Discussion Paper No. 1205

ISSN (Print) 0473-453X
ISSN (Online) 2435-0982

LEADING PATENT BREADTH, ENDOGENOUS QUALITY CHOICE, AND ECONOMIC GROWTH

Keishun Suzuki
Shin Kishimoto

March 2023

The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

Leading Patent Breadth, Endogenous Quality Choice, and Economic Growth*

Keishun Suzuki[†], Shin Kishimoto[‡]

March 13, 2023

Abstract

O'donoghue and Zweimüller (2004, J. of Econ. Growth), a seminal work, showed that broadening leading breadth in patent protection can stimulate innovation. However, the empirical literature has consistently found skeptical results on the positive effect. To fill the gap, we build another framework where the quality improvement size is derived as an interior solution. In our model, broadening leading breadth can negatively affect innovation because each innovator is incentivized to *free-ride* the other innovators' quality improvements. As a further analysis, we quantitatively investigate the growth effect of intervention in patent licensing negotiation using two different profit division rules derived from a cooperative game. We find that intervention in patent licensing negotiation increases the growth rate and stabilizes the economy.

Keywords: Patent protection, Leading breadth, Schumpeterian growth, Endogenous quality increments, Cooperative game.
JEL-Classification: C71, D45, O30.

*We thank many valuable comments from Ryousuke Shimizu, Tatsuro Iwaisako, Takeo Hori, Yuichi Furukawa, Akihiko Yanase, Real Arai, Kohei Hasui, and participants of the 2022 spring meeting of the Japanese Economic Association and the seminars of Aichi University and Osaka Metropolitan University. Specifically, we are grateful to Ryo Horii for his suggestions at the Joint Usage/Research Center at the Institute of Social and Economic Research (ISER), Osaka University. This research is financially supported by the Japan Society for the Promotion of Science (JSPS) KAKENHI Grant Numbers JP20K13449 (Suzuki) and JP20K01542 (Kishimoto). All remaining errors are our own.

[†]Graduate School of Social Sciences, Chiba University. 1-33, Yayoi-cho, Inage-ku, Chiba, Japan. Email: ksuzuki@chiba-u.jp.

[‡]Graduate School of Social Sciences, Chiba University. 1-33, Yayoi-cho, Inage-ku, Chiba, Japan. Email: skishimoto@chiba-u.jp.

1 Introduction

Research and development (R&D) is a cumulative process. Innovation builds on previous innovations, similar to economists extending previous studies when writing their papers. The cumulativeness creates an exciting but complex problem in the presence of the *leading breadth*, a scope of patent protection against competition from higher-quality products.[1] For a technology to be patentable, it must have a sufficient *novelty* compared to past technologies. However, the required novelty may be insufficient to avoid infringing existing patents. Generally, the minimum improvement size for patentability does not equal the size of improvement needed to avoid infringements of existing patents.[2]

Traditionally, strengthening patent protection had been recognized as an effective policy to promote technological progress because such policy reforms increase the rewards of innovation. Under a broader leading breadth, past innovators can claim patent infringement against new innovators. Therefore, like other dimensions of patent protection (e.g., the length of patent term, the lagging breadth, and the enforcement), the leading breadth enables patent holders to appropriate the returns from inventions. The traditional idea that more rewards induce more innovation is called the *Schumpeterian effect*. Many existing studies show that patent protection has been consistently strengthened worldwide (e.g., Ginarte and Park, 1997; Park, 2008; Papageorgiadis et al., 2014).

However, the empirical studies do not support the straightforward idea that strong patent protection enhances innovation. Lerner (2002), an inclusive survey paper, lists many empirical studies and states that "these papers generally cast doubt on claims that enhancing patent policy changes spurs innovative behavior." Boldrin and Levine (2009) also collect numerous empirical studies and summarize the findings as "they find weak or no empirical evidence that strengthening patent regimes increases innovation." This counterintuitive result is a longstanding question in the literature called the *patent puzzle*.

The leading breadth is no exception to the patent puzzle, although the literature on endogenous growth theory has paid little attention to it. O'donoghue and Zweimüller (2004) (from here on, O&Z) is a pioneer work incorporating leading breadth into a canonical quality-ladder model, like pension in the overlapping generations model. Specifically, a new innovator must pay the license fees to the past innovators but can eventually obtain the license fees from the future innovators after the subsequent generational innovations occur. This feature yields the so-called *backloading effect*, negatively affecting on the present discounted value of R&D by delaying the timing of the future rewards. O&Z shows that the growth effect of broadening leading breadth is always positive when the discount rate is sufficiently low such that the backloading

[1] This definition is from O'Donoghue et al. (1998). The concept of patent breadth includes both the lagging and leading breadths. While lagging breadth is a scope that protects an invention from imitation, leading breadth is a scope that protects an invention from future innovation.

[2] For the details, see Scotchmer (2004). She introduces a history of Maser and Laser to explain this point.

effect is smaller than the Schumpeterian effect. However, there is no significant evidence of the positive effect in empirical studies that focus on a specific policy reform that broadened the leading breadth in the U.S. and Japan (e.g., Hall and Ziedonis, 2001; Sakakibara and Branstetter, 2001).[3]

The Schumpeterian effect in the O&Z model arises from (i) the exogenous size of the quality improvement and (ii) the Bertrand competition where multiple firms on the quality ladder can form a price cartel. First, for simplicity, they consider a special case in which the quality improvement size is always the same as the minimum requirement for patenting (i.e., the legal lower bound of the quality improvement for the grant), which is exogenously given as a policy parameter. In other words, due to the corner solution, their model neglects the effect of broadening leading breadth on the quality improvement size. Second, in their model, a new innovator colludes with the licensors (i.e., the licensors do not produce their goods) to maximize the profit of their cartel in the Bertrand competition. Because a broader leading breadth increases the quality gap between the new innovator and the nearest rival outside the cartel, the new innovator can set a higher markup. This results in the broadening of leading breadth yielding a strong Schumpeterian effect in their model.

Our paper aims to explore the negative aspect of broadening leading breadth. We build a modified O&Z-type model in which each R&D firm chooses the quality improvement size as an interior solution. We find that the growth effect of broadening leading breadth can be negative by shrinking the quality improvement size. Our result is opposite to O&Z because the growth effect is still negative even when the discount rate is close to zero. It is well-known that high-income countries tend to have a lower discount rate (e.g., Wang et al., 2016; Falk et al., 2018). We contribute to the literature by solving the patent puzzle in developed countries, such as the U.S. and Japan.

The fundamental mechanism of the negative growth effect is the *free-ride* on the past innovators' quality improvement. In the O&Z-type models, some successive firms in a quality ladder form the cartel to enhance their monopolistic power. Our model assumes that the R&D success probability decreases in the quality improvement size as a realistic assumption. Then, when the past innovators in the cartel had sufficiently widened the quality gap between the nearest rival firm, R&D firms will shrink the quality improvement size to join the cartel quickly.

Our model predicts that the broadening leading breadth reduces each patent's contribution to economic growth. This seems consistent with the aggregate data in the U.S. because the

[3] Hall and Ziedonis (2001) focus on the doctrine of equivalents by the Court of Appeals for the Federal Circuit (CAFC) established in 1982 in the U.S. The doctrine of equivalents is a judicial treatment that even a new invention that does not coincide with the claims of past patents could be considered infringing on them if the new invention is intrinsically equivalent to them. As the doctrine of equivalents has expanded the scope of the leading breadth to the outside of the claims, an invention has the propensity to infringe on similar past patents. Sakakibara and Branstetter (2001) focus on the Japanese patent reform in 1988. This policy reform enabled patent applicants to include multiple claims in the same patent application. As claims define the technological scope of the patent, subsequent inventions are likely to infringe on past patents of similar inventions.

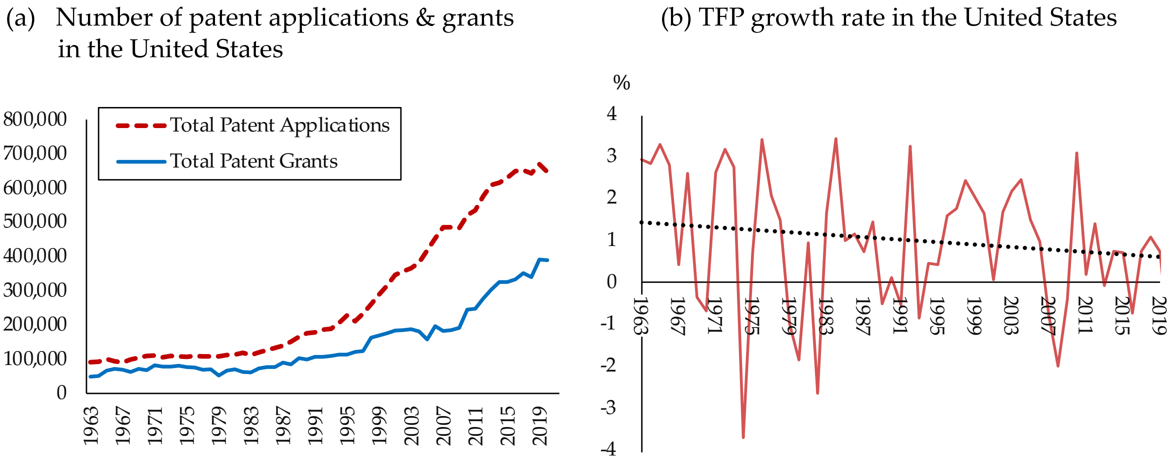

Figure 1: (a) Number of patent applications and grants in the United States and (b) TFP growth rate in the United States.

Source: The patent data is from the U.S. patent statistics chart (1963-2020) from the U.S. patent and trademark office. The TFP data is from the Federal Reserve Bank of San Francisco.

total factor productivity (TFP) growth rate in the U.S. has not increased over time, whereas the number of patent applications and grants has dramatically increased since the 1980s, as shown in Figure 1. Hall and Ziedonis (2001) focus on the U.S. policy reforms in the 1980s and find no significant evidence of accelerating the pace of innovation in the semiconductor industry where innovation is very cumulative. They attribute the change in patenting strategy of firms to the patent explosion since the 1980s. In high-tech industries where technologies are closely related to each other, firms must be licensed with many patents separately held by independent owners to commercialize their own technology. In private negotiations of patent licensing, the relative bargaining power of each patent holder matters (e.g., Sakakibara, 2010). Hall and Ziedonis (2001) emphasizes that firms have an incentive to "harvest" more patents from their R&D to improve their negotiating position. Their view suggests that the patent policy reforms have lowered the average quality of patents because firms began to apply for patents on trivial technologies.

1.1 Related literature

The recent literature on Schumpeterian growth theory has emphasized the role of patent licensing on economic growth (e.g., Chu, 2009; Chu et al., 2012; Chu and Pan, 2013; Niwa, 2016, 2018; Yang, 2018; Suzuki, 2020; Kishimoto and Suzuki, 2021; Klein, 2022).

Our paper closely relates to Klein (2022) that quantitatively finds that the broadening leading breadth reduces the growth rate. Although this result overlaps with ours, the specification is quite different because we define the leading breadth as the extent of intra-industry patent infringement as O&Z. In contrast to O&Z, Klein (2022) defines the leading breadth as the extent of *inter-industry* patent infringement.[4] Although we do not consider such inter-industry patent infringement, we allow a case in which a new innovator infringes the latest and older patents in the industry. Therefore, our model can complement Klein (2022) by showing that the negative growth effect of the broadening leading breadth can emerge in a different specification.

Furthermore, our paper relates to Chu (2009) that quantitatively evaluates how much reducing the backloading effect (e.g., lowering the license fees) increases the R&D in the O&Z-type model. The author finds that reducing the backloading effect significantly increases the R&D. However, it remains unclear how much such policies are possible because the license fees are determined in private negotiations between the relevant firms and are not directly controllable variables for outsiders. In reality, competition authorities or standard-setting organizations can intervene in the negotiations to lower the license fees (Section 5 details this point). We quantitatively analyze how such interventions affect the growth rate by calibrating the model to the U.S. economy. We find that, while the growth effect of the intervention is quantitatively small, it can stabilize the macroeconomy by eliminating the multiplicity of equilibria. Thus, we contribute to the literature by considering more specific policies and showing the macroeconomic effects.

