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ECONOMIC STIMULUS EFFECTS OF PRODUCT INNOVATION UNDER DEMAND STAGNATION

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Economic stimulus effects of product innovation under demand stagnation*

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Abstract

When confronting economic stagnation, innovation (product innovation in particular) is often cited as an effective stimulus because it is assumed to encourage household consumption and lead to higher demand. Using a secular stagnation model with wealth preference, we examine the effects of product innovation on employment and consumption. This study examines three types of product innovation, including quantity-augmenting-like innovation, addictive innovation, and variety expansion. The first works as if a larger quantity were consumed although the actual quantity remains the same, the second reduces the elasticity of the marginal utility of consumption, and the third increases the variety of consumption commodities. We find that the first and third reduce both consumption and employment, whereas the second expands them. It suggests that policy makers should carefully choose the type of product innovation to promote as an economic stimulus: addictive innovation stimulates business activity whereas quantity-augmenting-like innovation and variety expansion worsen stagnation.

JEL classification: O33, E31, E24

Keywords: demand stagnation, product innovation, consumer price index, economic stimulus

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1. Introduction

Many advanced nations have experienced long periods of stagnation with aggregate demand shortages, and innovation is often emphasized as an important economic stimulus along with fiscal and monetary policies.[1] The two kinds of innovation include process and product innovation. The former improves production efficiency, whereas the latter improves commodity quality and utility. Regarding process innovation, Ono (1994, 2001) proposed a model of secular stagnation related to wealth preference,[2] demonstrating that when confronting aggregate demand shortages, increases in productivity arising from process innovation decrease labor demand, worsen deflation, and consequently reduce employment and consumption. In contrast, product innovation increases the utility of consumption, and appears to enhance consumption and employment; however, we find that this is not necessarily the case.

This study uses Ono's secular stagnation model with liquidity (or wealth) preference to examine the effects of product innovation on consumption and employment, determining that different types of product innovation may increase or decrease consumption and employment because they increase not only the value of consumption, which encourages households to consume more, but also the value of money (financial assets), which encourages households to hold money.

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[1] For example, the action plan for a New Form of Capitalism in Japan, the creation of a new Department for Science Innovation and Technology in the UK, the CHIPS and Science Act in the US, the France 2030 intitiative, and the German Agency for Transfer and Innovation (DATI).

[2] Michau (2018) and Michaillat and Saez (2022) also treated models with wealth preference and determined that secular demand stagnation occurs because the marginal utility of wealth remains strictly positive. Illing, Ono, and Schlegl (2018); Mian, Straub, and Sufi (2021); and Hashimoto, Ono, and Schlegl (2023) considered wealth preference to analyze depressed aggregate demand.

Product innovation is an important issue in the literature regarding economic growth, but it has not been considered as a potential demand stimulus under demand stagnation.[3] Fiscal and monetary policies are much more broadly analyzed as demand stimulus policies. To fill this gap between theoretical research and policymaking, we examine the effects of product innovations that directly influence the utility of consumption on aggregate demand and employment.

We specify three types of product innovation. The first makes households feel as if they were consuming a larger quantity of a given commodity despite actually consuming the same quantity, which we refer to as quantity-augmenting-like product innovation. The second reduces the elasticity of the marginal utility of consumption, which we describe as addictive product innovation. The third is variety expansion, which enables households to consume a wider variation of commodities. Naturally, each form of innovation increases real consumption when full employment is achieved; however, we determine that only addictive product innovation increases real consumption and employment under demand stagnation. Variety expansion and quantity-augmenting-like product innovation exert the same influence as process innovation, decreasing real consumption and employment.

[3] Following Krugman (1980), product innovation has often been discussed in the context of "love of variety" and treated as a factor influencing firms' location choices and economic growth. Recent examples have included Baldwin and Martin (2004), Davis and Hashimoto (2016), Iwaisako and Tanaka (2017), and Kane and Peretto (2020). Nevertheless, product innovation has not been considered as demand stimulus.

2. A single-commodity economy

We first consider an economy with a single commodity x, where x represents the commodity's quantity. A representative household's utility of consumption is represented as follows:

$$U = u(x;\gamma);\quad u_x(\equiv \frac{\partial u}{\partial x}) > 0,\ u_{xx} < 0,\ u_\gamma > 0, \tag{1}$$

where γ is a product innovation parameter. Innovations typically require development costs and resources, which affect economic activity through changes in aggregate supply and demand, exerting a crowding-out effect without demand shortages and a demand-stimulus effect in the presence of demand shortages. However, to focus on the pure effect of product innovation on economic activity, we ignore these cost effects and analyze the effect of an exogenous rise in γ.

2-1. Firms

A representative firm is competitive and produces x using only labor with constant productivity θ. The nominal wage is W, the nominal price of x is P_x, and the firm will infinitely expand employment and output if $P_x\theta > W$ but will cease production if $P_x\theta < W$; therefore, to achieve equilibrium in the commodity market, the following must always hold:

$$P_x\theta = W. \tag{2}$$

2-2. Households

Real total consumption c is obtained as follows:

$$c = p_x x, \quad p_x = \frac{P_x}{P}, \tag{3}$$

where p_x is the real price of x, and P is the consumer price index (CPI). Using (3), we rewrite consumption utility U from equation (1) as follows:

$$U = u(\frac{c}{p_x}; \gamma).$$

Following Samuelson and Swamy (1974), price index P is defined as follows:

Definition: Price index P is the ratio of the (minimum) nominal costs of a given level of utility in two economic situations.

Therefore, when product innovation occurs and γ changes such that u increases, P must fall and change the real commodity price $p_x (= P_x/P)$ such that the level of real consumption (i.e., the real costs) c that yields the same utility remains unchanged. Mathematically, a rise in γ alters p_x so that U is unchanged for a given $c (= p_x x)$; hence, using the properties of $u(x; \gamma)$ from equation (1), we obtain the following:

$$0 = -u_x x \left(\frac{1}{p_x}\frac{\partial p_x}{\partial \gamma}\right) + u_\gamma \;\; \Rightarrow \;\; \frac{1}{p_x}\frac{\partial p_x}{\partial \gamma} = \frac{u_\gamma}{u_x x} > 0. \tag{4}$$

Intuitively, product innovation that increases utility raises the value of x so that P falls if the nominal price P_x is the same, and consequently, $p_x (= P_x/P)$ rises.

The household's lifetime utility is then represented as follows:

$$\int_0^\infty \big(U(c;\gamma) + v_T(m) + v_W(a)\big) e^{-\rho t} dt,$$

$$v_T{}' > 0, \;\; v_T{}'' < 0, \;\; v_T{}'(\infty) = 0, \;\; v_W{}' > 0, \;\; v_W{}'' < 0, \tag{5}$$

$$U(c;\gamma) \equiv u\left(\frac{c}{p_x};\gamma\right),$$

where ρ is the subjective discount rate, $v_T(m)$ is the utility of money m $(= M/P)$ for the transaction motive, and $v_W(a)$ is the utility of holding assets a related to wealth preference. Assets a include real money m and interest-bearing asset b with the nominal interest rate $R = r + \pi$, where r is the real interest rate and $\pi\left(= \dot{P}/P\right)$ is the CPI inflation rate. Households' labor endowment is normalized to unity but unemployment may emerge, and then the realized labor supply (= demand) n is less than 1. Therefore, the flow budget equation and asset constraint are

$$\dot{a} = ra + wn - Rm - c + z, \quad n \leq 1,$$
$$a = m + b, \tag{6}$$

where w is the real wage and z is government transfers.

The Hamiltonian function H of household behavior noted above is

$$H = U(c;\gamma) + v_T(m) + v_W(a) + \lambda[ra + wn - Rm - c + z],$$

where λ is the co-state variable, and p_x is invariant over time before and after γ unanticipatedly changes and makes p_x jump as shown in equation (4). Given that $x = c/p_x$, the first-order optimal condition is

$$\rho + \xi\frac{\dot{c}}{c} + \pi = \frac{v_T{}'(m)}{U_c(c;\gamma)} + \frac{v_W{}'(a)}{U_c(c;\gamma)} = R + \frac{v_W{}'(a)}{U_c(c;\gamma)},$$
$$\text{where } U_c(c;\gamma) = \frac{u_x(x;\gamma)}{p_x}, \quad \xi \equiv -\frac{cU_{cc}}{U_c} = -\frac{xu_{xx}}{u_x} > 0. \tag{7}$$

The left-hand side is the intertemporal marginal rate of substitution in consumption c, the middle is the marginal benefit of holding money m, and the right-hand side is the marginal benefit of holding interest-bearing asset b. The transversality condition is

$$\lim_{t\to\infty} \lambda a \exp(-\rho t) = 0. \tag{8}$$

2-3. Markets

The commodity market equilibrium is as follows:

$$x = \frac{c}{p_x} = \theta n, \tag{9}$$

where x and θn are demand and supply of commodity x. Given that p_x is constant over time, we obtain the following from equations (2) and (3):

$$\frac{\dot{P}_x}{P_x} = \frac{\dot{P}}{P} (\equiv \pi) = \frac{\dot{W}}{W}.$$

When full employment ($n = 1$) prevails, $x = \theta$, and W moves in parallel with P. In contrast, when confronting demand shortages, W falls slowly, depending on the deflationary gap, and P moves in parallel with W as follows:

$$\frac{\dot{W}}{W} (= \pi) = \alpha(n - 1) \quad \text{if} \quad n < 1, \tag{10}$$

where α is the adjustment speed of W. This is because workers do not accept rapid falls in W but welcome any increase in W, regardless of how rapid it is.

