The Advantage of Dual Discrimination in Lottery Contest Games\textsuperscript{1}
by
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Abstract
In a simple class of complete information lottery contests, the designer can combine two types of discrimination: a change of the contestants’ prize valuations subject to a balanced-budget constraint (direct discrimination), as well as a bias of the impact of their efforts (structural discrimination). Applying such dual discrimination, the designer reduces (increases) the higher (lower) prize value up to a minimal (maximal) level, but suitably increases (reduces) the corresponding prize share. Our main result establishes that this dual discrimination is advantageous yielding almost the maximal possible efforts - the highest valuation of the contested prize. The efforts in our setting can therefore be larger than those obtained under alternative modes of one-type favoritism. This is true in particular with respect to the optimal structural discrimination or head-starts in a simple lottery or in all-pay auction contest, (Franke et al. 2013, 2014). The optimal two-mode favoritism that combines structural discrimination and head starts, (Franke et al. 2016), can yield the maximal possible revenue in all-pay auctions. Our result establishes that the maximal efforts can also be induced in a simple lottery contest by resorting to our alternative mode of dual discrimination.

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\textbf{Keywords}: contest design, dual discrimination, direct discrimination, balanced-budget-constraint, structural discrimination.

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1. Introduction

In the vast contest literature that has numerous applications (internal labor market tournaments, promotional competitions, R&D races, rent-seeking, political and public policy competitions, litigation and sports), the most commonly assumed contest success function (CSF) is the simple lottery proposed by Tullock (1980), (see Konrad (2009) and references therein). In two-player contests, for $x_1 \geq 0$, $x_2 \geq 0$, and $\delta > 0$, this simple logit functions take the form:

\[
p_1(x_1, x_2) = \begin{cases} 
\frac{x_1}{x_1 + \delta x_2} & \text{if } x_1 + x_2 > 0 \\
0.5 & \text{if } x_1 = x_2 = 0
\end{cases}
\]

Usually, $x_1$ and $x_2$ are interpreted as the contestants’ efforts. However, $p_1$ has two possible interpretations. It can be interpreted as contestant 1’s winning probability of an indivisible prize or as his share in a divisible prize. In turn, the winning probability of contestant 2 or his share in the prize is equal to $p_2 = 1 - p_1$. Henceforth, we use the second interpretation, as in Corchon and Dahm (2010), Franke et al. (2013), Lee and Lee (2012) and Warneryd (1998). Nevertheless, although under this interpretation there is no uncertainty in the model and the contestants compete on the certain shares of a divisible prize, we preserve the terms “contest” and “CSF”. The asymmetry between the impact of the contestants’ efforts is captured by the parameter $\delta > 0$.

In our extended setting, we do enable the contest designer to control $\delta$, as first suggested in Lien (1986, 1990) and later by Clark and Riis (2000). This means that the designer can apply structural discrimination that affects the contestants’ shares in the contested prize (the same efforts may yield different shares, depending on the value of this parameter). By (1), a reduction in $\delta$ increases the bias in favor of contestant 1, who is assumed to be, with no loss of generality, the more motivated contestant (the one with the higher prize valuation). Furthermore, $0 < \delta < 1$ ($\delta > 1$) implies a bias in favor of contestant 1 (contestant 2). When $\delta = 1$ the contest is fair.

The empirical relevance of such discrimination in contests with a logit CSF is thoroughly discussed in Epstein et al. (2011) and Franke (2012). Franke et al. (2013) have justified structural discrimination on the grounds that it lends itself to a very appealing competitive-market interpretation. For a recent survey of discrimination in contests, see Mealem and Nitzan (2016). Note that under complete information on the prize valuations, the contest designer could set a price equal to the highest valuation.
and sell the prize to the contestant with that valuation. The analysis of discrimination in contests is therefore meaningful assuming that the designer must allow a viable competition, as apparently implicitly assumed in the literature on optimal design in contests. It turns out that such a competitive setting still allows the designer to extract the maximal possible revenue, namely the highest prize valuation.

In our contest environment, the designer’s ability to discriminate is enriched. In addition to structural discrimination, i.e., the control of $\delta$, the contest designer can affect the contestants’ incentives by directly changing their rewards in case of winning all the contested prize, thereby increasing or decreasing the gap between their prize valuations. Such a policy is usually based on a “give and take” mechanism in case of winning, which is henceforth referred to as direct discrimination. This form of discrimination has been introduced and studied in Mealem and Nitzan (2014, 2016), focusing on its comparative application in an all-pay-auction relative to a logit CSF, disregarding the possibility of structural discrimination.

A crucial element in this different type of discrimination is the balanced-budget constraint faced by the contest designer. This constraint, which limits the design of the optimal tax schedule, implies that when one contestant's share of the prize is subjected to a positive tax, the share of the prize won by the other contestant must be subjected to a negative tax, viz., the granting of a subsidy. The tax scheme consists then of two numbers (negative and positive) that are added to the contestants' initial valuations of the divisible prize. These numbers need to satisfy the requirement that the designer’s net expenditures are equal to zero in equilibrium. Of course, whether the constraint is satisfied or not depends both on the applied structural and direct discrimination; the former determining the contestants' shares in the prize and the latter the actual modified values of the prize.

Our main result establishes that in a simple lottery contest, dual favoritism that combines structural and direct discrimination is very effective: the maximal efforts can be increased to almost the initially highest prize valuation.\(^2\)

The maximal efforts in our setting are larger than those obtained in Fang (2002), where discrimination is not allowed, and larger than the efforts obtained in Franke et al. (2013, 2014) where only structural or head-starts discrimination is

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\(^2\) This is implied by our main result, assuming that the upper bound of the net value of the prize approaches infinity and the lower bound of the net value of the prize approaches zero.
allowed. In a simple lottery contest with our dual discrimination, total efforts can be larger than those obtained under any optimal single mode of discrimination and, in particular, discrimination applied in an $N$-player all-pay auction. Whereas using headstarts in addition to structural discrimination is not conductive to generate additional revenue in a simple lottery contest (Franke et al. 2016), in our dual discrimination lottery contests, the contest designer has an incentive to apply the optimal direct discrimination in addition to the optimal structural discrimination. As in the case of optimal two-mode favoritism that combines structural discrimination and head starts in an all-pay auction contest (Franke et al. 2016), the maximal possible revenue can be induced in a simple lottery contest by resorting to our alternative two-mode favoritism combining structural and direct discrimination.

Fang (2002) considered the unbiased simple class of $N$-player lottery contests assuming that the discretionary power of the contest designer is restricted to the exclusion of specific players. He showed that the exclusion principle, established in Baye et al. (1993) for an all-pay auction framework with complete information, is not valid in an unbiased lottery framework. Franke et al. (2013) extend this result to the biased lottery contest showing that it is never optimal for the designer to discourage strong contestants from participating in order to enhance competitive pressure among the remaining weaker contestants. Moreover, they have pointed out the existence of an additional inclusion principle: some weak contestants, who are inactive in the unbiased case of Fang (2002), are encouraged to become active. This implies that in their setting of structural discrimination, the designer will endogenously induce a more leveled playing field in comparison to the unbiased contest setting. This enhances competition and, in turn, increases the contestants’ exerted efforts. However, as long as $N > 2$, it is not optimal for the designer to completely level the playing ground so some weak contestants may still remain inactive, although at least three will always be active.

Franke et al. (2016) have recently shown that an optimally biased all-pay auction contest combining structural discrimination and head-starts always dominates the optimally biased lottery contest resorting to these two modes of favoritism; it yields larger efforts. This is in contrast to the outcome of the comparison between the unbiased versions of these contest models where the (unbiased) all-pay auction can yield less efforts when the exclusion principle applies (it is effort-enhancing to exclude the player with the highest prize valuation from participation, but the two
active weaker contestants may expend less efforts than all the active players in the lottery contest). Franke et al. (2016) also show that when the designer can apply structural discrimination, the exclusion principle of the all-pay auction becomes obsolete. The designer will always bias the all-pay auction such that the two strongest players are active and, moreover, compete on equal terms (the strongest player is not excluded, but his effectiveness is sufficiently weakened). All other players choose to be inactive. Applying a less extreme discrimination in the lottery contest induces more contestants (at least three) to participate. But the effect of increased competitiveness due to a higher number of active contestants cannot offset the effect of reduced competitiveness due to a less extreme discrimination and, consequently, the optimally biased all-pay auction yields larger efforts than the optimally biased simple lottery.

Finally note that in a simple lottery contest with our dual discrimination, even when \( N > 2 \), the maximal efforts can be increased to almost the initially highest prize valuation, where only two contestants are active in equilibrium; one of them must be the contestant with the highest prize valuation, but the second active player can be any of the other contestants. This result differs from the result obtained in Franke et al. (2013, Theorem 4.6), where at least three contestants are active in the equilibrium of the simple \( N \)-player contest with just structural discrimination. Furthermore, the exclusion principle of Baye et al. (1993) is not necessarily valid in our extended contest because the strongest player can be induced to be always active.

Interestingly, in the extreme case of our setting (see footnote 1), the individual with the lower prize valuation is offered the illusion of competing on a very large prize, albeit only a very small share of it can be won. The value of his prize is nevertheless positive and in fact, almost equal to the initial prize valuation of his rival, the individual with the higher prize valuation. The existence of effective incentives that induce participation in the contest together with the existence of an extreme illusion that results in efforts incurred by the individual with the lower prize valuation is a distinctive interesting feature of our contest. This feature is manifested in the examples presented in the next section.

This paper is organized as follows. After discussing the examples in Section 2, the main results are presented in Section 3. Section 4 is devoted to the extension to the \( N \)-player game. We conclude the paper in Section 5.
2. Illustration of direct discrimination

Our model of dual discrimination in a simple lottery contest is of particular relevance in certain applications. To illustrate the plausibility of direct discrimination with a balanced-budget constraint in contests, we present two applications. In the first one, the contest designer typically engages in a certain activity (some well-defined task or project) restricted to a certain budget. Although the budget is earmarked only for this activity, it can be used to manipulate and affect the incentives of the contestants (the contractors) who compete for the outsourced project. But a designer who engages in such manipulations and in particular, in discrimination, must satisfy the contest balanced-budget constraint that we assume in order to ensure the overall budget constraint is satisfied:

1. Municipal projects. A municipal authority is conducting a tender for a divisible project such as urban development including development of a sewage system, roads, sidewalks and gardening. Two companies compete for a share in the project. The municipal authority is restricted to a budget allocated, for example, by the federal government. Although the budget is allocated only to urban development, it can be used to influence the incentives of the competing contestants by applying the two possible modes of discrimination. In order to satisfy the overall budget constraint, a designer who resorts to structural and direct discrimination must also satisfy the assumed balanced budget constraint.

