricing improves consumer surplus and profits. In this area, the difference between the firms in terms of v_i is small. The following proposition summarizes the discussion.

Proposition 6. *Personalized pricing improves profits and consumer surplus if and only if for each* $i = 1, 2$,

$$2v_i + v_j > 3t,$$

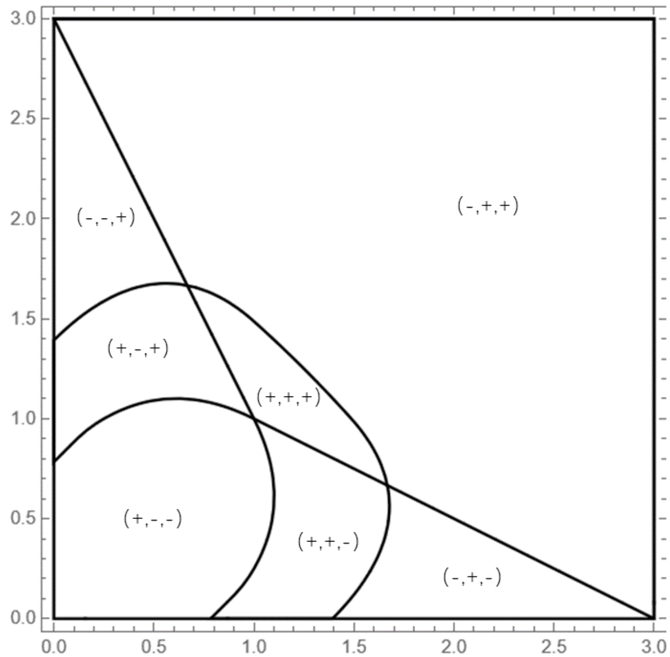

Figure 3: Parameter range when firm profits and consumer surplus improve
$(\cdot,\cdot,\cdot)$ indicates $(\text{sgn}(\Delta CS), \text{sgn}(\Delta\pi_1), \text{sgn}(\Delta\pi_2))$. Horizontal axis: v_1; Vertical axis: v_2.

$$\begin{cases} v_i < v_j + 3\sqrt{2}\sqrt{6t^2 - v_j^2} - 9t & \textit{if } v_j \leq t, \\ v_i < v_j + 3\sqrt{2}\sqrt{t(7t - 2v_j)} - 9t & \textit{if } v_j > t. \end{cases}$$

5 Conclusion

Given that advances in information technology have made personalized pricing a reality, we discuss the effect of personalized pricing on profits and welfare in a Hotelling duopoly model in which consumers can purchase from both firms. Our formulation complements Jeitschko et al. (2017) by considering personalized pricing and Rhodes and Zhou (2022) by adopting the standard Hotelling model.

We have the following results. Consumers benefit from personalized pricing only if no consumer purchases from both firms under uniform pricing. Under the necessary condition, consumer surplus improves if the additional intrinsic utility from the second product is smaller than a threshold value. Firms benefit from personalized pricing if at least some consumers purchase from both firms under uniform pricing or if the additional gain from the second product is larger than the transportation cost. There is a parameter range such that personalized pricing improves consumer surplus and firms' profits. The results contrast with the standard results in the personalized pricing literature based on Hotelling models and complement the findings in Jeitschko et al. (2017) and Rhodes and Zhou (2022).

Furthermore, we extend the model by endogenizing firms' pricing policies and obtain two types of equilibrium outcomes. First, both firms choose personalized pricing in any case; second, both firms choose uniform pricing when the additional intrinsic utility from the second product is in a parameter range in which the following two conditions hold (i) all consumers purchase from only one of the firms in equilibrium if the firms employ uniform pricing and (ii) some consumers purchase from both firms in equilibrium if only one of the firms employs personalized pricing. That is, in this parameter range, multiple equilibria can co-exist. The second type of equilibrium outcome is uncommon and novel in the context of personalized pricing, although several papers show that only one of the firms employs personalized pricing in asymmetric duopoly models (Ghose and Huang, 2009, Matsumura and Matsushima, 2015).

Those main results stem from the assumption that consumers purchase multiple items in the standard spatial competition model (Jeitschko et al., 2017). The key factors of those positive results are market demand expansions through personalized pricing. Therefore, we can conclude that the feasibility of personalized pricing is less likely to lead to worse outcomes for firms if personalized pricing expands the total demands of consumers. Also, we think that when we discuss the impact of personalized pricing on profits and welfare, we need to consider the propensity of consumers' multistore shopping (Gijsbrechts et al., 2008, Bell et al., 2011, Landsman and Stremersch, 2011).

References

Acquisti, A. and Varian, H. R. (2005). Conditioning prices on purchase history. Marketing Science, 24(3):367–381.

Adachi, T. and Tremblay, M. J. (2020). Business-to-business bargaining in two-sided markets. European Economic Review, 130:103591.

Anderson, S. P., Foros, Ø., and Kind, H. J. (2017). Product functionality, competition, and multipurchasing. International Economic Review, 58(1):183–210.

Bell, D. R., Corsten, D., and Knox, G. (2011). From point of purchase to path to purchase: How preshopping factors drive unplanned buying. Journal of Marketing, 75(1):31–45.

Carroni, E., Madio, L., and Shekhar, S. (2020). Superstars in two-sided markets: exclusives or not? TSE Working Papers 20-1083, Toulouse School of Economics (article).

Chen, Y. and Riordan, M. H. (2008). Price-increasing competition. RAND Journal of Economics, 39(4):1042–1058.

Chen, Z., Choe, C., Cong, J., and Matsushima, N. (2022). Data-driven mergers and personalization. RAND Journal of Economics, 53(1):3–31.

Chen, Z., Choe, C., and Matsushima, N. (2020). Competitive personalized pricing. Management Science, 66(9):4003–4023.

Choe, C., King, S., and Matsushima, N. (2018). Pricing with cookies: Behavior-based price discrimination and spatial competition. Management Science, 64(12):5669–5687.

Choe, C., Matsushima, N., and Tremblay, M. J. (2022). Behavior-based personalized pricing: When firms can share customer information. International Journal of Industrial Organization, 82:102846.

Choudhary, V., Ghose, A., Mukhopadhyay, T., and Rajan, U. (2005). Personalized pricing and quality differentiation. Management Science, 51(7):1120–1130.