1.2 Roadmap

The structure of our paper is as follows. Section 2 builds a modified O&Z-type model in which each R&D firm chooses the size of quality improvement as an interior solution. Section 3 solves the long-run equilibrium and shows that the R&D firms may choose a small improvement size to enjoy a free-ride on the quality improvements by the other innovators. Section 4 analytically shows that the broadening leading breadth may decrease the growth rate and discusses the socially optimal innovation size. Section 5 calibrates the U.S. economy and quantitatively shows that the broadening leading breadth decreases the growth rate. We also find that intervention in patent licensing negotiation has a growth-enhancing effect. Section 6 concludes.

2 A baseline model

We consider a canonical model in the literature of leading breadth (e.g., O'donoghue and Zweimüller, 2004; Chu, 2009). Following these studies, we incorporate the leading breadth of patent and profit division between several innovators into the quality-ladder model of Grossman and Helpman (1991, Ch.4).

[4] Klein (2022) assumes that an innovation infringes the patents of the leading firms in a fraction of $\phi \in (0, 1)$ of a unit mass of industries. The author quantitatively finds that an increasing ϕ (i.e., the broadening leading breadth) is growth-reducing.

2.1 Households

We consider the following closed economy in continuous time. The economy consists of a continuum of identical infinitely living households of measure $L > 0$. There is no population growth. A representative household has the following intertemporal utility function:

$$U_t = \int_0^\infty \exp(-\rho t) \ln c_t dt, \tag{1}$$

where ρ is the subjective discount rate and c_t is a consumption index of final goods at time t.

The economy has a unit-continuum of industries indexed by $h \in [0, 1]$, and the household consumes final goods across all industries. In each industry, there are some countable firms that produce final goods that are differentiated in terms of quality. Let $\mathcal{N}(h)$ be the set of the firms producing goods in industry h. Firm $k \in \mathcal{N}(h)$ produces a final good whose quality is $q_k(h)$.

The instantaneous period utility is given by

$$\ln c_t = \int_0^1 \ln \left(\sum_{k \in \mathcal{N}(h)} q_k(h) x_{kt}(h) \right) dh, \tag{2}$$

where $x_{kt}(h)$ is the consumption of firm k's good in industry h at time t. According to the additive specification in the abovementioned period utility, all goods in each industry are perfect substitutes for households.

Let e_t be the household's expenditure in time t. Then, it is given by

$$e_t = \int_0^1 \left(\sum_{k \in \mathcal{N}(h)} p_{kt}(h) x_{kt}(h) \right) dh,$$

where $p_{kt}(h)$ is the price of firm k's good in industry h at time t. Every household supplies a unit of labor inelastically and earns a wage in each period. The budget constraint for each period is given by

$$\dot{a}_t = r_t a_t + w_t - e_t,$$

where a_t is the value of assets (equities), r_t is the interest rate, and w_t is the wage rate.

We solve the utility maximization problem in two steps: static problem and dynamic problem. First, given instantaneous expenditure level e_t, the household maximizes the period utility function $\ln c_t$. Under the logarithmic utility function, the household spends the budget equally across $h \in [0, 1]$. Moreover, for each industry, the household chooses the good with the lowest quality-adjusted price. Assume that firm $\tilde{k}(h)$ produces such a good in industry h. Then, the individual demand in industry h at time t is $x_{\tilde{k}t}(h) = e_t / p_{\tilde{k}t}(h)$ and $x_{kt}(h) = 0$ for $k \in \mathcal{N}(h) \backslash \{\tilde{k}(h)\}$.

Second, we solve the dynamic maximization problem. Every household decides the expenditure in each period to maximize the intertemporal utility function, U_t, subject to the intertemporal budget constraint. Their indirect period utility function is given by $\ln c_t = \ln e_t - \ln P_t$, where P_t is the aggregate price index associated with the consumption index c_t, which is defined as

$$\ln P_t \equiv \int_0^1 \ln \left(\frac{p_{\tilde{k}t}(h)}{q_{\tilde{k}}(h)} \right) dh.$$

Each household spends to maximize the intertemporal utility. From the intertemporal utility maximization, the household's optimal time path for spending is $\dot{e}_t / e_t = r_t - \rho$. The aggregate expenditure is $E_t \equiv e_t L$. As L is constant, $\dot{E}_t / E_t = r_t - \rho$ also holds. We normalize the aggregate price index at each time so that $P_t = 1$. Then, $e_t = c_t$ holds for all time. Therefore, the familiar Euler equation $\dot{c}_t / c_t = r_t - \rho$ also holds.

The total demand for the good that has the quality $\tilde{k}$ in industry h is $X_{\tilde{k}t}(h) \equiv x_{\tilde{k}t}(h)L = E_t / p_{\tilde{k}t}(h)$. Hereafter, the notations omit t and h in cases where there is no risk of misunderstanding.

2.2 Industries

Consider an industry in which there are many generations of innovators.[5] More recent innovators can produce higher quality goods than older innovators. Let "firm 0" be the latest innovator at the time. Similarly, let "firm i" be the $(i+1)$-th latest innovator. To distinguish between past innovators and future innovators from the perspective of firm 0, let firm $-i$ be the i-th innovator after firm 0 has emerged in the industry. For simplicity, firm i or firm $-i$ is sometimes denoted by "i" or "$-i$." Panel (a) of Figure 2 illustrates the industry at a certain time.

We assume that each firm has the same linear production technology. They can produce one unit of their own good by devoting one unit of labor. However, their goods are differentiated in terms of quality. The quality of firm i's good is λ_i-times higher than that of firm $i+1$'s good (i.e., $q_i = \lambda_i q_{i+1}$), where $\lambda_i > 1$ is the quality improvement size between firm i and firm $i+1$. As discussed later, the size λ_i is endogenously determined by solving the firm i's maximization problem.

Patent breadth

Each firm has a patent for its own good. We assume that the lagging breadth is perfect in the sense that firm i can claim patent infringement against any other firm that produces a good with quality in $(q_{i+1}, q_i]$ (i.e., the range of the quality improved by firm i). For the leading breadth, we assume that firm i can claim patent infringement against any other firm that produces a

[5] More formally, we suppose that the cardinality $|\mathcal{N}(h)|$ is sufficiently large for all $h \in [0, 1]$.

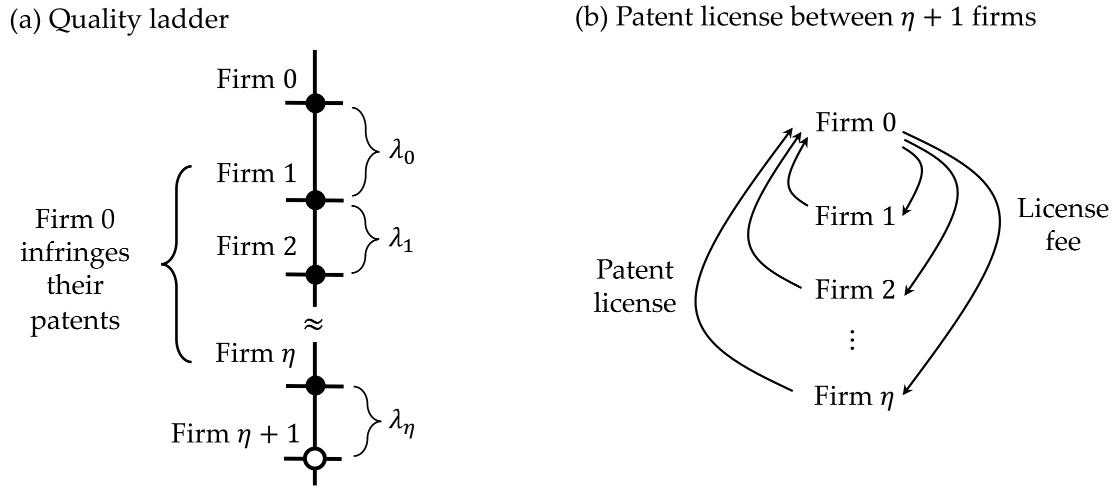

Figure 2: (a) Quality ladder and (b) Patent license

Notes: Firm 0 is the latest innovator in the industry. Although firm 0 has the patent for the highest quality good in the industry, because of the leading breadth, firm 0 infringes the patents of firm 1, firm 2, ..., and firm η. Therefore, to produce its own good, firm 0 must be licensed by all of them through the patent license negotiation (Panel (b)). λ_i is the firm i's quality improvement size (Panel (a)).

good with quality in $(q_i, q_{i-\eta}]$. This means that firm i's patent is certainly protected from the subsequent η innovations that occurred by firm $i-1$, firm $i-2$, ..., and firm $i-\eta$. We interpret the positive integer η as the degree of the leading breadth.

Our specification of the leading breadth is mathematically tractable because the optimal size of quality improvement (λ_i) can be analytically solved as an interior solution. Conversely, other specifications make it extremely difficult. For example, O&Z assumes that firm i's patent is protected from subsequent innovations because no other firm can produce goods with quality in $(q_i, Kq_i]$, where $K > 1$ is the parameter of the leading breadth. Unfortunately, their specification makes it almost impossible to analytically find the optimal size of quality improvement for each innovator as an interior solution (see the Online Appendix for the details). To avoid the complication, O&Z considers a corner solution in which all innovators make only the minimum quality improvements required to obtain a patent (i.e., $\lambda_i = P$ for all i), where $P > 1$ is given exogenously as a policy parameter. By setting $K = P^\alpha$ where $\alpha \in \mathbb{N}$, they eventually use α as the degree of the leading breadth instead of K, as η in our model.

By solving for the optimal λ_i as an interior solution, we can analyze the growth effect of the broadening leading breadth, including changes in the innovation size. O&Z neglects this channel because the innovation size is always constant in their model. As shown later, this channel is the main driver of the negative growth effect in our model. The Online Appendix demonstrates that our main result can continue to hold even when the scope of the leading breadth of firm i is given by $(q_i, Kq_i]$ and no minimum quality improvement is required to obtain a patent.

Bertrand competition

As illustrated in Panel (b) of Figure 2, due to the patent infringements, firm 0 cannot produce the good without being licensed by firm 1, firm 2, ..., and firm η. For notational ease, we define $\mathcal{L}_0 \equiv \{1, 2, \ldots, \eta\}$ as the set of the licensors of firm 0. If all firms in $\mathcal{L}_0$ license their own patents to firm 0, we call their consortium "pool" and denote the set by $\mathcal{P}_0 \equiv \{0\} \cup \mathcal{L}_0$.

All firms in the industry engage in Bertrand competition. Recall that their goods are perfect substitutes for all households. Therefore, firm 0 takes the *limit-pricing* strategy that charges a price low enough to exclude the nearest rival. Following O&Z, we assume that the competition authority allows a price cartel by $\mathcal{P}_0$. Namely, all firms in $\mathcal{L}_0$ do not produce their goods to consolidate the market power of firm 0. Then, the nearest competitor for firm 0 is firm $\eta + 1$. By the accumulation of the quality improvements, $q_0 = \left(\prod_{i=0}^{\eta} \lambda_i\right) \cdot q_{\eta+1}$ holds ($\prod$ is the symbol of the product). Recall that the lagging breadth prevents firm $\eta + 1$ from producing a good with higher quality than $q_{\eta+1}$. By the standard argument of the limit-pricing strategy, to exclude firm $\eta + 1$, firm 0 sets the following price:

$$\begin{aligned} p_0 &= \left(\frac{q_0}{q_{\eta+1}}\right) w \\ &= \left(\prod_{i=0}^{\eta} \lambda_i\right) w. \end{aligned} \tag{3}$$

This is the result of the Bertrand competition.[6]

We assume that the competition authority prohibits the pool from including another firm (e.g., firm $\eta + 1$) in $\mathcal{P}_0$ because it does not entail any patent license to firm 0 and aims to increase the markup. In other words, we assume that the competition authority does not allow the markup strictly higher than $\prod_{i=0}^{\eta} \lambda_i$.