The next application describes a situation where the balanced-budget constraint is due to a different reason. The constraint is no longer related to a fixed budget which is at the disposal of the designer for the purpose of carrying out a particular project. It is due to the fact that the two competing contestants are (at least partly) controlled by a parent company. The parent company may not prevent competition between its two subsidiaries by custom or by the law. However, despite the existing competition, the parent company still has the ability to enforce some overall financial discipline as well as the power to ensure that the designer's strength in manipulating the companies is limited. The control of the parent company on its two subsidiaries and its power in dealing with the designer, given the conflict of interests among them, explains its success in enforcing the balanced-budget constraint:

2. Portfolio distribution between two investment houses. In the capital market, the commission rate charged by an investment house is usually inversely related to the
size of the customer’s investment. Suppose that the average commission rate in the industry is \( \bar{z} \) and that a large client (e.g., a pension fund of some large employer) is interested in distributing its portfolio between two investment houses that are subsidiaries of some parent company. Despite the affinity among the two investment houses, they compete in the market.\(^3\) The first investment house has an established reputation while the second smaller house is relatively less known. Given the importance of the large customer, the preservation of the reputation of the first investment house (contestant 1) implies that it assigns a higher value to the investment of the large customer (the employer’s pension fund). Often, such a pension fund prefers to invest a large share of its portfolio in the reputable and usually larger investment house and this enables obtaining a commission rate lower than \( \bar{z} \). This implies that the pension fund actually “taxes” the larger and more reputable investment house relative to \( \bar{z} \). On the other hand, the investment house with the lower reputation usually receives the smaller share of the portfolio. However, the commission rate they charge are higher than \( \bar{z} \). The balanced-budget constraint is satisfied because of the market forces; The pension fund is interested in reducing the commission rate, and the parent company of the two investment houses is interested in increasing the commission rate.

3. The setting
In our contest there are two risk-neutral contestants,\(^4\) the high and low benefit contestants, 1 and 2. With no loss of generality, the initial prize valuations of the contestants, \( n_1 \) and \( n_2 \), satisfy the inequality \( n_1 \geq n_2 \) or \( k = \frac{n_1}{n_2} \geq 1 \) and that the contest designer is assumed to have full knowledge of the contestants’ prize valuations. In two-player contests, for \( x_1 \geq 0 \), \( x_2 \geq 0 \) and \( \delta > 0 \), the simple logit functions take the form:

\[^{3}\text{Two such competing investment houses, Psagot and Ofek, were subsidiaries of the leading Bank in Israel, Bank Leumi. Another example of two such companies is Gadish and Tagmulim, that were two subsidiaries of another major Israeli bank, Bank Hapoalim.}\]
\[^{4}\text{The case of multiple contestants is dealt with later on.}\]
and therefore \( p_2 = 1 - p_1 \).

Direct discrimination via differential taxation of the contested prize that affects the contestants’ actual prize valuations, \( n_1 \) and \( n_2 \), is a pair of (positive or negative) amounts, \( \varepsilon_1 \) and \( \varepsilon_2 \) that changes the prize valuations to \((n_1 + \varepsilon_1)\) and \((n_2 + \varepsilon_2)\) where \( 0 < n \leq n_1 + \varepsilon_1 \leq \bar{n} < \infty \). The lower and upper bounds of the contestants' actual net prize are the parameters \( n \) and \( \bar{n} \). We also assume that the contest designer faces a balanced-budget constraint, that is, \( \varepsilon_1 \) and \( \varepsilon_2 \) must also satisfy the requirement that:

\[
(2) \quad p_1 \varepsilon_1 + p_2 \varepsilon_2 = 0.
\]

Given the contestants’ fixed prize valuations and the CSF, the function that specifies the contestants’ shares given their efforts, \( p_i(x_1, x_2) \), the net payoff (surplus) of contestant \( i \) is:

\[
(3) \quad E(u_i) = p_i(x_1, x_2)(n_i + \varepsilon_i) - x_i \quad (i=1,2).
\]

In the optimal contest design setting, the objective function of the contest designer is:

\[
(4) \quad G = x_1 + x_2.
\]

The designer selects \( \delta \), \( \varepsilon_1 \) and \( \varepsilon_2 \). In this case, the two contestants maximize their net payoffs:

\[
(5) \quad E(u_1) = \frac{x_1}{x_1 + \delta x_2}(n_1 + \varepsilon_1) - x_1 \quad \text{and} \quad E(u_2) = \frac{\delta x_2}{x_1 + \delta x_2}(n_2 + \varepsilon_2) - x_2.
\]

If the designer selects \((\varepsilon_1, \varepsilon_2) = (0,0)\), then the optimal \( \delta \) is \( \delta = \frac{n_1}{n_2} \), and the corresponding efforts are equal to \( G = 0.25(n_1 + n_2) \), see Epstein et al. (2013). Therefore, in the following, we assume \((\varepsilon_1, \varepsilon_2) \neq (0,0)\) and from (2) we get that \( \varepsilon_1 \varepsilon_2 < 0 \). Let

\[
(6) \quad d = \frac{\frac{n_1 + \varepsilon_1}{n_2 + \varepsilon_2}}{\delta}.
\]

By the first order conditions (maximization of (5)),

\[
\left. \begin{align*}
\frac{\partial E}{\partial x_1} &= \frac{x_1}{x_1 + \delta x_2} - \frac{x_1^2}{(x_1 + \delta x_2)^2} (n_1 + \varepsilon_1) - x_1 = 0,
\frac{\partial E}{\partial x_2} &= \frac{\delta x_2}{x_1 + \delta x_2} - \frac{\delta x_2^2}{(x_1 + \delta x_2)^2} (n_2 + \varepsilon_2) - x_2 = 0.
\end{align*} \right\}
\]
\[ x_1^* = \frac{d(n_1 + \varepsilon_1)}{(d+1)^2} \quad \text{and} \quad x_2^* = \frac{d(n_2 + \varepsilon_2)}{(d+1)^2} \]

and, therefore,

\[ G = x_1^* + x_2^* = \frac{d(n_1 + \varepsilon_1 + n_2 + \varepsilon_2)}{(d+1)^2} \]

(8)

\[ p_1 = \frac{d}{d+1} \quad \text{and} \quad p_2 = \frac{1}{d+1} \]

and the balanced-budget constraint (2) takes the form

\[ p_1 \varepsilon_1 + p_2 \varepsilon_2 = \frac{d}{d+1} \varepsilon_1 + \frac{1}{d+1} \varepsilon_2 = 0 \]

(9)

or

\[ d \varepsilon_1 + \varepsilon_2 = 0 \]

(10)

By the balanced-budget constraint (11), \( d = -\frac{\varepsilon_2}{\varepsilon_1} \). Substituting \( d = -\frac{\varepsilon_2}{\varepsilon_1} \) in (7) we get

\[ G = x_1^* + x_2^* = -\frac{\varepsilon_1 \varepsilon_2 (n_1 + \varepsilon_1 + n_2 + \varepsilon_2)}{(\varepsilon_1 - \varepsilon_2)^2} \]

By (6) and \( d = -\frac{\varepsilon_2}{\varepsilon_1} \), we get that in equilibrium,

\[ \delta = -\left( \frac{n_1 + \varepsilon_1}{n_2 + \varepsilon_2} \right) \frac{\varepsilon_1}{\varepsilon_2} \]

(12)

and the positive net payoffs are:

\[ u_1^* = \frac{(n_1 + \varepsilon_1) d^2}{(d+1)^2} \quad \text{and} \quad u_2^* = \frac{(n_2 + \varepsilon_2)}{(d+1)^2} \]

(13)

Therefore, the designer objective function is:

\[ G(\varepsilon_1, \varepsilon_2) = -\frac{\varepsilon_1 \varepsilon_2 (n_1 + \varepsilon_1 + n_2 + \varepsilon_2)}{(\varepsilon_1 - \varepsilon_2)^2} \]

(14)

and his optimal strategy \((\varepsilon_1^*, \varepsilon_2^*)\) solves:

\[ \text{Max}_{(\varepsilon_1, \varepsilon_2) \in C} G(\varepsilon_1, \varepsilon_2) \]

(15)

where

\[ C = \{ (\varepsilon_1, \varepsilon_2) : \underline{n} \leq n_1 + \varepsilon_1 \leq \bar{n}, \underline{n} \leq n_2 + \varepsilon_2 \leq \bar{n}, \varepsilon_1, \varepsilon_2 < 0 \} \]

(16)
Proposition 1

Assume that \( n_1 > n_2, \, \bar{n} > 0 \) is sufficiently close to zero, and \( \bar{n} \) is sufficiently large. Then,

\[
G(\epsilon_1^*, \epsilon_2^*) = G(n - n_1, \bar{n} - n_2) = \frac{(\bar{n} + n)(n_1 - n)(\bar{n} - n_2)}{(n - n + n_1 - n_2)^2},
\]

\[
x_1^* = \frac{n(n_1 - n)(\bar{n} - n_2)}{(n - n + n_1 - n_2)^2}, \quad x_2^* = \frac{\bar{n}(n_1 - n)(\bar{n} - n_2)}{(n - n + n_1 - n_2)^2},
\]

\[
u_1^* = \frac{n(n - n_2)}{(n - n + n_1 - n_2)^2}, \quad u_2^* = \frac{\bar{n}(n_1 - n)}{(n - n + n_1 - n_2)^2}.
\]

To prove the above, let

\[(17) \quad C_1 = \{ (\epsilon_1, \epsilon_2) : n \leq n_1 + \epsilon_1 \leq \bar{n}, n_2 \leq n_2 + \epsilon_2 \leq \bar{n}, \epsilon_1 < 0 < \epsilon_2 \}\]

and

\[(18) \quad C_2 = \{ (\epsilon_1, \epsilon_2) : n \leq n_1 + \epsilon_1 \leq \bar{n}, n_2 \leq n_2 + \epsilon_2 \leq \bar{n}, \epsilon_2 < 0 < \epsilon_1 \}.
\]

Note that \( C = C_1 \cup C_2 \).