Cowan, S. and Yin, X. (2008). Competition can harm consumers. Australian Economic Papers, 47(3):264–271.

d'Aspremont, C., Gabszewicz, J. J., and Thisse, J.-F. (1979). On Hotelling's "stability in competition". Econometrica, 47(5):1145–1150.

de Palma, A., Leruth, L., and Regibeau, P. (1999). Partial compatibility with network externalities and double purchase. Information Economics and Policy, 11(2):209–227.

Esteves, R.-B. (2022). Can personalized pricing be a winning strategy in oligopolistic markets with heterogeneous demand customers? yes, it can. International Journal of Industrial Organization, 85:102874.

Esteves, R.-B. and Resende, J. (2016). Competitive targeted advertising with price discrimination. Marketing Science, 35(4):576–587.

Gabszewicz, J. J., Sonnac, N., and Wauthy, X. (2001). On price competition with complementary goods. Economics Letters, 70(3):431–437.

Gabszewicz, J. J. and Wauthy, X. (2003). The option of joint purchase in vertically differentiated markets. Economic Theory, 22(4):817–829.

Ghose, A. and Huang, K.-W. (2009). Personalized pricing and quality customization. Journal of Economics & Management Strategy, 18(4):1095–1135.

Gijsbrechts, E., Campo, K., and Nisol, P. (2008). Beyond promotion-based store switching: Antecedents and patterns of systematic multiple-store shopping. International Journal of Research in Marketing, 25(1):5–21.

Guo, L. (2006). Consumption flexibility, product configuration, and market competition. Marketing Science, 25(2):116–130.

Hajihashemi, B., Sayedi, A., and Shulman, J. D. (2022). The perils of personalized pricing with network effects. Marketing Science, 41(3):477–500.

Hannak, A., Soeller, G., Lazer, D., Mislove, A., and Wilson, C. (2014). Measuring price discrimination and steering on e-commerce web sites. Proceedings of the 2014 Conference on Internet Measurement Conference, pages 305–318.

Ishihara, A. and Oki, R. (2021). Asymmetric product line and multi-homing. SSRN Working Paper `https://ssrn.com/abstract=3550355`.

Jeitschko, T. D., Jung, Y., and Kim, J. (2017). Bundling and joint marketing by rival firms. Journal of Economics & Management Strategy, 26(3):571–589.

Jeitschko, T. D. and Tremblay, M. J. (2020). Platform competition with endogenous homing. International Economic Review, 61(3):1281–1305.

Jiang, B., Tian, L., and Zhou, B. (2019). Competition of content acquisition and distribution under consumer multipurchase. Journal of Marketing Research, 56(6):1066–1084.

Jullien, B., Reisinger, M., and Rey, P. (2022). Personalized pricing and distribution strategies. Management Science, Forthcoming. `https://doi.org/10.1287/mnsc.2022.4437`.

Kim, H. and Serfes, K. (2006). A location model with preference for variety. Journal of Industrial Economics, 54(4):569–595.

Kim, M. S., Kim, E., Hwang, S., Kim, J., and Kim, S. (2017). Willingness to pay for over-the-top services in china and korea. Telecommunications Policy, 41(3):197–207.

Kodera, T. (2015). Discriminatory pricing and spatial competition in two-sided media markets. B.E. Journal of Economic Analysis & Policy, 15(2):891–926.

Landsman, V. and Stremersch, S. (2011). Multihoming in two-sided markets: An empirical inquiry in the video game console industry. Journal of Marketing, 75(6):39–54.

Laussel, D. and Resende, J. (2022). When is product personalization profit-enhancing? a behavior-based discrimination model. Management Science, Forthcoming. `https://doi.org/10.1287/mnsc.2022.4298`.

Li, H. and Zhu, F. (2021). Information transparency, multihoming, and platform competition: A natural experiment in the daily deals market. Management Science, 67(7):4384–4407.

Liu, Q. and Serfes, K. (2013). Price discrimination in two-sided markets. Journal of Economics & Management Strategy, 22(4):768–786.

Matsumura, T. and Matsushima, N. (2015). Should firms employ personalized pricing? Journal of Economics & Management Strategy, 24(4):887–903.

Matsushima, N., Mizuno, T., and Pan, C. (2022). Personalized pricing with heterogeneous mismatch costs. ISER Discussion Paper No.1184, Osaka Univerisity, (Paper).

Melcher, N. (2021). CoD, Battlefield & Halo are launching within a month of each other; What motivates their fans and sets them apart? Newzoo (Article).

Mikians, J., Gyarmati, L., Erramilli, V., and Laoutaris, N. (2012). Detecting price and search discrimination on the internet. Proceedings of the 11th ACM Workshop on Hot Topics in Networks, pages 79–84.

Mussa, M. and Rosen, S. (1978). Monopoly and product quality. Journal of Economic Theory, 18(2):301–317.

Perloff, J. M. and Salop, S. C. (1985). Equilibrium with product differentiation. Review of Economic Studies, 52(1):107–120.

Rhodes, A. and Zhou, J. (2022). Personalized pricing and competition. SSRN Working Paper https://ssrn.com/abstract=4103763.

Shaffer, G. and Zhang, Z, J. (2002). Competitive one-to-one promotions. Management Science, 48(9):1143–1160.

Shaked, A. and Sutton, J. (1982). Relaxing price competition through product differentiation. Review of Economic Studies, 49(1):3–13.

Shekhar, S. (2022). Platform pricing choice: Exclusive deals or uniform prices. Review of Network Economics, 20(3):159–186.

Shiller, B. R. (2020). Approximating purchase propensities and reservation prices from broad consumer tracking. International Economic Review, 61(2):847–870.

Shiller, B. R. (2022). Discreet personalized pricing. Working Paper Series 133, Brandeis University.

Smith, A. N., Seiler, S., and Aggarwal, I. (2022). Optimal price targeting. Marketing Science, Forthcoming. https://doi.org/10.1287/mksc.2022.1387.

Thisse, J.-F. and Vives, X. (1988). On the strategic choice of spatial price policy. American Economic Review, 78(1):122–137.

Wu, C.-H. and Chiu, Y.-Y. (2022). Pricing and content development for online media platforms regarding consumer homing choices. European Journal of Operational Research, Forthcoming. https://doi.org/10.1016/j.ejor.2022.05.030.

Xu, Z. and Dukes, A. (2021). Personalization from customer data aggregation using list price. Management Science, 68(2):960–980.

