Fertility, Innovation, and Technological Spillovers in An Endogenous Growth Model with Overlapping Generations

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Abstract

Recent years witnessed a sharp rise in robot stock and industrial patents, in contrast to the reduction in fertility in many countries. To properly investigate these developments in the long run, we build a two-sector Diamond-type Overlapping Generations Model (OLG) with endogenous R&D-based growth, fertility, education, and skill differentials. Our paper contributes to the literature by (1) incorporating capital-complimentary technological change and education quality assessment to the OLG, (2) highlighting the significance of spillover effects from the high-skilled to the low-skilled sector, and (3) explaining the potential U-shape of skill premium. The model predicts fertility tends to decline further because parents must race with smarter machines by spending more on children's education; skill premium tends to rise to reward high-skilled workers, while the ratio of truly high-skilled individuals may decline in the near future. Our simulation exercise for the Japanese economy (from 1975) can confirm these trends with reality. Furthermore, depending on the strength of the spillovers, different states of equilibrium are realized, with high or low growth and high or low skill premium. An important implication from the paper is that policymakers should focus on enhancing the spillover effects by providing training to the low-skilled sector amid the rise of technology.

Keywords: overlapping generations, endogenous growth, endogenous fertility, quality education, innovation, technological spillover effects.

JEL Classification: E17, E23, I24, J13, J24, O31

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1 Introduction

Recent developments in automation technologies have transformed the production process and how people work. Machines are now replacing many repetitive tasks in manufacturing factories (Schwab, 2017). Society also has fewer people performing those tasks, necessitating the R&D in industrial robots that can replace humans (Acemoglu and Restrepo, 2022). In many industrialized countries, the increased stock of robots has moved in sync with the decrease in fertility rate, as depicted in Fig. 1. We believe this trend results from a reciprocal relationship between machines and fertility. In particular, in order to help their children win the race against the machines, parents tend to increase spending on education, which would be more affordable by having fewer babies. Consequently, the economy experiences less human labor, which necessitates the demand for more machines in production. The rise in education spending also increases the proportion of high-skilled labor, which can be used to create more intelligent robots with a broader range of variety. While it is true that machines are gradually replacing manual labor in many tasks, it is also important to acknowledge that people can adapt. In other words, while being suppressed by robots or machines, people can switch jobs to industries that have not been automated; or train themselves to use new technologies to improve their earnings. This paper looks into this possibility where a portion of technology can be transferred from the high-skilled to the low-skilled. The transfer can be understood in 2 mechanisms. The first is on the intensive margin, where technology spillovers increase workers' productivity. For example, thanks to the availability of the internet and smartphones (inventions of the high-tech sector), people can look for information, learn and train themselves, regardless of skills. The second is on the extensive margin, where technology generates new jobs in the low-skilled sector simultaneously with the jobs or tasks newly created in the high-skilled sector.

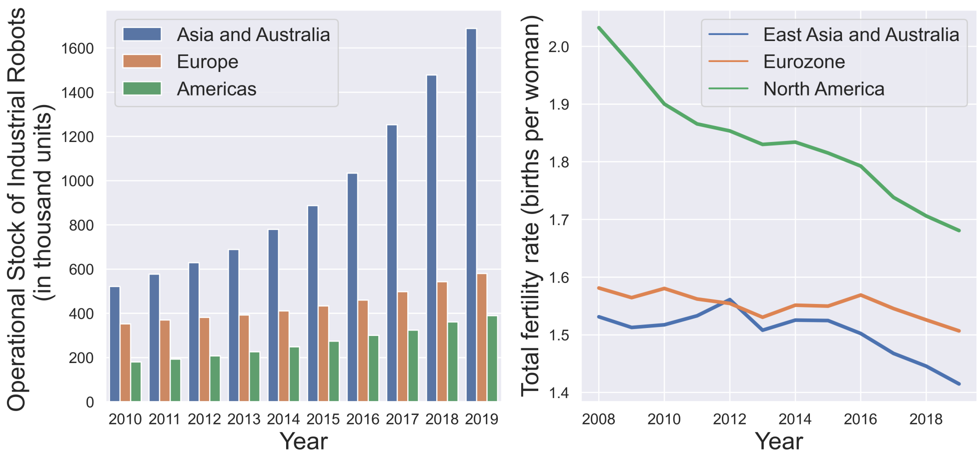

Figure 1: Operational Stock of Industrial Robots (Source: IFR) and Total Fertility Rate (Source: WB)

