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THE CONSUMPTION MULTIPLIER OF GOVERNMENT SPENDING: THE ROLE OF SUBSTITUTABILITY BETWEEN GOVERNMENT SPENDING AND LEISURE

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The Consumption Multiplier of Government Spending: The Role of Substitutability between Government Spending and Leisure [1]

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Abstract

It has been empirically observed that consumption responds positively to government spending shock, however, existing models with intertemporally-optimizing households do not easily reconcile this stylized fact. This paper aims to address this discrepancy between models and data, focusing on the non-separable preferences with respect to consumption, leisure, and government spending. We derive conditions for a positive consumption multiplier under the general utility function and find that consumption can respond positively when leisure and government spending are substitutes. Examples of government spending that would have such an effect include care for children and the elderly, education spending, highway and public transportation.

JEL Classification: E62; E32; E60

Keywords: Fiscal multiplier, Non-separable preferences, Substitutability

1 Introduction

Although many empirical studies indicate that consumption rises in response to increased government spending (e.g., Blanchard and Perotti (2002); Fatás and Mihov (2001); Mountford and Uhlig (2009); Fisher and Peters (2010); Mertens and Ravn (2010); Ben Zeev and Pappa (2017)), a positive consumption multiplier of government spending cannot be easily reconciled with existing models based on intertemporally-optimizing households. As shown in Baxter and King (1993), increased government spending diminishes the present value of disposable income due to higher taxes for financing government spending, resulting in a negative wealth effect that triggers a decline in consumption within a standard real business cycle model. This mechanism also applies to New-Keynesian models with sticky prices. Previous studies have proposed various modifications to these models in order to establish a positive consumption multiplier.[1]

One modification for obtaining a positive consumption multiplier is to assume non-separable preferences. Linnemann and Schabert (2004), Bouakez and Rebei (2007), Ganelli and Tervala (2009), Marattin and Palestini (2014) consider a non-separable utility function between consumption and government spending and show that the consumption multiplier can be positive when consumption and government spending are complements.[2] In contrast, Linnemann (2006) considers non-separable utility function between consumption and leisure and shows that the consumption multiplier can be positive when consumption and government spending are substitutes in a frictionless business cycle model for a certain functional form of preferences. However, Bilbiie (2009) shows that, for general utility function, the parameter restriction under which a positive consumption multiplier occurs in a frictionless business cycle model implies that either the utility function is not concave or that

[1] These modifications include productive government spending (Baxter and King (1993); Asimakopoulos, Lorusso, and Pieroni (2021)), hand-to-mouth households (Galí, López-Salido, and Vallés (2007)), deep habit (Ravn, Schmitt-Grohé, and Urbe (2006)), and others.

[2] In this context, the terms "substitutes" and "complements" are not used in the Hicks's definition but in the Edgeworth's definition. Let the utility function be $U(x_1, x_2)$. x_1 and x_2 are "Edgeworth substitutes" if $U_{x_1x_2} < 0$, "Edgeworth complements" if $U_{x_1x_2} > 0$, and "Edgeworth independent" if $U_{x_1x_2} = 0$. See, Karras (1994); Ni (1995).

consumption is inferior. [3]

In this paper, we consider a utility function that includes non-separability between leisure and government spending in a frictionless business cycle model. We clarify the conditions under which the consumption multiplier is positive for general utility function. According to previous studies, the consumption multiplier can be positive when consumption and government spending are complements and/or consumption and leisure are substitutes. We show that, in addition to these two conditions, the consumption multiplier can be positive when leisure and government spending are substitutes; under this condition, consumption need not be inferior.

2 The model

Our model is similar to a frictionless business cycle model used in Bilbiie (2009), except that government spending is contained in the utility function in a non-separable form.