Let $\tilde{\Pi}_0$ be the profit of firm 0. By $X_0 = E/p_0$ and (3), the profit becomes

$$\begin{aligned} \tilde{\Pi}_0 &= p_0 X_0 - w X_0 \\ &= \left(1 - \frac{1}{\prod_{i=0}^{\eta} \lambda_i}\right) E. \end{aligned} \tag{4}$$

2.3 Profit division in the pool

After the production, firm 0 shares $\tilde{\Pi}_0$ with the firms in $\mathcal{L}_0$ as the payments of the license fees. Let $\pi_i \geq 0$ be the net profit of firm $i \in \mathcal{P}_0$. We define $\mathbf{s} = (s_0, s_1, ..., s_\eta)$ as the profit-sharing

[6] To understand why this result holds, let p_0 and $p_{\eta+1}$ be the price of firm 0's good and the price of firm $\eta + 1$'s good, respectively. Each household purchases a good with the lowest quality-adjusted price in the industry, as all goods are perfect substitutes. Therefore, they purchase the firm 0's good when $q_0/p_0 \leq q_{\eta+1}/p_{\eta+1}$. As the production technology is linear, the lowest price that firm $\eta + 1$ can charge is the unit cost: $p_{\eta+1} = w$. Then, firm 0 can exclude firm $\eta + 1$ by charging (3).

vector where $s_i \equiv \pi_i / \tilde{\Pi}_0$ for all $i \in \mathcal{P}_0$. We assume that $\sum_{i=0}^{\eta} \pi_i = \tilde{\Pi}_0$ holds. Then, $\sum_{i=0}^{\eta} s_i = 1$ also holds. Let $\mathcal{S}$ be the set of the feasible profit-sharing vectors. More formally,

$$\mathcal{S} \equiv \left\{ \mathbf{s} \in [0,1]^{1+\eta} \,\middle|\, \sum_{i=0}^{\eta} s_i = 1 \right\}.$$

One inevitable problem when we discuss the profit division is how we consider the determination process of the profit-sharing vector. O&Z and other papers in the literature on the Schumpeterian growth theory have not formally considered how firms negotiate, and basically, they treat the license fees as exogenous for simplicity.[7]

We also treat the profit-sharing vector as exogenous in the baseline model for simplicity. However, in Appendix B, we formally consider how the firms in the pool share $\tilde{\Pi}_0$ by employing a game-theoretic framework. In the literature on game theory, several studies formulate license negotiations as cooperative games (e.g., Tauman and Watanabe, 2007; Watanabe and Muto, 2008; Kishimoto and Watanabe, 2017) and solve the license fees by applying the solution of the game. Appendix B will connect two literature streams that have been independently developed.

2.4 R&D

The potential firms can enter freely into the R&D.[8] The R&D function depends on the size of the quality improvement and the number of employing researchers. We assume that if an R&D firm chooses $\lambda > 1$ as the improvement size and employs z_t researchers in the time interval dt, the successful invention of the next-generation good happens with the probability $(\theta z_t / \lambda^{\beta}) dt$, where $\beta > 1$ represents the difficulty of a large-size innovation. Note that two or more firms never simultaneously succeed in their R&D because the probability of such an event is zero in the continuous-time models.[9] The R&D cost is $(w_t z_t) dt$, financed by issuing stocks.

Consider an industry where the leading firm on the ladder is firm 1. Then, if an R&D firm succeeds in inventing a superior good, it emerges in the industry as firm 0. Recall that λ_i is the quality improvement size of firm i. To avoid the complexity, we assume that the R&D firms take the sequence of the future quality improvements, $\{\lambda_{-i}\}_{i=1}^{\infty}$, as given.[10]

We define $V_i(\lambda_0)$ as the present discounted value of firm 0 when the subsequent innova-

[7] Kishimoto and Suzuki (2021) is an exception. They consider cooperative games that determine the license fee to describe how license fees are endogenously determined in the negotiation. A new innovator is the licensor and can earn the licensing revenue. In contrast, the present paper considers a different situation where a new innovator is the licensee and pays the license fees to other firms.

[8] All firms in the pool do not conduct R&D due to the Arrow's replacement effect. Their firm value is strictly positive. Therefore, their incentive to innovate is strictly weaker than the potential innovators with zero value.

[9] When we pick up two real numbers from a subinterval in $\mathbb{R}$ at random, the probability that their value is the same is zero.

[10] Without this assumption, we must consider a strategic interaction between the current innovators and the future innovators because λ_0 may affect the expected survival time of firm 0 via the changes of $\{\lambda_{-i}\}_{i=1}^{\infty}$. The analytical derivation of the optimal improvement size (λ_0) will be impossible.

tions have occurred i times after firm 0 entered the industry. The firm value is a function of λ_0 because the profit in (4) depends on it. $V_i(\lambda_0)$ changes to $V_{i+1}(\lambda_0)$ when the $(i+1)$-th subsequent innovation occurs after firm 0 entered the industry. It happens with the probability of $\theta z/(\lambda_{-i-1})^{\beta}$.

The optimization problem for each R&D firm is represented as

$$\max_{z\geq 0, \lambda_0 > 1} \frac{\theta z}{\lambda_0^{\beta}} V_0(\lambda_0) - wz, \tag{5}$$

where $V_0(\lambda_0)$ is the present discounted value of firm 0 when it succeeds in R&D. The objective function shows that, for λ_0, a trade-off exists between the value of R&D and the success probability. A larger λ_0 increases the firm value but decreases the success probability.

The linearity of R&D technology implies that the demand for researchers is unbounded above (i.e., $z \to \infty$) whenever $(\theta/\lambda_0^{\beta})V_0(\lambda_0) > w$ holds. However, it cannot happen in the labor market equilibrium. Therefore, the free-entry condition for R&D is given by

$$\frac{\theta}{\lambda_0^{\beta}} V_0(\lambda_0) \leq w. \tag{6}$$

If strict inequality holds in (6), then $z = 0$ holds because the R&D is not profitable. A positive and finite demand for researchers (i.e., $z \in (0, \infty)$) arises only if equality holds in (6). We focus on the latter case. Then, (6) works as the zero-profit condition for R&D. Although z is indeterminate in the optimization, it will be determined by the clearing condition of the labor market.

3 Long-run equilibria

In the decentralized equilibrium, all individuals solve their maximization problem, and all markets are clear at each period. We focus on a situation in which equality in the free-entry condition holds. We consider the following symmetric stationary situation:

A) the profit division $\mathbf{s} = (s_0, s_1, ..., s_{\eta})$ is constant over time;

B) any successful potential firm chooses the same size of the quality improvement (i.e., $\lambda_i = \lambda^*$ for all i);

C) all variables are symmetric across industries.

3.1 Labor market equilibrium

The fixed labor supply, $L > 0$, is devoted to production and R&D. Then, by the symmetry of industries, the labor market equilibrium condition at each time is

$$\frac{E}{\lambda^{1+\eta}w} + z = L. \tag{7}$$

3.2 Net profit stream

Let us consider the profit stream of firm 0. Suppose the subsequent innovations have occurred i times after firm 0 entered the industry. Then, firm $-i$ is the latest firm in the industry, and the set of the firms in the pool is $\mathcal{P}_{-i} = \{-i, -i+1, ..., 0, 1, ..., \eta - i\}$. By the argument of the limit-pricing strategy, firm $-i$ charges the price as follows:

$$p_{-i} = \left(\prod_{\ell=1}^{\eta-i} \lambda_\ell\right) \cdot \lambda_0 \cdot \left(\prod_{k=-i}^{-1} \lambda_k\right) w.$$

Then, the profit of firm $-i$ is

$$\tilde{\Pi}_{-i} = \left(1 - \frac{1}{\prod_{\ell=1}^{\eta-i} \lambda_\ell \cdot \lambda_0 \cdot \prod_{k=-i}^{-1} \lambda_k}\right) E.$$

By the stationary of $\mathbf{s}$, the stream of the net profit of firm 0 is $\{\pi_i\}_{i=0}^{\eta}$ where

$$\pi_i = s_i \tilde{\Pi}_{-i}. \tag{8}$$

Note that $\pi_i = 0$ for all $i > \eta$ because firm 0 is no longer a licensor of the latest firm at that time when the subsequent innovations have occurred $\eta + 1$ times after firm 0 entered the industry.

3.3 No-arbitrage condition

Consider a household with a stock of firm 0 whose value is $V_i(\lambda_0)$. In time interval dt, the household obtains the dividend $\pi_i dt$. $V_i(\lambda_0)$ changes to $V_{i+1}(\lambda_0)$ with the probability of $(\theta z/(\lambda_{-i-1})^\beta)dt$. We assume that there is a perfectly risk-free asset market, and the interest rate on the safe assets is r. Let $\dot{V}_i(\lambda_0)dt$ be the capital gain (or loss) of the stock. Then, standard arguments imply that the firm 0's value satisfies the following no-arbitrage condition:

$$rV_i(\lambda_0) = \pi_i + \dot{V}_i(\lambda_0) - \frac{\theta z}{(\lambda_{-i-1})^\beta}(V_i(\lambda_0) - V_{i+1}(\lambda_0)), \tag{9}$$

for $i = 0, ..., \eta$. Note that $V_i(\lambda_0) = 0$ for all $i > \eta$.

3.4 The balanced growth path

In the balanced growth path (BGP), the variables $\{A_t, w_t, E_t, V_{i,t}\}$ grow at the same constant speed for all $i = 0, ..., \eta$. Let the growth rate be g. Then, by the Euler equation, $g = r - \rho$ holds in the BGP. By using $\dot{V}_i / V_i = g$, the no-arbitrage condition (9) is rewritten as

$$\rho V_i(\lambda_0) = \pi_i - \frac{\theta z}{(\lambda_{-i-1})^{\beta}} (V_i(\lambda_0) - V_{i+1}(\lambda_0)), \tag{10}$$

for all t and $i = 0, ..., \eta$. By recursively solving the no-arbitrage conditions in (10), we can derive $V_0(\lambda_0)$ as follows:

$$V_0(\lambda_0) = \left(\frac{1}{\rho + \theta z / (\lambda_{-1})^{\beta}} \right) \left[\pi_0 + \sum_{i=1}^{\eta} \left(\pi_i \cdot \prod_{k=-i}^{-1} \psi_k \right) \right], \tag{11}$$

where ψ_k is a discount factor defined as

$$\psi_k \equiv \frac{\theta z / \lambda_k^{\beta}}{\rho + \theta z / \lambda_{k-1}^{\beta}}.$$

By the equality in (6), the optimal z is indeterminate in (5). Then, the problem in (5) can be simply rewritten as follows:

$$\max_{\lambda_0 > 1} \frac{V_0(\lambda_0)}{\lambda_0^{\beta}}. \tag{12}$$

By solving (12) and using the symmetry ($\lambda_i = \lambda^*$ for all i), we obtain the following Lemma.

Lemma 1. *In the symmetric equilibrium, the optimal quality improvement size is given by*

$$\lambda^* = \left(1 + \frac{1}{\beta} \right)^{1/(1+\eta)} \tag{13}$$

for any $\mathbf{s} \in \mathcal{S}$.

Proof. See Appendix A.1. □

Lemma 1 shows that the broadening patent breadth ($\eta \uparrow$) shrinks the optimal size of the quality improvement. Intuitively, the R&D firms have an incentive of *free-ride* on the quality improvements by the other innovators because the firm value depends on them, not only on λ_0. To confirm this, let us derive the Bertrand equilibrium price. By (3) and (13), in the symmetric equilibrium, the price becomes

$$\begin{aligned} p &= {\lambda^*}^{1+\eta} w \\ &= \left(1 + \frac{1}{\beta} \right) w. \end{aligned}$$

This shows that the markup does not depend on η.

