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REVISITING CES UTILITY FUNCTIONS FOR DISTRIBUTIONAL PREFERENCES: DO PEOPLE FACE THE EQUALITY–EFFICIENCY TRADE-OFF?

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Revisiting CES utility functions for distributional preferences: Do people face the equality–efficiency trade-off?

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Abstract

The constant elasticity of substitution (CES) function is widely used to model distributional preferences in modified dictator games. However, it has been pointed out that its parameter interpretations are inconsistent and problematic in applications. We constructed a model to address this issue by combining two formulations of the CES function. We demonstrated that the proposed model provides consistent interpretations of parameters. Our results clarified that the conventional interpretations of the standard CES function parameters for describing distributional preferences are inappropriate. Notably, "the equality–efficiency trade-off," a conventional interpretation of the substitution parameter, is unrelated to observed individual behaviors.

Keywords: CES, Distributional preferences, Equality, Efficiency, Parameter recovery

JEL codes: C91, D63, D64, D90

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1 Introduction

Modeling individual preferences is a central issue in understanding individuals' other-regarding behavior observed in experiments. One of the most employed modeling strategies is an outcome-based model that focuses on an individual's concerns about the distribution of payoffs among that individual and others (Fehr and Schmidt, 2006, Section 3.1; Cooper and Kagel, 2016, Section II). Among those strategies, the constant elasticity of substitution (CES) utility function of Arrow et al. (1961) has been shown to be a flexible and useful specification in modeling behaviors in modified dictator game experiments (Andreoni and Miller, 2002; Fisman, Kariv and Markovits, 2007).[1]

The attractiveness of the model based on CES utility is due to the theoretical flexibility that encompasses typical forms of other-regarding preferences such as Rawlsian inequality aversion, altruistic, and selfish preferences, each of which corresponds to extreme cases of the CES utility function. More importantly, the CES utility function encompasses these typical forms in a continuous manner using two parameters α—distribution parameter—and ρ—substitution parameter. The parameter α controls the relative importance of the own and others' payoffs. The parameter ρ controls the elasticity of substitution between these payoffs. Modeling that uses the CES utility function enables us to study the heterogeneous nature of individual preferences for "equality" and "efficiency." Indeed, there have been many papers documenting the heterogeneity of preferences and their association with other variables (Andreoni and Miller, 2002; Fisman, Kariv and Markovits, 2007; Fisman, Jakiela and Kariv, 2014, 2015; Fisman et al., 2015, 2022; Li, Dow and Kariv, 2017; Li et al., 2022).[2] The standard

[1] Cox, Friedman and Gjerstad (2007) employed a modified version of the CES utility function to model behaviors observed in several experiments. In this paper, we focus on the direct use of the standard CES utility function, specifically in modified dictator game experiments.

[2] Concern for the efficiency of distribution is also an essential element of the other-regarding preferences that have been studied in the literature (Charness and Rabin, 2002; Engelmann and Strobel, 2004, 2006; Fehr, Naef and Schmidt, 2006; Fehr and

formulation of the CES utility function is also used to study distributional preferences in different choice environments or contexts (Andreoni, 2007; Becker, Häger and Heufer, 2015; Brown, Meer and Williams, 2019; Erkut, 2022; Heufer, van Bruggen and Yang, 2020; Hong, Ding and Yao, 2015; Jakiela, 2013; Korenok, Millner and Razzolini, 2013; Müller, 2019; Porter and Adams, 2016; Robson, 2021).

Despite its appeal, the standard CES utility function is also known to have a theoretical difficulty. That is, in the limit of parameters corresponding to the Rawlsian maximin-type form (i.e., Leontief function form), the distribution parameter becomes ineffective in identifying differences in the utility level or behavior, and thus loses its natural interpretation. In the context of other-regarding preferences, this is a serious problem, because the distribution parameter is supposed to correspond to one of the central concerns, equality of the distribution behaviors. Although Thöni (2015) has articulated it as an essential problem in this context, this issue has not received much attention. On the other hand, in the field of macroeconomics, this theoretical problem is recognized from the very beginning (Arrow et al., 1961, p.231). It has been studied as a problem of how to normalize the CES production function (Klump, McAdam and Willman, 2012; Embrey, 2019). Several variants of the CES function have been proposed, which can be regarded as variations of the normalization constant of the input and output variables (Klump, McAdam and Willman, 2012, Section 2; Embrey, 2019, Section 4).[3]

Similar to theoretical studies, there has been a scarcity of studies on the parameter interpretation of the CES function in analyzing experimental data on distributional preferences. For example, Becker, Häger and Heufer (2015) briefly described potential problems of the standard CES function and attempted to analyze the data more appropriately by combining es-

Schmidt, 2006, Section 4.2; Cooper and Kagel, 2016, Section II.C).

[3]In this paper, we do not discuss in general which normalization is more desirable to use in the analysis of experimental data. Instead, we address the parameter interpretation by combining CES specifications, which have been shown to provide partial solutions in prior studies.

timation methods. However, Becker, Häger and Heufer (2015) does not clarify the problems with the conventional method as we do in the present paper.

Gauriot, Heger and Slonim (2020) is another such example, discussing the potentially inappropriate interpretation of the distribution parameter in conventional group-level analysis using the CES utility function or equivalent functional systems. Specifically, the authors proposed a novel utility function to generalize the CES function. However, its theoretical properties are less clear than the well-studied CES functions.[4] The authors also mainly discussed group-level rather than individual-level analyses. Finally, in Breitmoser (2013), the focus is on various stochastic modeling approaches using the standard CES utility function as opposed to the relationship between parameter values and observed behavior or preferences. Hence, despite its broad applications, the problem in interpreting parameter values of the CES utility function has not been resolved adequately in the context of distributional preferences.

This paper aims to address the interpretation of parameter values of the CES utility function in individual-level analysis. We adopt a strategy using two formulations from the theoretical studies of CES functions, the standard formulation of Arrow et al. (1961) and a variant of Senhadji (1997), combining them in a way that compensates for each other's disadvantages. We formulated a statistical model using a combined class of utility functions.

We reanalyzed the well-known data of Fisman, Kariv and Markovits (2007). The reanalysis showed that the problem of interpreting the distribution parameter is critical in this context, and that the proposed model

[4] The authors proposed to generalize the curvature parameter so that different values can be specified for the individual's and others' payoffs. However, as easily seen from some numerical simulations, it does not seem very promising in terms of solving the problem of interpretation of the distribution parameter. For example, in their formulation, it is possible to move the two curvature parameters, keeping $\alpha = 1/2$ fixed, to represent behaviors ranging from always spending equally to always allocating the most to oneself (or others). We believe this makes whatever α is expected to represent more ambiguous.

provides a more consistent interpretation than the standard CES model. Furthermore, the consistent interpretation of the distribution parameter revealed that the conventional interpretation of the substitution parameter—"the equality–efficiency trade-off"—may be misleading. In other words, "the equality–efficiency trade-off" interpretation of the substitution parameter is a mathematical artifact that stems from a shortcoming of the standard CES function in that it cannot characterize a specific range of behaviors. Hence, it is not primarily rooted in the behavior of the subjects.

We also conducted a parameter recovery analysis using simulations to examine the behavior of the model (Wilson and Collins, 2019). The results revealed that while both two parameter estimations were reasonably recovered over a broad range of possible parameter values, the estimated distribution parameter was more reliable than the estimated substitution parameter. The instability of the substitution parameter estimation depends mainly on the value of the distribution parameter rather than the value of the substitution parameter. Based on these facts, we present specific points that should be considered by researchers using this method.

The remainder of the paper is organized as follows. Section 2 provides details of the shortcoming of the standard CES utility function and describes our model, which characterizes a broader range of behaviors. Section 3 presents the results of a reanalysis of the well-known data of Fisman, Kariv and Markovits (2007) and Section 4 presents the parameter recovery simulation. Section 5 concludes.

