Equilibrium Yield Curve, the Phillips Curve, and Monetary Policy*

Very Preliminary and Do Not Cite or Circulate

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Abstract

This paper investigates the equilibrium yield curve in a model with optimal savings as a buffer stock. In the model, interest rates are set by a monetary policy rule, and income and inflation are assumed to consist of trend and cyclical components. Under the income and inflation processes estimated by US, UK, German, and Japanese data, a quantitative analysis accounts for a realistic upward sloping yield curve along with the positive correlation between income and inflation over the business cycle (i.e., the Phillips curve). A counterfactual analysis indicates that the economy with permanently low interest rates would be associated with flatter yield curves due to the changes in the monetary policy behavior near the zero lower bound.

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1 Introduction

Can we rationalize the shape of yield curve by consumers’ optimal behavior? Since the shape of yield curve is characterized by risk premiums on long-term bonds (i.e., term premiums), this question falls into the extensive literature to rationalize the level of risk premiums by consumers’ optimization.\(^1\) Rationalizing term premiums is, however, somewhat more challenging than other risk premiums in the following senses. First, since long-term bond prices are influenced by inflation dynamics, the model behavior must be consistent not only with real economic activity but also with inflation dynamics and their relationship with real economic variables. Namely, while yield curves are upward sloping on average (i.e., positive term premiums on average) in most advanced economies, term premiums are usually negative and small in a standard consumption based asset pricing model under the empirically observed income and inflation process, which is called the “bond premium puzzle” (Backus et al. (1989)). Second, the model must consider the policy behavior of central banks in addition to consumers’ optimization because the short end of yield curve is entirely set by the central bank in most countries. In particular, since the interest policy is recently constrained the zero lower bound (ZLB) in many countries, the model must explicitly incorporate those constraints in order to understand what the theory predicts about yield curves under a permanently low interest rate environment with the ZLB.

This paper tries to address those questions by analyzing the equilibrium yield curve in a model with optimal savings as a buffer stock (e.g., Deaton (1991)). In the model, consumers optimize their consumption path under an exogenous income and inflation process as well as nominal interest rates set by a monetary policy rule, and the equilibrium yield curve is derived by consumer’s optimal conditions. The contribution of this paper to the literature is twofold. First, it shows that the shape of yield curve is consistent with the empirically observed income and inflation dynamics including their co-movement (i.e., the Phillips curve). Namely, the model successfully accounts for a realistic upward sloping yield curve in the US, UK, Germany, and Japan under the income and inflation process estimated by data. Second, it shows that a monetary policy response to inflation is a key to accounting for the shape of yield curves. Given this importance of monetary policy behavior in shaping yield curves, a counterfactual simulation indicates that the equilibrium yield curve in a

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\(^1\)The most actively investigated issue in this literature is the equity premium puzzle. For an extensive survey on this literature, see Cochrane (2017).
permanently low interest rate environment (low-for-long, hereafter) would be significantly flattened due to changes in the monetary policy behavior near the ZLB of nominal interest rates.

While term premiums in most advanced economies are positive on average, the positive term premiums are not easy to be theoretically rationalized under the empirically observed co-movement between inflation and real economic activity. A main takeaway in the previous finance literature is that inflation and consumption growth should be *negatively* correlated to theoretically rationalize the positive term premiums. To see why, let us think about two-period bond price, \( Q_{2,t} \). Based on the Euler equation based asset pricing, the two-period bond price can be decomposed into the discounted value of expected one-period bond price and the term premium,

\[
Q_{2,t} = \mathbb{E}_t (Q_{1,t+1} M_{t,t+1}) \\
= \mathbb{E}_t (Q_{1,t+1}) / R_t + \text{cov}(\mathbb{E}_{t+1}(M_{t+1,t+2}), M_{t,t+1})
\]

where \( M_{t,t+1} \) is the nominal stochastic discount factor (SDF). This asset pricing formula implies that the term premium is positive if and only if autocorrelation of the SDF is negative, i.e., \( \text{cov}(\mathbb{E}_{t+1}(M_{t+1,t+2}), M_{t,t+1}) < 0 \). However, autocorrelation for consumption growth and inflation is positive in most countries, thus leading to the negative (and small) term premiums in a standard consumption-based asset pricing model (the bond premium puzzle) and making the negative cross-correlation between consumption growth and inflation necessary for positive term premiums. The intuition is simple. Assume that inflation and consumption growth are negatively correlated. Then, given that long-term bond prices decline in response to inflation, long-term bond prices and consumption growth are positively correlated. Hence, long-term bonds are poor hedge against consumption declines, thus leading bond investors to require positive term premiums. Since the negative correlation between consumption growth and inflation is empirically observed in most economies, some macro-finance model with *exogenous* consumption can account for positive term premiums (e.g., Piazzesi and Schneider (2007)). Although this result based on a model with *exogenous* consumption in the finance literature does not contain any inconsistencies per se, a macroeconomic model with *endogenous* consumption usually faces difficulty reconciling it with one of stylized facts in the empirical macroeconomics literature, namely the “Phillips curve.” While there are many variants of the Phillips curve in the literature, they basically establish the *positive* correlation between inflation and real economic activity including income, consumption, and
employment in data. Hence, given that a model with endogenous consumption is necessary for conducting policy experiments such as examining the equilibrium yield curve in the face of changes in monetary policy behaviors near the ZLB, it is quite challenging but important issue for the macro-finance literature to account for the positive term premiums induced by the negative correlation between inflation and consumption growth, while preserving the empirically observed positive correlation between inflation and real economic activity established in the Phillips curve literature.

This paper shows that decomposing the income process into a stationary and a non-stationary part is a key to reconciling those observations in finance and macroeconomic literature, which appear to be inconsistent with each other. In the empirical analysis of this paper, the income process is decomposed into a non-stationary and stationary part by the Bayesian estimation of an unobserved component model. The estimation result indicates that growth of the non-stationary part is negatively correlated with inflation while the stationary part is positively correlated with inflation. Hence, consumption growth is negatively correlated with inflation in the model because consumption is mainly driven by a non-stationary part of income under the permanent income hypothesis, thus leading to positive and large term premiums in the model. Along with the positive term premiums, income fluctuations over the business cycle are positively correlated with inflation just because of the positive correlation between the stationary part of income and inflation, which is consistent with the Phillips curve in the empirical macroeconomics literature. Those quantitative results are in contrast with the previous literature which investigates the equilibrium yield curve in macroeconomic models. For instance, Rudebusch and Swanson (2012) assume all variables are stationary in their model and account for positive term premiums by assuming the negative correlation between inflation and real economic activity over the business cycle, which is inconsistent with the empirical findings in the Phillips curve literature.