Zhou, B. and Zou, T. (2022). Competing for recommendations: The strategic impact of personalized product recommendations in online marketplaces. Marketing Science, Forthcoming. https://doi.org/10.1287/mksc.2022.1388.

Zhou, J. (2021). Mixed bundling in oligopoly markets. Journal of Economic Theory, 194:105257.

Appendix

A Endogenous choice of pricing policies

In the appendix, we consider the subgame in which firm 1 uses personalized pricing and firm 2 uses uniform pricing. The pricing sequence is that firm 2 sets its uniform pricing; after that, firm 1 sets personalized prices. We solve it by backward induction.

Firm 1's personalized pricing The utility from purchasing firm 1's product, firm 2's product, or both products is:

$$\begin{cases} U_1 = w - tx - p_1(x), & \text{purchasing from only firm 1,} \\ U_2 = w - t(1-x) - p_2, & \text{purchasing from only firm 2,} \\ U_{12} = 2w - V - t - p_1(x) - p_2, & \text{purchasing from firms 1 and 2.} \end{cases}$$

Let $v \equiv w - V$. Consumers purchase only from firm 1 if and only if $U_1 \geq U_2$ and $U_1 > U_{12}$; they purchase only from firm 2 if and only if $U_2 > U_1$ and $U_2 > U_{12}$; and they purchase from both firms if and only if $U_{12} \geq U_1$ and $U_{12} \geq U_2$. Substituting the utility functions into the inequalities, we get that 1) consumers purchase only from firm 1 when $p_1(x) \leq t(1-2x)+p_2$ and $p_2 > v - t(1-x)$; 2) consumers purchase only from firm 2 when $p_1(x) > t(1-2x)+p_2$ and $p_1(x) > v - tx$; 3) consumers purchase from both firms when $p_1(x) \leq v - tx$ and $p_2 \leq v - t(1-x)$. Firm 1 chooses the highest price that is acceptable for consumers locating at x, which means $p_1(x, p_2) = \max\{0, t(1-2x) + p_2, v - tx\}$. By organizing this price function, we derive the following conditions and firm 1's best response.

1. Firm 1 can offer personalized prices to all consumers if and only if $v > t$ or $p_2 > t$. In this case, consumers on $[0, \min\{(t + p_2 - v)/t, 1\})$ purchase only from firm 1, and consumers on $[\min\{(t + p_2 - v)/t, 1\}, 1]$ purchase from both firms. In the following, we omit the case in which $\min\{(t + p_2 - v)/t, 1\} = 1$ because this equation means that firm 2 is inactive.

2. Firm 1 offers positive personalized prices for consumers that locate on $[0,(t+p_2)/(2t)]$ if and only if $v \le t$ and $2v - t < p_2 \le t$. In this case, all consumers purchase from only one of the firms.

3. Firm 1 offers positive personalized prices for consumers that locate on $[0, v/t]$ if and only if $v \le t$ and $p_2 \le 2v - t$. In this case, consumers on $[0,(t+p_2-v)/t)$ purchase only from firm 1, consumers on $[(t+p_2-v)/t, v/t]$ purchase from both firms, and consumers on $(v/t, 1]$ purchase only from firm 2.

The personalized prices proposed by firms 1 are:

$$p_1(x,p_2) = \begin{cases} \begin{cases} t(1-2x)+p_2 & \text{for } x \in [0,(t+p_2-v)/t), \\ v - tx & \text{for } x \in [(t+p_2-v)/t,\ 1], \end{cases} & \text{if } \quad v > t \text{ or } p_2 > t, \\ \begin{cases} t(1-2x)+p_2 & \text{for } x \in [0,(t+p_2)/(2t)], \\ 0 & \text{for } x \in ((t+p_2)/(2t), 1], \end{cases} & \text{if } \quad v \le t \text{ and } \\ & \qquad 2v - t < p_2 \le t, \\ \begin{cases} t(1-2x)+p_2 & \text{for } x \in [0,(t+p_2-v)/t], \\ v - tx & \text{for } x \in ((t+p_2-v)/t, v/t), \\ 0 & \text{for } x \in [v/t, 1], \end{cases} & \text{if } \quad v \le t \text{ and } \\ & \qquad p_2 \le 2v - t. \end{cases}$$

Firm 2's uniform pricing We classify the case into the three subcases: (i) $v > t$ or $p_2 \ge t$; (ii) $v \le t$ and $2v - t < p_2 \le t$, and (iii) $v \le t$ and $p_2 \le 2v - t$.

When $v > t$ or $p_2 \ge t$ In this case, firm 1 offers positive personalized prices to all consumers, which means every consumer purchases from firm 1, and some of them also purchase from firm 2. Therefore, consumers at x who satisfy the condition that $U_{12} \ge U_1$, which means $x \ge (t - v + p_2)/t$, purchase from firm 2. The amount of consumers who purchase from firm 2 is

$$N_2 = \max\{\min\{\frac{v - p_2}{t}, 1\}, 0\}.$$

Then, firm 2 chooses its price p_2 to maximize its profit $\pi_2 = p_2 N_2$. Taking the first-order condition, $\partial \pi_2 / \partial p_2 = 0$, we derive the optimal price of firm 2:

$$p_2^* = \begin{cases} \frac{v}{2} & \text{if } t < v < 2t, \\ v - t & \text{if } v \geq 2t. \end{cases}$$

When $v \leq t$ **and** $2v - t < p_2 \leq t$ In this case, all consumers purchase from only one firm of the firms. Firm 2 can serve consumers on $((t + p_2)/(2t), 1]$. The amount of consumers who purchase from firm 2 is

$$N_2 = \max\{\min\{\frac{t - p_2}{2t}, 1\}, 0\}.$$

Then, firm 2 chooses p_2 to maximize its profit $\pi_2 = p_2 N_2$. Taking the first-order condition, $\partial \pi_2 / \partial p_2 = 0$, we derive the optimal price of firm 2:

$$p_2^* = \frac{t}{2}.$$

When $v \leq t$ **and** $p_2 \leq 2v - t$ In this case, consumers on $[0, (t + p_2 - v)/t)$ purchase only from firm 1, consumers on $[(t+p_2-v)/t, v/t]$ purchase from both firms, and consumers on $(v/t, 1]$ only purchase from firm 2. As $v \leq t$ and $p_2 \leq 2v - t$, p_2 must not be larger than v, so $1 - (t + p_2 - v)/t = (v - p_2)/t$ is always between 0 and 1. Similar to the first case, the amount of consumers who purchase from firm 2 is

$$N_2 = \frac{v - p_2}{t}.$$

Then, firm 2 chooses p_2 to maximize its profit $\pi_2 = p_2 N_2$. Taking the first-order condition, $\partial \pi_2 / \partial p_2 = 0$, we derive the optimal prices of firm 2:

$$p_2^* = \frac{v}{2}.$$

To sum up, the equilibrium outcome is as follows:

1. When $v \geq 2t$, the prices of firms 1 and 2 are $p_1^*(x) = v - tx$ and $p_2^* = v - t$. The resulting demands for firms 1 and 2 are $N_1^* = N_2^* = 1$. The profits are $\pi_1^* = v - t/2$ and $\pi_2^* = v - t$.

