Discussion Paper No. 1226

ISSN (Print) 0473-453X
ISSN (Online) 2435-0982

R&D SUBSIDIES, INNOVATION LOCATION, AND PRODUCTIVITY GROWTH

Colin Davis
Ken-ichi Hashimoto

December 2023

The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

R&D Subsidies, Innovation Location, and Productivity Growth

Colin Davis — Doshisha University*
Ken-ichi Hashimoto — Kobe University†

December 2023

Abstract

This paper studies how national research subsidies affect productivity growth and national welfare through adjustments in the geographic location of research and development (R&D) across countries. Our two-country framework features a tension in the firm-level innovation location decision between accessing technical knowledge and sourcing low-cost high-skilled labor. With trade costs and imperfect international knowledge diffusion, the larger country has a greater share of industry and tends to host a larger share of innovation. In this setting, we find that an R&D subsidy expands the implementing country's share of innovation and raises the rate of productivity growth. Although the non-implementing country experiences a welfare improvement, the rising cost of the policy generates a concave relationship between the R&D subsidy and the welfare of the implementing country, yielding an optimal R&D subsidy rate.

JEL Classifications: F12, F43, R11
Keywords: Optimal R&D Subsidies, Industry and Innovation Location patterns, Endogenous Productivity Growth, Endogenous Market Structure

*Institute for International Education, Doshisha University, Karasuma-Higashi-iru, Imadegawa-dori, Kamigyo, Kyoto, Japan, 602-8580, cdavis@mail.doshisha.ac.jp.
†Graduate School of Economics, Kobe University, 2-1 Rokkodai, Nada, Kobe, Japan, 657-8501, hashimoto@econ.kobe-u.ac.jp.
Acknowledgements: We are grateful for helpful comments from Taiji Furusawa, Tetsugen Haruyama, Keisaku Higashida, Yunfang Hu, Naoto Jinji, Ryoji Ohdoi, Takanori Shimizu, Mizuki Tsuboi, Akihito Yanase, Laixun Zhao, and participants of the 12th Spring Meeting of the Japan Society of International Economics, the Kansai Branch Meeting of the Japan Society of International Economic, and seminars at Kobe University and Matsuyama University. Financial support was provided by the Joint Usage/Research Center at ISER (Osaka University) and KIER (Kyoto University), and the Japan Society for the Promotion of Science: Grants-in-Aid for Scientific Research 20H05631, 20K01631, and 22K01511.

1 Introduction

Recent trends in research and development (R&D) policy at the national level have been driven by a recognition of the key role that innovation plays as an engine of economic growth. Indeed, expenditure on R&D reached a record 2.71% of world GDP in 2021 (World Bank, 2023). And, while R&D races in strategic industries across a small number of large countries have captured headlines (The Economist, 2022), casual observation finds that national R&D spending has increased broadly across the world. Among OECD member countries, for example, average R&D spending expanded from 1.51% to 2.16% of GDP between 2000 and 2019, with the number of countries spending more than 3% of GDP rising from 2 to 9 (OECD, 2022). Governments now recognize the need to support investment in R&D in an increasingly integrated global economy. As such, R&D subsidies have become a prominent policy tool.

This paper investigates how R&D subsidies affect innovation location patterns, long-run productivity growth, and national welfare. In particular, we construct a two-country endogenous growth and endogenous market structure framework (Smulders and van de Klundert, 1995; Peretto, 1996, 2018), in which governments offer firms R&D subsidies with the aim of attracting R&D investment and maximizing welfare. Following Davis and Hashimoto (2023), our analysis has two key features: monopolistically competitive firms that invest in process innovation to lower future production costs, and skill differentiated workers that choose between low-skilled employment in production and high-skilled employment in innovation.

In our framework, international trade costs and localized spillovers of technical knowledge from production to innovation present an important role for geography. Following Martin and Ottaviano (1999, 2001), iceberg trade costs generate a home market effect (Krugman, 1980) that causes industry to concentrate geographically in the country with the larger market. The concentration of industry strengthens knowledge spillovers, raising labor productivity in innovation in the larger country. And, as a result, the larger country hosts a greater share of innovation (Davis, 2013; Davis and Hashimoto, 2023). Importantly, firms balance the benefits of accessing the technical knowledge available in the larger country and sourcing high-skilled labor at lower cost in the smaller country, when selecting optimal locations for R&D investment. This trade-off in the firm-level innovation location decision has been emphasized in the business literature (Chung and Yeaple, 2008; Demirbag and Glaister, 2010; Lewin et al., 2009; Manning et al., 2008).

The long-run rate of productivity growth is closely linked with industry and innovation location patterns. As we have seen, an increase in the larger country's share of industry strengthens knowledge spillovers, improving labor productivity in innovation, but also raising high-skilled wages as the demand for high-skilled labor expands. In our framework, the wage effect dominates the productivity effect, and greater industry concentration raises the cost of process innovation, slowing productivity growth. In general, the empirical literature presents mixed results for the impact of industry concentration on economic growth. For example, Desmet and Rossi-Hansberg (2009, 2014) present evidence of a positive relationship between industry concentration and economic growth. Gardiner et al. (2011), however, report a negative relationship between economic growth and a number of measures for industry concentration.

We use the framework to study the effects of market integration on innovation location patterns and long-run productivity growth. First, a decrease in trade costs magnifies the home market effect, expanding the larger country's shares of industry and innovation. Knowledge spillovers improve and high-skilled wages rise in the larger country, while knowledge spillovers deteriorate and high-skilled wages fall in the smaller country. These opposing effects generate a convex relationship between the cost of innovation and trade costs, with greater trade integration initially increasing but then decreasing the long-run rate of productivity growth. Second, an improvement in the strength of international knowledge diffusion directly lowers the cost of innovation, increasing the rate of productivity growth. The results match with the theoretical results of Davis and Hashimoto (2023).

Turning to national R&D policy, we find that an R&D subsidy expands the implementing country's share of innovation and raises the rate of productivity growth. These results are consistent with empirical studies which find that R&D subsidies tend to increase national R&D employment (David et al., 2000; Hall and van Reenen, 2000; Garcia-Quevedo, 2004). In addition, there is growing evidence that fiscal incentives are generally effective for attracting the R&D activity of foreign multinational enterprises (Rodríguez-Pose and Wilkie, 2016). In our framework, the rising cost of supporting R&D policy generates a concave relationship between the R&D subsidy and the welfare of the implementing country, yielding an optimal R&D subsidy rate. In contrast, national R&D employment contracts in the non-implementing country. But, the faster rate of productivity growth ensures that the non-implementing country experiences a welfare improvement.

Our paper contributes to the endogenous growth literature that studies R&D subsidies in two-country endogenous growth frameworks. The analysis of Grossman and Helpman (1991) puts forward a positive role for R&D subsidies in the promotion of investment in innovation. These results are confirmed in the North-South product cycle literature which generally concludes that R&D subsidies in the North lead to faster economic growth (Currie et al., 1999; Glass and Saggi, 2001; Iwaisako et al., 2011). A closely related literature investigates how R&D subsidies influence the geography of innovation and considers the implications for R&D subsidy policy and economic growth. Impullitti (2010) introduces a quality-ladders growth model in which domestic and foreign firms compete for market leadership, and shows that fiercer competition from foreign firms raises the optimal R&D subsidy rate. And, Kondo (2013) develops an economic geography model of endogenous growth where industry and innovation agglomerate in a single country, and illustrates that greater trade integration reduces the intensity of R&D subsidy competition.

In a similar study, Davis and Hashimoto (2015) construct a two-country model of endogenous productivity growth with firms that invest in process innovation. With imperfect knowledge diffusion, the geographic concentration of relatively productive firms improves knowledge flows between firms, ensuring that the rate of economic growth is higher when industry is more concentrated in a single country. Thus, an R&D subsidy implemented in the larger (smaller) country tends to increase (decrease) the rate of economic growth. In contrast, in our framework the footloose structure of the firm, with production and innovation located independently in their lowest cost countries, combines with the positive link between national shares of innovation and high-skilled wages to generate a negative relationship between industry concentration and economic growth. In this setting, we show that while R&D subsidies always raise the rate of economic growth, rising costs in the implementing country lead to an optimal subsidy rate that maximizes national welfare.

The paper is organized as follows. Section 2 introduces our theoretical framework. In Section 3, we characterize long-run national labor allocations, the level of market entry, and the rate of productivity growth. Section 4 completes a policy analysis of trade costs, the degree of knowledge diffusion and national R&D subsidies. Section 5 concludes.

2 The Model

Consider an economy with two countries, home and foreign, and three types of economic activity, traditional production (Y), manufacturing (X), and process innovation (R). In each country, the labor force is composed of skill-differentiated workers that sort into low-skilled employment in production and high-skilled employment in innovation. The home country has a larger population, and with no international labor migration, the occupational choice of workers endogenously determines national supplies of low-skilled and high-skilled labor. Faced with trade costs and imperfect international knowledge diffusion, manufacturing firms locate production and process innovation independently in their respective lowest cost countries. In the following sections, we focus on the home country as we introduce the basic model setup. An asterisk is used to denote variables associated with the foreign country.