Finally, on the relationship between term premiums and the monetary policy, this paper conducts a counterfactual simulation to examine the equilibrium yield curves in the economy with a permanently low interest rate. The counterfactual simulation via comparative statics shows that if the economy faces a permanently low interest rate environment due to low growth and low inflation (the low-for-long) as argued by, for instance, the secular stagnation hypothesis, the equilibrium yield curve would not only shift downward but also significantly flatten mainly due to the changes in monetary policy behavior near the ZLB of nominal interest. Therefore, as we discuss in IMF (2017), this result of comparative statics implies that
the low-for-long economy may be associated with a higher financial stability risk due to the lack of bank profits adequate to build capital buffers, because the maturity transformation is one of main sources of their profits.

**Literature Review**

In terms of literature, this paper is closely related to the literature on the equilibrium yield curve in the consumption based asset pricing model. The early literature on this topic shows that replicating a realistic upward sloping yield curve is not an easy task under the empirically observed consumption and inflation process (e.g., Campbell (1986), Backus et al. (1989), and Boudoukh (1993)), and subsequently it is called the “Bond Premium Puzzle.” The seminal paper in this literature is Piazzesi and Schneider (2007). They investigate the equilibrium yield curve in a consumption based asset pricing model with the recursive preference and show that some key features in the U.S. term structure can be replicated under the negative correlation between consumption growth and inflation. Recently, Branger et al. (2016) investigates influence of the zero lower bound by a similar model. Those papers, however, assume an exogenous consumption path and do not analyze the consistency with macroeconomic variables including the Phillips curve. In addition, changes in a monetary policy behavior are difficult to analyze because interest rates in their model are endogenously determined by the Euler equation. Another strand of this literature is the one to account for the shape of yield curve by a general equilibrium model with endogenous income and inflation, namely a new Keynesian model (e.g., Rudebusch and Swanson (2008, 2012), De Paoli et al. (2010) Andreasen (2012), Dew-Becker (2014), and Swanson (2016)). In this literature, Ngo and Gourio (2016) and Nakata and Tanaka (2016) are closely related to this paper because they investigate the effect of the ZLB of interest rates on risk premiums. This paper takes a more stylized approach than theirs in the sense that income and inflation are determined by exogenous stochastic processes, but instead focuses more on the effects of optimal savings behavior and the empirical consistency including the Phillips curve. Furthermore, the stylized approach taken in this paper makes it possible to analyze the yield curve in the low-for-long economy while a new Keynesian model often faces inflation indeterminacy near the ZLB. Finally, den Haan (1995) and van Binsbergen et al. (2012) are also closely related to this paper in terms of motivation and modeling approach. They investigate term premiums in a model with the optimal savings behavior while they do not model the monetary policy rule and do not particularly mention the consistency with the Phillips curve. In terms
of the methodology, this paper uses a model with buffer-stock savings pioneered by Deaton (1991) and Carroll (1992). In the finance literature, Heaton and Lucas (1996, 1997) use this type of model to investigate equity premiums and portfolio choices, and Aoki et al. (2014) incorporate inflation into this model as this paper does and investigate money demand and portfolio choice. As far as I know, however, this is the first paper to apply this framework to the term structure of interest rates.

2 Model

The model is an endowment economy with optimal savings as a buffer stock. In the model, a representative consumer optimizes its consumption path under an exogenous income and inflation process as well as nominal interest rates set by a monetary policy rule. Then, the equilibrium yield curve is defined to be consistent with the conditions for consumers’ intertemporal optimization.

2.1 Budget Constraint

In every period, the consumer obtains real income, $Y_t$, as an endowment, and allocates it for consumption, $c_t$, and savings as a form of one-period nominal bond, $B_t$, or $n$-period nominal bond, $B_{n,t}$. Hence, the budget constraint for the consumer is formulated as

$$ P_t c_t + \frac{B_t}{R_t} + \sum_{n>1} Q_{n,t} B_{n,t} + \Phi \left( \frac{B_t}{R_t} \right) = P_t Y_t + B_{t-1} + \sum_{n>1} Q_{n-1,t} B_{n,t-1} $$  \hspace{1cm} (1)

where $P_t$ is a price level, $R_t$ is a nominal interest rate, and $Q_{n,t}$ a $n$-period bond price in period $t$. Here, a tiny cost for bond holdings, $\Phi(\cdot)$, satisfying

$$ \Phi' \left( \frac{B_t}{R_t} \right) > 0 \text{ and } \Phi'' \left( \frac{B_t}{R_t} \right) > 0 $$

is assumed to exist in order to avoid the divergence of bond holdings.

The consumer’s real income, $Y_t$, is assumed to consist of the non-stationary part, $y_t^*$, and the stationary part, $y_t$,

$$ \log(Y_t) = \log(y_t^*) + \log(y_t) $$  \hspace{1cm} (2)

and the growth rate of the non-stationary part, $g_t \equiv y_t^* / y_{t-1}^*$, and the stationary part, $y_t$, follow a stationary process with $\mathbb{E}(g_t) = g^*$ and $\mathbb{E}(\log(y_t)) = 0$. Hence, the growth rate of household’s income fluctuates around the potential growth, $g^*$, and the cyclical part, $y_t$,
fluctuates around the non-stationary part of income like the output gap. Similarly, inflation is defined as $\Pi_t \equiv P_t / P_{t-1}$, and is assumed to consist of the trend component, $\pi^*_t$, and the cyclical component, $\pi_t$, as in Cogley and Sbordone (2008),

$$\log(\Pi_t) = \log(\pi^*_t) + \log(\pi_t)$$  \hspace{1cm} (3)

where $\xi_t \equiv \pi^*_t / \pi^*_{t-1}$ and $\pi_t$ follow a stationary process.