Suppose that the representative household aims to maximize the expected present value of lifetime utility. The momentary utility function of the representative household at time t takes the general non-separable form:

$$U(C_t, L_t, G_t), \tag{1}$$

where, C_t represents consumption, L_t represents leisure ($L_t = \bar{T} - N_t$ where $\bar{T}$ is time endowment and N_t is hours worked), and G_t represents government spending. Assume that the utility function U satisfies $U_C > 0$, $U_L > 0$, $U_G > 0$, $U_{CC} < 0$, $U_{LL} < 0$, and the strong bordered Hessian condition with respect to (C, L), i.e., $U_{CC}U_L^2 + U_{LL}U_C^2 - 2U_{CL}U_CU_L < 0$.[4]

The household earns labor wage income and dividends from the firm. The government

[3] Bilbiie (2011) extend that framework to an environment with sticky prices, demonstrating that a positive consumption multiplier can occur under the conditions of concave utility and good normality restrictions when consumption and leisure are substitutes. Monacelli and Perotti (2008) also reach this conclusion by using particular functional form of utility function.

[4] See Appendix B for the necessity of this assumption.

spending is financed through lump-sum taxes on the household and issuing government bonds. The budget constraint for the representative household can be expressed as:

$$B_t + C_t = (1 + r_{t-1})B_{t-1} + W_t N_t + D_t - \tau_t, \tag{2}$$

where B_t represents one-period, risk-free government bonds, r_t represents the interest rate on government bonds, W_t represents wage, D_t represents dividends received from the firm, and τ_t represents lump-sum taxes.

The government's budget constraint is given by:

$$B_t = (1 + r_{t-1})B_{t-1} + G_t - \tau_t. \tag{3}$$

The government determines a stream of government spending exogenously.

Finally, the production function takes the form of:

$$Y_t = F(N_t), \tag{4}$$

where F is non-increasing return to scale, $F(0) = 0$, $F_N > 0$, $F_{NN} \leq 0$.

3 Analysis

Given government spending G_t, the representative household maximizes lifetime utility subject to the budget constraint.

The first-order conditions for the household's problem are given by

$$U_C(C_t, L_t, G_t) = \beta(1 + r_t)E_t U_C(C_{t+1}, L_{t+1}, G_{t+1}), \tag{5}$$

and

$$U_L(C_t, L_t, G_t) = U_C(C_t, L_t, G_t)W_t, \tag{6}$$

where β is the discount factor.

The optimal condition of the firm for labor is given by

$$W_t = F_N(N_t). \tag{7}$$

Following Bilbiie (2009), we log-linearize the optimality conditions in order to derive analytical results.[5] Let lowercase letters denote deviation from the steady state: that is, $x_t \equiv X_t - X$ where X represents steady state value of X_t.

Log-linearizing Eqs (6) and (7) yields

$$\frac{U_{LC}}{U_L}c_t + \frac{U_{LL}}{U_L}l_t + \frac{U_{LG}}{U_L}g_t = \frac{U_{CC}}{U_C}c_t + \frac{U_{CL}}{U_C}l_t + \frac{U_{CG}}{U_C}g_t + w_t, \tag{8}$$

and

$$w_t = \frac{F_{NN}}{F_N}n_t, \tag{9}$$

where the wage is defined as log deviations from steady state: that is, $w_t \equiv \ln(W_t/W)$.

Substituting Eq.(9) into Eq.(8), we have

$$\left(\frac{U_{LC}}{U_L} - \frac{U_{CC}}{U_C}\right)c_t = \left(\frac{F_{NN}}{F_N} + \frac{U_{LL}}{U_L} - \frac{U_{CL}}{U_C}\right)n_t + \left(\frac{U_{CG}}{U_C} - \frac{U_{LG}}{U_L}\right)g_t. \tag{10}$$

In addition, as $y_t = F_N n_t$ and $y_t = c_t + g_t$,

$$\left(\frac{U_{LC}}{U_L} - \frac{U_{CC}}{U_C}\right)c_t = \left(\frac{F_{NN}}{F_N} + \frac{U_{LL}}{U_L} - \frac{U_{CL}}{U_C}\right)\frac{c_t + g_t}{F_N} + \left(\frac{U_{CG}}{U_C} - \frac{U_{LG}}{U_L}\right)g_t. \tag{11}$$

[5] Analytical results without linearization are shown in Appendix B.