Firm equities have no value because firm profits are zero under linear technology and the net supply of bonds is assumed to be zero; therefore, the total supply of b is zero, represented as follows:

$$b = 0, \quad a = m.$$

The nominal money supply is M^s and its expansion rate is $\mu(= \dot{M}^s/M^s)$. The government transfers seigniorage μm^s to households:

$$z = \mu m^s.$$

From the money market equilibrium condition, $m^s(= M^s/P) = m$, we obtain $\dot{m}/m = \mu - \pi$. As $\dot{W}/W$ must be positive for the adjustment of W to support the full-employment steady state, the inflation rate must be non-negative; hence,

$$\mu \geq 0.$$

Considering that $a = m$, the previously noted dynamics of m and the dynamics of c represented by (7) formulate autonomous dynamics with respect to m and c as follows:

$$\frac{\dot{c}}{c} = \frac{1}{\xi}\left(\frac{v'(m)}{U_c(c;\gamma)} - \pi - \rho\right) \quad \text{where } v(m) \equiv v_T(m) + v_W(m),$$
$$\frac{\dot{m}}{m} = \mu - \pi. \tag{11}$$

3. Product innovation

Having established the model, we now examine the effects of product innovation. Before introducing the case of stagnation, we consider the standard case with full employment, where $n = 1$ and $x = \theta$ from (9), as a benchmark. From (4), a rise in γ raises the real price p_x; therefore, both real consumption $c(= p_x\theta)$ and the utility of consumption $u(\theta;\gamma)$ increase.

Proposition 1: *Under full employment, product innovation increases real consumption and the utility of consumption.*

3-1. Stagnation steady state

The full employment steady state mentioned above may not be reached, as mentioned below. In the full employment steady state, where $n = 1$, $\dot{c} = 0$, and $\dot{m} = 0$, from (7), (9) and (11), the following must hold:

$$\rho + \mu = \frac{v'\left(\frac{M^S}{P}\right)}{U_c(p_x\theta;\gamma)} \quad \text{where} \quad U_c(p_x\theta;\gamma) = \frac{u_x(\theta;\gamma)}{p_x}. \tag{12}$$

However, if the marginal utility of asset holding has a positive lower bound β, namely,

$$v'(\infty) \equiv v'_T(\infty) + v'_W(\infty) = v'_W(\infty) = \beta > 0,$$

where $v'_T(\infty) = 0$ from (5), and if the following condition is satisfied, namely,

$$(\rho \leq)\ \rho + \mu < \frac{\beta}{U_c(p_x\theta;\gamma)}\left(= \frac{p_x\beta}{u_x(\theta;\gamma)} < \frac{p_x v'\left(\frac{M^S}{P}\right)}{u_x(\theta;\gamma)} \text{ for any } P\right), \tag{13}$$

where $p_x = 1$ because P is initially set to equal P_x, there is no $P(= P_x)$ that satisfies (12), implying that a full-employment steady state is not achievable.[4] Note that this condition is valid for an economy with sufficiently high productivity θ.

If (13) is valid, the left-hand side of (12), representing the desire for consumption, is always smaller than the right-hand side, representing the desire for wealth (or money) accumulation or the wealth premium, when $x = \theta$. Therefore, household demand for commodity x decreases below θ, which leads to demand shortage and causes W and P to fall. In this case, W and P fall in a sluggish manner, represented by (10), and $v'(m)$ eventually reaches β. From (7), (9), (10), and (11), in the stagnation steady state, n satisfies the following:[5]

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[4] See Ono (1994, 2001) and Michau (2018), who determined the condition for the stagnation steady state to occur. Using parametric and non-parametric approaches, Ono, Ogawa and Yoshida (2014) empirically demonstrated that the utility of financial wealth is insatiable. Ono and Yamada (2018) used survey data and empirically validated a status preference for money that makes the marginal utility of money constant and causes secular demand stagnation to occur.

[5] See Ono (1994, 2001) for the proof of saddle stability in the neighborhood of the stagnation steady state.

$$\rho + \alpha(n-1) = \frac{p_x \beta}{u_x(\theta n; \gamma)} \left(= \frac{\beta}{U_c(c; \gamma)} \right). \tag{14}$$

Since (13) implies that the left-hand side of (14) is smaller than the right-hand side when $n = 1$ (or $x = \theta$), in order for the solution of $n \in (0,1)$ to exist, the former is larger than the latter when $n = 0$ (or $x = 0$). Therefore, the former is less inclined than the latter as n increases. These properties are mathematically represented as follows:

$$\rho > \alpha, \quad \Omega \equiv 1 + \frac{\alpha u_x^2 n}{u_{xx} \beta p_x x} > 0. \tag{15}$$

Note that deflation continues and m diverges to infinity in this steady state; however, the transversality condition (8) is valid as long as $\dot{m}/m (= \mu - \pi)$ is smaller than ρ.[6] Given that $\pi = \alpha(n-1)$, from (14), the transversality condition is equivalent to

$$\mu < \frac{p_x \beta}{u_x(\theta n; \gamma)},$$

where n is the solution of (14).

Totally differentiating (14) and applying (4), (9), and (15) to the result yields the effects of product innovation γ on total employment n and real consumption c as follows:

$$\begin{aligned} \Omega \frac{dn}{nd\gamma} &= \Omega \frac{dx}{xd\gamma} = -\frac{u_\gamma}{u_{xx} x^2} \left(\frac{x u_{x\gamma}}{u_\gamma} - 1 \right), \\ \Omega \frac{dn}{nd\gamma} < \Omega \frac{dc}{cd\gamma} &= \Omega \left(\frac{dx}{xd\gamma} + \frac{u_\gamma}{u_x x} \right) = -\frac{u_\gamma}{u_{xx} x^2} \left(\frac{x u_{x\gamma}}{u_\gamma} - \frac{x u_{xx}}{u_x} - 1 - \frac{u_x \alpha n}{\beta p_x} \right). \end{aligned} \tag{16}$$

The inequality in the second line of (16) implies the following property:

Lemma 1: *Suppose that unemployment occurs because of a shortage of demand. If product innovation increases employment, real consumption also increases.*

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[6] When $\mu < p_x\beta/u_x(\theta n;\gamma) < \rho < p_x\beta/u_x(\theta;\gamma) < \rho + \mu$, both the full-employment and stagnation steady states exist. See Ono and Ishida (2014) for details on the conditions for the two steady states to emerge.

3-2. Various product innovations

We consider two types of innovation; one functions as if consumption of the commodity increased, although the actual quantity is unchanged, whereas the other decreases the elasticity of the marginal utility of the commodity. As noted previously, the former is quantity-augmenting-like innovation, whereas the latter is termed addictive innovation. We apply these to (16) to examine their effects on total employment n and real consumption c (or utility U).

We first assume the following utility function:

$$u(x;\gamma) = h(\gamma x); \quad h'(\gamma x) > 0, \ h''(\gamma x) < 0. \tag{17}$$

This implies that innovation γ functions as if the consumption of the commodity increased by a factor of γ. From (17), we obtain the following properties:

$$xu_x = \gamma u_\gamma = \gamma x h'(\gamma x) > 0, \ \frac{xu_{xx}}{u_x} = \frac{xu_{x\gamma}}{u_\gamma} - 1 = \frac{\gamma x h''(\gamma x)}{h'(\gamma x)} < 0.$$

Applying these properties to (16) yields the following:

$$\begin{gathered} \Omega \frac{\gamma dn}{n d\gamma} = -1, \\ \Omega \frac{dc}{c d\gamma} = \Omega \left(\frac{dx}{x d\gamma} + \frac{u_\gamma}{u_x x} \right) = \frac{\alpha n u_x u_\gamma}{u_{xx} x^2 \beta p_x} < 0. \end{gathered} \tag{18}$$

Noting that $\Omega > 0$ from (15), these properties give the following proposition:

Proposition 2: *If unemployment occurs because of a shortage of demand, quantity-augmenting-like product innovation, which affects utility as shown in (17), worsens both total employment and real consumption.*

Intuitively, quantity-augmenting-like innovation increases the value of the commodity and the value of money by a factor of γ because the value of money is measured by how much value of the commodity can be purchased. Therefore, the marginal utility of consumption decreases if the quantity of the commodity is held constant, while the marginal utility of money is unchanged to be β. Consequently, the commodity is consumed less and total employment decreases, which worsens deflation and further reduces real consumption.

We next consider addictive innovation using the following utility function:[7]

$$u(x;\gamma) = \frac{x^{\gamma}-1}{\gamma} \quad \text{(where } \gamma < 1\text{);}$$

$$u_x = x^{\gamma-1} > 0,\ \gamma^2 u_\gamma = x^{\gamma}(\ln x^{\gamma} - 1) + 1, \tag{19}$$

$$\xi = -\frac{x u_{xx}}{u_x} = 1 - \gamma > 0,\ u_{x\gamma} = x^{\gamma-1}\ln x.$$

We regard $x(= c/p_x) = 1$ as the survival level of consumption and examine the case where $x > 1$ because an increase in innovation γ does not affect utility ($u_\gamma = 0$) when $x = 1$ and increases utility ($u_\gamma > 0$) when $x > 1$. We describe an increase in γ as addictive product innovation because it works in such a way that it reduces the elasticity of the marginal utility of consumption $\xi(= 1 - \gamma)$.