Lemma 1

Assume that \( n_1 > n_2, \, \bar{n} > 0 \) is sufficiently close to zero, and \( \bar{n} \) is sufficiently large. Then,

(a) \( \arg \max_{(\epsilon_1, \epsilon_2) \in C_1} G(\epsilon_1, \epsilon_2) = \{ (n - n_1, \bar{n} - n_2) \} \)

(b)

\[
\arg \max_{(\epsilon_1, \epsilon_2) \in C_2} G(\epsilon_1, \epsilon_2) = \begin{cases} 
\{ (\bar{n} - n_1, n - n_2) \} & \text{if } n_1 \leq 2n_2 - 3n \\
\{ (\bar{n} - n_1, n - n_2) \left( \frac{(n_1 + n)(n_2 - n)}{n_1 - 2n_2 + 3n} n - n_2 \right) \} & \text{if } n_1 > 2n_2 - 3n 
\end{cases}
\]

Proof of Lemma 1

We first show (a). Notice that

\[
\frac{\partial G}{\partial \epsilon_1} = \epsilon_2 \left[ \epsilon_1 (n + 3\epsilon_2) + \epsilon_3 (n + \epsilon_2) \right] \frac{\epsilon_1 - \epsilon_2}{(\epsilon_1 - \epsilon_2)^3}
\]

\[
\frac{\partial G}{\partial \epsilon_2} = \epsilon_1 \left[ \epsilon_2 (n + 3\epsilon_1) + \epsilon_3 (n + \epsilon_1) \right] \frac{\epsilon_2 - \epsilon_1}{(\epsilon_2 - \epsilon_1)^3}
\]

where \( n = n_1 + n_2 \). Thus for \( \epsilon_1 < 0 < \epsilon_2 \),
\[
\frac{\partial G}{\partial \varepsilon_1} < 0 \iff \varepsilon_1(n + 3\varepsilon_2) > -\varepsilon_2(n + \varepsilon_2)
\]

(20)

\[
\frac{\partial G}{\partial \varepsilon_2} > 0 \iff \varepsilon_2(n + 3\varepsilon_1) < -\varepsilon_1(n + \varepsilon_1)
\]

Therefore, for \(\varepsilon > 0\),

(21) \hspace{1cm} \arg \max_{\varepsilon_1 \geq 0} G(\varepsilon_1, \varepsilon_2) = \max \left\{ \left( n - n_1, -\varepsilon_1(n + \varepsilon_2) \right) \right\}

and for \(\varepsilon < 0\),

(22) \hspace{1cm} \arg \max_{\varepsilon_2 > 0} G(\varepsilon_1, \varepsilon_2) = \begin{cases} \min \left\{ \left( n - n_2, -\varepsilon_2(n + \varepsilon_1) \right) \right\} & \text{if } n + 3\varepsilon_1 > 0 \\ \varepsilon_1 = \frac{n - n_2}{n + 3\varepsilon_2} & \text{if } n + 3\varepsilon_1 \leq 0 \end{cases}

Let \((\bar{\varepsilon}_1, \bar{\varepsilon}_2) \in \arg \max_{(\varepsilon_1, \varepsilon_2) \in C} G(\varepsilon_1, \varepsilon_2)\). Then

(23) \hspace{1cm} \bar{\varepsilon}_1 = \max \left\{ n - n_1, -\varepsilon_1(n + \varepsilon_2) \right\} = n - n_1

because if \(\bar{\varepsilon}_2 = n - n_2\), then

(24) \hspace{1cm} (n - n_1) - \left( -\frac{\varepsilon_2(n + \varepsilon_2)}{n + 3\varepsilon_2} \right) = \frac{(n - n_2)(n + n_1)}{3n - 2n_2 + n_1} + n - n_1 > 0

when \(\frac{n}{n}\) is sufficiently large and if \(\bar{\varepsilon}_2 = -\frac{\varepsilon_1(n + \varepsilon_1)}{n + 3\varepsilon_1}\) and \(\bar{\varepsilon}_1 = -\frac{\varepsilon_2(n + \varepsilon_2)}{n + 3\varepsilon_2}\), then \((\bar{\varepsilon}_1, \bar{\varepsilon}_2) = (0, 0)\) or \((-0.5n, -0.5n)\). Therefore \((\bar{\varepsilon}_1, \bar{\varepsilon}_2) = (n - n_1, n - n_2)\) or

\(\left( n - n_1, -\varepsilon_1(n + \varepsilon_1) \right)\). However, because

(25) \hspace{1cm} n + 3\varepsilon_1 = n_2 - 2n_1 + 3n < 0

when \(\frac{n}{n}\) is sufficiently close to zero, we must have \((\bar{\varepsilon}_1, \bar{\varepsilon}_2) = (n - n_1, n - n_2)\).

We next show (b). Let \((\bar{\varepsilon}_1, \bar{\varepsilon}_2) \in \arg \max_{(\varepsilon_1, \varepsilon_2) \in C} G(\varepsilon_1, \varepsilon_2)\). By a similar argument,

\((\bar{\varepsilon}_1, \bar{\varepsilon}_2) = (n - n_1, n - n_2)\) or \(\left( -\frac{\varepsilon_2(n + \varepsilon_2)}{n + 3\varepsilon_2}, n - n_2 \right)\), but the latter solution is also possible if \(n + 3\varepsilon_2 = n_1 - 2n_2 + 3n > 0\).

Q.E.D
Proof of Proposition 1

Since \( n_1 > n_2 \), we have

\[
G(n - n_1, \overline{n} - n_2) - G(n - n_1, \overline{n} - n_2) = \frac{(n_1 - n_2)(n - n_2)(\overline{n} + n - n_1 - n_2)^2}{(n - n + n_1 - n_2)(\overline{n} - n_1 + n_2)} > 0
\]

If \( n_1 > 2n_2 - 3n \), then we can verify that

\[
\lim_{\overline{n} \to \infty} G(n - n_1, \overline{n} - n_2) - G\left( \frac{n_1 + n(n_2 - n)}{n_1 - 2n_2 + 3n}, n - n_2 \right)
\]

\[
= \frac{n(2n_1 + 4n_2) - 9n_2 + n_1(3n_1 - 4n_2)}{8n + 4n_1 - 4n_2} > \frac{n(7n_1 - 4n_2) - 9n_2 + 2n_1n_2}{8n + 4n_1 - 4n_2} > 0
\]

when \( n \) is sufficiently small. Therefore, we must have \( G(\varepsilon^*_1, \varepsilon^*_2) = G(n - n_1, \overline{n} - n_2) \)
when \( n \) is sufficiently small and \( \overline{n} \) is sufficiently large.

Q.E.D

In the next proposition, we obtain a sufficient condition for the values of \( \overline{n} \) and \( n \).

This condition is stated in terms of a new parameter \( r \) that relates the upper and lower bounds to the contestants' prize valuations in a simple specific way. The parameter \( r \) which characterizes the environment of the designer enables a more practical restatement of Proposition 1 by explicitly limiting the requirements that the upper bound is sufficiently large and the lower bound is sufficiently close to zero.

Proposition 2

Assume that \( n_1 > n_2 \), \( \overline{n} = n_1r \), \( n = \frac{n_2}{r} \), and \( r > 4.5 \). Then,

\[
G(\varepsilon^*_1, \varepsilon^*_2) = G(n - n_1, \overline{n} - n_2) = \frac{n_2 + n_1r^2}{(1 + r)^2},
\]

\[
x^*_1 = \frac{n_2}{(1 + r)^2}, \quad x^*_2 = \frac{n_1r^2}{(1 + r)^2},
\]

\[
u^*_1 = \frac{n_2r}{(1 + r)^2}, \quad u^*_2 = \frac{n_1r}{(1 + r)^2}.
\]

Proof of Proposition 2

We show that \( r > 4.5 \) suffices the assumption that \( n > 0 \) is sufficiently close to zero and \( \overline{n} \) is sufficiently large.
We first consider the proof of Lemma 1, where (24) and (25) use this assumption. The left-hand side of (24) is reduced to

\[
(n - n_1) - \left( -\frac{\bar{e}_2(n + \bar{e}_2)}{n + 3\bar{e}_2} \right) = \frac{(n, r - n_2)(n_1, r^2 - 2n_1 r - n_1 + 2n_2)}{r(3r + 1)n_1 - 2n_2}
\]

which is positive if \( r > 4.5 \). The left-hand side of (25) is reduced to

\[
n + 3\bar{e}_1 = (r + 3)\frac{n_2}{r} - 2n_i
\]

which is negative if \( r > 4.5 \).

We next consider the proof of Proposition 1. It is enough to show that if

\[
n_1 > 2n_2 - 3n = (2r - 3)\frac{n_2}{r} \quad \text{and} \quad r > 4.5
\]

then

\[
G(n - n_1, n - n_2) - G\left(\frac{(n_1 + n)(n_2 - n)}{n_1 - 2n_2 + 3n}, n - n_2\right) > 0.
\]

The left-hand side is reduced to

\[
\frac{(r-1)}{4r(r+1)^2[r(n_1 - n_2) + 2n_2]}, f(r)
\]

where

\[
f(r) = r^3n_1(3n_1 - 4n_2) + r^2n_1(n_1 + 2n_2) + r n_2(2n_1 - 5n_2) + n_2^2
\]

Note that \( n_1 > (2r - 3)\frac{n_2}{r} \) and \( r > 4.5 \) imply

\[
3n_1 - 4n_2 > (2r - 9)\frac{n_2}{r} > 0
\]

Thus, \( f''(r) > 0 \) and

\[
f'(r) > f'(1) = 11n_1^2 - 6n_1n_2 - 5n_2^2 > 0
\]

because \( n_1 > n_2 \). Consequently,

\[
f(r) > f(1) = 4(n_1^2 - n_2^2) > 0 \quad \text{if} \quad r > 4.5.
\]

**Q.E.D**

It is straightforward to verify that comparative statics with respect to the parameter \( r \) yields the following results:

\[
\frac{\partial x_1}{\partial r} = -\frac{2n_2}{(1+r)^3} < 0, \quad \frac{\partial x_2}{\partial r} = \frac{2n_1r}{(1+r)^3} > 0 \quad \text{and} \quad \frac{\partial G}{\partial r} = \frac{2(n_1r - n_2)}{(1+r)^3} > 0.
\]

The latter effect is due to the dominance of the positive effect of a change in \( r \) on the efforts of player 1 relative to its negative effect on the efforts.
of player 2. Also note that the equilibrium efforts are increasing in the prize valuations of the two players, however these efforts are smaller than \( n_1 \) as long as \( n_1 > n_2 \) and \( r \) is finite.