By (4) and (13), the profit of the pool is

$$\tilde{\Pi}_i = \left(\frac{1}{1+\beta}\right) E \equiv \tilde{\Pi} \quad \text{for all } i. \tag{14}$$

Using (8), (13), and (14), we can rewrite (11) as follows:

$$\frac{V_0}{E} = \left(\frac{1}{1+\beta}\right)\left(\frac{\Psi(z)}{\rho + \theta z/\lambda^{*\beta}}\right), \tag{15}$$

where function $\Psi(z)$ is defined as

$$\Psi(z) \equiv s_0 + \sum_{i=1}^{\eta} s_i \left(\underbrace{\frac{\theta z/\lambda^{*\beta}}{\rho + \theta z/\lambda^{*\beta}}}_{\equiv \psi^*(z) < 1}\right)^i. \tag{16}$$

Using (16), we characterize each profit-sharing vector in $\mathcal{S}$ as follows:

Definition 1. *Consider any two different profit-sharing vectors $\mathbf{s}', \mathbf{s}'' \in \mathcal{S}$. Holding $\psi^*(z)$ constant, if the change from $\mathbf{s}'$ to $\mathbf{s}''$ increases (decreases) $\Psi(z)$, then $\mathbf{s}''$ is more frontloaded (backloaded) than $\mathbf{s}'$ under $\psi^*(z)$.*

Intuitively, in a more frontloaded vector (e.g., s_0 is higher), firm 0 can receive the reward of the innovation in early timing. In contrast, in a more backloaded vector, firm 0 must wait for the arrival of the subsequent innovations for a long time to receive the reward of the innovation. The *most frontloaded* vector is $\mathbf{s}^{\mathbf{F}} \equiv (1, 0, ..., 0)$ because it maximizes $\Psi(z)$ for any given $\psi^*(z)$. Similarly, the *most backloaded* vector is $\mathbf{s}^{\mathbf{B}} \equiv (0, 0, ..., 1)$ because it minimizes $\Psi(z)$ for any given $\psi^*(z)$.

When the profit-sharing vector is sufficiently frontloaded, the graph of (15) is a downward-sloping curve as shown "FV" in Panel (a) of Figure 3. However, when the profit-sharing vector is sufficiently backloaded, the FV curve becomes an inverted-U shape as shown in Panel (b) of Figure 3.[11]

By using (6), (7), and (13), we obtain

$$\frac{V_0}{E} = \frac{\lambda^{*\beta}}{\theta(1+1/\beta)(L-z)}. \tag{17}$$

The graph of (17) is shown as the upward-sloping "LME" curve in Figure 3. Each intersection of two curves is a long-run equilibrium in the model. In this general equilibrium model, only the ratio of V_0 and E is determined.

[11] Appendix 1 in O&Z also discusses a similar point.

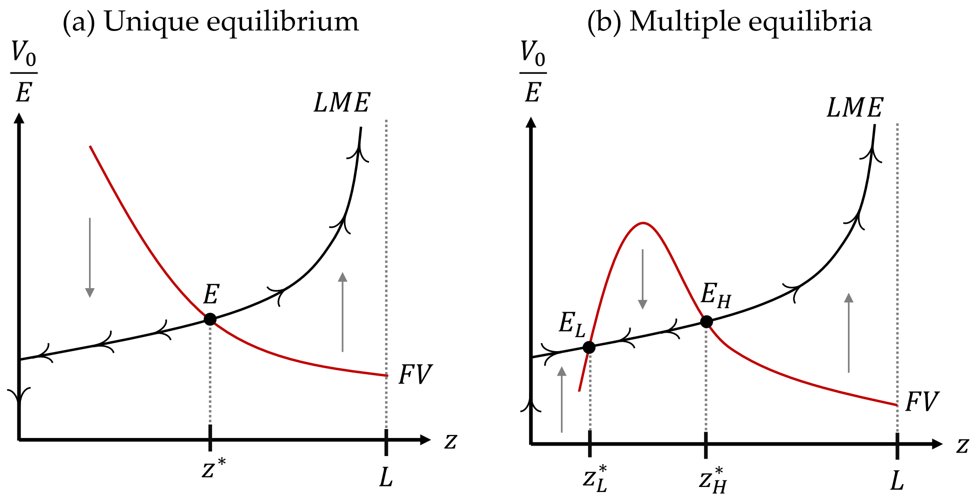

Figure 3: The long-run equilibria.

Notes: An intersection of the LME and FV curves is a long-run equilibrium. Panel (a) is a case of unique equilibrium, and Panel (b) is a case of multiple equilibria. When the profit-sharing vector is sufficiently backloaded, the economy may have multiple equilibria as Panel (b).

The long-run equilibrium is unique if the FV curve is higher than the LME curve at $z = 0$. By (15) and (17), we obtain a sufficient condition for the uniqueness of the long-run equilibrium as follows:

$$s_0 > \frac{\beta\rho}{\theta L}\left(1 + \frac{1}{\beta}\right)^{\beta/(1+\eta)}.$$

In other words, the long-run equilibrium is unique when the profit-sharing vector is sufficiently frontloaded. Then, the phase diagram in Panel (a) of Figure 3 is almost the same as the one in Grossman and Helpman (1991, Ch.4). The economy must lie on the LME curve. As the FV curve is derived by the no-arbitrage and BGP conditions, V_0 and E grow at g on the FV curve. $\dot{V}_0/V_0 < g = \dot{E}/E$ holds below the FV curve and $\dot{V}_0/V_0 > g = \dot{E}/E$ holds above the FV curve. If the economy is not on the FV curve, V_0/E changes over time. Therefore, point E in Panel (a) of Figure 3 is the unstable equilibrium. The unique equilibrium path is that the economy immediately jumps to point E at $t = 0$ and remains there forever.[12] Therefore, the economy has no transitional dynamics when the profit-sharing vector is sufficiently frontloaded.

The economy may have multiple equilibria when the profit-sharing vector is sufficiently backloaded. In this case, the expectation determines which equilibrium is realized. To understand the role of the expectation in the leading breadth model, consider a backloading profit-sharing vector. Then, to earn large net profits, firm 0 must wait for the arrival of the many subsequent innovations. Suppose the individuals have optimistic expectations such that they predict the speed of the technological advances is fast (i.e., z is high). The optimistic

[12]As shown in Grossman and Helpman (1991, Ch.4), other paths do not satisfy the transversality condition or rational expectations.

expectations increase the present value of innovation, V_0, because the waiting intervals of firm 0 become shorter. As a result, z rises; therefore, the initial optimistic expectations are self-fulfilled. In contrast, when individuals have pessimistic expectations about z, the present value of innovation is small, as the discounted sum of future profits is small. Then, the incentive to innovate weakens, and z truly falls. The initial pessimistic expectations are self-fulfilled in this case as well.

Under the multiple equilibria, the equilibrium path is not unique because all variables are jumpable in the model. Any equilibrium path can be selected depending on the expectation. To avoid the complexity, we focus on only the equilibrium paths that the economy initially jumps to either one of E_L and E_H and remains at the same point forever. O&Z conducts an equilibrium selection such that it neglects E_L. However, because the expectation is an essential macroeconomic feature generated by blocking patents, we do not neglect the pessimistic equilibrium paths. This enables us to investigate the effect on macroeconomic stability in Section 5.

4 Analytical discussion

First, this section investigates how the broadening leading breadth ($\eta \uparrow$) affects the growth rate. Second, we derive the socially optimal innovation size and resource allocation. Finally, we investigate the welfare effect of the broadening leading breadth.

4.1 Growth effect of the broadening leading breadth

In the BGP, the consumption index c_t also grows at g. The standard argument implies that, by (2), the growth rate is calculated as

$$g = \frac{\dot{c}}{c} = \underbrace{\frac{\theta z^*}{\lambda^{*\beta}}}_{\substack{\text{R\&D} \\ \text{success rate}}} \times \underbrace{\ln \lambda^*}_{\substack{\text{Quality} \\ \text{improvement} \\ \text{size}}} . \tag{18}$$

We can decompose the growth effect of the broadening leading breadth into the following three effects. First, the broadening leading breadth shrinks λ^* by (13). This decreases the growth rate via $\ln \lambda^*$ in (18). Second, the shrinking of λ^* raises the R&D success rate because the difficulty of quality improvement is lowered. This increases the growth rate via λ^* in the denominator of (18). Third, the broadening leading breadth changes z^* which is determined by (15) and (17). However, this channel is very complex because it is unclear how the FV curve moves.

Before the analyses, we impose the following assumption on the parameters:

Assumption 1. *The discount rate is sufficiently small such that*

$$\left(\frac{\theta L}{\beta}\right)\left(\frac{\beta}{1+\beta}\right)^{\beta/2} > \rho. \tag{19}$$

This parameter condition ensures that the growth rate in the decentralized economy is always positive.[13] Intuitively, if the discount rate is very large, the R&D investment is not profitable because the present value V_0 is very low. To build an innovation-driven growth model, we assume that the discount rate is sufficiently small.

We begin to analytically investigate the growth effect by establishing some Lemmas. First, we consider a special case in which the profit-sharing vector is the most frontloaded ($\mathbf{s} = \mathbf{s^F}$). Of course, this profit-sharing vector is unrealistic because the licensors would not be incentivized to license their patents without license fees. However, this extreme case helps us investigate the growth effect in more general cases.

Lemma 2. *Suppose that the profit-sharing vector is the most frontloaded ($\mathbf{s} = \mathbf{s^F}$). Then,*

(i) *the growth effect of the broadening leading breadth is not always positive. It is either one of the following two effects depending on the discount rate;*
  - *a non-monotonic growth effect (i.e., g and η are represented as an inverted U-shaped curve as shown in Figure 4) when the discount rate is moderate;*
  - *a negative growth effect for all $\eta \geq 1$ when the discount rate is sufficiently small.*

(ii) *the growth rate converges to zero as $\eta \to \infty$ regardless of the discount rate.*

Proof. See Appendix A.2. □

To investigate the growth effect in more general cases, we establish the following Lemma.

Lemma 3. *Let $g : \mathcal{S} \to \mathbb{R}$ be the function that assigns $\mathbf{s} \in \mathcal{S}$ to the growth rate in the BGP. Then, $g(\mathbf{s^F}) > g(\mathbf{s})$ holds for any $\mathbf{s} \neq \mathbf{s^F}$ and $\eta \geq 1$ because the most frontloaded profit-sharing vector ($\mathbf{s^F}$) maximizes z^*.*

Proof. By (16), the maximum value of $\Psi(z)$ is 1, as $\psi^* < 1$. Note that $\Psi(z) = 1$ holds only when $\mathbf{s} = \mathbf{s^F}$ and $\Psi(z) < 1$ holds when $\mathbf{s} \neq \mathbf{s^F}$. Therefore, by (15), the most frontloaded profit-sharing vector pushes the graph of the FV curve in Figure 3 up to the highest position. Then, $\mathbf{s} = \mathbf{s^F}$ maximizes z^* for all $\eta \geq 1$, as the LME curve does not depend on $s \in \mathcal{S}$. By Lemma 1, λ^* is independent of $\mathbf{s} \in \mathcal{S}$. As a result, we obtain the statement. □

[13] Strictly speaking, this condition ensures that the growth rate is positive in the case of $\mathbf{s} = \mathbf{s^F}$. As proved in Lemma 3, any other $\mathbf{s} \in \mathcal{S}$ yields a lower growth rate compared to $\mathbf{s^F}$. Therefore, if the growth rate is non-positive when $\mathbf{s} = \mathbf{s^F}$, then any $\mathbf{s} \in \mathcal{S}$ cannot yield a positive growth rate. Assumption 1 avoids such an uninteresting situation.

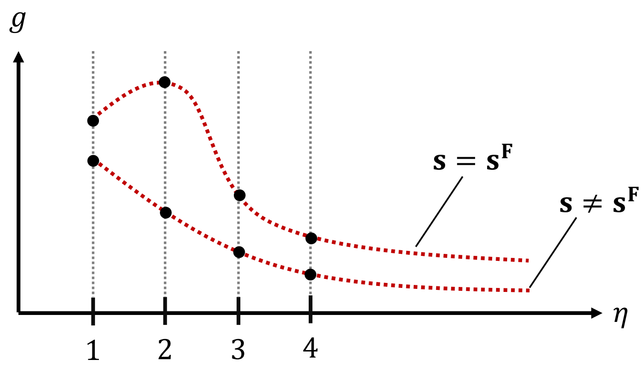

Figure 4: The growth rate (g) and the leading breadth (η)

Notes: Lemma 2 shows that g is not monotonically increasing in η and converges to zero as $\eta \to \infty$ when $\mathbf{s} = \mathbf{s^F}$. Lemma 3 shows that g is maximized when $\mathbf{s} = \mathbf{s^F}$. Therefore, for any $\mathbf{s} \in \mathcal{S}$, g is not monotonically increasing in η and converges to zero as $\eta \to \infty$.

Then, Lemmas 2 and 3 derive the following statement.