Applying (19) to (16) yields

$$(-u_{xx})x^2\gamma^2\Omega\frac{dn}{nd\gamma} = \psi(x;\gamma) \equiv x^{\gamma}[1 - (1-\gamma)\ln x^{\gamma}] - 1,$$

$$\psi(1;\gamma) = 0,\ \ \psi(\infty;\gamma) < 0,\ \ \psi_x(x;\gamma) = \gamma^2 x^{\gamma-1}[1 - (1-\gamma)\ln x]; \tag{20}$$

$$\Omega\frac{dn}{nd\gamma} < \Omega\frac{dc}{cd\gamma} = \frac{1}{(1-\gamma)x^{\gamma}}\left[\frac{x^{\gamma}-1}{\gamma} - \frac{u_\gamma u_x \alpha n}{\beta p_x}\right].$$

[7] Harrison et al. (2018) also assume a power utility function which displays constant relative risk aversion x^{γ} when analyzing the degree of addiction to cigarettes.

Noting that ψ is positive (and $dn/d\gamma > 0$) when $x(> 1)$ is close to 1 because $\psi(1;\gamma) = 0$ and $\psi_x(1;\gamma) = \gamma^2 > 0$, and negative when x is sufficiently large,[8] we summarize the above result as follows:

Proposition 3: *If the utility of consumption is given by (19) and unemployment owing to demand shortage appears, addictive product innovation may increase or decrease total employment and real consumption.*

We examine whether addictive product innovation increases employment and real consumption using consumption data for G7 countries. Because $x = 1$ is regarded as the survival level of consumption, as noted previously, the threshold x_ψ that satisfies $\psi(x_\psi;\gamma) = 0$ in (20) also represents the ratio of the threshold level to the survival level. By comparing this with the ratio of household consumption per capita to the poverty line for G7 countries, we examine whether addictive innovation leads to higher employment and real consumption.

Table 1: Threshold x_ψ below which $dn/d\gamma > 0$

$\xi(= 1 - \gamma)$	0.2	0.3	0.4	1.5	2
x_ψ	496	96.3	41.2	4.60	3.51

[8] This is obvious if $\gamma > 0$ because $\lim_{x\to\infty} x^\gamma = \infty$. However, if $\gamma < 0$, $\lim_{x\to\infty} x^\gamma = 0$. In this case, we apply l'Hôpital's rule and find $\psi(\infty;\gamma) = -1 < 0$.

Table 2: Ratio of household consumption per capita to the poverty line (G7 countries)

	US (2010)	UK (2011)	Germany (2011)	France (2011)	Canada (2010)	Italy (2011)	Japan (2015)
Household survey mean/poverty line	2.87	2.11	2.00	2.14	2.38	2.01	unavailable
Household final consumption/ poverty line	4.27	2.78	2.33	2.21	2.56	2.71	1.93

Source: Appendix 2 of Jolliffe and Prydz (2016). The authors do not provide data for Japan; therefore, we use the poverty line in the report of Japan's Ministry of Health, Labor and Welfare (in Japanese), p.25, given by https://www.mhlw.go.jp/toukei/list/dl/20-21-h28_rev2.pdf, and household final consumption expenditure per capita estimated from World Bank data, https://data.worldbank.org/indicator/NE.CON.PRVT.KN and https://data.worldbank.org/indicator/SP.POP.TOTL.

Table 1 presents the ratios of the threshold level to the survival level for various elasticities $\xi(= 1 - \gamma)$ because various levels of elasticity of the marginal utility of consumption are proposed in previous research, approximately 0.3 in microeconomic approaches such as Harrison et al. (2005), Anderson and Mellor (2008) and Meissner and Pfeiffer (2022), and 2 in the dynamic stochastic general equilibrium (DSGE) modeling literature.[9] Table 2 shows the ratio of average household consumption to the poverty line in G7 countries. The two tables reveal that the ratios in Table 2 are almost always smaller than x_ψ, and the only exception is the case in which we adopt $\xi = 2$ and the ratio of US household final consumption to the national poverty line. Therefore, using Lemma 1, we determine that addictive product innovation will almost always increase total employment and real consumption if the representative household's consumption equals the average consumption levels of G7 countries.

[9] Hall (1988) and Guvenen (2006) asserted that $1/\xi$ is much smaller than 1. Chiappori and Paiella (2011) argued that the median of relative risk aversion ξ is 2. This is because in the standard framework without wealth preference, a low relative risk aversion is inconsistent with a response of $\dot{c}/c$ to a change in the interest rate; however, in the presence of wealth preference, this problem does not appear. See Appendix A for details.

4. A two-commodity economy

We next turn to an economy with two commodities, x and y, to examine whether the results obtained in the single-commodity economy remain valid. We implicitly consider that x is the commodity for which product innovation occurs, while y is the composite of all other commodities.

4-1. The model

Firms are competitive and produce commodity $j\,(= x, y)$ with constant labor productivity θ_j. Their profit maximization behavior leads to

$$P_j\theta_j = W \text{ and } p_j\theta_j = w \text{ for } j = x, y; \quad \omega \equiv \frac{p_y}{p_x} = \frac{\theta_x}{\theta_y} = \text{constant}, \tag{21}$$

where P_j and $p_j(= P_j/P)$ are, respectively, the nominal and real prices of commodity j. Nominal wage W moves as mentioned in (10). Since P_x and P_y satisfy (21), they move in parallel with W, and consequently P does also, implying that $w(= W/P)$ is constant over time.

The utility of consumption is

$$U = u(x, y; \gamma),$$

which is assumed to be homothetic with respect to x and y. Given real consumption c, the representative household maximizes U subject to

$$p_x x + p_y y = c, \tag{22}$$

and satisfies

$$\frac{u_y(x,y;\gamma)}{u_x(x,y;\gamma)} = \frac{u_y\left(\frac{x}{y},1;\gamma\right)}{u_x\left(\frac{x}{y},1;\gamma\right)} = \omega = \frac{\theta_x}{\theta_y} \;\Rightarrow\; \frac{x}{y} = \zeta(\gamma), \tag{23}$$

because of homotheticity, where u_j implies the partial derivative of u with respect to variable $j\ (= x, y, \gamma)$ and ω is constant from (21). Therefore, we obtain

$$p_x x = \delta c, \quad p_y y = (1-\delta)c, \quad \zeta(\gamma)(= \tfrac{x}{y}) = \tfrac{\omega\delta}{1-\delta},$$
$$\delta = \delta(\gamma) = \tfrac{\zeta(\gamma)}{\zeta(\gamma)+\omega}, \quad \delta'(\gamma) = \tfrac{(1-\delta)^2}{\omega}\zeta'(\gamma), \tag{24}$$

where δ is the expenditure share of commodity x. U is then represented as a function of c and γ, which satisfies

$$U(c;\gamma) = u\left(\tfrac{\delta(\gamma)c}{p_x}, \tfrac{(1-\delta(\gamma))c}{\omega p_x}; \gamma\right),$$
$$U_c = \tfrac{u_x(x,y;\gamma)}{p_x} = \tfrac{u_y(x,y;\gamma)}{\omega p_x} > 0, \quad p_x^2 U_{cc} = \tfrac{\delta}{x}(u_{xx}x + u_{xy}y) < 0. \tag{25}$$

Given that the consumer price index P must be defined such that a change in γ does not affect U in (25) for a given $c(= C/P)$, by differentiating (25) where c is fixed and using (23), we find that $p_x(= P_x/P)$ satisfies

$$\frac{1}{p_x}\frac{\partial p_x}{\partial \gamma} = \delta \frac{u_\gamma}{u_x x} > 0. \tag{26}$$

Note that if $\delta = 1$ (i.e., the single-commodity case), (26) is equivalent to (4). The household's intertemporal behavior is the same as that in the single-commodity economy represented by (5) and (6) except that $U(c;\gamma)$ is given by (25). Thus, the intertemporal optimal condition is represented by (7) where U_c is given by (25). From (21), (24) and the market equilibria of the two commodities, we obtain the following:

$$x = n_x\theta_x, \quad y = n_y\theta_y,$$

and find that each sector's employment $n_j\ (j = x, y)$ and total employment $n(= n_x + n_y)$ are

$$n_x = \frac{x}{\theta_x} = \delta n, \ n_y = \frac{y}{\theta_y} = (1-\delta)n; \ n = \frac{c}{p_x\theta_x} = \frac{c}{p_y\theta_y}. \tag{27}$$

Before analyzing the case of unemployment, we consider the case of full employment ($n = 1$) as a benchmark. From (25) and (27), c and U are

$$c = p_x\theta_x = p_y\theta_y, \quad U = u\big(\theta_x\delta(\gamma), \theta_y\big(1-\delta(\gamma)\big);\gamma\big), \tag{28}$$

where $\delta(\gamma)$ is given by (24). Therefore, using (21), (23) and (26), we obtain

$$\frac{\gamma}{c}\frac{dc}{d\gamma} = \frac{\gamma}{p_x}\frac{\partial p_x}{\partial \gamma} > 0, \quad \frac{dU}{d\gamma} = u_\gamma > 0.$$

These properties imply that Proposition 1, which holds in the single-commodity economy, also holds in the two-commodity economy; that is, product innovation increases real consumption if full employment is achieved in the steady state.