It is important to note that automation processes, machine learning, or the Internet of Things (IoT) have only been developed and applied in the last 10 to 15 years. Therefore, it is difficult to predict precisely how technological advancement will affect the distribution of wealth in the long run. There are two missing elements in the current literature that require further investigation. First, due to the particular short-run of data, most seminal papers concerning the effects of automation on skill-induced income differentials work almost exclusively with the production function, such as Krusell et al. (2000), Acemoglu and Autor (2011), Lankisch et al. (2019), Acemoglu and Restrepo (2022). We think including a micro foundation in this framework is essential to explore long-run effects. Second, it is natural that production will become more skill-biased (Acemoglu and Restrepo, 2018) because people need to learn more sophisticated abilities to master or control the new machines/technologies. In this way, high-skill workers, mainly those with a university degree or higher, tend to gain certain benefits because they have more exposure and adaptability to these new skills. Low-skill workers, especially those without a college degree, on the other hand, can only perform manual tasks, and most of them do not have enough necessary skill sets to compete with high-skilled workers. As a result, as more machines are adopted more rapidly, the real wage of low-skill workers will be suppressed because these machines are naturally skill-biased, leading to a rise in the skill premium.

That being said, empirical studies show that interpreting the movement of skill premium can be tricky as it varies across countries (Parro, 2013). In the same period from 1990 to 2005, some countries, such as the US (Acemoglu and Restrepo, 2021), and the UK (Dupuy, 2007) experienced a surge in the skill premium, whereas others saw a reverse course, such as Japan (Hara et al., 2014; Kawauchi and Mori, 2014), France (de la Croix and Docquier, 2007), or South Korea (Lee, 2017). While litigation, unionization, and employment protection play decisive roles in affecting the skill premium, there is a possibility that such disparities among countries are the result of to which extent low-skilled workers benefit from technological advancements created in the high-skilled sector.

First, as shown in Lee and Clarke (2019), the high-tech sector has a positive job multiplier, where each of ten new high-skilled jobs generates around six non-tradeable service jobs that go to low-skilled workers. The new service jobs often include raw data inputters, janitors, construction workers, and security guards that do not require decent education and rely heavily on human-to-human interactions or situational adaptability (Autor and Dorn, 2013). Even inside high R&D intensive firms, low-skilled services are still needed and see their wage improved (Aghion et al., 2018). Second, many self-employed jobs nowadays, such as e-commerce retailers, video streamers, or influencers (grouped by the term "unincorporated entrepreneurship") require only inexpensive equipment such as a smartphone, a computer, and specialized software, of which prices are often free or reasonably low thanks to competition. One prime example is Japan. As depicted in Fig.2, the prices of information and technology-related commodities have decreased dramatically in Japan since the 1980s. Therefore, it is reasonable to assume that substantial spillovers can be expected because electronic and smart devices have become more affordable. As a result, in the risk of AI, it is easier for individuals to adapt and use these digital tools to improve job security Fossen and Sorgner (2018, 2022). Overall, in the presence of a trickle-down effect of technology, if knowledge transfer is viable and low-skilled workers know how to exploit the spillover effect from technology, they can improve their productivity and earnings, although by a smaller margin compared to the high-skilled individuals.

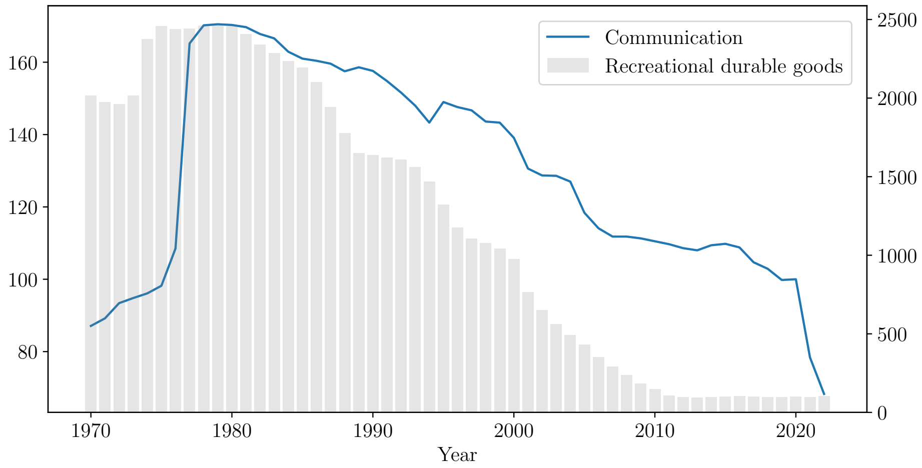

Figure 2: Price Indices of IT-related goods in Japan (Source: Statistics Bureau of Japan)

Note: Recreational durable goods include: computers, tablets, cameras, TV sets, etc. Communication includes mobile phones and related services. Price indices are 2020-based.