4. The \( N \) – player contest

With dual discrimination total efforts cannot exceed the prize valuation of contestant 1. That is,

\textbf{Proposition 3}

Under dual discrimination, in equilibrium, \( n_1 \geq G \).

\textbf{Proof of Proposition 3}

In equilibrium, the net payoff of every contestant under dual discrimination cannot be negative, since otherwise he can improve his situation and secure a zero net payoff by not taking part in the contest. In other words, for every contestant \( i \), \( u_i = p_i(n_i + \varepsilon_i) - x_i \geq 0 \). Summing over all the contestants, we get that

\[ \sum_{j=1}^{N} u_j = \sum_{j=1}^{N} \left[ p_j(n_j + \varepsilon_j) - x_j \right] \geq 0 \text{ or } \sum_{j=1}^{N} \left( p_j(n_j) + \sum_{j=1}^{N} (p_j \varepsilon_j) \right) - \sum_{j=1}^{N} x_j \geq 0. \]

Since \( G = \sum_{j=1}^{N} x_j \) and \( \sum_{j=1}^{N} (p_j \varepsilon_j) = 0 \), \( \sum_{j=1}^{N} (p_j n_j) \geq G \). Since \( n_1 \geq n_2 \geq \ldots \geq n_N \) and \( \sum_{j=1}^{N} p_j = 1 \), it must be true that \( n_1 \geq \sum_{j=1}^{N} (p_j n_j) \geq G \). \(\text{Q.E.D.}\)

\textbf{Proposition 4}

\(\text{This result is also valid for unbiased contests or for contests with alternative modes of discrimination, when discrimination is through modifying the efforts. The explanation is that in equilibrium, the net payoff of every contestant cannot be negative, since otherwise he can improve his situation and secure a zero net payoff by not taking part in the contest. In other words, for every contestant } i, \)

\[ u_i = p_i n_i - x_i \geq 0 \]. Summing over all the contestants, we get that \( \sum_{j=1}^{N} u_j = \sum_{j=1}^{N} (p_j n_j - x_j) \geq 0 \) or

\[ \sum_{j=1}^{N} (p_j n_j) \geq \sum_{j=1}^{N} x_j \). Since \( n_1 \geq n_2 \geq \ldots \geq n_N \) and \( \sum_{j=1}^{N} p_j = 1 \), it must be true that \( n_1 \geq \sum_{j=1}^{N} (p_j n_j) \geq G \).\]
If $N = 2$ and $n_i > n_2$, then the contest designer can always set $r$ such that the equilibrium efforts are sufficiently close to $n_1$. In this case $x_2^* \to n_i - n$, $u_1^* \to n$ and $u_2^* \to 0$.

**Proof of Proposition 4**

It can be verified that $G(e_1^*, e_2^*) = \frac{n_2 + n_i r^2}{(1 + r)^2} < n_1$ and $\lim_{r \to \infty} G = n_1$. Since $G$ is continuous and monotone in $r$, there exists $r$ such that $G$ is sufficiently close to $n_1$.

**Q.E.D**

By Proposition 3, we have shown that the contestants cannot be induced to exert larger efforts than $n_1$. Therefore, Proposition 4 can be extended to the case of any number of contestants $N$. In the more general multi-player contest, the designer has to reduce the stakes of $N - 2$ contestants to zero, making sure that contestant 1 with the highest stake is not included among them. That is,

**Corollary 1**

Given any number of contestants $N$, such that $\infty > n \geq n_1 \geq n_2, \ldots, n_N \geq n > 0$, the contest designer can always set $r$ such that the equilibrium efforts are sufficiently close to $n_1$.

**Proof of Corollary 1**

The proof is based on the following simple three-stage strategy that the designer applies:

1. **Stage 1**: The designer selects a contestant $j \in \{2, \ldots, N\}$.
2. **Stage 2**: For any contestant $i \neq \{1, j\}$, the designer chooses $e_i = -n_i$. That is, he reduces the initial prize valuations of $N - 2$ players to zero.
3. **Stage 3**: Applying the dual discrimination strategy with respect to the two contestants 1 and $j$, according to Proposition 4, the designer can induce efforts that are almost equal to $n_1$.

**Q.E.D.**
The explanation of our results is based on the following idea. First, assume without loss of generality that $j = 2$. On the one hand, the designer applies direct discrimination in favor of contestant 2 by reducing the stake of contestant 1 (the contestant with the initially higher prize value) to $n$ by choosing $\varepsilon_1$ and increases the stake of contestant 2 (the contestant with the initially lower prize value) to $\bar{n}$ by choosing $\varepsilon_2$. On the other hand, in order to satisfy the balanced-budget constraint, the designer must create an appropriate bias in favor of contestant 1 by selecting $\delta$, such that the balanced-budget constraint (10) satisfies equation (12):

$$
\delta = -\left(\frac{n_1 + \varepsilon_1}{n_2 + \varepsilon_2}\right)\frac{\varepsilon_1}{\varepsilon_2}
$$

Propositions 1 and 2 imply that dual discrimination, that is, $\varepsilon_1 = n - n_1$ and $\varepsilon_2 = \bar{n} - n_2$, is an effective strategy for increasing efforts. These efforts can be increased almost up to $n_1$, the initial higher prize valuation of contestant 1, provided that $r \to \infty$.

Propositions 1 and 2 imply that when the designer applies our two modes of discrimination in a simple lottery, each type has a positive “added value” that enhances the exertion of efforts relative to the situation where the designer resorts to just structural discrimination (Franke et al. (2013, 2014, 2016)). That is, the two modes of discrimination are supportive or “complementing” - their combination can yield larger efforts than those obtained by just structural discrimination. The advantage of combining these two types of discrimination relative to the use of just structural discrimination is due to the distinctive features of the contribution of each of these modes of discrimination to the exerted efforts as described below.

(i) Direct discrimination sufficiently increases the initially lower prize valuation while sufficiently reducing the initially higher prize valuation. This increases the sum of the contestants’ prize valuations to infinity when $r \to \infty$ and makes the ‘income effect’ (associated with a scheme that increases the sum of the final stakes from $(n_1 + n_2)$ to $(n_1 + \varepsilon_1 + n_2 + \varepsilon_2) = \bar{n} + n$) of this mode of discrimination the dominant effect.6

6 For a clarification of the meaning of the income effect associated with direct discrimination, see the discussion following Proposition 2 in Mealem and Nitzan (2014).
(ii) The maximal possible increase in the sum of the contestants’ prize valuations is not the result of direct discrimination alone. It is rendered possible by structural discrimination that makes sure that the balanced-budget constraint is satisfied. Specifically, structural discrimination counterbalances the above ‘income effect’ by almost completely favorably discriminating contestant 1, ensuring that his prize share converges to 1. The moderating effect described in (ii) is necessary to attain the maximal efforts. While structural discrimination has a ‘second order’ effect on efforts that moderates the income effect of direct discrimination, it also enables the dominance of this ‘first order’ income effect on efforts described in (i), namely, the increase in efforts due to the increase in the sum of the contestants’ prize valuations. The dominance of the effect of direct discrimination means that the more extreme this mode of discrimination, the higher the total efforts and this requires the extremity of structural discrimination.

Corollary 1 implies, in particular, that dual discrimination can be advantageous relative to just structural discrimination in the 2-player all-pay auction or the simple lottery contest.

**Corollary 2**

If \( N = 2 \) and \( n_1 > n_2 \), then there exists \( r \) such that dual discrimination yields efforts that are larger than the efforts obtained in the optimally biased 2-player all-pay auction (simple lottery) contest with structural discrimination. That is,

\[
\frac{n_2 + n_1 r^2}{(1 + r)^2} > 0.5(n_1 + n_2) > 0.25(n_1 + n_2) \quad \text{or} \quad r > \frac{k + 1 + \sqrt{2(k^2 + 1)}}{k - 1}.
\]

Note that our results imply that under the assumption that \( \bar{n} \) is sufficiently large and \( n \) is sufficiently close to zero \( (r \to \infty) \), the combination of the two modes of discrimination results in an outcome which is practically reasonable; the amount transferred between the contestants is finite. When the actual payment to contestant 2 (the tax taken from contestant 1) is equal to \( p_2 \varepsilon_2 \to n_1 \), since \( \delta \) converges to zero, that is, \( p_2 \) converges to zero, it seems that contestant 2 has no incentive to compete. But this is not the case because he plays a crucial role in yielding the almost maximal
possible efforts $n_1$. The designer sufficiently increases his prize valuation and by that induces him to exert efforts that are almost equal to $n_1$.

In the first example of a municipal project (see the examples discussed in section 2), despite the fact that both modes of discrimination can be extreme, their combined use still results in a balanced effect. The designer promises the small company a very large value in case it receives the entire project. The designer, by structural discrimination, ensures that the small company's share in the project is sufficiently small, such that the large reputable company wins almost the entire project. In this extreme and most effective case from the designer's point of view, the large company transfers the small company a reasonable finite amount which is almost equal to its initial valuation of the entire project.

In the second example of portfolio distribution between two investment houses in the extreme case the tax-subsidy transfer between the two investment houses under the optimal dual discrimination strategy is finite. In particular, by Proposition 1, it is equal almost to $n_1$, the value for investment house 1 of getting the entire portfolio of the large customer. Again, this result is plausible not because of the good will of the customer, but because it ensures him that the two investment houses exert almost the maximal efforts.

5. Conclusion
The enrichment of the “box of tools” of the contest designer by allowing him to exercise direct discrimination in addition to structural discrimination and head-starts clearly matters. We have shown that both structural and direct discrimination are effective in a lottery contest and therefore will be used by the designer. Under our lottery contest with dual discrimination, efforts are larger than those under any alternative contest game with optimal single favoritism and, in particular, optimal favoritism applied in an all-pay auction.