2.1 Household Preferences

Preferences are symmetric across countries, with dynastic households selecting expenditure-saving paths to maximize utility over an infinite time horizon. The intertemporal preferences of a household residing in the home country are

$$U = \int_0^\infty e^{-\rho t} \left(\alpha \ln C_X(t) + C_Y(t) \right) dt, \tag{1}$$

where $C_X(t)$ and $C_Y(t)$ are the consumptions of a manufacturing composite and a traditional good at time t, $\rho > 0$ is the subjective discount rate, and $\alpha > 0$. Intertemporal utility is maximized subject to the following flow budget constraint:

$$\dot{A}(t) = r(t)A(t) + I(t) - E(t) - T(t), \tag{2}$$

where $A(t)$ is household asset wealth, $r(t)$ is the interest rate, $I(t)$ is labor income, $E(t)$ is expenditure, $T(t)$ is a lump-sum tax on income, and a dot over a variable denotes time differentiation. The household's optimal expenditure-saving path is characterized by constant expenditure ($\dot{E}(t) = 0$), and accordingly $r(t) = \rho$ at all moments in time.

Given the per-period expenditure-saving decision, each household allocates constant shares of expenditure to the manufacturing composite and the traditional good: $P_X(t)C_X(t) = \alpha P_Y(t)$ and $P_Y(t)C_Y(t) = E(t) - \alpha P_Y(t)$, with $P_X(t)$ denoting the price index associated with manufacturing goods and $P_Y(t)$ denoting the price of the traditional good. The manufacturing composite and the corresponding price index have a constant elasticity of substitution (CES) formulation:

$$C_X(t) = \left(\int_0^{N(t)} q(i,t)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \qquad P_X(t) = \left(\int_0^{N(t)} p(i,t)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}, \tag{3}$$

where $N(t)$ is the total mass of product varieties produced in the manufacturing sectors of home and foreign at a given moment in time, and $\sigma > 1$ is the constant elasticity of substitution across product varieties. The price index and manufacturing composite can be used to derive the household demand for product variety i in the home country as

$$q(i,t) = \alpha p(i,t)^{-\sigma} P_X(t)^{\sigma-1} P_Y(t). \tag{4}$$

We derive similar demand conditions for households residing in the foreign country.

2.2 National Labor Supplies

In each country, skill is continuously distributed across workers according to a uniform distribution $F(z)$ with support $[0,1]$. Home and foreign skill have identical distributions, but different population sizes (Z and Z^*). And, at each moment in time, workers choose between employment in either production or innovation, with no costs incurred in changing occupations. A worker employed in production supplies one unit of low-skilled labor, regardless of skill level, and earns the low-skilled wage rate equal to unity. Alternatively, a worker employed in innovation supplies z units of high-skilled labor, and earns income $zw(t)$, where $w(t)$ is the high-skilled wage rate.

With positive employment in both production and innovation, there exists a threshold worker whose low-skilled and high-skilled incomes are equal ($w(t)z = 1$), and who is therefore indifferent between employment in production and innovation. Workers with skill levels $z \in [0, 1/w(t)]$ then find employment in production and workers with skill levels $z \in (1/w(t), 1]$ choose employment in innovation. Hence, aggregating across the population of households, the effective supplies of low-skilled and high-skilled labor in the home country become

$$L(w) = \int_0^{1/w(t)} dF(z)Z = \frac{Z}{w(t)}, \qquad H(w) = \int_{1/w(t)}^{1} z dF(z)Z = \frac{(w(t)^2 - 1)Z}{2w(t)^2}. \tag{5}$$

Given these effective labor supplies, the total labor income in the home country is $I(t)Z = L(t) + w(t)H(t)$. Foreign has analogous effective low-skilled and high-skilled labor supplies ($L^*(w^*, t)$ and $H^*(w^*, t)$), and total labor income equal to $I^*(t)Z^* = L^*(t) + w^*(t)H^*(t)$.

2.3 Production

The traditional sector features free trade between home and foreign ensuring a common international price when both countries produce traditional goods. Firms are perfectly competitive and produce one unit of output for each unit of low-skilled labor employed. Setting the traditional good as the model numeraire thus generates a unitary price and low-skilled wage rate in both countries: $P_Y(t) = P_Y^*(t) = 1$. Summing the home and foreign demands for traditional goods yields the world demand for low-skilled labor in traditional production as

$$L_Y(t) + L_Y^*(t) = (E(t) - \alpha)Z + (E^*(t) - \alpha)Z^*, \tag{6}$$

where we assume that the world demand for traditional goods is sufficiently large to ensure a positive level of low-skilled employment in traditional production in both countries at all moments in time; that is, $L_Y(t) > 0$ and $L_Y^*(t) > 0$.

The manufacturing sector is monopolistically competitive (Dixit and Stiglitz, 1977), with each firm employing low-skilled labor in the production of a unique differentiated product variety for supply to home and foreign households. The technology of firm i with production located in the home country is

$$x(i,t) = \theta(t)^{\gamma}(l_X(i,t) - \psi), \tag{7}$$

where $x(i,t)$ and $l_X(i,t)$ are firm-level output and low-skilled employment in production, $\theta(t)$ is a productivity coefficient that describes the current state of the firm's technology, $\gamma > 0$ is the productivity elasticity of output, and $\psi > 0$ is a fixed operating cost that is measured in units of low-skilled labor and is the same for all firms operating in home and foreign. While each firm employs a unique production process corresponding to its differentiated product variety, we assume that the productivity levels associated with production technologies are symmetric across firms, regardless of the location of production; that is, $\theta(t) = \theta^*(t)$.[1]

With CES preferences over manufacturing goods, firms maximize operating profit on sales by setting price equal to a constant markup over unit costs. More specifically, the optimal prices

[1]The model equilibrium features equal rates of productivity growth for all firms, regardless of their locations for production and innovation. Accordingly, assuming that initial productivity levels are equal across firms ensures a symmetric equilibrium; that is, $\theta(0) = \theta^*(0)$.

set for a manufacturing good produced in home and sold in the home and foreign markets are

$$p(t) = \frac{\sigma}{(\sigma - 1)\theta(t)^{\gamma}}, \qquad p^*(t) = \frac{\sigma\tau}{(\sigma - 1)\theta(t)^{\gamma}}, \tag{8}$$

where $\tau > 1$ is an iceberg trade cost, under which τ units must be shipped for every unit sold in the export market. Because all firms charge the same price, given our assumption of symmetric productivity levels, we now suppress the firm index i to simplify notation.

Equating the combined demands from home and foreign households (4) with firm-level supply ($x(t) = q(t)Z + \tau q^*(t)Z^*$), the optimal operating profit on sales ($\pi(t) = p(t)x(t) - l_X(t)$) associated with a firm locating production in the home country becomes

$$\pi(t) = \frac{\alpha p(t)^{1-\sigma}}{\sigma}\left(\frac{Z}{P_X(t)^{1-\sigma}} + \frac{\varphi Z^*}{P_X(t)^{*1-\sigma}}\right) - \psi, \tag{9}$$

where we have used $p^*(t) = \tau p(t)$, and $\varphi \equiv \tau^{1-\sigma}$ measures the freeness of international trade; that is, $d\varphi/d\tau < 0$, with no trade for $\varphi = 0$ and free trade for $\varphi = 1$.

2.4 Process Innovation

Manufacturing firms invest in process innovation to lower future production costs (Smulders and van de Klundert, 1995; Peretto, 1996, 2018). Specifically, the evolution of firm-level productivity for a firm with innovation located in the home country follows

$$\dot{\theta}(t) = \beta K(t) h_R(t), \tag{10}$$

where $\beta > 0$ is a parameter, $h_R(t)$ is firm-level employment of high-skilled labor, and $\beta K(t)$ is labor productivity in process innovation, with $K(t)$ capturing a knowledge spillover from production to innovation.

Adapting the specification of Baldwin and Forslid (2000), we model the knowledge spillover from production to innovation as a weighted average of the productivities of the technologies observable by the firm. For instance, knowledge spillovers are modeled as $K(t) = s_X(t)\theta(t) + \delta s_X^*(t)\theta^*(t)$ for a firm with R&D located in home, with the strength of technical spillovers between countries regulated by the degree of knowledge diffusion $\delta \in (0,1)$. Knowledge spillovers are completely local in nature for $\delta = 0$, but flow perfectly across countries for $\delta = 1$. Given the symmetry of firm productivity across the firms located in each country (i.e., $\theta(t) = \theta^*(t)$), the knowledge spillover can be simplified into two components: $K(t) \equiv k(t)\theta(t)$, where the productivity coefficient $\theta(t)$ captures the stock of technical knowledge embodied in a given firm's production technology. And, the regional strength of knowledge spillovers is determined by

$$k(t) = s_X(t) + \delta s_X^*(t). \tag{11}$$

Labor productivity in innovation is thus determined as the weighted-average productivity of observable firm-level knowledge stocks, with a stronger weighting for production technologies that are employed in proximity to the R&D department of the firm. This formulation for knowledge spillovers is consistent with broad empirical evidence concluding that knowledge spillovers diminish with distance (Bottazzi and Peri, 2003; Mancusi, 2008; Thompson, 2006).

Accounting for operating profit on sales and the cost of employing high-skilled labor in innovation, the total per-period profit of a firm with production and process innovation located in the home country is

$$\Pi(t) = \pi(t) - (1-\Delta)w(t)h_R(t), \tag{12}$$

where w is the high-skilled wage rate, and $\Delta \in (0,1)$ is the R&D subsidy rate provided to firms that locate process innovation in home. At each moment, firms set the level of high-skilled employment in innovation to maximize firm value, which equals the discounted profit stream associated with (12); that is, $V(t) = \int_t^\infty e^{-\int_t^{t'} r(i)di}\Pi(t')dt'$. The solution to this optimization problem is described by the following static and dynamic efficiency conditions:

$$p_R(t) = \frac{(1-\Delta)w(t)}{\beta k(t)\theta(t)}, \qquad p_R(t)r(t) - \dot{p}_R(t) = \frac{\partial \pi(t)}{\partial \theta(t)} = \frac{\alpha\gamma(\sigma-1)(Z+Z^*)}{\sigma\theta(t)N(t)}, \tag{13}$$

with $p_R(t)$ capturing the internal price for a mass of new process innovations developed by a firm in home at a given moment in time. Analogous conditions are derived for process innovation undertaken in the foreign country.