Eq.(11) can be rearranged as

$$\left\{\left(\frac{\dfrac{U_{LC}}{U_L}-\dfrac{U_{CC}}{U_C}}{\dfrac{F_{NN}}{F_N}+\dfrac{U_{LL}}{U_L}-\dfrac{U_{CL}}{U_C}}\right)F_N-1\right\}c_t=\left\{\left(\frac{\dfrac{U_{CG}}{U_C}-\dfrac{U_{LG}}{U_L}}{\dfrac{F_{NN}}{F_N}+\dfrac{U_{LL}}{U_L}-\dfrac{U_{CL}}{U_C}}\right)F_N+1\right\}g_t. \tag{12}$$

Solving Eq.(12) for c_t yields

$$c_t=\frac{\phi F_N+1}{\delta F_N-1}g_t, \tag{13}$$

where $\phi\equiv\left(\frac{U_{CG}}{U_C}-\frac{U_{LG}}{U_L}\right)/\left(\frac{F_{NN}}{F_N}+\frac{U_{LL}}{U_L}-\frac{U_{CL}}{U_C}\right)$ and $\delta\equiv\left(\frac{U_{LC}}{U_L}-\frac{U_{CC}}{U_C}\right)/\left(\frac{F_{NN}}{F_N}+\frac{U_{LL}}{U_L}-\frac{U_{CL}}{U_C}\right)$.

Using the intratemporal optimality condition $F_N = U_L/U_C$, we obtain the following proposition.

Proposition 1. *In response to increased government spending, consumption increases if and only if*

$$\frac{\phi(U_L/U_C)+1}{\delta(U_L/U_C)-1}>0. \tag{14}$$

In Bilbiie (2009), $\phi = 0$ as he (implicitly) assumes $U_{CG} = U_{LG} = 0$. Thus, $\delta U_L/U_C > 1$ is a necessary and sufficient condition for a positive consumption multiplier in Bilbiie (2009) (Theorem 1). In contrast, since we allow for nonzero ϕ, the necessary conditions for a positive consumption multiplier can be described as follows.

Corollary 1. *Under our assumptions for utility function and production function, the necessary condition for satisfying Eq.(14) is,*

1. $U_{CG} > 0$ *(consumption and government spending are complements),*
2. $U_{CL} < 0$ *(consumption and leisure are substitutes), and/or*
3. $U_{LG} < 0$ *(leisure and government spending are substitutes).*

As the necessary conditions $U_{CG} > 0$ and $U_{CL} < 0$ have already been shown in previous studies,[6] we focus on the condition $U_{LG} < 0$. In this case, the necessary and sufficient condition for a positive consumption multiplier is obtained as follows.

Proposition 2. *If $U_{CG} = U_{CL} = 0$, a necessary and sufficient condition for a positive consumption multiplier is,*

$$U_{LG} < U_C \left(\frac{F_{NN}}{F_N} + \frac{U_{LL}}{U_L} \right). \tag{15}$$

Proof. When $U_{CG} = U_{CL} = 0$,

$$\delta = \frac{-\frac{U_{CC}}{U_C}}{\frac{F_{NN}}{F_N} + \frac{U_{LL}}{U_L}},$$

$$\phi = \frac{-\frac{U_{LG}}{U_L}}{\frac{F_{NN}}{F_N} + \frac{U_{LL}}{U_L}}.$$

Since $U_{CC} < 0$ and $U_{LL} < 0$, $\delta < 0$ and the denominator of the left-hand side of Eq. (14) is negative. Thus, the numerator of the left-hand side of Eq.(14) must be negative for a positive consumption multiplier.