2. When $t \leq v < 2t$, the prices of firms 1 and 2 are

$$p_1^* = \begin{cases} t(1-2x) + v/2 & \text{for } x \leq 1 - v/(2t), \\ v - tx & \text{for } x \geq 1 - v/(2t), \end{cases} \quad \text{and} \quad p_2^* = v/2.$$

   The resulting demand for firms 1 and 2 are $N_1^* = 1$ and $N_2^* = v/(2t)$. The profits are $\pi_1^* = v(4t + v)/(8t)$ and $\pi_2^* = v^2/(4t)$.

3. When $\sqrt{2}t/2 \leq v < t$, the prices of firms 1 and 2 are

$$p_1^* = \begin{cases} t(1-2x) + v/2 & \text{for } x < 1 - v/2, \\ v - tx & \text{for } 1 - v/(2t) \leq x < v/t, \\ 0 & \text{for } x \geq v/t, \end{cases} \quad \text{and} \quad p_2^* = v/2.$$

   The resulting demands for firms 1 and 2 are $N_1^* = v/t$ and $N_2^* = v/(2t)$. The profits are $\pi_1^* = (4t^2 - 4tv + 5v^2)/(8t)$ and $\pi_2^* = v^2/(4t)$.

4. When $v \leq \sqrt{2}t/2$, the prices of firms 1 and 2 are

$$p_1^* = \begin{cases} t(1-2x) + t/2 & \text{for } x < 3/4, \\ 0 & \text{for } x \geq 3/4, \end{cases}, \quad \text{and} \quad p_2^* = \frac{t}{2}.$$

   The resulting demand for firms 1 and 2 are $N_1^* = 3/4$ and $N_2^* = 1/4$. The profits are $\pi_1^* = 9t/16$ and $\pi_2^* = t/8$.

The choices of pricing policies In the first stage, the firms choose their pricing policies simultaneously. Let firm i's profit be $\pi_i(a_i, a_j)$ $(i, j = 1, 2,\ j \neq i)$, where $a_i \in \{p, u\}$ represents firm i's pricing policy, where p means personalized pricing and u means uniform pricing. Then, we can indicate the profit function of each firm as follows (when there are multiple equilibria under (u, u) (when $2(2\sqrt{2}+1)t/7 \leq v \leq \sqrt{2}t$, see Section 3.2), we choose the higher profit $\pi_i(u, u) = t/2$):

$$\pi_i(p,p) = \begin{cases} v - \frac{t}{2} & \text{if } t < v, \\ \frac{t}{2} - \frac{v(t-v)}{t} & \text{if } t/2 < v \leq t, \\ \frac{t}{4} & \text{if } v \leq t/2. \end{cases} \quad \pi_i(p,u) = \begin{cases} v - \frac{t}{2} & \text{if } 2t \leq v, \\ \frac{v(4t+v)}{8t} & \text{if } t < v < 2t, \\ \frac{4t^2-4tv+5v^2}{8t} & \text{if } \frac{\sqrt{2}}{2}t < v \leq t, \\ \frac{9}{16}t & \text{if } v \leq \frac{\sqrt{2}}{2}t. \end{cases}$$

$$\pi_i(u,p) = \begin{cases} v - t & \text{if } 2t \leq v, \\ \frac{v^2}{4t} & \text{if } \frac{\sqrt{2}}{2}t < v < 2t, \\ \frac{1}{8}t & \text{if } v \leq \frac{\sqrt{2}}{2}t. \end{cases} \quad \pi_i(u,u) = \begin{cases} w - t & \text{if } 2t \leq v, \\ \frac{v^2}{4t} & \text{if } \sqrt{2}t < v < 2t, \\ \frac{t}{2} & \text{if } v \leq \sqrt{2}t. \end{cases}$$

Table 1: The choices of prcing policies

Firm 1 \ Firm 2	personalized	uniform
personalized	$(\pi_1(p,p),\ \pi_2(p,p))$	$(\pi_1(p,u),\ \pi_2(u,p))$
uniform	$(\pi_1(u,p),\ \pi_2(p,u))$	$(\pi_1(u,u),\ \pi_2(u,u))$

We compare each firm's profits under different pricing policies. No matter what the parameters are, $\pi_i(p, p) > \pi_i(u, p)$. On the other hand, $\pi_i(u, u) \geq \pi_i(p, u)$ if and only if $\sqrt{2}t/2 \leq v \leq 4t/5$. The discussion leads to Proposition 3.

B Equilibrium with and without consumers who purchase from both firms

We slightly extend the range of exogenous parameters in Jeitschko et al. (2017). We derive the conditions: (i) some consumers purchase from both firms; (ii) all consumers purchase from one of the firms. The location of consumers who are indifferent between choosing only firm 1 and choosing both firms satisfies $w_1 - tx_1^* - p_1 = w_1 + w_2 - V - t - p_1 - p_2$. Let $v_i = w_i - V$ $(i = 1, 2)$. By organizing this equation, we obtain $x_1^* = 1 - (v_2 - p_2)/t$.

Similarly, $x_2^* = (v_1 - p_1)/t$ is the location of consumers who are indifferent between choosing only firm 2 and choosing both firms. When some consumers purchase from both firms, $x_1^* < x_2^*$, which implies that $\Sigma_k p_k < v_1 + v_2 - t$. When all consumers purchase from one of the firms, $x_1^* \geq x_2^*$, which implies that $\Sigma_k p_k \geq v_1 + v_2 - t$.

B.1 Uniform pricing

Below, first, we show the equilibrium outcome in which all consumers purchase from one of the firms, which implies that $\Sigma_k p_k \geq v_1 + v_2 - t$. Second, we show the equilibrium outcome in which some consumers purchase from both firms, which implies that $\Sigma_k p_k < v_1 + v_2 - t$.