To make the model stationary, non-stationary variables should be detrended by $P_t$, $\pi^*_t$, and/or $y^*_t$, and the budget constraint should be reformulated by the detrended variables. First, the amount of nominal bond holdings are detrended as,

$$b_t = B_t / (P_t y^*_t \pi^*_t) \quad \text{and} \quad b_{n,t} = B_{n,t} / (P_t y^*_t \pi^*_t)^n.$$ 

where $b_t$ and $b_{n,t}$ are the detrended bond holdings for one-period bonds and $n$-period bonds. Nominal bond holdings are detrended by a price level and non-stationary part of income because they are cointegrated with those variables as on the balanced growth path in a standard growth model. In addition, nominal bond holdings should be detrended by the trend in inflation, $p\pi^*_t$, because the bond return is cointegrated with it. That is, the detrended nominal interest rate and $n$-period bond prices, $\tilde{R}_t$ and $\tilde{Q}_{n,t}$, are defined as,

$$\tilde{R}_t = R_t / \pi^*_t \quad \text{and} \quad \tilde{Q}_{n,t} = \pi^*_t Q_{n,t}.$$ 

Note that the $n$-period bond holdings, $B_{n,t}$, and prices, $Q_{n,t}$, should be detrended by the trend inflation powered by its maturity, $p\pi^*_t$, because all spot and forward rates up to its maturity should be detrended by the trend inflation. Finally, the detrend consumption, $\tilde{c}_t$, is defined by,

$$\tilde{c}_t = c_t / y^*_t$$

as in a standard neo-classical growth model. Then, the budget constraint is reformulated by those detrended variables as,

$$\tilde{c}_t + \frac{b_t}{\tilde{R}_t} + \sum_{n>1} \tilde{Q}_{n,t} b_{n,t} + \Phi \left( \frac{b_t}{\tilde{R}_t} \right) = 1 + \frac{b_{t-1}}{g_t \pi_t \xi_t} + \sum_{n>1} \frac{\tilde{Q}_{n-1,t} b_{n,t-1}}{g_t \pi_t \xi_t^n}$$  \hspace{1cm} (4)

by dividing both sides of the original budget constraint by $P_t$ and $y^*_t$.

2.2 Monetary Policy

As in a standard monetary model, the central bank sets the nominal interest rate, $R_t$, by a policy rule responding to inflation. Namely, the central bank is assumed to increase
(decrease) nominal interest rates in response to the positive (negative) inflation gap, $\pi_t \equiv \Pi_t/\pi_t^*$, and deviate the nominal interest rates from the neutral interest rate (the trend inflation plus the potential growth), $R_t^* \equiv \pi_t^* g^*$. That is, the central bank sets the nominal interest rate following a policy rule,

$$R_t = R_{t-1}^{\phi_r} \left[ R_t^* \left( \frac{\Pi_t}{\pi_t^*} \right)^{\phi_n} \right]^{1-\phi_r}. \quad (5)$$

Note that the nominal interest rate is assumed to depend on the last period’s interest rate, $R_{t-1}$, following the previous literature, suggesting that the central bank tends to smooth their policy changes in monetary policy. Here, $\phi_r$ and $\phi_n$ are parameters representing the degree of interest rate smoothing and responses to inflation gaps.

As in the budget constraint, the monetary policy rule is also reformulated by using the detrended variables as,

$$\tilde{R}_t = \left( \frac{\tilde{R}_{t-1}}{\xi_t} \right)^{\phi_r} \left[ g^* \pi_t^* \right]^{1-\phi_r} \quad (6)$$

by dividing the both sides of the monetary policy rule by the trend inflation, $\pi_t^*$.

### 2.3 Household’s optimization and Equilibrium Yield Curve

The household chooses their optimal consumption path so as to maximize their discounted lifetime utility. More specifically, the household maximize the following value function based on the Epstein-Zin-Weil preference,

$$V_t = \left\{ c_t^{1-\sigma} + \beta \mathbb{E}_t \left[ V_{t+1}^{1-\alpha} \mathbb{E}_t^{\frac{1-\alpha}{\sigma}} \right] \right\}^{\frac{1}{1-\sigma}} \quad (7)$$

subject to the budget constraint (1), exogenous real income and inflation, $Y_t$ and $\pi_t$, and the nominal interest rate set by the monetary policy rule (5). Here, $\sigma$ and $\alpha$ are parameters for the inverse of IES and the CRRA coefficient, respectively.

The equilibrium is characterized by by the Euler equation with respect to one-period bond holdings, $B_t$,

$$R_tE_t[M_{t,t+1}] = 1$$

and the Euler equations with respect to $n$-period bond holdings, $B_{n,t}$,

$$E_t[Q_{n-1,t+1}M_{t,t+1}] = Q_{n,t}, \quad \forall n > 1.$$
Here, $M_{t,t+1}$ is the nominal stochastic discount factor (SDF) from period $t$ to $t+1$, 

$$M_{t,t+1} = \frac{\beta}{\pi_{t+1}} \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \left[ \frac{V_{t+1}}{E_t \left( V_{t+1}^{1-\alpha} \right)^{1-\alpha}} \right]^{\sigma-\alpha}$$

The Euler equations are detrended in a similar way to the budget constraint.

Since the spot rate for each maturity, $R_{n,t}$, is defined as $R_{n,t} \equiv Q_{n,t}^{-\frac{1}{n}}$, the equilibrium yield curve is formulated by the asset prices for $n$-period bond, $Q_{n,t}$, in equilibrium. Also, term premiums, $\psi_{n,t}$, are defined as,

$$\psi_{n,t} = R_{n,t} - \hat{R}_{n,t}$$

where $\hat{R}_{n,t}$ is a $n$-period bond return for risk-neutral agents. As in the previous literature, the $n$-period bond prices and returns for risk-neutral agents, $\hat{Q}_{n,t}$ and $\hat{R}_{n,t}$ are defined as,

$$\frac{1}{R_t} E_t[\hat{Q}_{n-1,t+1}] = \hat{Q}_{n,t} \quad \text{and} \quad \hat{R}_{n,t} = \left( \frac{1}{\hat{Q}_{n,t}} \right)^\frac{1}{n}, \forall n > 1$$

To make the model quantitatively tractable, it is assumed that the supply of $n$-period bond is equal to zero in equilibrium without loss of generality, and consequently one-period nominal bonds are only choice of savings for consumers in equilibrium. Hence, the model consists of two endogenous, $(b_{t-1}, \tilde{R}_{t-1})$, and four exogenous state variables, $(y_t, \xi_t, y_t, \pi_t)$. In the later section, the model is solved quantitatively under the estimated process of income and inflation, and used for examining whether the equilibrium yield curve in the model can account for the empirically observed yield curve.