4-2. Stagnation steady state in the two-commodity economy

In the full-employment steady state, we still have (12) in which U_c is given by (25) and c satisfies (28). Therefore, if (13) holds, the full-employment steady state does not exist and the stagnation steady state represented by (14) in which U_c is given by (25) is reached. Then, using (27), we obtain

$$\rho + \alpha(n-1) = \frac{\beta}{U_c(c,\gamma)}, \ U_c(c,\gamma) = \frac{u_x\big(\delta(\gamma)\theta_x n, (1-\delta(\gamma))\theta_y n;\gamma\big)}{p_x}. \tag{29}$$

For the same reason that (15) is derived from (14), we have

$$\rho > \alpha, \quad \hat{\Omega} \equiv 1 + \frac{u_{xy}y}{u_{xx}x} + \frac{\alpha u_x^2 n}{u_{xx}\beta p_x x} > 0. \tag{30}$$

Note that these conditions are the same as in (15) if only commodity x exists (i.e., $y = 0$). From (26), (27), (29) and (30), we obtain

$$\hat{\Omega}\frac{dn}{nd\gamma}=\hat{\Omega}\frac{dx}{xd\gamma}=-\frac{\delta u_\gamma}{u_{xx}x^2}\left(\frac{xu_{x\gamma}+yu_{y\gamma}}{u_\gamma}-1\right),$$

$$\hat{\Omega}\frac{dc}{cd\gamma}=\hat{\Omega}\left(\frac{dx}{xd\gamma}+\frac{dp_x}{p_xd\gamma}\right)=-\frac{\delta u_\gamma}{u_{xx}x^2}\left(\frac{xu_{x\gamma}+yu_{y\gamma}}{u_\gamma}-\frac{xu_{xx}+yu_{xy}}{u_x}-1-\frac{u_x\alpha n}{\beta p_x}\right), \tag{31}$$

the derivation for which is presented in Appendix B. Note that if $y=0$, the two equations in (31) are the same as those in (16), which are valid in the single-commodity economy. From (26), (30) and (31), we find $(dc/c)/d\gamma>(dn/n)/d\gamma$, implying that Lemma 1 also holds in the two-commodity economy.

First, we consider quantity-augmenting-like product innovation for commodity x by assuming the following utility function:

$$u(x,y;\gamma)=h(\gamma x,y),$$

which satisfies the following:

$$xu_x=\gamma u_\gamma=\gamma xh_1(\gamma x,y)>0,\ \ u_y=h_2(\gamma x,y)>0,$$

$$\frac{xu_{xx}}{u_x}=\frac{xu_{x\gamma}}{u_\gamma}-1=\frac{\gamma xh_{11}}{h_1},\ \ \gamma u_{y\gamma}=xu_{xy}=\gamma xh_{12},$$

where h_i $(i=1,2)$ is the partial derivative with respect to the i-th factor of $h(\gamma x,y)$. By applying these properties to (31) and noting that u_{xx} and U_{cc} in (25) are negative while $\hat{\Omega}$ in (30) is positive, we obtain

$$\hat{\Omega}\frac{\gamma dn}{nd\gamma}=-\delta\left(1+\frac{yh_{12}}{\gamma xh_{11}}\right)=-\delta\left(1+\frac{u_{xy}y}{u_{xx}x}\right)<0,$$

$$\hat{\Omega}\frac{dc}{cd\gamma}=\frac{\delta\alpha nu_xu_\gamma}{u_{xx}x^2\beta p_x}<0.$$

This implies that Proposition 2 also holds in the two-commodity economy and quantity-augmenting-like product innovation worsens total employment and real consumption.

We next examine addictive product innovation using the following utility function:[10]

$$u(x,y;\gamma)=\frac{x^{\gamma}y^{\varepsilon}-1}{\gamma+\varepsilon}\quad\text{(where } \gamma\varepsilon>0,\ \gamma+\varepsilon<1\text{);}$$

$$u_{\gamma}=\frac{x^{\gamma}y^{\varepsilon}[(\gamma+\varepsilon)\ln x-1]+1}{(\gamma+\varepsilon)^2},\quad xu_{x\gamma}+yu_{y\gamma}=x^{\gamma}y^{\varepsilon}\ln x, \tag{32}$$

$$u_x=\frac{\gamma x^{\gamma-1}y^{\varepsilon}}{\gamma+\varepsilon},\ \ u_y=\frac{\varepsilon x^{\gamma}y^{\varepsilon-1}}{\gamma+\varepsilon},\ \ -\frac{xu_{xx}+yu_{xy}}{u_x}=1-(\gamma+\varepsilon),$$

where addictive product innovation is represented by an increase in γ. Applying u_x and u_y in (32) to (22) and (23) yields

$$p_x x=\frac{\gamma}{\gamma+\varepsilon}c,\quad p_y y=\frac{\varepsilon}{\gamma+\varepsilon}c,\quad y=\frac{\varepsilon}{\gamma\omega}x. \tag{33}$$

By substituting the values in (32) into (31) and using the third equation in (33), we obtain

$$\begin{aligned}&(-u_{xx})x^2(\gamma+\varepsilon)^2\hat{\Omega}\frac{dn}{\delta n d\gamma}=\hat{\psi}(x;\gamma),\\&\hat{\psi}(x;\gamma)\equiv x^{\gamma+\varepsilon}\left(\frac{\varepsilon}{\gamma\omega}\right)^{\varepsilon}\left[1-\big(1-(\gamma+\varepsilon)\big)\ln x^{\gamma+\varepsilon}\right]-1,\\&\hat{\psi}_x(x;\gamma)=\left(\frac{\varepsilon}{\gamma\omega}\right)^{\varepsilon}(\gamma+\varepsilon)^2x^{\gamma+\varepsilon-1}\left[1-\big(1-(\gamma+\varepsilon)\big)\ln x\right];\\&\hat{\Omega}\frac{dn}{nd\gamma}<\hat{\Omega}\frac{dc}{cd\gamma}=-\frac{\delta}{u_{xx}x^2}\left(\frac{x^{\gamma+\varepsilon}\left(\frac{\varepsilon}{\gamma\omega}\right)^{\varepsilon}-1}{\gamma+\varepsilon}-\frac{u_{\gamma}u_x\alpha n}{\beta p_x}\right).\end{aligned} \tag{34}$$

From (32) and (34) we derive the minimum x that holds when c equals the survival level, the threshold x above which an increase in γ leads to greater employment and real consumption, and the ratio of the two.

First, we obtain the minimum x. As mentioned at the outset of this section, x represents the product for which product innovation takes place while y represents the composite of all

[10] See Appendix D for the case where the first term of the numerator of u, which is $x^{\gamma}y^{\varepsilon}$ now, is replaced by a CES (constant elasticity of substitution) function with respect to x and y: $\left[\left(1-\frac{\varepsilon}{\nu}\right)x^{\eta}+\frac{\varepsilon}{\nu}y^{\eta}\right]^{\frac{\nu}{\eta}}$ in which $\nu=\gamma+\varepsilon$.

other products. By aligning the quantity units of x and y in such a way that the relative price ω is equal to one,[11] we can regard $y/x(= \varepsilon/(\gamma\omega))$ given in (33) as the quantity ratio of the composite to the innovative product that households rationally determine, which is naturally larger than 1; hence,

$$\left(\frac{y}{x} =\right)\frac{\varepsilon}{\gamma\omega} > 1, \quad \omega = 1. \tag{35}$$

By applying the third equation in (33) to u_γ in (32) and using (35), we obtain

$$(\gamma+\varepsilon)^2 u_\gamma = \hat{\varphi}(x;\gamma) \equiv x^{\gamma+\varepsilon}\left(\frac{\varepsilon}{\gamma\omega}\right)^{\varepsilon}[\ln x^{\gamma+\varepsilon} - 1] + 1,$$

$$\hat{\varphi}(1;\gamma) = 1 - \left(\frac{\varepsilon}{\gamma\omega}\right)^{\varepsilon} < 0, \quad \hat{\varphi}(\infty;\gamma) > 0,$$

$$\hat{\varphi}_x(x;\gamma) = (\gamma+\varepsilon)^2\left(\frac{\varepsilon}{\gamma\omega}\right)^{\varepsilon} x^{\gamma+\varepsilon-1}\ln x > 0 \ \text{ for } x > 1,$$

where x can only take values such that $u_\gamma > 0$. Therefore, x and its minimum x_φ satisfy

$$1 < x_\varphi < x \quad \text{where} \quad \hat{\varphi}(x_\varphi;\gamma) = 0 \ \ (\Leftrightarrow u_\gamma = 0). \tag{36}$$

We next obtain the threshold $x(= x_\psi)$. From (34), (35) and (36), $\hat{\psi}(x;\gamma)$, the threshold x_ψ at which $\hat{\psi}(x_\psi;\gamma) = 0$, and the minimum $x(= x_\varphi)$ given in (36) satisfy

$$\hat{\psi}(x_\psi;\gamma) = (x_\psi)^{\gamma+\varepsilon}\left(\frac{\varepsilon}{\gamma\omega}\right)^{\varepsilon}\left[1 - \big(1-(\gamma+\varepsilon)\big)\ln(x_\psi)^{\gamma+\varepsilon}\right] - 1 = 0,$$

$$\hat{\psi}(x_\varphi;\gamma) = (x_\varphi)^{\gamma+\varepsilon}\left(\frac{\varepsilon}{\gamma\omega}\right)^{\varepsilon}(\gamma+\varepsilon)^2\ln x_\varphi > 0, \quad \hat{\psi}(\infty;\gamma) < 0,$$

which implies

$$\hat{\psi} \gtreqless 0 \ \left(\Leftrightarrow \frac{dn}{d\gamma} \gtreqless 0\right) \ \text{ for } x \lesseqgtr x_\psi \quad \text{where } \ x_\varphi < x_\psi. \tag{37}$$

[11] In other words, the unit is the quantity of each commodity that is produced by a unit of labor.

From (37) and Lemma 1, we find that Proposition 3, which holds in the single-commodity economy, also holds in the two-commodity economy, indicating that addictive product innovation may increase or decrease total employment and real consumption.

