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INSURANCE AGAINST AGGREGATE SHOCKS

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Insurance against Aggregate Shocks

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Abstract

Although many studies in macroeconomics have examined the role of insurance in the presence of income risk, whether aggregate shocks are insurable has not been sufficiently investigated. We present a simple two-period general equilibrium model to show the conditions under which insurance against aggregate shocks works in an economy with constant-elasticity-substitution (CES) production technology and the Greenwood-Hercowitz-Huffman (GHH) utility function (Greenwood et al.,1988). Our theoretical investigation clarifies that only when agents are heterogeneous in their ability or initial wealth can aggregate shocks be insurable. From our quantitative investigation, we find that (i) agents with lower ability enjoy greater welfare improvement from insurance, and as agents' ability increases, the welfare improvement diminishes, (ii) agents enjoy greater welfare improvement when the damage from disasters is more severe and when the frequency of disasters is greater, and (iii) although the welfare improvement increases as agents' initial wealth increases, the impact of a difference in agents' initial wealth on the difference in the contribution of insurance is very moderate.

Keywords: aggregate shocks, heterogeneous agents, state-contingent claims, incomplete market.

JEL Classification Numbers: D52, G12

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1 Introduction

Can aggregate shocks be insured against? This question is crucial for humankind since the COVID-19 crisis. The COVID-19 pandemic caused worldwide lockdowns and disrupted daily life in societies. As a consequence, it damaged the world economy almost simultaneously.[1] We can say that the COVID-19 pandemic was an aggregate shock to the world. Although many studies have examined the role of insurance in the presence of income risk, whether aggregate shocks are insurable has not been sufficiently investigated. In particular, how much insurance against aggregate shocks can contribute to welfare improvement has not been studied thus far in the literature. This paper is the first attempt to fill this gap.

One can classify stochastic shocks into idiosyncratic shocks and aggregate shocks. Idiosyncratic shocks affect each agent randomly and independently. Therefore, an insurance contract for idiosyncratic shocks works well and increases agents' expected utility by smoothing consumption between stochastic states so that agents who received ex-post high income can finance agents who received ex-post low income. In contrast, aggregate shocks are unlikely to be insurable because when such shocks affect all agents in the same direction, no one seems to offer or accept an insurance contract. As such, most studies that analyzed the role of insurance in the context of macroeconomics have focused mainly on insurance against idiosyncratic shocks. Cochrane (1991) and Mace (1991) argued that if there is a complete insurance market for idiosyncratic shocks, each agent can achieve full risk sharing for the idiosyncratic shocks facing them. Then, individual consumption will no longer respond to idiosyncratic shocks. However, individual consumption will still move in the same direction in response to aggregate shocks since there is no way to diversify risks among agents.

The above arguments hold if agents in an economy are homogeneous. If agents are heterogeneous, aggregate shocks may have idiosyncratic effects on agent behavior, such as the endogenous labor supply, in response to the shocks. In this case, insurance may be effective against aggregate shocks. The objectives of this paper are to show the conditions under which insurance against aggregate shocks is effective in general equilibrium in an economy with a constant-elasticity-substitution (CES) production function and the Greenwood-Hercowitz-Huffman (GHH) utility function (Greenwood et al., 1988) and to quantitatively investigate how much insurance against aggregate shocks improves economic welfare (the insurance contribution). Although we consider a simple two-period general equilibrium model, we

[1] Barrett et al. (2021) reported the significant impacts of the COVID-19 pandemic on per capita GDP and total factor productivity (TFP) worldwide. Bloom et al. (2023) reported a decrease in TFP of 5% in 2020-21 in the United Kingdom.

obtain rich results both theoretically and quantitatively.

From our theoretical investigation, we clarify that only when agents are heterogeneous in their ability and initial wealth can aggregate shocks be insurable. Intuitively, the variation in their ability and initial wealth generates the variation in their total income, which causes idiosyncratic effects of aggregate shocks. More concretely, although all agents face the same wage rate, wage income is determined by the product of agent ability and the wage rate. While aggregate shocks affect the wage rate, a difference in agents' ability magnifies a difference in their wage income. Furthermore, whereas aggregate shocks affect the interest rate that all agents face, capital income is the product of the interest rate and initial wealth. Then, a difference in agents' initial wealth magnifies a difference in their capital income. These idiosyncratic effects of aggregate shocks make the marginal rate of substitution of consumption in the disaster state for consumption in the normal state uneven across agents. Then, the Pareto optimal equilibrium is unachievable in an economy without insurance. In other words, incomplete markets endogenously occur.

Our quantitative analyses also indicate significant results. First, by comparing indirect utility with insurance to that without insurance, we find that other things being equal, agents with lower ability enjoy greater welfare improvement from insurance, and as agents' ability increases, the welfare improvement is diminished. Second, agents enjoy greater welfare improvement when the damage from disasters (aggregate shocks) is more severe and when the frequency of disasters is greater. Third, although the welfare improvement increases as agents' initial wealth increases, the impact of a difference in agents' initial wealth on the difference in the contribution of insurance is very moderate.

Several researchers have examined the relation between insurance and aggregate shocks in the literature on general equilibrium theory. The most relevant study is by Krueger and Lustig (2010). They clarified the conditions under which, no matter whether an insurance market for idiosyncratic shocks exists, a no-trade equilibrium in the insurance market for aggregate shocks occurs, and the macroeconomic implications remain unchanged in an economy where agents face both idiosyncratic and aggregate shocks. In other words, they identified conditions that allow for the correct analysis of macro dynamics at the aggregate level under the assumption of a representative household, even in the presence of incompleteness in insurance markets for individual shocks.[2] These conditions can be summarized as follows: (1) the instantaneous utility function is homothetic, (2) idiosyncratic and aggregate shocks are independent, and (3) the capital income share is constant. Mailer and Mailar (2001) and

[2]Their result can be regarded as an extended version of the aggregation theorems that allow for the use of representative agent models, in line with the works of Gorman (1953, 1961) and Rubinstein (1974).

Mailer et al. (2005) demonstrated that one could analyze the dynamics of income and wealth distribution in an economy inhabited by a representative household by adopting settings that satisfy all the above conditions. The model presented by Clemens et al. (2020), an extension of the Mailer et al. (2005) model, explicitly introduced insurance against aggregate shocks, but Clemens et al. (2023), which is the published version of Clemens et al. (2020), discarded such insurance because the presence or absence of insurance against aggregate shocks does not affect the equilibrium allocation in their model because it meets all three conditions.

What is commonly shared in these studies is that limited attention has been given to the completeness or incompleteness of insurance markets against aggregate shocks. However, is insurance against aggregate shocks truly insignificant in macroeconomics? Again, this question is crucial for us because we all know the consequences of the COVID-19 crisis that negatively impacted almost the whole world. Since it is already known that insurance against aggregate shocks is not significant when the three conditions derived by Krueger and Lustig (2010) hold, we need to relax some of these conditions to investigate the role of insurance against aggregate shocks. To minimize deviations from previous research, we relax only condition (3). As previously described, only when agents are heterogeneous in their ability and initial wealth are aggregate shocks insurable. Although this theoretical finding is one of the contributions of our study, no-trade equilibrium in the insurance market for aggregate shocks occurs if the Cobb-Douglas production technology is employed in our model. This is because the Cobb-Douglas production technology satisfies Krueger and Lustig's third condition. We instead adopt a CES production technology. Our investigation is the first in the literature in that we address the role of insurance markets for aggregate shocks.

The remainder of this paper is organized as follows. In the next section, we present a two-period general equilibrium model in which state-contingent claims for aggregate shocks are introduced. Section 3 derives equilibrium, and section 4 explores whether insurance against aggregate shocks achieves the Pareto optimum and clarifies the mechanism that induces an incomplete market for aggregate shocks although they affect all agents in the same direction. In section 5, we perform quantitative analyses. Section 6 concludes this paper.

2 Model

Consider a two-period economy inhabited by a continuum of agents whose population is normalized to 1. Each agent is born with a certain endowment (initial wealth) in the first period, which varies between agents. The heterogeneity of agents originates not only from individual-specific endowments but also from their labor productivity. In the first period, they invest in real assets and/or purchase or issue state-contingent claims. In the second period, they acquire a return from real assets, clear state-contingent claims, and consume all the income.

2.1 Production

A representative firm produces final goods from capital and effective labor with a constant-elasticity-substitution (CES) production function:

$$\theta_i f(k,h) := \theta_i \left[bk^\sigma + (1-b)h^\sigma\right]^{\frac{1}{\sigma}}, \tag{1}$$

where k is aggregate capital, h is aggregate effective labor, and $b \in (0,1)$ and $\sigma \in (-\infty, 1)$ are parameters, where b becomes a capital share of the total output when the production function reduces to the Cobb-Dauglas production function as $\sigma \to 0$. $1/(1-\sigma)$ is the elasticity of substitution between capital and effective labor. θ_i $(i = 1,2)$ is the total factor productivity (TFP), which is a random variable such that $0 < \theta_1 < \theta_2$, being realized with probability $\pi_i \in (0,1)$ where $\pi_1 + \pi_2 = 1$. One may imagine that a disaster occurs when a low-productivity shock, θ_1, is realized.