Proposition 1. *Consider a profit-sharing vector $\mathbf{s} \in \mathcal{S}$. Then, by Lemma 2, the growth effect of the broadening leading breadth is not globally positive. The broadening leading breadth has either (i) a non-monotonic growth effect or (ii) a negative growth effect. Also, the growth rate converges to zero as $\eta \to \infty$.*

Proof. By Lemma 2, when $\mathbf{s} = \mathbf{s^F}$, the relationship between g and η is either a non-monotonic (an inverted U-shaped curve) or a negative (a downward-sloping curve). Moreover, the growth rate converges to zero as $\eta \to \infty$ when $\mathbf{s} = \mathbf{s^F}$. Recall that Lemma 3 shows that $\mathbf{s} \neq \mathbf{s^F}$ yields a lower growth rate than $\mathbf{s^F}$. Then, the relationship between g and η cannot be globally positive (an upward-sloping curve) when $\mathbf{s} \neq \mathbf{s^F}$. Otherwise, it contradicts Lemma 3. As a result, the statement in Proposition 1 holds. □

Proposition 1 shows that the broadening leading breadth does not always increase the growth rate. Although the relationship between g and η is generally ambiguous, we can show that, in a special case of $\rho \to 0$, we can establish the following statement.

Proposition 2. *For any $\mathbf{s} \in \mathcal{S}$ and $\eta \geq 1$, the growth effect is always negative when $\rho \to 0$.*

Proof. Assume that $\rho \to 0$. By (15) and (17), we can analytically solve the long-run equilibrium as $z^* = L/(1+\beta)$. As this is constant, the broadening leading breadth affects the growth effect only via λ^*. Note that $(\ln \lambda^*)/\lambda^{*\beta}$ is increasing in $\lambda^* \in (1, \bar{\lambda}]$ where $\bar{\lambda} \equiv (1 + 1/\beta)^{1/2}$ is the maximum size of the quality improvement derived by (13). Therefore, we obtain the result as the broadening leading breadth decreases λ^*. □

This result is opposite to O&Z because they show that the growth effect is positive when the discount rate is sufficiently small. In their model, the quality improvement size is constant, and the broadening leading breadth can unlimitedly increase the profit of the pool. This is the driver of the growth-enhancing effect in their model. Recall that the backloading effect, which is a negative growth effect, is weaker as ρ is smaller. Then, when the discount rate is sufficiently small, the former positive effect dominates the backloading effect.

In contrast, our model with endogenous quality improvement does not have the growth-enhancing effect via increased profit. Instead, we show that the free-ride behavior of the R&D firms (i.e., the shrinking of the quality improvement size) yields a new negative growth effect. This negative growth effect does not depend on the discount rate. Although there is a positive growth effect via the R&D success probability, the negative growth effect dominates it.

The Online Appendix demonstrates that this negative growth effect can be replicated in another specification of the leading breadth in which the scope does not depend on η. We formulate an overlapping generations (OLG) game between the R&D firms and show that their free-ride behavior can be rationalized as a Nash equilibrium in a special case.

4.2 Socially optimal innovation size and labor allocation

In this subsection, we investigate (i) whether the quality improvement size in the decentralized economy (13) is large or small from the perspective of welfare and (ii) whether the resource allocation in the decentralized equilibrium is socially optimal or not.

To address this problem, we first derive the first-best allocation in the economy. We evaluate the welfare of an economy that is in a steady state at $t = 0$ and stays there forever. By the utility function (2) and the labor market equilibrium condition (7), the representative household's period utility is rewritten as follows:

$$\ln c_t = g \cdot t + \ln(L - z) - \ln L.$$

Note that $(L - z)$ equals the aggregate output, as the production technology is one-to-one. Then, $(L - z)/L$ is per capita consumption in the equilibrium. By using this and integrating the lifetime utility function (1) with respect to time, we obtain the representative household's welfare as follows:

$$U = \frac{1}{\rho}\left[\frac{1}{\rho}\underbrace{\frac{\theta z \ln \lambda}{\lambda^{\beta}}}_{\text{Growth}} + \underbrace{\ln(L - z) - \ln L}_{\text{Consumption}}\right]. \tag{20}$$

A social planner maximizes (20) with respect to λ and z. Let $(\lambda^{\#}, z^{\#})$ be the solution. We assume that the interior solution exists (i.e., $\lambda^{\#} > 1, z^{\#} > 0$).[14] Then, by differentiating (20) with respect

[14]We consider an innovation-driven growth economy. Therefore, we avoid a trivial case in which a no-growth

to λ and z, we obtain the welfare-maximizing pair of the quality improvement size and the number of researchers as follows:

$$\lambda^{\#} = \exp\left(\frac{1}{\beta}\right),$$
$$z^{\#} = L - \left(\frac{\rho\beta}{\theta}\right)\exp(1). \tag{21}$$

Then, the following result holds.

Proposition 3. *In the decentralized economy,*

(i) $\lambda^{*} < \lambda^{\#}$ *holds: the quality improvement size is smaller than the socially optimal size;*

(ii) z^{*} *is either larger or smaller than the socially optimal level* $z^{\#}$.

Proof. See Appendix A.3. □

The first statement comes from the free-ride behavior of potential firms. From the perspective of welfare maximization, each potential firm should invest in R&D that will significantly improve quality. However, they are not incentivized to do that because the success probability of such a large R&D investment is very low. Under the leading breadth, each potential firm can join the pool in the industry even if the quality improvement size is small.

The second statement comes from the standard externalities in the model. First, each potential firm does not consider the positive externality of innovation on the households. Second, each potential firm does not care about the capital loss of the existing firms (i.e., a profit destruction effect). However, in addition to these normal externalities, the innovation size in the decentralized economy also yields the gap between z^{*} and $z^{\#}$. As $\lambda^{*} < \lambda^{\#}$, the R&D success probability is relatively high in the decentralized economy. It naturally works to increase z^{*}.

4.3 Welfare effect of the broadening leading breadth

How does the broadening leading breadth affect welfare? By (20), we can decompose the welfare effect into two channels: (i) via growth and (ii) via consumption. The analysis of the welfare effect is more complex than one of the growth effect because a higher growth rate does not mean that the welfare is also high.

Although we will quantitatively show that the welfare effect can be negative under plausible parameters in the next section, we can analytically show the negative welfare effect in the special cases as follows:

economy is socially optimal by assuming that $z^{\#} > 0$. The parameter assumption for $z^{\#} > 0$ is stricter than one in Assumption 1 because $\exp(2) > (1 + 1/\beta)^{\beta}$ holds for all $\beta > 0$. Therefore, the assumption for $z^{\#} > 0$ derives the parameter condition in Assumption 1.

Proposition 4. *For any* $\mathbf{s} \in \mathcal{S}$ *and* $\eta \geq 1$*, the welfare effect of the broadening leading breadth is negative when* $\rho \to 0$*. In the case of* $\mathbf{s} = \mathbf{s^F}$*, the welfare effect is not globally positive. In particular, there exists* $\tilde{\eta}$ *such that the welfare effect is negative in* $\eta \geq \tilde{\eta} + 1$*.*

Proof. Proposition 2 shows that, for any $\mathbf{s} \in \mathcal{S}$ and $\eta \geq 1$, the growth effect is always negative, but z^* is independent of η. Therefore, by (20), the welfare effect is always negative. In the case of $\mathbf{s} = \mathbf{s^F}$, we showed the existence of $\tilde{\eta}$ such that the growth effect is negative in $\eta \geq \tilde{\eta} + 1$. As z^* is increasing in η in the case of $\mathbf{s} = \mathbf{s^F}$, the welfare effect is negative in $\eta \geq \tilde{\eta} + 1$. □

5 Further analyses: a quantitative illustration

5.1 Two theoretically grounded profit-sharing vectors

This section quantitatively investigates the growth effect of the broadening leading breadth by calibrating the U.S. economy. So far, we have treated a profit-sharing vector as exogenous. Although this simplification is commonly used in the O&Z-type models, in the quantitative analysis, we need to specify the profit-sharing vector. Because we cannot directly observe the profit-sharing vector in reality, it is difficult to find the plausible profit-sharing vector.

As a benchmark, we use the most backloaded vector and the equal-sharing vector defined as follows:

$$\mathbf{s^B} \equiv (0, 0, ..., 1)\,.$$
$$\mathbf{s^E} \equiv \left(\frac{1}{1+\eta}, \frac{1}{1+\eta}, ..., \frac{1}{1+\eta}\right).$$

We can provide the theoretical basis for adopting these two profit-sharing vectors by cooperative game theory. In Appendix B, we formulate a game of pool formation and profit division between the firms in an industry and derive $\mathbf{s^B}$ and $\mathbf{s^E}$ as two different solution concepts.

First, Appendix B shows that $\mathbf{s^B}$ is uniquely given by applying the *core* to the game. In cooperative game theory, the core is widely used as a standard solution concept. The core is defined based on the incentives for negotiating firms to gain more profits. We regard a profit allocation in the core representing license fees determined through negotiation of the firms in the industry without any intervention from the outside.

Although $\mathbf{s^B}$ is highly unfair to firm 0, it seems a natural result that the patent licensing negotiation yields an exploitative licensing fee when the bargaining power of the licensee is very weak. Sakakibara (2010) empirically shows that the relative bargaining powers between firms significantly affect the license fees determined in their private negotiation. Patent holders have the right to refuse the request for the license because the patents are the private property of each of them. Thus, the relative bargaining power of the patent holder tends to be high, and the license fees that the new innovator pays to the past innovators would be expensive. For example, the Federal Trade Commission in the United States has recently insisted that Qualcomm has illegally charged an excessive license fee in the market for smartphone chips.

However, in reality, standard-setting organizations (SSOs) intervene in the license negotiation to prevent the patent holders from refusing the license and charging an unfairly high license fee. SSOs often request firms holding standard-essential industry patents to license under the *fair, reasonable, and non-discriminatory* (FRAND) conditions. We use $\mathbf{s^B}$ as the benchmark result of the private negotiation without any intervention.

Second, Appendix B shows that we have $\mathbf{s^E}$ by applying the *Shapley value* to the game. The Shapley value is a normative solution concept introduced by Shapley (1953). In the literature, Layne-Farrar et al. (2007) and Dewatripont and Legros (2013) use the Shapley value as a benchmark for FRAND conditions in SSOs. We interpret the equal sharing vector $\mathbf{s^E}$ as the result of a negotiation with the intervention that aims to correct unfair license fees.

Specifying two profit-sharing vectors enables us to investigate the growth effect of the intervention in the license negotiation.

5.2 The long-run equilibria with and without intervention

Intervention case ($\mathbf{s^B}$)

Suppose that the license negotiation is not intervened. Then, the oldest firm in the pool obtains the entire profit as the license fee. This means firm 0 can obtain the license fee after the subsequent innovations occur η times. Then, (16) becomes

$$\Psi(z) = \left(\frac{\theta z / \lambda^{*\beta}}{\rho + \theta z / \lambda^{*\beta}} \right)^{\eta}.$$

By using this and (15), the FV curve becomes as follows:

$$\frac{V_0^*}{E} = \left(\frac{1}{1+\beta} \right) \frac{\left(\theta z / \lambda^{*\beta}\right)^{\eta}}{\left(\rho + \theta z / \lambda^{*\beta}\right)^{1+\eta}}. \tag{22}$$

Therefore, the FV curve is an inverted-U shape as shown in Panel (b) Figure 3. As a result, multiple equilibria can emerge under no intervention.

No-intervention case ($\mathbf{s^E}$)

Next, we assume that the license negotiation is intervened. Then, each firm in the pool equally obtains the same payoff following the Shapley value. In this case, (16) becomes

$$\begin{aligned}\Psi(z) &= \left(\frac{1}{1+\eta}\right)\left[1+\sum_{i=1}^{\eta}\left(\frac{\theta z/\lambda^{*\beta}}{\rho+\theta z/\lambda^{*\beta}}\right)^{i}\right] \\ &= \left(\frac{1}{1+\eta}\right)\left[1-\left(\frac{\theta z/\lambda^{*\beta}}{\rho+\theta z/\lambda^{*\beta}}\right)^{1+\eta}\right].\end{aligned}$$

By using this and (15), the FV curve becomes as follows:

$$\frac{V_0^*}{E} = \frac{1}{\rho}\left(\frac{1}{1+\eta}\right)\left(\frac{1}{1+\beta}\right)\left[1-\left(\frac{\theta z/\lambda^{*\beta}}{\rho+\theta z/\lambda^{*\beta}}\right)^{1+\eta}\right]. \tag{23}$$

This is decreasing in z. Therefore, the FV curve is downward-sloping as shown in Panel (a) Figure 3. As a result, the long-run equilibrium is unique under intervention. The discussion yields the following statement:

Proposition 5. *Consider the intervention ($\mathbf{s^B} \to \mathbf{s^E}$) described in Definition 5 in Appendix B. The intervention in the licensing negotiation stabilizes the economy.*

5.3 A quantitative analyses

We quantitatively investigate whether patent protection against future innovation enhances economic growth. As the benchmark scenario, we consider a situation in which the license negotiation is intervened, and therefore, firm 0 shares its profit with other firms according to the Shapley value.