B.1.1 Equilibrium when all consumers purchase from one of the firms

In this section, assume that $\Sigma_k p_k \geq \Sigma_k v_k - t$. The number of consumers who belong to firm i, N_i $(i = 1, 2)$, is:

$$N_i = \max\{\min\{\hat{N}_i, 1\}, 0\}, \text{ where } \hat{N}_i = \frac{1}{2} + \frac{v_i - v_j - (p_i - p_j)}{2t}. \tag{1}$$

Firm i $(i = 1, 2)$ chooses its price p_i to maximize its profit $\pi_i = p_i N_i$. Solving the first-order conditions, $\partial \pi_i / \partial p_i$ $(i = 1, 2)$, we obtain the optimal prices, number of consumers belonging to firm i, and resulting profits:

$$p_i^* = t + \frac{v_i - v_j}{3}, \quad N_i^* = \frac{1}{2} + \frac{v_i - v_j}{6t}, \quad \pi_i^* = 2t\left(\frac{1}{2} + \frac{v_i - v_j}{6t}\right)^2. \tag{2}$$

We need to check the condition that the firms have no incentive to deviate from the prices. After some calculus (available in Section B.1.2), we obtain the condition $(i, j = 1, 2,\ j \neq i)$:

$$\begin{cases} v_i \leq 3t - (\frac{3\sqrt{2}}{2} - 1)v_j & \text{if } v_j \leq 2t, \\ v_i \leq 3t + v_j - 3\sqrt{2}\sqrt{t(v_j - t)} & \text{if } 2t < v_j \leq 3t. \end{cases} \tag{3}$$

The condition is in equation (3) in Proposition 5.

B.1.2 Equilibrium when some consumers purchase from both firms

In this section, suppose that $\Sigma_k p_k < v_1 + v_2 - t$. The number of consumers who belong to firm i, $N_i, i = 1, 2$, is:

$$N_i = \max\{\min\{\hat{N}_i, 1\}, 0\}, \text{ where } \hat{N}_i = \frac{v_i - p_i}{t}. \tag{4}$$

Because some consumers purchase from both firms, $\Sigma_k N_k > 1$. Firm i chooses its price p_i to maximize its profit $\pi_i = p_i N_i$. Solving the first-order conditions, $\partial\pi_i/\partial p_i$ $(i = 1, 2)$, we obtain the optimal prices, number of consumers belonging to firm i, and resulting profits:

$$p_i^{**} = \begin{cases} \frac{v_i}{2} & \text{if } v_i < 2t, \\ v_i - t & \text{if } v_i \geq 2t, \end{cases} \quad N_i^{**} = \begin{cases} \frac{v_i}{2t} & \text{if } v_i < 2t, \\ 1 & \text{if } v_i \geq 2t, \end{cases} \quad \pi_i^{**} = \begin{cases} \frac{v_i^2}{4t} & \text{if } v_i < 2t, \\ v_i - t & \text{if } v_i \geq 2t. \end{cases} \tag{5}$$

We need to check the condition that the firms have no incentive to deviate from the prices. After some calculus (available in the last part of this section), we obtain the condition:

$$\begin{cases} v_i > (1 + \sqrt{2})(t - \frac{v_j}{2}) & \text{if } v_j > (6 - 4\sqrt{2})t, \\ v_i > 3t + \frac{1}{2}v_j - 2\sqrt{tv_j} & \text{if } v_j \leq (6 - 4\sqrt{2})t. \end{cases} \tag{6}$$

The condition is in equation (2) in Proposition 4.

Deviation incentives when all consumers purchase from one of the firms We show the condition that firms have no incentive to deviate from their prices. First, $p_i = p_i^*$ satisfies the first- and second-order conditions under the condition that $\Sigma_k p_k \geq \Sigma_k v_k - t$. Thus, p_i^* is always the optimal price for firm i when all consumers purchase from one of the firms. Therefore, we check the situation that firm i decreases its price to the level such that $p_i + p_j^* < \Sigma_k v_k - t$; that is, some consumers purchase from both firms. The demand for firm i is N_i in equation (4). Solving the first-order condition, the optimal deviation price of firm i is:

$$p_i^{D*} = \begin{cases} \frac{v_i}{2} & \text{if } v_i < 2t \text{ and } t < \frac{5}{12}v_i + \frac{1}{3}v_j, \\ \frac{4}{3}v_i + \frac{2}{3}v_j - 2t & \text{if } v_i < 2t \text{ and } t \geq \frac{5}{12}v_i + \frac{1}{3}v_j, \\ v_i - t & \text{if } v_i \geq 2t \text{ and } t < \frac{1}{3}v_i + \frac{2}{3}v_j, \\ \frac{4}{3}v_i + \frac{2}{3}v_j - 2t & \text{if } v_i \geq 2t \text{ and } t \geq \frac{1}{3}v_i + \frac{2}{3}v_j, \end{cases}$$

and the maximizing profit of firm i is

$$\pi_i^{D*} = \begin{cases} \frac{v_i^2}{4t} & \text{if } v_i < 2t \text{ and } t < \frac{5}{12}v_i + \frac{1}{3}v_j, \\ (\frac{4}{3}v_i + \frac{2}{3}v_j - 2t)(2 - \frac{v_i + 2v_j}{3t}) & \text{if } v_i < 2t \text{ and } t \geq \frac{5}{12}v_i + \frac{1}{3}v_j, \\ v_i - t & \text{if } v_i \geq 2t \text{ and } t < \frac{1}{3}v_i + \frac{2}{3}v_j, \\ \frac{4}{3}v_i + \frac{2}{3}v_j - 2t & \text{if } v_i \geq 2t \text{ and } t \geq \frac{1}{3}v_i + \frac{2}{3}v_j. \end{cases}$$

Comparing the deviation profit with the optimal profit π_i^*, we obtain the condition that firm i has no incentive to deviate from the optimal price p_i^* as in equation (2).