Under homothetic utility, the ratio x_ψ/x_φ is equal to the consumption ratio of threshold and survival levels. Table 3 lists the ratios calculated for different pairs of (γ, ε). Note that γ and ε satisfy (35) and the elasticity of the marginal utility of real consumption ξ is $1 - (\gamma + \varepsilon)$ because substituting them into $u(x, y; \gamma)$ in (32) yields

$$U(c;\gamma) = \frac{\left(\frac{\gamma}{\gamma+\varepsilon}\right)^{\gamma+\varepsilon}\left(\frac{\varepsilon}{\gamma\omega}\right)^{\varepsilon}\left(\frac{c}{p_X}\right)^{\gamma+\varepsilon} - 1}{\gamma+\varepsilon}, \quad \xi \equiv -\frac{U_{cc}c}{U_c} = 1 - (\gamma + \varepsilon) > 0.$$

The parameter values in Table 3 satisfy these properties.[12]

Table 3: Survival consumption level and threshold (the two-commodity case)

$\xi(= (1 - (\gamma + \varepsilon))$	0.2			0.3			0.4		
ε/γ	1	10	100	1	10	100	1	10	100
x_ψ/x_φ	496	162	150	96.3	30.8	28.5	41.2	13.4	12.4

Comparing x_ψ/x_φ in Table 3 with the G7 countries' data in Table 2 reveals that the threshold ratios x_ψ/x_φ presented in Table 3 are all higher than the ratio of household consumption per capita to the poverty line for G7 countries in Table 2, which implies

[12] We do not examine the case in which the elasticity of the marginal utility of consumption $1 - (\gamma + \varepsilon)$ is larger than 1 in the two-commodity case. This is because the effect of a rise in γ on the demand ratio of the two commodities is quite unrealistic. See Appendix A for details.

$dn/d\gamma > 0$ for those values of x_ψ/x_φ. Therefore, using Lemma 1, we find that addictive innovation increases both employment and real consumption in the two-commodity economy as well as in the single-commodity economy if the representative household's consumption equals the average consumption levels of G7 countries.

5. Variety expansion

Finally, we consider the effect of product innovation that expands the variety of consumption commodities on employment and real consumption.

5-1. CES utility

We first treat the case where the utility of consumption is one of constant elasticity of substitution (CES) functions as follows:

$$U = u(x, \gamma) = h\left(\left(\int_0^\gamma x_j^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}}\right), \quad \sigma > 1; \;\; h' > 0, \; h'' < 0, \tag{38}$$

where γ represents the variety of commodities, which is exogenous, σ is the elasticity of substitution, and x_j is the quantity of commodity $j \in [0, \gamma]$.

By assuming common labor productivity θ for all j and symmetric preference for x_j, we find that real consumption c is

$$c = \int_0^\gamma p_j x_j dj = \gamma p_x x, \quad (p_x, x) = (p_j, x_j) \;\text{ for } j \in [0, \gamma], \tag{39}$$

where p_j is the real price of commodity j while p_x and x are respectively the common real price and quantity. Therefore, from (38) and (39), U satisfies

$$U = u(x; \gamma) = \; h\left(\gamma^{\frac{\sigma}{\sigma-1}} x\right) = h\left(\frac{\gamma^{\frac{1}{\sigma-1}}}{p_x} c\right), \quad U_c = \frac{\gamma^{\frac{1}{\sigma-1}}}{p_x} h'\left(\frac{\gamma^{\frac{1}{\sigma-1}}}{p_x} c\right). \tag{40}$$

Comparing (40) with (17) and replacing $\gamma^{\sigma/(\sigma-1)}$ in (40) by γ in (17) suggests that variety expansion γ should function in the same way as quantity-augmenting-like innovation, discussed in subsection 3-1, by the magnitude of $\gamma^{\sigma/(\sigma-1)}$.[13]

To see this, we first note that U must be invariant for the same c before and after a jump of γ if p_x is properly defined. Therefore, from (40), we obtain

$$U_\gamma = \frac{\partial\left(\frac{\gamma^{\frac{1}{\sigma-1}}}{p_x}\right)}{\partial\gamma} c h'\left(\frac{\gamma^{\frac{1}{\sigma-1}}}{p_x} c\right) = 0 \;\Rightarrow\; \frac{\partial\left(\frac{\gamma^{\frac{1}{\sigma-1}}}{p_x}\right)}{\partial\gamma} = 0 \;\left(\rightarrow \frac{\partial p_x}{\partial\gamma} > 0\right),\;\; U_{c\gamma} = 0. \tag{41}$$

Furthermore, the household's intertemporal behavior is the same as that in the single-commodity economy represented by (5) and (6), except that $U(c;\gamma)$ is given by (40). Thus, the intertemporal optimal condition is represented by (7) where U_c is presented by (40). Given that labor productivity is θ for all $j \in [0,\gamma]$, using (39), we find that each variety's employment n_j and total employment $n(=\gamma n_j)$ satisfy

$$n_j = \frac{x}{\theta}, \quad n = \frac{\gamma x}{\theta} = \frac{c}{p_x\theta}. \tag{42}$$

If full employment ($n = 1$) prevails, from (40) and (42) c and U satisfy

$$c = p_x\theta, \quad U = h\left(\theta\gamma^{\frac{1}{\sigma-1}}\right).$$

Noting that $\partial p_x/\partial\gamma > 0$ from (41), we find that the above equations imply

$$\gamma\uparrow \Rightarrow c\uparrow, U\uparrow \text{ under full employment.}$$

In this state, real money m must satisfy (12); hence,

$$\rho + \mu = \frac{v'(m)}{U_c(p_x\theta;\gamma)} > \frac{\beta}{U_c(p_x\theta;\gamma)} \quad \text{where } U_c(p_x\theta;\gamma) = \frac{\gamma^{\frac{1}{\sigma-1}}}{p_x} h'\left(\theta\gamma^{\frac{1}{\sigma-1}}\right).$$

[13] In particular, if $\sigma \rightarrow \infty$, implying that all varieties are perfect substitutes, $\gamma^{\sigma/(\sigma-1)} = \gamma$; hence, the variety expansion naturally becomes the same as quantity-augmenting-like innovation.

Therefore, if (13) is valid where U_c is replaced by the above value, there is no full-employment steady state and aggregate demand stagnation emerges in the steady state.

In the stagnation steady state, we have condition (14) in which U_c is given by (40). Therefore, from (42), the following is obtained:

$$\rho + \alpha(n-1) = \frac{\beta}{U_c}, \quad \text{where } n = \frac{c}{p_x \theta}. \tag{43}$$

From (41), an increase in γ does not vary U_c, implying that the wealth premium β/U_c given by the right-hand side is unaffected by a change in γ if c is the same. However, less employment is required (i.e., $n = c/(p_x\theta)$ decreases) to achieve the same c because an increase in γ raises p_x, as shown in (41). Therefore, deflation worsens, which lowers the desire for consumption $\rho + \alpha(n-1)$ given by the left-hand side, and real consumption and employment decrease. From (43), it is also apparent that an increase in γ, which raises p_x, works in the same way as process innovation represented by an increase in labor productivity θ. Thus, this variety expansion works in the same way as quantity-augmenting-like innovation examined in subsection 3-2.

In summary, we obtain the following proposition:

Proposition 4: *If the utility of consumption is of the CES type given by (38), product innovation that expands the variety γ increases real consumption under full employment. In the presence of aggregate demand shortage, however, it decreases employment and real consumption.*

Note that the above result is valid, whether under monopolistic competition or under perfect competition.[14]

Using the same CES utility function of consumption and utility of money, Johdo (2008) analyzed the effects of a production subsidy on employment and real consumption, assuming that the market is monopolistically competitive with free entry and that firms are symmetric and require both variable and fixed production costs. In this setup, Johdo concluded that a production subsidy expands variety and increases employment and real consumption, contradicting Proposition 4, if the elasticity of substitution σ is sufficiently large. This finding is valid not because the variety expands but because new entrant firms require fixed costs, which functions in the same manner as government purchases and stimulates employment. If there is no fixed cost and the variety is exogenously determined, variety expansion decreases employment and real consumption, as stated in Proposition 4. Variety expansion itself worsens employment and real consumption, regardless of the magnitude of $\sigma(> 1)$.

5-2. A general case

Quantity-augmenting-like innovation in the two-commodity case, which is analyzed in section 4-2, can be regarded as an introduction of commodity x (i.e., variety expansion) to an economy with only y. Given that the consumption utility function is $u(x, y; \gamma) = h(\gamma x, y)$, the case of $\gamma = 0$ implies that x does not exist, and an increase in γ from zero indicates that commodity x is newly introduced. Noting that Proposition 2 holds regardless

[14] The nominal price of each commodity $P_j(= P_x)$ equals the marginal cost W/θ under perfect competition whereas it equals $[\sigma/(\sigma - 1)]W/\theta$ under monopolistic competition. However, this difference does not affect the present analysis because the variety is exogenously given. .

of the level of γ, we find that an introduction of commodity x into an economy with only y lowers real consumption and employment.

Analogously, using the following homothetic function of consumption utility with N commodities:

$$U = h(x_1, x_2, \dots, x_{N-1}, \gamma x_N),$$

this subsection examines the effects of an increase in γ on real consumption and employment and finds that Proposition 2 is valid in the N-commodity economy; hence, the introduction (i.e., variety expansion) of commodity N reduces real consumption and total employment. This means that regardless of whether or not the consumption utility function is of the CES type, variety expansion always worsens business activity under homothetic utility in the presence of aggregate demand shortages.