2 Theory

In this paper, we propose a modified CES utility model to analyze distributional preferences based on the experimental methodology of Fisman, Kariv and Markovits (2007, henceforth FKM). In their methodology, each subject participates in a modified dictator game[5] and is asked to allocate mone-

[5]The modified dictator game was developed by Andreoni and Miller (2002) to test whether the redistribution decisions were consistent with the utility maximization model.

tary payoffs between themselves and another anonymous subject. Each allocation decision is made by choosing a pair of monetary payoffs (x_1, x_2) from the budget line $p_1x_1 + p_2x_2 = 1$, where x_1 and x_2 correspond to the payoffs to the self (the subject) and the other (an anonymous subject), respectively. The ratio p_2/p_1 represents the price of giving. Each subject makes 50 allocation decisions defined by 50 different prices of giving.

The experimental data are analyzed by using the utility maximization model, which is a classical economic model of consumer behavior.[6] In the utility maximization model, it is assumed that there is a utility function $U(x_1, x_2)$ and that the payoff allocation (x_1, x_2) is generated from the following optimization problem:

$$\begin{aligned} \max_{x_1,x_2} \quad & U(x_1, x_2) \\ \text{s.t.} \quad & p_1x_1 + p_2x_2 \leqq 1 \\ & x_1, x_2 \geqq 0. \end{aligned} \tag{1}$$

As seen in the above formulation, a crucial methodological decision for properly modeling the redistribution behavior is the specification of the parametric class of the utility function, which is explained in the following subsection.

2.1 Utility function specification

In this subsection, we first describe the two conventional formulations and their shortcomings. Then, using these two conventional specifications, we propose a new class of parametric utility functions.

In FKM, the standard CES function originally derived by Arrow et al. (1961) was used as a class of utility functions. The standard CES utility

[6]In the literature, a conventional data analysis procedure consists of two consecutive parts. The first part is the revealed preference analysis (Afriat, 1967, 1972; Chambers and Echenique, 2016), and the second is the econometric analysis described in the present paper. However, in principle, these two procedures can be used independently. Therefore, for simplicity, this paper focuses only on the econometric analysis.

function is defined as follows:

$$U(x_1, x_2) = (\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho})^{-\frac{1}{\rho}} \tag{2}$$

where $\alpha \in [0,1]$ is the *distribution parameter* and $\rho \in [-1, \infty) \setminus \{0\}$ is the *substitution parameter.* The distribution parameter α controls the relative importance of the payoffs. The substitution parameter ρ controls the curvature of its indifference curves and characterizes the *elasticity of substitution* $\sigma \in (0, \infty)$ as

$$\sigma = \frac{1}{\rho + 1}. \tag{3}$$

The standard CES utility function has three typical forms in the limit of the substitution parameter. Specifically, the utility function (2) tends to a *symmetric* Leontief utility function $U(x_1, x_2) = \min\{x_1, x_2\}$ as $\sigma \to 0$ ($\rho \to \infty$), a Cobb–Douglas utility function $U(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$ as $\sigma \to 1$ ($\rho \to 0$), and a linear utility function $U(x_1, x_2) = \alpha x_1 + (1-\alpha)x_2$ as $\sigma \to \infty$ ($\rho \to -1$). Figure 1 visualizes three limits.

It is known that special care must be taken when using CES-type functions for modeling (Arrow et al., 1961, p.231). As seen in the first case, about the limit of the function, which is visualized in the left panel of Figure 1, the distribution parameter becomes less effective at describing individual behaviors as σ approaches 0. In other words, as σ approaches 0, the ability of α to represent the relative importance of the payoffs decreases. This is an essential problem when studying altruistic preferences (Thöni, 2015). Indeed, the ineffectiveness of the distribution parameter means that the behavioral tendency to give some of one's money to others may not be represented by the parameter, even if it is present in the observed allocation decisions.

Fortunately, there is another formulation of the CES function that overcomes this well-known shortcoming. In Senhadji (1997), a reformulated CES function was defined as follows:

$$U(x_1, x_2) = (\alpha^{1+\rho} x_1^{-\rho} + (1-\alpha)^{1+\rho} x_2^{-\rho})^{-\frac{1}{\rho}} \tag{4}$$

where $\alpha \in [0, 1]$, $\rho \in [-1, \infty) \setminus \{0\}$. Figure 2 visualizes three limits of this formulation. The reformulated CES utility function (4) tends to a Leontief utility function $U(x_1, x_2) = \min\{x_1/\alpha, x_2/(1-\alpha)\}$ as $\sigma \to 0$ ($\rho \to \infty$) where σ is defined as (3). Moreover, for $\sigma \to 1$ ($\rho \to 0$), the limit is $U(x_1, x_2) = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\, x_1^{\alpha} x_2^{1-\alpha}$, which is a monotonic transformation of the standard CES function (2) case. Hence, in this formulation, we can circumvent the well-known deficit of the standard CES function. However, on the contrary, this function approaches the *symmetric* linear utility function $U(x_1, x_2) = x_1 + x_2$ as $\sigma \to \infty$ ($\rho \to -1$), as visualized in the right panel of Figure 2. Therefore, unfortunately, the distribution parameter becomes ineffective in describing individual behaviors as σ approaches ∞, which would also be an essential problem when studying altruistic preferences for the same reason discussed above.

Therefore, in this study, the class of utility functions was formulated by the following combined definition of the above CES functions:

$$U(x_1, x_2) = \begin{cases} (\alpha^{1+\rho} x_1^{-\rho} + (1-\alpha)^{1+\rho} x_2^{-\rho})^{-\frac{1}{\rho}} & \text{if } \rho > 0 \ (\iff \sigma < 1) \\ \alpha \log(x_1) + (1-\alpha)\log(x_2) & \text{if } \rho = 0 \ (\iff \sigma = 1) \\ (\alpha x_1^{-\rho} + (1-\alpha) x_2^{-\rho})^{-\frac{1}{\rho}} & \text{if } \rho < 0 \ (\iff \sigma > 1) \end{cases} \quad (5)$$

where $\alpha \in [0, 1]$, $\rho \in [-1, \infty)$, and $\sigma = 1/(\rho + 1)$. The validity of this formulation is clear. As mentioned above, each of the two conventional formulations has its advantages and disadvantages in the exact opposite limit of the parameter. Therefore, if we combine the two properly, we can utilize the advantages of the two formulations while avoiding the disadvantages. Moreover, the combined formulation (5) has a well-defined and well-known demand function, including the case $\sigma \to 1$. Indeed, in both of the conventional formulations (2) and (4), the corresponding demand functions are exactly the same as those derived from the Cobb–Douglas utility function.[7] A visualization of indifference curves corresponding to

[7] In Thöni (2015), another reformulation was proposed as a remedy for the short-

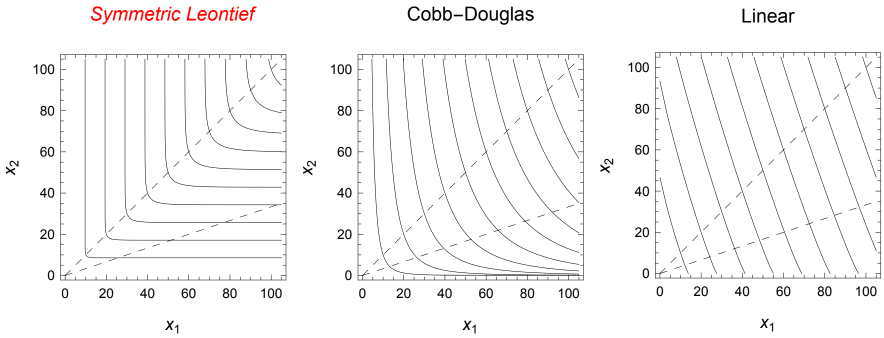

Figure 1: The standard CES function of Arrow et al. (1961)

Notes: Indifference curves of standard CES utility functions with $\alpha = 0.75$. Each panel depicts an approximation of a limiting case: $\sigma = 0.1$, $\sigma = 1 + 10^{-3}$, and $\sigma = 10$ are depicted in the left, middle, and right panels, respectively. Dashed lines depict equations $x_1 = x_2$ and $x_1/\alpha = x_2/(1-\alpha)$. Note that the Leontief case (the left panel) approximates the *symmetric* Leontief function while we set the distribution parameter *asymmetric*.

