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Inventor Mobility, Knowledge Diffusion, and Growth

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Abstract

This paper develops an endogenous growth model that incorporates a frictional inventor market and examines the allocation of inventors across firms, knowledge diffusion, and its impact on growth. In our model, inventors play dual roles: they engage in in-house R&D and transfer knowledge from previous employers to new ones when changing jobs. Using an administrative panel dataset on German inventors matched to their employing establishments and patents, we find that, relative to general workers, inventors are more likely to transition to less productive establishments and suffer a higher wage growth via the transition. We also find that the knowledge base of establishments measured by patents grows faster when a significant proportion of their inventors originate from establishments possessing a larger knowledge base. We then calibrate the model to reflect these empirical findings and examine the effects of innovation policy. While subsidies to frontier firms discourage knowledge diffusion from these firms to technologically lagging firms, these subsidies also encourage innovation within frontier firms. The former negative effect dominates in the short term, but the latter positive effect dominates in the long run.

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1 Introduction

Inventors play an essential role in both innovation within firms and knowledge diffusion between firms, which are important sources of economic growth. As Arrow (1962) stated "mobility of personnel among firms provides a way of spreading information," the mobility of inventors between firms has been considered an essential source of knowledge diffusion between firms.[1] Thus, policies related to labor markets for inventors are likely to have a significant impact on the firm productivity and economic growth.

This study provides an endogenous growth model to analyze the market for inventors and its impact on firm productivity and economic growth. In our model, inventors play dual roles: (i) they participate in in-house R&D efforts, enhancing the firm's technology, and (ii) they facilitate the transfer of knowledge from their former employers to their new ones when they change jobs. To quantify the model, we utilize data on inventors and patents linked to administrative labor market career information about inventors and their employing establishments in Germany. With these data, we document three novel empirical observations regarding inventors' job transitions, wage changes accompanying these transitions, and the influence of inventor inflows on the future innovation activity of recruiting firms. Finally, we discipline the model to align with these empirical findings and explore the consequences of inventor labor market policies.

In the theoretical part, we introduce an endogenous growth model that features the labor market where inventors and firms interact, and knowledge spills over across firms via inventor job transitions. Heterogeneous firms offer job openings, considering the knowledge diffusion from the inventors' prior employers, and inventors and firms match randomly in a frictional labor market. We focus on on-the-job search given our interest in how the inter-firm mobility of inventors influences knowledge spillover. This model is the first endogenous economic growth model that considers the endogenous job flows of inventors across firms and the knowledge diffusion through the job flows of inventors. The model's strength lies in its ability to endogenously generate both net and gross job flow of inventors and knowledge spillovers resulting from these flows, which are responsive to economic conditions and policy changes.

The empirical section documents novel findings for the job flows of inventors and their consequences. First, we examine the patterns of the mobility of inventors — defined

[1] For evidence from recent studies, see, Jaffe et al. (1993); Almeida and Kogut (1999); Song et al. (2003); Hoisl (2007); Rosenkopf and Almeida (2003); Breschi and Lissoni (2009); Singh and Agrawal (2011); Kaiser et al. (2015); Rahko (2017); Braunerhjelm et al. (2020).

as workers who have created patents — using inventor biography data from Germany, which links labor market biographies and their employing establishments recorded in the German social security data to patent register data. We find that a large proportion of inventors move to less productive establishments. This result is robust to the use of different productivity measures: establishment size, average wage, and number of patent citations. This job flow pattern of inventors is in contrast to general workers, who are more likely to move to more productive firms, as established in previous literature (e.g., Haltiwanger et al. (2018)). Moreover, we find that the wages of inventors grow more than those of general workers when they change jobs. This finding suggests that firms compensate for knowledge diffusion when they hire new inventors.

Then, we investigate how inventors' job flows influence the knowledge base of establishments, as measured by patents. We find that when a larger proportion of inventors comes from establishments with a more extensive knowledge base, the knowledge base of establishments grows faster over the next three to five years. Furthermore, we apply an instrumental variable to inventor flows and obtain significant results with the same sign as in the OLS specification. These results suggest the presence of knowledge diffusion through inventor flows.

In the quantitative section of this paper, we calibrate the model to match the key characteristics of the joint distribution of German inventors and firm dynamics observed in the microdata. We show that the calibrated model fits the target and non-target moments well, confirming that the model is well-suited to study counterfactual exercises.

We initially apply the calibrated model to conduct comparative statics analyses on matching efficiency. The model suggests that a decrease in matching efficiency, followed by a reduction in inventor mobility, leads to a decline in the economic growth rate. According to INV-BIO data, inventor mobility in Germany has been diminishing since the 1990s. Similarly, Akcigit and Goldschlag (2023a) report a decline in inventor mobility in the U.S. beginning in the early 2000s. Consequently, our model offers a framework for understanding the relationship between the observed decrease in inventor mobility and the deceleration of aggregate productivity growth in developed countries over recent decades.

Finally, we analyze the transition dynamics in our model to evaluate the effects of labor market policies on inventors. A key issue for those overseeing innovation policy is identifying which firms should be granted subsidies. In this context, we investigate the transition from an initial Balanced Growth Path (BGP) without subsidies to a new BGP with subsidies directed at technologically frontier firms. Frontier firms are characterized as those ranking in the upper half of the productivity distribution, with weighting based on the number of inventors. In the short term, subsidies to frontier firms reduce aggregate output by impeding the mobility of inventors from these leading firms to less advanced ones, thus hampering knowledge transfer. In contrast, over the long term, this policy boosts aggregate output by accelerating the growth rate at the technological frontier. Therefore, the impact of targeted subsidies on specific groups of firms hinges on whether policymakers focus on short-term or long-term economic effects.

Related Literature. Our paper is related to the literature on endogenous growth theory, particularly the diffusion of technology and knowledge, including Luttmer (2007), Lucas (2009), Lucas and Moll (2014), Perla and Tonetti (2014), Akcigit et al. (2018), Buera and Oberfield (2020), Shi and Hopenhayn (2020), Benhabib et al. (2021), and Prato (2022). Buera and Lucas (2018) surveys this topic. Perla and Tonetti (2014) and Lucas and Moll (2014) advanced the literature by modeling agents who choose to invest in technology diffusion. This approach enables the investigation of incentives, externalities, and welfare-improving policies. Our formulation of the knowledge diffusion function is based on the semi-endogenous growth model proposed by Buera and Oberfield (2020), which investigates international knowledge diffusion. Their model provides a micro-foundation for the knowledge diffusion function and expresses knowledge diffusion as a synergy of novel ideas and insights drawn from others. Similar to ours, Benhabib et al. (2021) and Shi and Hopenhayn (2020) address the interaction between R&D innovation and knowledge diffusion. In particular, our model applies the firms' innovation process formulated by Benhabib et al. (2021) to generate a realistic stationary productivity distribution. In the technology diffusion literature, our work is most closely related to Akcigit et al. (2018), who explicitly model inventors and analyze their role in knowledge diffusion among inventors. We depart from this literature by focusing on knowledge diffusion among firms due to inventor mobility. Moreover, we introduce a new perspective to this literature by incorporating labor market frictions, emphasizing the interaction between inventors and firms.

An expansive body of empirical research supports the concept of knowledge spillovers facilitated by job transitions of inventors. In one of the first such studies, Almeida and Kogut (1999) show that locations with greater intraregional labor mobility between firms tend to have more localized knowledge flows. Song et al. (2003) illustrate that mobile inventors build upon ideas from their previous firm more often than other inventors at the hiring firm. Rosenkopf and Almeida (2003) analyzes firm pairs, showing that those with higher labor mobility also have greater subsequent knowledge flow. These pioneering studies have inspired further research to facilitate our understanding of the connection between job transitions of inventors and knowledge spillovers (Hoisl, 2007; Breschi and Lissoni, 2009; Singh and Agrawal, 2011; Kaiser et al., 2015; Rahko, 2017; Braunerhjelm et al., 2020). Mawdsley and Somaya (2016) provide a review of these studies[2]. While most studies in this field struggle with limitations related to drawing causal inferences, some papers address the endogeneity problem. Singh and Agrawal (2011) employ a difference-in-differences approach to compare pre-move and post-move citation rates for poached inventors' previous and comparable control patents, concluding that acquiring firms intensify their use of inventions from the inventors' previous employers. Kaiser et al. (2015) use lagged mobility and industry mobility averages as instrumental variables for inventor mobility, uncovering a significantly positive impact of incoming inventors on their new employers' patent activity. Our paper is the first to integrate these insights into an endogenous growth model, emphasizing the interaction of inventor mobility and knowledge diffusion across firms. Furthermore, our study is novel in that it compares the patterns of job changes and the associated wage changes between inventors and general workers, providing evidence that suggests knowledge transmission and compensation for it.

Our paper also relates to the literature on frictional labor markets. In particular, our study benefits from recent developments in the modeling of multi-worker firms and on-the-job search, including Schaal (2017), Elsby and Gottfries (2021), and Bilal et al. (2023). The contemporary presence of on-the-job search and a non-constant return to scale revenue function in employment makes, in general, the firm problem intractable because we need to track the distribution of wages within each firm. To address the intractability, we assume that a firm posts a privately efficient number of vacancies, following Bilal et al. (2023). This assumption reduces the state variables to firm productivity and the number of inventors, thereby rendering the model tractable. Based on Bilal et al. (2022), Bilal et al. (2023) presented an endogenous growth model where the productivity distribution of incumbents determines the productivity of entrant firms. This model introduces an

[2] A related area of study is the relationship between geography and knowledge diffusion. Early research by Jaffe et al. (1993) suggested a higher probability of cited patents originating from the same location as the citing ones. Breschi and Lissoni (2009) further improved this approach by introducing inventor mobility as a control, revealing that spatial proximity's effect on knowledge diffusion is cut by more than half. This suggests that the critical role of geography in knowledge transfer primarily results from inventors seldom relocating across regions.

endogenous growth rate akin to Luttmer (2007). However, their model abstracts away the knowledge spillovers through worker mobility and its implication for economic growth.

Herkenhoff et al. (2018) and Shi (2023) explore models wherein knowledge diffuses among firms or colleagues via worker mobility. However, these studies consider models where firms employ only one or two workers at most. In contrast, our model allows firms to hire an arbitrary number of inventors unless it is profitable. Furthermore, while these papers examine more generalized workers, we restrict our focus to inventors and investigate the impact on economic growth.

The rest of the paper proceeds as follows. Section 2 describes the theory. Section 3 introduces the data and empirical results. Section 4 presents the calibration of the model and the quantitative policy counterfactual. Section 5 concludes.

2 Model

This section introduces an endogenous growth model featuring the role of the labor market, where inventors and firms match, and knowledge diffusion across firms due to inventors' job flows. Time is continuous, and the horizon is infinite. Inventors play two roles: (i) they engage in R&D activities in the firm to which they belong; (ii) when they switch jobs, they transfer knowledge from their previous employer to the new one, thereby enhancing productivity—this is referred to as knowledge diffusion. Inventors are homogeneous, except in terms of which firm they belong to[3]. We focus on on-the-job search since we are interested in the effect of the inter-firm mobility of inventors on knowledge diffusion. Inventors and firms are randomly matched in a frictional labor market. Firms make hiring decision by internalizing the marginal benefits of their contributions through internal R&D and knowledge diffusion. As we will discuss later, the state variable of a firm is summed up to the productivity of the firm, z, and the number of inventors employed, n. We construct a BGP equilibrium where aggregate variables grow at a constant rate g and inventor and firms' productivity distributions are stationary. Section 4 presents the transition dynamics.

[3]Since this study considers constant wage contracts, wages can differ even among inventors who belong to the same firm. However, as we will discuss later, it is not necessary to track the distribution of wages within firms when characterizing the equilibrium

Household

The representative household is composed of $\mathtt{n}$ individuals who supply inelastically one unit of time to the labor market for inventor. The size of the population is constant. Individuals work as inventors and receive wage payments from their firms. There is full insurance within the family, and thus the household problem can be split into a choice of aggregate consumption and a stage where the consumption is distributed across household members. The latter stage is irrelevant to labor market dynamics, so we focus on the former. The household discounts the future at the rate ρ. It derives utility from consumption, which we assume is logarithmic:

$$\int_0^\infty e^{-\rho t} \log \hat{C}(t) dt.$$

Variables with hats indicate that they are variables before detrending. We assume that the household trades shares in a mutual fund that owns all firms in the economy and trades a risk-free bond in zero net supply. As is standard, this implies that firms discount future payoffs at a constant risk-free rate $r(t) = \rho + g(t)$ in equilibrium on a BGP.

Production Technology

There is a unit mass of a continuum of firms. These firms produce a homogeneous product. Each firm has heterogeneous productivity $\hat{z}$. For simplicity, firm output equals firm productivity. As we will discuss later, in equilibrium in this model, firm productivity support is finite, and a maximum value of firm productivity exists. Let $\bar{z}(t)$ denote the maximum productivity of any firm, which we interpret as the technology frontier.

Matching Technology

Each firm employs a continuum of inventors n. Firms and inventors meet in a frictional labor market. Let $\hat{Z}(t)$ denote the aggregate productivity. A firm pays a cost $c(v)\hat{Z}(t)$ to post v vacancies. The cost function $c(v)$ is increasing and concave, and satisfies $c(0) = 0$ and $c'(0) = 0$. We focus on on-the-job search and assume that firms cannot lay off inventors, and inventors cannot voluntarily quit their jobs. Therefore, there are no unemployed inventors. Each vacancy randomly matches at a rate of A with an inventor who is working at other firms. For simplicity, we assume that the vacancy matching rate A is exogenous and does not depend on labor market tightness. An inventor meets a firm at rate $A\mathrm{v}$

where v is the total number of vacancies. An inventor incurs no cost of the search. As the vacancy cost is multiplied by $\hat{Z}(t)$, the vacancy cost grows as the economy grows. The rationale for this assumption is that as the economy grows, the price of resources for the vacancy (e.g., wages for human resources) also grows at the same rate.

Evolution of Firms' Productivity

We assume that firms' productivity changes due to the following three reasons: (i) innovation, (ii) knowledge diffusion, and (iii) leapfrog.

Innovation. The productivity of firms, with productivity $\hat{z}$ and inventor count n, increases by $\gamma(n)\hat{z}$, where $\gamma(\cdot)$ is an increasing and concave function. Consequently, the rate of productivity growth attributed to in-house R&D innovation is higher for firms that employ a larger number of inventors.

Knowledge diffusion. When a firm with productivity $\hat{z}$ poaches an inventor from a firm with productivity $\hat{z}'$, the poaching firm's productivity increases by $\alpha(\hat{z}'/\hat{z})\hat{Z}(t)$ where $\alpha(\cdot)$ is an increasing and concave function. Therefore, a firm gains more knowledge when it poaches an inventor from a firm with higher relative productivity. While better insights lead to higher growth, the concavity of $\alpha(\cdot)$ implies that if the productivity difference between the poaching and poached firm is large, it becomes difficult for the poaching firms to utilize that knowledge.

Leapfrog. Finally, following Benhabib et al. (2021), we assume that firms can leapfrog to the frontier of the productivity distribution $\bar{z}(t)$ with an arrival rate $\eta > 0$. The possibility that firms can leap to the technology frontier represents an opportunity for the innovation process to yield significant insights rather than just steady incremental progress. This assumption establishes a stationary distribution with an upper bound on productivity for each period. The existence of this upper bound in the productivity distribution is crucial, as it ensures that the effect of knowledge diffusion does not become overly pronounced.

Contractual Environment

When a model includes random search, on-the-job search, and a non-constant return-to-scale revenue function in employment, the firm's problem generally becomes intractable. This is because it is necessary to track the entire wage distribution within and across firms to compute optimal retention and vacancy policies. Following Bilal et al. (2022), we make two assumptions regarding the contractual environment. These assumptions ensure that the state vector consists only of the firm size and productivity.

Assumption 1. (**Bertrand Competition**) When a vacancy posted by a poaching firm matches with an inventor employed at another firm, the two firms engage in Bertrand competition through a sequential auction. First, the poaching firm makes a take-it-or-leave-it wage offer. Then, the targeted firm makes a take-it-or-leave-it counteroffer to the worker. Finally, the inventor decides whether to stay or to move.

Assumption 2. (**Privately Efficient Vacancy Posting**) The firm posts the number of vacancies that maximizes the sum of the firm's and its inventors' values.

While Assumption 1 is standard in the on-the-job search literature, Assumption 2 might be viewed as somewhat stringent. This latter assumption is necessary to simplify the analytical characterization and quantitative analysis of the model. Under these assumptions, decisions made by both the firms and the inventors are privately efficient, as though they were maximizing their total joint value. Consequently, the state variables of the joint value function are reduced to the firm size and productivity. Therefore, there is no need to track the wage distribution within and across firms to determine equilibrium allocations.

Distributions and Aggregate Variables

Let $\hat{F}(\hat{z}, n, t)$ be the cumulative distribution function of firms such that

$$1 = \int d\hat{F}(\hat{z}, n, t).$$

We assume that the total mass of firms in the economy is normalized to one. The distribution should also satisfy the inventor market clearing condition:

$$\mathtt{n} = \int n d\hat{F}(\hat{z}, n, t).$$

Let $\hat{v}(\hat{z}, n, t)$ be the amount of vacancy a firm $(\hat{z}, n)$ post at time t. The total mass of vacancy is given by

$$\mathtt{v(t)} = \int \hat{v}(\hat{z}, n, t) d\hat{F}(\hat{z}, n, t).$$

Because firms produce a homogeneous product, the aggregate output is given by

$$\hat{Z}(t) = \int \hat{z} d\hat{F}(\hat{z}, n, t).$$

Let $\hat{f}(\hat{z}, n, t)$ be a density of $\hat{F}(\hat{z}, n, t)$. Let define employment-weighted density

$$\hat{f}_n(\hat{z}, n, t) \equiv \frac{n\hat{f}(\hat{z}, n, t)}{\mathrm{n}}$$

and $\hat{F}_n(\hat{z}, n, t)$ the corresponding cumulative distribution. Also, let define vacancy-weighted distributions

$$\hat{f}_v(\hat{z}, n, t) = \frac{\hat{v}(\hat{z}, n, t)\hat{f}(\hat{z}, n, t)}{\mathrm{v}}$$

and $\hat{F}_v(\hat{z}, n, t)$ the corresponding cumulative distribution.