Deviation incentives when some consumers purchase from both firms We show the condition that the firms have no incentive to deviate from their prices. First, $p_i = p_i^{**}$ satisfies the first- and second-order conditions under the condition that $\Sigma_k p_k < v_1 + v_2 - t$. Thus, p_i^{**} is always the optimal price for firm i when some consumers purchase from both firms. Therefore, we check the situation that firm i raises its price to the level such that $p_i + p_j^{**} \geq v_1 + v_2 - t$; that is, all consumers purchase from one of the firms. The demand for firm i is N_i in equation (1). Solving the first-order condition, the optimal deviation price of firm i is:

$$p_i^{D**} = \begin{cases} v_i + \frac{1}{2}v_j - t & \text{if } v_j < 2t \text{ and } t < \frac{2v_i + 3v_j}{6}, \\ \frac{2v_i - v_j + 2t}{4} & \text{if } v_j < 2t \text{ and } t \geq \frac{2v_i + 3v_j}{6}, \\ v_i & \text{if } v_j \geq 2t. \end{cases}$$

and the deviation profit of firm i is

$$\pi_i^{D**} = \begin{cases} \left(v_i + \frac{1}{2}v_j - t\right)\left(1 - \frac{v_j}{2t}\right) & \text{if } v_j < 2t \text{ and } t < \frac{2v_i + 3v_j}{6}, \\ \frac{1}{2t}\left(\frac{2v_i - v_j + 2t}{4}\right)^2 & \text{if } v_j < 2t \text{ and } t \geq \frac{2v_i + 3v_j}{6}, \\ 0 & \text{if } v_j \geq 2t. \end{cases}$$

Comparing the deviation profit with the optimal profit π_i^{**}, we obtain the condition that firm i has no incentive to deviate from the optimal price p_i^{**} as in equation (5).

B.1.3 Welfare

All consumers purchase from one of the firms When no consumers purchase multiple units, consumer and total surpluses are:

$$\begin{aligned} CS^{U*} &= \frac{(v_1 - v_2)^2 + 18t(2V + v_1 + v_2) - 45t^2}{36t} \\ TS^{U*} &= \frac{5(v_1 - v_2)^2 + 18t(2V + v_1 + v_2) - 9t^2}{36t}. \end{aligned}$$

Some consumers purchase from both firms When some consumers purchase from both firms, consumer and total surpluses are:

$$CS^{U**} = \begin{cases} \frac{v_1^2 + v_2^2}{8t} + V & \text{if } v_1 < 2t \text{ and } v_2 < 2t, \\ \frac{v_2^2}{8t} + V + \frac{t}{2} & \text{if } v_1 \geq 2t \text{ and } v_2 < 2t, \\ \frac{v_1^2}{8t} + V + \frac{t}{2} & \text{if } v_1 < 2t \text{ and } v_2 \geq 2t, \\ V + t & \text{if } v_1 \geq 2t \text{ and } v_2 \geq 2t, \end{cases}$$

$$TS^{U**} = \begin{cases} \dfrac{3(v_1^2+v_2^2)}{8t} + V & \text{if } v_1 < 2t \text{ and } v_2 < 2t, \\ \dfrac{3v_2^2}{8t} + V + v_1 - \dfrac{t}{2} & \text{if } v_1 \geq 2t \text{ and } v_2 < 2t, \\ \dfrac{3v_1^2}{8t} + V + v_2 - \dfrac{t}{2} & \text{if } v_1 < 2t \text{ and } v_2 \geq 2t, \\ V + v_1 + v_2 - t & \text{if } v_1 \geq 2t \text{ and } v_2 \geq 2t. \end{cases}$$

B.2 Personalized pricing

First, we derive the condition that some consumers purchase from both firms. Given that consumers at x purchase from firm j at a positive personalized price, they also buy from firm i if and only if:

$$\begin{aligned} & w_i + w_j - V - t - p_i(x) - p_j(x) \geq w_j - td_j(x) - p_j(x) \\ \Rightarrow \quad & p_i(x) \leq v_i - t(1 - d_j(x)), \end{aligned}$$

where $d_j(x)$ is the distance between firm j and the consumer at x. If firm j cannot attract consumers at x at a nonnegative personalized price, it does not supply to them and sets $p_j(x) = 0$. Then, firm i's personalized price is acceptable for consumers at x if and only if:

$$w_i - td_i(x) - p_i(x) \geq w_j - td_j(x) - 0 \quad \Rightarrow \quad p_i(x) \leq v_i - v_j + t(d_j(x) - d_i(x)).$$

We obtain the following result: The schedules of personalized prices depend on v_i and t. Concretely,

1. both firms can offer personalized prices that induce all consumers to purchase from both firms if and only if $v_i > t$, $\forall i = 1, 2$;

2. firm i offers positive personalized prices for consumers such that $d_i(x) < v_i/t$ if and only if $v_i \leq t$ and $v_i + v_j > t$. In this case, consumers on $(1 - v_2/t, v_1/t)$ purchase from both firms;

3. no firm can offer personalized prices that induce consumers to buy from both firms if and only if $v_1 + v_2 \leq t$.

The personalized prices of firms 1 and 2 are:

$$p_1(x) = \begin{cases} \max\{v_1 - tx, 0\} & \text{if } v_2 > t, \\ \begin{array}{ll} \max\{v_1 - v_2 + t(1-2x), 0\} & \text{for } x \in [0, 1 - v_2/t] \\ \max\{v_1 - tx, 0\} & \text{for } x \in [1 - v_2/t, 1] \end{array} & \text{if } v_2 \leq t \text{ and } v_1 + v_2 > t, \\ \max\{v_1 - v_2 + t(1-2x), 0\} & \text{if } v_1 + v_2 \leq t, \end{cases}$$

$$p_2(x) = \begin{cases} \max\{v_2 - t(1-x), 0\} & \text{if } v_1 > t, \\ \begin{array}{ll} \max\{v_2 - v_1 + t(2x-1), 0\} & \text{for } x \in [v_1/t, 1] \\ \max\{v_2 - t(1-x), 0\} & \text{for } x \in [0, v_1/t] \end{array} & \text{if } v_1 \leq t \text{ and } v_1 + v_2 > t, \\ \max\{v_2 - v_1 + t(2x-1), 0\} & \text{if } v_1 + v_2 \leq t. \end{cases}$$

In this case, total surplus is:

$$TS^P = \begin{cases} v_1 + v_2 + V - t & \text{if } v_1 > t \text{ and } v_2 > t, \\ \dfrac{v_1^2}{2t} + v_2 + V - \dfrac{t}{2} & \text{if } v_1 \leq t \text{ and } v_2 > t, \\ v_1 + \dfrac{v_2^2}{2t} + V - \dfrac{t}{2} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ \dfrac{v_1^2 + v_2^2 + 2tV}{2t} & \text{if } v_1 \leq t,\ v_2 \leq t \text{ and } v_1 + v_2 > t, \\ \dfrac{(v_1 - v_2)^2 + 2t(v_1 + v_2 + 2V) - t^2}{4t} & \text{if } v_1 + v_2 \leq t. \end{cases}$$