To explore the possibility of technology spillover and analyze the long-term effects of automation where machines complement high-skilled labor more than low-skilled, we build a modified Diamond (1965)-type OLG model. Household education and fertility choices are endogenized based on the frameworks from Galor and Weil (2000); de la Croix and Doepke (2003); Hirazawa and Yakita (2017). To account for the skill differentials, we use the two-skill type canonical model in de la Croix (2012). However, the original model does not include an evaluation of education quality, which is crucial since acquiring advanced skills is becoming more important in a highly-automated society. Our contribution to this line of literature is to introduce a technology-discounted education, where the quality of any educational expenses is subject to the state of machine intelligence at that time. Regarding intergenerational transfer, our model assumes that parents' expenditure is the sole source of education funding, determining whether an individual will become high-skilled or low-skilled. This result is theoretically and empirically supported by Anger and Heineck (2010); Becker et al. (2018).

For the production function, it is essential to acknowledge the existence of two literature trends. The first is the skill-biased technological change led by Autor et al. (1998); Acemoglu (2002) where capital tends to substitute labor (in a task-based approach), which reduces the demand for low-skill labor and, in effect, raises relative demand for skilled labor. The second trend is the capital-complimentary technological change where Krusell et al. (2000) argues that the skill premium rises because capital favors those with high skills, shifting the demand for high-skill workers directly. We based our assumptions on the latter, where capital complements high-skilled labor more than low-skilled labor, similar to Kimura and Yasui (2007); Chen (2010)[1]. In the end, the rise of machines and sophisticated software should benefit those who can exploit them by way of programming and be able to perform skills-demanding tasks. In this regard, our framework is similar to Lankisch et al. (2019); Gasteiger and Prettner (2022). However, to account for the adaptability and technology spillovers in the low-skilled sector, we use a 2-sector production function in an endogenous-growth OLG framework similar to Eicher (1999); Kitagawa and Shibata (2005); Sachs et al. (2015). It still preserves the capital-skill complementary aspect as in Lankisch et al. (2019) but allows for a more accessible and tractable analysis. Finally, since a high-skilled population also affects the progression of machines' evolution, we endogenize growth by integrating an R&D sector that develops new varieties of machines (Romer, 1990; Jones, 1995). This endogenous growth feature has been implemented in an OLG framework, notably Chou and Shy (1991); Hashimoto and Tabata (2016); Futagami and Konishi (2019); Prettner and Strulik (2020) where past technology and researchers are the engines of innovation. The bottom line is that our model captures the integrated and endogenous effects of fertility, innovation, and skill differentials while considering their long-term implications on growth.

The contributions of our paper are as follows. First, the model presented here is one of a few attempts to capture a rich economic environment with 2-sector endogenous growth, fertility, and education decisions inside an overlapping generations framework. Second, we contribute to the current line of literature by considering an evaluation of education investment (where quality education is reflected by a race with the rise in machine variety) and spillover effects from the high-skilled sector to the low-skilled sector. By doing so, interesting dynamics during convergence to the steady state emerges that are worth examining. Third, the model captures Japan's economic and demographic development reasonably well. As a result, we can draw some important implications for the progression of Japanese society in the future.

Similar to the unified growth framework (Galor, 2005), our results highlight that endogenous fertility and education are essential to capture the long-run effects of technologies because households adjust their demographic choices, even more so in an environment where machines compete with low-skilled individuals. Furthermore, the spillover effect from the high to low-skilled sector plays an important role. An economy with high spillovers generates a higher growth rate and wealth, narrowing the gap between high-skilled and low-skilled. In contrast, a low-spillover economy keeps widening the skill premium by allowing no technology advancement to be tricked down to the low-skilled sector. Consequently, when fertility and education are considered, such an environment is more likely to hamper growth in the long run since there will be substantial shortages in the supply of high-skilled individuals (of which a portion comes from low-skilled families) and in the supply of capital (since low-skilled workers' low savings cannot contribute

[1]A comparable production function of this type has been used by Galor and Weil (1996)

sufficiently to the stock of capital input).