Capital and labor are paid their marginal products because the markets of production factors are competitive. Then, we have

$$w(\theta_i) := \theta_i (1-b)\left[bk^\sigma + (1-b)h^\sigma\right]^{\frac{1-\sigma}{\sigma}} h^{\sigma-1} \tag{2}$$

and

$$r(\theta_i) := \theta_i b\left[bk^\sigma + (1-b)h^\sigma\right]^{\frac{1-\sigma}{\sigma}} k^{\sigma-1}, \tag{3}$$

where $w(\theta_i)$ and $r(\theta_i)$ are the wage and interest rates, respectively, when θ_i is realized.

2.2 Agents

Consider an agent, say, agent $j \in \Omega$ where Ω is the whole set of agents, who is endowed with the GHH type of utility function. Agent j maximizes the expected utility as follows.

$$\max \sum_{i=1}^{2} \pi_i \frac{x_j(\theta_i)^{1-\gamma} - 1}{1-\gamma} \tag{4}$$

subject to

$$a_j + \sum_{i=1}^{2} p_i m_j(\theta_i) \leq I_j, \tag{5}$$

and

$$c_j(\theta_i) \leq r(\theta_i)a_j + e_j l_j(\theta_i) w(\theta_i) + m_j(\theta_i) \tag{6}$$

for $i \in \{1, 2\}$. $x_j(\theta_i)$ in Eq. (4) is composed of consumption felicity and labor infelicity such that $x_j(\theta_i) := c_j(\theta_i) - \Psi l_j(\theta_i)^{1+\chi}/(1+\chi)$ with $\Psi > 0$, where $c_j(\theta_i)$ and $l_j(\theta_i)$ are consumption and labor, respectively, and $\chi \in (0, \infty)$ is the inverse of Frisch elasticity of labor supply. If $\gamma = 1$, $(x_j(\theta_i)^{1-\gamma} - 1)/(1-\gamma)$ is reduced to $\ln x_j(\theta_i)$. Inequality (5) is the budget constraint in the first period where a_j is a real asset, $m_j(\theta_i)$ is a state-contingent claim with p_i being its price, and I_j is agent j's initial endowment. The aggregation of a_j becomes aggregate capital in the economy. Short sales of a_j (i.e., $a_j < 0$) are allowed. Agents who purchase (sell) one unit of state-contingent claims at a price of p_i in the first period are paid (pay) one unit of final goods in the second period. Inequality (6) is the budget constraint in period 2 when θ_i is realized, where e_j is agent j's ability and $e_j l_j(\theta_i)$ is individual effective labor.

2.3 First-order conditions

The first-order conditions are given by

$$\pi_i \left(c_j(\theta_i) - \Psi \frac{l_j(\theta_i)^{1+\chi}}{1+\chi} \right)^{-\gamma} = \lambda_j^i, \tag{7}$$

$$(1 - p_i r(\theta_i))\, \lambda_j^i - p_i r(\theta_{i'}) \lambda_j^{i'} = 0, \tag{8}$$

and

$$-\pi_i \left(c_j(\theta_i) - \Psi \frac{l_j(\theta_i)^{1+\chi}}{1+\chi} \right)^{-\gamma} \Psi l_j(\theta_i)^{\chi} + \lambda_j^i e_j w(\theta_i) = 0, \tag{9}$$

where λ_j^i is the Lagrange multiplier with $(i, i') = (1, 2)$ or $(2, 1)$. Eqs. (7), (8), and (9) are the first-order conditions with respect to consumption, state-contingent claims, and labor, respectively.

Eqs. (7) and (9) yield

$$l_j(\theta_i) = \left[\frac{e_j w(\theta_i)}{\Psi} \right]^{\frac{1}{\chi}}, \tag{10}$$

which is the individual labor supply.

Lemma 1. *The following two equations hold:*

$$p_1 r(\theta_1) + p_2 r(\theta_2) = 1 \tag{11}$$

and

$$\frac{\lambda_j^2}{\lambda_j^1} = \frac{p_2}{p_1}. \tag{12}$$

Proof. Eqs. (11) and (12) directly follow from Eq. (8). □

There are two equations in Eq. (8) for the combination of $(i, i') = (1, 2)$ or $(2, 1)$. For λ_j^1 and λ_j^2 to exist, however, Eq. (11) must hold. In this case, one of the two equations in Eq. (8) is redundant, and they reduce to one equation, Eq. (12). This situation occurs because agents have three investment opportunities for the two stochastic states: one real asset and two state-contingent claims. Due to this situation, agents' rational decisions on how much they purchase and sell state-contingent claims become indeterminate.

From Eqs. (7) and (12), it follows that

$$(p_2\pi_1)^{-\frac{1}{\gamma}} \left(c_j(\theta_1) - \Psi \frac{l_j(\theta_1)^{1+\chi}}{1+\chi} \right) = (p_1\pi_2)^{-\frac{1}{\gamma}} \left(c_j(\theta_2) - \Psi \frac{l_j(\theta_2)^{1+\chi}}{1+\chi} \right). \tag{13}$$

Furthermore, Eqs. (6) and (10) yield

$$c_j(\theta_i) - \Psi \frac{l_j(\theta_i)^{1+\chi}}{1+\chi} = r(\theta_i) a_j + \frac{\chi \Psi^{-\frac{1}{\chi}}}{1+\chi} \left[e_j w(\theta_i) \right]^{\frac{1+\chi}{\chi}} + m_j(\theta_i). \tag{14}$$

3 Equilibrium

A competitive equilibrium is given by prices, $\{p_i, r(\theta_i), w(\theta_i)\}$ for $i = 1, 2$, and allocation, $\{(a_j, m_j(\theta_i), c_j(\theta_i), l_j(\theta_i))\}$ for $i = 1, 2$ and $j \in \Omega$, such that given $\{p_i, r(\theta_i), w(\theta_i)\}$ each agent solves their utility maximization problem, given $\{r(\theta_i), w(\theta_i)\}$ the representative firm solves its profit maximization problem, and capital, state-contingent claim, and labor markets all clear.

3.1 Capital and insurance markets

The insurance market clearing condition is given by

$$\int_{j \in \Omega} m_j(\theta_i) dj = 0 \tag{15}$$

for $i = 1$ and 2. The aggregate capital is obtained as follows:

$$k = \int_{j \in \Omega} a_j dj. \tag{16}$$

From Eqs. (5), (15), and (16), it follows that

$$k = I, \tag{17}$$

where $I := \int_{j\in\Omega} I_j dj$ is the aggregate initial endowment. Eq. (17) is a supply of capital and Eq. (3) yields a demand for capital. Then, given effective labor, h, the capital market clearing condition is given by

$$r(\theta_i) = \theta_i b \left[bI^\sigma + (1-b)h^\sigma\right]^{\frac{1-\sigma}{\sigma}} I^{\sigma-1}. \tag{18}$$

3.2 Labor market

From Eq. (10), the supply of aggregate effective labor is obtained as follows:

$$h(\theta_i) = \int_{j\in\Omega} e_j l_j(\theta_i) dj = \Psi^{-\frac{1}{\chi}} e^{\frac{1+\chi}{\chi}} w(\theta_i)^{\frac{1}{\chi}}, \tag{19}$$

where $e := \left[\int_{j\in\Omega} e_j^{\frac{1+\chi}{\chi}} dj\right]^{\frac{\chi}{1+\chi}}$. Note that aggregate effective labor, h, is subject to the realization of productivity shock, θ_i. Since given aggregate capital, k, Eq. (2) yields a demand for aggregate effective labor, inserting Eq. (19) in Eq. (2) produces the labor market clearing condition as follows:

$$w(\theta_i) = \theta_i(1-b)\left[bk^\sigma + (1-b)(\Psi^{-\frac{1}{\chi}} e^{\frac{1+\chi}{\chi}} w(\theta_i)^{\frac{1}{\chi}})^\sigma\right]^{\frac{1-\sigma}{\sigma}} (\Psi^{-\frac{1}{\chi}} e^{\frac{1+\chi}{\chi}} w(\theta_i)^{\frac{1}{\chi}})^{\sigma-1}. \tag{20}$$