Condition for Successful Poaching

Define the poaching indicator function $\hat{\mathbb{1}}_P$ that takes 1 if the poaching successes and takes 0 otherwise. Let $\hat{\Omega}(\hat{z}, n, t)$ denote the joint value of an organization composed of a firm with productivity $\hat{z}$ and its n inventors at time t. Then, the poaching indicator function is expressed as

$$\hat{\mathbb{1}}_P(\hat{z}, n, \hat{z}', n', t) = \begin{cases} 1 & \text{if} \quad \hat{\Omega}_n(\hat{z}, n, t) + \alpha(\hat{z}'/\hat{z})\hat{Z}(t)\hat{\Omega}_z(\hat{z}, n, t) > \hat{\Omega}_n(\hat{z}', n', t) \\ 0 & \text{otherwise} \end{cases}$$

The first term $\hat{\Omega}_n(\hat{z}, n, t)$ is the derivative of the joint value with respect to n, which represents the change in the joint value resulting from an increase in the stock of inventors. This term captures the marginal contribution of the inventor to the firm's in-house R&D activity. The term $\alpha(\hat{z}'/\hat{z})Z\hat{\Omega}_z(\hat{z}, n, t)$ represents the change in the joint value resulting from an increase in firm productivity when the firm hires a new inventor. This term emerges because hiring a new inventor facilitates the transfer of ideas from the firm where the inventor previously worked. The poaching of the inventor is successful if the total marginal value of the inventor for the poaching firm $(\hat{z}, n)$ exceeds the value for the poached firm $(\hat{z}', n')$.

Hamilton-Jacobi-Bellman Equation

The following Hamilton-Jacobi-Bellman (HJB) equation determine the joint value $\hat{\Omega}(\hat{z}, n, t)$:

$$
\begin{aligned}
r(t)\hat{\Omega}(\hat{z}, n, t) - \frac{\partial \hat{\Omega}(\hat{z}, n, t)}{\partial t} = \max_{\hat{v} \geq 0} \; & \hat{z} - c(\hat{v})\hat{Z}(t) \\
& + \underbrace{A\hat{v} \int \left[\hat{\Omega}_n(\hat{z}, n, t) + \alpha(\hat{z}'/\hat{z})\hat{Z}(t)\hat{\Omega}_z(\hat{z}, n, t) - \hat{\Omega}_n(\hat{z}', n', t)\right]^+ d\hat{F}_n(\hat{z}', n', t)}_{\text{Poaching Hire}} \\
& + \underbrace{\gamma(n)\hat{z}\hat{\Omega}_z(\hat{z}, n, t)}_{\text{In-house R\&D}} \\
& + \underbrace{\eta\left[\hat{\Omega}(\overline{z}, n, t) - \hat{\Omega}(\hat{z}, n, t)\right]}_{\text{Leapfrog}}
\end{aligned} \tag{1}
$$

When a firm (z, n) hires a new inventor, the total value increases by $\hat{\Omega}_n(\hat{z}, n, t) + \alpha(\hat{z}'/\hat{z})\hat{Z}\hat{\Omega}_z(\hat{z}, n, t) - \hat{\Omega}_n(\hat{z}', n', t)$. The first and second term is the gain in value to the firm and incumbent inventors due to the new hire. The third term is the value the firm and incumbent inventors give the new inventor, which equals the highest value its former employer would pay to retain them. As mentioned earlier, the poaching is successful if this difference is positive.

Conversely, an incumbent inventor may quit and move to a higher marginal value firm. The firm and remaining inventors will lose $\hat{\Omega}_n(z, n, t)$ and are thus prepared to increase the inventor's value by $\hat{\Omega}_n(z, n, t)$ to retain them. Knowing this, the external firm hires the inventor by offering the inventor exactly $\hat{\Omega}_n(z, n, t)$. Therefore, the joint value of the firm, remaining inventors, and poached inventor are unchanged, and no "Poached Quit" term appears in (1).

Kolmogorov Forward Equation

The Kolmogorov forward equation (KFE) describes the evolution of the firm's distribution across productivity and the number of inventors. To characterize the KFE, we derive the drifts for changes in firm-level productivity and the number of inventors. The drift for the firm-level productivity change is given by

$$\hat{\mu}_z(\hat{z},n,t) \equiv \underbrace{\gamma(n)\hat{z}}_{\text{In-house R\&D}} \underbrace{+ A\hat{v}(\hat{z},n,t)\hat{Z}(t)\int \hat{\mathbb{1}}_P(\hat{z},n,\hat{z}',n',t)\alpha(\hat{z}'/\hat{z})d\hat{F}_n(\hat{z}',n',t)}_{\text{Knowledge diffusion}}. \tag{2}$$

The first term on the right-hand side represents productivity growth due to in-house R&D. The second term accounts for the firms' productivity growth resulting from knowledge diffusion. When firms post $\hat{v}$ vacancies, these vacancies match with $A\hat{v}$ inventors. Owing to the randomness of the matchings, the original firms of these inventors are taken from the inventor-weighted firm distribution $\hat{F}_n$. If a poaching attempt is successful ($\hat{\mathbb{1}}_P(\hat{z},n,\hat{z}',n',t)=1$), the productivity of the poaching firm increases by $\alpha(\hat{z}'/\hat{z})\hat{Z}(t)$. Note that our definition of $\hat{\mu}_z(\hat{z},n,t)$ does not include changes in productivity due to leapfrogging, and we need to include an additional term to incorporate leapfrogging in the KFE equation, which we will describe below.

The drift for the change in the number of inventors is determined by

$$\hat{\mu}_n(\hat{z},n,t) \equiv \underbrace{A\hat{v}(\hat{z},n,t)\int \hat{\mathbb{1}}_P(\hat{z},n,\hat{z}',n',t)d\hat{F}_n(z',n',t)}_{\text{Poaching hire}} - \underbrace{A\mathbf{v}\frac{n}{\mathbf{n}}\int \hat{\mathbb{1}}_P(\hat{z}',n',\hat{z},n,t)d\hat{F}_v(\hat{z}',n',t)}_{\text{Poached by other firms}} \tag{3}$$

The first term on the right-hand side illustrates the increase in the number of inventors owing to poaching hires from other firms, while the second term represents a decrease in the number of inventors as they are poached by other firms.

Given the above definition of $\hat{\mu}_z(\hat{z},n,t)$ and $\hat{\mu}_n(\hat{z},n,t)$, the KFE is presented as

$$\frac{\partial}{\partial t}\hat{f}(\hat{z},n,t) = -\underbrace{\frac{\partial}{\partial n}\Big(\hat{\mu}_n(\hat{z},n,t)\hat{f}(\hat{z},n,t)\Big)}_{N\text{ of inventor change}} - \underbrace{\frac{\partial}{\partial \hat{z}}\Big(\hat{\mu}_z(\hat{z},n,t)\hat{f}(\hat{z},n,t)\Big)}_{\text{Productivity change}} \underbrace{-\eta\hat{f}(\hat{z},n,t) + \eta\int_0^{\overline{z}}\hat{f}(\hat{z},n,t)d\hat{z}\hat{\boldsymbol{\delta}}(\overline{z})}_{\text{Leapfrog}} \tag{4}$$

where $\hat{\boldsymbol{\delta}}(\overline{z})$ is the Dirac delta function, which is zero everywhere except $\hat{z}=\overline{z}$ where it is infinite and satisfies $\int\hat{\boldsymbol{\delta}}(\overline{z})dz=1$.

Technology Frontier

Here, we argue that the technology frontier is finite, and we characterize its growth rate. If $\bar{z}(0) < \infty$, then $\bar{z}(t)$ will remain finite for all t. This is because it evolves from the firms' productivity growth in the interval infinitesimally close to $\bar{z}(t)$, and the firm's growth rate is finite. Furthermore, the growth rate of the technology frontier is determined by the productivity growth rate of firms that possess the highest growth rate among those at the technology frontier. This is because these firms will be at the technology frontier in the next instant. The following lemma formally characterizes the productivity growth rate of the technology frontier:

Lemma 1. (Growth Rate of the Technology Frontier) *If* $\bar{z}(0) < \infty$, *then* $\bar{z}(t) < \infty \ \forall t < \infty$ *and*

$$g(t) \equiv \frac{\bar{z}'(t)}{\bar{z}(t)} = \max_{n \in \{n \mid \hat{f}(\bar{z}(t),n,t)>0\}} \frac{\hat{\mu}_z(\bar{z}(t), n, t)}{\bar{z}(t)}$$

Normalization

In the following, we examine economies in a BGP equilibrium, where the distribution remains constant when appropriately scaled, and aggregate output experiences constant growth. It is convenient to transform this system into a set of stationary equations for computing BGP equilibria. While we could standardize using any variable that grows at the same rate as the aggregate economy, it is expedient to normalize variables relative to the technology frontier $\bar{z}(t)$. Define the normalized values and functions as follows:

$$z \equiv \hat{z}/\bar{z}(t)$$

$$Z(t) \equiv \hat{Z}(t)/\bar{z}(t)$$

$$\Omega(z, n, t) = \Omega(\hat{z}/\bar{z}(t), n, t) \equiv \hat{\Omega}(\hat{z}, n, t)/\bar{z}(t) \tag{5}$$

$$F(z, n, t) = F(\hat{z}/\bar{z}(t), n, t) \equiv \hat{F}(\hat{z}, n, t) \tag{6}$$

$$\mathbb{1}_P(z, n, z', n', t) = \mathbb{1}_P(\hat{z}/\bar{z}(t), n, \hat{z}'/\bar{z}(t), n', t) \equiv \hat{\mathbb{1}}_P(\hat{z}, n, \hat{z}', n', t) \tag{7}$$

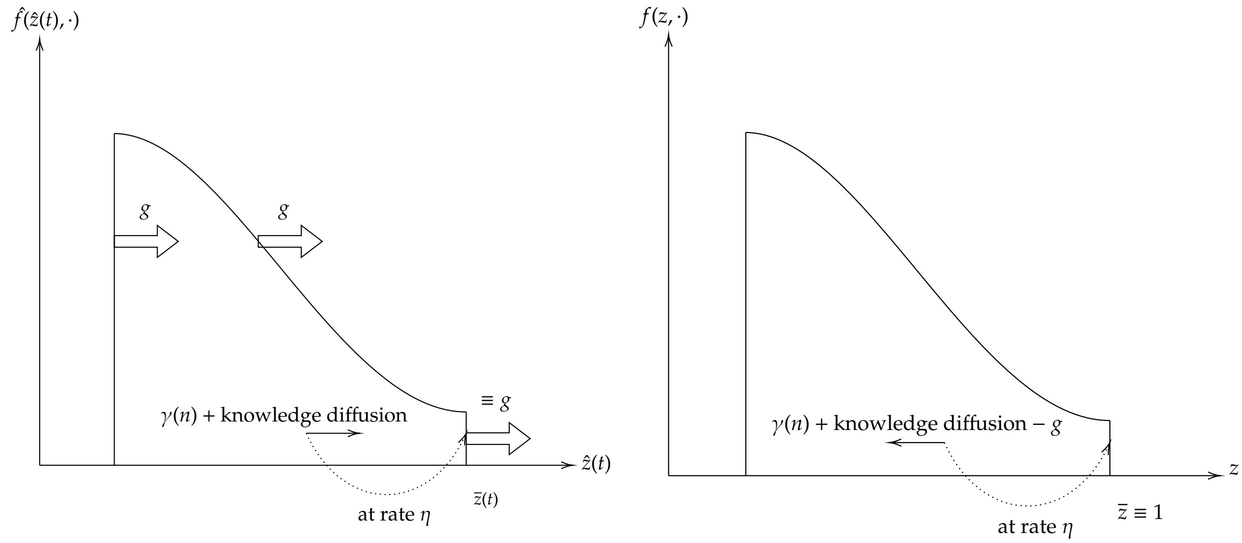
$$v(z, n, z', n', t) = v(\hat{z}/\bar{z}(t), n, \hat{z}/\bar{z}(t), n', t) \equiv \hat{v}(\hat{z}, n, \hat{z}', n', t) \tag{8}$$

The technology frontier is normalized to $\bar{z}(t)/\bar{z}(t) = 1$. The above normalizations make the value functions, productivity distributions, and growth rates stationary.

See the Figure 1 for an illustration of the original and detrended distributions. Noth-

Figure 1: Marginal Density for Productivity of Firms

(a) Original (b) Detrended



Notes: Illustration of original and detrended marginal distribution for firms' productivity on the BGP.

ing prevents the distribution from spreading without knowledge diffusion, driving the productivity variance to infinity. However, because of knowledge diffusion, as the distribution extends, productivity growth due to knowledge diffusion increases, and these forces compress the distribution.

Balanced Growth Path

Now, we describe a BGP equilibrium where aggregate productivity grows at a constant rate, and distributions are stationary. Define the growth rate of aggregate productivity to be $g_Z(t) \equiv \hat{Z}'(t)/\hat{Z}(t)$. That is, $g_Z(t) = g_Z$ and $F(z, n, t) = F(z, n)$ for all t. Aggregate output is given by

$$\begin{aligned}\hat{Z}(t) &= \int \hat{z} d\hat{F}(\hat{z}, n, t) \\ &= \bar{z}(t) \int z dF(z, n, t)\end{aligned}$$

On a BGP, the detrended productivity distribution is constant: $F(z, n, t) = F(z, n)$. Therefore, $g_Z = \hat{Z}'(t)/\hat{Z}(t) = \bar{z}'(t)/\bar{z}(t) = g$, and we obtain the following lemma:

Lemma 2. (Growth Rate of the Technology Frontier and Aggregate Productivity) *On a BGP, the aggregate productivity growth rate equals the technology frontier's growth rate. That is,* $g_Z = g$.

The following definition summarizes the characteristics of our BGP equilibrium.

Definition 1. (**Balanced Growth Path**) A *BGP equilibrium* consists of: (i) a joint value function $\Omega(z,n)$; (ii) a vacancy policy $v(z,n)$; (iii) a stationary distribution of firms $f(z,n)$; (iv) vacancy- and employment-weighted distributions $f_v(z,n)$ and $f_n(z,n)$; (v) poaching indicator function $\mathbb{1}_P(z,n,z',n')$; (vi) the aggregate productivity Z and the total vacancies v, and (vii) the economic growth rate g such that

1. The joint value $\Omega(z,n)$ satisfies the HJB equation

$$\begin{aligned}\rho\Omega(z,n) =& z - c(v(z,n))Z \\ &+ Av(z,n)\int\left[\Omega_n(z,n) + \alpha(z'/z)Z\Omega_z(z,n) - \Omega_n(z',n')\right]^+ dF_n(z',n') \\ &+ (\gamma(n)-g)\, z\Omega_z(z,n) \\ &+ \eta\left[\Omega(1,n) - \Omega(z,n)\right]\end{aligned}$$

2. The vacancy policy $v(z,n)$ satisfies the first order condition

$$c_v(v(z,n))Z = A\int\left[\Omega_n(z,n) + \alpha(z'/z)Z\Omega_z(z,n) - \Omega_n(z',n')\right]^+ dF_n(z',n') \tag{9}$$

3. A density function $f(z,n)$ satisfies the KFE equation

$$0 = -\frac{\partial}{\partial n}\left(\mu_n(z,n)f(z,n)\right) - \frac{\partial}{\partial z}\left(\mu_z(z,n)f(z,n)\right) - \eta f(z,n) + \eta\int_0^1 f(z',n)dz'\boldsymbol{\delta}(1)$$

where the drift of the number of employed inventors $\mu_n(z,n)$ and productivity $\mu_z(z,n)$ are given by

$$\begin{aligned}\mu_n(z,n) \equiv& Av(z,n)\int \mathbb{1}_P(z,n,z',n')dF_n(z',n') - A\mathrm{v}\frac{n}{\mathrm{n}}\int \mathbb{1}_P(z',n',z,n)dF_v(z',n') \\ \mu_z(z,n) \equiv& (\gamma(n)-g)\, z + Av(z,n)Z\int \mathbb{1}_P(z,n,z',n')\alpha(z'/z)dF_n(z',n')\end{aligned}$$

4. Vacancy- and employment-weighted distributions are consistent:

$$f_v(z,n) = \frac{v(z,n)f(z,n)}{\mathrm{v}}$$
$$f_n(z,n) = \frac{nf(z,n)}{\mathrm{n}}$$

5. Poaching indicator function $\mathbb{1}_P(z,n,z',n')$ is given by

$$\mathbb{1}_P(z,n,z',n') = \begin{cases} 1 & \text{if} \quad \Omega_n(z,n) + \alpha(z'/z)Z\Omega_z(z,n) > \Omega_n(z',n') \\ 0 & \text{otherwise} \end{cases}$$

6. The aggregate productivity Z and the total vacancies v rate are given by

$$Z = \int z dF(z,n)$$
$$\mathrm{v} = \int v(z,n)dF(z,n)$$

7. The inventor market clearing condition is satisfied:

$$\mathrm{n} = \int n dF(z,n)$$

Appendix A comprehensively derivates the normalized system. We also establish some properties of the joint value function in the Appendix. In it, we show that the following properties hold: (i) Ω is increasing in productivity: $\Omega_z > 0$; (ii) Ω is increasing in the number of inventors: $\Omega_n > 0$.

The equilibrium of the model is solved numerically in Section 4. Before that, we turn to the description of the empirical results.

3 Data and Empirical Findings

In this section, we investigate the job flows of inventors — workers who have created patents — between establishments using inventor biography data from Germany. The results provide motivation for our model, and we use these results to discipline the numerical model, as explained in Section4.