2.5 Production Location Patterns

Firms are free to shift production between countries, at no cost, with the aim of maximizing profit on sales (Martin and Rogers, 1995; Martin and Ottaviano, 1999). As such, when there is positive employment in the manufacturing sectors of each country, operating profit on sales is the same for all firms, regardless of where they locate production; that is, $\pi(t) = \pi^*(t)$. We use operating profit on sales (9) and the price indices for home and foreign (3) to solve for the equilibrium share of firms locating production in home as

$$s_X(t) \equiv \frac{n(t)}{N(t)} = \frac{Z - \varphi Z^*}{(1-\varphi)(Z+Z^*)}, \tag{14}$$

with $n(t)$ and $n^*(t)$ measuring the masses of firms with manufacturing located in home and foreign. As national shares of production are fully determined by model parameters, s_X is constant; that is, $\dot{s}_X(t) = 0$. In addition, following the results of Krugman (1980), the manufacturing sector is subject to a home market effect, with the larger home country ($Z > Z^*$) hosting a greater share of production ($s_X(t) > 1/2$). A decrease in the trade cost magnifies the home market effect, generating a positive relationship between $s_X(t)$ and φ (Baldwin et al., 2004).

Substituting national manufacturing shares (14) back into operating profit on sales (9) with the price indices (3), and reorganizing the result, equilibrium firm-level employment in production, regardless of the country of location, is obtained as

$$l_X(t) = l_X^*(t) = \frac{\alpha(\sigma-1)(Z+Z^*)}{\sigma N(t)} + \psi, \tag{15}$$

where we have used the production function (7).

2.6 Innovation Location Patterns

The footloose nature of innovation activity ensures that the internal price of new process innovations equalizes across countries when there is innovation located in both countries; that is, $p_R(t) = p_R^*(t)$. This price condition is used with (11) and (13) to obtain the home share of production that equates innovation costs across countries:

$$s_X(t) = \frac{(1-\Delta)w(t) - \delta(1-\Delta^*)w^*(t)}{(1-\delta)((1-\Delta)w(t) + (1-\Delta^*)w^*(t))}, \tag{16}$$

where $\Delta^* \in (0,1)$ is the R&D subsidy rate provided to firms locating process innovation in foreign. From this expression, it is clear that if $Z \geq Z^*$, we have $s_X \geq 1/2$, and thus $w(t)(1-\Delta) \geq w^*(t)(1-\Delta^*)$, when both countries host process innovation and have positive high-skilled employment levels.

Substituting (16) and (11) back into (13) gives the equilibrium price associated with a mass of process innovations developed in either home or foreign: $p_R(t) = p_R^*(t) = c(t)/\theta(t)$, where

$$c(t) = \frac{(1-\Delta)w(t)}{\beta k} = \frac{(1-\Delta^*)w^*(t)}{\beta k^*} = \frac{(1-\Delta)w(t) + (1-\Delta^*)w^*(t)}{\beta(1+\delta)}, \tag{17}$$

represents the national cost component of process innovation, and is equalized across countries. Given the high-skilled wages of each country, an improvement in the degree of knowledge diffusion or an increase in the national R&D subsidy of either country results in lower innovation costs. Using (13) and (17), we find that the firm-level cost of high-skilled employment in innovation is common across countries:

$$(1-\Delta)w(t)h_R(t) = (1-\Delta^*)w^*(t)h_R^*(t) = \frac{\alpha\gamma(\sigma-1)(Z+Z^*)}{\sigma N(t)} - \left(\rho - \frac{\dot{w}(t)}{w(t)}\right)c(t), \quad (18)$$

with $\dot{w}(t)/w(t) = \dot{w}^*(t)/w^*(t)$, given the constant location pattern for production ($\dot{s}_X(t) = 0$).

National shares of aggregate innovation output ($N(t)\dot{\theta}(t)$) are determined proportionately with high-skilled labor supplies. Denoting the masses of firms locating process innovation in home and foreign by $m(t)$ and $m^*(t)$, at each moment in time the national innovation outputs of home and foreign are $m(t)\dot{\theta}(t)$ and $m^*(t)\dot{\theta}(t)$. Then, combining (10) and (17) yields the share of innovation output produced by the home country as

$$s_R(t) \equiv \frac{m(t)\dot{\theta}(t)}{m(t)\dot{\theta}(t) + m^*(t)\dot{\theta}(t)} = \frac{(1-\Delta)w(t)H(t)}{(1-\Delta)w(t)H(t) + (1-\Delta^*)w^*(t)H^*(t)}. \quad (19)$$

This expression shows that national shares of innovation output depend on R&D subsidies and high-skilled employment levels.

It emerges from a study of national labor market dynamics that innovation location patterns jump immediately to equilibrium through adjustments in high-skilled wages, for a given level of market entry (N), when regional innovation costs equalize across countries ($c(t) = c^*(t)$). This intermediate result is outlined in the following lemma.

Lemma 1 *With the free movement of process innovation between countries, the high-skilled wage rates ($w(t)$ and $w^*(t)$) jump immediately to their steady-state values for a given level of market entry (N), generating an equilibrium location pattern for process innovation ($s_R(t)$).*
Proof: See Appendix A.

Our analysis proceeds using the equilibrium values for the high-skilled wage rates with $\dot{w}(t) = \dot{w}^*(t) = 0$ and the home country share of process innovation $\dot{s}_R(t) = 0$.

2.7 Government

The government of each country maintains a balanced fiscal budget as it covers the cost of subsidizing domestic investment in process innovation through a lump-sum tax ($T(t)$) on household income. Aggregating across the households residing in home and foreign, we have

$$\Delta w(t)H(t) = T(t)Z, \qquad \Delta^* w^*(t)H^*(t) = T^*(t)Z^*, \tag{20}$$

where $w(t)H(t)$ and $w^*(t)H^*(t)$ represent the total values of investment in R&D in each country. Lump-sum taxes are set to balance fiscal budgets at all moments in time, where for simplicity we assume that governments cannot issue bonds.

2.8 Market Entry and Exit

The model is closed by deriving a free market entry and exit condition for the manufacturing sector. Guided by the analysis of Novshek and Sonnenchein (1987), we consider a stable market entry and exit process in which firm value responds correctly to changes in the level of market entry (N) through adjustments in per-period profits. Specifically, in Appendix A we show that $1 > \gamma(\sigma - 1)$ is a sufficient condition for $d\Pi(t)/dN(t) < 0$. As there are no costs incurred in product development, firms enter the market when firm value is positive ($V(t) > 0$), inducing a fall in profit. Similarly, firms exit when firm value is negative ($V(t) < 0$), causing a rise in profit. This process of market entry and exit drives firm value immediately and permanently to zero ($\dot{V}(t) = V(t) = 0$), yielding the following zero profit conditions for the manufacturing sectors of home and foreign ($\Pi(t) = 0$):

$$l_X(t) = (\sigma - 1)(1 - \Delta)w(t)h_R(t) + \sigma\psi = (\sigma - 1)(1 - \Delta^*)w^*(t)h_R^*(t) + \sigma\psi. \tag{21}$$

Therefore, with free market entry and exit, asset wealth is reduced to zero ($A(t)Z + A^*(t)Z^* = V(t)N(t) = 0$), and household expenditure equals total wage income at all moments in time:

$(E(t) + T(t))Z = I(t)Z$ and $(E^*(t) + T^*(t))Z^* = I^*(t)Z^*$.

3 Long-run Equilibrium

This section characterizes long-run innovation location patterns and productivity growth. Specifically, we study steady-state equilibria with constant intersectoral labor allocations; that is, equilibria for which the high-skilled wage rates are constant: $\dot{w} = \dot{w}^* = 0$. Hereafter, we suppress the time script (t) to simplify notation.

3.1 National Labor Allocations

Following the frameworks of Davis (2013) and Davis and Hashimoto (2023), national labor allocations are obtained through two implicit conditions for the high-skilled wage rates: a *share locus* and an *investment locus*.

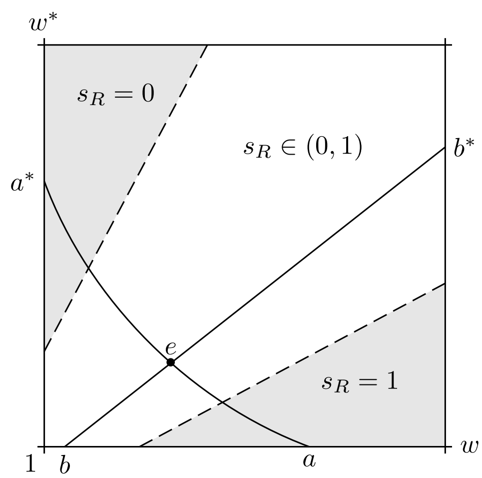
First, the share locus is obtained by matching the home country share of production (s_X) required to equate operating profits (14) and process innovation costs (16) across countries:

$$\frac{w^*}{w} = \left(\frac{1-\Delta}{1-\Delta^*}\right)\left(\frac{(\delta-\varphi)Z + (1-\delta\varphi)Z^*}{(1-\delta\varphi)Z + (\delta-\varphi)Z^*}\right). \tag{22}$$

National production shares are fully determined by exogenous population sizes (Z and Z^*) and trade costs (φ) through a standard home market effect (Krugman 1980). As such, observing the strength of knowledge spillovers (k and k^*) and the R&D subsidy rates (Δ and Δ^*) associated with each country, firms shift process innovation between locations until the national component of innovation costs is equalized across countries ($c = c^*$). The relative high-skilled employment levels that are consistent with equalized innovation costs are described by the high-skilled wage ratio (w^*/w) that arises along the share locus. As depicted by the bb^* line in Figure 1, the share locus has a strictly positive slope.