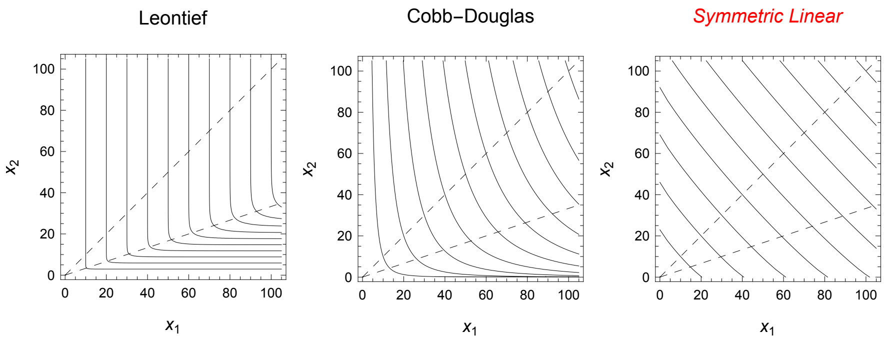

Figure 2: The CES function of Senhadji (1997)

Notes: Indifference curves of the reformulated CES utility functions of Senhadji (1997) with $\alpha = 0.75$. Each panel depicts an approximation of a limiting case: $\sigma = 0.1$, $\sigma = 1 - 10^{-3}$, and $\sigma = 10$ are depicted in the left, middle, and right panels, respectively. Dashed lines depict equations $x_1 = x_2$ and $x_1/\alpha = x_2/(1-\alpha)$. Note that the linear case (the right panel) approximates the *symmetric* linear function while we set the distribution parameter *asymmetric*.

this extended formulation would be a figure made from the above two figures in the following order: left of Figure 2, middle of Figure 2, middle of Figure 1, and right of Figure 1.

In the analysis, the share form of the demand function, which is derived from the utility maximization model (1) with the utility function specified by (5), is used as in FKM:

$$s = p_1 x_1 = \begin{cases} \frac{\left(\frac{\alpha}{1-\alpha}\right)}{\left(\frac{\alpha}{1-\alpha}\right)+\left(\frac{p_2}{p_1}\right)^{1-\sigma}} & \text{if } \sigma \leqq 1 \\ \frac{\left(\frac{\alpha}{1-\alpha}\right)^{\sigma}}{\left(\frac{\alpha}{1-\alpha}\right)^{\sigma}+\left(\frac{p_2}{p_1}\right)^{1-\sigma}} & \text{otherwise} \end{cases} \tag{6}$$

where $\alpha \in [0,1]$ and $\sigma \in (0,\infty)$. Note that the amount of wealth is normalized to one.

One can easily see that the distribution parameter α is always effective in describing individual behaviors regardless of the elasticity of substitution σ, including its limits as σ tends to $0, 1$, or ∞. Moreover, in this formulation, the behavioral interpretation for the distribution parameter α is clear for all three limiting cases. First, in the Leontief function case, i.e., $\sigma \to 0$, the distribution parameter characterizes the ratio of payoffs as $x_1/x_2 = \alpha/(1-\alpha)$. Second, in the Cobb–Douglas function case ($\sigma = 1$), it characterizes the ratio of expenditure shares, i.e., $p_1x_1/p_2x_2 = \alpha/(1-\alpha)$. Finally, in the linear function case, i.e., $\sigma \to \infty$, it characterizes the degenerated allocation for self as $p_1x_1 = 1$ if $p_1/p_2 \leqq \alpha/(1-\alpha)$ and $p_1x_1 = 0$ otherwise.[8]

This gives us a good theoretical reason to interpret the distribution parameter as a measure of preference for equality. Indeed, if we focus

comings of (2) and (4). However, the proposed formulation has another property that when $\sigma \to 1$, the demand function approaches that of the Cobb–Douglas utility function with the distribution parameter $\alpha^2/\big(\alpha^2 + (1-\alpha)^2\big)$, where $\alpha \in [0,1]$. Note that this distribution parameter is equal to α only if $\alpha = 0, 0.5, 1$. Therefore, to maintain the classical interpretation of the limiting case in the center, i.e., when $\sigma \to 1$, we adopt the combined formulation (5).

[8]In the linear case, if $p_1/p_2 = \alpha/(1-\alpha)$, then any choice satisfying the budget constraint is optimal.

on $\alpha = 1/2$, we see that, in every case identified above, the behavior generally treats some quantity as equal depending on the price—in the case of Leontief, it is the amount of payoffs; in the case of Cobb–Douglas, the amount of spending; and in the linear case, the opportunity to occupy payoffs. These seem to belong to a special case of "proportional equality," the traditional framework since Aristotle for discussing equality in relation to distributive justice (Gosepath, 2021, Section 2.2). More precisely, if we set $\alpha = 1/2$, then

$$\frac{x_1}{x_2} = \left(\frac{p_2}{p_1}\right)^{\sigma}, \tag{7}$$

hence, it satisfies the form of "proportional equality." From this point of view, the elasticity of substitution σ controls how strongly the decision maker considers the relative price information as a valid signal to make the distribution equal. As we will see in Section 3, this consistent interpretation of the distribution parameter reveals problems with "the equality–efficiency trade-off" interpretation of the substitution parameter ρ.

Finally, the limitations of this formulation should also be noted. As can easily be seen from the definition of the utility function (5), in the selfish case: $\alpha = 1$, the (elasticity of) substitution parameter, ρ (or σ), becomes irrelevant.[9] In other words, the model does not satisfy identifiability in the sense that more than one parameter pair corresponds to the same behavior corresponding to the selfish case. This theoretical shortcoming is not a problem posed only by our extension, but is a drawback associated with the use of CES-type functions in general. In the following, we also report the practical implications of these shortcomings to clarify the limitations.

2.2 Statistical model specification

In this study, to estimate the two parameters of the extended CES utility function, a Bayesian statistical model was formulated. The model involves

[9]Although the exact opposite case ($\alpha = 0$) raises similar issues, here, we only discuss $\alpha = 1$, which typically appears in the context of altruistic preferences.

novelties to Bayesian modeling but are otherwise straightforward extensions of standard methods such as Andreoni and Miller (2002) and FKM. Specifically, an error term distribution of the share form demand and prior distributions for the parameters were specified as described below.

2.2.1 Residual error term distribution

A truncated normal distribution was used for the residual error term. Hence, it is assumed that an observation of the expenditure share is distributed on the unit interval in proportion to the normal distribution with location s and scale σ_s. This formulation leads to a natural interpretation that the expenditure share in the allocation decision is likely to be around the theoretical share s.[10]

2.2.2 Prior distributions for parameters α, σ and σ_s

For the distribution parameter α, the uniform distribution on the unit interval is used as a non-informative prior distribution.

For the elasticity of substitution σ, the prior distribution was specified on $\log(\sigma)$ rather than on σ. Figure 3 visualizes the reason for this decision: the symmetric treatment of the three limiting cases. As we see in the previous subsection, in the original scale (i.e., σ), the Leontief utility function is placed at zero and the linear utility function at infinity, treating both asymmetrically. On the other hand, this reparameterization allows us to treat them symmetrically by placing the Leontief utility function at negative infinity and the linear utility function at positive infinity, with the

[10]In the literature, a censored normal distribution, i.e., the two-limit Tobit model, has been used for the distribution of the error term. However, the two-limit Tobit model can lead to unnatural interpretations when the scale σ_s is moderately large. For example, when $s = 0.8$ and $\sigma_s = 0.1$, the expenditure share is more likely to be in the right corner ($s = 1$) instead of a point closer to the theoretical share $s = 0.8$. For this reason, we specified the error term with the truncated normal distribution instead of the censored distribution. Note that the former is also a standard specification in related literature (Andreoni, Gillen and Harbaugh, 2013; Ahn et al., 2014; Echenique, Imai and Saito, 2020, Online Appendix).

Cobb–Douglas utility function at the center. This symmetry is visible if we see the parameters in the $(\sigma, \rho+1)$-plane as in Figure 3.