### 3 Quantitative Analysis

This section conducts a quantitative analysis based on the model in Section 2. Namely, the quantitative analysis asks: Can the model quantitatively rationalize the features of yield curve under the estimated process of income and inflation? What is the role of monetary policy in shaping yield curves? The outline of the quantitative analysis is as follows. First, after specifying the functional forms of the income and inflation processes, the parameters for those processes are estimated by a Bayesian method of an unobservable variable model using the US, UK, German and Japanese data. Then, given the estimated income and inflation processes, the consumer’s optimal policy functions are quantitatively computed by a recursive
method. The equilibrium yield curve in the model is computed by the optimal policy function of consumption, and examined whether it can account for a realistic upward sloping yield curve in data and what the economic intuition behind the shape of the equilibrium yield curve. Finally, a counterfactual policy experiment is conducted to predict the shape of yield curve in the economy with permanently low interest rates.

3.1 Estimation of Income and Inflation Process

In the model, income and inflation are assumed to consist of a trend and cyclical part as described in (2) and (3), and both parts jointly follow an exogenous process. The goal of this subsection is to decompose the income and inflation process in the US, UK, Germany and Japan into the trend and cyclical part, and simultaneously estimate the parameters for the joint process by a Bayesian method. The stochastic process estimated in this subsection will be used for calibrating the income and inflation process in the next section.

3.1.1 Econometric Specification and Data

The functional form for the income and inflation process is specified as follows. First, the cyclical part of income and inflation, \( y_t \) and \( \pi_t \), jointly follow a VAR(1). The VAR is supposed to describe a joint stationary process of income and inflation over the business cycle, and thus expected to capture the Phillips curve. Then, the growth rate of the non-stationary income, \( g_t \equiv y_t^* / y_{t-1}^* \), is assumed to follow an AR(1) processes, but the shock is assumed to be possibly correlated with the shock to inflation. That is, \( X_t \equiv [\log(y_t), \log(\pi_t), \log(g_t)]' \) jointly follows a VAR(1):

\[
X_{t+1} = A_0 + A_1 X_t + \varepsilon_{X,t+1}, \quad \varepsilon_{X,t+1} \sim N(0, \Sigma_X)
\]

where:

\[
A_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ (1 - \rho_y)g^* \end{bmatrix}, \quad A_1 = \begin{bmatrix} \rho_y & \rho_{y\pi} & 0 \\ \rho_{\pi y} & \rho_{\pi\pi} & 0 \\ 0 & 0 & \rho_g \end{bmatrix}, \quad \Sigma_X = \begin{bmatrix} \sigma_{yy} & \sigma_{y\pi} & 0 \\ \sigma_{\pi y} & \sigma_{\pi\pi} & \sigma_{g\pi} \\ 0 & \sigma_{g\pi} & \sigma_{gg} \end{bmatrix},
\]

where the average of income growth rate is equal to \( g^* \), which can be interpreted as an potential growth rate in the economy. Finally, growth of trend inflation, \( \xi_t \equiv \pi_t^* / \pi_{t-1}^* \), independently follows AR(1):

\[
\log(\xi_t) = \rho_\xi \log(\xi_{t-1}) + \varepsilon_{\xi,t} \quad \text{where} \quad \varepsilon_{\xi,t} \sim iidN(0, \sigma_\xi)
\]
In the above specification of inflation process, inflation non-neutrality argued in the Phillips curve literature is captured by the following two sorts of parameters. First, the lagged income gap, $y_{t-1}$, and the lagged inflation gap, $\pi_{t-1}$, are assumed to possibly influence $\pi_t$ and $y_t$ in the VAR specification. The sign of the effects, $\rho_{\pi y}$ and $\rho_{yy}$, is expected to be positive based on the Phillips curve literature, but their sign and magnitude are estimated by data with flat prior. Second, the real income shock to to the stationary part is assumed to possibly correlated with the shock to the cyclical part of inflation, i.e., $\sigma_{\pi x} \neq 0$. Again, the correlation is expected to be positive based on the Phillips curve literature, but its sign and magnitude are also estimated by data with flat prior.

In the Bayesian estimation of parameters, the prior distributions are set as follows. First, for identifying between trend and cycle in inflation, it is assumed that: (1) the prior distribution for the AR(1) parameter of trend inflation is centered around a very high persistence (Beta[0.95, 0.03]), and (2) its volatility is very small and the ratio of volatility for the trend inflation to that for the cyclical inflation, $\sigma_\xi/\sigma_\pi$, is 1.0%. That is, the trend inflation is identified by defining it as a very persistent and less volatile component of inflation by assumption. Second, for identifying between trend and cycle in income, a tight prior (Beta[0.80, 0.05]) is assigned for the AR(1) parameter of the stationary part of income, $\rho_y$, suggesting that the trend and cyclical part of income is identified by defining the cyclical part of income as a stationary process near the business cycle frequency. Third, improper and flat prior distributions (i.e., Uniform [-1,1]) are applied to $\rho_{yx}, \rho_{\pi x}, \sigma_{\pi x}, \sigma_{\pi y}$, implying that those parameters are estimated purely by data without any restrictive prior assumptions. Since those are the key parameters to determine the comovement between inflation and real economic activity, the estimated comovement is also interpreted as the one estimated purely by data without any prior restrictions. Finally, other prior distributions are set to conventional ones.

With the above prior distributions, 11 parameters ($\rho_{\pi y}, \rho_{yx}, \rho_{\pi x}, \rho_{xx}, \rho_{\pi x}, \sigma_{\pi y}, \sigma_{\pi x}, \sigma_{\pi}, \sigma_{\xi}$) are estimated by a Bayesian method using the US, UK, Germany and Japanese data for real income growth, $\Delta(Y_t)$, and changes in inflation, $\Delta \log(\Pi_t)$. That is, the observation equations in the state space representation are:

$$\Delta \log(\Pi_t^{Data}) = \xi_t + \pi_t - \pi_{t-1}$$
$$\Delta \log(Y_t^{Data}) = g_t + y_t - y_{t-1}$$

The personal disposable income and the retail price indexes are used as the corresponding
data for \( Y_t \) and \( \Pi_t \) in the model.\(^2\) The sample periods are 1957Q2 - 2017Q4 for UK, 1959Q2 - 2017Q4 for US, and 1975Q2 - 2017Q4 for Germany and Japan.