Second, the investment locus is derived by matching the world supply of low-skilled labor with the total demands from traditional production and manufacturing: $L + L^* = L_Y + L_Y^* + Nl_X$.

Figure 1: Equilibrium Labor Allocations



Substituting $(E+T)Z = L + wH$ and $(E^*+T^*)Z^* = L^* + w^*H^*$ with (6), (15), (13), (20), and (21) into the low-skilled labor market clearing condition yields

$$(1-\Delta)wH + (1-\Delta^*)w^*H^* = \frac{\alpha(\gamma(\sigma-1)\psi - \rho c)(Z+Z^*)}{\sigma(\psi - \rho c)}. \tag{23}$$

This expression equates the industry-level cost of investment in innovation on the lefthand side with industry-level operating profit ($N\pi$) on the righthand side, and is thus an aggregate free market entry condition for the manufacturing sector. As illustrated by the aa^* curve in Figure 1, the investment locus has a strictly negative slope, indicating that home and foreign are substitutes in the firm-level location decision for process innovation (see Appendix B).

A long-run equilibrium with positive levels of high-skilled employment in both home and foreign is shown by the intersection of the investment locus and the share locus at point e in Figure 1. The dashed lines delineate the range of high-skilled wages over which innovation costs equalize across countries, and both home and foreign have positive shares of production and innovation. In the shaded areas, the balance of knowledge spillovers and national R&D subsidies ensures that a single country hosts all production and innovation due to strictly lower innovation costs: production and innovation concentrate fully in home in the bottom-right shaded area, and concentrate fully in foreign in the top-left shaded area. Hence, from (16), an interior solution, with positive levels of high-skilled employment in both countries ($s_R \in (0,1)$), requires that the share locus lie within the following range: $\delta(1-\Delta)/(1-\Delta^*) \leq w^*/w \leq (1-\Delta)/(\delta(1-\Delta^*))$.

3.2 Market Entry and Long-run Productivity Growth

With national labor allocations determined by the investment locus and the share locus, we now derive the long-run level of market entry and the productivity growth rate. Beginning with the level of market entry, we use (15), (13), and (21) to obtain

$$N = \frac{\alpha(1-\gamma(\sigma-1))(Z+Z^*)}{\sigma(\psi-\rho c)}. \tag{24}$$

Because $1 > \gamma(\sigma-1)$ is a sufficient condition for stable market entry and exit ($\partial\Pi/\partial N < 0$), we find that $\psi > \rho c$ is required for a positive measure of manufacturing firms. In addition, the level of market entry adjusts proportionately with changes in the overall size of the manufacturing industry, as measured by $\alpha(Z+Z^*)$. Importantly, an increase in the cost of process innovation (c) has a positive effect on market entry ($dN/dc = \rho N/(\psi-\rho c) > 0$). Intuitively, a rise in c causes firms to reduce employment in process innovation, thereby lowering per-period high-skilled labor costs ($(1-\Delta)wh_R$), expanding per-period profit ($\partial\Pi/\partial c > 0$), and inducing new firms to enter the market.

Turning to the long-run rate of productivity growth, we combine (10), (13), and (24) to derive

$$g \equiv \frac{\dot{\theta}}{\theta} = \frac{\alpha\gamma(\sigma-1)(Z+Z^*)}{\sigma c N} - \rho = \frac{\gamma(\sigma-1)\psi - \rho c}{(1-\gamma(\sigma-1))c}. \tag{25}$$

Thus, we find that $\gamma(\sigma-1)\psi > \rho c$ is required for a positive level of high-skilled employment (23) and a positive productivity growth rate (25). The long-run rate of productivity growth is not biased by a scale effect, as changes in the overall size of the population ($Z+Z^*$) are fully absorbed by adjustments in market entry (N). Naturally, an increase in the innovation cost (c) has a negative effect on firm-level investment in process innovation, lowering the rate of productivity growth ($dg/dc = -(g+\rho)\psi/((\psi-\rho c)c) < 0$).

3.3 National Welfare

National welfare is measured using the average steady-state utility of a household. Together (1), (2), (3), (8), (10), (5), (14), (20), and (25) yield average household utility for the residents of home and foreign as follows:

$$U(0) = B + \frac{\alpha}{\rho(\sigma-1)} \ln \frac{(1+\varphi)Z}{(Z+Z^*)} N + \frac{\alpha\gamma g}{\rho^2} + \frac{L + (1-\Delta)wH}{\rho Z}, \tag{26}$$

$$U^*(0) = B + \frac{\alpha}{\rho(\sigma-1)} \ln \frac{(1+\varphi)Z^*}{(Z+Z^*)} N + \frac{\alpha\gamma g}{\rho^2} + \frac{L^* + (1-\Delta^*)w^*H^*}{\rho Z^*}, \tag{27}$$

where $B = -\alpha/\rho(1 - \ln(\alpha(\sigma-1)/\sigma)$, and we have normalized initial firm-level productivity to one: $\theta(0) = \theta^*(0) = 1$. In our framework, welfare is linked with R&D location patterns through three channels: the level of market entry (N), the rate of productivity growth (g), and per-period expenditure, which equals after-tax labor income ($E = L + (1-\Delta)wH$).

4 Policy Analysis

This section considers the effects of economic policy on high-skilled employment, market entry, productivity growth, and national welfare levels. We focus on trade costs (φ), the degree of knowledge diffusion (δ), and R&D subsidies in the home country (Δ).

4.1 Benchmark Parameter Values

Our policy analysis includes numerical evaluations of how national welfare levels respond to adjustments in policy variables. Here we introduce the benchmark parameter set adopted for the numerical analysis. Beginning with the demand side parameters, the discount rate is set to $\rho = 0.02$ following Jones et al. (1993). We assume a value of $\sigma = 4$ for the elasticity of substitution between product varieties, generating a price-cost markup of $\sigma/(\sigma-1) = 1.33$, which is within the range of estimates reported by De Loecker et al. (2020). Fixed household expenditure on manufacturing goods is set to $\alpha = 1.2$. On the supply side, we assume that the productivity elasticity of output is $\gamma = 0.15$, the fixed operating cost is $\psi = 0.16$, and the innovation cost parameter is $\beta = 0.7$. We fix the trade cost to $\tau = 1.7$, following Anderson and van Wincoop (2004) and Novy (2013), generating a value of $\varphi = 0.2$ for the freeness of trade. The degree of knowledge diffusion is set to $\delta = 0.15$ to match the mid-range estimates of Bloom et al. (2013). The populations of home and foreign are $Z = 10.5$ and $Z^* = 10$, and the benchmark values for the research subsidies of home and foreign are $\Delta = \Delta^* = 0.15$. Under the benchmark parameter set the home share of production is $s_X = 0.52$, and home and foreign high-skilled wages are $w = 1.11$ and $w^* = 1.06$. The innovation cost, the level of market entry, and the rate of productivity growth are $c = 2.29$, $N = 29.61$, and $g = 0.21$. Lastly, national welfare levels are $U = 50.56$ and $U^* = 50.75$.

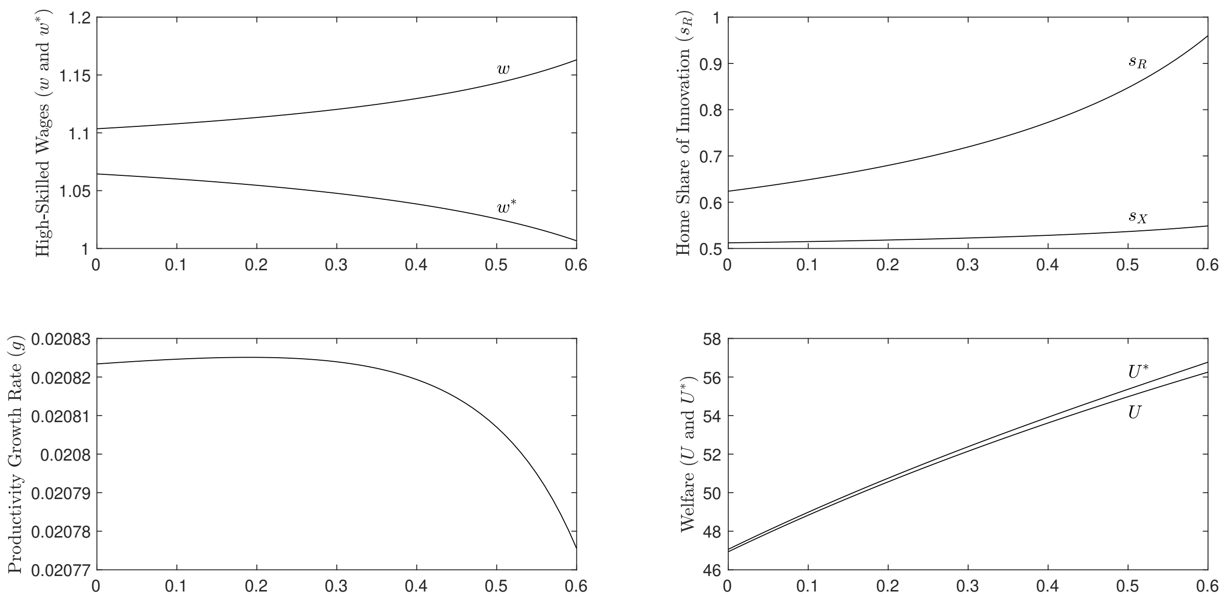
4.2 Trade Costs

Beginning with the impacts of trade integration, we investigate how a change in the cost of trade affects productivity growth, and obtain the following proposition.