The condition for the numerator of the left-hand side of Eq. (14) to be negative is,

$$\left(\frac{-\frac{U_{LG}}{U_L}}{\frac{F_{NN}}{F_N} + \frac{U_{LL}}{U_L}} \right) \frac{U_L}{U_C} + 1 < 0,$$

$$\left(\frac{-\frac{U_{LG}}{U_L}}{\frac{F_{NN}}{F_N} + \frac{U_{LL}}{U_L}} \right) \frac{U_L}{U_C} < -1,$$

$$-\frac{U_{LG}}{U_L} \frac{U_L}{U_C} > -\left(\frac{F_{NN}}{F_N} + \frac{U_{LL}}{U_L} \right),$$

$$U_{LG} < U_C \left(\frac{F_{NN}}{F_N} + \frac{U_{LL}}{U_L} \right),$$

where the reversal of the inequality in the third line is due to the fact that the term

[6] See, Linnemann and Schabert (2004); Bouakez and Rebei (2007); Ganelli and Tervala (2009) for $U_{CG} > 0$ and Bilbiie (2009, 2011) for $U_{CL} < 0$.

$\left(\frac{F_{NN}}{F_N} + \frac{U_{LL}}{U_L}\right)$ is negative. $\square$

Bilbiie (2009) shows that the condition for a positive consumption multiplier implies that consumption is inferior. However, our condition Eq. (15) in proposition 2 guarantees that consumption is not inferior. See Appendix A for details.

4 Implications and Discussion

In this section, we discuss the intuitive mechanism, economic implications, and empirical plausibility of the finding that $U_{LG} < 0$ can lead to a positive consumption multiplier. To focus on the impact of U_{LG}, we assume $U_{CG} = U_{CL} = 0$ throughout this section.

4.1 *Why is the consumption multiplier positive when $U_{LG} < 0$?*

Let us first give an intuitive explanation that the consumption multiplier can be positive if $U_{LG} < 0$. Suppose government spending increases by one dollar. The household will be taxed on the present value of one dollar, which reduces consumption and leisure through a negative wealth effect. Less leisure means more labor, so the decrease in consumption is canceled out to some extent by the increase in output. As $U_{LL} < 0$ and $F_{NN} \leq 0$, the marginal utility of leisure is higher, and the marginal productivity of labor is lower than that prior to the increase in government spending. Thus, output increases by less than one dollar when $U_{LG} = 0$. As a result, consumption must fall because of resource constraints. This is why the output multiplier of government spending is below one, and the consumption multiplier is negative in a frictionless business cycle model. The condition for a positive consumption multiplier is equivalent to the output multiplier that is greater than one. If $U_{LG} < 0$, government spending reduces the marginal utility of leisure (or marginal "disutility" of labor), it is possible that output would increase by more than one dollar, allowing for consumption to rise. In other words, if government spending and leisure are substitutes, increased government spending enhances the incentive to work and allows for more room to increase consumption by increasing the output multiplier.

4.2 *Is $U_{LG} < 0$ empirically plausible?*

While there are many empirical studies that examine the substitutability between consumption and government spending (Kormendi (1983); Aschauer (1985); Karras (1994); Ni (1995); Amano and Wirjanto (1998); Bouakez and Rebei (2007); Fève, Matheron, and Sahuc (2013); Sims and Wolff (2018); Bouakez, Rachedi, and Santoro (2023)), there are few empirical studies for the substitutability between leisure and government spending. Conway (1997), one of the few exceptions, examines the substitutability between leisure and government spending using the U.S. data from the University of Michigan's Panel Study of Income Dynamics (PSID) by estimating a linear labor supply function based on a non-separable utility function with respect to consumption, leisure, and government spending. As a result, Conway (1997) finds a positive substitution effect of government spending on leisure (or complementary effect on labor supply) for men and unmarried women in the U.S.[7] Thus, $U_{LG} < 0$ is empirically plausible.