### 3.1.2 Estimation Results

Table 2 shows the prior distributions and the estimated posterior mean and 90 percent confidence intervals. Several comments are in order. First, the stationary part of income (income gap) are positively correlated with inflation. In all countries except for Japan, the lagged income gap (inflation) is positively correlated with inflation (income gap) (i.e., \( \rho_{y\pi} > 0 \) and \( \rho_{\pi\pi} > 0 \)), and the shock to income gap is also positively correlated with the shock to inflation (i.e., \( \sigma_{\pi\pi} > 0 \)). In Japan, while the correlation between lagged income gap and inflation is almost zero, the shocks to income gap and inflation are positively correlated. Figure 1 shows the scatter plots of estimated inflation gap and income gap, clearly pointing out a positive correlation between them. Second, the shock to the non-stationary part of income is negatively correlated with the shock to inflation (i.e., \( \sigma_{\pi\pi} < 0 \)). Figure 2 shows the scatter plots of estimated inflation gap and growth of non-stationary part of income, clearly pointing out a negative correlation between them. In sum, the estimation result implies that the stationary part of income is positively correlated with inflation while the non-stationary part of income is negatively correlated with inflation. It is worth noting that improper and flat prior distributions (i.e., Uniform [-1,1]) are applied to \( \rho_{y\pi}, \rho_{\pi\pi}, \sigma_{\pi\pi}, \) and \( \sigma_{\pi\pi} \), implying that the estimated comovement between inflation and income is interpreted as the one estimated purely by data without any prior restrictions.

These estimation results are in line with the past empirical literature on macroeconomic fluctuations but rare to be incorporated into DSGE models. First, the positive correlation between income gap and inflation is consistent with the literature on the Phillips curve. Second, the estimation result in Table 2 is consistent with the past VAR literature on the identification of supply and demand shock. For instance, Blanchard and Quah (1989) identify the supply and demand shock by assuming that the supply shock has permanent effects on output while the demand shock does not, and shows that the supply and demand shock

\(^2\)Due mainly to the availability of long time series data, I use the following specific inflation series in each country. US: PCE deflator (chain price index), UK: Long Term Price Indicator of Consumer Goods and Services, DE: Consumer Price Index, JP: General Consumer Price Index Excluding Fresh Food. Also, the real income data for US and UK are taken from the national statistics while those for Germany and Japan are taken from the real income index compiled at the Dallas Fed.
has a negative and positive effect on unemployment rates, respectively. Given the negative correlation between inflation and unemployment rates in the Phillips curve literature, their result implies that the stationary part of output is positively correlated with inflation while the non-stationary part of output is negatively correlated with inflation, which is consistent with the estimated income and inflation processes in Figure 2. Third, the income and inflation comovement identified in this subsection is not common in a DSGE literature. As pointed out by Rudebusch and Swanson (2012), a positive non-stationary supply shock usually induces a positive response of inflation in a standard DSGE model, thus making it difficult for DSGE models to replicate those income and inflation comovement observed in the data. Replicating the observed income and inflation comovement by a micro-founded DSGE model is an interesting and important issue, but I leave it for future research and focus on its asset pricing implications by taking it as given in the following analysis.

3.2 Simulation Exercise for Term Structure

In the simulation exercise, the policy functions are computed by the policy function iteration with discretized grids (Coleman, 1991). Then, the equilibrium yield curve is computed by putting estimated $g_t, y_t, \xi_t$ and $\pi_t$ into the policy functions for each country. Furthermore, for comparison purposes, the equilibrium yield curve in the trend-stationary model is also computed by: (1) Define the deviations from the HP filter trend of income as $y_t$, (2) Estimate the joint process of $y_t$ and $\pi_t$ assuming $g_t = g^*$, and (3) Solve the model under the process of $y_t$ and $\pi_t$, and compute the equilibrium yield curve in each country. Note that the trend stationary setting is more common the literature.

3.2.1 Calibration

The process of income and inflation by country is approximated by a Markov chain with discretized grids. Namely, the VAR for $X_t \equiv \left[ \log(y_t), \log(\pi_t), \log(g_t) \right]'$, is approximated as follows. First, discretize $y_t, \pi_t$, and $g_t$ by the discrete grids of $G_y, G_\pi$, and $G_g$. Then, approximate the VAR(1) by a first-order Markov chain over $G_X \equiv G_y \times G_\pi \times G_g$ by the method in Terry and Knotek (2011). The AR(1) process for growth of trend inflation, $\xi_t$, is also approximated by a Markov chain.

Most other parameter values are calibrated to standard values. For preference parameters, the discount rate $\beta$ and the inverse of IES, $\sigma$, is set to 0.9985 and 2.0, which are
standard values in the literature. For the risk averseness parameter, \( \alpha \), several values are examined and choose a different value for different countries. The steady state value for bond holdings, \( b^* \), is set to 4.8 based on the average asset-income ratio in the U.S., but it barely influences the quantitative results. For the cost of bond holding, first, its functional form is assumed to be quadratic,

\[
\phi \left( \frac{b_t}{\bar{R}_t} \right) = \frac{\phi_b}{2} \left( \frac{b_t}{\bar{R}_t} - \frac{b^*}{\bar{R}^*} \right)^2 \bar{R}_t,
\]

and set the parameter, \( \phi_b \), to an arbitrary small number, 0.001, just for avoiding divergence of bond holdings. Finally, for the parameters of the monetary policy rule, the degree of response to inflation, \( \phi_\pi \) and the interest rate smoothing, \( \phi_r \), are set to 2.0 and 0.8 based on the monetary economics literature. In the robustness check, other values for the degree of response to inflation is examined to investigate the effects of a monetary policy rule on the equilibrium yield curve. Table 1 summarizes the calibration values in the baseline case.

### 3.2.2 Baseline Result

Figure 3 shows the equilibrium yield curves in the U.S. and Japan along with the average level of yield curves in actual data (the lines with circle). The figure indicates that the model can account for realistic upward sloping yield curves in all countries, and that the average level of term premiums is quantitatively close to the actual data for US and UK. The term premiums are higher for the case of higher risk averseness as expected. Furthermore, the equilibrium yield curve in a trend-stationary model is very flat, implying that term premiums are very small and close to zero.