Whereas using head-starts in addition to structural discrimination is not conductive to generate additional revenue in a simple lottery contest, (Franke et al. 2016), in our dual discrimination lottery contests, the contest designer has an incentive to apply the optimal direct discrimination in addition to the optimal structural discrimination. The optimal two-mode favoritism that combines structural discrimination and head starts can yield the maximal possible revenue in all-pay
auctions, (Franke et al. 2016). Our main result establishes that the maximal efforts can also be induced in a simple lottery contest by resorting to our alternative mode of dual discrimination. The largest possible efforts are equal to the initially highest prize valuation, when $r \to \infty$. Hence, dual discrimination is indeed advantageous provided that it is based on the appropriate structural bias and head-starts in an all-pay auction contest and on the appropriate structural bias and direct discrimination in a simple lottery contest.

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Government loan guarantees and the credit decision-making structure

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Abstract. Governments can alleviate the problem of banks denying credit to high risk borrowers and excluding weaker sectors from borrowing by introducing state-guaranteed loan programs. The main contribution of this paper is the elucidation of the importance of the bank's credit decision-making structure in ensuring overall effectiveness of loan guarantees. In particular, the government can use the guarantee as an instrument for credit inducement and for affecting the bank's decision-making system i.e., its degree of centralization, bias towards approval of loans and reliance on objective loan-specific information.

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1. Introduction.

Governments intervene in the credit market in order to provide loans in cases where private markets will not. They act in response to difficulties faced by certain sectors in obtaining credit. These sectors include households who seek mortgages, small businesses, minorities,
women and developing regions or industries. The reluctance of banks to grant credit is due to the high risk associated with lending to these sectors. In this study we analyze a bank's credit decision structure focusing on the effect of a government loan guarantee on credit allocation. We wish to contribute to a better understanding of the micro foundations of macro-economic phenomena in which bank lending plays a crucial role. The emphasis is on clarifying the importance of the decision-making structure for determining the success of such a government guarantee in achieving its goal of appropriate lending inducement. In fact, the government can determine the marginal credit-effectiveness of a loan guarantee, and can affect the degree of centralization of the bank’s decision-making system, the bias towards approval of loans and the extent of reliance on objective information relevant to any specific loan.

A large number of countries have government-sponsored credit guarantee schemes, including the majority of OECD countries. We learn from Green (2003), that there are over 2,250 guarantee schemes in more than 100 countries. The OECD in its 2012 report 7 discusses the design of Government Credit Guarantee Schemes (CGS) and Mutual Credit Guarantee schemes, pointing to the fact that, following the 2008 economic crisis, many existing schemes were expanded and new schemes were set up in an effort to overcome the economic crisis. The OECD report shows that the volume of credit guarantee schemes can reach 7.3% of GDP as in the case of Japan. However, there is great heterogeneity in the design of scheme mechanisms. Specifically, of interest to this study is the fact that credit assessments and credit decisions can be made by the public entity that provides the guarantee, the lending institution or both. In this sense alone there is variation in the level of centralization of the credit decision within different credit schemes. In Austria, Bulgaria, The Czech Republic, France, Germany, Hungary, Italy, Netherlands, Poland, Romania, Russia, Slovenia, Spain and Turkey, only one of the above institutions makes the credit decision. Whereas in Belgium, Estonia, France, Greece, Latvia, Lithuania, Luxemburg and Portugal both institutions make the credit decision. The difference in decision-making centralization arises from variations in the types of schemes as well as legal issues, nonetheless illustrating the difference in decision design at the national level. Furthermore, each organization, the public entity and the private lender, will have their own organizational design for which data is sparsely available. However, as the OECD report states: “The design of CGSs is crucial for their effectiveness and sustainability”. In this study we focus on the decision structure of the organization and its effect on the CGS.

Specific examples of government loan guarantee schemes are Germany, where government guarantees were provided for loans given by savings banks until 2001 (Gropp, Guettler and Saadi 2015). The U.S. government has in the past used different institutions for its individual loan programs whose purpose is to increase lending through banks, e.g., government sponsored secondary market mortgage institutions, such as Fannie Mae and Freddie Mac and notably, the Small Business Administration, which provides government guarantees to private financial intermediaries. In the U.K. a loan guarantee scheme provides access to credit to

7 OECD Centre for Entrepreneurship, SME and Entrepreneurship Financing: The Role of Credit Guarantee Schemes and Mutual Guarantee Societies in supporting finance for small and medium-sized enterprises, 30-Jan-2013, CFE/SME(2012)1/FINAL.
small firms suffering from credit rationing. Over the period 1998-2001 Japanese banks gave
government loans to SMEs (Uesugi, Sakai and Yamashiro, 2010) and in 2008 the Japanese
government set up the Emergency Credit Guarantee Program (Ono, Uesugi and Yasuda 2013).
In both studies on Japanese Government loan schemes, the authors found that the credit
programs were successful at increasing lending to the firms that participated. Cowan, Drexler
and Yaoez (2015) find that the Chilean partial credit guarantee scheme increased lending to
SMEs but also increased default rates due to adverse selection created by the scheme.

Loan guarantees reduce the risk faced by the lender. But they may have an undesirable
effect of applying unsatisfactory loan screening methods and decision structures to loan
requests. Moreover, Honohan (2010) reviews and discusses the goals and costs of partial
credit guarantee schemes exposing the difficulty in estimating the social benefit from such
schemes, despite their popularity. In this study we define effectiveness of a guarantee scheme
as the maximal increase in lending. While this is the primary goal of a credit guarantee
scheme, there are additional criteria of effectiveness that concern the economy. For a credit
guarantee scheme to be successful at increasing growth in the economy the increase in
lending should be channeled to companies that are most likely to experience growth and
further investment. Furthermore, it should be apportioned to firms that are most likely to be
profitable and pay back their loans. Otherwise the scheme will become extremely costly and
hence ineffective from a cost-benefit point of view.

The objective of the present study is to examine the effect of a government’s loan
guarantee on lending and on the design of the bank’s decision-making system i.e., its degree
of centralization, bias towards approval of loans and extent of reliance on objective loan-
specific information. This objective is carried out by focusing on the bank's credit decision,
viz., whether to approve or reject a specific loan request. The decision is analyzed applying
the uncertain dichotomous choice setting that stresses the role of the decision-making
structure, namely, the decision rule that aggregates the committee members' credit decisions.
The question of how a bank's organizational structure affects its credit decisions has been
discussed within the context of credit availability for SMEs. Notably, Berger and Udell
(2002) argue that different types of lending (relationship lending vs. transactional lending)
require different organizational structures for banks. More specifically, since small borrowers
typically generate soft information, they will succeed more at obtaining credit from less
hierarchical banks were loan officers can make credit decisions on their own. Stein (2002)
discusses the effect of two specific (centralized vs. decentralized) such designs on the share of
small business lending. Canales and Nanda (2012) studies the organizational structure that
provides better lending terms for small businesses, finding that decentralized banks provide
larger loans to small businesses. These findings are further supported by Cotugno, Mnoferrà
and Sampagnaro (2013) where hierarchical distance is shown to be negatively related to credit
availability. However, the decision structure not only affects the likelihood of loan approval,
but also determines the quality of the loan decision. Liberti and Mian (2009) find that greater

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8 For credit scoring methods see Mester (1997), Altman and Saunders (1997) and Allen, Delong and
Saunders (2004) and for literature analyzing the design of appropriate cut-off methods for credit
decisions when a credit score is provided see Andersson (2004). Much less research has been devoted
to the design of the decision-making structure in banks.
hierarchical distance between the agent who collects information and the loan officer who makes the loan decision leads to less reliance on subjective information and more reliance on objective information. Meissner (2005) studies the effect of the number of votes needed to approve loans using historical data from New England focusing on the approval of loans with private gains and emphasizing the effect on good lending practices. Graham, Harvey and Puri (2015) examine the decision process and use of information as it is reflected in the delegation of financial decisions within firms. In the following we set out to address the question of how the organizational structure of banks determines the effectiveness of credit guarantee schemes.

Our theoretical framework is that of group decision-making in a committee of fixed size that is subject to human fallibility. This field of study has attracted a great deal of attention. Nitzan and Paroush (1982, 1985), Grofman, Owen and Feld (1983) and Shapely and Grofman (1984) laid the theoretical foundations of the uncertain binary choice model. Following previous results, Ben-Yashar and Nitzan (1997) defined the optimal decision rule in an extended setting which allows asymmetric choice. The loan guarantee framework allows us to demonstrate how these results can be applied to resolving a specific issue faced by the government, leading to new insights into the decision rule. In our banking application of this model, a credit committee is appointed by the bank's board of directors. In the bank setting this committee can be interpreted as a group of decision makers who meet in order to vote on loan approval, a structure of management, central offices and branches all of whom are part of a chain of decision makers on loan approval, or a credit scoring model. The task of the committee is to approve or reject a loan request while trying to reach the correct decision concerning loan approval. Each committee member has expertise in determining whether or not a loan should be granted. The decisions of the credit committee members are aggregated by using a decision rule that yields a final decision regarding the approval or rejection of the requested loan. In our setting the government can use a loan guarantee as an effective tool to increase lending. Our first main theorem shows that the structure of decision making in the bank determines the effectiveness of a loan guarantee in increasing lending to high-risk borrowers. More specifically, for a given guarantee level, it is shown that the government can expect the maximal increase in lending by varying the guarantee when the simple majority rule is used to aggregate the decisions of the committee members and the minimal increase when the committee applies a centralized or a decentralized decision rule. This implies that if the government can control both the loan guarantee and the decision-making rule applied by the bank or, alternatively, it can set only the guarantee, but is aware of the reaction function of the bank to the guarantee, the government can exploit its advantage and set the guarantee that

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9 Sah (1991) and Sah and Stiglitz (1986, 1988) applied the asymmetric model to study the architecture of economic systems and, in particular to compare the performance of hierarchies and polyarchies. Other studies analyzed the optimal decision rule under constraints, e.g., Ben-Yashar, Kraus and Khuller (2001) and Ben-Yashar and Kraus (2002), the optimal decision rule in polytomous choice, Ben-Yashar and Paroush (2001), and the optimal allocation of committee members, Ben-Yashar and Danziger (2011). Since the seminal work of Austen-Smith and Banks (1996), much attention has been also devoted to the role of strategic decisions, see for example, Ben-Yashar and Milchtaich (2007). Also see Dietrich and List (2013).