3.3 Existence and uniqueness of equilibrium wage and interest rates

By inserting Eq. (19) in Eq. (18) and Eq. (17) in Eq. (20), we have

$$\begin{aligned} r(\theta_i) &= \theta_i b \left[bI^\sigma + (1-b)(\Psi^{-\frac{1}{\chi}} e^{\frac{1+\chi}{\chi}} w(\theta_i)^{\frac{1}{\chi}})^\sigma\right]^{\frac{1-\sigma}{\sigma}} I^{\sigma-1} \\ &= \theta_i b \left[b + (1-b)(\Psi^{-\frac{1}{\chi}} e^{\frac{1+\chi}{\chi}} w(\theta_i)^{\frac{1}{\chi}}/I)^\sigma\right]^{\frac{1-\sigma}{\sigma}} \end{aligned} \tag{21}$$

and

$$w(\theta_i) = \theta_i(1-b)\left[bI^\sigma + (1-b)(\Psi^{-\frac{1}{\chi}}e^{\frac{1+\chi}{\chi}}w(\theta_i)^{\frac{1}{\chi}})^\sigma\right]^{\frac{1-\sigma}{\sigma}}(\Psi^{-\frac{1}{\chi}}e^{\frac{1+\chi}{\chi}}w(\theta_i)^{\frac{1}{\chi}})^{\sigma-1}$$
$$= \theta_i(1-b)\left[b(\Psi^{-\frac{1}{\chi}}e^{\frac{1+\chi}{\chi}}w(\theta_i)^{\frac{1}{\chi}}/I)^{-\sigma} + (1-b)\right]^{\frac{1-\sigma}{\sigma}}, \tag{22}$$

respectively. Given θ_i, Eqs. (21) and (22) determine the equilibrium wage and interest rates. Since the right-hand side of Eq. (22) is a decreasing function, it is straightforward to show that the equilibrium wage rate is uniquely determined. Furthermore, once the equilibrium wage rate is determined, the equilibrium interest rate is also uniquely determined by Eq. (21). Henceforth, we denote the wage and interest rates in equilibrium by $w^*(\theta_i)$ and $r^*(\theta_i)$. It holds that $w^*(\theta_1) < w^*(\theta_2)$ because if θ_i increases, the demand for labor increases, as shown in Eq. (2). Additionally, $r^*(\theta_1) < r^*(\theta_2)$ because $r^*(\theta_i)$ increases with $w^*(\theta_i)$ and θ_i, as shown in Eq. (21).

Although it is difficult to obtain the equilibrium wage and interest rates explicitly from Eqs. (21) and (22), some comparative statistics are useful for understanding the situations of the capital and labor markets. First, Eqs. (2) and (19) prove that $\partial w^*(\theta_i)/\partial\Psi > 0$ and $\partial h^*(\theta_i)/\partial\Psi < 0$. From Eq. (19), it follows that the increase in a labor reluctant parameter, Ψ, shifts the effective labor supply curve to the left, and thus, the wage rate increases and the effective labor decreases in equilibrium. Second, from Eq. (21) and $\partial h^*(\theta_i)/\partial\Psi < 0$, we can prove that $\partial r(\theta_i)/\partial\Psi < 0$ because if the supply of effective labor decreases, the marginal product of capital decreases as the complementarity between capital and effective labor is present in the CES production function. Third, it is straightforward that $\partial r(\theta_i)/\partial I < 0$ and $\partial w(\theta_i)/\partial I > 0$ by considering the marginal products of capital and effective labor, respectively. All the above consequences hold regardless of the realized values of θ_i.

Here, we present a lemma useful for the analysis in what follows.

Lemma 2. *It holds that*

$$\frac{r^*(\theta_i)}{w^*(\theta_i)^{\frac{1+\chi}{\chi}}} = \left(\frac{b}{1-b}\right)\left(\frac{\Psi^{-\frac{1}{\chi}}e^{\frac{1+\chi}{\chi}}}{I}\right)^{1-\sigma} w^*(\theta_i)^{-\frac{\sigma}{\chi}}. \tag{23}$$

Proof. Eq. (23) follows from Eqs. (21) and (22). □

3.4 Aggregation

By using Eqs. (15)-(17), we aggregate Eq. (14) across agents to obtain the following equation:

$$\int_{j\in\Omega}\left(c_j(\theta_i)-\Psi\frac{l_j(\theta_i)^{1+\chi}}{1+\chi}\right)dj=r^*(\theta_i)I+\frac{\chi\Psi^{-\frac{1}{\chi}}}{1+\chi}\left[ew^*(\theta_i)\right]^{\frac{1+\chi}{\chi}} \tag{24}$$

for $i\in\{1,2\}$, where we have used the equilibrium wage and interest rates. Aggregating both sides of Eq. (13) and substituting Eq. (24) into the resulting equation, we have

$$p_2\pi_1\left(r^*(\theta_2)I+\frac{\chi\Psi^{-\frac{1}{\chi}}}{1+\chi}\left[ew^*(\theta_2)\right]^{\frac{1+\chi}{\chi}}\right)^{\gamma}=p_1\pi_2\left(r^*(\theta_1)I+\frac{\chi\Psi^{-\frac{1}{\chi}}}{1+\chi}\left[ew^*(\theta_1)\right]^{\frac{1+\chi}{\chi}}\right)^{\gamma}. \tag{25}$$

Proposition 1 below presents the price of the state-contingent claim.

Proposition 1. *The price of the state-contingent claim in equilibrium, p_i^*, is given by*

$$p_i^*=\frac{\pi_i\left(r^*(\theta_{i'})I+\frac{\chi\Psi^{-\frac{1}{\chi}}}{1+\chi}\left[ew^*(\theta_{i'})\right]^{\frac{1+\chi}{\chi}}\right)^{\gamma}}{\sum_{(i,i')=(1,2)}^{(2,1)}\pi_i r^*(\theta_i)\left(r^*(\theta_{i'})I+\frac{\chi\Psi^{-\frac{1}{\chi}}}{1+\chi}\left[ew^*(\theta_{i'})\right]^{\frac{1+\chi}{\chi}}\right)^{\gamma}} \tag{26}$$

where $(i,i')=(1,2)$ *or* $(2,1)$.

Proof. Eq. (26) follows from Eqs. (11) and (25). □

Two remarks concerning Proposition 1 are in order. First, given the same probabilities for states 1 and 2, i.e., $\pi_1^*=\pi_2^*$, $p_1^*>p_2^*$ holds because $r^*(\theta_2)>r^*(\theta_1)$ and $w^*(\theta_2)>w^*(\theta_1)$. We obtain this outcome because agents acquire a higher income in state 2 than in state 1, and thus, agents who benefit from state 2 demand the state-contingent claim of state 1 more than that of state 2 to smooth consumption between the two stochastic states. Second, if the probability with which state 1 occurs is very small, the outcome of $p_1^*>p_2^*$ does not necessarily hold because if the probability is so small, agents care much less about it. In this case, $p_1^*<p_2^*$ is more likely to occur even though the income in state 2 is higher than that in state 1.

For the exposition below, we define two variables such that

$$B_j=\frac{\chi\Psi^{-\frac{1}{\chi}}}{1+\chi}e_j^{\frac{1+\chi}{\chi}}\quad\text{and}\quad B=\frac{\chi\Psi^{-\frac{1}{\chi}}}{1+\chi}e^{\frac{1+\chi}{\chi}}.$$

Note that $\int_{j\in\Omega}B_jdj=B$, which is the average of B_j. Since one can regard B_j as another measure of agents' ability, we also call B_j and B agents' ability and its average, respectively, unless there is confusion. According to Eqs. (5), (11), (13), and (14), it follows that

$$
\begin{aligned}
& r^*(\theta_2)m_j(\theta_1) - r^*(\theta_1)m_j(\theta_2) \\
&= \frac{\left([p_1^*\pi_2]^{-\frac{1}{\gamma}} r^*(\theta_2) - [p_2^*\pi_1]^{-\frac{1}{\gamma}} r^*(\theta_1)\right) I_j + \left([p_1^*\pi_2]^{-\frac{1}{\gamma}} w^*(\theta_2)^{\frac{1+\chi}{\chi}} - [p_2^*\pi_1]^{-\frac{1}{\gamma}} w^*(\theta_1)^{\frac{1+\chi}{\chi}}\right) B_j}{[p_1^*\pi_2]^{-\frac{1}{\gamma}} p_1^* + [p_2^*\pi_1]^{-\frac{1}{\gamma}} p_2^*}.
\end{aligned}
\tag{27}
$$

By aggregating Eq. (27), we obtain

$$
\left([p_1^*\pi_2]^{-\frac{1}{\gamma}} r^*(\theta_2) - [p_2^*\pi_1]^{-\frac{1}{\gamma}} r^*(\theta_1)\right) I + \left([p_1^*\pi_2]^{-\frac{1}{\gamma}} w^*(\theta_2)^{\frac{1+\chi}{\chi}} - [p_2^*\pi_1]^{-\frac{1}{\gamma}} w^*(\theta_1)^{\frac{1+\chi}{\chi}}\right) B = 0.
\tag{28}
$$