3.1 Data

Our analyses utilize two administrative data sets, "Linked Inventor Biography Data 1980–2014" (INV-BIO) and "Sample of Integrated Labor Market Biographies" (Stichprobe der Integrierten Arbeitsmarktbiografien — SIAB).[4]

The INV-BIO data combines labor market biographies recorded in the German social security data (Integrated Employment Biographies — IEB) with patent register data from the European Patent Office (EPO). This data set tracks information about 152,350 inventors who have registered their patents to the EPO from 1980 to 2014. The information includes their unique ID, age, gender, level of education, daily wage, and the number of citations received by the patents associated with each inventor in the EPO's records. The data also contains information about the establishments employing the inventors, such as the establishment ID, the total number of their employees, and the mean daily wage of their full-time employees. The advantage over patent-based datasets used in previous studies (e.g., EPO patent data by Akcigit et al. (2018)) is that we can use social security information to keep track of inventors' flows even when they are not creating patents.

The SIAB data is a 2% random sample from IEB. This data set contains the same information about individuals and their employing establishments as INV-BIO, except for patent-related information. In the absence of the patent data, we identify inventors in SIAB using a 3-digit occupation code, as described in Section 3.3. The data set covers 3,322,316 individuals from 1980 to 2019.

Since merging datasets is not allowed, we use the two datasets separately for each analysis: when comparing the movement patterns of inventors and other workers in Section 3.3, we use SIAB, which includes both, but otherwise we use INVBIO, a dataset focused exclusively on information about inventors.

3.2 Inventor Flows in INV-BIO

First, we adopt an approach similar to Haltiwanger et al. (2018) to characterize inventor flows using INV-BIO. We assign each establishment to a percentile rank according to patent information or productivity measure. We then compute the transition probabilities of inventor flows between these ranks.[5]

[4]More detailed information is presented in Online Appendix B.1.

[5]Establishments could be classified into different percentiles based on the measure each year. The ranks of the origin and destination establishments are determined based on the measure from the previous year, preceding the movement of inventors.

We utilize three different measures as proxies for the knowledge quality or productivity level.[6] The first measure is based on the forward citations for patents that establishments have created. Measuring patent quality through forward citations is widely employed in the literature about patent creation (e.g., Pakes (1986); Hall et al. (2001); Akcigit et al. (2018)). In particular, Akcigit et al. (2018) measures the idea quality of inventor teams based on the number of forward citations their patent receives. Similarly, our measure for an establishment e in year t, z_{et} is given by:

$$z_{et} = \frac{\sum_{j=-2}^{0} \text{citations}_{et+j}}{3}, \tag{10}$$
$$\text{where citations}_{et} = \sum_{i} \text{citations}_{it} \times \frac{n_{ie}}{n_i}.$$

citations_{it} denotes the count of forward citations that occur five years after year t for patent i, which is created by a team including inventors employed at establishment e. Note that the team developing the patent can consist of inventors from different establishments. n_{ie} represents the number of inventors at establishment e in the team, while n_i represents the total number of inventors in the team, including those affiliated with different establishments. We multiply citations_{it} by n_{ie}/n_i to adjust for the contribution made by inventors from establishments other than e.[7] Therefore, citations_{et} is the count of five-year forward citations for patents that establishment e created, adjusting for the contributions of other establishments. Following Akcigit et al. (2018), we use the three-year backward average as the measure. The other measures are the number of employees (establishment size) and the mean wage of full-time employees, following standard practice in the literature, as summarized by Moscarini and Postel-Vinay (2018).[8]

Table 1 shows the transition probabilities of inventor flows from origin to destination.[9]

[6] We assume that the knowledge quality and productivity level are positively correlated. In fact, the three measures are positively correlated with each other as described in Online Appendix B.1.

[7] In other words, we start by dividing the count of forward citations by the total number of inventors involved in the team for each inventor's patents. Afterward, we aggregate these values for the inventors who are employed at establishment e.

[8] On-the-job search models with heterogeneous productivity firms (e.g., Postel-Vinay and Robin (2002)) predict that more productive firms offer higher wages and attract more workers, leading to their growth in size.

[9] Online Appendix B.1 shows the distribution of inventors according to each of the three measures. It reveals a notable concentration of inventors within specific establishments. Irrespective of the type of measure, more than half of the inventors are found in establishments ranked above the 80th percentile, and only approximately 10 percent belong to establishments below the 50th percentile. This aligns with Akcigit

Table 1: Transition Probabilities of Inventor Flows

(A) Rank by Citation/Inventor

Share of flows (%)		Destination establishment rank				
		≤ 50%	50-60	60-70	70-80	80-100
	≤ 50%	2.3	0.2	0.3	0.4	4.3
Origin	50-60	1.7	0.2	0.2	0.3	3.0
establishment	60-70	1.9	0.2	0.3	0.3	3.6
rank	70-80	2.2	0.2	0.2	0.4	4.2
	80-100	19.5	2.0	2.4	3.4	46.7

(B) Rank by Establishment Size

Share of flows (%)		Destination establishment rank				
		≤ 50%	50-60	60-70	70-80	80-100
	≤ 50%	2.6	0.9	0.7	0.8	6.3
Origin	50-60	0.4	0.5	0.6	0.4	2.3
establishment	60-70	0.5	0.2	0.7	0.9	3.0
rank	70-80	0.6	0.3	0.4	1.3	4.8
	80-100	5.8	2.5	3.4	4.7	55.7

Notes: Detailed description is presented below the panel (C) in the next page.

There is a substantial movement of inventors from higher ranks to lower ranks, denoted by the red-colored cells. The sum of values in these red cells amounts to 33.7% in panel (A), 18.8% in panel (B), and 28.5% in panel (C). This suggests that a large portion of inventors move from higher-ranked establishments to lower-ranked ones.[10] Online Appendix B.1 shows that this pattern is observable even when the sample is limited to job flows accompanied by wage increases.

This pattern is not found in previous literature on worker flows. For example, Haltiwanger et al. (2018) construct transition probabilities of worker flows based on the mean wage of firms, and they observe a higher probability of flows to higher ranks compared to lower ranks. This discrepancy suggests that the tendency for many flows to lower ranks is a distinctive characteristic specific to inventors.

and Goldschlag (2023b)'s finding that inventors are concentrated in large incumbents in the U.S.

[10] Another observable pattern is that the values along the diagonal are considerably high, particularly in the bottom right of each panel: 49.9% in panel (A), 60.8% in panel (B), and 50.8% in panel (C). This indicates that many inventors tend to move within the same rank, especially within the highest rank. This can be observed in the literature on worker flows (e.g., Haltiwanger et al. (2018)).

Table 2: Inventor Flows across Establishments (cont.)

(C) Rank by Mean Wage

Share of flows (%)		Destination establishment rank				
		≤50%	50-60	60-70	70-80	80-100
	≤ 50%	2.9	1.1	1.0	1.1	3.5
Origin	50-60	0.8	0.9	1.1	0.9	2.3
establishment	60-70	1.0	0.9	2.0	2.1	4.1
rank	70-80	1.4	1.0	1.7	3.6	7.3
	80-100	5.5	3.6	5.4	7.2	37.6

Notes: This table shows transition probabilities of inventor flows across percentiles of establishments. The inventors staying in the same establishment are excluded. The percentile rank in panel (A) is based on the three-year backward average of forward patent citation counts. Panel (B) is based on the number of employees, and panel (C) is based on the mean wage of full-time employees. Establishments could be classified into different percentiles based on these measures each year. The ranks of the origin and destination establishments are determined based on the measure from the previous year, preceding the movement of inventors. The sample encompasses data from 1980 to 2014. The values in the table represent the proportion of inventor flows in each cell in relation to the total flows in INV-BIO.

3.3 Inventor Flows in Comparison with Worker Flows

Next, we compare inventor flows with worker flows. We utilize the SIAB for the comparison since INV-BIO lacks information on workers other than inventors.

To identify inventors within SIAB, we use a 3-digit occupation code. We find that the majority of inventors in INV-BIO are affiliated with specific occupations, each with their corresponding shares: research and development (20.2%), machine-building and operations (19.8%), mathematics, biology, and physics (19.1%), and mechatronics, energy, and electronics (18.8%). These four occupations account for nearly 80% of the inventors in INV-BIO. We thus consider workers in these four occupations within SIAB to likely be inventors.

Table 2 presents a comparison of summary statistics between the two data sets. The mean daily wage of workers in the four occupations (identified inventors) in SIAB falls between that of workers in SIAB and that of inventors in INV-BIO. Furthermore, the proportion of female workers among the identified inventors lies between the two groups. These findings suggest that our identified inventors include both actual inventors and a portion of non-inventor workers. Therefore, the result of the subsequent comparison between workers and identified inventors should be considered conservative due to the presence of attenuation bias.

Table 2: Identified Inventors in SIAB and Inventors in INV-BIO

Summary statistics (1980 - 2014)		SIAB Workers	SIAB Identified inventors	INV-BIO Inventors
Daily wage, Euro	Mean	59.0	78.9	156.2
	S.D.	47.2	52.1	30.0
Age	Mean	38.7	38.4	42.4
	S.D.	12.9	12.4	9.0
Females, %		45.2	14.8	5.7
N of obs., thousand		21,344	2,871	420

Notes: This table compares the summary statistics between workers in SIAB and the inventors in INV-BIO. Identified inventors in SIAB are workers who work in the following four occupations: "research and development", "machine-building and operations", "mathematics, biology, and physics", and "mechatronics, energy, and electronics." The worker in the table includes the identified inventors. The summary statistics are calculated using a pooled sample with daily wage, age, and gender filled in.

We estimate the following Probit model:

$$P(D_{it} = 1) = \Phi(\beta_0 + \beta_1 I_{it} + \beta_2 X_{it}) \quad (11)$$

Individual i is the job changer without an unemployment spell. I_{it} serves as a dummy for the inventors, taking a value of one if individual i works in one of the four occupations in year t, and zero otherwise. D_{it} equals one if individual i moves from a more productive establishment to a less productive one in year t, and zero if the move is to a more productive establishment. Note that we investigate the moving between establishments rather than ranks here. For constructing D_{it}, we use the number of employees or mean wage as a proxy for productivity. The vector of control variables, X_{it}, includes age, a square of age, gender, and educational attainment. To avoid the incidental parameter problem, we estimate the model without incorporating fixed effects.[11] The function Φ is the cumulative distribution function of the standard normal distribution.

The coefficient of our interest is β_1. The positive β_1 implies that inventors are more likely to move to less productive establishments than other workers. Standard errors (SEs) are clustered by destination establishment and year, supposing the presence of persistent establishment and year specific shocks.

Table 3 presents results. The first column uses the establishment size as the productivity measure, and the second column uses the mean wage as the measure. The results show

[11] The estimated value of β_1 in the linear model with the fixed effects is also significantly positive as in Table 3. See Online Appendix B.2.

Table 3: Estimation Result for Inventor Flows

	(1) $P(D_{it} = 1)$				(2) $\Delta \log w_{it}$	
	Whole sample		Sample with wage ↑			
I_{it}	.077***	.036***	.052***	.012***	.017***	.021***
	(.004)	(.004)	(.004)	(.004)	(.005)	(.004)
D_{it}					-.078***	-.084***
					(.006)	(.005)
$D_{it} \times I_{it}$					.016***	-.002
					(.006)	(.006)
Control	√	√	√	√	√	√
Fixed Effects					√	√
Measure for D_{it}	Size	Mean wage	Size	Mean wage	Size	Mean wage
N	3,572,567	3,533,344	2,082,939	2,060,714	859,888	859,861
Adj. R^2	.019	.016	.005	.003	.13	.13

Notes: Control variables include age, a square of age, gender, and educational attainment. Fixed effects include year, year × industry, and destination establishment fixed effects. I_{it} equals to one if individual i works in one of the four occupations ("research and development", "machine-building and operations", "mathematics, biology, and physics", and "mechatronics, energy, and electronics") in year t, and zero otherwise. D_{it} equals one if individual i moves to a less productive establishment in year t, and zero otherwise. The productivity measure is based on the establishment size or the mean wage in year $t-1$. The sample spans from 1980 to 2019. SEs clustered by year and establishments are reported in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

that the probability of an inventor transitioning to an establishment with fewer employees or a lower mean wage is higher than other workers, implying that inventors are more likely to move to a less productive establishment than other workers. Moving to the third and fourth columns, we narrow down the sample to job changers who experience wage increases, and the coefficients for I_{it} are still significantly positive. This suggests that many inventors move to less productive establishments and experience wage increases.

To further examine the association between the direction of flows and wages, we run the following regression:

$$\log w_{it} - \log w_{it-1} = \beta_0 + \beta_1 D_{it} + \beta_2 I_{it} + \beta_3 D_{it} I_{it} + \beta_4 X_{it} + \alpha + \varepsilon_{it} \tag{12}$$

The variable w_{it} represents the daily wage of individual i after a job change, while w_{it-1} represents the wage before the job change. The vector of fixed effects α includes year, year × industry, and destination establishment fixed effects. The definition of other variables remains the same as in the equation (11).

The last two columns of Table 3 show the estimation results. The coefficients for I_{et} are significantly positive, indicating that inventors experience greater wage increases by around 2% through job changes than other general workers.

The coefficients of D_{it} are negative, meaning that workers tend to experience fewer wage increases when moving to less productive establishments. However, the coefficients for $D_{it} \times I_{it}$ are significantly positive in the fifth column. The positive coefficient implies that inventors experience fewer wage decreases by moving to less productive establishments than general workers.

Knowledge transfer with inventor mobility has the potential to explain these results. That is, an inventor who worked in a high-productivity establishment can transfer that knowledge to a low-productivity establishment when changing jobs. Therefore, establishments are more willing to poach inventors from more productive establishments than other general workers and compensate inventors for the benefits.

3.4 Empirical Evidence of Knowledge Diffusion

The result in the previous section suggests the presence of knowledge diffusion via inventor flows. This section further investigates how the inventor flows influence the productivity growth of establishments.

Our specification is given by:

$$\log z_{et+j} - \log z_{et} = \beta_0 + \beta_1 H\text{-}Share_{et} + \beta_2 X_{et} + \alpha_e + \alpha_t + \varepsilon_{et} \tag{13}$$

z_{et} is the knowledge quality of establishment e in year t, as defined in (10) of Section 3.2. We use three, four, or five-year forward citations for z_{et}. The variable $H\text{-}Share_{et}$ represents the percentage share of inventor inflows from establishments with higher knowledge base measured by patent catitions, to total inventor inflows to establishment e. $\log n_{et}$ is the log of the number of inventors. The vector of control variables X_{et} includes the log of the establishment size, number of inventors, mean wage, and z_{et}. The vector of fixed effects α includes year, year $\times$ industry, and establishment fixed effects. The equation (13) is estimated using INV-BIO from 1980 to 2019. Standard errors (SEs) are clustered by destination establishment and year.

The results are reported in Table 4. The table shows that when more inventors come from productive establishments, the knowledge growth of the poaching establishments is

Table 4: Estimation Result for Knowledge Growth

	$\log z_{et+j} - \log z_{et}$				
	$j = 3$	$j = 4$	$j = 5$	$j = 3$	$j = 3$
H-Share$_{et}$ (%)	.0022***	.0028***	.0027***	.0024***	.0024***
	(.0004)	(.0004)	(.0004)	(.0003)	(.0003)
Control	√	√	√	√	√
Fixed Effects	√	√	√	√	√
Citation	3y fwd	3y fwd	3y fwd	4y fwd	5y fwd
N	24,625	22,270	19,982	26,791	26,451
Adj R^2	.21	.27	.36	.23	.25

Notes: Control variables are z_{et}, log of a number of employees and mean wage. Fixed effects include year, year × industry, and establishment fixed effects. The sample spans from 1980 to 2014. z_{et} is the forward citation measure (backward 3-year moving average). *H-Share*$_{et}$ is the share of the number of inventors who moved from establishments with a higher $z_{e't-1}$ at $t-1$. If there are no inflows, *H-Share*$_{et}$ is set to zero. SEs clustered by year and establishment are reported in parenthesis. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$, .

higher over a period of three, four, or five years.

However, the coefficient of *H-Share*$_{et}$ is susceptible to the endogeneity problem. The unobservable expectation for $\log z_{et+j} - \log z_{et}$ can be correlated with the realized $\log z_{et+j} - \log z_{et}$ and *H-Share*$_{et}$. To address this issue of omitted variable bias[12], we employ an instrumental variable (IV) strategy. In this approach, we utilize the patent citation rank for establishments in their states from the previous year (referred to as *Regional Rank*$_{et-1}$) as an instrument for *H-Share*$_{et}$. In the first stage, the *Regional Rank*$_{et-1}$ is expected to be highly correlated with *H-Share*$_{et}$. A lower knowledge rank indicates a higher number of establishments with greater knowledge level in the state. Consequently, the share of inventors poached from these highly knowledgeable establishments (*H-Share*$_{et}$) is more likely to be higher. The instrument can be considered to satisfy the exclusion condition when taking into account the fixed effects and control variables. To address the issue of mean reversion in knowledge quality, we add z_{et-1} as one of the control variables.

Table 5 shows the result using the IV. In the first stage, we find a significant correlation between *Regional Rank*$_{et-1}$ and *H-Share*$_{et}$. Specifically, if the knowledge level is relatively lower within the state (indicated by a higher value of *Regional Rank*$_{et-1}$), *H-Share*$_{et}$ tends to

[12]This omitted variable bias can have either an upward or a downward effect. If there is an expectation of high productivity growth, establishments may offer higher wages to attract more inventors from highly productive establishments, leading to an upward bias. On the other hand, if lower productivity growth is anticipated, establishments may attempt to offset the lower growth by poaching inventors from more productive establishments, resulting in a downward bias.