4.3 *What kind of government spending brings $U_{LG} < 0$?*

Conway (1997) also gives examples of government spending that has substitution effects on leisure (or complimentary effects on labor). First, public transportation and highways would have a complementary effect on labor supply by lowering the transportation costs for working. Second, educational spending may improve opportunities for rewarding work or change preferences for work. Third, care for children and the elderly could substitute home production and also have a complementary effect on labor. All of these would have substitution effects on leisure.

[7] Conway (1997) uses Hicks's definition of substitutes/complements, which differs from the definition of Edgeworth used in our study. However, we confirm that $U_{LG} < 0$ is also basically satisfied under the estimated parameters values and utility function in Conway (1997).

For clarity, let us consider the following specific form of the utility function:

$$U = \log C_t + \gamma \log(\bar{T} - H_t - N_t), \tag{16}$$

where $H_t = \bar{H} - \theta G_t$ is hours of home production and/or commuting time, and $L_t = \bar{T} - H_t - N_t$. $\bar{H}$ is home production/commuting time in the absence of government spending. γ is the scale parameter for the utility of leisure. We assume that the household cannot choose their own home production/commuting time and that these are determined exogenously through government spending. When $\theta > 0$, government spending contributes to increasing "disposable time," defined here as total time $\bar{T}$ minus H_t. The household can increase either leisure or labor (in the firm) for the increased amount of disposable time. As the marginal utility of leisure decreases as leisure increases, the relationship between government spending and leisure will be substitutes when $\theta > 0$.

Here we assume that the production function is constant returns to scale,

$$Y_t = N_t. \tag{17}$$

Under these settings, $U_C = 1/C$, $U_{CC} = -1/C^2$, $U_L = \gamma(1/L)$, $U_{LL} = -\gamma(1/L^2)$, $F_N = 1$, $F_{NN} = 0$, $U_{CG} = 0$, $U_{LC} = 0$, $U_{LG} = -\gamma\theta(1/L^2)$. As $U_L = U_C$ from the intratemporal optimality condition, δ and ϕ in Proposition 1 can be expressed as follows;

$$\begin{aligned} \delta &= -\gamma, \\ \phi &= -\theta. \end{aligned}$$

Thus, the necessary and sufficient condition for a positive consumption multiplier is,

$$\theta > 1.$$

In summary, government spending that increases disposable time has a substitution effect on leisure, and if the effect is sufficiently large, the consumption multiplier can be positive.

5 Conclusion

This paper examines the conditions for a positive response of consumption to increased government spending under general non-separable preference among consumption, leisure, and government spending. Consequently, we find that a positive consumption multiplier can occur when leisure and government spending are substitutes. We do not claim that this condition alone can solve the discrepancy between data and existing models for consumption multipliers, but we believe that it is one factor worth considering.

Although this paper attempts to explain the stylized fact that the consumption multiplier is usually positive, it also has useful implications for the policy debate on the kind of spending that can raise the fiscal multiplier. Typically, a public investment that raises the marginal productivity (or Total Factor Productivity, TFP) of a firm, as shown in Baxter and King (1993), is thought to yield a large fiscal multiplier. Our findings show that not only policies that increase a firm's productivity but also spending that increases a household's disposable time can enhance the efficacy of fiscal policy.

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Appendix A. Non-inferiority

Bilbiie (2009) shows that a positive consumption multiplier can occur if and only if consumption is inferior when $U_{CG} = U_{LG} = 0$ and $U_{CL} \neq 0$. In this section, we show that a positive consumption multiplier can occur even if consumption is not inferior when $U_{LG} < 0$.

To focus on the effects of the substitutability between leisure and government spending, we consider the following utility function in which only leisure and government spending are non-separable:

$$U(C_t) + V(L_t, G_t), \tag{A.1}$$

where $U_C > 0$, $V_L > 0$, and $V_G > 0$. Here we assume strictly quasi-concavity, $U_{CC} < 0$ and $V_{LL} < 0$. In this expression, V_L, V_{LL}, and V_{LG} correspond to U_L, U_{LL}, and U_{LG} in the main text, respectively. The utility function (A.1) implies $U_{CG} = U_{CL} = 0$.