For given real prices p_i for $i \in [1, N]$ and real consumption c, the intratemporal rational behavior of a representative household satisfies

$$h_i(x_1, x_2, \dots, x_{N-1}, \gamma x_N) = \lambda p_i \quad \text{for } i \in [1, N-1],$$

$$h_N(x_1, x_2, \dots, x_{N-1}, \gamma x_N) = \lambda \frac{p_N}{\gamma}, \tag{44}$$

$$c = \sum_{i=1}^{N} p_i x_i,$$

where h_i denotes the partial derivative of the utility function h with respect to the i-th factor and λ is the Lagrange multiplier of c. Given that δ_i denotes the expenditure share of commodity i, utility U is rewritten as follows:

$$p_i x_i = \delta_i c \quad \text{for } i \in [1, N],$$

$$U = h\left(\frac{\delta_1 c}{p_1}, \dots, \frac{\delta_{N-1} c}{p_{N-1}}, \gamma \frac{\delta_N c}{p_N}\right), \tag{45}$$

$$\sum_{i=1}^{N} \delta_i = 1, \ \ \sum_{i=1}^{N} d\delta_i = 0.$$

Firms are competitive and labor productivity $\theta_i(= x_i/n_i)$ for $i \in [1, N]$ is constant; therefore, all the relative prices are constant and satisfy

$$p_i\theta_i = p_j\theta_j \text{ and } \frac{dp_i}{p_i} = \frac{dp_j}{p_j} \text{ for any } i, j \in [1, N]. \tag{46}$$

From (45) and (46), employment n_i in sector i and total employment n are

$$n_i = \frac{x_i}{\theta_i} = \frac{\delta_i c}{p_i\theta_i}, \quad n = \frac{c}{p_i\theta_i}. \tag{47}$$

CPI is defined so that a change in γ does not vary U for given c. Therefore, partially differentiating the second equation of (45) with respect to γ, and applying (44), the properties of δ_i and $\delta_i{}'$ given by (45), and (46) to the result yields

$$\begin{aligned} U_\gamma &= h_N \frac{\delta_N c}{p_N} - \sum_{i=1}^{N-1} h_i \frac{\delta_i c}{p_i}\left(\frac{\partial p_i/\partial\gamma}{p_i}\right) - h_N \frac{\gamma\delta_N c}{p_N}\left(\frac{\partial p_N/\partial\gamma}{p_N}\right) \\ &= \lambda c\left(\frac{\delta_N}{\gamma} - \frac{\partial p_N/\partial\gamma}{p_N}\right) = 0, \end{aligned} \tag{48}$$

which implies that $p_i(= P_i/P)$ for any $i \in [1, N]$ satisfies

$$\frac{\gamma}{p_i}\frac{\partial p_i}{\partial\gamma} = \delta_N > 0 \text{ for any } i \in [1, N]. \tag{49}$$

If full employment prevails (i.e., $n = 1$), from (45), (47), (48) and (49), we obtain

$$\frac{dU}{d\gamma} = U_c\frac{dc}{d\gamma} = U_c\frac{\partial p_i}{\partial\gamma}\theta_i > 0.$$

Proposition 1 is valid and variety expansion always increases real consumption and consumption utility.

If aggregate demand stagnation occurs in the steady state, the household's intertemporal optimization is the same as that in equations (14), (29), and (43), which is given as follows:

$$\rho + \alpha(n-1) = \frac{\beta}{U_c}, \quad U_c = \sum_{i=1}^{N-1}\frac{h_i}{p_i}\delta_i + \frac{h_N}{p_N}\gamma\delta_N = \lambda, \tag{50}$$

where total employment n is given by (47) and the second equation is from (44) and (45). Furthermore, $U_c(= \lambda)$ satisfies the following property:

Lemma 2: *If the utility of consumption is homothetic, then*

$$U_{c\gamma}\left(=\frac{\partial\lambda}{\partial\gamma}\right)=0.$$

Proof. See Appendix C.

From Lemma 2, an increase in γ does not affect the right-hand side of the first equation in (50) while it raises p_i from (49), decreases total employment n given by (47) for the same c, and lowers the left-hand side of the first equation in (50). Specifically, the right-hand side represents the desire for wealth accumulation, which is unaffected, and the left-hand side represents the desire for consumption, which decreases. Consequently, real consumption c decreases, which results in a reduction in total employment $n(=c/(p_i\theta_i))$, where p_i rises, as shown by (49). This result is formally restated as follows:

Proposition 5: *In the presence of aggregate demand shortage, quantity-augmenting-like innovation on a commodity decreases real consumption and total employment in the N-commodity economy. This also implies that an introduction of a new commodity (i.e., variety expansion) decreases real consumption and total employment.*

Therefore, under full employment, variety expansion increases real consumption, whereas in the presence of aggregate demand shortage, variety expansion generally decreases real consumption and total employment, and diminishes business activity as long as the consumption utility is homothetic. This holds true, regardless of whether the commodities are substitutes or complements to one another.

6. Utility with three product innovations: An example

Let us finally present an example of consumption utility that includes the influences of quantity-augmenting-like product innovation, addictive product innovation and variety expansion at the same time. It is the following utility function in the case where there are initially two commodities, x and y, and a new commodity z that has not yet been supplied:

$$U = u(x, y, z; \zeta, \nu, \gamma) = \frac{(\zeta x)^{\nu-\varepsilon} y^{\varepsilon} + (\gamma z)^{\nu} - 1}{\nu};$$
$$(\nu - \varepsilon)\varepsilon > 0,\ \nu < 1,\ \zeta > 0,\ \gamma \geq 0. \tag{51}$$

Apparently, this function is homothetic with respect to x, y and z. Note that when $\gamma = 0$ and $\zeta = 1$, it is the same as the utility function in (32).[15]

Assuming that initially $\gamma = 0$ and $\zeta = 1$, an increase in ζ implies quantity-augmenting-like product innovation, an increase in ν implies addictive product innovation, and an increase in γ from zero implies an introduction of commodity z. As discussed in subsections 3-2 and 4-2, addictive innovation, represented by an increase in ν, increases real consumption and total employment as long as household consumption is around the average of G7 countries. From Proposition 5, the other two decrease real consumption and total employment. Therefore, if there is a choice of product innovation to be encouraged, the one that increases ν (namely, addictive product innovation) should be chosen for the purpose of stimulating business activity.

[15] See Appendix D for the case where the first term of the numerator of u is a CES function with respect to x and y: $\left[\left(1 - \frac{\varepsilon}{\nu}\right)(\zeta x)^{\eta} + \frac{\varepsilon}{\nu} y^{\eta}\right]^{\frac{\nu}{\eta}}$. Note that this function turns to be the same as that in (51) if $\eta = 0$.

7. Concluding remarks

Product innovation raises the utility of consumption and is expected to increase consumption. Under full employment this assumption holds; therefore, product innovation is often cited as an economic stimulus even when a country is confronting unemployment due to a shortage of aggregate demand. However, it increases not only the value of consumption but also the value of money, which is measured by the commodities it can buy. The former stimulates the desire for consumption and the latter urges people to hold money if they have wealth preference. Consequently, product innovation may increase or decrease real consumption and employment under aggregate demand stagnation, depending on the relative magnitude of the two.

We consider three types of product innovation, including quantity-augmenting-like product innovation, addictive product innovation, and variety expansion. The first works as if households were consuming a larger quantity of the commodity for which the innovation takes place, although they actually consume the same quantity. The second decreases the elasticity of the marginal utility of consumption. The third expands the variety of consumption commodities.

Quantity-augmenting-like product innovation proportionally increases the value of the commodity and the value of money. Consequently, the marginal utility of consumption becomes smaller than that of wealth because the elasticity of the marginal utility of consumption is greater than that of the marginal utility of wealth, decreasing real consumption. Employment also declines because the product innovation decreases the quantity of the commodity that yields the same real consumption.

Addictive product innovation directly increases both the utility and marginal utility of consumption, of which the former also increases the value of money because the same amount of money can buy the commodity that gives higher utility. Therefore, it is generally ambiguous which raises its own marginal utility more, consumption or money, implying that the effects on real consumption and employment are also ambiguous. However, for a plausible value of the elasticity of the marginal utility of consumption, addictive innovation increases both employment and real consumption.

Finally, variety expansion functions in the same manner as quantity-augmenting-like product innovation. Therefore, it reduces both real consumption and total employment.

In conclusion, addictive product innovation should be encouraged as an economic stimulus, while quantity-augmenting-like product innovation or variety expansion should not be encouraged, because addictive product innovation stimulates aggregate demand and expands employment while the remaining two aggravate stagnation.

Appendices

Appendix A: The elasticity of the marginal utility of consumption

Without wealth preferences, as assumed in the standard model, the Ramsey equation in (7) reduces to

$$\rho + \xi \frac{\dot{c}}{c} + \pi = \frac{v_T{}'(m)}{U_c(c;\gamma)} = R.$$

Therefore, a change in R will vastly vary $\dot{c}/c$ if ξ is small. This is why the DSGE literature insists that ξ must be sufficiently large. However, in the presence of wealth preference, the Ramsey equation (7) is as follows:

$$\rho + \xi \frac{\dot{c}}{c} + \pi = R + \frac{v_W{}'(a)}{U_c(c;\gamma)}.$$

Therefore, regardless of the value of ξ, a change in R can be absorbed by a change in the wealth premium $v_W{}'(a)/U_c(c;\gamma)$ without affecting $\dot{c}/c$.