The prior distribution for $\log(\sigma)$ was set to the Student's t-distribution with degrees of freedom $\nu = 4$. The reason for this choice lies in Figure 4. The Student's t-distribution prior on $\log(\sigma)$ (left panel) can treat each of the three limits of the CES utility function more uniformly than can the *uniform* prior (right panel). In other words, the uniform distribution is not *non-informative* here because the uniform distribution on $\log(\sigma)$ unintentionally emphasizes the Leontief and linear functions, as seen in the right panel. The Cauchy distribution (the Student's t-distribution with degree of freedom $\nu = 1$) and the normal distribution (the Student's t-distribution with degree of freedom $\nu = \infty$) can be candidates for the prior distribution. However, the degree of freedom $\nu = 4$ is adopted here to make the statistical model weakly informative and as parsimonious as possible (Gelman, Simpson and Betancourt, 2017).

Finally, for the scale parameter σ_s of the residual error term, the exponential distribution with rate parameter $\lambda = 0.5$ was adopted as a prior distribution. The rationale for this choice is as follows. First, note that because demand is modeled in the share form (6), which can be expressed within a unit interval, an error term with a scale of $\sigma_s > 1.5$ is sufficient to mimic the uniformly random choice model proposed in Bronars (1987). Furthermore, the 0.95-quantile and 0.99-quantile of the exponential distribution with the rate parameter $\lambda = 0.5$ are about 1.5 and 2.3, respectively. Therefore, this prior distribution assigns a positive probability value even to models that can be considered as almost completely random choice behavior, and eventually, it does not seem to be very restrictive. This choice allows us to keep the model as parsimonious as possible (Gelman, Simpson and Betancourt, 2017).

2.3 Rescaling $\log(\sigma)$ to τ

Although the model was formulated by three parameters $\alpha, \log(\sigma)$, and σ_s, a different scale was used to analyze the data. In particular, $\log(\sigma)$ was treated by converting to the value of F_4: the cumulative distribution

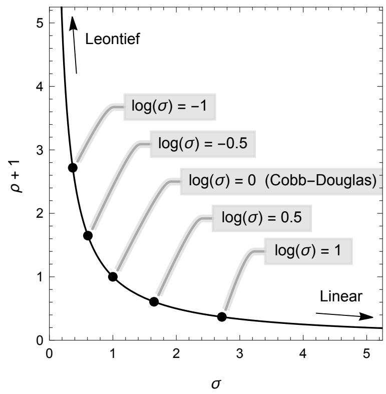

Figure 3: The symmetric reparameterization $\log(\sigma)$

Notes: Equation $\rho + 1 = 1/\sigma$ and points $(\sigma, \rho + 1)$ are depicted. The plotted points correspond to symmetric values $\log(\sigma) \in \{-1, -0.5, 0, 0.5, 1\}$.

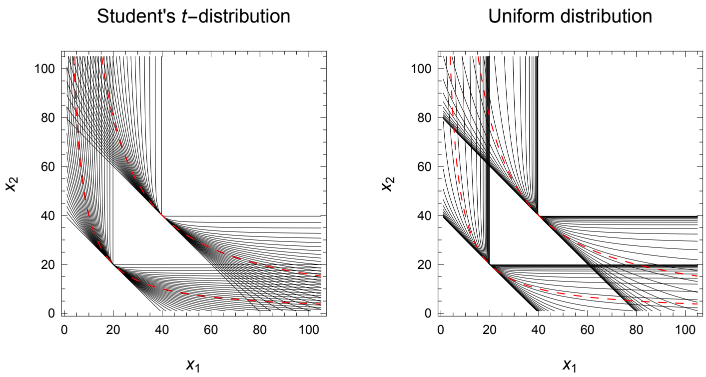

Figure 4: Prior distributions on $\log(\sigma)$

Notes: Indifference curves of the extended CES functions with $\alpha = 0.5$ and 30 different values of $\log(\sigma)$ (solid lines) and the Cobb–Douglas utility function (dotted lines). **Left:** 30 different values of $\log(\sigma)$ are the r-quantile points of the Student's t-distribution with degree of freedom $\nu = 4$ where r takes a value corresponding to one of the points that divide interval $[0.005, 0.995]$ into 29 equal length intervals. **Right:** 30 different values of $\log(\sigma)$ are the $k/29$-quantile points ($k = 0, 1, \ldots, 29$) of the uniform distribution on the closed interval between the 0.005-quantile and the 0.995-quantile points of the Student's t-distribution with $\nu = 4$. Note that if we use the uniform distribution on a broader interval, it gives more emphasis to the Leontief and linear functions.

function of the Student's t-distribution with $\nu = 4$.[11] Let τ be the *rescaled substitution parameter* defined as follows:

$$\tau = F_4\big(\log(\sigma)\big) = \frac{1}{2} + \frac{\log(\sigma)\big(\log(\sigma)^2 + 6\big)}{2\big(\log(\sigma)^2 + 4\big)^{\frac{3}{2}}} \tag{8}$$

where $\log(\sigma) \in (-\infty, \infty)$. Note that τ is defined by using a one-to-one correspondence from ρ: the Leontief limit $\rho \to \infty$ corresponds to $\tau \to 0$, the Cobb–Douglas limit $\rho \to 0$ corresponds to $\tau = 1/2$, and the linear limit $\rho \to -1$ corresponds to $\tau \to 1$. Eventually, data analysis was conducted based on the parameters α, τ, and σ_s.

This rescaling has two methodological advantages. First, because τ is in the unit interval, it is normalized to be on the same scale as α. Using the same scale helps us compare the variability of α and τ in the analysis. Second, this rescaling enables a well-balanced treatment of the three limits of the CES utility function. Indeed, as can be seen in Figure 4, if parameter $\log(\sigma)$ is weighted by the Student's t-distribution with $\nu = 4$, then the three limits of the CES utility function are more balanced than the original scale that corresponds to a uniform weight on $\log(\sigma)$. That is to say, the cumulative distribution function F_4 can map the three limits of the CES utility function on the unit interval in a balanced way.[12]

3 Reanalysis of Fisman, Kariv and Markovits (2007)

We reanalyzed FKM's replication data (Fisman, Kariv and Markovits, 2019).[13] For comparison, we also estimated the standard CES version of

[11] See, for example, Shaw (2006).

[12] Note that the prior distribution set to $\log(\sigma)$ can also be interpreted as a uniform prior distribution on the parameter τ. In other words, τ is a scale to normalize and, at the same time, balance the three limiting cases of the CES utility function. Note that our Bayesian method can be modified to a maximum likelihood method or a least squares method by using some optimization methods (Gelman et al., 2013, Section 4.5). However, such modifications are not essential for the purposes of this paper.

[13] We used only the data of the two-person budget set experiment in FKM.

our model, that is, a model specified by the standard CES utility function (2) and the statistical assumptions described in Subsection 2.2.

Parameter estimation is conducted by simulating the posterior distribution of parameters by Markov chain Monte Carlo (MCMC) methods. The analysis is implemented in R (R Core Team, 2016) using MCMC methods in Stan (Carpenter et al., 2017; Stan Development Team, 2022).

3.1 Meaningfulness of the distribution parameter

First, Figure 5 shows that our extended CES model is more appropriate than the standard CES model for estimating the distribution parameter α. When compared with the observed choices depicted in the top panel of each column, the posterior distribution of α and its estimate for our extended CES model (the vertical dashed line depicted in the panel labeled "Extended") seem to be more relevant than the standard CES version (labeled "Standard"). More specifically, the posterior distribution of the standard CES version of our model spreads if the observed choices are concentrated on the diagonal line, and shrinks around $\alpha = 1$ if the observed choice is not on the diagonal line. This is caused directly by the vanishing of the distribution parameter in the standard CES utility function if $\rho \approx \infty$ (or $\tau \approx 0$). By contrast, the extended CES model helps us interpret the distribution parameter α consistently over individuals.