The budget constraint of the household can be rewritten as

$$C_t + W_t L_t \leq E_t,$$

where E_t represents *full income* given by $E_t = W_t\bar{T} + (1 + r_{t-1})B_{t-1} + D_t - \tau_t - B_t$.

Inferior goods have negative income elasticity at given prices. The demand functions with respect to consumption and leisure for a given full income level E are derived by solving the following static optimization problem:

$$\max \ \ U(C) + V(L, G) \ \ \text{s.t.} \ \ C + WL \leq E.$$

Solving this static optimization problem, we obtain as

$$WU_C(C) - V_L(L, G) = 0, \tag{A.2}$$

and

$$C + WL - E = 0. \tag{A.3}$$

Applying the implicit function theorem, we can examine the effect of changes in full income level E on consumption demand. Given the price of leisure W, the total differentiation of Eqs. (A.2) and (A.3) yields

$$(WU_{CC})\frac{\partial C}{\partial E} - V_{LL}\frac{\partial L}{\partial E} = 0, \tag{A.4}$$

and

$$\frac{\partial C}{\partial E} + W\frac{\partial L}{\partial E} = 1. \tag{A.5}$$

Solving Eqs. (A.4) and (A.5),

$$\frac{\partial C}{\partial E} = \frac{V_{LL}}{W^2 U_{CC} + V_{LL}} > 0, \quad \frac{\partial L}{\partial E} = \left(\frac{W^{-1}V_{LL} + WU_{CC}}{U_{CC}}\right)^{-1} > 0,$$

necessarily hold from $U_{CC} < 0$ and $V_{LL} < 0$. $\frac{\partial C}{\partial E} > 0$ and $\frac{\partial L}{\partial E} > 0$ imply that neither consumption nor leisure is inferior.

Bilbiie (2009) shows that the condition for a positive consumption multiplier implies that consumption is inferior. However, our condition Eq. (15) in proposition 2 guarantees that consumption is not inferior.

Appendix B. Analytical Results without Linearization

The strong bordered Hessian condition of U with respect to (C, L):

$$U_{CC}U_L^2 + U_{LL}U_C^2 - 2U_{CL}U_CU_L < 0, \tag{A.6}$$

is known as a sufficient condition for strict quasi-concavity of U with respect to (C, L). As shown in Katzner (1968), the converse relationship does not hold, i.e., there exists a function that violates the above inequality, but it is still strictly quasi-concave. Debreu (1972) also shows that the condition (A.6) is a necessary and sufficient condition for differentiability of a demand function. To obtain analytical results, we need to differentiate C_t by G_t, so this condition must be satisfied. The quasi-concavity of U with respect to G is not necessary because G is determined exogenously for the household.

Recall the optimal conditions for the household (6) and the firm (7):

$$U_L(C_t, L_t, G_t) = U_C(C_t, L_t, G_t)W_t,$$
$$W_t = F_N(N_t).$$

As $Y_t = F(N_t)$ and $N_t = \bar{T} - L_t$, we obtain

$$U_L(C_t, L_t, G_t) = U_C(C_t, L_t, G_t)F_N(\bar{T} - L_t), \tag{A.7}$$

$$F(\bar{T} - L_t) = C_t + G_t. \tag{A.8}$$

Define the function $H(C, L, G) = (H^1(C, L, G), H^2(C, L, G))$ as follows:

$$H^1(C, L, G) = U_L(C, L, G) - U_C(C, L, G)F_N(\bar{T} - L),$$
$$H^2(C, L, G) = F(\bar{T} - L) - C - G.$$

From equations (A.7) and (A.8), the following relation regarding H^1 and H^2 holds:

$$H^1(C_t, L_t, G_t) = H^2(C_t, L_t, G_t) = 0.$$

To check whether the implicit function theorem is applicable for (C, L), let us calculate the Jacobian of H with respect to (C, L):