We must set the structural five parameters $(\eta, \rho, L, \theta, \beta)$. For η, we use the fragmentation index used by Ziedonis (2004) and Entezarkheir (2017). This measure is defined as

$$F_i = 1 - \sum_{j=1}^{J}\left(\frac{cite_{ij}}{cite_i}\right)^2, \; i \neq j$$

where $cite_{ij}$ is the number of citations made by firm i in its patent documents to the patents of firm j and $cite_i$ is the count of all the citations made by firm i to other firms' patents. $F_i = 0$ holds if every citation is to the patents of one firm. Let us assume that firm i equally cites the other firms' patents (i.e., $cite_{ij} = c$ holds for all j). Then, $Jc = cite_i$ holds. Entezarkheir (2017) reports that the mean of the index is 0.7. Then, we obtain $J \simeq 3.33$. This means that, on average, a patent cites three different firms' patents. The number of citations measures how many other patents are technologically close to the patent. Therefore, we adopt $\eta = 3$. For the

		Baseline $\eta = 3$	$\eta = 4$	$\eta = 5$	$\eta = 6$	$\eta = 7$	$\eta = 8$
	λ^*	1.057	1.046	1.038	1.032	1.028	1.025
With intervention ($\mathbf{s^E}$)	g	2.00%	1.66%	1.41%	1.23%	1.08%	0.96%
	ι	0.359	0.372	0.380	0.385	0.387	0.388
	z	0.0632	0.0627	0.0621	0.0615	0.0610	0.0604
No intervention ($\mathbf{s^B}$)	g_H	1.93%	1.58%	1.33%	1.14%	0.99%	0.86%
	ι_H	0.346	0.354	0.357	0.357	0.354	0.349
	z_H	0.0609	0.0597	0.0584	0.0570	0.0557	0.0542
	g_L	0.025%	0.034%	0.042%	0.049%	0.055%	0.061%
	ι_L	0.005	0.008	0.011	0.015	0.020	0.025
	z_L	0.0008	0.0013	0.0018	0.0024	0.0031	0.0038
	$\overline{g}$	0.98%	0.81%	0.69%	0.59%	0.52%	0.46%

Table 1: The growth effect of broadening leading breadth ($\eta \uparrow$).

Notes: As shown in Lemma 1, λ^* does not depend on $\mathbf{s} \in \mathcal{S}$. The long-run equilibrium is unique when $\mathbf{s} = \mathbf{s^E}$ but two equilibria emerge when $\mathbf{s} = \mathbf{s^B}$. The subscript H means the optimistic equilibrium, while L means the pessimistic equilibrium. $\iota_m \equiv \theta z_m / \lambda^\beta$ is the R&D success probability in each equilibrium ($m = H, L$). $\overline{g}$ is the average growth rate defined as $\overline{g} \equiv (g_H + g_L)/2$.

discount rate, we use a conventional value $\rho = 0.01$. We set $L = 0.334$, as the U.S. population is around 0.344 billion. According to Norrbin (1993) and Basu (1996), the empirically plausible range of markup of price over marginal cost is in $[1.05, 1.4]$. From (3), the markup in the model is $(\lambda^*)^{1+\eta} = 1 + 1/\beta$. We set $\beta = 4$ so that the markup becomes a moderate value in the range ($1 + 1/\beta = 1.25$). Finally, we set $\theta = 7.1$ so that $g \simeq 0.02$, which is the average growth rate of the U.S. economy.

Table 1 is the result. The broadening leading breadth decreases the growth rate when the negotiation is intervened ($\mathbf{s} = \mathbf{s^E}$). The negative growth effect comes from a reduction in λ^* and z^*. By (18), if we hold z^* constant, there is an inverted-U relationship between the growth rate and λ^*, and the growth rate is maximized at $\lambda^* = \exp(1/\beta)$. In our numerical example, $\exp(1/\beta) \simeq 1.284$ is larger than λ^* reported in Table 1. This means that the reduction of λ^* works to decrease the growth rate in the numerical example. In addition, by (18), the reduction in z^* also decreases the growth rate. As η is larger, the share $1/(1+\eta)$ in $\mathbf{s^E}$ decreases while the lifetime of firm 0 increases. Because the future revenues are discounted, the broadening leading breadth also decreases the present value of R&D in the intervention case. As a result, the FV curve (23) shifts downward, and the innovation rate z decreases. Hence, the broadening leading breadth discourages the incentive for R&D.

When the negotiation is not intervened ($\mathbf{s} = \mathbf{s^B}$), the growth effect differs between the two equilibria. The broadening leading breadth decreases the growth rate in the optimistic equilibrium (g_H) but increases the growth rate in the pessimistic equilibrium (g_L). The average growth rate, defined as $(g_H + g_L)/2$, decreases in η. Therefore, it seems reasonable to consider that, even with no intervention, the broadening leading breadth would have a negative growth effect. The intuition of the negative growth effect is the same as in the intervention case. When $\mathbf{s} = \mathbf{s^B}$, as η is larger, firm 0 must wait a long time (i.e., η times of future innovations) to obtain the license fee $\tilde{\Pi}$. This decreases the present value of R&D and shifts the FV curve (22) downward. As a result, the broadening leading breadth decreases the innovation rate z_H. In other words, the broadening leading breadth enhances the backloading effect in both cases.

6 Conclusion

This paper investigated how broadening leading breadth affects economic growth in a Schumpeterian growth model with endogenous quality improvement. In our model, the optimal size of the quality improvement is determined as an interior solution, unlike the previous studies. We found that the growth effect of broadening leading breadth is not always positive. In particular, an extremely broadening leading breadth is harmful to growth.

The fundamental mechanism of the negative growth effect is the free-ride behaviors of innovators. In previous studies, the increase in the number of firms in the price cartel increases the markup because the quality improvement is always constant. The Schumpeterian effect was the main driver of the positive growth effect. However, this channel vanishes in our model because the R&D firms shrink their sizes of quality improvement. In other words, the R&D firms can receive stronger positive externalities from many future innovators. Therefore, when λ is endogenous, and the potential firms would like to free-ride, the broadening leading breadth may decrease λ because of the strong positive externalities.

We also found that intervention in patent licensing negotiation stimulates innovation. Without any intervention, the patent license fee tends to be high because the negotiation power of the licensor is strong. We formulated a cooperative game played by all firms on a quality ladder and showed that the outcome of the profit division, derived as the core of the game, becomes a unique unfair distribution. Following the literature, we considered intervention in the negotiation such that the firms are forced to distribute the profit based on the Shapley value. Our numerical example also showed that the intervention stabilizes the economy by eliminating the pessimistic equilibrium.

Appendix A. Technical details of the baseline model

A.1 Proof of Lemma 1

Substituting (8) and (11) into (12), the maximization problem can be rewritten as follows:

$$\max_{\lambda_0>1} \frac{1}{\lambda_0^{\beta}} \left[s_0 \left(1 - \frac{1}{\lambda_0 \cdot \prod_{\ell=1}^{\eta} \lambda_\ell} \right) E + \sum_{i=1}^{\eta} \left[s_i \left(1 - \frac{1}{\prod_{\ell=1}^{\eta-i} \lambda_\ell \cdot \lambda_0 \cdot \prod_{k=-i}^{-1} \mathbb{E}(\lambda_k)} \right) E \cdot \prod_{k=-i}^{-1} \mathbb{E}(\psi_k) \right] \right].$$

The first-order condition is

$$\begin{aligned}
&-\frac{\beta}{\lambda_0^{\beta+1}} \left[s_0 \left(1 - \frac{1}{\lambda_0 \cdot \prod_{\ell=1}^{\eta} \lambda_\ell} \right) + \sum_{i=1}^{\eta} \left[s_i \left(1 - \frac{1}{\prod_{\ell=1}^{\eta-i} \lambda_\ell \cdot \lambda_0 \cdot \prod_{k=-i}^{-1} \mathbb{E}(\lambda_k)} \right) \cdot \prod_{k=-i}^{-1} \mathbb{E}(\psi_k) \right] \right] \\
&\quad + \frac{1}{\lambda_0^{\beta}} \left[\frac{s_0}{\lambda_0^2 \cdot \prod_{\ell=1}^{\eta} \lambda_\ell} + \sum_{i=1}^{\eta} \left[s_i \left(\frac{1}{\prod_{\ell=1}^{\eta-i} \lambda_\ell \cdot \lambda_0^2 \cdot \prod_{k=-i}^{-1} \mathbb{E}(\lambda_k)} \right) \cdot \prod_{k=-i}^{-1} \mathbb{E}(\psi_k) \right] \right] = 0 \\
&\Leftrightarrow \beta \left[s_0 \left(1 - \frac{1}{\lambda_0 \cdot \prod_{\ell=1}^{\eta} \lambda_\ell} \right) + \sum_{i=1}^{\eta} \left[s_i \left(1 - \frac{1}{\prod_{\ell=1}^{\eta-i} \lambda_\ell \cdot \lambda_0 \cdot \prod_{k=-i}^{-1} \mathbb{E}(\lambda_k)} \right) \cdot \prod_{k=-i}^{-1} \mathbb{E}(\psi_k) \right] \right] \\
&\quad - \frac{s_0}{\lambda_0 \cdot \prod_{\ell=1}^{\eta} \lambda_\ell} - \sum_{i=1}^{\eta} \left[s_i \left(\frac{1}{\prod_{\ell=1}^{\eta-i} \lambda_\ell \cdot \lambda_0 \cdot \prod_{k=-i}^{-1} \mathbb{E}(\lambda_k)} \right) \cdot \prod_{k=-i}^{-1} \mathbb{E}(\psi_k) \right] = 0.
\end{aligned}$$

Using the symmetry ($\lambda_i = \lambda_0, \forall i$), we obtain

$$\begin{aligned}
&\beta \left[s_0 \left(1 - \frac{1}{\lambda_0^{\eta+1}} \right) + \sum_{i=1}^{\eta} \left[s_i \left(1 - \frac{1}{\lambda_0^{\eta+1}} \right) \cdot \psi^*(z)^i \right] \right] - \frac{s_0}{\lambda_0^{\eta+1}} - \sum_{i=1}^{\eta} \left(\frac{s_i}{\lambda_0^{\eta+1}} \psi^*(z)^i \right) = 0 \\
&\Leftrightarrow s_0 \left[\beta - (1+\beta) \frac{1}{\lambda_0^{\eta+1}} \right] + \left[\beta - (1+\beta) \frac{1}{\lambda_0^{\eta+1}} \right] \sum_{i=1}^{\eta} s_i \psi^*(z)^i = 0 \\
&\Leftrightarrow \left[\beta - (1+\beta) \frac{1}{\lambda_0^{\eta+1}} \right] \underbrace{\left(s_0 + \sum_{i=1}^{\eta} s_i \psi^*(z)^i \right)}_{+} = 0,
\end{aligned}$$

where $\psi^*(z)$ is defined in (16). By solving $\beta - (1+\beta)/\lambda_0^{\eta+1} = 0$, we obtain the result. □

A.2 Proof of Lemma 2

Suppose the profit-sharing vector is the most frontloaded ($\mathbf{s} = \mathbf{s^F}$). Then, by (15) and (17), we obtain the interior solution of z as follows:

$$z^* = \left(\frac{1}{1+\beta} \right) \left[L - \left(\frac{\rho\beta}{\theta} \right) \lambda^{*\beta} \right]. \tag{24}$$

Note that $z^* > 0$ holds by Assumption 1. Then, the growth rate is calculated as follows:

$$\begin{aligned} g &= \left(\frac{1}{1+\beta}\right)\left(\frac{\theta L}{\lambda^{*\beta}} - \rho\beta\right)\ln\lambda^* \\ &= \frac{\ln(1+1/\beta)}{1+\beta}\underbrace{\left[\theta L\left(\frac{\beta}{1+\beta}\right)^{\beta/(1+\eta)} - \rho\beta\right]}_{\substack{\text{positive and}\\ \text{increasing in } \eta}}\underbrace{\left(\frac{1}{1+\eta}\right)}_{\substack{\text{decreasing}\\ \text{in } \eta}}. \end{aligned}$$

Note that the square bracket in the above equation is increasing in η, as $\beta/(1+\beta) < 1$. Additionally, the growth rate converges to zero as $\eta \to \infty$, as the square bracket converges to a finite value. Thus, we obtain the second statement in Lemma 2.