Proposition 2. *The state-contingent claims purchased or sold by agent j in equilibrium, $m_j^*(\theta_1)$ and $m_j^*(\theta_2)$, satisfy the following equation:*

$$
r^*(\theta_2)m_j^*(\theta_1) - r^*(\theta_1)m_j^*(\theta_2) = \frac{\left([p_2^*\pi_1]^{-\frac{1}{\gamma}} r^*(\theta_1) - [p_1^*\pi_2]^{-\frac{1}{\gamma}} r^*(\theta_2)\right)(B_j I - B I_j)}{\left([p_1^*\pi_2]^{-\frac{1}{\gamma}} p_1^* + [p_2^*\pi_1]^{-\frac{1}{\gamma}} p_2^*\right) B}.
\tag{29}
$$

Proof. Eq. (29) follows from Eqs. (27) and (28). □

If agents are homogeneous in Eq. (29), i.e., if $B_j = B$ and $I_j = I$ for all j, it holds that $m_j^*(\theta_1)/r^*(\theta_1) = m_j^*(\theta_2)/r^*(\theta_2)$. This equation and Eq. (11) imply that purchasing state-contingent claims and holding real assets are completely indifferent. Then, in any case, agents acquire a return, $r^*(\theta_i)$, when θ_i is realized. In this case, one naturally considers that a no-trade equilibrium ($m_j^*(\theta_1) = m_j^*(\theta_2) = 0$ for all $j \in \Omega$) occurs for the state-contingent claims. Therefore, for insurance against aggregate shocks to be effective, some agents should be heterogeneous such that $B_j \neq B$ or $I_j \neq I$.

Corollary 1. *Eq. (29) is equivalent to*

$$
\begin{aligned}
& r^*(\theta_2)m_j^*(\theta_1) - r^*(\theta_1)m_j^*(\theta_2) \\
&= \frac{\left(\frac{b}{1-b}\right)\left(\frac{1+\chi}{\chi} B\right)^{1-\sigma} \left(w^*(\theta_1)w^*(\theta_2)\right)^{\frac{1+\chi}{\chi}} \left(w^*(\theta_1)^{-\frac{\sigma}{\chi}} - w^*(\theta_2)^{-\frac{\sigma}{\chi}}\right)(B_j I - B I_j)}{I^{1-\sigma}\left(I + p_1^* B w^*(\theta_1)^{\frac{1+\chi}{\chi}} + p_2^* B w^*(\theta_2)^{\frac{1+\chi}{\chi}}\right)}.
\end{aligned}
\tag{30}
$$

Proof. See Appendix A.

Note from Corollary 1 that if the production technology is of the Cobb-Douglas type, i.e.,

$\sigma \to 0$, or if the labor supply is inelastic, i.e., $\chi \to \infty$, it holds that $m_j^*(\theta_1)/r^*(\theta_1) = m_j^*(\theta_2)/r^*(\theta_2)$, and again, a no-trade equilibrium is a natural outcome as in the case of homogeneous agents.

4 Optimality

The fact that aggregate shocks are insurable contradicts our intuition from the first welfare theorem. Does the insurance plan for aggregate shocks that satisfies Eq. (29) or (30) achieve Pareto optimal equilibrium? If so, what is the mechanism through which the insurance plan works for aggregate shocks? To elaborate on these points, we begin by considering a social planner problem.

4.1 Social planner problem

Suppose that a social planner solves the following social welfare maximization problem:

$$\max \int_{j\in\Omega} \varphi^j \left(\sum_{i=1}^{2} \pi_i \frac{x_j(\theta_i)^{1-\gamma} - 1}{1-\gamma} \right) dj \tag{31}$$

subject to

$$\int_{j\in\Omega} c_j(\theta_i)dj \leq \theta_i \left[bI^\sigma + (1-b) \left(\int_{j\in\Omega} e_j l_j(\theta_i) dj \right)^\sigma \right]^{\frac{1}{\sigma}} \tag{32}$$

for $i \in \{1, 2\}$, where $\varphi^j \in (0, \infty)$ is the weight parameter on each agent's expected utility. We verify that there are prices that support the social planner problem's first-order conditions being consistent with competitive equilibrium.

The first-order conditions of the social planner problem with respect to $c_j(\theta_i)$ and $l_j(\theta_i)$ are given by

$$\varphi^j \pi_i \left(c_j(\theta_i) - \Psi \frac{l_j(\theta_i)^{1+\chi}}{1+\chi} \right)^{-\gamma} = \lambda^i \tag{33}$$

and

$$\begin{aligned} &- \varphi^j \pi_i \left(c_j(\theta_i) - \Psi \frac{l_j(\theta_i)^{1+\chi}}{1+\chi} \right)^{-\gamma} \Psi l_j(\theta_i)^\chi \\ &+ \lambda^i e_j \theta_i (1-b) \left[bI^\sigma + (1-b) \left(\int_{j\in\Omega} e_j l_j(\theta_i) dj \right)^\sigma \right]^{\frac{1-\sigma}{\sigma}} \left(\int_{j\in\Omega} e_j l_j(\theta_i) dj \right)^{\sigma-1} = 0, \end{aligned} \tag{34}$$

respectively, where λ^i is the Lagrange multiplier. By setting $\lambda^i = p_i$ and from Eq. (33), we obtain the following equation:

$$p_2\pi_1\left(c_j(\theta_1)-\Psi\frac{l_j(\theta_1)^{1+\chi}}{1+\chi}\right)^{-\gamma}=p_1\pi_2\left(c_j(\theta_2)-\Psi\frac{l_j(\theta_2)^{1+\chi}}{1+\chi}\right)^{-\gamma}, \tag{35}$$

which is identical to Eq. (13). Since $h(\theta_i)=\int_{j\in\Omega}e_jl_j(\theta_i)dj$, we can let

$$w(\theta_i)=\theta_i(1-b)\left[bI^{\sigma}+(1-b)\left(\int_{j\in\Omega}e_jl_j(\theta_i)dj\right)^{\sigma}\right]^{\frac{1-\sigma}{\sigma}}\left(\int_{j\in\Omega}e_jl_j(\theta_i)dj\right)^{\sigma-1} \tag{36}$$

in Eq. (34). Then, Eqs. (33) and (34) yield

$$l_j(\theta_i)=\left[\frac{e_jw(\theta_i)}{\Psi}\right]^{\frac{1}{\chi}}, \tag{37}$$

which is also identical to Eq. (10). These consequences mean that competitive equilibrium with state-contingent claims achieves the Pareto optimal outcome.

4.2 Source of incomplete markets

We have found that state-contingent claims are crucial in achieving the Pareto optimum in our model when aggregate shocks affect the economy. In this section, we focus on the mechanism that induces an incomplete market for aggregate shocks, although they affect all agents in the same direction.

Remark 1. *It follows from Eq. (33) that for agent j, the marginal rate of substitution of consumption in state 1 for consumption in state 2 is given by $[\pi_1x_j(\theta_1)^{-\gamma}]/[\pi_2x_j(\theta_2)^{-\gamma}]=\lambda^1/\lambda^2$ in the Pareto optimum, which means that the marginal rate of substitution being equal across agents is the necessary condition for the Pareto optimum.*

Based on Remark 1, we examine whether each agent's marginal rate of substitution without state-contingent claims is uneven across them. Consider a situation in which there are no state-contingent claims. Since Eq. (10) holds regardless of the presence of state-contingent claims, Eq. (14) with $a_j=I_j$ becomes

$$x_j(\theta_i)=c_j(\theta_i)-\Psi\frac{l_j(\theta_i)^{1+\chi}}{1+\chi}=r(\theta_i)I_j+\frac{\chi\Psi^{-\frac{1}{\chi}}}{1+\chi}\left[e_jw(\theta_i)\right]^{\frac{1+\chi}{\chi}}. \tag{38}$$

From Eqs. (2), (3), and (19), it follows that

$$w(\theta_i) = \left(\frac{1-b}{b}\right)^{\frac{\chi}{1+\chi-\sigma}} \left(\Psi^{-\frac{1}{\chi}} e^{\frac{1+\chi}{\chi}}\right)^{\frac{(\sigma-1)\chi}{1+\chi-\sigma}} I^{\frac{(1-\sigma)\chi}{1+\chi-\sigma}} r(\theta_i)^{\frac{\chi}{1+\chi-\sigma}}. \tag{39}$$

Substituting Eq. (39) into Eq. (38), we can rewrite Eq. (38) as

$$x_j(\theta_i) = c_j(\theta_i) - \Psi\frac{l_j(\theta_i)^{1+\chi}}{1+\chi} = r(\theta_i)I_j + r(\theta_i)^{\frac{1+\chi}{1+\chi-\sigma}}\delta_j, \tag{40}$$

where

$$\delta_j := \frac{\chi\Psi^{-\frac{1}{\chi}}}{1+\chi} e_j^{\frac{1+\chi}{\chi}} \left(\frac{1-b}{b}\right)^{\frac{1+\chi}{1+\chi-\sigma}} \left(\Psi^{-\frac{1}{\chi}} e^{\frac{1+\chi}{\chi}}\right)^{\frac{(\sigma-1)(1+\chi)}{1+\chi-\sigma}} I^{\frac{(1-\sigma)(1+\chi)}{1+\chi-\sigma}}. \tag{41}$$

The marginal rate of substitution without state-contingent claims is defined as

$$R(I_j, \delta_j) := \frac{\pi_1 \left(r(\theta_1)I_j + r(\theta_1)^{\frac{1+\chi}{1+\chi-\sigma}}\delta_j\right)^{-\gamma}}{\pi_2 \left(r(\theta_2)I_j + r(\theta_2)^{\frac{1+\chi}{1+\chi-\sigma}}\delta_j\right)^{-\gamma}}. \tag{42}$$

According to Remark 1 and Eq. (42), if $R(I_j, \delta_j)$ is independent of j, state-contingent claims are unnecessary for the economy to achieve the Pareto optimum.