Proposition 1 *An increase in the freeness of trade (φ) raises the larger home country's share of R&D, expanding high-skilled employment in the home country and contracting high-skilled employment in the smaller foreign country. Productivity growth (g) is concave in φ with a maximum occurring at $\varphi = \bar{\varphi}$, where $\bar{\varphi}$ generates $IZ/w = I^*Z^*/w^*$.*
Proof: See Appendix C.

Referencing Figure 1, a fall in trade costs (τ) magnifies the home market effect, causing the share locus (22) to shift rightwards as a greater share of firms (14) choose to locate production in the home country ($ds_X/d\varphi > 0$). Knowledge spillovers (11) from production into innovation then strengthen in home and weaken in foreign, inducing firms to relocate process innovation in home ($ds_R/d\varphi > 0$). As such, the economy moves rightwards along the investment locus (23), with high-skilled employment expanding in home ($dw/d\varphi > 0$) and contracting in foreign ($dw^*/d\varphi < 0$). With the high-skilled wage rising in home and falling in foreign,

Figure 2: Freeness of Trade (φ)



These figures are produced using the following parameter set: α = 1.2 β = 0.7, δ = 0.15, γ = 0.15, φ = 0.2, ψ = 0.16, ρ = 0.02, σ = 4, Z = 10.5, Z^* = 10, Δ = 0.15, and Δ^* = 0.15. This parameter set yields $w = 1.11$, $w^* = 1.06$, $s_X = 0.52$, $c = 2.29$, $N = 29.61$, $g = 0.021$, $U = 50.56$, and $U^* = 50.75$.

the effect of improved trade integration and the cost of innovation depends on the tension between the rates of change in the high-skilled wage $(1/w)(dw/d\varphi)$ and the regional knowledge spillover $(1/k)(dk/d\varphi)$. Focusing on the home country, for $(1/c)(dc/d\varphi) = (1/w)(dw/d\varphi) - (1/k)(dk/d\varphi) < 0$, knowledge spillovers increase at a faster rate than high-skilled wages and innovation costs fall. Alternatively, for $(1/c)(dc/d\varphi) = (1/w)(dw/d\varphi) - (1/k)(dk/d\varphi) > 0$, knowledge spillovers increase at a slower rate than high-skilled wages and innovation costs rise. In Appendix C, we show that relative adjustments in high-skilled wages and knowledge spillovers generate a convex relationship between trade integration and innovation costs (c) with $\varphi = \bar{\varphi}$ generating a minimum at $\bar{c}$, where the respective increases and decreases in home and foreign high-skilled wages balance ($IZ/w = I^*Z^*/w^*$). Because trade costs only affect market entry (N) and productivity growth (g) through adjustments in innovation costs, improved trade integration reduces N and increases g for $\varphi < \bar{\varphi}$, but expands N and decreases g for $\varphi > \bar{\varphi}$.

The implications of trade integration for national welfare can be examined by taking the total derivatives of (26) and (27) with respect to the freeness of trade (φ):

$$\frac{dU}{d\varphi} = \frac{\alpha}{\rho(\sigma-1)(1+\varphi)} - \left(\frac{1}{(\sigma-1)(\psi-\rho c)} + \frac{\gamma}{\rho c}\right)\frac{\alpha g}{\rho}\frac{dc}{d\varphi} + \frac{(1-\Delta)wH - \Delta L}{\rho Z w}\frac{dw}{d\varphi},$$
$$\frac{dU^*}{d\varphi} = \frac{\alpha}{\rho(\sigma-1)(1+\varphi)} - \left(\frac{1}{(\sigma-1)(\psi-\rho c)} + \frac{\gamma}{\rho c}\right)\frac{\alpha g}{\rho}\frac{dc}{d\varphi} + \frac{(1-\Delta^*)w^*H^* - \Delta^* L^*}{\rho Z^* w^*}\frac{dw^*}{d\varphi}.$$

As shown by the first term on the righthand side, households benefit from lower prices on imported product varieties when trade costs fall. The second term describes the combined market entry and productivity growth channels. Although these two channels are generally in opposition, we find that the productivity growth channel always dominates, with a positive sign for the second term when innovation costs fall ($dc/d\varphi < 0$) and the rate of productivity growth increases (i.e., when $\varphi < \bar{\varphi}$), and a negative sign when innovation costs rise ($dc/d\varphi > 0$) and the rate of productivity growth decreases (i.e., when $\varphi > \bar{\varphi}$). The third term expresses the adjustment in after-tax labor income, and may be positive or negative depending on the size of the R&D subsidy. Notwithstanding that the balance of the effects outlined above is generally ambiguous, we can identify a positive impact of trade integration on national welfare when trade costs are relatively high ($\varphi < \bar{\varphi}$), and the home R&D subsidy is sufficiently small while the foreign subsidy is sufficiently large. Naturally, as changes in trade costs have no effect on high-skilled employment when the home and foreign populations are equal ($Z = Z^*$), we have $dU/d\varphi = dU^*/d\varphi = 1/(1+\varphi) > 0$. As such, greater trade integration is beneficial for both countries in the symmetric case.

A numerical evaluation of the effects of trade integration on high-skilled wages, R&D location patterns, productivity growth, and national welfare levels is presented in Figure 2. Specifically, we consider an increase in the freeness of trade over the range $\varphi \in (0, 0.6)$, for which an interior solution exists with positive high-skilled employment in both countries. The plots show that high-skilled wages rise in home and fall in foreign with a decrease in trade costs, following the results of Proposition 1. Accordingly, the home country's share of R&D increases. The threshold value $\bar{\varphi} = 0.19$ for the freeness of trade minimizes the cost of innovation (c) and the level of market entry (N) and maximizes the rate of productivity growth (g). The plots illustrate that home welfare (U) and foreign welfare (U^*) are both increasing in the freeness of trade for the benchmark parameter set.

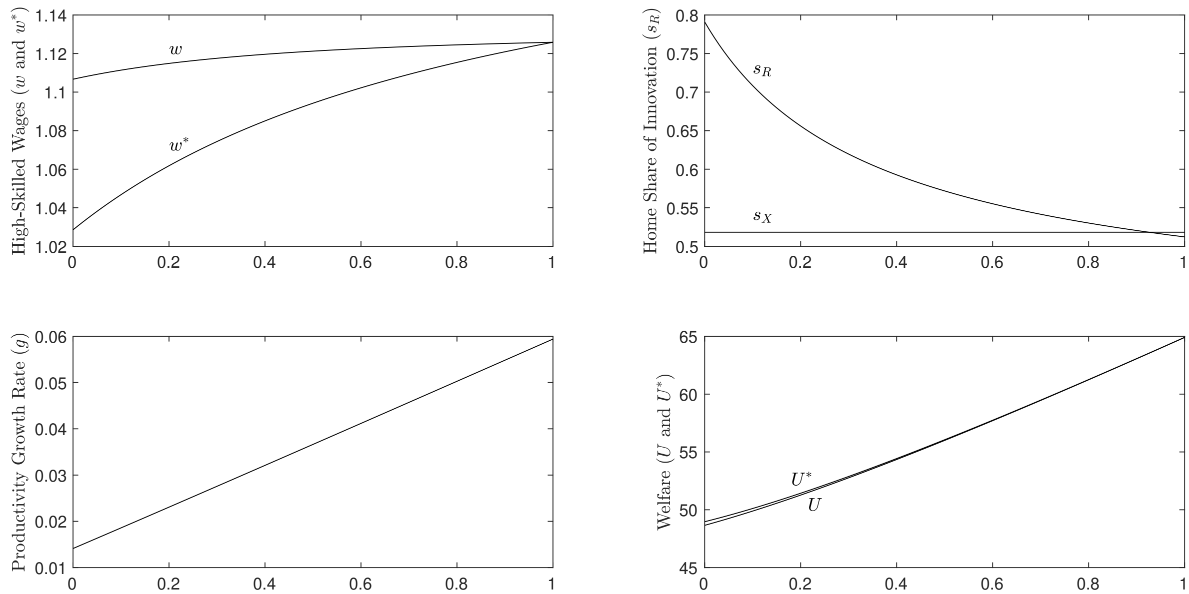
4.3 Knowledge Diffusion

Next, we consider the effects of an improvement in the degree of knowledge diffusion. The results are summarized in the following proposition:

Proposition 2 *An increase in the degree of knowledge diffusion (δ) has an ambiguous effect on the geographic location of innovation, as high-skilled labor may rise or fall in the larger home country, while high-skilled employment expands in the smaller foreign country. The rate of productivity growth increases.*
Proof: See Appendix D.