Furthermore, when the utility function is given by (32) and $\gamma < 0$, the effect of a rise in γ on the demand for x is highly unrealistic, as shown below. Under this utility function, x and y satisfy the third equation in (33); hence,

$$\frac{x}{y} = \frac{\gamma\omega}{\varepsilon}.$$

This demonstrates that in the case where $\gamma > 0$, $\varepsilon > 0$; thus, $\xi = 1 - (\gamma + \varepsilon) < 1$, which we examine in the text, a rise in γ increases demand for x. Intuitively, an increase in the utility of x and a decrease in the elasticity (or the curvature) of the marginal utility of x, $-u_{xx}x/u_x = 1 - \gamma$ from (32), both of which are due to a rise in γ, naturally raise demand for x. However, in the case where $\gamma < 0$, $\varepsilon < 0$; thus, $\xi = 1 - (\gamma + \varepsilon) > 1$, a rise in γ (or equivalently, a decrease in $|\gamma|$) reduces demand for x. Intuitively, a rise in γ increases the utility of x and decreases the elasticity (or the curvature) of the marginal utility of x, and nevertheless, it decreases demand for x, which is unrealistic. Therefore, we ignore the case where $\gamma < 0$, $\varepsilon < 0$ and $\xi > 1$ in the text.

Appendix B: Proof of equation (31)

From (24) and (29), we obtain

$$\hat{\Omega}\frac{dn}{nd\gamma} = \frac{1}{u_{xx}x^2}\left(\delta u_\gamma - xu_{x\gamma} - (u_{xx} - \frac{u_{y\gamma}}{\omega})x^2\frac{\delta'}{\delta}\right). \tag{A1}$$

Partially differentiating (23) with respect to x and γ yields

$$\frac{u_y(x,y;\gamma)}{u_x^2(x,y;\gamma)}\left(\frac{u_{yx}(x,y;\gamma)}{\omega} - u_{xx}(x,y;\gamma)\right) = \frac{u_y\left(\frac{x}{y},1;\gamma\right)}{yu_x^2\left(\frac{x}{y},1;\gamma\right)}\left(\frac{u_{yx}\left(\frac{x}{y},1;\gamma\right)}{\omega} - u_{xx}\left(\frac{x}{y},1;\gamma\right)\right),$$

$$\frac{u_y(x,y;\gamma)}{u_x^2(x,y;\gamma)}\left(\frac{u_{y\gamma}(x,y;\gamma)}{\omega} - u_{x\gamma}(x,y;\gamma)\right) = \frac{u_y\left(\frac{x}{y},1;\gamma\right)}{u_x^2\left(\frac{x}{y},1;\gamma\right)}\left(\frac{u_{y\gamma}\left(\frac{x}{y},1;\gamma\right)}{\omega} - u_{x\gamma}\left(\frac{x}{y},1;\gamma\right)\right),$$

$$-\left(\frac{u_{yx}\left(\frac{x}{y},1;\gamma\right)}{\omega} - u_{xx}\left(\frac{x}{y},1;\gamma\right)\right)\zeta' = \frac{u_{y\gamma}\left(\frac{x}{y},1;\gamma\right)}{\omega} - u_{x\gamma}\left(\frac{x}{y},1;\gamma\right),$$

from which we find

$$u_{xx}(x,y;\gamma) - \frac{u_{yx}(x,y;\gamma)}{\omega} = \left(\frac{u_{y\gamma}(x,y;\gamma)}{\omega} - u_{x\gamma}(x,y;\gamma)\right)\frac{1}{y\zeta'}. \tag{A2}$$

Applying δ' in (24) and (A2) to (A1) and gives

$$\hat{\Omega}\frac{dn}{nd\gamma} = \frac{1}{u_{xx}x^2}\left(\delta u_\gamma - \frac{(1-\delta)xu_{y\gamma}}{\omega} - \delta x u_{x\gamma}\right).$$

Given that $(1-\delta)x/\omega = \delta y$ from (27), the above equation implies the first equation in (31).

Since $c = p_x\theta_x n$ from the last equation in (27), $(dp_x/d\gamma)/p_x$ is given by (26), and $\hat{\Omega}$ is given in (30), from the first equation in (31), we obtain the second equation in (31).

Appendix C: Proof of Lemma 2

Given that function h is homothetic, the first-order optimal conditions (44) lead to

$$\frac{h_i(x_1,\ldots,x_{N-1},X_N)}{h_N(x_1,\ldots,x_{N-1},X_N)} = \frac{h_i(x_1/X_N,\ldots,x_{N-1}/X_N,1)}{h_N(x_1/X_N,\ldots,x_{N-1}/X_N,1)} = \frac{\gamma p_i}{p_N}, \quad X_N = \gamma x_N. \tag{A3}$$

Partial derivatives of the first equation in (A3) with respect to X_N and x_j for $j \in [1, N-1]$ are

$$\frac{h_{iN}}{h_N} - \frac{h_i h_{NN}}{h_N^2} = \sum_{j=1}^{N-1}\left(\frac{\hat{h}_{ij}}{\hat{h}_N} - \frac{\hat{h}_i\hat{h}_{Nj}}{\hat{h}_N^2}\right)\left(-\frac{x_j}{X_N^2}\right),$$

$$\frac{h_{ij}}{h_N} - \frac{h_i h_{Nj}}{h_N^2} = \left(\frac{\hat{h}_{ij}}{\hat{h}_N} - \frac{\hat{h}_i\hat{h}_{Nj}}{\hat{h}_N^2}\right)\frac{1}{X_N}, \quad \text{for } j \in [1, N-1] \tag{A4}$$

$$\text{where} \quad \hat{h} = h(x_1/X_N,\ldots,x_{N-1}/X_N,1).$$

From (45) and (A3),

$$\frac{x_j}{X_N} = \frac{(\frac{p_N}{\gamma})\delta_j}{p_j\delta_N} \quad \text{for} \;\; j \in [1, N-1].$$

Using this equation and (A4), we find

$$\sum_{j=1}^{N}\left(U_{ij} - U_{Nj}\right)\delta_j = 0,$$
$$\text{where} \;\; U_{ij} = \frac{h_{ij}}{p_i p_j}, \; U_{Nj} = \frac{h_{Nj}}{(p_N/\gamma)p_j}, \; U_{NN} = \frac{h_{NN}}{(p_N/\gamma)^2}. \tag{A5}$$

Note that this property is obtained because $h(x_1, \ldots, X_N)$ is homothetic.

Partially differentiating (44) with respect to γ and applying (49) to the result produces the following:

$$H\begin{pmatrix} \frac{\partial x_1}{\partial \gamma} \\ \vdots \\ \frac{\partial x_{N-1}}{\partial \gamma} \\ \frac{\partial X_N}{\partial \gamma} \\ \frac{\partial \lambda}{\partial \gamma} \end{pmatrix} = \begin{pmatrix} \lambda \frac{\partial p_1}{\partial \gamma} \\ \vdots \\ \lambda \frac{\partial p_{N-1}}{\partial \gamma} \\ \frac{\lambda}{\gamma}\left(\frac{\partial p_N}{\partial \gamma} - \frac{p_N}{\gamma}\right) \\ 0 \end{pmatrix} = \frac{\lambda}{\gamma}\begin{pmatrix} \delta_N p_1 \\ \vdots \\ \delta_N p_{N-1} \\ (\delta_N - 1)\frac{p_N}{\gamma} \\ 0 \end{pmatrix},$$
$$\text{where} \quad H = \begin{pmatrix} h_{11} & \cdots & h_{1N-1} & h_{1N} & -p_1 \\ \vdots & & & & \vdots \\ h_{N-11} & \cdots & h_{N-1N-1} & h_{N-1N} & -p_{N-1} \\ h_{N1} & \cdots & h_{NN-1} & h_{NN} & -\frac{p_N}{\gamma} \\ -p_1 & \cdots & -p_{N-1} & -\frac{p_N}{\gamma} & 0 \end{pmatrix}. \tag{A6}$$

By applying Cramer's rule to (A6), we obtain $\gamma\partial\lambda/\lambda\partial\gamma$:

$$\left(\frac{\gamma}{\lambda}\right)\frac{\partial \lambda}{\partial \gamma} = \frac{\det(H_\lambda)}{\det(H)}, \tag{A7}$$

where we rewrite $\det(H_\lambda)$ using the definition of U_{ij} in (A5) as follows:

$$\det(H_\lambda) = \begin{vmatrix} h_{11} & \cdots & h_{1N-1} & h_{1N} & \delta_N p_1 \\ \vdots & & & & \vdots \\ h_{N-11} & \cdots & h_{N-1N-1} & h_{N-1N} & \delta_N p_{N-1} \\ h_{N1} & \cdots & h_{NN-1} & h_{NN} & (\delta_N - 1)\frac{p_N}{\gamma} \\ -p_1 & \cdots & -p_{N-1} & -\frac{p_N}{\gamma} & 0 \end{vmatrix} = \left(\frac{p_N}{\gamma}\right)^2 \left(\prod_{j=1}^{N-1} p_j^2\right)\Gamma,$$

$$\text{where } \Gamma = \begin{vmatrix} U_{11} & \cdots & U_{1N-1} & U_{1N} & \delta_N \\ \vdots & & & & \vdots \\ U_{N-11} & \cdots & U_{N-1N-1} & U_{N-1N} & \delta_N \\ U_{N1} & \cdots & U_{NN-1} & U_{NN} & \delta_N - 1 \\ -1 & \cdots & -1 & -1 & 0 \end{vmatrix}.$$

Subtracting column N from columns $1, \dots, N-1$ in Γ and multiplying row j by δ_j for $j \in [1, N]$ yields

$$\frac{\Gamma}{\prod_{j=1}^{N} \delta_j} = \begin{vmatrix} (U_{11} - U_{1N})\delta_1 & \cdots & (U_{1N-1} - U_{1N})\delta_1 & U_{1N}\delta_1 & \delta_N\delta_1 \\ \vdots & & & & \vdots \\ (U_{N-11} - U_{N-1N})\delta_{N-1} & \cdots & (U_{N-1N-1} - U_{N-1N})\delta_{N-1} & U_{N-1N}\delta_{N-1} & \delta_N\delta_{N-1} \\ (U_{N1} - U_{NN})\delta_N & \cdots & (U_{NN-1} - U_{NN})\delta_N & U_{NN}\delta_N & (\delta_N - 1)\delta_N \\ 0 & \cdots & 0 & -1 & 0 \end{vmatrix}.$$