When the production technology is of the Cobb-Douglas type ($\sigma \to 0$) or when the labor supply is inelastic ($\chi \to \infty$), we can compute the marginal rate of substitution as $R(I_j, \delta_j) = [\pi_1 r(\theta_1)^{-\gamma}] / [\pi_2 r(\theta_2)^{-\gamma}]$, which is independent of j. In this case, the marginal rate of substitution is even across agents, and thus, the economy achieves Pareto optimal equilibrium without state-contingent claims. This outcome is consistent with the case in which we take $\sigma \to 0$ or $\chi \to \infty$ in Eq. (30) of Corollary 1, and state-contingent claims do not play any role in insuring against aggregate shocks in competitive equilibrium. Furthermore, when agents are homogeneous in their ability and initial wealth such that $e_j = e$ and $I_j = I$ for all $j \in \Omega$, it follows that $R(I_j, \delta_j) = R(I, \delta)$, where δ is defined such that one replaces e_i with e in Eq. (41). In this case, $R(I_j, \delta_j)$ is also independent of j, and thus, the economy achieves Pareto optimal equilibrium without state-contingent claims. This outcome is consistent with the case in which B_j and I_j in Eq. (30) are replaced by B and I, respectively. Again, in this case, state-contingent claims do not play any role in insuring against aggregate shocks in competitive equilibrium.

The above assessments imply that if agents are heterogeneous in their ability and initial wealth in an economy with generic CES production technology and an endogenous labor supply, the marginal rate of substitution of consumption in state 1 for consumption in state

2 is uneven without state-contingent claims. In this case, a situation of incomplete markets endogenously occurs, and the Pareto optimal outcome is unachievable.

4.3 Insurance design

Since agents have three investment opportunities for the two stochastic states, how much they purchase or sell state-contingent claims is indeterminate. Whereas their optimal program is given by Eq. (29) or Eq. (30) such that the market clearing condition (15) should be satisfied, an extra equation is necessary to pin down the state-contingent claims that agents purchase or sell. We consider typical insurance designs in this section.

The sign of σ plays a crucial role in the insurance plan because it determines whether $w^*(\theta_1)^{-\frac{\sigma}{\chi}} - w^*(\theta_2)^{-\frac{\sigma}{\chi}}$ is positive or negative in Eq. (30). Many empirical studies have suggested various values of the elasticity of substitution between capital and labor (e.g., Klump et al., 2007, 2012; Chirinko, 2008; León-Ledesma et al., 2010). Among others, Gechert et al. (2022) performed a meta-analysis with 3186 observations of the elasticity of substitution obtained from 121 prior studies. They reported that the mean of the elasticity of substitution between capital and labor is 0.3.[3] According to their suggestion, it follows that $1/(1-\sigma) = 0.3$, which means $\sigma = -7/3 < 0$. Then, we assume that σ is negative.

Assumption 1. $\sigma < 0$.

Under Assumption 1, since $w^*(\theta_1) < w^*(\theta_2)$, it holds that $w^*(\theta_1)^{-\frac{\sigma}{\chi}} - w^*(\theta_2)^{-\frac{\sigma}{\chi}} < 0$.

Example 1

One of the possible ways to discern the amount of state-contingent claims is to have the net purchase of the state-contingent claims zero in the first-period budget constraint (5) so that the following equation holds:

$$p_1 m_j(\theta_1) + p_2 m_j(\theta_2) = 0. \tag{43}$$

[3]Gechert et al. (2022) conducted a meta-analysis, correcting for publication bias, using variation across industries, and including information on the first-order condition for capital.

Under this insurance design, each agent's net purchase of state-contingent claims in the first period is 0. From Eqs. (30) and (43) with Eq. (11), it follows that

$$m_j^*(\theta_1) = \frac{p_2^*\left(\frac{b}{1-b}\right)\left(\frac{1+\chi}{\chi}B\right)^{1-\sigma}(w^*(\theta_1)w^*(\theta_2))^{\frac{1+\chi}{\chi}}\left(w^*(\theta_1)^{-\frac{\sigma}{\chi}} - w^*(\theta_2)^{-\frac{\sigma}{\chi}}\right)(B_jI - BI_j)}{I^{1-\sigma}\left(I + p_1^*Bw^*(\theta_1)^{\frac{1+\chi}{\chi}} + p_2^*Bw^*(\theta_2)^{\frac{1+\chi}{\chi}}\right)} \tag{44}$$

and

$$m_j^*(\theta_2) = -\frac{p_1^*\left(\frac{b}{1-b}\right)\left(\frac{1+\chi}{\chi}B\right)^{1-\sigma}(w^*(\theta_1)w^*(\theta_2))^{\frac{1+\chi}{\chi}}\left(w^*(\theta_1)^{-\frac{\sigma}{\chi}} - w^*(\theta_2)^{-\frac{\sigma}{\chi}}\right)(B_jI - BI_j)}{I^{1-\sigma}\left(I + p_1^*Bw^*(\theta_1)^{\frac{1+\chi}{\chi}} + p_2^*Bw^*(\theta_2)^{\frac{1+\chi}{\chi}}\right)}. \tag{45}$$

Under the insurance design embodied by Eqs. (44) and (45), the insurance market clearing condition (15) is satisfied because $\int_{j\in\Omega}(B_jI - BI_j)\,dj = 0$. Whether $m_j^*(\theta_i)$ is positive or negative depends upon the sign of $B_jI - BI_j$. Suppose that Assumption 1 holds. Then, if $B_j - B > (B/I)(I_j - I)$, it holds that $m_j^*(\theta_1) < 0$ and $m_j^*(\theta_2) > 0$. An agent with $B_j - B > (B/I)(I_j - I)$ sells the state-contingent claim for state 1 (disaster state) and purchases the state-contingent claim for state 2 (normal state) in the first period. As agents' ability increases and initial wealth decreases, they are more likely to desire to receive payments in the normal state while compensating for damages in the disaster state. Conversely, if $B_j - B < (B/I)(I_j - I)$, it holds that $m_j^*(\theta_1) > 0$ and $m_j^*(\theta_2) < 0$. An agent with $B_j - B < (B/I)(I_j - I)$ sells the state-contingent claim for the normal state and purchases the state-contingent claim for the disaster state. As agents' ability decreases and initial wealth increases, they are more likely to desire compensation in the disaster state while paying in the normal state.

Example 2

There are other insurance designs that are satisfied with Eq. (29) or (30). The following design indicates that agents are prepared for the disaster state without holding the sate-contingent claim for state 2.

$$m_j^*(\theta_1) = \frac{\left(\frac{b}{1-b}\right)\left(\frac{1+\chi}{\chi}B\right)^{1-\sigma}(w^*(\theta_1)w^*(\theta_2))^{\frac{1+\chi}{\chi}}\left(w^*(\theta_1)^{-\frac{\sigma}{\chi}} - w^*(\theta_2)^{-\frac{\sigma}{\chi}}\right)(B_jI - BI_j)}{I^{1-\sigma}r^*(\theta_2)\left[I + p_1^*Bw^*(\theta_1)^{\frac{1+\chi}{\chi}} + p_2^*Bw^*(\theta_2)^{\frac{1+\chi}{\chi}}\right]} \tag{46}$$

and

$$m_j^*(\theta_2) = 0. \tag{47}$$

Table 1: Parameter Values

Parameter	Value	Source/Target
Elasticity of substitution between capital and effective labor	$\sigma = -7/3$	Gechert et al. (2022)
Capital share when $\sigma \to 0$	$b = 0.33$	RBC literature
Relative risk aversion with respect to $x_j(\theta_i)$	$\gamma = 1.1$	normalization
Inverse of Frisch elasticity	$\chi = 0.4545$	Clemens et al. (2023)
Parameter of labor reluctance	$\Psi = 0.043$	See Appendix B
Parameter of agent ability distribution	$n = 3.2034$	$n = 1.001(1+\chi)/\chi$
Parameter of agent ability distribution	$\alpha = 0.5557$	See Appendix B
TFP in disaster state	$\theta_1 = 0.01511$	$w(\theta_1) = 0.1w(\theta_2)$
TFP in normal state	$\theta_2 = 0.3719$	See Appendix B

Again, suppose that Assumption 1 holds. Then, if $B_j - B > (B/I)(I_j - I)$, it holds that $m_j^*(\theta_1) < 0$. An agent with $B_j - B > (B/I)(I_j - I)$ sells the state-contingent claim for state 1 (disaster state). As agents' ability increases and the initial endowment decreases, they are more likely to compensate for damages in the disaster state. Conversely, if $B_j - B < (B/I)(I_j - I)$, it holds that $m_j^*(\theta_1) > 0$. An agent with $B_j - B < (B/I)(I_j - I)$ purchases the state-contingent claim for state 1. As agents' ability decreases and the initial endowment increases, they are more likely to desire compensation in the disaster state.