An increase in the strength of international knowledge spillovers influences the location of R&D activity through two channels. First, the direct effect of lower innovation costs ($\partial c/\partial\delta = -c/(1+\delta) < 0$) is an increase in the return to investment in process innovation that shifts the investment locus upwards in Figure 1. This investment channel places upward pressure on high-skilled employment in both countries. The second channel is a relative knowledge spillover effect. Because foreign has a smaller share of industry ($s_X > 1/2$), an increase in δ generates a larger improvement in knowledge spillovers for R&D located in foreign than in home ($\partial k^*/\partial\delta > \partial k/\partial\delta$). This relative knowledge spillover effect leads firms to relocate process innovation, putting downward pressure on high-skilled employment in home and upward pressure on high-skilled employment in foreign. The investment and knowledge spillover channels align for foreign, causing an expansion in high-skilled employment ($dw^*/d\delta > 0$). In contrast, for home the opposing directions of the two channels imply that high-skilled employment may rise or fall in response to an improvement in knowledge diffusion. Given the ambiguous effect on high-skilled employment, the home country's share of R&D (s_R) may rise or fall in response to

Figure 3: Degree of Knowledge Diffusion (δ)



These figures are produced using the following parameter set: α = 1.2 β = 0.7, δ = 0.15, γ = 0.15, φ = 0.2, ψ = 0.16, ρ = 0.02, σ = 4, Z = 10.5, Z^* = 10, Δ = 0.15, and Δ^* = 0.15. This parameter set yields $w = 1.11$, $w^* = 1.06$, $s_X = 0.52$, $c = 2.29$, $N = 29.61$, $g = 0.021$, $U = 50.56$, and $U^* = 50.75$.

the increase in the degree of knowledge diffusion. Overall, however, we find that the cost of innovation decreases ($dc/d\delta < 0$), ensuring that the level of market entry falls ($(dN/dc)(dc/d\delta) < 0$) and the rate of productivity growth accelerates ($(dg/dc)(dc/d\delta) > 0$).

We take the total derivatives of (26) and (27) with respect to δ to address the effects of changes in an increase in the degree of knowledge diffusion on national welfare:

$$\frac{dU}{d\delta} = -\left(\frac{1}{(\sigma-1)(\psi-\rho c)} + \frac{\gamma}{\rho c}\right)\frac{\alpha g}{\rho}\frac{dc}{d\delta} + \frac{(1-\Delta)wH - \Delta L}{\rho Z w}\frac{dw}{d\delta},$$
$$\frac{dU^*}{d\delta} = -\left(\frac{1}{(\sigma-1)(\psi-\rho c)} + \frac{\gamma}{\rho c}\right)\frac{\alpha g}{\rho}\frac{dc}{d\delta} + \frac{(1-\Delta^*)w^*H^* - \Delta^* L^*}{\rho Z^* w^*}\frac{dw^*}{d\delta}.$$

The first term is the positive balance of the market entry and productivity growth channels, showing that the fall in innovation costs associated with an improvement in knowledge spillovers has a positive affect on household welfare through an increase in the rate of productivity growth. The second term captures the adjustment in after-tax labor income, and may be positive or negative depending on the size of the R&D subsidy. In the symmetric case with equal population sizes ($Z = Z^*$), there is no relative knowledge spillover effect and the upward shift in the investment locus causes high-skilled employment to expand in both home and foreign. Then, for sufficiently low values for the home and foreign R&D subsidies, the adjustment in after-tax income is positive and the increase in the degree of knowledge diffusion generates welfare improvements for both countries ($dU/d\delta > 0$ and $dU^*/d\delta > 0$).

Figure 3 provides a numerical evaluation of the effects of an increase in the degree of knowledge diffusion on high-skilled wages, R&D location patterns, productivity growth, and national welfare levels over the range $\delta \in (0, 1)$. For the benchmark parameter set, a strengthening of international knowledge spillovers raises the high-skilled wages of home and foreign, with w and w^* eventually converging as national levels of high-skilled employment equalize when there is perfect knowledge diffusion across countries. The home country's share of innovation falls with an increase in knowledge diffusion, but does not converge to $s_R = 0.5$, as the home country always maintains a knowledge spillover advantage due to its larger share of production ($s_X > 1/2$). The decrease in innovation costs lowers the level of market entry and raises the rate of productivity growth following the results of Proposition 2. And, the plots indicate that home welfare (U) and foreign welfare (U^*) are increasing in the degree of knowledge diffusion for the benchmark parameter set.

4.4 R&D Subsidies

We now turn to a study of how national R&D subsidies affect innovation location patterns and productivity growth. The results are summarized in the following proposition.

Proposition 3 *An increase in the R&D subsidy rate raises the R&D share of the implementing country, expanding its high-skilled employment, while contracting high-skilled employment in the non-implementing country. The rate of productivity growth increases.*
Proof: See Appendix E.

The intuition behind the results summarized in Proposition 3 can be understood by considering, for example, an increase in the R&D subsidy of the home. Returning to Figure 1, the share locus shifts rightwards, as a measure of firms relocated process innovation from foreign to home to take advantage of lower R&D costs ($ds_R/d\Delta > 0$). Accordingly, high-skilled employment expands in home and contracts in foreign, causing the relative high-skilled wage rate (w/w^*) to rise until innovation costs are again equalized across countries. In addition, the increase in the R&D subsidy rate expands aggregate investment in innovation, causing the investment locus to shift rightwards, putting upward pressure on high-skilled employment in both countries. In the home country, the shifts in the share locus and the investment locus lead to greater high-skilled employment and investment in process innovation ($dw/d\Delta > 0$). In the foreign country, however, the shift in the share locus tends to reduce high-skilled employment while the shift in the investment locus tends to expand high-skilled employment. In Appendix E, we show that the effect of the shift in the share locus always dominates, and high-skilled employment in the foreign country necessary falls ($dw^*/d\Delta < 0$).

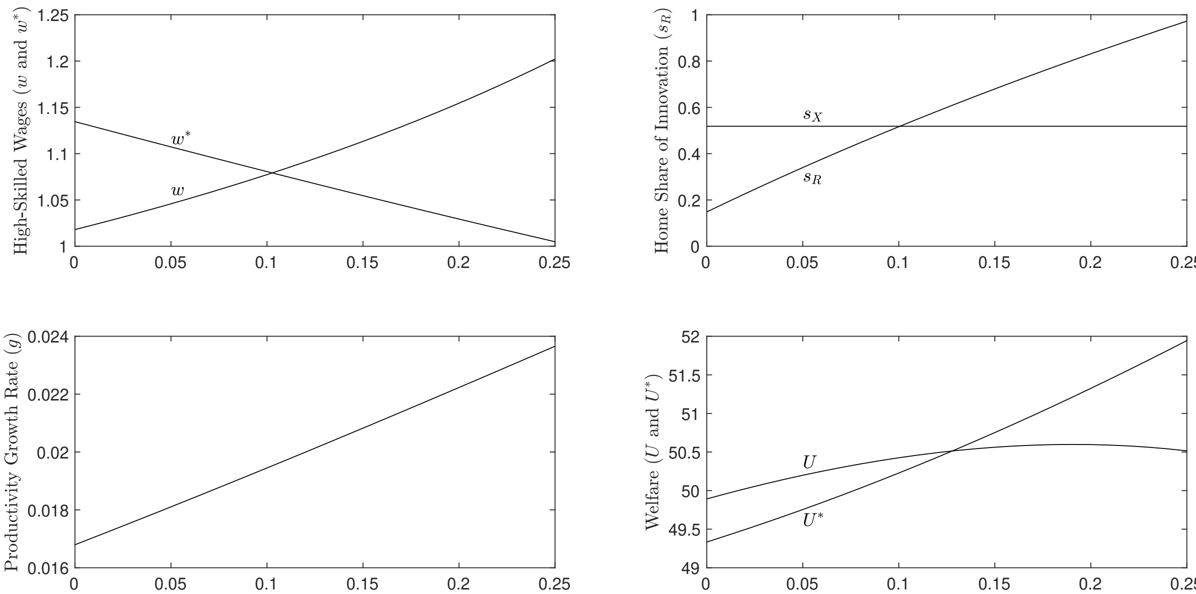
Turning to the implications of R&D subsidy policy for productivity growth, the increase in the R&D subsidy directly lowers the cost of innovation, while the rise in the home high-skilled wage tends to increase the innovation cost, and the fall in the foreign high-skilled wage tends to decrease it. In Appendix E, we show that the overall effect of an increase in the R&D subsidy rate of either country is to lower the cost of innovation for all firms, regardless of where they locate process innovation. Accordingly, the level market entry falls ($dN/d\Delta = (dN/dc)(dc/d\Delta) < 0$), and the long-run rate of productivity growth rises ($dg/d\Delta = (dg/dc)(dc/d\Delta) > 0$).

We observe the welfare effects of an increase in the home R&D subsidy by taking the derivatives of (26) and (27) with respect to Δ:

$$\frac{dU}{d\Delta} = -\left(\frac{1}{(\sigma-1)(\psi-\rho c)} + \frac{\gamma}{\rho c}\right)\frac{\alpha g}{\rho}\frac{dc}{d\Delta} + \frac{(1-\Delta)wH - \Delta L}{\rho Z c}\frac{dc}{d\Delta} - \frac{\Delta L}{(1-\Delta)\rho Z},$$
$$\frac{dU^*}{d\Delta} = -\left(\frac{1}{(\sigma-1)(\psi-\rho c)} + \frac{\gamma}{\rho c}\right)\frac{\alpha g}{\rho}\frac{dc}{d\Delta} + \frac{(1-\Delta^*)w^*H^* - \Delta^* L^*}{\rho Z^* c}\frac{dc}{d\Delta}.$$

The first term once again describes the balance between the market entry and productivity growth

Figure 4: Home R&D Subsidy (Δ)



These figures are produced using the following parameter set: α = 1.2 β = 0.7, δ = 0.15, γ = 0.15, φ = 0.2, ψ = 0.16, ρ = 0.02, σ = 4, Z = 10.5, Z^* = 10, Δ = 0.15, and Δ^* = 0.15. This parameter set yields $w = 1.11$, $w^* = 1.06$, $s_X = 0.52$, $c = 2.29$, $N = 29.61$, $g = 0.021$, $U = 50.56$, and $U^* = 50.75$.

channels and is positive as the R&D subsidy reduces the cost of innovation ($dc/d\Delta < 0$). The second term shows the adjustment in after-tax income, and may be positive or negative depending on the size of the R&D subsidy. The third term in the first expression measures the direct cost to home households of subsidizing the R&D subsidy through a lump-sum tax.