Why can the model accounts for an upward sloping yield curve even under the Phillips curve? Since the negative correlation between consumption growth and inflation is a necessary condition for obtaining positive term premiums, this question can be rephrased as: why can the model replicate the negative correlation between consumption growth and inflation even though a cyclical part of income and inflation are positively correlated in the model over the business cycle? Given the estimation result in the previous section, the answer to this question is clear and simple. On the one hand, the stationary part of income entails the Phillips curve because \( \sigma_{g\pi} > 0 \). On the other hand, the non-stationary part of income is negatively correlated with inflation. Under the permanent income hypothesis, consumption growth is more influenced by the non-stationary part of income than the stationary one, suggesting that the estimation result \( \sigma_{g\pi} < 0 \) induces the negative correlation between
consumption growth and inflation in the model. That is, the quantitative result here simply implies that the difference between the stationary and non-stationary income in its relation to inflation is a key to understanding the equilibrium yield curve. This result is, however, in quite contrast with the previous literature. For instance, Rudebusch and Swanson (2012) uses a standard new Keynesian model to investigate the equilibrium yield curve and shows that the model can account for an upward sloping yield curve under the realistic calibration. In their calibration, however, they focus only on the detrended stationary values and assume that a cyclical part of income is negatively correlated with inflation in order to replicate the negative correlation between consumption growth and inflation, which contradicts the empirical literature on the Phillips curve. As shown in the trend-stationary case in my simulation, if we assume a realistic comovement between income and inflation, it is still challenging to replicate the positive and large term premiums.

Figure 4 shows the real yield curves (the dashed line) along with the nominal yield curves (the bold line) for the baseline case in the model. The figure indicates that a most part of nominal term premiums is accounted for by real term premiums in the US, and that term premiums associated with inflation risk premiums do not play a significant role for explaining the upward sloping yield curve. This result of decomposition is in contrast with the previous theoretical finance literature, which argues that inflation risk premiums rather than real term premiums are a main part of nominal term premiums (e.g., Piazzesi and Schneider (2007)), but it is consistent with the empirical finance literature on term premiums. For instance, Abrahams et al. (2016) decompose nominal term premiums in the U.S. by estimating the affine term structure model and show that real term premiums are much larger than inflation risk premiums.

The response of monetary policy to inflation plays a key role for replicating the positive and large real term premiums. To understand why, let us think about the 2-period real bond price, \( q_{2,t} \). By the Euler equation with respect to real bonds in each maturity, the 2-period real bond price is decomposed as,

\[
q_{2,t} = \mathbb{E}_t (q_{1,t+1} m_{t,t+1}) \\
= \mathbb{E}_t (q_{1,t+1})/r_t + \text{cov}(1/r_{t+1}, m_{t,t+1})
\]

Note that this result does not mean that there are little inflation risk premiums. Inflation risk premiums are positive and significant in this model, but there are almost the same level of inflation risk premiums for the short- and the long-term interest rates, suggesting that term premiums associated with inflation risk premiums are not large.

[^3]
where \( m_t \) is the real SDF and \( r_t \) is a real interest rate in period \( t \). This equation implies that real term premiums are positive if and only if \( \text{cov}(1/r_{t+1}, m_{t,t+1}) < 0 \). In the model, real interest rate increases in response to inflation because the central bank is assumed to increase nominal interest rates by more than inflation. That is, the monetary policy rule is satisfied with “Taylor principle” with \( \phi_\pi > 1 \) as in the literature of monetary economics. Furthermore, the real SDF, \( m_t \), also increases in response to inflation because consumption growth and inflation are negatively correlated in the model. Taken together those two observations, real interest rates and the real SDF are positively correlated, thus leading to \( \text{cov}(1/r_{t+1}, m_{t,t+1}) < 0 \) and positive real term premiums. Intuitively, if the real interest rate increases in response to inflation, the real bond is not a good hedge against a decline in consumption caused by inflationary shocks, thus leading to positive real term premiums.

The significant effect of monetary policy responses on term premiums is confirmed by comparative statics with respect to the degree of response of monetary policy to inflation. Figure 5 shows that the equilibrium yield curve for the baseline (\( \phi_\pi = 2.0 \)) along with the case of more (and less) aggressive response to inflation (\( \phi_\pi = 3.0 \) and 1.0). The figure indicates that when the central bank responds to inflation more aggressively, the equilibrium yield curve would become steeper. The economic intuition behind this result is simple: The more aggressively the central bank respond to inflation, the more negatively correlated nominal bond prices (i.e., inverse of nominal interest rates) and inflation are. Given the negative correlation between consumption growth and inflation, the result implies that term premiums become larger when the central bank responds to inflation more aggressively, suggesting that differences in the term structure of nominal interest rate across countries are possibly explained by differences in the monetary policy rule.

### 3.2.3 Term Structure of Volatility and Macroeconomic Moments

The previous subsection shows that the model can successfully account for the level of term premiums under the observed income and inflation process. This subsection examines two additional points for verifying the validity of the model. The first point is the term structure of interest rate volatility. The model can account for the \textit{average} of long-term interest rates relative to short-term interest rates, but it does not necessarily mean that it can account for the \textit{volatility} of long-term interest rates. The second point to be examined in this subsection is the macroeconomic moments for consumption growth. As shown in Section 2, since term premiums are mostly determined by consumption growth and its relations to inflation, it is
necessary to check if macroeconomic moments for consumption growth do not outrageously deviate from the data.

First, for term structure of interest rate volatility, long-term interest rates have slightly lower volatility than short-term interest rates in data (Bold black lines with circles in Figure 6). From a theoretical perspective, this slightly lower volatility for long term interest rates is puzzling: Theoretical models predict much lower volatility for long term interest rates than we observe in data. The economic intuition for “the excess volatility of long term interest rates” in data is simple. Long term interest rates are basically an average of current and expected future short-term interest rates. Hence, if short term interest rates stationary and follow a mean reverting process, long term interest rates do not significantly respond to fluctuations in short term interest rates and so have much smaller volatility.

Figure 6 shows the relative volatility of long term interest rates in the model (bold blue lines). The figure indicates that while the relative volatility for long term rates is still a little bit lower than data (bold lines with circles), the model can successfully replicate a relatively larger volatility for long term interest rates. A key to understanding the large volatility for long term interest rates in the model is the time-varying trend inflation, $\pi_t^*$. The relative volatility for the case of fixed trend inflation, which are shown as dashed blue lines in the figure, indicates that if the trend inflation is fixed at a constant value as in a standard monetary model, the relative volatility for long term interest rates would be much lower. The reason why the time varying trend inflation boosts the volatility of long term interest rates is that it changes the steady state level of interest rates. That is, when the trend inflation changes, the short- and long-term interest rates move by almost the same amount because people expect that the future path of short term interest rates also shifts by the same amount. Hence, the volatility of long-term interest rates caused by changes in trend inflation is the same magnitude as that of short-term interest rates. Figure 6 also shows that the term structure of volatility for interest rates detrended by the HP filter (dashed blue lines with circles). They indicate that volatility of detrended long-term interest rates relative to volatility of detrended short-term interest rates is much smaller and close to the case without trend inflation, suggesting that non-stationary trend components such as trend inflation generate the excess volatility of long-term interest rates in data.