10 Banks are also known to widely use credit scoring models, in which case a credit committee member can be interpreted as a criterion in the credit scoring model.
induces the maximal lending or the maximal marginal effect of the guarantee on lending that implies maximal reliance on objective loan-specific information. Our second theorem illustrates this possibility for the particular environment where the risk of the projects faced by borrowers is distributed uniformly.

Our findings stress the importance of the decision-making structure of the financial institution used by the government for its guarantee program and specifically provide a theoretic basis for the reduced effectiveness of such programs in organizations with centralized decision structures. These theoretic findings concur with the empirical literature on government guarantee schemes that have been widely used to increase lending to small and medium sized enterprises. A detailed analysis of the organization and success of such schemes can be found in Green (2003). These schemes are found to be efficient in increasing lending especially in emerging and industrialized countries. According to Green (2003) one explanation can be found in the design and implementation of guarantee schemes. Specifically the degree of centralization of the lending organization is a factor that determines efficiency. Green (2003) finds that in developing countries, which tend to be over centralized in the sense that a central office makes the final decision on loan approval, the schemes are less effective. In particular, our findings emphasize that a loan guarantee affects not only the final credit decision, but also the bank's decision-making structure.

2. The model.

An entrepreneur who has no wealth can apply to the bank for a loan of 1 unit, which if granted, allows him to proceed with a project that requires 1 unit of investment. The loan can be used only for the purpose of investing in the project. A project returns either $Y$, which is fully observable, with probability $P_Y > 0$ or zero with probability $(1 - P_Y)$. The entrepreneur knows the characteristics of his project, so from his point of view, the expected return from his proposed project is $P_Y Y$. The probability $P_Y$ represents the project's risk level whereby a low risk project is associated with a high $P_Y$. The bank's success depends on the realization of the return $Y$ on the project, that is, on the probability $P_Y$. However, a project's $P_Y$ is unknown to the bank. For the bank it is a random variable that varies according to a commonly known distribution function.

The bank must decide whether to approve or reject the entrepreneur's loan application, taking into account that the gross cost of lending 1 unit is equal to $C \geq 1$. Setting the risk-free rate to zero, loan repayment is $R > 1$ ensuring that both the bank and the entrepreneur are able to participate in the program such that $Y > R > C$. Government intervention is represented by a

\footnote{\textit{R} must be such that the entrepreneur participates in the loan program providing a positive expected income, i.e., $P_Y (Y - R) > 0$ and above the bank's cost of funds, otherwise the bank will never extend a loan.}
guarantee, $g^{12}$. The guarantee $g$ is the amount by which the bank is reimbursed by the government in the event that the entrepreneur cannot repay the loan and it is set such that, $0 \leq g \leq R$. There are two types of loans, good loans (1) and bad loans (-1). A correct decision is to approve (1) a good loan and to reject (-1) a bad loan. A good loan is a loan that finances a project with a probability of success $P_y \tau > \tau$, where $\tau$ is the threshold probability of success that determines what is a correct decision from the bank's standpoint, i.e., the bank has a positive expected income from a loan. The threshold probability $\tau$ is determined by the parameters known to the bank, $g$, $C$, and $R$, such that for cases where $P_y > \tau$, $\tau = \frac{C - g}{R - g}$, a loan provides the bank with expected income, $P_y R + (1 - P_y) g - C > 0$. Hence, given the distribution function $f(P_y)$ of $P_y$, the a-priori probability of a good loan, $\alpha$, is determined as follows: $\alpha = \int_{\tau} f(P_y) dP_y$. Since $\frac{\partial \tau}{\partial g} < 0^{13}$, it follows that $\frac{\partial \alpha}{\partial g} > 0$. That is, an increase in the size of the guarantee lowers the threshold for good loans resulting in a larger a-priori probability that the bank faces a good loan.

Since the probability $P_y$ is unknown to the bank, the bank’s board of directors appoints a credit committee of $n=2k+1$ members whose task is to approve or reject a loan application by assessing whether $P_y > \tau$ or not. The common objective of all the credit committee members is to make the correct decision concerning loan approval.\(^{14}\) Each member’s decision regarding the type of loan (good or bad) is based on his specific information, such as past experience in lending to the entrepreneur, the entrepreneur’s leverage and other attributes of the entrepreneur and of the loan application. A credit committee member’s decisional skill is represented by $p$, $1/2 < p < 1$, which represents his probability of approving a good loan and rejecting a bad one. We assume that the committee members have homogeneous skills and that decisional skills are statistically independent across credit committee members. A final decision is reached by applying a decisive decision rule, which is a function that assigns 1 (approval) or -1 (rejection) to any set of decisions made by the members of the credit committee. The assumption of homogeneous decisional skills is very common in the literature since along time skills tend to become homogeneous due to deliberation and effective learning processes, see, for example, Ben Yashar and Nitzan (2016).

It is plausible to resort to qualified majority rules since, by the main result in Nitzan and Paroush (1982) and Ben Yashar and Nitzan (1997), if individuals have an identical decisional skill $p$, the optimal decision rule is a qualified majority rule. A qualified majority rule is

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\(^{12}\) The guarantee $g$, can be interpreted as a percentage of loan repayment by assigning the value 1 to the loan repayment $R$, in which case the loan size is less than 1.

\(^{13}\) Note that, $\frac{\partial \left(\frac{C - g}{R - g}\right)}{\partial g} = \frac{1}{(R - g)^2} (C - R) < 0$.

\(^{14}\) In our setting, we can disregard the typical problems that arise in a classical social choice setting where preferences are heterogeneous (e.g., the difficulty of attaining a social compromise, (Young 1988, 1995) and the problem of majority tyranny, (Baharad and Nitzan 2002).
represented by an integer $q$, the quota required for the decision to be 1. That is, the committee decision is 1, if and only if the number of the credit committee members who support approval is larger than or equal to $q$. Note that $q=k+1$ represents the simple majority rule, $q=n$ represents a centralized decision rule, whereby the approval of all the credit committee members is required to approve a loan and $q=1$ represents a decentralized decision rule, whereby the approval of only one credit committee member is required to approve a loan. In the trivial cases, the decision is made without consulting the committee when $q=0$ (always approve) or when $q=n+1$ (never approve). Note that although the credit committee members are assumed to be equally skilled, this does not imply that the optimal decision rule is the simple majority rule. In fact, as explained below and in the next section, despite the simplifying assumption, the set of potentially optimal decision rules is the spectrum of all possible qualified majority rules. The particular optimal qualified majority rule hinges on the environmental biases, viz., the a-prior probabilities and the net income from the possible states of the loan (good or bad).

3. The effect of government intervention on loan approval.

Given the parameter $q$ that represents the qualified majority rule used by the credit committee, let us denote the probabilities that the committee approves a good loan and rejects a bad loan by $T(q:1)$, and $T(q:-1)$, respectively. Hence, the probability that a loan request is approved by the credit committee is denoted by $Pr(1:q)$ where

$$Pr(1:q) = \alpha T(q:1) + (1-\alpha)(1-T(q:-1))$$

(1)

Note that $1-T(q:-1)$ is the probability that the approved decision is incorrect.

We can establish that the guarantee enables the government to increase the probability that a loan is approved. Namely, the probability of approval increases with the magnitude of the government guarantee, that is: $rac{\partial Pr(1:q)}{\partial g} > 0$. This can be shown by recalling that given the decision rule $q$, the decision to approve a loan request requires the support of at least $q$ committee members. Furthermore,

$$T(q:1) = \sum_{j=q}^{n} \binom{n}{j} p^j (1-p)^{n-j}$$

(2)

and
\[
(1 - T(q : -1)) = \sum_{j=q}^{n} \binom{n}{j} (1 - p)^j p^{n-j}
\]

(3)

Also,

\[
\frac{\partial \Pr(1:q)}{\partial g} = \frac{\partial \alpha}{\partial g} T(q : 1) - \frac{\partial \alpha}{\partial g} (1 - T(q : -1)) = \frac{\partial \alpha}{\partial g} (T(q : 1) - (1 - T(q : -1)))
\]

\[
= \frac{\partial \alpha}{\partial g} \sum_{j=q}^{n} \Delta_j
\]

(4)

where \( \Delta_j = \binom{n}{j} [p^j (1 - p)^{n-j} - (1 - p)^j p^{n-j}] \).

Since (a) \( \frac{\partial \alpha}{\partial g} > 0 \).

(b) \( \forall j > \frac{n}{2}, \Delta_j > 0. \)

(c) \( \forall j = a, \Delta_a = -\Delta_{n-a}. \)

If \( q > \frac{n}{2} \), then by (a) and (b), \( \frac{\partial \Pr(1:q)}{\partial g} > 0 \). If \( q < \frac{n}{2} \), then by (c), we know that,

\[
\sum_{j=q}^{n} \Delta_j = \sum_{j=n-q+1}^{n} \Delta_j
\]

Since \( n-q+1 > n/2 \), this last term is positive by (b), and with (a),

\[
\frac{\partial \Pr(1:q)}{\partial g} > 0.
\]

Hence, the probability of approving a loan increases with government intervention due to the fact that the a-priori probability that a loan request is good increases when the threshold of good projects is reduced. The lower threshold is achieved by the government guarantee that reduces the loss to the bank in the event of a failed project. The implication is that, from the

\[ \Delta_j > 0 \iff (p^j (1 - p)^{n-j}) > (p^{n-j} (1 - p)^j) \iff j > n - j > \frac{n}{2}. \] (Note that under the model's assumptions, \( p > 1/2 \) and hence \( \frac{n}{2} > j \).

\[ \Delta_a = -\Delta_{n-a}. \] Hence, \( \Delta_{n-a} = \sum_{a}^{n} p^{n-a} (1-p)^a - (1-p)^a p^{n-a} \) and \( \Delta_a = \sum_{a}^{n} (1-p)^a p^{n-a} - (1-p)^a p^{n-a} \).
bank's point of view, it now faces a larger proportion of good loans. Hence, some loans that would have been rejected before the introduction of the guarantee are now approved.