Table 5: Estimation Result for Knowledge Growth using IV

	$\log z_{et+j} - \log z_{et}$				
	$j = 3$	$j = 4$	$j = 5$	$j = 3$	$j = 3$
H-Share$_{et}$ (%)	.092***	.101***	.103***	.084***	.088***
	(.007)	(.007)	(.009)	(.005)	(.006)
Control	√	√	√	√	√
Fixed Effects	√	√	√	√	√
Citation	3y fwd	3y fwd	3y fwd	4y fwd	5y fwd
First Stage IV					
Regional Rank$_{et-1}$	24.4***	22.7***	22.8***	30.0***	29.3***
	(1.7)	(1.5)	(1.8)	(1.7)	(1.7)
N	22,213	20,052	17,996	23,137	23,609
F statistic	204.8	232.6	155.2	302.7	286.2

Notes: Control variables are the log of the establishment size, number of inventors, mean wage, z_{et}, and z_{et-1}. The sample spans from 1980 to 2014. z_{et} is the the forward citation measure at t. *H-Share*$_{et}$ is the share of the number of inventors who moved from establishments with a higher z at $t-1$. *Regional Rank*$_{et-1}$ is the establishment's rank of z_{et-1} among all establishments in the state (16 states). A higher rank means lower z_{et-1}. The rank is normalized so that the maximum is equal to 1. SEs clustered by year and establishment are reported in parenthesis. *** $p < 0.01$, *** $p < 0.05$. The first stage F value is the Cragg-Donald Wald F statistic.

be higher.

The impact of poaching from more knowledgeable establishments is highly significant and even larger than the results obtained from the OLS in Table 4. In the first column in Table 5 using the three-year forward citations, 1% increase in the share of inventors from more productive establishments increases citations by around 10% relative to the unconditional mean. In sum, our results suggest that establishments can enhance their knowledge growth by recruiting inventors from high-productivity establishments.

4 Quantitative Analysis

This section quantifies the effects of inventor job flows and knowledge transfers on innovation and productivity and studies the effects of counterfactual policy. To do this, we calibrate the model from Section 2 to match the data desrcibed in Section 3. We subsequently demonstrate that the calibrated model closely fits the data for targeted moments, and we use it to examine the effects of policies related to the labor market for inventors.

4.1 Stochastic Process for In-House R&D Ability and Functional Forms

In this section, we introduce a stochastic process to characterize the unpredictable nature of in-house R&D ability, following the model presented in Benhabib et al. (2021). We set the functional form of the in-house R&D function to $\gamma(n, i) = \overline{\gamma}_i n^{\delta}$, where the index i represents the in-house R&D ability, which can either be high (h) or low (l). The innovation ability i follows a two-state Markov process. The R&D capacity is greater when in the h state than in the l state ($\overline{\gamma}_h > \overline{\gamma}_l$). The transition intensity (the rate at which the R&D ability changes) from the l to h state is denoted by λ_l, and the transition intensity from the h to l state is denoted by λ_h. We employ this two-state Markov process to formulate the stochastic innovation process because it allows the composition of firms at the technology frontier to change over time, which is essential for the existence of a stationary distribution on a BGP.

We will conduct our numerical exercises by calibrating the model with high transition rates. The characteristics of the stochastic process with conditional draws are similar to those with unconditional draws when the switching rates are high, as estimated in our calibrations.

For other functional forms, we assume that the vacancy cost function is $c(v) = \frac{\overline{c}}{\phi+1} v^{\phi+1}$. The knowledge diffusion rate function is $\alpha(z'/z) = \overline{\alpha}\,(z'/z)^{\beta}$ such that the knowledge diffusion rate is increasing in the productivity of the poached firm relative to the poaching firm.

4.2 Calibration

We calibrate the model along a BGP equilibrium to match features of the allocation of inventors across establishments and characteristics of inventor job flows between establishments.

Externally Set or Normalized

We normalize or set to standard values six parameters, as summarized in the Panel A of Table 6. The discount rate ρ implies an annual real interest rate of 5%. The first-order condition for vacancies implies that we cannot identify $\overline{c}$ and A separately, so we normalize $\overline{c}$. We normalize the productivity of technology frontier $\overline{z}$ and the measure of firms $\mathtt{m}$ to 1 without loss of generality. We also set l-type R&D coefficient $\overline{\gamma}_l$ to zero without loss of

Table 6: Parameter Values

Parameter	Description	Value
— Panel A. Externally Set or Normalized —		
ρ	Discount Rate	0.0041
$\bar{z}$	Frontier Productivity	1
$\mathtt{m}$	Measure of Firms	1
$\bar{c}$	Vacancy Cost Coefficient	100
ϕ	Vacancy Cost Elasticity	3.45
$\overline{\gamma}_l$	l-type R&D Coefficient	0
— Panel B. Direct Match to Data —		
$\mathtt{n}$	Measure of Inventors	5
λ_h	Jump Intensity: $h \to l$	0.02
λ_l	Jump Intensity: $l \to h$	0.01
— Panel C. SMM Calibration —		
β	Diffusion Curvature	0.33
$\overline{\alpha}$	Diffusion Rate	0.0012
$\overline{\gamma}_h$	h-type R&D Coefficient	0.0006
η	Leapfrog	0.0001
δ	R&D Curvature	0.35
A	Matching Efficiency	0.26

Notes: List of model parameters and calibrated values. In Panel C, all parameters are calibrated jointly for the SMM calibration.

generality[13]. We use the value of the vacancy cost elasticity calibrated by Bilal et al. (2022).

Direct Match to Data

We set three parameters to directly match the moments from German inventor data, as summarized in Panel B of Table 6. The measure of inventor $\mathtt{n}$ is determined by the average number of inventors per establishment, given a unit measure of firms.

The transition rate of innovation ability λ_h and λ_l match the estimations from the two-state Markov transition matrix for the growth rate of the knowledge in establishments

[13] See Benhabib et al. (2021)

near the technology frontier in the spirit of Benhabib et al. (2021). Details are as follows. The knowledge in establishments is measured by patent citation, and we designate establishments within the top 10% of the 5-year forward citation measure, Z_{et} in Section 3.2, as the frontier each year. Among these, establishments exhibiting positive growth rates of the citations are categorized as being in the high state, whereas the remaining are seen as being in the low state. Based on this, we estimate the transition matrix.

Internal Calibration Using SMM

We estimate the six key parameters of the model listed in the Panel C of Table 6. These parameters are captured by the vector: $\Theta = \left\{\beta, \overline{\alpha}, \overline{\gamma}_h, \eta, \delta, A\right\}$ and estimated by minimizing the objective function

$$\mathcal{L}(\Theta) = (\hat{m} - m(\Theta))' W^{-1} (\hat{m} - m(\Theta))$$

where $\hat{m}$ is a vector of empirical moments and $m(\Theta)$ are their model counterparts. The diagonal components of matrix W have the same weights. All non-diagonal components are zero. In the case of distributional information, the weights are adjusted so that the weights of the entire distribution add up to one. For example, the unconditional distribution of inventors is characterized by five quantile points and therefore weights 1/5. All non-diagonal components are zero. [14]

4.3 Results

Table 7: Targeted Moments

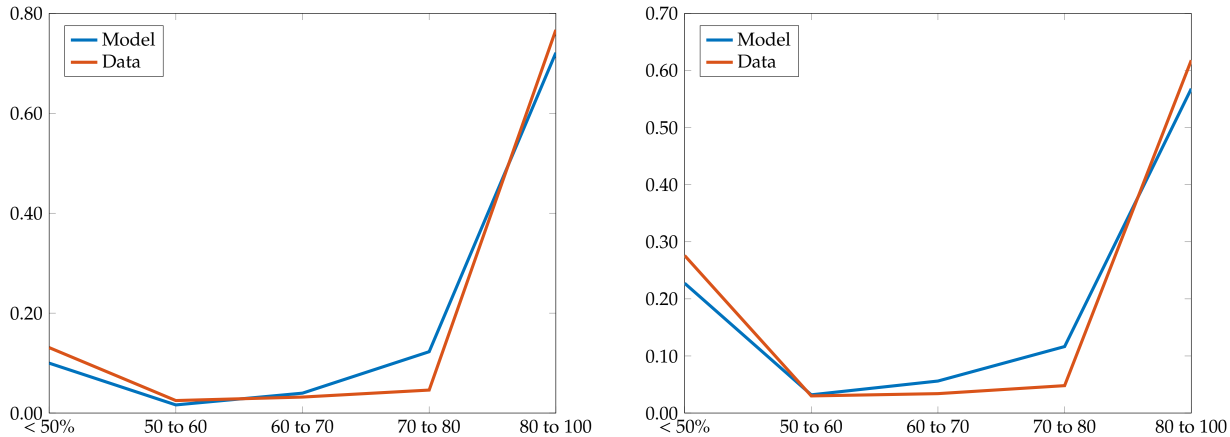
Moments	Data	Model
EE rate (%, monthly)	1.17	1.13
Growth rate (%, monthly)	0.16	0.13
Distribution of inventor by firm ranking	Figure 2a	
Distribution of inventor flow by poaching firm ranking	Figure 2b	

Notes: EE rate and growth rate are both monthly frequencies; EE rate is calculated by dividing the number of job changers by the number of inventors in the INV-BIO data.

[14]Prior to estimation, we did the first 100 iterations for each of the six parameters and excluded regions that did not converge. Then the parameter space we explore is as follows: β ranges from 0.1 to 0.7, $\overline{\alpha}$ ranges from 0.001 to 0.003, $\overline{\gamma}_h$ ranges from 0.0001 to 0.001, η ranges from 0.00005 to 0.0003, δ ranges from 0.2 to 0.5, and A ranges from 0.05 to 0.3. For each of these parameter spaces, we take 20 grids and compute 2000 Halton grids.

Figure 2: Inventor Distributions by Firm Productivity

(a) Density Weighted by the Mass of Inventors (b) Density Weighted by Inventor Inflows



Notes: The data in (a) and (b) are plots of the distribution in Table B.3. (A) and the marginal distribution of poaching firms in Table 1 (B). The corresponding model values are calculated using the productivity of the firms, the inventor and their joint density $f(\hat{z}, n, t)$ under the calibrated parameters.

Table 7 and Figure 2 summarize the target moments and parameters. Not only do the macro moments (EE rate and growth rate) in Table 7 provide a good fit, but Figure 2 also shows that the joint distribution of inventors and firms is well replicated. Overall, despite over-identification, the fit of the moments within the internal calibration is reasonably good.

Although the parameters are calibrated jointly, we will discuss the most relevant moments for each parameter. First, the EE rate primarily provides information about the matching efficiency A, which governs the size of the job flow. The growth rate has information mainly on γ_h. Both α and β are related to knowledge diffusion, where β controls for the sensitivity of knowledge diffusion to the difference in productivity between poaching firms and incumbents, and α adjusts the average size of knowledge diffusion. These parameters predominantly determine poaching firms' distribution over productivity in Figure 2b. Finally, Figure 2a, representing the relationship between productivity and inventors, primarily informs δ and η.

Next, we discuss the fit of the calibrated model for a important non-targeted regression result. In our empirical analysis, using equation (13), we find that the greater the share of inventor inflows from higher knowledge firms, the greater the productivity gains of the poaching firms. We examine whether the model can replicate this relationship. The corresponding equation of the model is given by

$$\hat{\mu}_z(\hat{z}, n, t) = \beta_0 + \beta_1 \textit{H-Share} + \beta_2 \log n + \varepsilon$$

where $\hat{\mu}_z(\hat{z}, n, t)$ is the growth rate of productivity and *H-Share* is the fraction of $\hat{\mu}_n(\hat{z}, n, t)$ who transitioned from firms with higher productivity. We compute the coefficients using weighted least squares, with the density function of the firms $f(\hat{z}, n, t)$ in each grid as the sample. The model-implied coefficient β_1 is 0.003, well within the range of the OLS and IV estimates in Table 4, thereby successfully producing a reasonable quantitative magnitude.

4.4 Quantitative Exercises

The previous section demonstrated that the calibrated model accurately aligns with the data for both targeted and non-targeted moments. Consequently, the model is well-suited for conducting counterfactual analyses. Initially, we will assess the impact of changes in matching efficiency in frictional labor markets for inventors. Subsequently, we will investigate the ramifications of a hypothetical policy intervention. In this policy analysis, we suggest a hypothetical policy that offers subsidies to frontier firms, aiming to foster innovation within these entities.

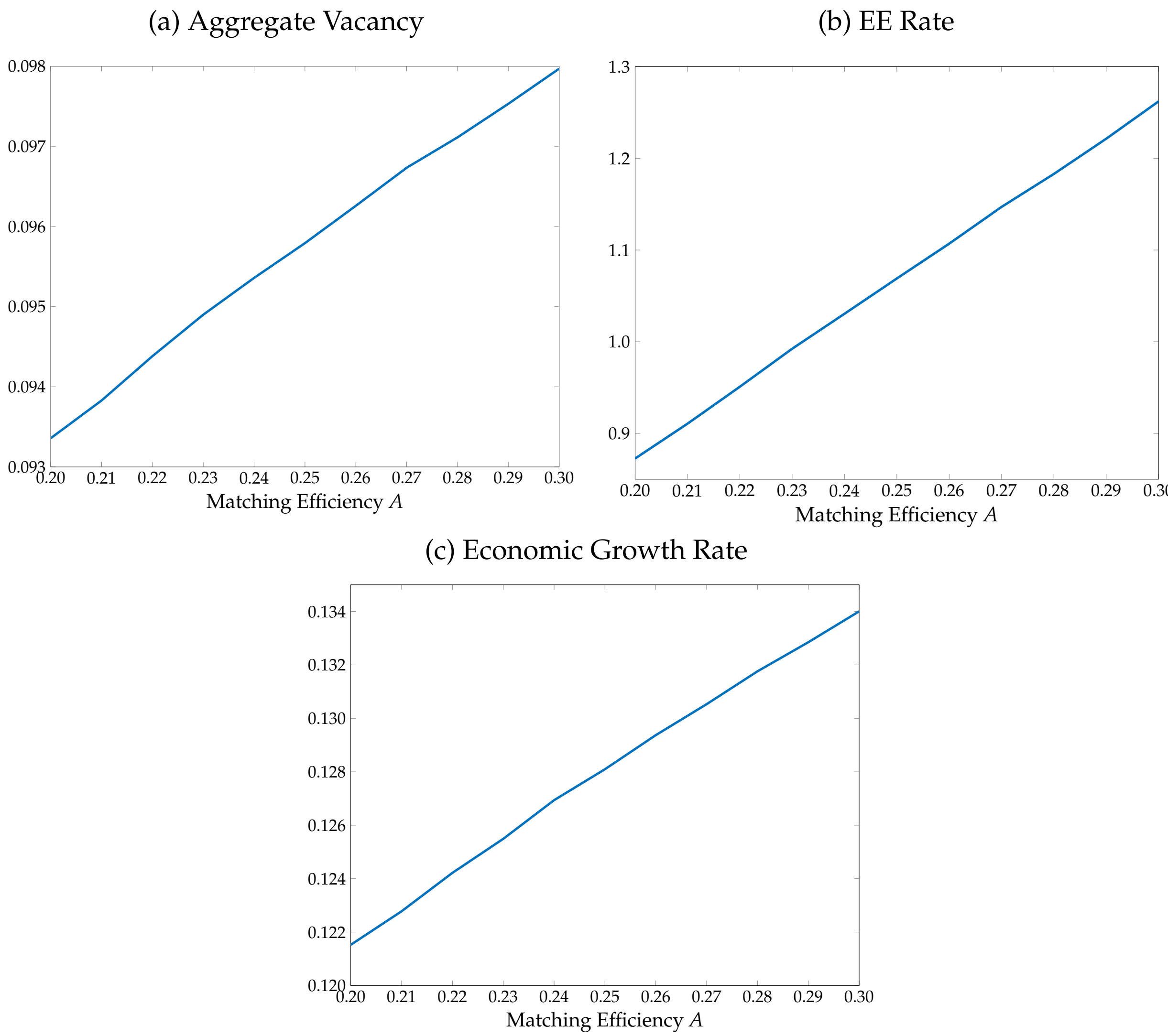
Quantifying the Impact of Inventor Market Frictions

How do changes in frictions within the market for inventors impact the economy? Frictions in this market can emerge from a variety of sources, such as search and matching frictions, regulations related to labor, and agreements between employees and firms. To assess the impact of shifts in market frictions for inventors, we conduct a comparative static analysis focusing on the matching efficiency parameter A.

As Figure 3 (a) shows, in our model, an increase in matching efficiency increases the vacancy posting of firms. Also, as Figure 3 (b) shows, higher matching efficiency and more vacancy postings lead to higher job-to-job transition rates for the inventor.