Notably, if the insurance is designed such that Eq. (29) or (30) holds together with the market clearing conditions, any insurance design can achieve the Pareto optimum, although the optimal insurance design is indeterminate. The quantitative analysis in the next section visualizes the role of the state-contingent claims in achieving the Pareto optimum.

5 Quantitative analysis

To see the effect that the state-contingent claims for aggregate shocks have on indirect utility, we perform numerical exercises in this section.

5.1 Parameters

The parameter values that we use in the numerical analysis are listed in Table 1. As discussed in the previous section, Gechert et al. (2022) suggested that the elasticity of substitution between capital and labor is $1/(1-\sigma) = 0.30$. Following their report, we set $\sigma = -7/3$. We set $b = 0.33$, which becomes a capital share of the output assumed in the standard real business cycles literature when the production function reduces to the Cobb-Douglas production function as $\sigma \to 0$. γ is related to the coefficient of relative risk aversion, which

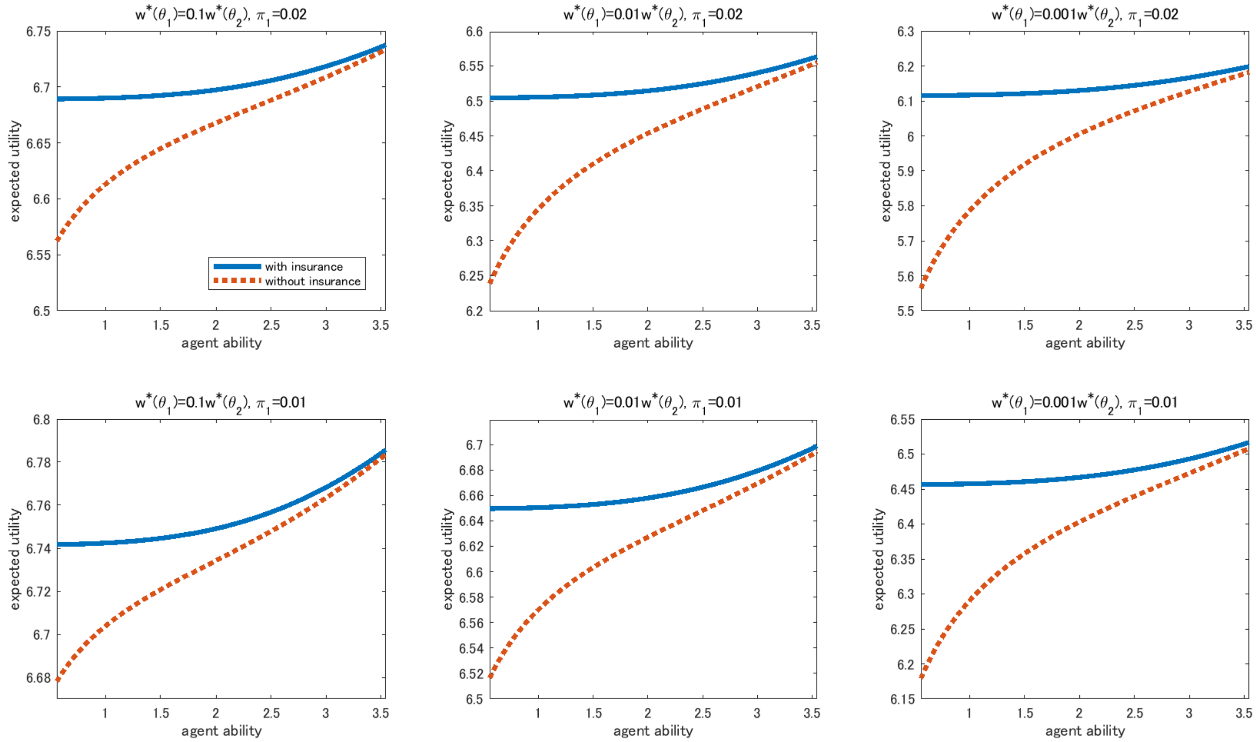

Figure 1: Agent ability versus expected utility

is given by $\gamma c_j(\theta_i)/x_j(\theta_i)$ in our model. There is mixed evidence for the coefficient of relative risk aversion and it is subject to agent ability in our setting. Thus, it is difficult to pin down γ. Greenwood et al. (1988) used two alternative values, $\gamma = 1.001$ and $\gamma = 2.0$. In our exercise, we employ a value relatively closer to the former and set $\gamma = 1.1$, which leads to the case of near-log utility.[4]

We assume that individual-specific labor productivity is $e_j = j$ and follows a Pareto distribution, whose cumulative distribution function, $G(j)$, is given by

$$G(j) = \begin{cases} 1 - \left(\frac{\alpha}{j}\right)^n & \text{if } \alpha < j \\ 0 & \text{otherwise,} \end{cases} \tag{48}$$

where $\alpha > 0$. We assume that $n > (1+\chi)/\chi$ so that e can be well defined. From Eq. (48), the density function of j is $G'(j) = n\alpha^n j^{-n-1}$; thus, e can be computed as

$$e = \left[\int_\alpha^\infty j^{\frac{1+\chi}{\chi}} \cdot n\alpha^n j^{-n-1} dj\right]^{\frac{\chi}{1+\chi}} = \left[\frac{n\chi}{n\chi - (1+\chi)}\right]^{\frac{\chi}{1+\chi}} \alpha. \tag{49}$$

[4]Although we examined some other alternative values of γ, the main results were essentially unchanged.

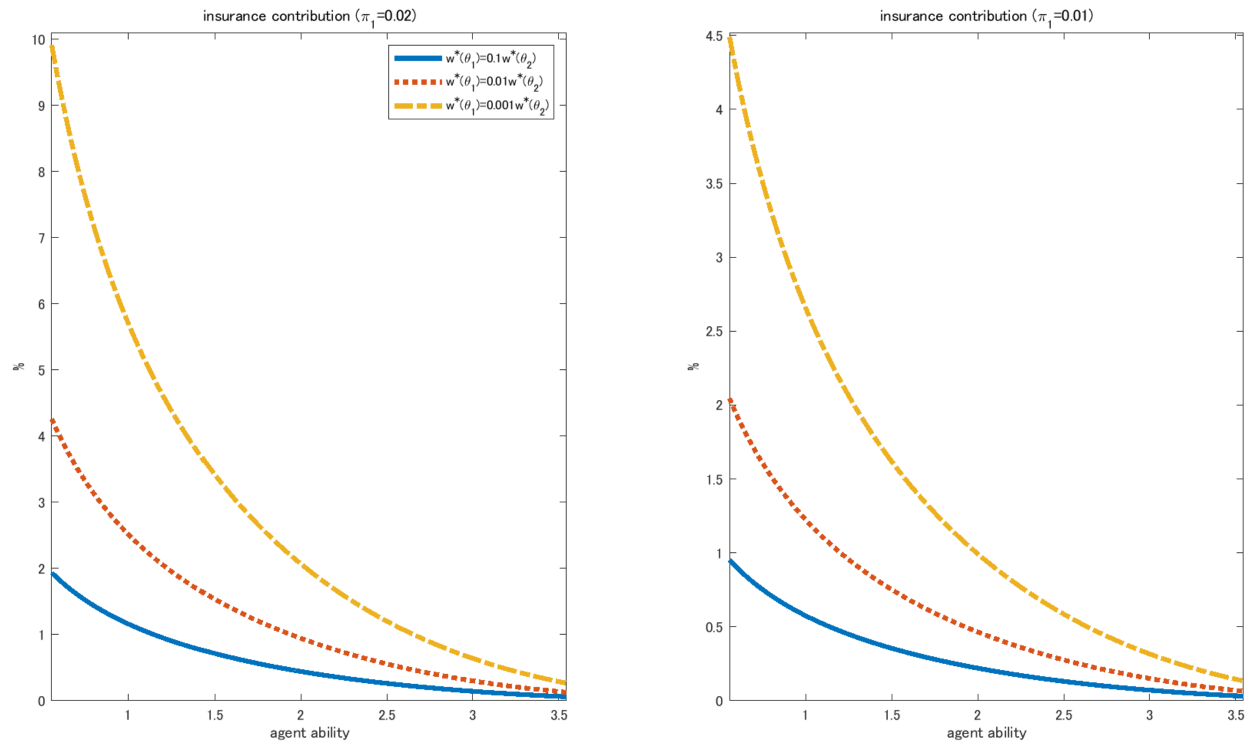

Figure 2: Agent ability versus insurance contribution to expected utility (%)

Accounting for the average working hours in the United States, Clemens et al. (2023) set $\chi = 0.4545$. Thus, we use the same value. The values of n and α determine the average labor productivity, but n should be greater than $(1+\chi)/\chi$. Then, we set $n = 1.001(1+\chi)/\chi$. We still must determine the values of Ψ, α, θ_1, and θ_2. To determine Ψ, α, and θ_2, we use the data for per worker GDP, per worker capital, and the annual average working hours, which are obtained from the Penn World Table, version 10.01. We elaborate on the details in Appendix C.