The profit of each firm is:

$$\pi_1^P = \begin{cases} v_1 - \dfrac{t}{2} & \text{if } v_1 > t \text{ and } v_2 > t, \\ \dfrac{v_1^2}{2t} & \text{if } v_1 \leq t \text{ and } v_2 > t, \\ v_1 + \dfrac{v_2(v_2 - 2t)}{2t} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ \dfrac{v_1^2 + v_2^2 - 2tv_2 + t^2}{2t} & \text{if } v_1 \leq t,\ v_2 \leq t \text{ and } v_1 + v_2 > t, \\ \dfrac{(v_1 - v_2 + t)^2}{4t} & \text{if } v_1 + v_2 \leq t. \end{cases}$$

$$\pi_2^P = \begin{cases} v_2 - \dfrac{t}{2} & \text{if } v_1 > t \text{ and } v_2 > t, \\ v_2 + \dfrac{v_1(v_1 - 2t)}{2t} & \text{if } v_1 \leq t \text{ and } v_2 > t, \\ \dfrac{v_2^2}{2t} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ \dfrac{v_1^2 + v_2^2 - 2tv_1 + t^2}{2t} & \text{if } v_1 \leq t,\ v_2 \leq t \text{ and } v_1 + v_2 > t, \\ \dfrac{(v_2 - v_1 + t)^2}{4t} & \text{if } v_1 + v_2 \leq t. \end{cases}$$

By calculating $CS = TS - \pi_1 - \pi_2$, we obtain consumer surplus as follows:

$$CS^P = \begin{cases} V & \text{if } v_1 > t \text{ and } v_2 > t, \\ v_1 + V - \dfrac{v_1^2}{2t} - \dfrac{t}{2} & \text{if } v_1 \leq t \text{ and } v_2 > t, \\ v_2 + V - \dfrac{v_2^2}{2t} - \dfrac{t}{2} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ v_1 + v_2 + V - \dfrac{v_1^2 + v_2^2}{2t} - t & \text{if } v_1 \leq t,\ v_2 \leq t \text{ and } v_1 + v_2 > t, \\ \dfrac{2t(v_1 + v_2 + 2V) - (v_1 - v_2)^2 - 3t^2}{4t} & \text{if } v_1 + v_2 \leq t. \end{cases}$$

B.3 Comparison

We compare the outcomes in the cases of personalized pricing and uniform pricing. The differences between the values under personalized pricing and those under uniform pricing when all consumers purchase from only one of the firms are as follows:

$$\Delta TS^S = \begin{cases} \dfrac{v_1 + v_2}{2} - \dfrac{5(v_1 - v_2)^2}{36t} - \dfrac{3}{4}t & \text{if } v_1 > t \text{ and } v_2 > t, \\ \dfrac{13v_1^2 + 10v_1v_2 - 5v_2^2 - 18t(v_1 - v_2) - 9t^2}{36t} & \text{if } v_1 \leq t \text{ and } v_2 > t, \\ \dfrac{13v_2^2 + 10v_1v_2 - 5v_1^2 - 18t(v_2 - v_1) - 9t^2}{36t} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ \dfrac{13v_1^2 + 10v_1v_2 + 13v_2^2 - 18t(v_1 + v_2) + 9t^2}{36t} & \text{if } v_1 \leq t,\ v_2 \leq t \text{ and } v_1 + v_2 > t, \\ \dfrac{(v_1 - v_2)^2}{9t} & \text{if } v_1 + v_2 \leq t, \end{cases}$$

$$\Delta CS^S = \begin{cases} -\dfrac{(v_1 - v_2)^2 + 18t(v_1 + v_2) - 45t^2}{36t} & \text{if } v_1 > t \text{ and } v_2 > t, \\ -\dfrac{19v_1^2 - 2v_1v_2 + v_2^2 - 18t(v_1 - v_2) - 27t^2}{36t} & \text{if } v_1 \leq t \text{ and } v_2 > t, \\ -\dfrac{19v_2^2 - 2v_1v_2 + v_1^2 - 18t(v_2 - v_1) - 27t^2}{36t} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ -\dfrac{19v_1^2 - 2v_1v_2 + 19v_2^2 - 18t(v_1 + v_2) - 9t^2}{36t} & \text{if } v_1 \leq t,\, v_2 \leq t \text{ and } v_1 + v_2 > t, \\ -\dfrac{5(v_1 - v_2)^2 - 9t^2}{18t} & \text{if } v_1 + v_2 \leq t, \end{cases}$$

$$\Delta \pi_1^S = \begin{cases} -\dfrac{(3t + v_1 - v_2)^2}{18t} + v_1 - \dfrac{t}{2} & \text{if } v_1 > t \text{ and } v_2 > t, \\ -\dfrac{(3t + v_1 - v_2)^2 - 9v_1^2}{18t} & \text{if } v_1 \leq t \text{ and } v_2 > t, \\ -\dfrac{v_1^2 - 2v_1v_2 - 8v_2^2 - 12t(v_1 - v_2) + 9t^2}{18t} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ \dfrac{4v_1^2 + v_1v_2 + 4v_2^2 - 3t(v_1 + 2v_2)}{9t} & \text{if } v_1 \leq t,\, v_2 \leq t \text{ and } v_1 + v_2 > t, \\ \dfrac{7(v_1 - v_2)^2 + 6t(v_1 - v_2) - 9t^2}{36t} & \text{if } v_1 + v_2 \leq t, \end{cases}$$

$$\Delta \pi_2^S = \begin{cases} -\dfrac{(3t + v_2 - v_1)^2}{18t} + v_2 - \dfrac{t}{2} & \text{if } v_1 > t \text{ and } v_2 > t, \\ -\dfrac{v_2^2 - 2v_1v_2 - 8v_1^2 - 12t(v_2 - v_1) + 9t^2}{18t} & \text{if } v_1 \leq t \text{ and } v_2 > t, \\ -\dfrac{(3t + v_2 - v_1)^2 - 9v_2^2}{18t} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ \dfrac{4v_2^2 + v_1v_2 + 4v_1^2 - 3t(v_2 + 2v_1)}{9t} & \text{if } v_1 \leq t,\, v_2 \leq t \text{ and } v_1 + v_2 > t, \\ \dfrac{7(v_2 - v_1)^2 + 6t(v_2 - v_1) - 9t^2}{36t} & \text{if } v_1 + v_2 \leq t. \end{cases}$$