As matching efficiency increases, the economic growth rate increases (Figure 3 (c)) through the following mechanism. First, as the job-to-job transition rate increases, knowledge diffusion becomes more active. As a result, the dispersion of firm productivity decreases (Figure 4 (b)). Note that high-productivity firms grow mainly through in-house R&D, while low-productivity firms grow mainly through knowledge diffusion (See equation (2)). Since lower variance in firm productivity reduces growth through knowledge

Figure 3: Comparative Statics: Aggregate Variables

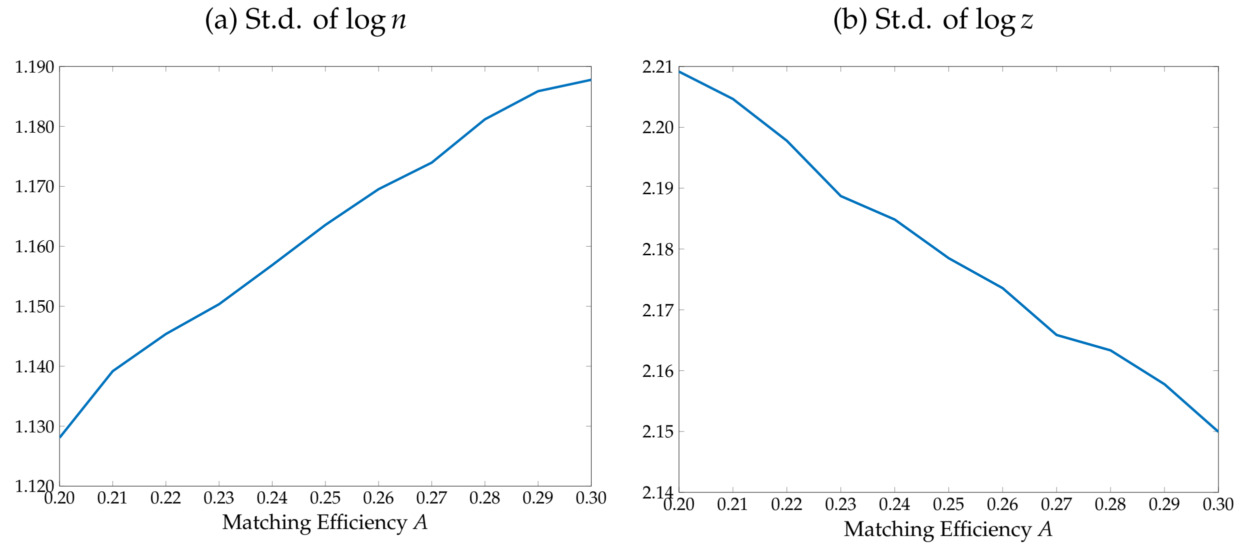


Notes: The figure display the comparative statics of varying matching efficiency A.

diffusion, it leads to a relative increase in inventor hiring by more productive firms. As a result, more inventors are attracted to firms in the technology frontier. As shown in Lemma 2, the economic growth rate in a BGP is determined by the productivity growth rate of frontier firms. Therefore, the economic growth rate also increases (Figure 3 (c)).

Our model links the observed decrease in inventor mobility in Germany and the US to the low economic growth in developed countries in recent years, as documented in the secular stagnation literature (e.g., Summers (2014); Eggertsson et al. (2019); Akcigit and Ates (2021)). In the INV-BIO data, we find a significant decline in the inventor job

Figure 4: Comparative Statics: Distribution



Notes: The figure display the comparative statics of varying matching efficiency A.

Table 8: The Change in the Second-Order Moments of the Distribution in the Data

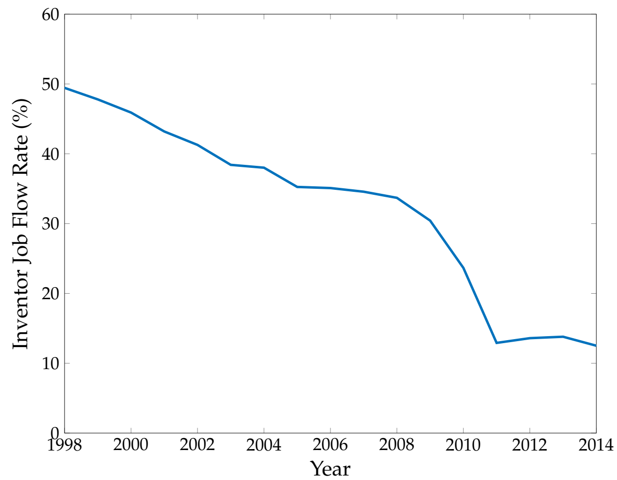
	1998	2014
Coefficient of variation of N of inventors	0.93	1.12
Coefficient of variation of productivity	2.93	0.27

Notes: The first column shows the change in the coefficient of variation of the number of inventors working in each establishment in the INV-BIO data. We compare the values for 1998, the first year of Figure 5, and 2014, the last year. Similarly, the second column shows the change in the coefficient of variation of the establishments' innovativeness measured by patent citation.

flow rate over time (Figure 5). Similarly, Akcigit and Goldschlag (2023a) document that inventor mobility has decreased since the early 2000s in the US. Our model indicates that the decline in the matching efficiency, which subsequently leads to a reduction in job-to-job transitions, contributes to a lower economic growth rate. Thus, our model provides a framework linking the observed decrease in inventor mobility to the slowdown in aggregate productivity growth in developed countries over the past few decades.

In our model, changes in the distribution of firms and inventors play an essential role in the mechanism linking inventor mobility and economic growth rates, and indeed, the changes in the distributional characteristics of the model are consistent with the data. The first row of Table 8 shows the coefficient of variation of the number of inventors working in each establishment. Here, we compare the values for 1998, the first year of Figure 5,

Figure 5: Inventor Job Flow Rate in Germany



Notes: This figure shows the job flow rate for German inventors, calculated using INV-BIO data.

and 2014, the last year. Similarly, the second row shows the coefficient of variation of the establishments' innovativeness measured by patent citation. The direction of these changes in the data is consistent with the direction of change in the variance of z and the variance of n in our model (Figure 4) when the matching efficiency A decreases.

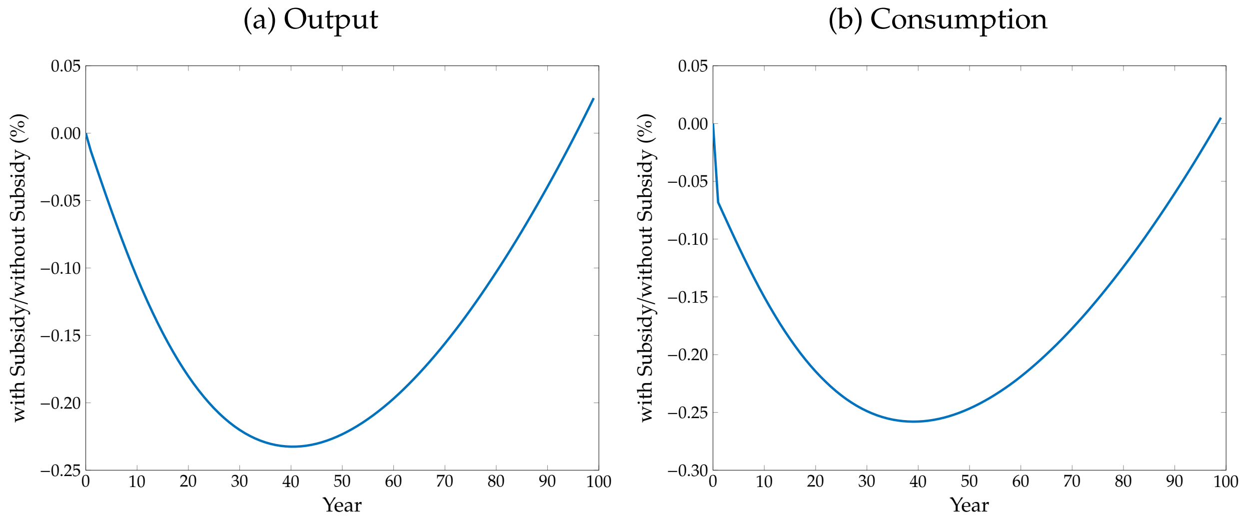
Policy Exercises: Subsidy for Firms Near the Technology Frontier

This section analyzes the consequences of subsidies for firms near the technology frontier. Policy-makers may consider encouraging R&D activity by offering subsidies to technologically progressive firms. Alternatively, they may wish to subsidize technologically lagging firms to promote knowledge diffusion through the movement of inventors. To find the more beneficial policy, we examine the transition from an initial BGP without subsidy to a new BGP with a subsidy aimed at technologically frontier firms.

We investigate the transition from an initial BGP with no subsidy to a new BGP with a subsidy rate 10% for frontier firms. We define frontier firms as those with productivity z in the top half of the distribution, weighted by the number of inventors[15]. We assume subsidies are financed by a constant rate tax on the bottom half of the distribution. This exercise imposes a 17% tax on the production of these remaining firms. Subsidies and

[15] In the distribution weighted by the number of inventors, the firms with productivity z in the top half of the distribution correspond to approximately the top 10% of firms with productivity in the unweighted distribution.

Figure 6: Transition Dynamics



Notes: The figures display transitional dynamics on implementing a counterfactual subsidy for technologically frontier firms. Panel (a) shows the path for aggregate output relative to the old aggregate output. Panel (b) shows the path for aggregate consumption.

taxation changes are permanent. The agents do not anticipate the policy change until $t = 0$, and they are perfect foresight after $t = 1$.

The left panel of Figure 6 displays the output path relative to the baseline balanced growth path (BGP). Aggregate output decreases during the initial 40 years, but shows an increase in the long run. In the short run, this policy hinders job flows from technology-leading firms to laggard firms, thereby impeding knowledge diffusion to the latter. However, over the long term, it enhances the growth rate of the technology frontier, which ultimately fosters positive impacts on knowledge diffusion to laggard firms. Consequently, the rate of economic growth increases in the long term.

Lastly, we calculate the policy change's welfare effects by analyzing the economy's transitional dynamics, applying a discount rate ρ to future periods following policy implementation. Our results indicate a decline in welfare of 0.14%, measured in terms of consumption equivalent. This outcome is primarily driven by a short-term decline in aggregate productivity. However, this effect is largely offset by a long-term increase in productivity. Our analysis suggests that the effectiveness of a policy is dependent on the policymaker's time horizon.

5 Conclusion

This paper explores labor markets where inventors and firms interact, focusing on the implications for the distribution of inventors across firms, knowledge diffusion, and productivity growth. To examine these dynamics, we construct an endogenous growth model that incorporates frictions in labor markets for inventors. In our model, inventors (i) contribute to in-house R&D efforts, enhancing the firm's technology, and (ii) facilitate knowledge transfer from their previous employers to their new ones when they change jobs. Heterogeneous firms create job openings, considering the knowledge transfer from the inventors' prior employers. To quantify this framework, we utilize data on inventors and patents connected to administrative labor market career information about individuals and their employing establishments in Germany. We find three empirical findings: (i) inventors are more likely to transition to less productive establishments compared to general workers, (ii) inventors face significant increases in wages when changing jobs compared to general workers, and (iii) the number of patent citations increases more rapidly when a higher proportion of their inventors originates from higher productivity establishments. We calibrate our model to align with these empirical findings and demonstrate that the calibrated model closely fits both target and non-target moments, confirming its suitability for conducting counterfactual exercises. We then examine the transition from an initial Balanced Growth Path (BGP) without a subsidy to a new BGP with a subsidy targeting technologically frontier firms. In the short term, this subsidy decreases aggregate output by discouraging inventor mobility from frontier firms to laggard firms, thereby hindering knowledge diffusion. However, in the long term, the impact of the subsidy on output is reversed, as it enhances the growth rate of the technological frontier.

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Appendix

A Theoretical Derivations

A.1 Normalizing the Distribution

Differentiate (6) with respect to $\hat{z}$ and n and use the definition $z \equiv \hat{z}/\bar{z}(t)$,

$$\frac{1}{\bar{z}(t)}\frac{\partial}{\partial z}\frac{\partial}{\partial n}F(\hat{z}/\bar{z}(t), n, t) = \frac{\partial}{\partial \hat{z}}\frac{\partial}{\partial n}\hat{F}(\hat{z}, n, t)$$

$$f(z, n, t) = \bar{z}(t)\hat{f}(\hat{z}, n, t) \tag{14}$$

Differentiate (14) with respect to t and use the chain rule to obtain the transformation of the time derivative:

$$\frac{\partial}{\partial t}f(\hat{z}/\bar{z}(t), n, t) - \frac{\hat{z}}{\bar{z}(t)}\frac{\bar{z}'(t)}{\bar{z}(t)}\frac{\partial}{\partial z}f(\hat{z}/\bar{z}(t), n, t) = \bar{z}'(t)\hat{f}(\hat{z}, n, t) + \bar{z}(t)\frac{\partial}{\partial t}\hat{f}(\hat{z}, n, t).$$

Define the growth rates of the technology frontier as $g(t) \equiv \bar{z}'(t)/\bar{z}(t)$. Use the definition of $g(t)$ and the definition $z \equiv \hat{z}/\bar{z}$(t),

$$\frac{\partial}{\partial t}f(z, n, t) - zg(t)\frac{\partial}{\partial z}f(z, n, t) = g(t)\bar{z}(t)\hat{f}(\hat{z}, n, t) + \bar{z}(t)\frac{\partial}{\partial t}\hat{f}(\hat{z}, n, t)$$

Use (14),

$$\frac{\partial}{\partial t}f(z, n, t) - zg(t)\frac{\partial}{\partial z}f(z, n, t) - g(t)f(z, n, t) = \bar{z}(t)\frac{\partial}{\partial t}\hat{f}(\hat{z}, n, t). \tag{15}$$

Next, we derive the law of motion for firm-level detrended productivity. Note that

$$\begin{aligned}\frac{d}{dt}z &= \frac{d}{dt}\left(\frac{\hat{z}}{\bar{z}(t)}\right)\\ &= \frac{\frac{d}{dt}\hat{z}}{\bar{z}(t)} - \frac{\hat{z}}{\bar{z}(t)}\frac{\bar{z}'(t)}{\bar{z}(t)}\\ &= \frac{\frac{d}{dt}\hat{z}}{\bar{z}(t)} - g(t)z\end{aligned}$$

Therefore, the drift of detrended productivity is

$$\mu_z(z,n,t) = \frac{\hat{\mu}_z(z,n,t)}{\bar{z}(t)} - g(t)z \tag{16}$$

Use (2), and then, use (6), (7) , and (8)[16],

$$\mu_z(z,n,t) = (\gamma(n) - g(t))\, z + Av(z,n,t)Z(t) \int \mathbb{1}_P(z,n,z',n',t)\alpha(z'/z)dF_n(z',n',t).$$

Similarly, we derive the law of motion for firm-level employment growth, which is a function of detrended variables. Substitute (6), (7), and (8) into (3),

$$\begin{aligned}\mu_n(z,n,t) \equiv& \hat{\mu}_n(\hat{z},n,t)\\ =& Av(z,n,t) \int \mathbb{1}_P(z,n,z',n',t)dF_n(z',n',t) - A\mathbf{v}\frac{n}{\mathbf{n}} \int \mathbb{1}_P(x,n,x',n',t)dF_v(z',n',t).\end{aligned} \tag{17}$$

Multiply (4) by $\bar{z}(t)$,

$$\begin{aligned}\bar{z}(t)\frac{\partial}{\partial t}\hat{f}(\hat{z},n,t) =& -\frac{\partial}{\partial n}\left(\hat{\mu}_n(\hat{z},n,t)\bar{z}(t)\hat{f}(\hat{z},n,t)\right) - \frac{\partial}{\partial \hat{z}}\left(\hat{\mu}_z(\hat{z},n,t)\bar{z}(t)\hat{f}(\hat{z},n,t)\right)\\ & - \eta\bar{z}(t)\hat{f}(\hat{z},n,t) + \eta\int_0^{\bar{z}} \bar{z}(t)\hat{f}(\hat{z},n,t)d\hat{z}\boldsymbol{\delta}(\bar{z})\end{aligned}$$

Use (14), (16), and (17),[17]

$$\begin{aligned}\frac{\partial}{\partial t}f(z,n,t) =& zg(t)\frac{\partial}{\partial z}f(z,n,t) + g(t)f(z,n,t) - \frac{\partial}{\partial n}\left(\mu_n(z,n,t)f(z,n,t)\right) - \frac{\partial}{\partial z}\left(\left(\mu_z(z,n,t) - g(t)z\right)f(z,n,t)\right)\\ & - \eta f(z,n,t) + \eta\int_0^1 f(z,n,t)dz\boldsymbol{\delta}(1)\end{aligned}$$

[16]For any set of functions h and $\hat{h}$ such that $h(z,n) = \hat{h}(\hat{z},n)$, using change of variables, $\int\int \hat{h}(\hat{z},n)\hat{f}(\hat{z},n)d\hat{z}dn = \int\int h(z,n)zf(z,n)(1/z)dzdn = \int\int h(z,n)f(z,n)dzdn$. Therefore, $\int \hat{h}(\hat{z},n)d\hat{F}(\hat{z},n) = \int h(z,n)dF(z,n)$.

[17]For any set of functions h and $\hat{h}$ such that $h(z,n) = \hat{h}(\hat{z},n)$, $\frac{\partial}{\partial \hat{z}}\hat{h}(\hat{z},n) = \frac{\partial}{\partial z}h(z,n)\frac{dz}{d\hat{z}} = \frac{\partial}{\partial z}h(z,n)\cdot\frac{1}{\bar{z}(t)}$.