3.2 "Equality–efficiency trade-off"

Here, we point out that interpreting the CES parameter ρ as "the equality–efficiency trade-off" is inappropriate. In the literature, the parameter ρ is often taken to represent "the equality–efficiency trade-off" that individual subjects face when distributing payoffs. This interpretation stems from the fact that in the standard CES model, the case $\rho \to \infty$ (or $\tau \to 0$) corresponds to the *symmetric* Leontief function. According to the standard CES model, as ρ positively diverges, choices are concentrated around the diagonal line $x_2 = x_1$, regardless of relative prices (see Figure 1). Thus, the equality of payoff allocations increases, while at the same time, the effi-

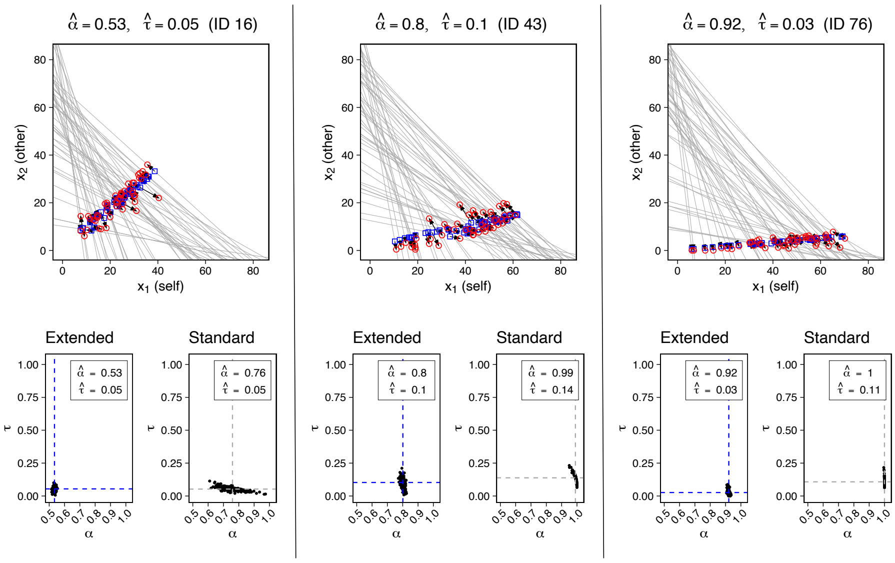

Figure 5: Meaningfulness of the extended CES model

Notes: Each column corresponds to an individual subject, whose ID is presented on the top panels. The posterior mean estimates $\hat{\alpha}$ and $\hat{\tau}$ using our extended model are also presented on the top panels. **Top:** The observed choices of an individual subject (red circle), the estimated choices based on our model (blue square), and the residuals defined by the differences between the two (black arrow). **Bottom:** The posterior samples of (α, τ) based on our model (labeled "Extended") and the standard CES version of the model (labeled "Standard"), i.e., a model specified with the utility function (2) and the statistical assumptions described in Subsection 2.2. The dashed lines indicate posterior mean estimates, which are also presented at the top-right of each panel.

ciency of payoff allocations decreases. However, Figure 5 shows that these subjects did not face such a "trade-off." While the τ estimates do not differ much between these subjects in both the standard and extended CES models, the observed behavioral tendencies for minimizing the differences (maximizing equality) vary considerably between subjects—obviously, ID-16 allocates equality-oriented, while ID-76 behaves quite selfishly, if not perfectly, although they both give up efficiency. It is still reasonable to interpret a large ρ (or a small τ) as low-efficiency orientation as conventionally assumed. However, it is not appropriate to interpret a large ρ as a high-equality orientation because the value of ρ solely does not discriminate between equality-oriented ID-16 and selfish ID-76. The mathematical shortcoming of the standard CES function—for $\rho \to \infty$ (or $\tau \to 0$), the effect of α vanishes, and the representation of the various behavioral patterns degenerates to the equality orientation—has led to the mistaken understanding of ρ as the "equality–efficiency trade-off." Note that although our extension of the CES function resolves α's degeneracy, it is still not plausible to interpret a large ρ as a high-equality orientation.

Let us now consider the interpretation of the two parameters of the extended CES model. In the extended CES model, τ represents intensity of maximizing efficiency by appropriately controlling the strength of the behavioral reaction to price changes regardless of the value of α, excluding the extreme case $\alpha = 1$. As noted in section 2.1, focusing on $\alpha = 1/2$, the three extreme cases of Leontief, Cobb–Douglas, and linear are special cases of "proportional equality," so the distribution parameter α seems appropriate as a measure of preference for equality. From this fact, we can also say that τ is a parameter that organizes the various forms of equality-focused behavior in terms of their different responses to price changes (or differences in the intensity of maximizing efficiency). Therefore, the extended CES model maps the distributional behavior of subjects—including egalitarian, non-egalitarian, and these intermediate—using two *separate* axes of "equality" and "efficiency."[14]

[14] It is reasonable to assume that this trade-off exists in distributional behavior if one

Finally, although the CES parameters are a map that organizes various distributional behaviors, this is not a complete list of possible behavioral patterns. Which of these various behaviors is socially more desirable is not self-evident. Indeed, the desirability issue seems to have a substantial relationship with one of the main issues in political philosophy (Gosepath, 2021, Section 3). Thus, in general, one should be cautious about judging small (or large) values of CES parameters as desirable or just a priori.

3.3 Comparison with the standard CES specification

Figure 6 shows estimation differences between our model and the standard CES version across all subjects. As intended in the utility function specification, the estimation differences for the distribution parameter α are prevalent for the case $\hat{\tau} < 0.5$. Moreover, in most of these cases, the estimated α of our model is considerably smaller than that of the standard CES model. The reason for this result can be inferred from Figure 5. If a subject behaves so that the flexible CES estimate is around $\alpha = 0.5$ (as in the left panel of Figure 5), then the posterior mean of the standard CES model must be higher than 0.5 because such behavior is compatible with a broad range of $\alpha > 0.5$ for the standard CES model. On the other hand, if a subject behaves so that the flexible CES estimate has $\alpha > 0.5$ (as in the middle and right panels of Figure 5), then the posterior mean of the standard CES model must be higher than that value because such behavior is only compatible with some narrow range around $\alpha \approx 1$ for the standard CES model. Thus, by using the standard CES specification, each of these

intentionally designs a set of choices such that subjects face an "equality–efficiency trade-off," Engelmann and Strobel (2004, 2006) are examples of such experiments. However, whether this trade-off represents a typical real-world constraint is context-dependent and debatable. For example, it has long been argued that equality does not necessarily sacrifice (to a large extent) efficiency (Okun, 2015, Section 3 and 4; Herzog, 2021, Section 4.2). Andersen and Maibom (2020) have noted that "empirical knowledge on the precise form and slope of the trade-off is scant." They have also argued for the trade-off between equality (expressed using the Gini coefficient) and efficiency (described as average income per capita) using cross-country data. However, even in this existent case, it has been shown that many countries would not face this trade-off.

subjects would be considered more selfish than their behavioral tendencies would indicate.

For the case $\hat{\tau} > 0.5$, the estimation differences on α are much less prevalent. This also reflects our intention because the utility function in our model is defined by the standard CES function when $\tau > 0.5$. However, note that even for $\hat{\tau} > 0.5$, we see differences in the estimates, which are concentrated around $\alpha = 1$. More precisely, for some cases such that $\hat{\tau} > 0.5$ and $\hat{\alpha} \approx 1$, the estimated τ are smaller in our model than in the standard CES model. Figure 7 shows the reason behind this result. As can be seen in the right panel, in the standard CES model, the posterior distribution of τ is not consistent with the range $\tau < 0.5$ because most of the parameters (α, τ) in that range correspond to a *symmetric* Leontief function (i.e., a tendency to maximize equality) that is far from the observed behavior presented in the left panel (i.e., a tendency to minimize equality). On the other hand, in our model, the posterior distribution of τ is consistent with the range $\tau < 0.5$ because the parameter α represents intensity of minimizing inequality regardless of the value τ. Therefore, the posterior distribution of τ becomes broader, and its posterior mean becomes smaller in our model than in the standard CES model.

3.4 Inferential uncertainty

First, note that the wide posterior distribution of τ, shown in Figure 7, is not a drawback of our model compared with the standard CES model. Rather, our model adequately represents the uncertainty because the observed behavior is consistent with a wide range of τ. In the standard CES model, on the other hand, the observed behavior is mainly consistent with $\tau > 0.5$, but this is primarily due to the ineffectiveness of α in the $\tau < 0.5$ area, rather than an observation of some clear tendency to respond to price changes.