Sometimes, however, given its budget framework, the government can only marginally change the loan guarantee. The structure of decision-making in the credit committee, i.e., the decision rule used to aggregate the decisions of the credit committee members is of crucial importance in determining the magnitude of the marginal effect of government intervention on the probability of loan approval. Our next result determines the decision rule that induces the maximal increase in lending in response to a marginal change in the guarantee $g$ that has been chosen by the government.

**Theorem 1.** The effect of a change in the government guarantee on the probability of approving a loan varies symmetrically with the parameter $q$ representing the qualified-majority rule applied by the credit committee. The change in the probability of loan approval is maximal at the simple majority rule and is minimal at the extreme centralized and decentralized qualified-majority rules. That is,

$$\frac{\partial \Pr(1: q = k + 1)}{\partial g} > \frac{\partial \Pr(1: q = k + 1 + i)}{\partial g} = \frac{\partial \Pr(1: q = k + 1 - i)}{\partial g} \quad \text{and}$$

$$\left(\frac{\partial \Pr(1: q = k + 1 + i)}{\partial g}\right)_{i \geq 1} < 0, \text{ where } i \text{ is a positive integer.}$$

**Proof.** Recall that

$$\frac{\partial \Pr(1: q)}{\partial g} = \frac{\partial \alpha}{\partial g} \sum_{j=q}^{n} \Delta_j,$$

where

$$\Delta_j = \left(\begin{array}{c} n \\ j \end{array}\right) p^j (1-p)^{n-j} - (1-p)^j p^{n-j}.\left(\begin{array}{c} n \\ j \end{array}\right).$$

and (a) $\frac{\partial \alpha}{\partial g} > 0$.

(b) $\forall j > \frac{n}{2}$, $\Delta_j > 0$.

(c) $\forall j = a$, $\Delta_a = -\Delta_{n-a}$.

By (b) $\sum_{j=k+1}^{n} \Delta_j > \sum_{j=k+1+1}^{n} \Delta_j$, hence,

$$\frac{\partial \Pr(1: k + 1)}{\partial g} > \frac{\partial \Pr(1: k + 1 + i)}{\partial g}, \text{ where } i \text{ is a positive integer.}$$
By (c) above, if \( q < \frac{n}{2} \) then \( \sum_{j=q}^{n} \Delta_j = \sum_{j=n-q+1}^{n} \Delta_j \). Therefore,

\[
\frac{\partial \Pr(1:q)}{\partial g} = \frac{\partial \Pr(1:n-q+1)}{\partial g}.
\]

Specifically, this is true when \( q = k+1-i \). Substituting \( n = 2k+1 \) yields, \( n-(k+1-i)+1 = k+1+i \).

Hence, \( \frac{\partial \Pr(1:k+1-i)}{\partial g} = \frac{\partial \Pr(1:k+1+i)}{\partial g} \). Furthermore, \( \sum_{j=k+1+i}^{n} \Delta_j \) decreases with \( i \).

Therefore, \( \left( \frac{\partial \Pr(1:k+1+i)}{\partial g} \right) \left( \frac{\partial g}{\partial i} \right) < 0 \).

QED

THEOREM 1 implies that the structure of decision making in the credit committee determines the effectiveness of the marginal loan-guarantee program. The government can expect the greatest increase in the probability of loan approval when the simple majority rule is used to aggregate the decisions of the credit committee members. That is, the largest increase in lending corresponding to an increase in the guarantee is achieved when a simple majority rule is used. There is symmetry in the attainable effectiveness when moving away from the simple majority rule towards centralized and decentralized decision-making structures. In other words, as a bank is either more centralized or more decentralized in its credit committees' decision-making structure, a given increase in the guarantee will achieve a lower increase in lending and the government will have to offer a higher increase in the guarantee in order to achieve a target increase in lending. This is necessary since greater centralization (decentralization) requires more (less) support of decision makers and therefore it becomes more difficult to achieve a meaningful marginal effect.

Expanding on this result, note that the most extreme decision rules are the centralized decision rule and the decentralized decision rule. Under the former rule all decision makers are required to vote in favor of approving the loan while under the latter rule only one favorable decision maker is required to approve a loan. An increase in the guarantee produces an increase in the a priori probability. In the extreme case where all members need to vote in favor, a marginal change in the a priori probability will have only a small effect, since most projects are rejected. Similarly, in the extreme case where only one member is required to vote in favor of approving a loan, a marginal change in the a priori probability of loan approval will have only a small effect on the probability of loan approval since most projects are approved. Hence, in extreme cases of centralized and decentralized decision rules, large changes in the a priori probabilities are required, in order to affect the probability of loan approval. Conversely, in the less extreme cases, a small change in the a priori probability has a more meaningful effect, viz., it will produce a significant effect on the decision to approve a
loan. A comprehensive discussion of extreme decision rules can be found in Ben-Yashar and Nitzan (2001) and Sah and Stiglitz (1986).

4. The effect of government intervention on the bank’s decision structure.

The decision rule applied by the bank has a crucial impact on the success of the government’s guarantee program. However, if the bank applies the optimal decision-making structure, the government must take into consideration that the guarantee may alter the bank’s optimal decision rule. This in turn may affect the success that can be expected from the guarantee in terms of loan approval.

Without loss of generality, it is assumed that rejection of a loan request (good or bad) is associated with zero income for the bank. In the case of a particular good loan, where \( P_y > \tau \), the bank’s net expected income from that particular loan, \( B(1) \), is the difference between the expected income from approving it and from rejecting it. That is,

\[
B(1) = P_y R + (1 - P_y)g - C - 0 > 0
\]

(5)

In the case of a particular bad loan, where \( P_y < \tau \), the bank’s net expected income from the loan, \( B(-1) \), is the difference between the expected income from rejecting it and from accepting it. That is,

\[
B(-1) = 0 - \left( P_y R + (1 - P_y)g - C \right) > 0
\]

(6)

The optimal decision rule from the bank’s point of view, which maximizes its expected income from its decision, is a qualified-majority rule, represented by \( \hat{q} \), see, Nitzan and Paroush (1982, 1985) and Ben-Yashar and Nitzan (1997), where

\[
\hat{q} = \frac{n}{2} - \frac{\delta + \lambda}{2 \ln \left( \frac{p}{1 - p} \right)}
\]

(7)

and \( \lambda = \ln \frac{\alpha}{1 - \alpha} \), \( \delta = \ln \frac{EB(1)}{EB(-1)} \) where \( EB(1) \) and \( EB(-1) \) are the expected values of \( B(1) \) and \( B(-1) \), respectively.
Note that, $\lambda$ and $\delta$ are bias components that determine the extent of the optimal bias towards approving or rejecting the loan. Recall that both of these biases are affected by the guarantee $g$ set by the government. This framework of endogenous biases constitutes a significant extension of the above literature, where the biases are exogenous to the optimal decision rule. Note that, $\lambda$ reflects the asymmetry in the priors of the two types of loans (a good loan and a bad loan) and $\delta$ reflects the asymmetry of the net expected incomes associated with the two types of loans. The biases contain information that is independent of the decisions of the individual committee members. Hence, the final decision concerning loan approval is based on two distinct types of information. The first type is loan specific (e.g., leverage of a specific borrower, the borrower's history, projected earnings, etc.) that is known to the individual committee members. This type of information determines for each committee member whether to vote in favor of loan acceptance. It is objective information in the sense that the government does not control it. The second type of information contained in the biases is general information concerning the environment in which the committee makes the decision. The environment reflects combined characteristics of the loan requests such as the percentage of good loans and the expected income from the pool of loans. Accordingly, when the board of directors relies heavily on the biases when choosing a decision structure, it reduces the reliance on the objective loan-specific information used by the committee members in approving a loan. This is crucial for financial stability which relies on appropriate use of all information and which can deteriorate when financial decisions are detached from fundamental information concerning borrowers.

In the symmetric case where $EB(1)=EB(-1)$ and $\alpha=1/2$, the bias elements vanish and the optimal decision rule is the simple majority rule, $\hat{q}=n/2$. In this case a decision is based only on objective (uncontrolled) information concerning the specific loan request as known to the individual committee members. If $\delta + \lambda > 0$, the bias is in favor of approving the loan request and, therefore, $\hat{q}<n/2$, i.e., less than half of the credit committee members are required to decide in favor of the loan in order for an approval decision to be made thereby reducing the importance of loan specific information that may be known to individual decision makers concerning a specific loan. The decentralized structure presents the extreme case where the bias is very large, and only one credit committee member is required to make a positive decision. In this case, the final decision is certainly based more on the biases and less on the specific loan-related information known to the individual committee members. When $\delta + \lambda < 0$, the bias is in favor of rejecting the loan request, and therefore $\hat{q}>n/2$. In the centralized structure, we observe the extreme situation in which all credit committee members must decide in favor of the loan in order for the loan to be approved thereby extremely reducing the importance of loan specific information that may be known to individual decision makers concerning a specific loan (since the requirement for the rejection of a loan request is minimal). In this case too, the final decision is, again, certainly based more on the biases and less on the specific loan-related information known to the individual committee members.

17 There are two trivial cases where the decision is made without consulting the credit committee, that is, either always approve or never approve a loan, based only on the biases.
4.1 An illustration: The Standard Uniform Distribution.

To illustrate the usefulness of our setting, henceforth let us assume that the distribution of $P_y$ is uniform. The functions $f_1(P_y)$ and $f_2(P_y)$ denote the conditional distribution functions of $P_y$ given that loans are good and bad, respectively. In this case we find that if $g$ increases, the optimal structure of the bank’s credit committee becomes more decentralized and hence more lenient toward approval of a loan, i.e., a smaller proportion of decision makers is necessary for approval of the loan. In the following proposition we focus on the optimal structure of the bank’s credit committee\(^{18}\).