Figure 4 presents a numerical evaluation of the effects of the home R&D subsidy on high-skilled wages, R&D location patterns, productivity growth, and national welfare levels over the range $\Delta \in (0, 0.25)$, for which an interior equilibrium exists with positive high-skilled employment in home and foreign. The home high-skilled wage increases and the foreign high-skilled wage decreases with the home R&D subsidy, following the results of Proposition 3. The home country's share of R&D thus increases across the observed policy range. With the cost of innovation (c) falling, the level of market entry contracts and the rate of productivity growth rises. Over the benchmark parameter set, the increase in the home R&D subsidy generates an improvement in foreign welfare, but yields a concave relationship between home welfare and the home R&D subsidy, with a maximum at the optimal subsidy rate of $\bar{\Delta} = 0.191$.

5 Concluding Remarks

R&D subsidies have become a prominent national policy tool in recent years. In this paper, we develop a two-country framework to investigate how national R&D subsidies affect productivity growth and national welfare through adjustments in the geographic location of R&D. The framework has two key features. Monopolistically competitive firms that invest in process innovation with the aim of reducing future production costs. And, an occupational choice for workers between employment in production and employment in innovation that determines national supplies of low-skilled and high-skilled labor. Our analysis focuses on the tension in the firm-level location decision for innovation between accessing technical knowledge and sourcing low-cost high-skilled labor. Assuming that technical knowledge accumulates within firm-level production technologies, locating innovation in proximity to industry increases knowledge spillovers and raises labor productivity in innovation. The geographic concentration of innovation raises the cost of employing high-skilled labor. however, by increasing high-skilled wages.

Firms locate production and innovation separately in the countries that offer the lowest respective costs. Trade costs ensure that the country with the larger market, as measured by population, hosts a greater share of industry through a standard home market effect. And, limited international knowledge diffusion implies stronger knowledge spillovers from production to innovation in the country with the greatest share of industry. In this setting, a reduction in trade costs expands the larger country's share of industry and innovation, but the impact on the rate of productivity growth depends on whether the benefit of stronger knowledge spillovers or the cost of greater high-skilled wages dominates. An improvement in international knowledge diffusion tends to expand the smaller country's share of innovation, and increases the rate of productivity growth. Investigating the effects of R&D subsidy policy, we find that an R&D subsidy expands the implementing country's share of innovation and raises the productivity growth rate. While the non-implementing country enjoys a welfare improvement, a tension between the welfare benefits of faster productivity growth and higher labor income, and the rising aggregate cost of the R&D subsidy generates a concave relationship between the R&D subsidy and the welfare of the implementing country, yielding an optimal R&D subsidy rate.

Appendix A

In this appendix, we first show that for a given the level of market entry (N), national labor markets jump immediately to equilibrium if process innovation is footloose between countries. Noting that (14) that $\dot{s}_X = 0$ ensures $\dot{w}/w = \dot{w}^*/w^*$, we reorganize (18) to obtain a differential equation for the dynamics of the home country high-skilled wage rate:

$$\dot{w} = \left((1-\Delta)wH + (1-\Delta^*)w^*H^* - \frac{\alpha\gamma(\sigma-1)(Z+Z^*)}{\sigma} + \rho c N \right) \frac{w}{cN}, \tag{A1}$$

where we have used $(1-\Delta)wh_R = (1-\Delta)wH/N + (1-\Delta^*)w^*H^*/N$. Holding market entry ($N$) constant, we evaluate the partial derivative of (A1) with respect to w around the steady-state at $\dot{w} = 0$ to obtain

$$\frac{\partial \dot{w}}{\partial w} = \frac{(1-\Delta)IZ + (1-\Delta^*)I^*Z^*}{cN} + \rho > 0,$$

where $dw^*/dw = w^*/w$ for $c = c^*$, since $w^* = w(1-\Delta)k^*/((1-\Delta)^*k)$. As the high-skilled wage rate is a control variable, we thus find that w and w^* jump immediately and permanently to their steady-state values, for a given level of market entry, as outlined in Lemma 1.

Second, we take use (19) to derive the following differential equation for the home country share of process innovation:

$$\dot{s}_R = s_R(1-s_R)\left(\frac{IZ}{wH} - \frac{I^*Z^*}{w^*H^*}\right)\frac{\dot{w}}{w} = 0,$$

where we have used $\dot{w}/w = \dot{w}^*/w^* = 0$. Accordingly, as high-skilled wage rates jump immediately to their steady values, the home country share also jumps to its equilibrium value (19).

Third, we demonstrate that firm-level profit (Π) is decreasing in market entry (N) for $1 > \gamma(\sigma-1)$. Setting $\dot{w} = 0$ yields the following relationship between the level of market entry and the equilibrium high-skilled wage rate in the national labor market:

$$N = \frac{\alpha\gamma(\sigma-1)(Z+Z^*)}{\sigma\rho c} - \frac{(1-\Delta)wH + (1-\Delta^*)w^*H^*}{\rho c}. \tag{A2}$$

Taking the total derivative (A2) yields $dw/dN = -\rho cw/(\rho N + (1-\Delta)IZ + (1-\Delta^*)) < 0$, where we have used $dw^*/dw = w^*/w$ and $dc/dw = c/w$. Then, combining (9), (12), (15), and (A2) with $(1-\Delta)wh_R = (1-\Delta)wH/N + (1-\Delta^*)w^*H^*/N$, we obtain per-period profit for a given level of market entry as $\Pi = \alpha(1-\gamma(\sigma-1))(Z+Z^*)/(\sigma N) - \psi + \rho c$. Hence, we have

$$\frac{d\Pi}{dN} = -\frac{\alpha(1-\gamma(\sigma-1))(Z+Z^*)}{\sigma N^2} - \frac{(\rho c)^2}{\rho N + (1-\Delta)IZ + (1-\Delta^*)I^*Z^*},$$

and we conclude that $1 > \gamma(\sigma-1)$ is a sufficient condition for $d\Pi/dN < 0$.

Appendix B

Taking the total derivatives of (22) and (23), we obtain the slopes of the share locus and the investment locus respectively as $dw^*/dw = w^*/w > 0$ and

$$\frac{dw^*}{dw} = -\frac{(1-\Delta)\left[\beta(1+\delta)(\psi-pc)IZ + \psi N\rho w\right]w^*}{(1-\Delta^*)\left[\beta(1+\delta)(\psi-pc)I^*Z^* + \psi N\rho w^*\right]w} < 0.$$

Appendix C

Comparative statics for changes in trade costs are evaluated by taking the total derivatives of the investment locus (23) and the share locus (22) with respect to φ:

$$\begin{aligned}\frac{dw}{d\varphi} &= -\left(\frac{I^*Z^*}{w^*} + \frac{\psi N\rho}{(\psi-pc)\beta(1+\delta)}\right)\frac{(1-\Delta)(1-\delta^2)(Z-Z^*)}{(1-\varphi)^2(Z+Z^*)k^2|J|} > 0,\\ \frac{dw^*}{d\varphi} &= \left(\frac{IZ}{w} + \frac{\psi N\rho}{(\psi-pc)\beta(1+\delta)}\right)\frac{(1-\Delta)^2(1-\delta^2)(Z-Z^*)}{(1-\Delta^*)(1-\varphi)^2(Z+Z^*)k^2|J|} < 0,\end{aligned}$$

where $|J| = -(1-\Delta)IZ/w^2 - (1-\Delta^*)I^*Z^*/w^2 - \psi N\rho c/((\psi-\rho c)w^2) < 0$. Together, these comparative statics ensure that an improvement in the freeness of trade increases the home country share of innovation; that is, taking the total derivative of (19) yields:

$$\frac{ds_R}{d\varphi} = s_R(1-s_R)\left(\frac{IZ}{wH}\left(\frac{1}{w}\frac{dw}{d\varphi}\right) - \frac{I^*Z^*}{w^*H^*}\left(\frac{1}{w^*}\frac{dw^*}{d\varphi}\right)\right) > 0.$$

In addition, the effect of a change in the freeness of trade on the innovation cost (c) is

$$\frac{dc}{d\varphi} = \left(\frac{IZ}{w} - \frac{I^*Z^*}{w^*}\right)\frac{(1-\Delta)^2(1-\delta^2)(Z-Z^*)}{(1-\varphi)^2(Z+Z^*)k^2|J|} \gtreqless 0.$$

Examining how the first term in parenthesis adjusts with changes in the freeness of trade, we have $(L^*/w^{*2})(dw^*/d\varphi) - (L/w^2)(dw/d\varphi) < 0$. The innovation cost is therefore convex in φ with a minimum at $IZ/w = I^*Z^*/w^*$.