The results in Figure 6 have several implications both the finance and macroeconomic literatures. For the finance literature, the above result implies that the volatility in term premiums does not play a significant role to account for the excess volatility puzzle of long-
term interest rates. This is interpreted as good news for this literature. For instance, some empirical models including van Binsbergen et al. (2012) are struggled to fit the data when they try to account for both level and volatility of long term interest rates only by term premiums, but the above result implies that the large volatility of long term interest rate can attribute to trend inflation rather than term premiums. For the macroeconomic literature, the above result implies that interest rates should be modeled as non-stationary variables. The macroeconomic literature models interest rates as stationary and mean reverting variables for tractability even though empirical time series literature usually treats short- and long-term interest rates are non-stationary and cointegrated with each other. The above result implies that macroeconomic models possibly replicate such a cointegrated relationship between short- and long-term interest rates by assuming that trend inflation is time-varying.

Next, we ask: Are Macroeconomic Moments Consistent with Data? For this purpose, macroeconomic moments for quarterly, yearly, and 2-year consumption growth are examined. Table 3 shows that the moments are broadly consistent with data, particularly for longer-term growth. It is worth noting that the moments for longer-term consumption growth are more relevant to term premiums for longer-maturities. Hence, the model can account for both the shape of yield curves and key macroeconomic moments for term premiums simultaneously, suggesting that the model successfully rationalizes the upward sloping yield curves in data.

3.3 Equilibrium Yield Curve in the Low-for-Long Economy

What does the model predict about the equilibrium yield curve in the economy with permanently low interest rates (low-for-long economy)? Amid the recent low growth and low inflation in advanced economies, the low-for-long economy is a real risk for many countries. IMF (2017) shows that commercial banks would face lower profitability in the low-for-long economy for several reasons, thus possibly posing a risk to financial stability due to the lack of enough profits to build capital buffers. Hence, given the maturity transformation is a key source of bank profits, the risk to financial stability would be more severe in the low-for-long economy.

---

4This result is consistent with an empirical work by Fuhrer (1996), which shows that the excess volatility for long term interest rates can be accounted for by changes in monetary policy behavior including targeted inflation. It is also consistent with the recent empirical work by Kurmann and Otrok (2013), which shows that most fluctuations of term spreads are accounted for by the fact that short term interest rates are more volatile than long-term interest rates.
economy if the economy faces a flatter equilibrium yield curve there.

This subsection tries to address the above question by conducting counterfactual simulations for the low-for-long economy. In the model, the low-for-long economy is described by setting \( \log(g^*) = 0 \) and \( \log(\pi_t^*) = 0 \). In addition, the zero lower bound of nominal interest rate is introduced to consider the changes of monetary policy behavior around there. More specifically, the monetary policy rule is changed as,

\[
R_t = \max \left\{ 1.0, R_{t-1}^{\phi_r} \left[ \pi_t^* g^* \left( \frac{\Pi_t}{\pi_t^*} \right)^{\phi_r} \right]^{1-\phi_r} \right\},
\]

and set \( g^* = 1 \) and \( \pi_t^* = 1 \) for all \( t \). Under those assumptions, the optimal policy functions for consumption and the equilibrium yield curves are computed and compared with the baseline case through comparative statics.

Figure 7 shows the nominal and real yield curves in equilibrium for the low-for-long economy (the blue bold and dashed lines) along with those in the baseline economy (the black bold and dashed lines) in the U.S. and U.K. The figure indicates that the low-for-long economy would face a flatter yield curve in addition to lower level of interest rates. Both in the U.S. and U.K., the model predicts that the term spreads for 5-year and 10-year bonds would be almost half in the low-for-long economy. Furthermore, the figure shows that the decline in real term premiums account for most of the decline in nominal term premiums, and that inflation risk premiums are almost unchanged even in the low-for-long economy.

A key to understanding the result is the change in responses of real interest rates to inflation under the low-for-long economy due to the ZLB. Above the ZLB, the real return of nominal bonds, \( R_t/\pi_{t+1} \), increases in response to an inflation hike because the monetary policy rule in the model satisfies the Taylor principle. On the other hand, around the ZLB, the real return of nominal bonds decreases rather than increases in response to an inflation hike because the central bank cannot aggressively respond to inflation due to the ZLB. Hence, the positive correlation between the real SDF and real interest rates would be weakened around the ZLB, thus leading to lower real term premiums in the low-for-long economy. Intuitively, a long-term bond becomes more like insurance rather than a risky asset in the low-for-long economy because inflation decreases consumption growth but at the same time increases real bond prices, thus holding bonds is a good hedge for a risk associated with inflation around the ZLB.

The above result implies that a permanently low interest rate environment potentially poses a risk to financial stability. While IMF (2017) shows that commercial banks are
expected to face lower profitability even without the flattening of yield curves, the flatter yield curve predicted by the comparative statics implies that the low profitability problem in the low-for-long economy might be more severe than expected, given that the maturity transformation is a key source of their profits. Since sustainable profits are necessary for commercial banks to build capital buffers, the low profitability problem in the low-for-long environment potentially poses a risk to financial stability. Hence, while the low-for-long economy is still only one of risks in a limited number of advanced economies, it should be beneficial for commercial banks and policy makers to keep in mind about the risk to financial stability in the low-for-long economy.

4 Conclusion

This paper examines equilibrium yield curves by question by analyzing the equilibrium yield curve in a model with optimal savings as a buffer stock. In the model, consumers optimize their consumption path under an exogenous income and inflation process as well as nominal interest rates set by a monetary policy rule, and the equilibrium yield curve is defined as a result of consumer’s optimal conditions. The contribution of this paper to the literature is twofold. First, it shows that the shape of yield curve is consistent with the empirically observed income and inflation dynamics including their co-movement (i.e., the Phillips curve). Namely, the model successfully accounts for a realistic upward sloping yield curve in the US and Japan under the income and inflation process estimated by data. Second, it shows that an endogenous monetary policy response to inflation is a key to accounting for the shape of yield curves. Given this importance of monetary policy behavior in shaping yield curves, a counterfactual simulation implies that the equilibrium yield curve in a permanently low interest rate environment would be significantly flattened due to changes in the monetary policy behavior around the ZLB, implying that a permanently low interest rate environment potentially poses a risk to financial stability.