However, it is also true that this variability is a problem that needs to be addressed as long as we use the CES utility function framework in the context of altruistic preferences. Indeed, as can be seen from definition (5), for any value of τ, $\alpha = 1$ corresponds to a pure selfish preference that only

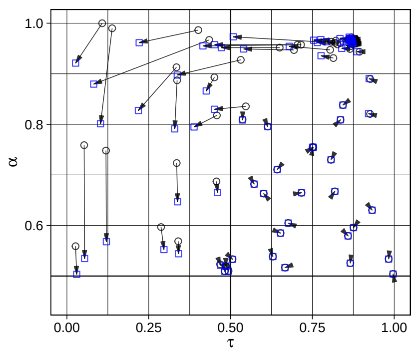

Figure 6: Estimation differences between models

Notes: Posterior mean estimates of our model (blue square) and the standard CES version of it (black circle), and differences between the two estimations for each subject (black arrow).

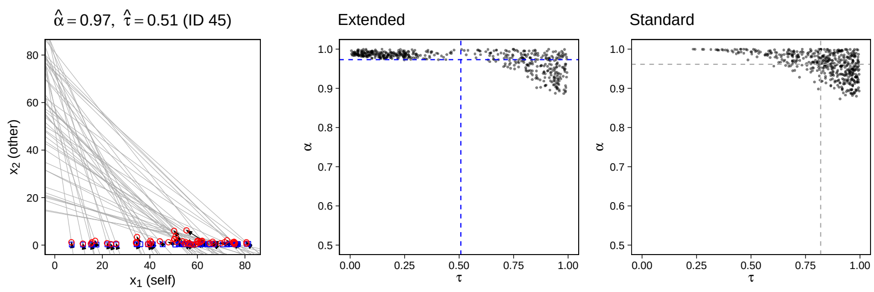

Figure 7: Estimation differences when $\hat{\alpha} \approx 1$

Notes: A case such that $\hat{\alpha} \approx 1$ causes smaller $\hat{\tau}$ in our model than in the standard CES model. **Left:** The observed choices of an individual subject (red circle), the estimated choices based on our model (blue square), and the residuals defined by the differences between these two (black arrow). The posterior mean estimates of α and τ are also indicated on the top. **Middle and Right:** The posterior samples of (α, τ) based on our model (labeled as "Extended") and the standard CES version of the model (labeled as "Standard"). The dashed lines indicate posterior mean estimates.

considers x_1 (the payoff to self) and ignores x_2 (the payoff to the other). Hence, when $\alpha = 1$, the value of τ can be anything, and thus, it is not identifiable. Moreover, in the context of distributional preferences, some selfish preference-type subject may correspond to $\alpha \approx 1$ in the CES utility function framework. Therefore, in such cases, an estimate of τ tends to be noisy because many values of τ are consistent with the observed behavior that indicates $\alpha \approx 1$. Figure 8 visualizes this point. As shown in the figure on the left, there are a non-negligible number of subjects who show $\hat{\alpha} \approx 1$ in this experiment; and in the figure on the right, the uncertainty in the posterior distribution of τ is large for those subjects.

Finally, Figure 8 suggests two more patterns of inferential uncertainty of the model. First, the uncertainty of α is smaller than that of τ. This can be seen from the fact that the vertical bars of each marker tend to be relatively shorter than the horizontal bars.[15] Second, the uncertainty is larger when the estimated scale of residual error $\hat{\sigma_s}$ is large. This can be seen from the fact that the bars of each marker tend to be relatively longer for the red triangles than for the blue circles.

In the next subsection, we present simulation results to confirm these general patterns of our model's inferential uncertainty in a more systematic way, and to provide practical guidelines for using this methodology in the context of distributional preferences.

4 Parameter recovery simulation

To see general patterns of our model's inferential uncertainty, we conducted a parameter recovery simulation, which is a simple yet important practice for any model-based behavior analysis (Wilson and Collins, 2019).

We conducted a simulation study by generating synthetic data sets fixing parameters α, τ, and σ_s. In particular, α and τ are fixed so that

[15] Note that the scales of the axes are different in the right panel. In particular, if the length of the vertical and horizontal bars on a marker is the same, it means that the former is actually smaller than the latter.

the overall simulation covers a valid range such that $\alpha \geqq 0.5$ and $\tau \in [0,1]$.[16] Moreover, σ_s was fixed at $\sigma_s = 0.01, 0.05, 0.1$, and 0.15 to cover a typical range obtained by our estimation in the FKM data. For any fixed parameters α, τ, and σ_s, we generated 200 data sets, each consisting of 50 choices. Hence, in total, we generated $6 \times 11 \times 4 \times 200 = 52,800$ data sets in the entire simulation. In the following, we focus on the cases $\sigma_s = 0.01$ and $\sigma_s = 0.1$. Results for other cases are presented in the Appendix.

Each choice is generated by first computing the share demand from equation (6) and then generating a noisy share demand from the truncated normal distribution described in 2.2.1 in subsection 2.2.[17] Note that for comparison purposes, we used an identical price system with 50 specific budget constraints throughout the entire simulation.

4.1 Parameter identification (Case $\sigma_s = 0.01$)

First, by focusing on the case of $\sigma_s = 0.01$, we observe the impact of the parameter identification problem. Figure 9 shows simulation results for this case.

The first thing to note is that for most of the true parameters, we can recover their values by our inference procedure. In particular, we are able to recover the distribution parameter α at a level that can be used in practice. This is good news because the distribution parameter is a more fundamental parameter in the context of distributional preferences. Moreover, the rescaled substitution parameter τ can also be recovered in large part of the entire combinations of true parameter values. However, note that the estimation of τ is relatively more variable than that of α in each combination.

[16] More precisely, α was set at $\alpha = 0.5, 0.6, 0.7, 0.8, 0.9$, and 0.99, and τ was set at $\tau = 0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$, and 0.99. Note that we ignore $\alpha < 0.5$ because it is symmetric to the case $\alpha > 0.5$.

[17] Parameter σ is computed from fixed τ by using the quantile function of the Student's t-distribution with $\nu = 4$ and the exponential function. See the definition of τ in (8) in subsection 2.3.

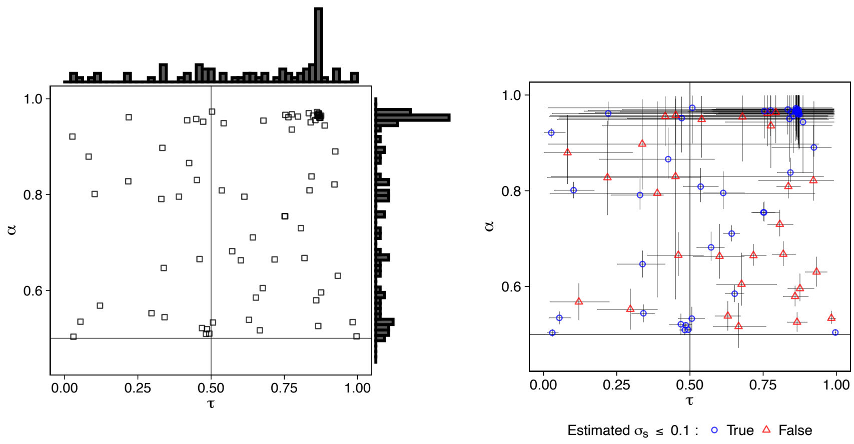

Figure 8: Scatterplot of the extended CES estimates

Notes: **Left:** Posterior mean estimate of (τ, α) and marginal histograms. **Right:** Posterior mean estimate (blue circle or red triangle) with 0.025- and 0.975-quantile of the marginal posterior distributions of α and τ (bars on a marker).