$$H^* \equiv \begin{vmatrix} H_C^1 & H_L^1 \\ H_C^2 & H_L^2 \end{vmatrix} = U_{LL} - 2U_{CL}F_N + U_{CC}(F_N)^2 + U_C F_{NN}$$
$$= U_C F_{NN} - \begin{vmatrix} U_{CC} & U_{CL} & 1 \\ U_{LC} & U_{LL} & F_N \\ 1 & F_N & 0 \end{vmatrix} = U_C F_{NN} - (U_C)^{-2} \begin{vmatrix} U_{CC} & U_{CL} & U_C \\ U_{LC} & U_{LL} & U_L \\ U_C & U_L & 0 \end{vmatrix} < 0,$$

where the last inequality comes from the strong bordered Hessian condition (A.6). Therefore, the implicit function theorem is applicable and both C_t and L_t can be represented as continuously differentiable functions of G_t. We denote those functions as $C_t(G_t), L_t(G_t)$, respectively. Using these functions, H^1 and H^2 can be written as

$$H^1(C_t(G_t), L_t(G_t), G_t) = U_L(C_t(G_t), L_t(G_t), G) - U_C(C_t(G_t), L_t(G_t), G_t)F_N(\bar{T} - L_t(G_t)) = 0,$$
$$H^2(C_t(G_t), L_t(G_t), G_t) = F(\bar{T} - L_t(G_t)) - C_t(G_t) - G_t = 0.$$

Differentiating the above equations with respect to G_t, we obtain

$$\begin{pmatrix} H_C^1 & H_L^1 \\ H_C^2 & H_L^2 \end{pmatrix} \begin{pmatrix} C_t' \\ L_t' \end{pmatrix} = \begin{pmatrix} -U_{LG} + U_{CG}F_N \\ 1 \end{pmatrix}.$$

Thus,

$$\begin{aligned} \begin{pmatrix} C_t' \\ L_t' \end{pmatrix} &= \begin{pmatrix} H_C^1 & H_L^1 \\ H_C^2 & H_L^2 \end{pmatrix}^{-1} \begin{pmatrix} -U_{LG} + U_{CG}F_N \\ 1 \end{pmatrix} \\ &= \frac{1}{H^*} \begin{pmatrix} H_L^2 & -H_L^1 \\ -H_C^2 & H_C^1 \end{pmatrix} \begin{pmatrix} -U_{LG} + U_{CG}F_N \\ 1 \end{pmatrix}, \end{aligned}$$

which implies that,

$$C_t'(G_t) = \frac{-U_{LL} + U_{CL}F_N + U_{LG}F_N - U_{CG}(F_N)^2 - U_C F_{NN}}{H^*}. \tag{A.9}$$

Since H^* is negative, the necessary conditions of $C_t' > 0$ (i.e., C_t increase in response to G_t) are

1. $U_{CG} > 0$ (consumption and government spending are complements),
2. $U_{CL} < 0$ (consumption and leisure are substitutes), and/or
3. $U_{LG} < 0$ (leisure and government spending are substitutes).

Therefore, Corollary 1 is verified.

Suppose that $U_{CG} = U_{CL} = 0$. (A.9) can be rewritten as follows:

$$C_t'(G_t) = \frac{-U_{LL} + U_{LG}F_N - U_C F_{NN}}{H^*}.$$

Since $H^* < 0$, the following relationship holds:

$$C_t'(G_t) > 0 \Leftrightarrow U_{LG} < \frac{U_{LL}}{F_N} + \frac{U_C F_{NN}}{F_N}.$$

Using the intratemporal optimality condition $F_N = U_L/U_C$, we obtain

$$C_t'(G_t) > 0 \Leftrightarrow U_{LG} < U_C \left(\frac{F_{NN}}{F_N} + \frac{U_{LL}}{U_L} \right).$$

Thus, Proposition 2 is also verified.