The paper proceeds as follows. In the next section, we present the theoretical framework for the model. Section 3 analyzes the equilibrium dynamics and long-run steady states for variables of interest. In section 4, we perform numerical simulations to highlight the results. The benchmark model is calibrated to the Japanese economy, followed by counterfactual simulations to investigate the influence of technological spillovers. Finally, section 5 concludes, discusses the important implications of the model, and suggests future extensions.

2 The Model

2.1 Final Good Sector

Production is akin to Eicher (1999), Chen (2010), and Sachs et al. (2015). There are two final goods, Y_t^L, which uses primitive technology requiring only low-skilled labor L, and the high-tech good Y_t^H, which uses high-skilled labor H_t^Y augmented by machines X and advanced technology A. Here, the technology is represented by an expansion of machine variety. The production function for each sector is

$$Y_t^L = A_{t-1}^{\nu}\theta L_t, \tag{1}$$

$$Y_t^H = (H_t^Y)^{1-\alpha}\int_0^{A_t}(X_t^i)^{\alpha}di, \tag{2}$$

where θ indicates how effectively the primitive sector can utilize A_{t-1}^{ν}. We called this "degree of absorption", which is also helpful during the simulation process. Another innovation of the paper is to consider a spillover of technologies, or ideas, from the high-tech to the primitive sector, represented byA_{t-1}^{ν}. Due to obsolescence, only A_{t-1}, the latest technology that can be competitively produced, is available for free use. This assumption is natural because monopolistic power in the high-tech sector tends to evaporate after one period, effectively making it a public good thanks to competition. The parameter $\nu \in [0, 1]$, on the other hand, represents the "degree of spillover" – to what extent the technology used by the high-tech sector can be educated to the low-skilled sector. If $\nu = 1$, all the latest competitively produced ideas can be transferred to the low-skilled sector. A $\nu = 0$ indicates that no technological progress will be available to the lower sector, or a negative ν implies a negative spillover effect on this sector. To keep the model simple, ν is assumed to be 1 for the remainder of the analysis. We will briefly relax this assumption in the numerical simulation section to see its implications. On the other hand, the high-tech sector uses high-skilled labor H_t with a continuum of machine-making firms X_t^i, indexed from 0 to A_t. This treatment is similar to the intermediate-good sector often seen in the endogenous growth framework. Unlike its primitive counterpart, this sector can immediately exploit and gain beneficial productivity from the latest technological developments.

The aggregate production function is simply

$$Y_t = Y_t^L + Y_t^H, \tag{3}$$

which exhibits constant returns to scale. Assume that the two goods have the same price and are normalized to 1; the profit of the final good sector is:

$$\pi_t^Y = Y_t - w_t^H H_t^Y - w_t^L L_t - \int_0^{A_t} p_t^i X_t^i di.$$

Input prices are determined as

$$w_t^H = \frac{\partial \pi_t^Y}{\partial H_t^Y} = (1-\alpha)(H_t^Y)^{-\alpha} \int_0^{A_t} (X_t^i)^\alpha di, \tag{4}$$

$$w_t^L = A_{t-1}^\nu \theta, \tag{5}$$

$$p_t^i = \alpha (H_t^Y)^{1-\alpha} (X_t^i)^{\alpha-1}. \tag{6}$$

This specification assumes that when the variety of machines increases, technologies mainly benefit high-skilled workers first, as the wage for high-skilled (4) increases with machines' variety. In contrast, low-skilled workers must wait until such technologies are readily available to the masses under perfect competition to utilize them – a "low-hanging fruit" problem (Cowen, 2011). In other words, technological advancement is complementary to high-skilled labor. This implication, therefore, still preserves the qualitative importance similar to Prettner and Strulik (2020), without too many complications in the production function.