Because $\frac{\partial}{\partial z}\left(zg(t)f(z,n,t)\right) = zg(t)\frac{\partial}{\partial z}f(z,n,t) + g(t)f(z,n,t)$, we get

$$\frac{\partial}{\partial t}f(z,n,t) = -\frac{\partial}{\partial n}\left(\mu_n(z,n,t)f(z,n,t)\right) - \frac{\partial}{\partial z}\left(\mu_z(z,n,t)f(z,n,t)\right) - \eta f(z,n,t) + \eta\int_0^1 f(z,n,t)dz\boldsymbol{\delta}(1)$$

A.2 Normalizing the Value Function

Differentiate (5) with respect to $\hat{z}$ and rearrange,

$$\frac{\partial}{\partial z}\Omega(z,n,t) = \frac{\partial}{\partial z}\hat{\Omega}(\hat{z},n,t). \tag{18}$$

Differentiate (5) with respect to n and rearrange,

$$\frac{\partial}{\partial n}\Omega(z,n,t) = \frac{\partial}{\partial n}\hat{\Omega}(\hat{z},n,t)/\overline{z}(t). \tag{19}$$

Rearrange and differentiate (5) with respect to t,

$$\frac{\partial}{\partial t}\hat{\Omega}(\hat{z},n,t) = \overline{z}'(t)\Omega(\hat{z}/\overline{z}(t),n,t) - \hat{z}\frac{\overline{z}'(t)}{\overline{z}(t)}\frac{\partial}{\partial z}\Omega(\hat{z}/\overline{z}(t),n,t) + \overline{z}(t)\frac{\partial}{\partial t}\Omega(\hat{z}/\overline{z}(t),n,t).$$

Divided by $\overline{z}(t)$ and use the definition of $g(t) \equiv \overline{z}'(t)/\overline{z}(t)$ and the definition $z \equiv \hat{z}/\overline{z}(\mathrm{t})$,

$$\frac{1}{\overline{z}(t)}\frac{\partial}{\partial t}\hat{\Omega}(\hat{z},n,t) = g(t)\Omega(z,n,t) - g(t)z\frac{\partial}{\partial z}\Omega(z,n,t) + \frac{\partial}{\partial t}\Omega(z,n,t). \tag{20}$$

Divide (1) by $\overline{z}(t)$ and then substitute (18), (19), and (20), and use (6),

$$\begin{aligned}(r(t)-g(t))\,\Omega(z,n,t) + \frac{\partial}{\partial t}\Omega(z,n,t) = &\max_{v\geq 0}\; z - c(v)Z(t)\\ &+ Av\int\left[\Omega_n(z,n,t) + \alpha(z'/z)Z(t)\Omega_z(z,n,t) - \Omega_n(z',n',t)\right]^+ dF_n(z',n',t)\\ &+ (\gamma(n)-g(t))\,z\Omega_z(z,n,t)\\ &+ \eta\left[\Omega(1,n,t) - \Omega(z,n,t)\right]\end{aligned}$$

Use $r(t) = \rho + g(t)$,

$$\begin{aligned}\rho\Omega(z,n,t) + \frac{\partial}{\partial t}\Omega(z,n,t) = \max_{v\geq 0}\; & z - c(v)Z(t) \\ & + Av\int \left[\Omega_n(z,n,t) + \alpha(z'/z)Z(t)\Omega_z(z,n,t) - \Omega_n(z',n',t)\right]^+ dF_n(z',n',t) \\ & + (\gamma(n) - g(t))\, z\Omega_z(z,n,t) \\ & + \eta\left[\Omega(1,n,t) - \Omega(z,n,t)\right]\end{aligned}$$

A.3 $\Omega_z(z,n) > 0$

Rewrite the problem in terms of $x = \log z$. Denote with an abuse of notation, $\Omega(x,n) = \Omega(e^z, n)$ and $v(x,n) = v(e^z, n)$. Also, denote $\hat{\alpha}(x,x') = \alpha(\exp(x'-x))/\exp(x)$. Then, the HJB equation becomes

$$\begin{aligned}\rho\Omega(x,n) = & \exp(x) - c(v(x,n))Z \\ & + Av(x,n)\int \left[\Omega_n(x,n) + \hat{\alpha}(x,x')Z\Omega_x(x,n) - \Omega_n(x',n')\right]^+ dF_n(x',n') \\ & + (\gamma(n) - g)\,\Omega_x(x,n) \\ & + \eta\left[\Omega(0,n) - \Omega(x,n)\right].\end{aligned} \tag{21}$$

Denote $\zeta(x,n) = \Omega_x(x,n)$. Differentiate the Bellman equation (21) w.r.t. x and use the envelope theorem,

$$\begin{aligned}\rho\zeta(x,n) = & \exp(x) \\ & Av(x,n)\frac{\partial}{\partial x}\int \left[\Omega_n(x,n) + \hat{\alpha}(x,x')Z\Omega_x(x,n) - \Omega_n(x',n')\right]^+ dF_n(x',n') \\ & + (\gamma(n) - g)\,\zeta_x(x,n) \\ & - \eta\zeta(x,n).\end{aligned} \tag{22}$$

Let define the poaching indicator function as

$$\mathbb{1}_P(x,n,x',n') = \begin{cases} 1 & \text{if} \quad \Omega_n(x,n) + \hat{\alpha}(x,x')Z\Omega_x(x,n) - \Omega_n(x',n') > 0 \\ 0 & \text{otherwise} \end{cases}$$

Then,

$$\begin{aligned}
&\frac{\partial}{\partial x}\int \left[\Omega_n(x,n)+\hat{\alpha}(x,x')Z\Omega_x(x,n)-\Omega_n(x',n')\right]^+ dF_n(x',n') \\
&= \int \frac{\partial}{\partial x}\left[\Omega_n(x,n)+\hat{\alpha}(x,x')Z\Omega_x(x,n)-\Omega_n(x',n')\right]^+ dF_n(x',n') \\
&= \int \mathbb{1}_P(x,n,x',n')\frac{\partial}{\partial x}\left[\Omega_n(x,n)+\hat{\alpha}(x,x')Z\Omega_x(x,n)-\Omega_n(x',n')\right] dF_n(x',n') \\
&= \int \mathbb{1}_P(x,n,x',n')\left[\zeta_n(x,n)+\hat{\alpha}_x(x,x')Z\zeta_x(x,n)+\hat{\alpha}(x,x')Z\zeta_x(x,n)\right] dF_n(x',n')
\end{aligned}$$

The second equality follows because for any differentiable function $f(x)$,

$$\frac{\partial}{\partial x}\left[f(x)\right]^+ = \begin{cases} f'(x) & \text{if} \quad f(x)>0 \\ \text{not differentialble} & \text{if} \quad f(x)=0 \\ 0 & \text{if} \quad f(x)<0 \end{cases}$$

and at $f(x)=0$, the derivative $\frac{\partial}{\partial x}\left[f(x)\right]^+$ is bounded by $\frac{\partial}{\partial x}\left[f(x)\right]^+ \in \left[\min\{0,f'(x)\},\max\{0,f'(x)\}\right]$, and the measure of (x',n') that satisfies $\Omega_n(x,n)+\hat{\alpha}(x,x')Z\Omega_x(x,n)-\Omega_n(x',n')=0$ is zero for any (x,n). Therefore, (22) is rewritten as

$$\begin{aligned}
\rho\zeta(x,n) = &\exp(x) \\
&+ Av(x,n)\int \mathbb{1}_P(x,n,x',n')dF_n(x',n')\zeta_n(x,n) \\
&+ Av(x,n)Z\int \mathbb{1}_P(x,n,x',n')\hat{\alpha}_x(x,x')dF_n(x',n')\zeta(x,n) \\
&+ Av(x,n)Z\int \mathbb{1}_P(x,n,x',n'))\hat{\alpha}(x,x')dF_n(x',n')\zeta_x(x,n) \\
&+ \left(\gamma(n)-g\right)\zeta_x(x,n) \\
&- \eta\zeta(x,n)
\end{aligned}$$

$$\begin{aligned}
&\left(\rho+\eta-Av(x,n)Z\int \mathbb{1}_P(x,n,x',n')\hat{\alpha}_x(x,x')dF_n(x',n')\right)\zeta(x,n) \\
&= \exp(x) \\
&\quad + Av(x,n)\int \mathbb{1}_P(x,n,x',n')dF_n(x',n')\zeta_n(x,n)
\end{aligned}$$

$$+\left(\gamma(n) + Av(x,n)Z\int \mathbb{1}_P(x,n,x',n')\hat{\alpha}(x,x')dF_n(x',n') - g\right)\zeta_x(x,n)$$

Now, define the "effective discount rate"

$$R(x,n) = \rho + \eta - Av(x,n)Z\int \mathbb{1}_P(x,n,x',n')\hat{\alpha}_x(x,x')dF_n(x',n')$$

Define the stochastic process

$$\begin{aligned} dx_t &= \left\{\gamma(n_t) + Av(x_t,n_t)Z\int \mathbb{1}_P(x,n,x',n')\hat{\alpha}\left(x_t,x'\right)dF_n\left(x',n'\right) - g\right\}dt \\ dn_t &= \left\{Av(x_t,n_t)\int \mathbb{1}_P(x,n,x',n')dF_n\left(x',n'\right)\right\}dt \end{aligned} \tag{23}$$

We can now use the Feynman–Kac formula (Pham (2009)) to go back to the sequential formulation:

$$\zeta(z,n) = \mathbb{E}\left[\int_0^\infty e^{-\int_0^t R(x_\tau,n_\tau)d\tau}\exp(x)dt \mid x_0 = z, n_0 = n, \{x_t,n_t\} \text{ follows (23)}\right]$$

Because $\exp(x)$ is positive, $\zeta(z,n)$ is positive. This concludes the proof.

A.4 $\Omega_n(z,n) > 0$

Denote $\zeta(x,n) = \Omega_x(x,n)$ and $\hat{\alpha}(x,x') = \alpha(\exp(x'-x))/\exp(x)$ Differentiate the Bellman equation (21) w.r.t. n,

$$\begin{aligned} \rho\zeta(x,n) = &+ Av(x,n)\int \mathbb{1}_P(x,n,x',n')dF_n(x',n')\zeta_n(x,n) \\ &+ Av(x,n)Z\int \mathbb{1}_P(x,n,x',n')\hat{\alpha}(x,x')dF_n(x',n')\zeta_x(x,n) \\ &+ (\gamma(n) - g)\,\zeta_x(x,n) \\ &+ \gamma'(n)\Omega_x(x,n) \\ &+ \eta\left[\zeta(0,n) - \zeta(x,n)\right] \end{aligned}$$

$$\rho\zeta(x,n) = \gamma'(n)\Omega_x(x,n)$$

$$+ Av(x,n)\int \mathbb{1}_P(x,n,x',n')dF_n(x',n')\zeta_n(x,n)$$
$$+\left(\gamma(n) + Av(x,n)Z\int \mathbb{1}_P(x,n,x',n')\hat{\alpha}(x,x')dF_n(x',n') - g\right)\zeta_x(x,n)$$
$$+ \eta\left[\zeta(0,n) - \zeta(x,n)\right]$$

Define the stochastic process

$$\begin{aligned} dx_t &= \left\{\gamma(n_t) + Av(x_t,n_t)Z\int \mathbb{1}_P(x,n,x',n')\hat{\alpha}\left(x_t,x'\right)dF_n\left(x',n'\right) - g\right\}dt + (0 - x_t)\,dH_t \\ dn_t &= \left\{Av(x_t,n)\int \mathbb{1}_P(x,n,x',n')dF_n\left(x',n'\right)\right\}dt \end{aligned} \tag{24}$$

where H_t is a compensated Poisson process of intensity η. Again, we can use the Feynman–Kac formula to go back to the sequential formulation:

$$\zeta(z,n) = \mathbb{E}\left[\int_0^\infty e^{-\rho}\gamma'(n)\Omega_x(x,n)dt \mid x_0 = z, n_0 = n, \{x_t,n_t\} \text{ follows (24)}\right]$$

Because $\gamma'(n) > 0$ by assumption and $\Omega_x(x,n)$ is positive from the previous proof, $\zeta(z,n)$ is positive. This completes the proof.

B Empirical Appendix

B.1 Data

Our analyses utilize two administrative data sets, ”Linked Inventor Biography Data 1980-2014” (INV-BIO) and ”Sample of Integrated Labor Market Biographies” (Stichprobe der Integrierten Arbeitsmarktbiografien - SIAB). Both data sets are constructed by the Institute for Employment Research (IAB).

The SIAB data is a 2% random sample from (Integrated Employment Biographies - IEB). The IEB combines data from five different sources, each of which may contain information from various administrative procedures. It comprises all individuals in Germany who hold at least one of the following employment statuses: employment subject to social security, marginal part-time employment, receipt of benefits according to the German Social Code III or II, official registration as a job seeker at the German Federal Employment Agency, and (planned) participation in programs of active labor market policies (Dauth

Table B.1: Summary Statistics

(A) INV-BIO

Establishment level variables	Mean	S.D.	N of est. (thus.)
N of inventors (n_{et})	4.9	18.5	119
N of employees	688.9	2150.6	119
Mean daily wage, Euro	121.6	55.5	119
N of three-year forward citations for patents (three-year backward average, z_{et})	11.3	69.2	119
Share of inventors moving from higher productivity est. (H-Share$_{et}$), %	61.2	49.5	119
Total inventor inflows	1.67	5.30	119

(B) SIAB

Worker level variables	Mean	S.D.	N of workers (thus.)
Dummy for moving to less productive est. (D_{it})			
based on est. size	0.50	-	4,669
based on mean wage	0.52	-	4,583
Dummy for the identified inventors (I_{it})	0.10	-	5,691
Daily Wage, Euro	44.4	42.1	5,691
Age	33.7	12.9	5,691
Share of Women, %	47.3	-	5,691

and Eppelsheimer 2020 for more detail).

The patent information contained in the INV-BIO dataset is sourced from register data recorded in PATSTAT, which includes bibliographical, procedural, and legal status information on patent applications handled by the European Patent Office. Additionally, data from DPMAregister, the online patent register of the German Patent and Trademark Office, is incorporated to enhance the PATSTAT data extract. The DPMAregister provides exclusive records of national patent applications that are not transferred to the European Patent Office or filed under the PCT (Patent Cooperation Treaty) route. As a result, the INV-BIO dataset comprises inventors who are listed on patent filings at the European Patent Office (EPO) between 1999 and 2011 and have been successfully linked with IEB (Dorner et al. 2018 for more detail).

Table B.1 shows the summary statistics for INV-BIO and SIAB, respectively. Table B.2 shows the correlation between the three measures used as the proxy for knowledge quality or productivity level in Section 3.2 and 3.3. We can observe positive correlations between them.

Table B.2: Correlation between Three Measures

Correlation	z_{et}	Est. size	Mean Wage
z_{et}	1.00		
Est. size	0.44	1.00	
Mean wage	0.11	0.08	1.00

Table B.3: Distribution of Inventors

(A) Rank by Citation

Establishment percentile rank	≤50%	50-60	60-70	70-80	80-100
Share of Inventors (%)	13.1	2.5	3.2	4.6	76.7

(B) Rank by establishment size

Establishment percentile rank	≤50%	50-60	60-70	70-80	80-100
Share of Inventors (%)	9.3	3.9	5.2	7.5	74.0

(C) Rank by Mean Wage

Establishment percentile rank	≤50%	50-60	60-70	70-80	80-100
Share of Inventors (%)	10.2	6.6	10.5	14.8	57.8

Notes: This table shows the distribution of inventors across percentiles of establishments. The percentile in panel (A) is based on the three-year backward average of forward patent citation counts. Panel (B) is based on the establishment size, and panel (C) is based on the mean wage of full-time employees. Establishments could be classified into different percentiles based on these measures each year. The sample encompasses data from 1980 to 2014. The values in the table represent the proportion of inventors within each percentile in relation to the total number of inventors in INV-BIO.

B.2 Robustness Check of Empirical Analyses

The table from B.4 to B.6 shows the result of the robustness check for our empirical result. Table B.4 shows the transition matrix of inventor flows with wage increases, suggesting many flows from more productive establishments to less productive ones, even in this sample.

Instead of Probit model in Section 3.3, we estimate the following equation to control for fixed effects,

$$D_{it} = \beta_0 + \beta_1 I_{it} + \beta_2 X_{it} + \alpha_e + \alpha_t + \varepsilon_{it} \tag{25}$$

Definitions of variables are the same as in Section 3.3. Table B.5 shows that inventors are more likely to move to less productive establishments conditional on fixed effects.

Table B.4: Transition Probabilities of Inventor Flows with wage increase

(A) Rank by Citation/Inventor

Share of flows (%)		Destination establishment rank				
		≤ 50%	50-60	60-70	70-80	80-100
	≤ 50%	2.7	0.2	0.3	0.4	4.1
Origin	50-60	2.1	0.2	0.2	0.3	3.1
establishment	60-70	2.3	0.2	0.3	0.4	3.6
rank	70-80	2.7	0.2	0.3	0.4	3.6
	80-100	19.0	1.8	2.1	3.1	45.9

(B) Rank by Establishment Size

Share of flows (%)		Destination establishment rank				
		≤ 50%	50-60	60-70	70-80	80-100
	≤ 50%	3.8	1.2	0.9	0.9	6.6
Origin	50-60	0.4	0.8	0.9	0.5	2.1
establishment	60-70	0.4	0.2	1.1	1.2	2.8
rank	70-80	0.5	0.3	0.4	1.9	4.9
	80-100	3.5	1.5	2.1	3.1	58.2

(C) Rank by Mean Wage

Share of flows (%)		Destination establishment rank				
		≤ 50%	50-60	60-70	70-80	80-100
	≤ 50%	4.4	1.5	1.3	1.4	4.4
Origin	50-60	0.9	1.3	1.6	1.1	2.5
establishment	60-70	0.8	0.9	2.6	2.9	4.1
rank	70-80	0.7	0.6	1.6	4.8	7.4
	80-100	2.0	2.6	2.8	4.9	42.4

Table B.5: Estimation Result for Inventor Flows (Linear Model)

	D_{it}			
	Whole sample		Sample with wage ↑	
I_{it}	.008***	.010**	.009***	.014***
	(.003)	(.004)	(.003)	(.004)
Control	√	√	√	√
Fixed Effects	√	√	√	√
Measure for D_{it}	Size	Mean wage	Size	Mean wage
N	2,938,537	2,959,368	1,609,460	1,617,613
Adj. R^2	.25	.22	.21	.20

C Quantitative Appendix

C.1 Numerical Solution to Joint Value HJB Equation

Change of variables

We want to solve

$$
\begin{aligned}
\rho\Omega(z,n,i) = \max_{v\geq 0} & \, z - c(v)Z \\
& + Av\int \left[\Omega_n(z,n,i) + \alpha(z'/z)Z\Omega_z(z,n,i) - \Omega_n(z',n',i')\right]^+ dF_n(z',n',i) \\
& + (\gamma(n,i) - g)\, z\Omega_z(z,n,i) \\
& + \eta\left[\Omega(1,n,i) - \Omega(z,n,i)\right] \\
& + \lambda_i\left[\Omega(z,n,-i) - \Omega(z,n,i)\right]
\end{aligned}
$$