**Proposition 1.** $\frac{\partial q}{\partial g} < 0$.

**Proof.**

$$EB(1) = \int_{C-g}^{R-g} (P_y R + (1 - P_y) g - C) f_1(P_y) dP_y = (R - g) \int_{C-g}^{R-g} f_1(P_y) dP_y + g - C$$

$$= (R - g) \frac{1}{2} \left( 1 + \frac{C - g}{R - g} \right) + g - C = \frac{1}{2} (R - C).$$

$$EB(-1) = - \int_{0}^{C-g} (P_y R + (1 - P_y) g - C) f_2(P_y) dP_y = - \left[ (R - g) \int_{0}^{C-g} f_2(P_y) dP_y + g - C \right]$$

$$= - \left[ (R - g) \frac{1}{2} \left( \frac{C - g}{R - g} \right) + g - C \right] = \frac{1}{2} (C - g).$$

It can be shown that $\frac{\partial EB(1)}{\partial g} = 0$, $\frac{\partial EB(-1)}{\partial g} < 0$ and it has been shown that $\frac{\partial \alpha}{\partial g} > 0$. Since,

$\delta = \ln \left( \frac{EB(1)}{EB(-1)} \right)$ and $\lambda = \ln \left( \frac{\alpha}{1 - \alpha} \right)$,

$$\frac{\partial \delta}{\partial g} = \frac{1}{EB(1)EB(-1)} \left( \frac{\partial EB(1)}{\partial g} EB(-1) - \frac{\partial EB(-1)}{\partial g} EB(1) \right) > 0$$

and

$$\frac{\partial \lambda}{\partial g} = \frac{1}{\alpha(1 - \alpha)} \frac{\partial \alpha}{\partial g} > 0$$

\(^{18}\) Note that whereas in Theorem 1, the focus is on the effect of the government guarantee on the probability of approving a loan, given a decision rule, in Proposition 1 the focus is on the effect of the guarantee on the optimal decision rule.
Therefore, \[
\frac{\partial \hat{d}}{\partial g} = -\frac{1}{2 \ln \left( \frac{p}{1-p} \right)} \frac{\partial (\delta + \lambda)}{\partial g} < 0.
\]

PROPOSITION 1 implies that, when the optimal decision rule is used and the guarantee is increased, fewer credit committee members are required to be in favor of a loan in order for the loan to be approved.

When a guarantee is introduced, the biases \(\lambda\) and \(\delta\) change and hence the optimal rule is updated. In other words, government intervention affects the way in which the decisions of the credit committee members should be aggregated. Let us assume that \(R > C > \frac{R}{2}\), namely the bank has relatively high lending costs that create a negative bias in the decision rule favoring loan rejection. The government can introduce a guarantee that weakens and perhaps eliminates the negative bias, increasing the probability of loan approval. However, if the guarantee is very high the government may find that it has created an unwarranted positive bias causing loan approval to be based too much on the bias and insufficiently on loan-specific information known to individual committee members.

Since the biases can reduce the reliance on objective loan-specific information known to individual committee members, the government may wish to prevent such insufficient reliance on information in the decision process, by setting the guarantee \(g\), such that the sum of the biases equals zero. By doing so the government induces the bank to use the simple majority rule. Notice that any alternative qualified majority rule relies less on the objective information because it reduces the number of decisive decision makers, i.e., the minimal number of committee members whose decision determines the committee decision, either in favor or against loan approval. The following result determines the guarantee, \(g\) that results in the selection of the simple majority rule and, hence, maximal reliance on the loan-specific information known to individual committee members in the loan approval decision.

THEOREM 2. The simple majority rule becomes the optimal rule for the bank, if the government sets the guarantee \(g = 2C - R\). In this case, the biases \(\lambda\) and \(\delta\) are equal to zero.

\[19\text{For example, both under the extreme decentralized and centralized rules mentioned above, the number of decisive individuals is one. In the former case any individual can ensure the approval of a loan whereas in the latter case any individual can ensure the rejection of a loan.}\]
Proof. We need to show that if $g = 2c - R$, then $\lambda = \delta = 0$ and hence the simple majority rule is the optimal one. If $g = 2c - R$, then $r = \left( \frac{c-g}{R-g} \right) = \left( \frac{R-c}{2(R-c)} \right) = \frac{1}{2}$. Hence, $\alpha = 0.5$. In this case, since the distribution of $P_y$ is assumed to be uniform,

$$EB(1) = \int_0^1 \left( P_y R + (1 - P_y)(2c - R) - C \right) f_1(P_y) dP_y$$

$$= 2(R - C) \int_0^1 P_y f_1(P_y) dP_y + C - R = 2(R - C) \frac{3}{4} + C - R = \frac{1}{2} (R - C).$$

And,

$$EB(-1) = -\int_0^1 \left( P_y R + (1 - P_y)(2c - R) - C \right) f_2(P_y) dP_y$$

$$= -2(R - C) - \frac{1}{4} - C + R = \frac{1}{2} (R - C).$$

We have shown that $\alpha = 0.5$, and $EB(1) = EB(-1)$, hence, $\lambda = \delta = 0$.

QED

We have illustrated how the government can set the guarantee at a level that induces the bank to choose the simple majority rule which results in the maximal marginal effect on the probability of loan approval while possibly preventing insufficient reliance on valuable information known to individual committee members concerning a specific loan request. In practice, a bank may have a minimum guarantee level at which it will be willing to participate in the credit guarantee scheme, as pointed out by Honohan (2010). In this case, it will not be possible for the government to introduce a guarantee that steers the bank towards the simple majority rule and information loss due to the biases will not be fully avoided.

In general, the optimal decision rule chosen by the bank to approve a loan request is based on the biases as well as on other objective information known to the committee members. Consider the case $\frac{R}{2} > c$, where the bank has relatively low lending costs that create a positive bias in the decision rule favoring loan approval.\(^{20}\) The introduction of the guarantee in this case further strengthens this bias, again, causing loan approval to be based too much on the bias and insufficiently on loan-specific information which results in an increase of the probability of loan approval. A sufficiently high guarantee, which results in a high probability

\(^{20}\) Systemic risk in the banking sector is reduced using risk-based minimum capital requirements which translate into higher costs for banks that undertake riskier loans. Within the framework of our model, using such a measure to increase the bank's cost, $C$, lowers the bank's positive bias towards approving risky loans.
of loan approval, may from the bank's point of view justify even an extreme decision structure, viz., automatic approval of loan requests in which case the credit committee is abolished. In this case no objective valuable information is used and approval of loans is based only on the biases. However, the government and the public it represents may view things differently than the bank, giving rise to a moral hazard problem. This is because an increase in the probability of loan approval that results from a more lenient committee may imply insufficient reliance on objective (uncontrolled) information as well as approval of riskier loans that are more likely to default making the guarantee more costly for the government. Note that the government faces a cost only in cases where the lender defaults and cannot repay his loan. This moral hazard problem has been documented by Gropp, Gruendel and Guettler (2014) who show that the removal of government guarantees caused German savings banks to reduce credit risk by cutting off the riskiest borrowers from credit, hence demonstrating the association between government guarantees and credit risk. Moreover, they find that reduction in risk was due to tightening of lending standards and importantly for our results, credit risk decreased more in banks for which the value of the guarantee was higher prior to discontinuing the guarantee.

A wider economic consequence of reliance on the government guarantee and the positive bias it creates increasing risky lending, is the possibility of a financial crisis in the economy. Brunnermeier (2009) explains the various mechanisms through which sub-prime lending and its securitization, including the use of inadequate information for credit rating, led to the U.S. crisis in 2007-2008. Mian and Sufi (2009) show that the expansion in the supply of mortgage credit in the U.S., led to a rapid increase in house prices from 2001 to 2005 and subsequent defaults from 2005 to 2007. They find areas in the U.S. where applicants were denied credit and later on were able to obtain mortgages. Subsequently house prices increased sharply followed by a large increase in default rates. Mian and Sufi (2011) show that a significant fraction of both the sharp rise in U.S. household leverage and the increase in defaults from 2006 to 2008 can be explained by homeowners borrowing against the increase in home equity. Thus, the government's decision on the guarantee involves a trade-off between increasing the scale of risky lending and insufficient reliance on objective loan-specific information.

5. Conclusions.

In this paper we have extended and applied the results in Ben Yashar and Nitzan (1997) and Nitzan and Paroush (1982) to the case of loan guarantees that are used by governments to overcome shortcomings in the credit market. By using a framework of endogenous biases instead of exogenous ones and adding to previous results, we are able to suggest new insights into such government programs. Notice that the new approach of endogenous biases has been demonstrated assuming homogenous decisional skills that are independent of the general environment that reflects the characteristics of the loan requests. The advantage of these simplifying assumptions is based not on the robustness of the results to more general settings, but on their effective instrumental role in illustrating the interrelationship between government loan guarantees and the bank's credit decision-making structure.

The structure of decision-making in banks has been shown to be a crucial factor in determining the effect of government loan programs on the extent of lending. In essence our results point to the conclusion that, when operating a loan-guarantee program, governments
marginally varying the loan guarantee can achieve the largest increase in lending and the maximal reliance on objective relevant information when facing banks that have neither centralized nor decentralized decision-making structures. This has important policy implications for governments planning such programs and taking into account their anticipated impact on weaker borrowers.

If the government is aware of the relationship between the parameters of the decision structure and the optimal qualified majority rule applied by the credit committee, then it can exploit its advantage to set the most effective loan guarantee that induces maximal lending. This policy will result in extreme decentralization, whereby only one credit committee member is required to make an approval decision or in the extreme case where the credit committee is not consulted for loan approval. The government may face therefore a trade-off between increasing the scale of lending and insufficient reliance on important information.

Our results have interesting implications regarding the effect of the guarantee on risk. The uncertain dichotomous choice decision model we have used allows us to explain such phenomena as sub-prime loans and information loss in the lending-decision process. This suggests a useful theoretical framework for demonstrating the moral hazard associated with government guarantees and clarifies how the government can prevent this specific problem through its choice of a guarantee. On the one hand, the bank is affected by the guarantee such that its threshold is lowered, causing approval of riskier loans. On the other hand, the decision rule is affected by the guarantee such that valuable information may not be taken into account when the decision whether to approve a loan or reject it relies more heavily on the biases and less on the objective relevant information known to the members of the credit committee. The possible dilemma faced by the government due to these two effects of the guarantee can be solved by applying an objective function that takes into consideration the positive effect of the guarantee on lending as well as the negative effect of the guarantee on risk due to insufficient reliance on loan-specific information and on the cost faced by the government due to the guarantee. A solution of the optimization problem based on this objective function may allow the government to choose the guarantee that serves the public in the best way.

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