2.2 Machine Sector

For simplicity, we assume that all capital is used to make machines. The machine-making sector is comprised of competitive vintage firms and monopolistic novel firms. Every period, several novel firms arise with new types of machines that augment high-skilled labor, and by doing so, they gain a monopolistic profit temporarily. The structure of an OLG model allows us to assume that a firm can only retain its monopolistic power for one period (approximately 25 – 30 years). After that, the product can be competitively produced and earn zero profit. This is particularly true given that new ideas (especially in software development) can be replicated given enough time after the products are first introduced on the market. The production is linear so that one unit of machine requires one unit of capital i where

$$X_t^i = K_t^i, \tag{7}$$

so the profit function of producing machine i is

$$\pi_t^i = p_t^i X_t^i - R_t K_t^i. \tag{8}$$

where K_t^i is the amount of capital needed to produce X_t^i, p_t^i is the price of selling X_t^i units of machine i, and R_t is the capital rental rate firms pay to the capital owner.

For vintage firm $i \in [0, A_{t-1}]$, machines are competitively produced such that profit earned is zero where

$$p_t^i = R_t \text{ for } i \in [0, A_{t-1}].$$

Equating (6) to R_t yields the output of these firms as

$$X_t^i = H_t^Y \left(\frac{\alpha}{R_t}\right)^{\frac{1}{1-\alpha}} \text{ for } i \in [0, A_{t-1}]. \tag{9}$$

For novel firms, $i \in (A_{t-1}, A_t]$, machines are monopolistically produced such that their inventors earn some monopolistic profit, obtained by plugging (8) into (6)

$$\pi_t^i = \alpha (H_t^Y)^{1-\alpha} (X_t^i)^\alpha - R_t X_t. \tag{10}$$

Maximizing profit implies the FOC

$$\frac{\partial \pi_t^i}{\partial X_t^i} = 0 \Leftrightarrow \alpha^2 (H_t^Y)^{1-\alpha} (X_t^i)^{\alpha-1} = R_t,$$

so their price and output are set as

$$p_t^i = \frac{R_t}{\alpha}, \qquad X_t^i = H_t^Y \left(\frac{\alpha^2}{R_t}\right)^{\frac{1}{1-\alpha}} \text{ for } i \in (A_{t-1}, A_t]. \tag{11}$$

Thus, profit per invention is obtained by plugging (11) into (10)

$$\pi_t^i = \left(\frac{R_t}{\alpha} - R_t\right) X_t^i = (1-\alpha)\alpha^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}} H_t^Y. \tag{12}$$

The total profit of this sector is

$$\pi_t = \triangle A_{t-1} (1-\alpha)\alpha^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}} H_t^Y,$$

where $\triangle A_{t-1} = A_t - A_{t-1}$. All profits will be transferred in the form of dividends to the capital owners (the old and retired individuals) to fund innovation.

2.3 R&D sector

We assume that the R&D sector innovates and creates new varieties of machines by employing researchers (Romer, 1990). To achieve an analytical result, we use Jones (1995)'s specification, assuming that the production of new machine variety follows

$$\triangle A_{t-1} = A_t - A_{t-1} = \delta A_{t-1}^\phi H_t^R, \tag{13}$$

where $\phi \in [0, 1]$ represents the past research's externality. As is common, we assume that $\phi < 1$ captures the duplication of effort (new knowledge does not necessarily add substantial improvement to the existing stock) and eliminates the scale effects from past research ideas. The profit gained from selling inventions of the R&D sector are

$$\pi_t^A = \triangle A_{t-1}\pi_t - w_t^R H_t^R. \tag{14}$$

Substituting (13) into (14), we have the problem for an innovator

$$\max_{H_t^R} \pi_t^A := \delta A_{t-1}^{\phi} H_t^R (1-\alpha)\alpha^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}} H_t^Y - w_t^R H_t^R. \tag{15}$$

$$\text{s.t. } H_t^R \geq 0 \tag{16}$$

Assuming that δ, A_{t-1} is large enough such that the solution for H_t^R is interior and $\pi_t^A > 0$, the optimal choice of researchers is taken by differentiating (15) with respect to H_t^R and set it to zero so that

$$\delta A_{t-1}^{\phi}(1-\alpha)\alpha^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}} H_t^Y = w_t^R,$$

which implies that

$$H_t^Y = \frac{w_t^R}{\delta(1-\alpha)\alpha^{1/(1-\alpha)} A_{t-1}^{\phi} \left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}}}. \tag{17}$$