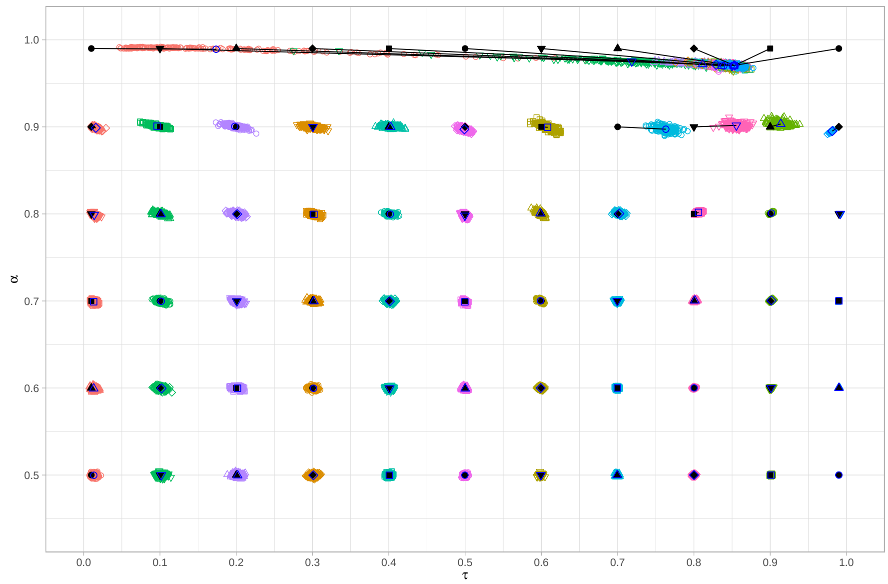

Figure 9: Estimation results of the simulation with $\sigma_s = 0.01$

Notes: For each true parameter (τ, α), the true parameter value (black closed marker), 200 corresponding simulation results (colored open markers), and mean of the 200 simulations (blue open marker) are depicted. Note that true parameter values and corresponding means calculated by 200 simulations are connected by black lines.

On the other hand, as expected from definition (5), the parameter identification problem for τ is severe when $\alpha = 0.99$. This is a common feature for any CES specification and hence, inevitable. Even when $\alpha = 0.9$, identifying τ is difficult for cases such that $\tau \geqq 0.7$. It should also be noted that this latter difficulty stems from not only the specification of the CES, but also the finite nature of the experimental method. Specifically, because the budget system consists of a finite number of price systems, there is a limited amount of information to identify subtle differences in τ.

4.2 Inferential uncertainty (Case $\sigma_s = 0.1$)

Finally, we focus on a more realistic case, $\sigma_s = 0.1$, to examine the characteristics of the inferential uncertainty of our estimation procedure. Figure 10 shows the result. The first thing to note is that, as expected in the previous section, the inferential uncertainty is larger when the scale of residual error σ_s is large. As in the previous case, α can be recovered more precisely than τ. Indeed, if we note that the horizontal and vertical axes in this figure are drawn on the same scale, we see that the α estimate is less likely to overlap with its neighbor 0.1 away when compared with τ. We also see that while the estimates for τ are not so precise, they do not overlap significantly with parameters that are 0.2 apart, which suggests that they hold reasonable information. However, it should be noted that the bias and variability of the estimates become more severe for $\alpha = 0.9$ and 0.99.

Hence, to observe this uncertainty more quantitatively, Figure 11 summarizes the same results in terms of inferential error. For both parameters, the magnitude of estimation error largely depends on the value of α rather than that of τ. In general, the estimation error is relatively smaller for α than for τ, but even for α, distinguishing subtle differences seems difficult when $\alpha \approx 1$. However, estimation for α is reasonably reliable to distinguish large differences. Moreover, estimation for τ is also reliable when we focus on the cases where the value of α is not extreme such as $\alpha \leqq 0.8$. Note that in the FKM data, a non-negligible number of subjects were in that reliable range, and the variation of τ in those subjects was not trivial (see Figure 8). Hence, even though these parameter estimates are not perfect, we may be able to extract valid and meaningful information about behavioral differences and the implications of these differences.

4.3 Discussion of the Results

Overall, these results suggest that special care should be taken when using this experimental procedure in practice, especially when analyzing the value of τ. Especially when $\alpha \approx 1$, it is inevitable that we consider these limitations in the analysis if we aim to obtain consistent results among different subjects. In the context of distributional preferences, such difficult cases are not merely logical possibilities because most experiments must involve some proportion of selfish preference-type subjects. For example, in such cases, τ can be dropped in the subsequent analysis if α is above a certain threshold, e.g., $\alpha > 0.8$ in the current settings. It is also desirable to conduct a sensitivity analysis of the threshold value. However, this threshold will depend on the purpose of the analysis, the set of the price systems, and the setting of the statistical model, so that no generic threshold is likely to exist. Therefore, it would be desirable to conduct simulations and set the threshold in advance for each individual study.

5 Conclusion

Prior work has developed an experimental procedure that allows for collecting rich individual-level data to elicit distributional preferences (Andreoni and Miller, 2002; Fisman, Kariv and Markovits, 2007). While this procedure has been adopted in several application studies, it has also been pointed out that the estimated distribution parameter is not meaningful over a range of possible parameter values (Thöni, 2015). This study extended the basic estimation method and demonstrated its properties to provide some guidelines for its use in the context of distributional preferences.

We extended the class of CES utility functions by using a combination

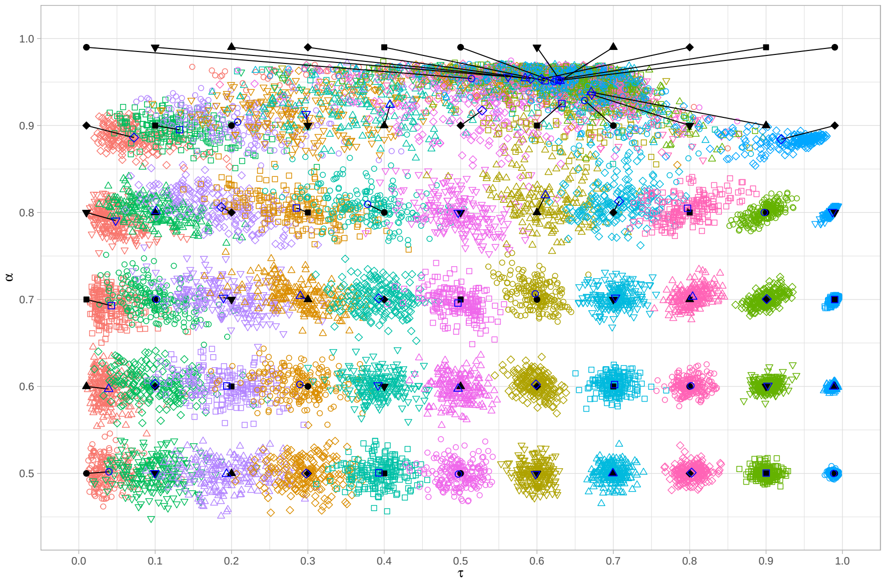

Figure 10: Estimation results of the simulation with $\sigma_s = 0.1$

Notes: For each true parameter (τ, α), the true parameter value (black closed marker), 200 corresponding simulation results (colored open markers), and mean of the 200 simulations (blue open marker) are depicted. Note that true parameter values and corresponding means calculated by 200 simulations are connected by black lines.

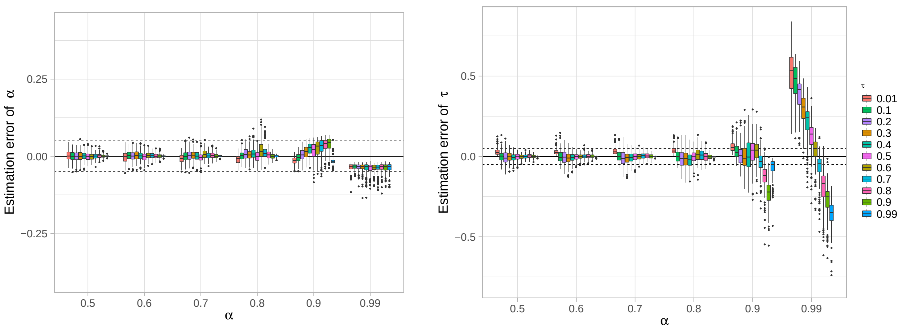

Figure 11: Estimation error of the simulation with $\sigma_s = 0.1$

Notes: **Left:** Estimation error of α. **Right:** Estimation error of τ. In each panel, each box corresponds to 200 estimation errors defined by the estimated values minus true parameter value α or τ. Dashed horizontal lines indicate ± 0.05. Note that the scale of the vertical axis of the left panel is doubled compared with the right panel.

of known CES specifications to make interpretation of the distribution parameter, α, align better with individual behavior. Based on the extended CES utility specification, we developed a parameter estimation method using the Bayesian statistical modeling approach that also gives a more balanced scale for the substitution parameter, τ, than the original scale, ρ.