As in our quantitative exercise, let $c(v) = \frac{\bar{c}}{\phi+1}v^{\phi+1}$. The first order condition for vacancies is

$$
\bar{c}v(z,n,i)^{\phi}Z = A\int \left[\Omega_n(z,n,i) + \alpha(z'/z)Z\Omega_z(z,n,i) - \Omega_n(z',n',i')\right]^+ dF_n(z',n',i')
$$

$$
v(z,n,i) = \left\{\frac{A}{\bar{c}Z}\int \left[\Omega_n(z,n,i) + \alpha(z'/z)Z\Omega_z(z,n,i) - \Omega_n(z',n',i')\right]^+ dF_n(z',n',i')\right\}^{\frac{1}{\phi}}
$$

Consider a change of variables. Let $\tilde{z} = \log z, \tilde{n} = \log n$. Now, define $\tilde{\Omega}(\tilde{z}, \tilde{n}, i) = \Omega\left(e^{\tilde{z}}, e^{\tilde{n}}, i\right) = \Omega(z, n, i)$. Applying the chain rule to $\Omega(z, n, i) = \widetilde{\Omega}(\log z, \log n, i)$, and rearranging:

$$\Omega_n(z, n, i)n = \widetilde{\Omega}_{\tilde{n}}(\widetilde{z}, \tilde{n}, i)$$
$$\Omega_z(z, n, i)z = \widetilde{\Omega}_{\tilde{z}}(\widetilde{z}, \tilde{n}, i)$$

or equivalently,

$$\Omega_n(z, n, i) = \frac{\widetilde{\Omega}_{\tilde{n}}(\widetilde{z}, \tilde{n}, i)}{e^{\tilde{n}}}$$
$$\Omega_z(z, n, i) = \frac{\widetilde{\Omega}_{\tilde{z}}(\widetilde{z}, \tilde{n}, i)}{e^{\tilde{z}}}$$

As in our quantitative exercise, let $\alpha(z'/z) = \overline{\alpha}\,(z'/z)^{\beta}$. Then, we can rewrite it as

$$\alpha(z'/z) = \overline{\alpha}\left(e^{\tilde{z}'}/e^{\tilde{z}}\right)^{\beta}$$
$$= \overline{\alpha}e^{\beta(\tilde{z}' - \tilde{z})}$$

As in our quantitative exercise, let $\gamma(n, i) = \overline{\gamma}_i n^{\delta}$. Then, we can rewrite it as

$$\gamma(n, i) = \overline{\gamma}_i e^{\delta\tilde{n}}$$

Define $\tilde{F}(\tilde{z}, \tilde{n}, i) = F(e^{\tilde{z}}, e^{\tilde{n}}, i) = F(z, n, i)$. The relationship between density of $F(z, n, i)$ and $\tilde{F}(\tilde{z}, \tilde{n}, i)$ is given by

$$f(z, n, i) = \frac{\partial}{\partial z}\frac{\partial}{\partial n}F(z, n, i)$$
$$= \frac{\partial}{\partial z}\frac{\partial}{\partial n}\tilde{F}(\log z, \log n, i)$$
$$= \frac{1}{zn}\tilde{f}(\tilde{z}, \tilde{n}, i)$$

When we change the variables from (z, n) to $(\tilde{z}, \tilde{n})$, the Jacobian is zn. Therefore, for any set of functions h and $\tilde{h}$ such that $h(z, n, i) = \tilde{h}(\tilde{z}, \tilde{n}, i)$,

$$\int \tilde{h}(\tilde{z}, \tilde{n}, i)d\tilde{F}(\tilde{z}, \tilde{n}, i) = \int h(z, n)dF(z, n, i)$$

The total output can be expressed as

$$\mathtt{Z} = \int e^{\tilde{z}} d\tilde{F}(\tilde{z}, \tilde{n}, i)$$

The total mass of the inventor can be expressed as

$$\mathtt{n} = \int e^{\tilde{n}} d\tilde{F}(\tilde{z}, \tilde{n}, i)$$

Let define

$$\tilde{f}_{\tilde{n}}(\tilde{z}, \tilde{n}, i) = \frac{e^{\tilde{n}} \tilde{f}(\tilde{z}, \tilde{n}, i)}{\mathtt{n}}$$

and $\tilde{F}_{\tilde{n}}(\tilde{z}, \tilde{n}, i)$ the corresponding cumulative distribution. Then, the inventor-weighted distribution $f_{\tilde{n}}(z, n, i)$ can be rewritten as

$$\begin{aligned} f_n(z, n, i) &= \frac{n f(z, n, i)}{\mathtt{n}} \\ &= \frac{1}{z} \cdot \frac{1}{n} \frac{e^{\tilde{n}} \tilde{f}(\tilde{z}, \tilde{n}, i)}{\mathtt{n}} \\ &= \frac{1}{z} \cdot \frac{1}{n} \tilde{f}_{\tilde{n}}(\tilde{z}, \tilde{n}, i) \end{aligned}$$

Therefore, for any set of functions h and $\tilde{h}$ such that $h(z, n, i) = \tilde{h}(\tilde{z}, \tilde{n}, i)$,

$$\int \tilde{h}(\tilde{z}, \tilde{n}, i) d\tilde{F}_{\tilde{n}}(\tilde{z}, \tilde{n}, i) = \int h(z, n) dF_n(z, n, i)$$

Define $\tilde{v}(\tilde{z}, \tilde{n}, i) = v(e^{\tilde{z}}, e^{\tilde{n}}, i) = v(z, n, i)$. Then,

$$\tilde{v}(\tilde{z}, \tilde{n}, i) = \left\{ \frac{A}{\bar{c}Z} \int \left[\widetilde{\Omega}_{\tilde{n}}(\widetilde{z}, \tilde{n}, i)/e^{\tilde{n}} + \overline{\alpha} e^{\beta(\tilde{z}' - \tilde{z})} Z \widetilde{\Omega}_{\tilde{z}}(\widetilde{z}, \tilde{n}, i)/e^{\tilde{z}} - \widetilde{\Omega}_{\tilde{n}}(\widetilde{z}', \tilde{n}', i')/e^{\tilde{n}} \right]^{+} d\tilde{F}_{\tilde{n}}(\tilde{z}', \tilde{n}', i') \right\}^{\frac{1}{\phi}}$$

The Bellman equation can be rewritten as

$$\begin{aligned} \rho \tilde{\Omega}(\tilde{z}, \tilde{n}, i) = e^{\tilde{z}} &- \frac{\bar{c}}{\phi + 1} \tilde{v}(\tilde{z}, \tilde{n}, i)^{\phi+1} Z \\ &+ A \tilde{v}(\tilde{z}, \tilde{n}, i) \int \left[\widetilde{\Omega}_{\tilde{n}}(\widetilde{z}, \tilde{n}, i)/e^{\tilde{n}} + \overline{\alpha} e^{\beta(\tilde{z}' - \tilde{z})} Z \widetilde{\Omega}_{\tilde{z}}(\widetilde{z}, \tilde{n}, i)/e^{\tilde{z}} - \widetilde{\Omega}_{\tilde{n}}(\widetilde{z}', \tilde{n}', i')/e^{\tilde{n}'} \right]^{+} d\tilde{F}_{\tilde{n}}(\tilde{z}', \tilde{n}', i') \\ &+ \left(\overline{\gamma}_i e^{\delta \tilde{n}} - g \right) \widetilde{\Omega}_{\tilde{z}}(\widetilde{z}, \tilde{n}, i) \end{aligned}$$

$$+\ \eta\left[\tilde{\Omega}(0,\tilde{n},i)-\tilde{\Omega}(\tilde{z},\tilde{n},i)\right]$$
$$+\ \lambda_i\left[\tilde{\Omega}(\tilde{z},\tilde{n},-i)-\tilde{\Omega}(\tilde{z},\tilde{n},i)\right]$$

Let define poaching indicator function with transformed variable $\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')$ as

$$\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')=\begin{cases}1 & \text{if}\quad \widetilde{\Omega}_{\tilde{n}}(\widetilde{z},\tilde{n},i)/e^{\tilde{n}}+\overline{\alpha}e^{\beta(\tilde{z}'-\tilde{z})}Z\widetilde{\Omega}_{\tilde{z}}(\widetilde{z},\tilde{n},i)/e^{\tilde{z}}>\widetilde{\Omega}_{\tilde{n}}(\widetilde{z}',\tilde{n}',i')/e^{\tilde{n}}\\ 0 & \text{otherwise}\end{cases}$$

The Bellman equation can be rewritten as

$$\begin{aligned}\rho\tilde{\Omega}(\tilde{z},\tilde{n},i)=e^{\tilde{z}}-\frac{\overline{c}}{\phi+1}\tilde{v}(\tilde{z},\tilde{n},i)^{\phi+1}Z\\
&+A\tilde{v}(\tilde{z},\tilde{n},i)/e^{\tilde{n}}\int\left[\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')\right]d\tilde{F}_{\tilde{n}}(\tilde{z}',\tilde{n}',i')\widetilde{\Omega}_{\tilde{n}}(\widetilde{z},\tilde{n},i)\\
&+A\tilde{v}(\tilde{z},\tilde{n},i)\overline{\alpha}Z/e^{\tilde{z}}\int\left[\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')e^{\beta(\tilde{z}'-\tilde{z})}\right]d\tilde{F}_{\tilde{n}}(\tilde{z}',\tilde{n}',i')\widetilde{\Omega}_{\tilde{z}}(\widetilde{z},\tilde{n},i)\\
&-A\tilde{v}(\tilde{z},\tilde{n},i)\int\left[\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')\widetilde{\Omega}_{\tilde{n}}(\widetilde{z}',\tilde{n}',i')/e^{\tilde{n}'}\right]d\tilde{F}_{\tilde{n}}(\tilde{z}',\tilde{n}',i')\\
&+\left(\overline{\gamma}_ie^{\delta\tilde{n}}-g\right)\widetilde{\Omega}_{\tilde{z}}(\widetilde{z},\tilde{n},i)\\
&+\eta\left[\tilde{\Omega}(0,\tilde{n},i)-\tilde{\Omega}(\tilde{z},\tilde{n},i)\right]\\
&+\lambda_i\left[\tilde{\Omega}(\tilde{z},\tilde{n},-i)-\tilde{\Omega}(\tilde{z},\tilde{n},i)\right]\end{aligned}$$

$$\begin{aligned}\rho\tilde{\Omega}(\tilde{z},\tilde{n},i)=e^{\tilde{z}}-\frac{\overline{c}}{\phi+1}\tilde{v}(\tilde{z},\tilde{n},i)^{\phi+1}Z-A\tilde{v}(\tilde{z},\tilde{n},i)\int\left[\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')\widetilde{\Omega}_{\tilde{n}}(\widetilde{z}',\tilde{n}',i')/e^{\tilde{n}'}\right]d\tilde{F}_{\tilde{n}}(\tilde{z}',\tilde{n}',i')\\
&+A\tilde{v}(\tilde{z},\tilde{n},i)/e^{\tilde{n}}\int\left[\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')\right]d\tilde{F}_{\tilde{n}}(\tilde{z}',\tilde{n}',i')\widetilde{\Omega}_{\tilde{n}}(\widetilde{z},\tilde{n},i)\\
&+\left(\overline{\gamma}_ie^{\delta\tilde{n}}+A\tilde{v}(\tilde{z},\tilde{n},i)\overline{\alpha}Z/e^{\tilde{z}}\int\left[\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')e^{\beta(\tilde{z}'-\tilde{z})}\right]d\tilde{F}_{\tilde{n}}(\tilde{z}',\tilde{n}',i')-g\right)\widetilde{\Omega}_{\tilde{z}}(\widetilde{z},\tilde{n},i)\\
&+\eta\left[\tilde{\Omega}(0,\tilde{n},i)-\tilde{\Omega}(\tilde{z},\tilde{n},i)\right]\\
&+\lambda_i\left[\tilde{\Omega}(\tilde{z},\tilde{n},-i)-\tilde{\Omega}(\tilde{z},\tilde{n},i)\right]\end{aligned}$$

Implicit method

We solve the Bellman equation using an implicit method. Let Δ denote step-size and τ the iteration of the algorithm. Then given $\tilde{\Omega}^{\tau-1}(\tilde{z},\tilde{n},i)$, the implicit method gives an update

$$
\begin{aligned}
&\frac{1}{\Delta}\left[\tilde{\Omega}^{\tau}(\tilde{z},\tilde{n},i)-\tilde{\Omega}^{\tau-1}(\tilde{z},\tilde{n},i)\right]+\rho\tilde{\Omega}(\tilde{z},\tilde{n},i)= \\
&e^{\tilde{z}}-\frac{\bar{c}}{\phi+1}\tilde{v}(\tilde{z},\tilde{n},i)^{\phi+1}Z-A\tilde{v}(\tilde{z},\tilde{n},i)\int\left[\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')\widetilde{\Omega}_{\tilde{n}}^{\tau-1}(\tilde{z}',\tilde{n}',i')/e^{\tilde{n}'}\right]d\tilde{F}_{\tilde{n}}(\tilde{z}',\tilde{n}',i') \\
&+A\tilde{v}(\tilde{z},\tilde{n},i)/e^{\tilde{n}}\int\left[\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')\right]d\tilde{F}_{\tilde{n}}(\tilde{z}',\tilde{n}',i')\widetilde{\Omega}_{\tilde{n}}^{\tau}(\tilde{z},\tilde{n},i) \\
&+\left(\overline{\gamma}_i e^{\delta\tilde{n}}+A\tilde{v}(\tilde{z},\tilde{n},i)\overline{\alpha}Z/e^{\tilde{z}}\int\left[\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')e^{\beta(\tilde{z}'-\tilde{z})}\right]d\tilde{F}_{\tilde{n}}(\tilde{z}',\tilde{n}',i')-g\right)\widetilde{\Omega}_{\tilde{z}}^{\tau}(\tilde{z},\tilde{n},i) \\
&+\eta\left[\tilde{\Omega}^{\tau}(0,\tilde{n},i)-\tilde{\Omega}^{\tau}(\tilde{z},\tilde{n},i)\right] \\
&+\lambda_i\left[\tilde{\Omega}^{\tau}(\tilde{z},\tilde{n},-i)-\tilde{\Omega}^{\tau}(\tilde{z},\tilde{n},i)\right]
\end{aligned}
$$

Rearranging this expression:

$$
\begin{aligned}
&\left(\frac{1}{\Delta}+\rho+\eta\right)\tilde{\Omega}^{\tau}(\tilde{z},\tilde{n},i) \\
&-A\tilde{v}(\tilde{z},\tilde{n},i)/e^{\tilde{n}}\int\left[\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')\right]d\tilde{F}_{\tilde{n}}(\tilde{z}',\tilde{n}',i')\widetilde{\Omega}_{\tilde{n}}^{\tau}(\tilde{z},\tilde{n},i) \\
&-\left(\overline{\gamma}_i e^{\delta\tilde{n}}+A\tilde{v}(\tilde{z},\tilde{n},i)\overline{\alpha}Z/e^{\tilde{z}}\int\left[\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')e^{\beta(\tilde{z}'-\tilde{z})}\right]d\tilde{F}_{\tilde{n}}(\tilde{z}',\tilde{n}',i')-g\right)\widetilde{\Omega}_{\tilde{z}}^{\tau}(\tilde{z},\tilde{n},i) \\
&-\lambda_i\left[\tilde{\Omega}^{\tau}(\tilde{z},\tilde{n},-i)-\tilde{\Omega}^{\tau}(\tilde{z},\tilde{n},i)\right] \\
=\;&e^{\tilde{z}}+\eta\tilde{\Omega}^{\tau}(0,\tilde{n},i)-\frac{\bar{c}}{\phi+1}\tilde{v}(\tilde{z},\tilde{n},i)^{\phi+1}Z \\
&-A\tilde{v}(\tilde{z},\tilde{n},i)\int\left[\tilde{\mathbb{1}}_P(\tilde{z},\tilde{n},i,\tilde{z}',\tilde{n}',i')\widetilde{\Omega}_{\tilde{n}}^{\tau-1}(\tilde{z}',\tilde{n}',i')/e^{\tilde{n}'}\right]d\tilde{F}_{\tilde{n}}(\tilde{z}',\tilde{n}',i') \\
&+\frac{1}{\Delta}\tilde{\Omega}^{\tau-1}(\tilde{z},\tilde{n},i)
\end{aligned}
$$

We now discretize $\tilde{n}$ on an evenly spaced $N_{\tilde{n}} \times 1$ grid and $\tilde{z}$ on an evenly spaced $N_{\tilde{z}} \times 1$. Stack these according to:

$$\begin{pmatrix} \widetilde{z}_1, \tilde{n}_1, h \\ \widetilde{z}_2, \tilde{n}_1, h \\ \vdots \\ \widetilde{z}_{N_{\tilde{z}}}, \tilde{n}_1, h \\ \vdots \\ \widetilde{z}_1, \tilde{n}_{N_{\tilde{n}}}, h \\ \vdots \\ \widetilde{z}_{N_{\tilde{z}}}, \tilde{n}_{N_{\tilde{n}}}, h \\ \widetilde{z}_1, \tilde{n}_1, l \\ \vdots \\ \widetilde{z}_{N_{\tilde{z}}}, \tilde{n}_{N_{\tilde{n}}}, l \end{pmatrix}$$

The above equation can be rewritten in vector form as:

$$\left(\frac{1}{\Delta} + \rho + \eta\right)\tilde{\Omega}^{\tau} - \mu_n \widetilde{\Omega}^{\tau}_{\tilde{n}} - \mu_z \widetilde{\Omega}^{\tau}_{\tilde{z}} - \Lambda\tilde{\Omega}^{\tau} = \pi + \frac{1}{\Delta}\tilde{\Omega}^{\tau-1} \tag{26}$$

where

- the element of $N_{\tilde{z}} \times N_{\tilde{n}} \times 2$ vector $\tilde{\Omega}^{\tau}$ consists of $\tilde{\Omega}^{\tau}(\tilde{z}, \tilde{n}, i)$,
- the element of $N_{\tilde{z}} \times N_{\tilde{n}} \times 2$ vector $\tilde{\Omega}^{\tau}_{\tilde{n}}$ consists of $\tilde{\Omega}^{\tau}_{\tilde{n}}(\tilde{z}, \tilde{n}, i)$,
- the element of $N_{\tilde{z}} \times N_{\tilde{n}} \times 2$ vector $\tilde{\Omega}^{\tau}_{\tilde{z}}$ consists of $\tilde{\Omega}^{\tau}_{\tilde{z}}(\tilde{z}, \tilde{n}, i)$,
- the element of $N_{\tilde{z}} \times N_{\tilde{n}} \times 2$ vector μ_n consists of

$$A\tilde{v}(\tilde{z}, \tilde{n}, i)/e^{\tilde{n}} \int \left[\tilde{\mathbb{1}}_P(\tilde{z}, \tilde{n}, i, \tilde{z}', \tilde{n}', i')\right] d\tilde{F}_{\tilde{n}}(\tilde{z}', \tilde{n}', i'),$$