Appendix D

We obtain comparative statics for a change in the degree of knowledge diffusion using the total derivatives of the investment locus (23) and the share locus (22) with respect to δ:

$$\frac{dw}{d\delta} = \frac{\beta(1+\varphi)(Z-Z^*)I^*Z^*c}{(1-\varphi)(Z+Z^*)ww^*k|J|} - \frac{(1-s_X)\psi N\rho c}{(\psi-\rho c)wk|J|} \gtreqless 0,$$
$$\frac{dw^*}{d\delta} = -\frac{\beta(1+\varphi)(Z-Z^*)IZcw^*}{(1-\varphi)(Z+Z^*)w^3k^*|J|} - \frac{s_X\psi N\rho cw^*}{(\psi-\rho c)w^2k^*|J|} > 0,$$

where $|J| = -(1-\Delta)IZ/w^2 - (1-\Delta^*)I^*Z^*/w^2 - \psi N\rho c/((\psi-\rho c)w^2) < 0$. The total derivative of the home country's share of innovation (19) yields

$$\frac{ds_R}{d\delta} = s_R(1-s_R)\left(\frac{IZ}{wH}\left(\frac{1}{w}\frac{dw}{d\delta}\right) - \frac{I^*Z^*}{w^*H^*}\left(\frac{1}{w^*}\frac{dw^*}{d\delta}\right)\right).$$

Thus, the effect of an increase in δ on s_R is generally ambiguous. However, we find that a rise in the degree of knowledge diffusion leads to a fall in the cost of innovation:

$$\frac{dc}{d\delta} = \left(\frac{s_X^*IZ}{w} + \frac{s_XI^*Z^*}{w^*}\right)\frac{\beta c^2}{w^2|J|} < 0.$$

Appendix E

Comparative statics for changes in R&D subsidies are obtained by taking the total derivatives of the investment locus (23) and the share locus (22) with respect to Δ and Δ^*:

$$\frac{dw}{d\Delta} = -\left(H + \frac{(1-\Delta^*)I^*Z^*}{(1-\Delta)w} + \frac{\psi N\rho c}{(\psi-\rho c)(1-\Delta)w}\right)\frac{1}{|J|} > 0,$$
$$\frac{dw^*}{d\Delta} = \frac{w^*L}{w^2|J|} < 0,$$
$$\frac{dw}{d\Delta^*} = \frac{L^*}{w|J|} < 0,$$
$$\frac{dw^*}{d\Delta^*} = -\left(H^* + \frac{(1-\Delta)IZ}{(1-\Delta^*)w^*} + \frac{\psi N\rho c}{(\psi-\rho c)(1-\Delta^*)w^*}\right)\frac{w^{*2}}{w^2|J|} > 0,$$

where $|J| = -(1-\Delta)IZ/w^2 - (1-\Delta^*)I^*Z^*/w^2 - \psi N\rho c/((\psi-\rho c)w^2) < 0$. Taking the total derivative of (19) with respect to the home R&D subsidy, we have

$$\frac{ds_R}{d\Delta} = -\frac{(1-\Delta^*)L\Omega}{((1-\Delta)wH + (1-\Delta^*)w^*H^*)^2w^2|J|} > 0,$$

where $\Omega \equiv ((1-\Delta)wH + (1-\Delta^*)w^*H^*)I^*Z^* + w^*H^*\psi N\rho c/(\psi-\rho c) > 0$. In addition, we find that an increase in the R&D subsidy of either country lowers innovation costs:

$$\frac{dc}{d\Delta} = \frac{cL}{w^2|J|} < 0, \qquad \frac{dc}{d\Delta^*} = \frac{cL^*}{w^2|J|} < 0.$$

References

[1] Anderson, J., van Wincoop, E. (2004). Trade costs. *Journal of Economic Literature* 42, 691-751.

[2] Baldwin, R., Forslid R. (2000). The core-periphery model and endogenous growth: Stabilizing and destabilizing integration. *Economica* 67, 307-324.

[3] Baldwin, R., Martin, P., Ottaviano G., Robert-Nicoud, F. (2004). *Economic Geography and Public Policy*, Princeton, NJ: Princeton University Press.

[4] Bloom, N., Schankerman, M., van Reenen, J. (2013). Identifying technology spillovers and product market rivalry. *Econometrica* 81, 1347-1393.

[5] Bottazzi, L., Peri, G. (2003). Innovation and spillovers in regions: evidence from European patent data. *European Economic Review* 47, 687-710.

[6] Chung, W., Yeaple, S. (2008). International knowledge sourcing: evidence from U.S. firms expanding abroad. *Strategic Management Journal* 29, 1207-1224.

[7] Currie, D., Levine, P., Pearlman, J., Chui, M. (1999). Phases of imitation and innovation in a North-South endogenous growth model. *Oxford Economic Papers* 51, 60-88.

[8] David, P., Hall, B., Toole, A. (2000) Is public R&D a complement or substitute for private R&D? A review of the econometric evidence. *Research Policy* 29, 497-529.

[9] Davis, C. (2013). Regional integration and innovation offshoring with occupational choice and endogenous growth. *Journal of Economics* 180, 59-79.

[10] Davis, C., Hashimoto, K. (2015). R&D subsidies, international knowledge diffusion, and fully endogenous productivity growth. *Macroeconomic Dynamics* 19, 1816-1838.

[11] Davis, C., Hashimoto, K. (2023). Innovation offshoring and reshoring with fully endogenous growth. *Southern Economic Journal* 90, 90-120.

[12] De Loecker, J., Eeckhout, J., Unger, G. (2020). The rise of market power and the macroeconomic implications. *Quarterly Journal of Economics* 135, 561-644.

[13] Demirbag, M., Glaister, K. (2010). Factors determining offshore location choice for R&D projects: a comparative study of developed and emerging regions. *Journal of Management Studies* 47, 1534-1560.

[14] Desmet, K., Rossi-Hansberg, E. (2009). Spatial growth and industry age. *Journal of Economic Theory* 144, 2477-2502.

[15] Desmet, K., Rossi-Hansberg, E. (2014). Spatial development. *American Economic Review* 104, 1211-1243.

[16] Dixit, A., Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. *American Economic Review* 67, 297-308.

[17] García-Quevedo, J. (2004). Do public subsidies complement business R&D? A meta-analysis of econometric evidence. *Kyklos* 57, 87-102.

[18] Gardiner, B., Martin, R., Tyler, P. (2011). Does spatial agglomeration increase national growth? Some evidence from Europe. *Journal of Economic Geography* 11, 979-1006.

[19] Glass, A., Saggi, K. (2001). Innovation and wage effects of international outsourcing. *European Economic Review* 45, 67-86/

[20] Grossman, G., Helpman, E. (1991). *Innovation and Growth in the Global Economy*. Cambridge: MIT Press.

[21] Hall, B., van Reenen, J. (2000) How effective are fiscal incentives for R&D? A review of the evidence. *Research Policy* 29, 449-469.

[22] Impulliti, G. (2010). International Competition and U.S. R&D Subsidies: A Quantitative Welfare Analysis. *International Economic Review* 51, 1127-1158.

[23] Iwaisako, T., Tanaka, H., Futagami, K. (2011). A welfare analysis of global patent protection in a model with endogenous innovation and foreign direct investment. *European Economic Review* 58, 1137-1151.

[24] Jones, L., Manuelli, R., Rossi, P. (1993). Optimal taxation in models of endogenous growth. *Journal of Political Economy* 101, 485-517.

[25] Kondo, H. (2013). International R&D subsidy competition, industrial agglomeration and growth. *Journal of International Economics* 89, 233-251.

[26] Krugman, P. (1980). Scale economies, product differentiation, and the pattern of trade. *American Economic Review* 70, 950-959.

[27] Lewin, A., Massini, S., Peeters, C. (2009). Why are companies offshoring innovation? The emerging global race for talent. *Journal of International Business Studies* 40, 901-925.

[28] Mancusi, M. (2008). International spillovers and absorptive capacity: a cross-country sector analysis based on patents and citations. *Journal of International Economics* 76, 155-165.

[29] Manning, S., Massini, S., Lewin, A. (2008). A dynamic perspective on next generation offshoring: the global source of science and engineering talent. *Academy of Management Perspectives* 22, 35-54.

[30] Martin, P., Ottaviano, G. (1999). Growing locations: Industry location in a model of endogenous growth. *European Economic Review* 43, 281-302.

[31] Martin, P., Ottaviano, G. (2001). Growth and agglomeration. *International Economic Review* 42, 947-968.

[32] Martin, P., Rogers, C. (1995). Industrial location and public infrastructure. *Journal of International Economics* 39, 335-351.

[33] Novshek, W., Sonnenchein, H. (1987). General equilibrium with free entry: A synthetic approach to the theory of perfect competition. *Journal of Economic Literature* 115, 1281-1306.

[34] Novy, D. (2013). Gravity Redux: Measuring international trade costs with panel data. *Economic Inquiry* 51, 101-121.

[35] OECD (2023), Gross domestic spending on R&D (indicator). Retrieved from doi: 10.1787/d8b068b4-en (Accessed on February 12, 2023).

[36] Peretto, P. (1996). Sunk costs, market structure, and growth. *International Economic Review* 37, 895-923.

[37] Peretto, P. (2018). Robust endogenous growth. *European Economic Review* 108, 49-77.

[38] Rodríguez-Pose, A., Wilkie, C. (2016). Context and the role of policies to attract foreign R&D in Europe. *European Planning Studies* 24, 2014-2035.

[39] Smulders, S., van de Klundert, T. (1995). Imperfect competition, concentration, and growth with firm-specific R&D. *European Economic Review* 39, 139-160.

[40] The Economist. (2022). China and the West are in a race to foster innovation. *The Economist*, October 13.

[41] Thompson, P. (2006). Patent citations and the geography of knowledge spillovers: evidence from investor- and examinar-added citations. *The Review of Economics and Statistics* 88, 383-388.

[42] The World Bank (2023). *Research and development expenditure (% of GDP)*. Retrieved from https://data.worldbank.org/indicator/GB.XPD.RSDV.GD.ZS (Accessed on December 20, 2023).