Our reanalysis of data from previous studies demonstrates that the extended estimation method gives a straightforward interpretation of the distribution parameter rather than the conventional method. This is due to our extended CES specification that separately controls the distributional share and the substitution reaction of individual behavior. Moreover, this makes clear that the substitution parameter does not necessarily represent "the equality–efficiency trade-off" because this interpretation is primarily based on the limitation of the standard CES specification, and not on the restriction of individual behavior. Note that our conclusion is not that individuals never face such trade-offs. Instead, our results suggest that to examine whether people face the trade-off, we should address it more elaborately in the research design. Note that the existence of "the equality–efficiency trade-off" is context-dependent and debatable (see footnote 14).

In our parameter recovery simulation study, we also found that the distribution parameter is more reliable to use in further analyses than the substitution parameter. The estimates for the substitution parameter may hold reasonable information if we use them with special care. More precisely, τ may have valid information when the distribution parameter α is not close to 1. Hence, for example, one possible option is to drop τ if α is above some threshold value, and conduct sensitivity analysis on the threshold value. Because the threshold depends on the study design, it is desirable to perform simulation analysis in advance. Note that, in the context of distributional preferences, such difficult cases should not be mere logical possibilities because most experiments should involve some selfish preference-type subjects. In the Fisman, Kariv and Markovits (2007) data, there was a non-trivial variation of τ (see Figure 8). Hence, even though these parameter estimates are imperfect, we may be able to extract valid and meaningful information about behavioral differences and their implications. Finally, we reemphasize here that this type of simulation study is a simple and important practice for any model-based behavioral analysis (Wilson and Collins, 2019).

While this study extends the basic estimation method and provides a more appropriate interpretation with some guidelines for its use, further cautions are worth noting. First, as noted in section 3, we should take care in interpreting the distribution parameter beyond the "equality" interpretation. The equality-focused behavior represented by setting $\alpha = 1/2$ is a subset of the broad notion of "proportional equality," which can contain egalitarian and non-egalitarian ideas of justice (Gosepath, 2021, p. 8). Therefore, in particular, we should be cautious in making a priori judgments that small (or large) values of the distribution parameter are desirable or just. Note that this experiment focuses on a local distributional decision between a few people in an abstract setting. Hence, we should be aware that concepts discussed in political philosophy, which focuses on more global and institutional problems in more specific contexts, may not directly apply to this experimental setting. Future work should examine appropriate meanings and contexts of "equality" and "efficiency" represented by the CES parameters rather than assuming these a priori. Second, our model cannot be a remedy for potential "experimenter demand effects" (Zizzo, 2010). Hence, we should pay attention to when and how to use this experimental procedure, even when we use our extended data analysis method. Finally, because our focus in this study is on CES utility functions and their parameters, it is obvious that some other variables remained unchanged in the method and can be improved. In particular, future work will consider other aspects of the model, such as the residual error term or the class of utility function in the first place, to make the model more general and flexible, better representing subjects' behavior. Future work should also examine the set of price systems of the experiment to make informative, cost-efficient, and purpose-oriented inferences.

Appendix

We present the simulation results for the cases $\sigma_s = 0.05$ (Figure A1) and $\sigma_s = 0.15$ (Figure A2). The general patterns are similar to the cases $\sigma_s = 0.01$ and $\sigma_s = 0.1$. In particular, the estimation error is relatively smaller for α than for τ. Limitations of the estimation are more severe when noise is large, such as $\sigma_s = 0.15$. That is, even for α, distinguishing subtle differences seems difficult when $\alpha \approx 1$. Therefore, as discussed in the main text, to extract valid and meaningful information about behavioral differences and their implications, they should be used with special care.

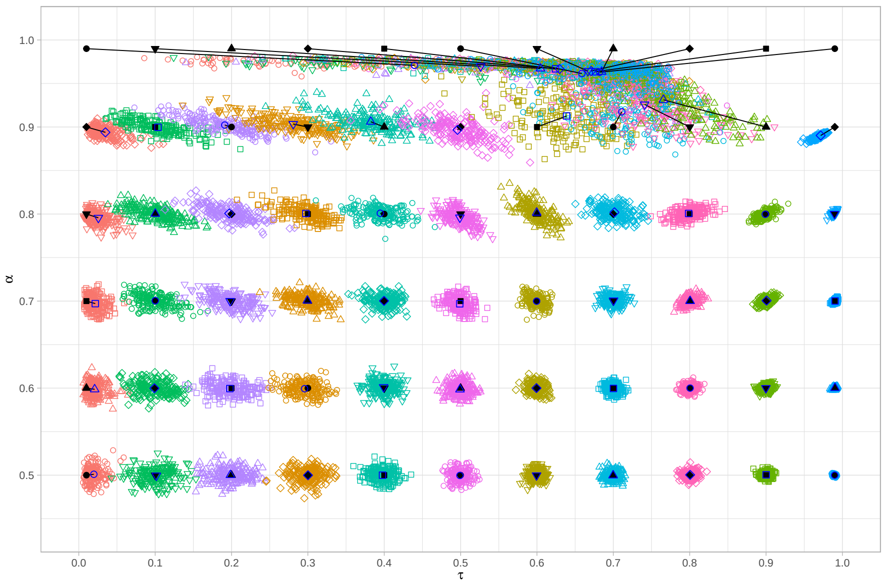

Figure A1: Estimation error of the simulation with $\sigma_s = 0.05$

Notes: For each true parameter (τ, α), the true parameter value (black closed marker), 200 corresponding simulation results (colored open markers), and mean of the 200 simulations (blue open marker) are depicted. Note that true parameter values and corresponding means calculated by 200 simulations are connected by black lines.

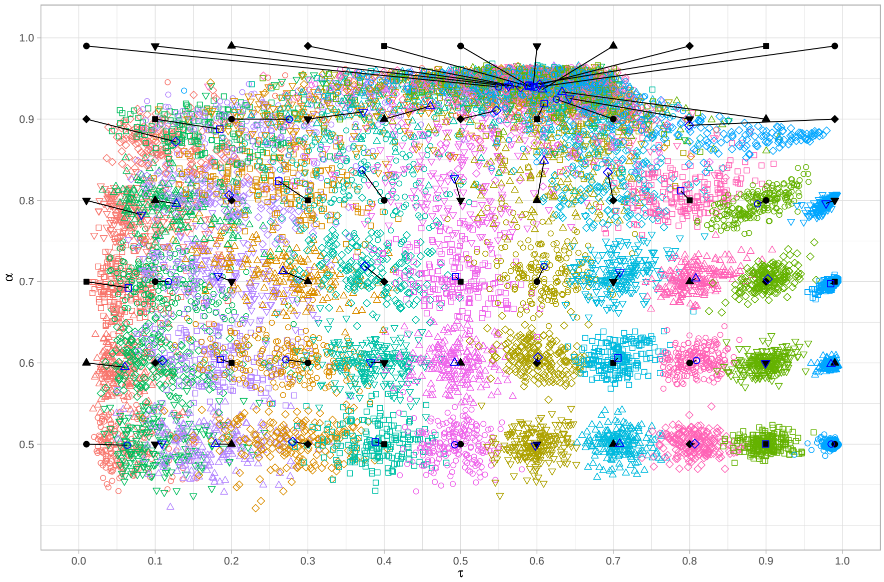

Figure A2: Estimation error of the simulation with $\sigma_s = 0.15$

Notes: For each true parameter (τ, α), the true parameter value (black closed marker), 200 corresponding simulation results (colored open markers), and mean of the 200 simulations (blue open marker) are depicted. Note that true parameter values and corresponding means calculated by 200 simulations are connected by black lines.

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