In this case, the variable H^R does not appear in the first-order condition, which means that the optimal value H^R does not directly depend on nor affect the condition for profit maximization. So long as the number of research labor is positively employed, any additional units of such research labor will contribute to the maximization of the innovator's profit. The productivity of H^Y is non-linear, but H^R is, while the wage paid to high-skilled workers is the same, so at first, it is optimal to employ H^Y rather than employing the first unit of research labor. Until the marginal productivity of H^Y is equal to wage (in other words, the amount of H^Y demanded by equation (17)), then the rest of the high-skilled labor will be allocated to research labor. For the rest of the analysis in this paper, we set initial conditions of δ, A_{t-1} high enough so that the high-skilled labor will be allocated to both final good production and research activities. As H^Y and H^R are both drawn from the pool of high-skilled labor H, Eq.(17) will help us first determine the optimal allocation of H_t^Y; then we can derive the research labor H_t^R given H_t at any given time.

2.4 Household

Following a simplified version of de la Croix (2012), we assume that there are two types of agents, low-skilled (L) and high-skilled (H) indexed by j so that $j \in \{L, H\}$. Parents care about the number of children (quantity) and their educational level (quality). An agent lives for three periods: childhood, adulthood, and old age. All decisions are made during adulthood when they optimize their consumption path and the quality and quantity of children. During childhood, agents receive educational investment from parents, which will decide the type of worker they become. Savings during adulthood will be spent to finance consumption of old age since they do not work in this period. The time structure of an agent born in time t follows Table 1.

Table 1: The model's time structure

Generation i & Period t	t=1	2	3	$\rightarrow \ldots t=\infty$
i=0	old			
1	adults + kids	old		
2		adults + kids	old	
3			adults + kids	old
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\downarrow$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

An agent of type j maximizes the following utility function

$$\max_{c_t^j, e_t^j, n_t^j} U_t^j = \ln(c_t^j) + \beta \ln(d_{t+1}^j) + \gamma \ln(\Pi_t^j \cdot n_t^j), \tag{18}$$

where c_t^j, d_{t+1}^j are consumptions of parents of type j when young and old. The parameters β and γ are subjective discount factors weighting consumption and having children (altruism), n_t^j is the number of children, and Π_t^j denotes the probability that a child becomes a high-skilled worker. Given a level of education investment e_t^j received from parents, that probability is determined by

$$\Pi_t^j = \mu^j \left(\frac{e_t^j + \bar{e}}{A_t} \right)^\eta, \tag{19}$$

where $\bar{e}$ is the default education available from the society so that a child still receives at least some certain amount of education even if there is no investment from parents. The parameter μ represents the intergenerational transmission of human capital, specifically $\mu^H > \mu^L$. Intuitively, children of high-skilled parents will be more likely to become high-skilled than children from low-skilled families. One innovation in this paper is that the value of education is weighted by the number of robots' variety A_t. This specification is important because when robots become more capable of doing jobs that used to be done by humans, workers must learn more skills to stay ahead. Intuitively, it reflects the quality of education expenses. Finally, parameter $\eta \in (0, 1)$ governs the curvature of the skill conversion.

Household budget when young is

$$w_t^j(1-\rho n_t^j) = c_t^j + s_t^j + e_t^j n_t^j + \varepsilon n_t^j, \tag{20}$$

where $w_t^j, c_t^j, s_t^j, n_t^j$ are the wage rate, consumption, saving, the number of children, ε and e_t^j are the good cost of childrearing and education cost per child of a parent of type j. The parameter $\rho \in [0,1]$ is the time cost of having children. The budget during old age is

$$d_{t+1}^j = R_{t+1} s_t^j. \tag{21}$$

The problem for a household of type j is to maximize (18) subject to (19), (20), (21) by choosing the optimal s_t^j, n_t^j, e_t^j. The solutions are summarized as follows.

Proposition 1 (Household problem)**.** The optimal decisions for an agent of type j are

$$c_t^j = \frac{1}{1+\beta+\gamma} w_t^j, \tag{22}$$

$$s_t^j = \frac{\beta}{1+\beta+\gamma} w_t^j, \tag{23}$$

$$d_{t+1}^j = \frac{\beta}{1+\beta+\gamma} R_{t+1} w_t^j. \tag{24}$$

If $w_t^j \leq \dfrac{\bar{e}-\eta\varepsilon}{\eta\rho}$

$$e_t^j = 0, \tag{25}$$

$$n_t^j = \frac{\gamma w_t^j}{(1+\beta+\gamma)(\rho w_t^j + \varepsilon)}. \tag{26}$$