By differentiating g with respect to η, we obtain

$$\frac{dg}{d\eta} \gtreqless 0$$

$$\Leftrightarrow \rho \gtreqless \underbrace{\theta L\left(\frac{\beta}{1+\beta}\right)^{\beta/(1+\eta)}}_{\text{increasing in } \eta}\underbrace{\left[\frac{1}{\beta} - \frac{1}{1+\eta}\ln\left(1+\frac{1}{\beta}\right)\right]}_{\text{positive and increasing in } \eta} \equiv \Omega(\eta). \tag{25}$$

Note that the square bracket in the RHS of (25) is positive. This can be easily shown by using the fact that $(1+1/\beta)^\beta$ is increasing in $\beta > 0$ and is strictly lower than $\exp(1) \simeq 2.71828$. Then, $\Omega(\eta)$ is strictly increasing in η and $\lim_{\eta\to\infty}\Omega(\eta) = \theta L/\beta$. By this fact and Assumption 1, the following two cases arise depending on the value of the discount rate. The first case is that ρ is sufficiently small such that $\rho < \Omega(\eta) \Leftrightarrow dg/d\eta < 0$ holds for all $\eta \geq 1$. In this case, the broadening leading breadth always decreases the growth rate.

The second case is that ρ is a moderate value. Suppose that $\Omega(\tilde{\eta}) < \rho < \Omega(\tilde{\eta}+1)$ where $\tilde{\eta} \geq 2$ is a threshold of η. Then, by (25), we obtain $dg/d\eta > 0$ for $\eta = 1, 2, ..., \tilde{\eta}$. This means that the broadening leading breadth in $\eta \in \{1, 2, ..., \tilde{\eta}-1\}$ increases the growth rate. However, as $\rho < \Omega(\eta) \Leftrightarrow dg/d\eta < 0$ holds for all $\eta \geq \tilde{\eta}+1$, the broadening leading breadth in $\eta \geq \tilde{\eta}+1$ decreases the growth rate. Note that the growth effect is ambiguous when $\tilde{\eta} \to \tilde{\eta}+1$. As a result, we obtain the first statement in Lemma 2. □

A.3 Proof of Proposition 3

The first statement immediately holds by comparing (13) with (21).

Let us consider the second statement. In the decentralized economy, z^* or z_L^* can be zero depending on the parameters. To understand this, consider the vertical intercepts of the two curves in Figure 3. By (15) and (17), the two curves intersect on the vertical line (i.e., $z^* = 0$ or

$z_L^* = 0$ holds) if the following parameter condition holds:

$$\frac{s_0}{\rho(1+\beta)} = \frac{\lambda^*}{\theta(1+1/\beta)L}.$$

Note that the parameter condition does not contradict Assumption 1. As a result, $z^* < z^\#$ can hold, as $z^\# > 0$.

Next, we consider whether $z^* > z^\#$ can hold or not. Lemma 3 shows that $\mathbf{s}^{\mathbf{F}}$ maximizes z^*. Moreover, the value of (24) is maximized when $\eta \to \infty$. Therefore, the highest value of z^* that is achievable in the decentralized economy is given by

$$z^*_{\max} \equiv \left(\frac{1}{1+\beta}\right)\left(L - \frac{\rho\beta}{\theta}\right).$$

Assumption 1 ensures that $z^*_{\max} > z^\#$. As a result, we obtain the second statement.

Appendix B. The game of pool formation and profit division

This Appendix formulates a game of pool formation and profit division. Then, we derive the core and the Shapley value.

B.1 Basic setup

Consider an industry in which firm 0 is the latest firm as Figure 2. Let $\mathcal{N} \equiv \{0, 1, ..., \omega\}$ be the set of the firms in the industry. This means that, in the industry, the oldest innovator is firm ω, and innovations have occurred ω times. We assume that the number of firms in the industry is sufficiently large:

Assumption 2. *$|\mathcal{N}| = 1 + \omega > 2\eta$.*[15]

Each event in the game occurs in the following order:

(i) **Pool formation**. The firms in $\mathcal{N}$ can form several *coalitions*. If all the firms in a coalition enter into certain contracts and the coalition satisfies certain conditions, we call the coalition *pool*, which will be formally defined later.

(ii) **Competition**. All pools engage in the Bertrand competition. Due to the property of the Bertrand competition on the quality ladder, only one pool earns a positive profit.

(iii) **Profit division**. The pool that won the Bertrand competition shares the profit with the firms in the pool.

[15] $|\mathcal{N}|$ is the cardinality of $\mathcal{N}$ (i.e., the number of firms).

A coalition, denoted by T, is a non-empty subset of $\mathcal{N}$. Every firm can form a coalition consisting only of itself. Note that some coalitions may be unable to produce goods due to a lack of patent licenses or violation of the prohibition by the competition authority. Such a coalition becomes *inactive* and cannot participate in the Bertrand competition. Conversely, there may be an *active* coalition that can produce goods. To clarify this concept, we formally define the pool as follows.

Definition 2. *Consider a coalition of firms in which the youngest firm is firm i, and let $\mathcal{P}_i$ be the coalition (i.e., $\mathcal{P}_i \subseteq \mathcal{N}$ and $\min \mathcal{P}_i = i$). Then, $\mathcal{P}_i$ is* **pool** *if it satisfies (i) and (ii) and every firm in $\mathcal{P}_i$ agrees on (iii) to (vi):*

(i) $\mathcal{P}_i$ **includes all the licensors of firm** *i: $\mathcal{L}_i \subset \mathcal{P}_i$.*

- $\mathcal{L}_i \equiv \{i+1, i+2, ..., \min\{i+\eta, \omega\}\}$ *is the set of firms that have patents infringed by firm i.*

(ii) **Regulation by the competition authority***: $\mathcal{P}_i \cap (\mathcal{N} \backslash \mathcal{L}_i) = \{i\}$.*

- *The competition authority prohibits $\mathcal{P}_i$ from including any firm, not in $\mathcal{L}_i$ because such a firm does not license the patent to firm i and it just aims to expand the cartel size.*
- $\mathcal{P}_i = \{i\} \cup \mathcal{L}_i$ *holds by (i) and (ii).*

(iii) **Exclusive patent license**.

- *Every firm in $\mathcal{L}_i$ grants the patent licenses to firm i to enable it to produce the good.*
- *Every firm in $\mathcal{P}_i$ refuses the patent license to any firm in $\mathcal{N} \backslash \mathcal{P}_i$.*

(iv) **Price cartel**.

- *Every firm in $\mathcal{P}_i \backslash \{i\}$ does not produce its own good to increase the market power of firm i as much as possible.*

(v) **Profit division**.

- *If $\mathcal{P}_i$ earns a positive profit, firm i shares it with $\mathcal{L}_i$ in accordance with the profit-sharing vector already determined when $\mathcal{P}_i$ formed.*
- *No firm transfers the profit to the firms in $\mathcal{N} \backslash \mathcal{P}_i$.*

(vi) **Relationship Length**.

- *All firms in $\mathcal{P}_i$ continue the relationship until a new innovator enters the industry.*

For each $i \in \mathcal{N}$, we call $\mathcal{P}_i$ the **profitable pool** if $\mathcal{P}_i$ obtains a positive profit in the Bertrand competition regardless of what pools are formed by the firms in $\mathcal{N} \backslash \mathcal{P}_i$. Then, by this definition, we can show the following statement:

Lemma 4. *$\mathcal{P}_i$ is the profitable pool if and only if $i \in \{0, 1, ..., \eta\}$.*

Proof. Consider a pool $\mathcal{P}_i$ where $i \in \{\eta+1, \eta+2, ..., \omega\}$. Then, $\mathcal{P}_i$ loses to a superior pool $\mathcal{P}_\ell$ where $\ell \in \{0, 1, ..., i-\eta-1\}$ in the Bertrand competition. Therefore, $\mathcal{P}_i$ is not the profitable pool by definition. Next, consider a pool $\mathcal{P}_i$ where $i \in \{0, 1, ..., \eta\}$. Note that the firms in $\{0, 1, ..., \eta\} \backslash \mathcal{P}_i$ cannot form the pool by Definition 2 (i). This means that $\mathcal{P}_i$ can produce the good with the highest quality in the industry. Therefore, $\mathcal{P}_i$ always earns a positive profit in the Bertrand competition. □

Following the baseline model, we consider a symmetric situation where all firms in the industry have equally chosen λ^* given by (13). Then, by conditions (i) to (iv) in Definition 2, any profitable pool always earns the profit $\tilde{\Pi}$ defined in (14) because any profitable pool has $\eta+1$ firms and the competition authority does not allow the markup strictly higher than $(\lambda^*)^{1+\eta}$. All firms regard $\tilde{\Pi}$ as a constant because they take E, an aggregate variable determined in the BGP, as given. On the other hand, as shown in the proof of Lemma 4, the non-profitable pools gain nothing when the superior pools form. Thus, following von Neumann and Morgenstern (1944), we define the profit of the non-profitable pool as zero from a pessimistic viewpoint.

The function $v : 2^{\mathcal{N}} \to \mathbb{R}$, called the characteristic function in cooperative game theory, assigns a profit that each coalition $T \subseteq \mathcal{N}$ definitely gains in the Bertrand competition. Note that the coalitions, not the pools, gain no profit because of a lack of patent licenses or violation of the prohibition by the competition authority. Then, the characteristic function is given as[16]

$$v(T) = \tilde{\Pi} \text{ if } T = \mathcal{P}_i \text{ for } i \in \{0, 1, \ldots, \eta\}, \text{ and}$$
$$v(T) = 0 \text{ otherwise.}$$

Let $i \in \{0, 1, \ldots, \eta\}$, and assume that the profitable pool $\mathcal{P}_i$ forms. Then, $\mathcal{P}_i$ earns the profit $\tilde{\Pi}$, and each firm $j \in \mathcal{N} \setminus \mathcal{P}_i$ gains nothing. Thus, by condition (v) in Definition 2, a vector $\boldsymbol{\pi} \equiv (\pi_j)_{j\in\mathcal{N}}$ of the net profit of the firms in the industry is defined as

$$\sum_{j\in\mathcal{P}_i} \pi_j = \tilde{\Pi}, \text{ and}$$
$$\pi_j = 0 \text{ for all } j \notin \mathcal{P}_i.$$

B.2 The core of the game

We define the concept of *blocking* in the game as follows:

Definition 3. *A coalition $T \subseteq \mathcal{N}$* **blocks** *$\boldsymbol{\pi}^*$ through $\boldsymbol{\pi} = (\pi_j)_{j\in\mathcal{N}}$ if the following properties hold.*

- **Feasibility**: $\sum_{j\in T} \pi_j \leq v(T)$.
- **Improvement**: $\pi_j > \pi_j^*$ *holds for all* $j \in T$.

[16]By convention, we define $v(\emptyset) = 0$.