- the element of $N_{\tilde{z}} \times N_{\tilde{n}} \times 2$ vector μ_z consists of

$$\overline{\gamma}_i e^{\delta\tilde{n}} + A\tilde{v}(\tilde{z}, \tilde{n}, i)\overline{\alpha}Z/e^{\tilde{z}} \int \left[\tilde{\mathbb{1}}_P(\tilde{z}, \tilde{n}, i, \tilde{z}', \tilde{n}', i')e^{\beta(\tilde{z}'-\tilde{z})}\right] d\tilde{F}_{\tilde{n}}(\tilde{z}', \tilde{n}', i') - g,$$

  and

- the element of $N_{\tilde{z}} \times N_{\tilde{n}} \times 2$ vector π consists of

$$
\begin{aligned}
& e^{\tilde{z}} + \eta \tilde{\Omega}^{\tau}(0, \tilde{n}, i) - \frac{\bar{c}}{\phi + 1} \tilde{v}(\tilde{z}, \tilde{n}, i)^{\phi+1} Z \\
& - A\tilde{v}(\tilde{z}, \tilde{n}, i) \int \left[\tilde{\mathbb{1}}_P(\tilde{z}, \tilde{n}, i, \tilde{z}', \tilde{n}', i') \widetilde{\Omega}_{\tilde{n}}^{\tau-1}(\widetilde{z}', \tilde{n}', i') / e^{\tilde{n}'} \right] d\tilde{F}_{\tilde{n}}(\tilde{z}', \tilde{n}', i')
\end{aligned}
$$

$(N_{\tilde{z}} \times N_{\tilde{n}} \times 2) \times (N_{\tilde{z}} \times N_{\tilde{n}} \times 2)$ matrix Λ is

$$
\Lambda = \begin{pmatrix} -\lambda_h I_{N_{\tilde{z}} \times N_{\tilde{n}}} & \lambda_h I_{N_{\tilde{z}} \times N_{\tilde{n}}} \\ \lambda_l I_{N_{\tilde{z}} \times N_{\tilde{n}}} & -\lambda_l I_{N_{\tilde{z}} \times N_{\tilde{n}}} \end{pmatrix}
$$

where $I_{N_{\tilde{z}} \times N_{\tilde{n}}}$ is $N_{\tilde{z}} \times N_{\tilde{n}}$ identity matrix. Let $D_{\tilde{n}}$ be the $(N_{\tilde{z}} \times N_{\tilde{n}} \times 2) \times (N_{\tilde{z}} \times N_{\tilde{n}} \times 2)$ matrix that, when pre-multiplying $\tilde{\Omega}^{\tau}$, gives an approximation of $\tilde{\Omega}^{\tau}_{\tilde{n}}$. Analogously, define $D_{\tilde{n}}$:

$$
\begin{aligned}
\widetilde{\Omega}^{\tau}_{\tilde{n}} &= D_{\tilde{n}} \widetilde{\Omega}^{\tau} \\
\widetilde{\Omega}^{\tau}_{\tilde{z}} &= D_{\tilde{z}} \widetilde{\Omega}^{\tau}
\end{aligned}
$$

To compute the derivative matrices $D_{\tilde{n}}$ and $D_{\tilde{z}}$, we follow an upwind scheme. That is, we use a forward approximation when the drift of the state variable is positive and a backward approximation when the drift of the state is negative. Using these, we can write (26) as

$$
\left[\left(\frac{1}{\Delta} + \rho + \eta \right) - \mu_n D_{\tilde{n}} - \mu_z D_{\tilde{z}} - \Lambda \right] \tilde{\Omega}^{\tau} = \pi + \frac{1}{\Delta} \tilde{\Omega}^{\tau-1}.
$$

The implicit method works by updating $\widetilde{\Omega}^{\tau}$ through the above equation.

C.2 Numerical Solution to Kolmogorov Forward Equation

The total mass of the inventor can be expressed as

$$
\mathrm{v} = \int \tilde{v}(\tilde{z}, \tilde{n}, i) d\tilde{F}(\tilde{z}, \tilde{n}, i)
$$

Let define

$$
\tilde{f}_{\tilde{v}}(\tilde{z}, \tilde{n}, i) = \frac{\tilde{v}(\tilde{z}, \tilde{n}, i) \tilde{f}(\tilde{z}, \tilde{n}, i)}{\mathrm{v}}
$$

and $\tilde{F}_{\tilde{v}}(\tilde{z}, \tilde{n}, i)$ the corresponding cumulative distribution. We construct the Kolmogorov forward equation in terms of the transformed variables.

$$\begin{aligned}
0 = &- \frac{\partial}{\partial \tilde{n}} \left(\tilde{\mu}_{\tilde{n}}(\tilde{z}, \tilde{n}, i) \tilde{f}(\tilde{z}, \tilde{n}, i) \right) - \frac{\partial}{\partial \tilde{z}} \left(\tilde{\mu}_{\tilde{z}}(\tilde{z}, \tilde{n}, i) \tilde{f}(\tilde{z}, \tilde{n}, i) \right) \\
&- \eta \tilde{f}(\tilde{z}, \tilde{n}, i) + \eta \int_0^1 \tilde{f}(\tilde{z}', \tilde{n}, i) dz' \boldsymbol{\delta}(0) \\
&- \lambda_i \tilde{f}(\tilde{z}, \tilde{n}, i) + \lambda_{-i} \tilde{f}(\tilde{z}, \tilde{n}, -i)
\end{aligned}$$

where [18]

$$\tilde{\mu}_{\tilde{n}}(\tilde{z}, \tilde{n}, i) = A \frac{\tilde{v}(\tilde{z}, \tilde{n}, i)}{e^{\tilde{n}}} \int \left[\tilde{\mathbb{1}}_P(\tilde{z}, \tilde{n}, i, \tilde{z}', \tilde{n}', i') \right] d\tilde{F}_{\tilde{n}}(\tilde{z}', \tilde{n}', i') - A \frac{\mathbf{v}}{\mathbf{n}} \int \tilde{\mathbb{1}}_P(\tilde{z}', \tilde{n}', i', \tilde{z}, \tilde{n}, i) d\tilde{F}_{\tilde{v}}(\tilde{z}', \tilde{n}', i')$$

$$\tilde{\mu}_{\tilde{z}}(\tilde{z}, \tilde{n}, i) = \overline{\gamma}_i e^{\delta \tilde{n}} + A \tilde{v}(\tilde{z}, \tilde{n}, i) \overline{\alpha} Z / e^{\tilde{z}} \int \left[\tilde{\mathbb{1}}_P(\tilde{z}, \tilde{n}, i, \tilde{z}', \tilde{n}', i') e^{\beta(\tilde{z}' - \tilde{z})} \right] d\tilde{F}_{\tilde{n}}(\tilde{z}', \tilde{n}', i') - g$$

We can vectorize this in the same way as above, and obtain

$$0 = -D_{\tilde{n}} \tilde{\mu}_{\tilde{n}} \tilde{f} - D_{\tilde{z}} \tilde{\mu}_{\tilde{z}} \tilde{f} - \eta \tilde{f} + \eta \tilde{f}_0 + \Lambda' \tilde{f}$$

where

- the element of $N_{\tilde{z}} \times N_{\tilde{n}} \times 2$ vector $\tilde{f}$ is the stacked as the value function,
- the element of $N_{\tilde{z}} \times N_{\tilde{n}} \times 2$ vector $\tilde{\mu}_{\tilde{n}}$ consists of $\tilde{\mu}_{\tilde{n}}(\tilde{z}, \tilde{n}, i)$,
- the element of $N_{\tilde{z}} \times N_{\tilde{n}} \times 2$ vector $\tilde{\mu}_{\tilde{z}}$ consists of $\tilde{\mu}_{\tilde{z}}(\tilde{z}, \tilde{n}, i)$, and
- the element of $N_{\tilde{z}} \times N_{\tilde{n}} \times 2$ vector $\tilde{f}_0$ consists of $\int_0^1 \tilde{f}(\tilde{z}', \tilde{n}, i) dz' \boldsymbol{\delta}(0)$[19].

To construct the derivative matrices, we use a backward approximation when the drift of the state variable is positive, and a forward approximation when the drift of the state is negative. This expression can be rearranged to yield

$$\left(-D_{\tilde{n}} \tilde{\mu}_{\tilde{n}} - D_{\tilde{z}} \tilde{\mu}_{\tilde{z}} - \eta + \Lambda' \right) \tilde{f} = -\eta \tilde{f}_0$$

and the distribution of individuals is updated according to the above equation.

[18]Note that $\frac{dn/n}{dt} = \frac{d \log n}{dt} = \frac{d\tilde{n}}{dt}$ and $\frac{dz/z}{dt} = \frac{d \log z}{dt} = \frac{d\tilde{z}}{dt}$.

[19]Due to the Dirac delta function $\boldsymbol{\delta}(0)$, the elements of $\tilde{f}_0$ can take positive value only if $\tilde{z} = 0$.

C.3 Solving the Transition Path

We illustrate how to solve the transition path. We solve the transition path in terms of transformed variables and later recover the non-transformed values over the transition. Finally, we explain how to calculate the consumption-equivalent welfare gain from the policy change.

Perfect Foresight Equilibrium

First, we define the perfect foresight equilibrium, which differs from the balanced growth path equilibrium in that it has a time notation and, the HJB equation and KFE have time derivative terms. Note that the perfect foresight equilibrium is detrended by the productivity of the technology frontier $\bar{z}(t)$ at each period.

Definition 2. (Perfect Foresight Equilibrium) A *perfect foresight equilibrium* consists of: (i) a joint value function $\Omega(z, n, t)$; (ii) a vacancy policy $v(z, n, t)$; (iii) a stationary distribution of firms $f(z, n, t)$; (iv) vacancy- and employment-weighted distributions $f_v(z, n, t)$ and $f_n(z, n, t)$; (v) poaching indicator function $\mathbb{1}_P(z, n, z', n', t)$; (vi) the aggregate productivity $Z(t)$, the aggregate consumption $C(t)$, and the total vacancies $\mathrm{v}(t)$, and (vii) the economic growth rate $g(t)$ such that

1. The joint value $\Omega(z, n, t)$ satisfies the HJB equation

$$\begin{aligned}
&\rho\Omega(z,n,t) + \frac{\partial}{\partial t}\Omega(z,n,t) = \\
&\quad z - c(v(z,n,t))Z(t) \\
&\quad + Av(z,n,t)\int \left[\Omega_n(z,n,t) + \alpha(z'/z)Z(t)\Omega_z(z,n,t) - \Omega_n(z',n',t)\right]^+ dF_n(z',n',t) \\
&\quad + (\gamma(n) - g(t))\, z\Omega_z(z,n,t) \\
&\quad + \eta\left[\Omega(1,n,t) - \Omega(z,n,t)\right]
\end{aligned}$$

2. The vacancy policy $v(z, n, t)$ satisfies the first order condition

$$c_v(v(z,n,t))Z = A\int \left[\Omega_n(z,n,t) + \alpha(z'/z)Z\Omega_z(z,n,t) - \Omega_n(z',n',t)\right]^+ dF_n(z',n',t)$$

3. A density function $f(z,n,t)$ satisfies the KFE equation

$$\frac{\partial}{\partial t} f(z,n,t) = -\frac{\partial}{\partial n}\left(\mu_n(z,n,t) f(z,n,t)\right) - \frac{\partial}{\partial z}\left(\mu_z(z,n,t) f(z,n,t)\right) - \eta f(z,n,t) + \eta \int_0^1 f(z,n,t) dz \boldsymbol{\delta}(1)$$

where

$$\mu_z(z,n,t) \equiv (\gamma(n) - g)\, z + Av(z,n,t) Z \int \mathbb{1}_P(z,n,z',n',t)\alpha(z'/z) dF_n(z',n',t)$$
$$\mu_n(z,n,t) \equiv Av(z,n,t) \int \mathbb{1}_P(z,n,z',n',t) dF_n(z',n',t) - A\mathrm{v}\frac{n}{\mathrm{n}} \int \mathbb{1}_P(z',n',z,n,t) dF_v(z',n',t)$$

4. Vacancy- and employment-weighted distributions are consistent:

$$f_v(z,n,t) = \frac{v(z,n,t) f(z,n,t)}{\mathrm{v}(t)}$$
$$f_n(z,n,t) = \frac{n f(z,n,t)}{\mathrm{n}(t)}$$

5. Poaching indicator function $\mathbb{1}_P(z,n,z',n',t)$ is

$$\mathbb{1}_P(z,n,z',n',t) = \begin{cases} 1 & \text{if} \quad \Omega_n(z,n,t) + \alpha(z'/z) Z \Omega_z(z,n,t) > \Omega_n(z',n',t) \\ 0 & \text{otherwise} \end{cases}$$

6. The aggregate productivity $Z(t)$, aggregate consumption $C(t)$, and the total vacancies $\mathrm{v}(t)$ satisfy rate satisfy

$$Z(t) = \int z dF(z,n,t)$$
$$C(t) = \left\{ 1 - \int c(v(z,n,t)) dF(z,n,t) \right\} Z(t)$$
$$\mathrm{v}(t) = \int v(z,n,t) dF(z,n,t)$$

7. The inventor market clearing condition is satisfied:

$$\mathrm{n}(t) = \int n dF(z,n,t)$$

Now, we recover the non-transformed values over the transition from the transformed values. Without loss of generality, let normalize the productivity of the technology frontier at time $t = 0$ to 1: $\bar{z}(0) = 1$. Then,

$$\begin{aligned}\bar{z}(t) &= \exp\left(\int_0^t g(\tau)d\tau\right) \\ \hat{Z}(t) &= \bar{z}(t)Z(t) \\ \hat{C}(t) &= \bar{z}(t)C(t)\end{aligned}$$

Solution Algorithm

To solve the transition path, we guess a path for optimal behavior of firms over a discretized grid for productivity, number of inventors and time. Subsequently, we iterate on

1. Given a path for the distribution of firms and inventors, update optimal behavior of firms and inventors backwards in time;
2. Given a path for behavior, update the evolution of the distribution of firms and inventor forward in time
3. If the updated path for the distributions and behavior are close enough to the original path, stop. Otherwise return to 1.

Recover Non-Transformed Values

Now, we recover the non-transformed values over the transition from the transformed values. Without loss of generality, let normalize the productivity of the technology frontier at time $t = 0$ to 1: $\bar{z}(0) = 1$. Then,

$$\begin{aligned}\bar{z}(t) &= \exp\left(\int_0^t g(\tau)d\tau\right) \\ \hat{Z}(t) &= \bar{z}(t)Z(t) \\ \hat{C}(t) &= \bar{z}(t)C(t)\end{aligned}$$

Consumption-Equivalent Welfare Gains

Definition 3. (Consumption-Equivalent Welfare Gains from Policy Change) Consider an economy without policy change and the associated consumption path $\left\{\hat{C}(t)\right\}_{t\geq 0}$. *The* *consumption equivalent welfare gains from policy change* is the scalar $\mathcal{L}$ such that the consumer is indifferent between the consumption path $\left\{\mathcal{L} \times \hat{C}(t)\right\}_{t \geq 0}$ and the consumption path generated by the policy change.

Let $V\left(\left\{\hat{C}(t)\right\}_{t \geq 0}\right)$ define the welfare:

$$V\left(\left\{\hat{C}(t)\right\}_{t \geq 0}\right) \equiv \int_0^\infty e^{-\rho t} \log \hat{C}(t) dt.$$

Then,

$$\begin{aligned} V\left(\mathcal{L} \times \left\{\hat{C}(t)\right\}_{t \geq 0}\right) &= \int_0^\infty e^{-\rho t} \log\left(\mathcal{L} \times \hat{C}(t)\right) dt \\ &= \int_0^\infty e^{-\rho t} \log \mathcal{L} dt + \int_0^\infty e^{-\rho t} \log \hat{C}(t) dt \\ &= \frac{\log \mathcal{L}}{\rho} + \int_0^\infty e^{-\rho t} \log \hat{C}(t) dt \\ &= \frac{\log \mathcal{L}}{\rho} + V\left(\left\{\hat{C}(t)\right\}_{t \geq 0}\right) \end{aligned}$$

This implies

$$\frac{\log \mathcal{L}}{\rho} = V\left(\mathcal{L} \times \left\{\hat{C}(t)\right\}_{t \geq 0}\right) - V\left(\left\{\hat{C}(t)\right\}_{t \geq 0}\right)$$

or equivalently,

$$\mathcal{L} = \exp\left[\rho\left\{V\left(\mathcal{L} \times \left\{\hat{C}(t)\right\}_{t \geq 0}\right) - V\left(\left\{\hat{C}(t)\right\}_{t \geq 0}\right)\right\}\right]$$

Let $\left\{\hat{C}'(t)\right\}_{t \geq 0}$ denote the consumption path generated by policy change. Then, the consumption equivalent welfare gains from policy change are calculated as

$$\mathcal{L} = \exp\left[\rho\left\{V\left(\left\{\hat{C}'(t)\right\}_{t \geq 0}\right) - V\left(\left\{\hat{C}(t)\right\}_{t \geq 0}\right)\right\}\right]$$