Otherwise, if $w_t^j > \dfrac{\bar{e}-\eta\varepsilon}{\eta\rho}$

$$e_t^j = \frac{\eta(\rho w_t^j + \varepsilon) - \bar{e}}{1-\eta}, \tag{27}$$

$$n_t^j = \frac{(1-\eta)\gamma w_t^j}{(1+\beta+\gamma)(\rho w_t^j - \bar{e} + \varepsilon)}. \tag{28}$$

Proof. See Appendix A. ■

The results are standard compared to other endogenous fertility and education models. For parents with sufficient income, education expenses will increase with earnings. On the other hand, for parents with low enough income, it is optimal to not spend at all on their children's education. Regarding fertility, we can state the following result.

Proposition 2. For sufficiently high-income parents, fertility is negatively affected by income if $0 < \varepsilon < \bar{e}$, and if parents' income is below the threshold level, fertility increases with income.

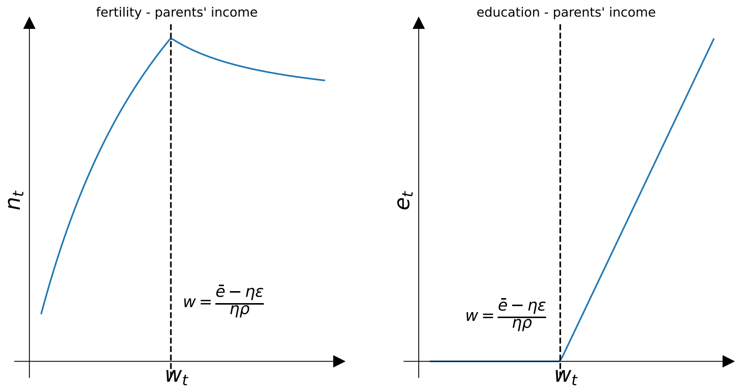

Figure 3: Children quantity-quality trade-off

Proof. Differentiating (28) with respect to w_t^j for $w_t^j > \dfrac{\bar{e} - \eta\varepsilon}{\eta\rho}$ yields

$$\frac{\partial n_t^j}{\partial w_t^j} = \frac{(1-\eta)\gamma}{(1+\beta+\gamma)} \cdot \frac{\varepsilon - \bar{e}}{(\rho w_t^j + \varepsilon - \bar{e})^2} < 0 \text{ as long as } \varepsilon < \bar{e}.$$

Differentiating (26) with respect to w_t^j for $w_t^j \leq \dfrac{\bar{e} - \eta\varepsilon}{\eta\rho}$ yields

$$\frac{\partial n_t^j}{\partial w_t^j} = \frac{\gamma}{(1+\beta+\gamma)} \cdot \frac{\varepsilon}{(\rho w_t^j + \varepsilon)^2} > 0 \text{ as long as } \varepsilon > 0.$$

■

We impose the following condition on parameters to ensure the qualitative result of Proposition 2.

Assumption 1. *The good cost of children is positive and smaller than the default education* $0 < \varepsilon < \bar{e}$.

Assumption 1 is essential to guarantee that the model exhibits the children's quality-quantity trade-off for parents with sufficiently high income, as depicted in figure 3. Under a low enough wage rate, the choice of education is at the corner ($e_t = 0$), and there is a positive relationship between fertility and income. Very low-income parents cannot afford childcare and tend to have fewer children. As earning improves, so long as they do not have to spend on education, parents can finance the childcare cost to have more children. This part of the result is qualitatively similar to Galor (2005). However, such a relationship is reversed after the income level surpasses a certain threshold where the education choice is an interior solution ($e_t > 0$). The child quantity-quality trade-off begins. Parents sacrifice to have fewer children in order to spend more education on each one of them. For parents in this income range, the portion of time forgone to raise children is relatively more expensive than for lower-income parents. Such opportunity costs and pressure from education expenses create an inverted relationship between income and fertility for high-earning parents. This result, in general, is qualitatively consistent with Doepke (2004); Strulik et al. (2013).

From here, it is convenient and easy to show that the fertility choice is always bounded from above.