Note that when at least some consumers purchase from both firms under uniform pricing, v_1 and v_2 cannot be less than t at the same time. Differences between the values under personalized pricing and those under uniform pricing are:

$$\Delta TS^M = \begin{cases} 0 & \text{if } v_1 \geq 2t \text{ and } v_2 \geq 2t, \\ v_2 - \dfrac{3v_2^2}{8t} - \dfrac{t}{2} & \text{if } v_1 \geq 2t \text{ and } t < v_2 < 2t, \\ \dfrac{v_2^2}{8t} & \text{if } v_1 \geq 2t \text{ and } v_2 \leq t, \\ v_1 - \dfrac{3v_1^2}{8t} - \dfrac{t}{2} & \text{if } t < v_1 < 2t \text{ and } v_2 \geq 2t, \\ v_1 + v_2 - \dfrac{3(v_1^2 + v_2^2)}{8t} - t & \text{if } t < v_1 < 2t \text{ and } t < v_2 < 2t, \\ \dfrac{v_2^2 - 3v_1^2 + 8tv_1 - 4t^2}{8t} & \text{if } t < v_1 < 2t \text{ and } v_2 \leq t, \\ \dfrac{v_1^2}{8t} & \text{if } v_1 \leq t \text{ and } v_2 \geq 2t, \\ \dfrac{v_1^2 - 3v_2^2 + 8tv_2 - 4t^2}{8t} & \text{if } v_1 \leq t \text{ and } t < v_2 < 2t, \end{cases}$$

$$\Delta CS^M = \begin{cases} -t & \text{if } v_1 \geq 2t \text{ and } v_2 \geq 2t, \\ -\dfrac{v_2^2 + 4t^2}{8t} & \text{if } v_1 \geq 2t \text{ and } t < v_2 < 2t, \\ v_2 - \dfrac{5v_2^2}{8t} - t & \text{if } v_1 \geq 2t \text{ and } v_2 \leq t, \\ -\dfrac{v_1^2 + 4t^2}{8t} & \text{if } t < v_1 < 2t \text{ and } v_2 \geq 2t, \\ -\dfrac{v_1^2 + v_2^2}{8t} & \text{if } t < v_1 < 2t \text{ and } t < v_2 < 2t, \\ -\dfrac{v_1^2 + 5v_2^2 - 8tv_2 + 4t^2}{8t} & \text{if } t < v_1 < 2t \text{ and } v_2 \leq t, \\ v_1 - \dfrac{5v_1^2}{8t} - t & \text{if } v_1 \leq t \text{ and } v_2 \geq 2t, \\ -\dfrac{v_2^2 + 5v_1^2 - 8tv_1 + 4t^2}{8t} & \text{if } v_1 \leq t \text{ and } t < v_2 < 2t, \end{cases}$$

$$\Delta\pi_1^M = \begin{cases} \dfrac{t}{2} & \text{if } v_1 \geq 2t \text{ and } v_2 > t, \\ -v_2 + \dfrac{v_2^2}{2t} + t & \text{if } v_1 \geq 2t \text{ and } v_2 \leq t, \\ v_1 - \dfrac{v_1^2}{4t} - \dfrac{t}{2} & \text{if } t < v_1 < 2t \text{ and } v_2 > t, \\ -\dfrac{v_1^2 - 2v_2^2 - 4t(v_1 - v_2)}{4t} & \text{if } t < v_1 < 2t \text{ and } v_2 \leq t, \\ \dfrac{v_1^2}{4t} & \text{if } v_1 \leq t \text{ and } v_2 > t, \end{cases}$$

$$\Delta\pi_2^M = \begin{cases} \dfrac{t}{2} & \text{if } v_1 > t \text{ and } v_2 \geq 2t, \\ v_2 - \dfrac{v_2^2}{4t} - \dfrac{t}{2} & \text{if } v_1 > t \text{ and } t < v_2 < 2t, \\ \dfrac{v_2^2}{4t} & \text{if } v_1 > t \text{ and } v_2 \leq t, \\ -v_1 + \dfrac{v_1^2}{2t} + t & \text{if } v_1 \leq t \text{ and } v_2 \geq 2t, \\ -\dfrac{v_2^2 - 2v_1^2 - 4t(v_2 - v_1)}{4t} & \text{if } v_1 \leq t \text{ and } t < v_2 < 2t. \end{cases}$$

By comparing social welfare, we obtain the condition that all players in this market are improved by personalized pricing. Suppose that all consumers purchase from only one of the firms under uniform pricing, then the profit of each firm improves if and only if, for each $i = 1, 2$,

$$\begin{cases} 7(v_i - v_j) > (6\sqrt{2} - 3)t & \text{if } v_j \leq \dfrac{5 - 3\sqrt{2}}{7}t, \\ 8v_i + v_j - 3\sqrt{t^2 + 10tv_j - 7v_j^2} > 3t & \text{if } \dfrac{5 - 3\sqrt{2}}{7}t < v_j \leq \dfrac{1}{4}t, \\ v_i - v_j + 3\sqrt{3t^2 + v_j^2} > 6t & \text{if } \dfrac{1}{4}t < v_j \leq t, \\ 2v_i + v_j > 3t & \text{if } v_j > t. \end{cases}$$

Consumer surplus improves if and only if

$$\begin{cases} v_i < v_j + 3\sqrt{2}\sqrt{6t^2 - v_j^2} - 9t & \text{if } v_j \leq t, \\ v_i < v_j + 3\sqrt{2}\sqrt{t(7t - 2v_j)} - 9t & \text{if } v_j > t. \end{cases}$$

Total surplus improves except when $v_1 = v_2 \leq \frac{t}{2}$. Those conditions are in Proposition 5.

In particular, both firm profits and consumer surplus improve if and only if for each $i = 1, 2$,

$$2v_i + v_j > 3t \quad and \quad \begin{cases} v_i < v_j + 3\sqrt{2}\sqrt{6t^2 - v_j^2} - 9t & \text{if } v_j \leq t, \\ v_i < v_j + 3\sqrt{2}\sqrt{t(7t - 2v_j)} - 9t & \text{if } v_j > t. \end{cases}$$

The condition is in Proposition 6.