There are several avenues for the future works. First, while this paper assumes the exogenous income and inflation processes, a promising but demanding proposal is to model those variables as endogenous variables in a general equilibrium model and account for key features of equilibrium yield curves at the same time. Since this paper shows some necessary conditions for a stationary and non-stationary part of income, those conditions are good staring points for modeling. Second, while this paper focuses only on the U.S. and Japan,
the analysis can be expanded to cover other countries. Such analyses across countries help understand what causes the differences in term structure of interest rates across countries.

References


**Appendix: Quantitative Methodology**

[To be added...]
Table 1: Calibration Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate, $\beta$</td>
<td>0.9985</td>
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<tr>
<td>Inverse of IES, $\sigma$</td>
<td>2.0</td>
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<tr>
<td>Cost for bond holdings, $\phi_b$</td>
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<tr>
<td>Steady-state savings, $b^s$</td>
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</tr>
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<td>Interest rate smoothing, $\phi_r$</td>
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Figure 1: Phillips Curve: $\log(y_t)$ and $\log(\pi_t)$

Note: This figure shows the scatter plots for estimated inflation gaps and income gaps.
Table 2: Estimation Results

<table>
<thead>
<tr>
<th>Name</th>
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<th>Posterior</th>
<th>US</th>
<th>UK</th>
<th>Germany</th>
<th>Japan</th>
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<td>0.59</td>
<td>0.48</td>
<td>0.71</td>
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<td></td>
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<td>(0.50,0.25)</td>
<td>[0.36,0.71]</td>
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<td>[0.22,0.74]</td>
<td>[0.60,0.83]</td>
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<td>0.48</td>
<td>0.30</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.50,0.25)</td>
<td>[0.33,0.66]</td>
<td>[0.37,0.59]</td>
<td>[0.14,0.45]</td>
<td>[0.44,0.76]</td>
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<td></td>
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<tr>
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<td></td>
<td></td>
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<td>[0.73,0.88]</td>
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<tr>
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<td>0.94</td>
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<tr>
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<td></td>
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<td>1.71</td>
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<tr>
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<td>$\sigma_{gg}$</td>
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<td>0.27</td>
<td>0.29</td>
<td>0.19</td>
<td>0.09</td>
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<tr>
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<td>Inv.Gamma</td>
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<td>0.09</td>
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<td></td>
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<td>$\sigma_{\pi y}$</td>
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<td>0.02</td>
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<td>[-0.58,0.60]</td>
</tr>
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</table>

Note: This table shows the prior distributions and the estimated posterior mean and 90 percent confidence intervals. The parameters are estimated by a Bayesian method using $\Delta \log(II)$ and $\Delta \log(Y_t)$ in US, UK, Germany, and Japan as observable variables. Inflation series in each country are US: PCE deflator (chain price index), UK: Long Term Price Indicator of Consumer Goods and Services, DE: Consumer Price Index, JP: General Consumer Price Index Excluding Fresh Food. Also, the real income data for US and UK are taken from the national statistics while those for Germany and Japan are taken from the real income index compiled at the Dallas Fed. The sample periods are 1957Q2 - 2017Q4 for UK, 1959Q2 - 2017Q4 for US, and 1975Q2 - 2017Q4 for Germany and Japan.
Figure 2: Income Growth and Inflation: $\log(g_t)$ and $\log(\pi_t)$

Note: This figure shows the scatter plots for estimated inflation gaps and growth of non-stationary part of income.

Figure 3: Equilibrium Yield Curve with Different Risk Averseness

Note: This figure shows the equilibrium yield curves.
Figure 4: Equilibrium Yield Curve (Nominal and Real)

Note: This figure shows the equilibrium yield curves in the U.S. and the U.K. for the base line case. The bold line shows the nominal yield curves while the dashed line shows the real yield curve.

Figure 5: Equilibrium Yield Curve with Different Monetary Policy Rules

Note: This figure shows the equilibrium yield curves in the U.S. and the U.K. with different monetary policy rules. The bold line shows the baseline case ($\phi_\pi = 2.0$) while the dashed lines shows the case of high and low degree of response to inflation ($\phi_\pi = 3.0$ and 1.0).
Figure 6: Relative Volatility for Each Maturity

![Graph showing relative volatility for each maturity for U.S. and U.K. with model, model (w/o trend inflation), data, and data (detrend by HP filter).]

Note: This figure shows the relative volatility of interest rates in data and the model. The bold lines show the case of time varying trend inflation (baseline) while the dashed lines show the case of fixed trend inflation.

Table 3: Macroeconomic Moments for Consumption Growth

<table>
<thead>
<tr>
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<th>U.S.</th>
<th>U.K.</th>
</tr>
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<tbody>
<tr>
<td></td>
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<td>Data</td>
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<tr>
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<td></td>
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<tr>
<td>$std(\Delta c_t)$</td>
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</tr>
<tr>
<td>$corr(\Delta c_t, \Delta y_t)$</td>
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<td>0.53</td>
</tr>
<tr>
<td>$corr(\Delta c_t, \pi_t)$</td>
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<td>-0.36</td>
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<td>$corr(\Delta c_{t-1}, \Delta c_t)$</td>
<td>0.11</td>
<td>0.32</td>
</tr>
<tr>
<td>Yearly growth</td>
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<td></td>
</tr>
<tr>
<td>$std(\Delta c_t)$</td>
<td>0.59</td>
<td>0.47</td>
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<td>$corr(\Delta c_t, \Delta y_t)$</td>
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<td>$corr(\Delta c_t, \pi_t)$</td>
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<td>2-year growth</td>
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<td>$std(\Delta c_t)$</td>
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<td>$corr(\Delta c_t, \pi_t)$</td>
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Note: This table shows macroeconomic moments for quarterly, yearly, and 2-year consumption growth.
Figure 7: Equilibrium Yield Curve (Baseline versus Low-for-Long)

Note: This figure shows the equilibrium nominal and real yield curves in the U.S. and Japan for the baseline case (black bold and dashed lines) and for the low-for-long economy (the blue bold and dashed lines with circles).