Adding rows $1, \dots, N-1$ to row N and applying (A5) and $\sum_{j=1}^{N} \delta_j = 1$ to the result produces

$$\frac{\Gamma}{\prod_1^N \delta_j} = \begin{vmatrix} (U_{11} - U_{1N})\delta_1 & \cdots & (U_{1N-1} - U_{1N})\delta_1 & U_{1N}\delta_1 & \delta_N\delta_1 \\ \vdots & & & & \vdots \\ (U_{N-11} - U_{N-1N})\delta_{N-1} & \cdots & (U_{N-1N-1} - U_{N-1N})\delta_{N-1} & U_{N-1N}\delta_{N-1} & \delta_N\delta_{N-1} \\ 0 & \cdots & 0 & \sum_{i=1}^{N} U_{iN}\delta_i & 0 \\ 0 & \cdots & 0 & -1 & 0 \end{vmatrix}.$$

Note that zeros in columns $1, \dots, N-1$ of row N are obtained because of homotheticity, while the zero in column $N+1$ of row N is due to quantity-augmenting-like innovation. Finally, multiplying the last row by $\sum_{i=1}^{n} U_{iN}\delta_i$ and adding the result to row N makes all values of row N equal 0, implying $\Gamma = 0$ and $\det(H_\lambda) = 0$. Therefore, the value given by (A7) satisfies

$$\left(\frac{\gamma}{\lambda}\right)\frac{\partial\lambda}{\partial\gamma} = \frac{\det(H_\lambda)}{\det(H)} = 0.$$

Appendix D:

We replace the Cobb-Douglas function $(\zeta x)^{\nu-\varepsilon}y^{\varepsilon}$ in the numerator of (51) by a more general CES function as follows:

$$U = u(x, y, z; \zeta, \nu, \gamma) = \frac{\left[\left(1-\frac{\varepsilon}{\nu}\right)(\zeta x)^{\eta} + \frac{\varepsilon}{\nu}y^{\eta}\right]^{\frac{\nu}{\eta}} + (\gamma z)^{\nu} - 1}{\nu}, \tag{A8}$$

where $\eta < 1,\ (\nu - \varepsilon)\varepsilon > 0,\ \nu < 1,\ \zeta > 0,\ \gamma \geq 0,$

and show that the results obtained in the text hold true under this function as well. Note that this function turns to be the same as the function in (51) when $\eta = 0$.

First, it is readily found that this function is homothetic with respect to x, y and z. Therefore, from Proposition 5, quantity-augmenting-like innovation, represented by an increase in ζ, and an introduction of commodity z, represented by an increase in γ from zero, decrease real consumption and employment.

Let us next examine the effect of addictive product innovation, which is represented by an increase in ν when $\gamma = 0$ and $\zeta = 1$. In this case, u in (A8) reduces to

$$u(x, y; \nu) = \frac{\left[\left(1-\frac{\varepsilon}{\nu}\right)x^{\eta} + \frac{\varepsilon}{\nu}y^{\eta}\right]^{\frac{\nu}{\eta}} - 1}{\nu},$$

which is a CES version of u in (32). Note that it is equivalent to u in (32) when $\eta = 0$. It satisfies

$$\nu^2 u_\nu = \left(x^\eta + \frac{\epsilon}{\nu}(y^\eta - x^\eta)\right)^{\frac{\nu}{\eta}}$$

$$\times\left[\frac{\nu}{\eta}\ln\left(x^\eta + \frac{\epsilon}{\nu}(y^\eta - x^\eta)\right) - \frac{\frac{\epsilon}{\eta}(y^\eta - x^\eta)}{\left(x^\eta + \frac{\epsilon}{\nu}(y^\eta - x^\eta)\right)} - 1\right] + 1,$$

$$\nu(xu_{\nu x} + yu_{\nu y})$$

$$= \left(x^\eta + \frac{\epsilon}{\nu}(y^\eta - x^\eta)\right)^{\frac{\nu}{\eta}}\left[\frac{\nu}{\eta}\ln\left(x^\eta + \frac{\epsilon}{\nu}(y^\eta - x^\eta)\right) - \frac{\frac{\epsilon}{\eta}(y^\eta - x^\eta)}{\left(x^\eta + \frac{\epsilon}{\nu}(y^\eta - x^\eta)\right)}\right], \quad \text{(A9)}$$

$$u_x = \left(1 - \frac{\epsilon}{\nu}\right)x^{\eta-1}\left(x^\eta + \frac{\epsilon}{\nu}(y^\eta - x^\eta)\right)^{\frac{\nu}{\eta}-1},$$

$$u_y = \frac{\epsilon}{\nu}y^{\eta-1}\left(x^\eta + \frac{\epsilon}{\nu}(y^\eta - x^\eta)\right)^{\frac{\nu}{\eta}-1},$$

Thus, (23) and (35) turn to

$$\frac{u_y}{u_x} = \omega_{yx} = \frac{\frac{\epsilon}{\nu}}{1-\frac{\epsilon}{\nu}}\left(\frac{x}{y}\right)^{1-\eta} \Rightarrow \frac{y}{x} = \left(\frac{1}{\left(\frac{\nu}{\epsilon}-1\right)\omega_{yx}}\right)^{\frac{1}{1-\eta}} \equiv \Upsilon > 1, \quad \omega_{yx} = 1. \quad \text{(A10)}$$

Given that $y = \Upsilon x$ from (A10), substituting Υx to y in (A9) and applying the results to the first equation of (31) leads to

$$\frac{dn}{d\nu} \gtreqless 0 \Leftrightarrow \tilde{\psi}(x;\nu) \equiv \nu^2\left(xu_{\nu x} + \Upsilon x u_{\nu y} - u_\nu\right) \gtreqless 0,$$

$$\tilde{\psi}(x;\nu) = -1 + x^\nu\left(1 - \frac{\epsilon}{\nu}(1-\Upsilon^\eta)\right)^{\frac{\nu}{\eta}}$$

$$\times\left\{1 - (1-\nu)\left[\ln\left(x^\nu\left(1 - \frac{\epsilon}{\nu}(1-\Upsilon^\eta)\right)^{\frac{\nu}{\eta}}\right) + \frac{\epsilon\left(\frac{1-\Upsilon^\eta}{\eta}\right)}{1-\frac{\epsilon}{\nu}(1-\Upsilon^\eta)}\right]\right\}.$$

Note that when $\eta = 0$, $\tilde{\psi}(x;\nu)$ is equal to $\hat{\psi}(x;\gamma)$ in (34) where γ replaces ν. The threshold x in the case where u is given by (A8), denoted by $\tilde{x}_\psi$, satisfies $\tilde{\psi}(\tilde{x}_\psi;\nu) = 0$, while the minimum x, denoted by $\tilde{x}_\varphi$, satisfies $\tilde{\varphi}(\tilde{x}_\varphi;\nu) = 0$, where $\tilde{\varphi}(x;\nu)$ is obtained by substituting Υx to y in the first equation of (A9) as follows:

$$\nu^2 u_\nu = \tilde{\varphi}(x;\nu) \equiv x^\nu \left(1 - \frac{\epsilon}{\nu}(1 - \Upsilon^\eta)\right)^{\frac{\nu}{\eta}}$$

$$\times \left\{\frac{\nu}{\eta} \ln\left(\left(1 - \frac{\epsilon}{\nu}(1 - \Upsilon^\eta)\right) x^\eta\right) + \frac{\frac{\epsilon}{\eta}(1 - \Upsilon^\eta)}{\left(1 - \frac{\epsilon}{\nu}(1 - \Upsilon^\eta)\right)} - 1\right\} + 1\ (> 0).$$

We can easily find that when $\eta = 0$, $\tilde{x}_\psi$ and $\tilde{x}_\varphi$ are equal to x_ψ and x_φ in (36) and (37), respectively.

Table 4 summarizes the ratio of the two values, $\tilde{x}_\psi / \tilde{x}_\varphi$, for various ξ, $\varepsilon/(\nu - \epsilon)$ and η. When $\eta = 0$, implying that u is given by (51), the values are the same as those in Table 3. In fact, when $\eta = 0.001$, they are the same as those in Table 3. Moreover, the ratio does not change much for various values of η and is always lower than the values in Table 2. Therefore, the result obtained in the text –i.e., addictive innovation increases both employment and real consumption, is quite robust for various values of η.

Table 4: Survival consumption level and threshold (the case of CES utility)

$\xi(= 1 - \nu)$		0.2			0.3			0.4		
$\varepsilon/(\nu - \epsilon)$		1	10	100	1	10	100	1	10	100
$\frac{\tilde{x}_\psi}{\tilde{x}_\varphi}$	$\eta = 0.9$	496	183	183	96.3	34.7	34.7	41.2	14.9	14.9
	$\eta = 0.1$	496	162	151	96.3	30.8	28.7	41.2	13.4	12.5
	$\eta = 0.001$	496	162	150	96.3	30.8	28.5	41.2	13.4	12.4
	$\eta = -0.1$	496	162	149	96.3	30.8	28.3	41.2	13.4	12.3
	$\eta = -10$	496	238	151	96.3	44.9	28.7	41.2	18.9	12.5
	$\eta = -50$	496	330	190	96.3	62.6	36.1	41.2	26.2	15.4

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