5.2 Results

We examine what would happen if the wage rate in a disaster state became 10% of that in a normal state, i.e., $w^*(\theta_1) = 0.1w^*(\theta_2)$ as a benchmark case, which might be extreme but could occur. Figure 2 presents the relationship between agent ability and indirect utility with agents' initial wealth remaining as the average per capita capital from 2011 to 2019. The two panels in the left column show the benchmark case. The panels in the middle and right columns present the cases in which $w^*(\theta_1) = 0.01w^*(\theta_2)$ and $w^*(\theta_1) = 0.01w^*(\theta_2)$ for robustness checks, respectively. The upper and lower rows show the cases in which the

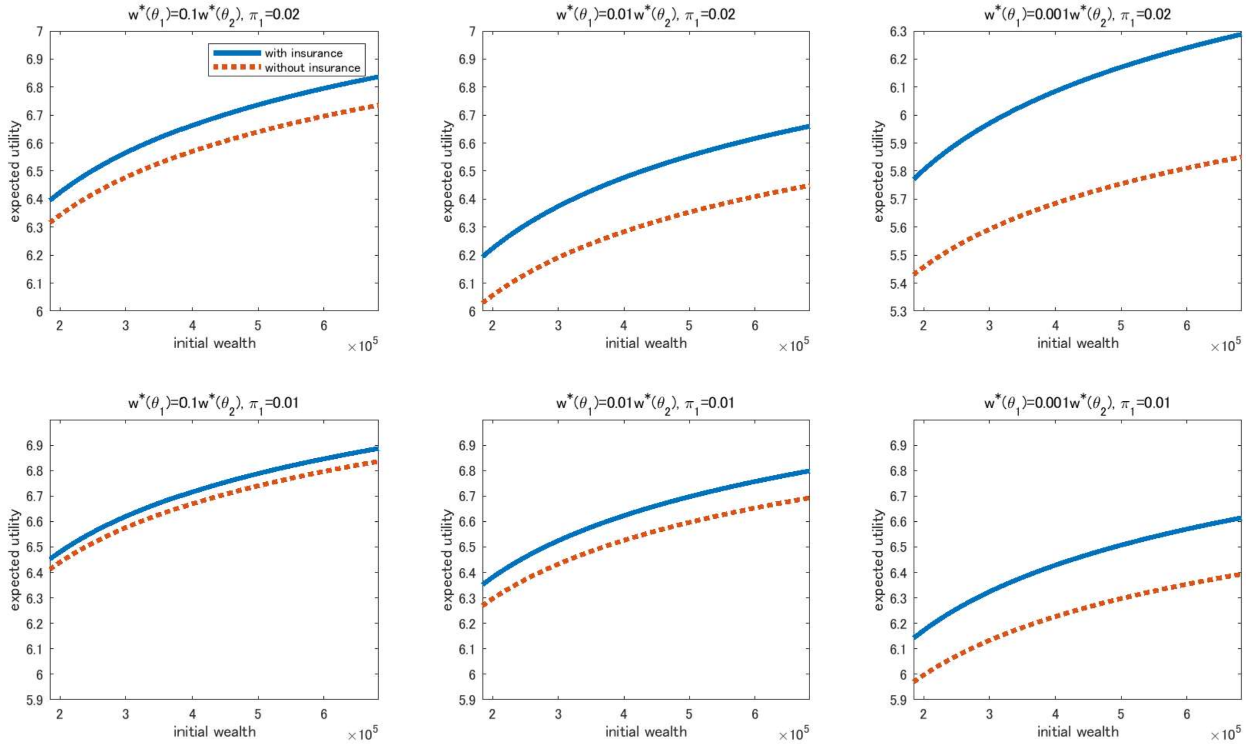

Figure 3: Initial wealth versus expected utility

probabilities of disaster occurrence are $\pi_1 = 0.02$ and $\pi_1 = 0.01$, respectively. Although we assume a two-period model, one may regard the two periods as a year. Then, one can consider that a disaster occurs once every fifty years if $\pi_1 = 0.02$ (since $1/0.02 = 50$) and once a hundred years if $\pi_1 = 0.01$ (since $1/0.01 = 100$). The solid (dotted) line in Figure 1 indicates the indirect utility when an aggregate shock is (not) insured against.

As shown in Figure 1, the indirect utility without insurance is less than that with insurance in all cases despite the agent's ability. Whereas our theoretical investigation naturally expects this outcome, the numerical results show that labor allocation may not be optimal when an aggregate shock occurs in an economy with heterogeneous agents. It is natural that as an agent's ability increases, the indirect utility with and without insurance increases. It is also natural that as the wage rate in a disaster state decreases, the indirect utility decreases. Furthermore, as the probability of disaster occurrence decreases, the indirect utility increases. Notably, the contribution of insurance to indirect utility is greater for agents with lower ability than for agents with higher ability. In particular, the contribution of insurance almost vanishes for agents with sufficiently high ability. We elaborate on this point in Figure 2 by observing the percentage change in indirect utility from the case without insurance to

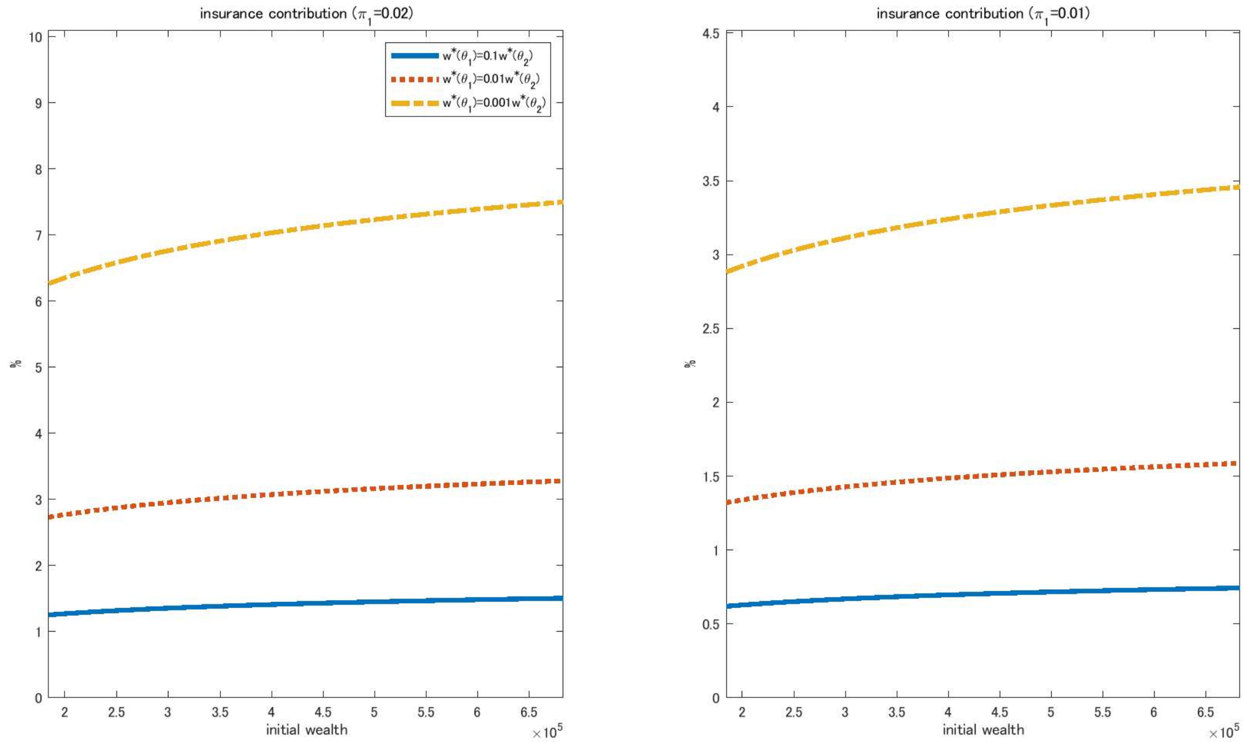

Figure 4: Initial wealth versus insurance contribution to expected utility (%)

that with insurance.