Using Definition 3, we define the core of the game as follows:

Definition 4. *The* **core** *is the set of $\boldsymbol{\pi}$ that is not blocked by any coalition.*

By Definition 3, any net profit vector $\boldsymbol{\pi}^*$ such that $\pi_j^* < 0$ for some firm $j \in \mathcal{N}$ is blocked by $\{j\}$ because $v(\{j\}) = \pi_j = 0$ holds. Thus, without loss of generality, we assume that the core is a subset of the set of net profit vectors π such that $\pi_j \geq 0$ for each $j \in \mathcal{N}$. With this setup, we can show the following statement:

Proposition 6. *Let $i \in \{0, 1, \dots, \eta\}$, and assume that $\mathcal{P}_i$ forms. Then, the net profit vector that belongs to the core of the game is uniquely given by $\boldsymbol{\pi}^C = (\pi_j^C)_{j \in \mathcal{N}}$ such that*

$$\pi_\eta^C = \tilde{\Pi} \text{ and } \pi_j^C = 0 \text{ for all } j \in \mathcal{N} \backslash \{\eta\}.$$

Proof. Let $\mathcal{P}_i$ be the profitable pool where $i \in \{0, 1, ..., \eta\}$. Consider a net profit vector $\boldsymbol{\pi}$ such that $\pi_\eta < \tilde{\Pi}$ holds. Then, $\mathcal{P}_i \backslash \{\eta\}$ has some firms that obtain a positive net profit. We define $\overline{\pi} \equiv \sum_{k=i}^{\eta-1} \pi_k$ and $\underline{\pi} \equiv \sum_{k=\eta+1}^{i+\eta} \pi_k$. Suppose that $\overline{\pi} > 0$. Then, $\mathcal{P}_\eta$ can block $\boldsymbol{\pi}$ through $\boldsymbol{\pi}'$ such that

$$\pi_j' = 0 \text{ for all } j \in \{0, 1, ..., \eta - 1\},$$
$$\pi_j' = \pi_j + \frac{\overline{\pi}}{1+\eta} \text{ for all } j \in \{\eta, \eta+1, \eta+2, ..., 2\eta\}, \text{ and}$$
$$\pi_j' = \pi_j \text{ for all } j \in \{2\eta, 2\eta+1, ..., \omega\}.$$

Next, suppose that $\underline{\pi} > 0$. Then, $\mathcal{P}_0$ can block $\boldsymbol{\pi}$ through $\boldsymbol{\pi}''$ such that

$$\pi_j'' = \pi_j + \frac{\underline{\pi}}{1+\eta} \text{ for all } j \in \{0, 1, ..., \eta\}, \text{ and}$$
$$\pi_j'' = 0 \text{ for all } j \in \{\eta+1, \eta+2, ..., \omega\}.$$

As a result, a net profit vector $\boldsymbol{\pi}$ such that $\pi_\eta < \tilde{\Pi}$ holds cannot belong to the core.

Finally, consider the net profit vector $\boldsymbol{\pi}^C$ given in the statement. Suppose that a coalition T can block $\boldsymbol{\pi}^C$ through $\boldsymbol{\pi}$. If $v(T) = 0$, then by the definition of $\boldsymbol{\pi}^C$ and the feasibility in Definition 3, $\sum_{j \in T} \pi_j \leq v(T) = 0 \leq \sum_{j \in T} \pi_j^C$; thus, $\boldsymbol{\pi}$ does not satisfy the improvement in Definition 3. Therefore, T that can block $\boldsymbol{\pi}^C$ must be the profitable pool (i.e., $v(T) = \tilde{\Pi} > 0$). This means that, by Lemma 4, T contains firm η. Then, by the improvement in Definition 3, $\sum_{j \in T} \pi_j > \sum_{j \in T} \pi_j^C = \tilde{\Pi} = v(T)$, which contradicts the feasibility in Definition 3. Thus, no coalition can block $\boldsymbol{\pi}^C$. □

Proposition 6 shows that firm η obtains all the profit, and the rest does not obtain a positive net profit in the core of the game. Intuitively, firm η is the **pivotal firm** in $\mathcal{N}$ because any pool without firm η is not profitable. Therefore, the other firms in the profitable pool do not have any negotiation power.[17]

By Lemma 4 and Proposition 6, it is unclear whether $\mathcal{P}_0$ is chosen as the profitable pool in $\{\mathcal{P}_0, \mathcal{P}_1, ..., \mathcal{P}_\eta\}$. Because they equally yield $\pi_\eta = \tilde{\Pi}$, the pivot (i.e., firm η) is indifferent between them. To deal with the problem, we assume that $\lambda_i = \lambda_{i+1} + \varepsilon$ for all $i \in \mathcal{N}$, where $\varepsilon > 0$ is infinitesimal. Although it does not change any result in the baseline model because $\varepsilon > 0$ is a negligibly small number, $\mathcal{P}_0$ earns the largest profit in $\{\mathcal{P}_0, \mathcal{P}_1, ..., \mathcal{P}_\eta\}$. Note that firm η will no longer be pivotal when the next innovation occurs. Because this is the last opportunity for firm η to obtain a positive net profit, firm η will choose $\mathcal{P}_0$.

B.3 The Shapley value

Following the literature, we also consider the next intervention as an approximation of the license under FRAND conditions:

Definition 5. *The* **intervention** *in the licensing negotiation is that*

(i) *All firms in $\mathcal{L}_0$ are prohibited from refusing firm 0's offer of the patent licenses;*
   - *The property implies that $\mathcal{P}_0 = \{0\} \cup \mathcal{L}_0$ is the profitable pool in the industry.*

(ii) *The license fees should be fair in the sense that all firms in $\mathcal{P}_0$ must share the profit $\tilde{\Pi}$ following the Shapley value.*

Definition 6. *Let $\mathcal{P}_0$ be the profitable pool. The* **Shapley value** *π_j^S for each $j \in \mathcal{P}_0$ is given as follows:*

$$\pi_j^S \equiv \sum_{T \subseteq \mathcal{P}_0 \setminus \{j\}} \frac{t!(\eta - t)!}{(1+\eta)!} (v(\{j\} \cup T) - v(T)),$$

where t is the cardinality of a coalition T (i.e., $t = |T|$).

For each $j \in \mathcal{P}_0$ and for each $T \subseteq (\mathcal{P}_0 \setminus \{j\})$, $v(\{j\} \cup T) - v(T)$ is called the **marginal contribution** of firm j to coalition T. In the formation process for $\mathcal{P}_0$, when a coalition T already forms and a firm j joins that coalition, the firm demands and is promised to gain its marginal contribution to T. Suppose the order in which each firm joins is determined with equal probability. Then, the Shapley value for each firm $j \in \mathcal{P}_0$ is the average of j's marginal contribution to the formation of $\mathcal{P}_0$. In this sense, the Shapley value is a fair profit allocation among $\mathcal{P}_0$.

[17] Even if firm $i (\neq \eta)$ in the profitable pool suggests leaving the pool to decrease the profit of the profitable pool, firm η can form another pool that does not contain firm i but yields the same profit $\tilde{\Pi}$ in the Bertrand competition.

Proposition 7. *Suppose that $\mathcal{P}_0$ forms. The net profit vector applied to the Shapley value of the game is uniquely given as $\boldsymbol{\pi}^S = (\pi_j^S)_{j\in\mathcal{N}}$ such that*

$$\pi_j^S = \frac{\tilde{\Pi}}{1+\eta} \text{ for all } j \in \mathcal{P}_0 \text{ and } \pi_j^S = 0 \text{ for all } j \notin \mathcal{P}_0.$$

Proof. Note that because $\mathcal{P}_0$ forms under the intervention, each firm $j \notin \mathcal{P}_0$ gains nothing; that is, under the profit allocation based on the Shapley value, $\pi_j^S = 0$ for each $j \notin \mathcal{P}_0$. Furthermore, as mentioned above, $v(T) = 0$ holds for each $T \subsetneq \mathcal{P}_0$ due to the patent infringements. Thus, when and only when firm $j \in \mathcal{P}_0$ joins $\mathcal{P}_0 \backslash \{j\}$, the marginal contribution of firm j is positive (i.e., $T = \mathcal{P}_0 \backslash \{j\}$ if and only if $v(\{j\} \cup T) - v(T) = v(\mathcal{P}_0) = \tilde{\Pi} > 0$). Therefore, we obtain the result. □

References

[1] Basu, S. (1996). "Procyclical productivity: increasing returns or cyclical utilization?" The Quarterly Journal of Economics, 111(3), 719-751.

[2] Chu, A. C. (2009). "Effects of blocking patents on R&D: A quantitative DGE analysis." Journal of Economic Growth, 14(1), 55-78.

[3] Chu, A. C., Cozzi, G., & Galli, S. (2012). "Does intellectual monopoly stimulate or stifle innovation?" European Economic Review, 56(4), 727-746.

[4] Chu, A. C., & Pan, S. (2013). "The escape-infringement effect of blocking patents on innovation and economic growth." Macroeconomic Dynamics, 17(4), 955-969.

[5] Dewatripont, M., & Legros, P. (2013). " ‘Essential’ Patents, FRAND Royalties and Technological Standards." The Journal of Industrial Economics, 61(4), 913-937.

[6] Entezarkheir, M. (2017). "Patent thickets, defensive patenting, and induced R&D: an empirical analysis of the costs and potential benefits of fragmentation in patent ownership." Empirical Economics, 52(2), 599-634.

[7] Falk, A., Becker, A., Dohmen, T., Enke, B., Huffman, D., & Sunde, U. (2018). "Global evidence on economic preferences." The Quarterly Journal of Economics, 133(4), 1645-1692.

[8] Grossman, G., & Helpman, E. (1991). "Innovation and growth in the global economy." The MIT Press, Cambridge, Massachusetts.

[9] Hall, B. H., & Ziedonis, R. H. (2001). "The patent paradox revisited: an empirical study of patenting in the US semiconductor industry, 1979-1995." Rand Journal of Economics, 32(1), 101-128.

[10] Kishimoto, S., & Suzuki, K. (2021). "Growth and welfare effects of intervention into patent license negotiation." MPRA Paper.

[11] Kishimoto, S., & Watanabe, N. (2017). "The kernel of a patent licensing game: The optimal number of licensees." Mathematical Social Sciences, 86, 37-50.

[12] Klein, M. A. (2022). "The reward and contract theories of patents in a model of endogenous growth." European Economic Review, 147, 104178.

[13] Layne-Farrar, A., Padilla, A. J., & Schmalensee, R. (2007). "Pricing patents for licensing in standard-setting organizations: Making sense of FRAND commitments." Antitrust Law Journal, 74(3), 671-706.

[14] von Neumann, J., & Morgenstern, O. (1944). "Theory of games and economic behavior." 1st Edition. Princeton University Press, Princeton, NJ, [2nd Edn. 1947], [3rd Edn. 1953].

[15] Niwa, S. (2016). "Patent claims and economic growth." Economic Modelling, 54, 377-381.

[16] Niwa, S. (2018). "Effects of a blocking patent on R&D with endogenous survival activities." Journal of Economics, 125(3), 263-277.

[17] Norrbin, S. C. (1993)."The relationship between price and marginal cost in U.S. industry: A contradiction," Journal of Political Economy, 101(6), 1149-1164.

[18] O'donoghue, T., & Zweimüller, J. (2004). "Patents in a model of endogenous growth." Journal of Economic Growth, 9(1), 81-123.

[19] Sakakibara, M. (2010). "An empirical analysis of pricing in patent licensing contracts." Industrial and Corporate Change, 19(3), 927-945.

[20] Sakakibara, M., & Branstetter, L. (2001). "Do Stronger Patents Induce More Innovation? Evidence from the 1988 Japanese Patent Law Reforms." RAND Journal of Economics, 32(1), 77-100.

[21] Shapley, L. S. (1953). "A value for n-person games." Contributions to the Theory of Games, 2(28), 307-317.

[22] Suzuki, K. (2020). "Patent protection, optimal licensing, and innovation with endogenous entry." Macroeconomics Dynamics, 24(8), 2033-2059.

[23] Tauman, Y., & Watanabe, N. (2007). "The Shapley value of a patent licensing game: the asymptotic equivalence to non-cooperative results." Economic Theory, 30(1), 135-149.

[24] Watanabe, N., & Muto, S. (2008). "Stable profit-sharing in a patent licensing game: General bargaining outcomes." International Journal of Game Theory, 37, 505-523.

[25] Wang, M., Rieger, M. O., & Hens, T. (2016). "How time preferences differ: Evidence from 53 countries." Journal of Economic Psychology, 52, 115-135.

[26] Yang, Y. (2018). "On the optimality of IPR protection with blocking patents." Review of Economic Dynamics, 27, 205-230.

[27] Ziedonis, R. H. (2004). "Don't fence me in: Fragmented markets for technology and the patent acquisition strategies of firms." Management Science, 50(6), 804-820.