Suppose that $EU^\iota := \sum_{i=1}^{2} \pi_i[(x_j(\theta_i)^\iota)^{1-\gamma} - 1]/(1-\gamma)$ ($\iota = a, b$) is an indirect utility where a (b) stands for "insured" ("uninsured") and $x_j(\theta_i)^\iota$ is the value determined in equilibrium. Figure 2 presents the percentage contribution of insurance to indirect utility, measured by $100 \times (EU^a - EU^b)/EU^b$. As shown in Figure 2, when the agent's ability is low, there are significant insurance contributions. In particular, at the lowest agent ability ($j = 0.5557$), the insurance contribution approximately ranges from 2% to 10% in the once-in-fifty-years case ($\pi_1 = 0.02$) and from 1% to 4.5% in the once-in-a-hundred-years case. As expected from the analysis in Figure 1, however, the insurance contribution shrinks as the agent's ability becomes high. Observing the three cases in each panel, we find that greater damage to the wage rate induces a greater insurance contribution. Comparing the left and right panels, we note that a greater frequency of disasters causes a greater insurance contribution.

We next examine the relationship between agents' initial wealth and indirect utility. As seen in Figure 3, we observe that the indirect utility increases with initial wealth in all panels. Whereas the indirect utility uninsured is less than that insured as expected, we note from the comparison between the upper and the lower rows that a higher frequency of disasters produces greater welfare improvement. Furthermore, severe disaster damage induces a greater contribution of insurance, which we find by examining the three columns from left to right. We can confirm the last by observing the three cases in each panel in Figure 4, which presents the percentage contribution of insurance. Figure 4 also indicates that the percentage contribution moderately increases with agents' initial wealth.

6 Concluding remarks

The COVID-19 pandemic has damaged the world economy. It is presumed that this kind of crisis occurs once a hundred years. Thus, this experience is rare for ordinary people whose longevity is at most 80 or 90 years. Although we have hardly investigated insurance against aggregate shocks in the macroeconomics literature thus far, this crisis made us notice that it is crucial to consider such insurance. Our model suggests that an ex-post redistribution performed by insurance can induce an ex-ante welfare improvement without reallocating resources before a disaster (Example 1). Under this insurance program, agents with lower ability can enjoy greater welfare improvement than agents with higher ability even if the insurance program reallocates resources from agents with lower ability to agents with greater ability if a disaster does not occur.

A caveat limits our ability to see the results obtained in this paper. We investigated a two-period model. Although rich results have been derived, we do not know what would happen if we extended the time horizon to infinity, as in usual dynamic macroeconomic models. In particular, it is interesting to study the effect of insurance against aggregate shocks when there is capital accumulation. This topic is worthy of future research.

Appendices

Appendix A

We first derive Eq. (29) from Eq. (30). By using Eq. (23) in Lemma 2, the right-hand side of Eq. (30) can be rewritten as

$$r^*(\theta_2)m_j^*(\theta_1) - r^*(\theta_1)m_j^*(\theta_2) = \frac{\left(r^*(\theta_1)w^*(\theta_2)^{\frac{1+\chi}{\chi}} - r^*(\theta_2)w^*(\theta_1)^{\frac{1+\chi}{\chi}}\right)(B_jI - BI_j)}{I + p_1^*Bw^*(\theta_1)^{\frac{1+\chi}{\chi}} + p_2^*Bw^*(\theta_2)^{\frac{1+\chi}{\chi}}}. \quad \text{(A.1)}$$

Eliminating I from the denominator in the right-hand side of Eq. (A.1) by using Eq. (28) and applying Eq. (11) in Lemma 1 yield Eq. (29). To derive Eq. (30) from Eq, (29), we follow the opposite of the above procedure. □

Appendix B

To pin down the values of Ψ and α, we use the actual data for per worker GDP, per worker capital, and persons' working hours. We assembled the data for real GDP (rgdpna), real capital (rnna), the number of workers (emp), and persons' annual average working hours (avh) from the Penn World Table, 10.01 from 1950 to 2019, and prepared per worker real GDP and per worker real capital for the same period.

The stochastic states are irrelevant for determining Ψ and α, so we omit θ_i from the variables unless stated otherwise but add a time subscript, t, for further exposition. We assume that the annual average working hours in year t are $\tilde{H}_t := \int_{j\in\Omega} l_{t,j}dj$. From Eqs. (10) and (48), we obtain

$$\tilde{H}_t := \int_{j\in\Omega} l_{t,j}dj = \left(\frac{n\chi}{n\chi-1}\right)\left(\frac{\alpha w_t}{\Psi}\right)^{\frac{1}{\chi}},$$

or equivalently,

$$w_t = \left(\frac{\Psi}{\alpha}\right)\left(\frac{n\chi-1}{n\chi}\tilde{H}_t\right)^{\chi}. \tag{B.1}$$

Since $h_t = \int_{j\in\Omega} e_j l_{t,j}dj$ is computed as $h_t = (w_t/\Psi)^{\frac{1}{\chi}} e^{\frac{1+\chi}{\chi}}$, it follows from Eqs. (49) and (B.1) that

$$h_t = \tilde{h}_t\alpha, \tag{B.2}$$

where $\tilde{h}_t = \tilde{H}_t(n\chi-1)/(n\chi-1-\chi)$. Therefore, Eq. (B.1) is rewritten as

$$w_t = \frac{\Psi}{\alpha^{1+\chi}}\left(\frac{n\chi-1-\chi}{n\chi}\alpha\tilde{h}_t\right)^{\chi}. \tag{B.3}$$

From Eqs. (1) and (2), we have

$$w_t = \frac{(1-b)y_t h_t^{\sigma-1}}{bk_t^{\sigma}+(1-b)h_t^{\sigma}}, \tag{B.4}$$

where y_t is per worker GDP and k_t is per worker capital. Eliminating w_t from Eqs. (B.3) and (B.4) with (B.2), we have

$$Z_t(\Psi\alpha^{-\sigma}) + V\Psi = y_t\tilde{h}_t^{-1-\chi}, \tag{B.5}$$

where

$$Z_t := \left(\frac{b}{1-b}\right)\left(\frac{n\chi - 1 - \chi}{n\chi}\right)^{\chi} k_t^{\sigma}\tilde{h}_t^{-\sigma} \tag{B.6}$$

and

$$V := \left(\frac{n\chi - 1 - \chi}{n\chi}\right)^{\chi}. \tag{B.7}$$

From Eqs. (B.5)-(B.7), it follows that

$$\Psi\alpha^{-\sigma} = \left(\frac{Var(y_t\tilde{h}_t^{-1-\chi})}{Var(Z_t)}\right)^{\frac{1}{2}} \tag{B.8}$$

and

$$\Psi = \frac{1}{V}\left(\overline{y_t\tilde{h}_t^{-1-\chi}} - \left(\frac{Var(y_t\tilde{h}_t^{-1-\chi})}{Var(Z_t)}\right)^{\frac{1}{2}}\overline{Z_t}\right), \tag{B.9}$$

where the variance is taken for the period 1950-2019 and the overline represents the mean of variables over the same period. Eqs. (B.8) and (B.9) yield

$$\alpha = \left[\frac{1}{V}\left(\overline{y_t\tilde{h}_t^{-1-\chi}} - \left(\frac{Var(y_t\tilde{h}_t^{-1-\chi})}{Var(Z_t)}\right)^{\frac{1}{2}}\overline{Z_t}\right)\left(\frac{Var(y_t\tilde{h}_t^{-1-\chi})}{Var(Z_t)}\right)^{-\frac{1}{2}}\right]^{\frac{1}{\sigma}}. \tag{B.10}$$

We use Eqs. (B.9) and (B.10) as the calibrated values of Ψ and α, respectively.

Next, we determine θ_2. Once one has obtained α, it follows from Eqs. (1) and (B.2) that

$$y_t = \theta_2 W_t, \tag{B.11}$$

where $W_t := \left(bk_t^{\sigma} + (1-b)\alpha^{\sigma}\tilde{h}_t^{\sigma}\right)^{1/\sigma}$. To determine θ_2, we first prepared time series of y_t and W_t from 1950-2019 and created θ_2 year by year. We assumed that the United States was in a normal state (i.e., state 2) from 2011 to 2019. Then, we assembled the θ_2 values over the period 2011-2019 to average them over the same period. We used the average value of θ_2. For $I = k$ in Eqs. (21) and (22), we averaged per capita capital over the period 2011-2019. Now that we have had parameter values other than θ_1, we obtain $w(\theta_2)$ and $r(\theta_2)$ endogenously from Eqs. (21) and (22). We set θ_1 by using Eq. (22) such that $w(\theta_1) = 0.1w(\theta_2)$, as noted in Table 1, to investigate a counterfactual situation. We also examine two other alternative cases in which $0.01w(\theta_2)$ and $0.001w(\theta_2)$ for robustness checks.

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