Lemma 1. *For high-income parents, the maximum fertility choice is*

$$n^H_{\max} = \lim_{w^H_t \to \bar{w}} = \frac{\gamma}{(1+\beta+\gamma)\rho}\left(1 - \eta\frac{\varepsilon}{\bar{e}}\right),$$

and the minimum fertility choice is

$$n^H_{\min} = \lim_{w^H_t \to \infty} = \frac{\gamma}{(1+\beta+\gamma)\rho}(1-\eta).$$

By Assumption 1, it is guaranteed that $n^H_{\min} < n^H_{\max}$.

3 Equilibrium

3.1 Intertemporal Equilibrium

To determine the allocation of high-skilled (which proportion goes to final-good production, which proportion goes to R&D), we impose a non-arbitrage condition across wages for high-skilled

$$w^R_t = w^H_t. \tag{29}$$

The labor market clears where

$$L_t = (1-\rho n^L_t)N^L_t, \tag{30}$$

$$H_t = (1-\rho n^H_t)N^H_t. \tag{31}$$

The high-skilled labor market clears such that

$$H^Y_t + H^R_t = H_t. \tag{32}$$

And the capital market clears as

$$\int_0^{A_t} X^i_t di = \int_0^{A_{t-1}} X^i_t di + \int_{A_{t-1}}^{A_t} X^i_t di = K_t. \tag{33}$$

Let us solve the capital market clearing condition first. Using (9) and (11), we can write (33) as

$$H^Y_t \left(\frac{\alpha}{R_t}\right)^{\frac{1}{1-\alpha}} \left[A_{t-1} + \triangle A_{t-1}\alpha^{\frac{1}{1-\alpha}}\right] = K_t. \tag{34}$$

From here, we obtain the endogenized interest rate as

$$R_t = \alpha \left[\frac{K_t}{H_t^Y \left(A_{t-1} + \triangle A_{t-1} \alpha^{\frac{1}{1-\alpha}} \right)} \right]^{\alpha-1}. \tag{35}$$

Similarly, the total output from the intermediate sector is derived as

$$\int_0^{A_t} (X_t^i)^\alpha di = (H_t^Y)^\alpha \left(\frac{\alpha}{R_t} \right)^{\frac{\alpha}{1-\alpha}} \left[A_{t-1} + \triangle A_{t-1} \alpha^{\frac{\alpha}{1-\alpha}} \right]. \tag{36}$$

We now solve the labor market. First, the non-arbitrage condition implies that

$$w_t^R = w_t^H = (1-\alpha)(H_t^Y)^{-\alpha} \int_0^{A_t} (X_t^i)^\alpha di.$$

Using the result from (36), the wage for high-skilled becomes

$$w_t^H = (1-\alpha) \left(\frac{\alpha}{R_t} \right)^{\frac{\alpha}{1-\alpha}} \left[A_{t-1} + \triangle A_{t-1} \alpha^{\frac{\alpha}{1-\alpha}} \right]. \tag{37}$$

We apply this result to (17) to obtain the allocation of high-skilled labor in the final-good production as

$$H_t^Y = \frac{\left[A_{t-1} + \triangle A_{t-1} \alpha^{\frac{\alpha}{1-\alpha}} \right]}{\alpha^{\frac{1}{1-\alpha}} \delta A_{t-1}^\phi}. \tag{38}$$

The skilled-labor market clearing conditions (32) implies that

$$H_t^R = H_t - \frac{\left[A_{t-1} + \triangle A_{t-1} \alpha^{\frac{\alpha}{1-\alpha}} \right]}{\alpha^{\frac{1}{1-\alpha}} \delta A_{t-1}^\phi}. \tag{39}$$

The final missing piece is the labor market, where H_t must be determined. Assuming $w_t^H > \dfrac{\bar{e} - \eta\varepsilon}{\eta\rho}$ from now on, plugging (28) into (31) yields

$$H_t = N_t^H \left[1 - \frac{(1-\eta)\rho\gamma w_t^H}{(1+\beta+\gamma)(\rho w_t^H - \bar{e} + \varepsilon)} \right]. \tag{40}$$

Inserting the wage rate at (37), we have

$$H_t = N_t^H \left[1 - \frac{\xi}{\rho - \frac{\bar{e}-\varepsilon}{(1-\alpha)\left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}} \left[A_{t-1} + \triangle A_{t-1} \alpha^{\frac{\alpha}{1-\alpha}} \right]}} \right], \tag{41}$$

where $\xi = \dfrac{(1-\eta)\rho\gamma}{1+\beta+\gamma}$.

Substituting (41) into (39), the rate of innovation at (13) can be characterized by the following nonlinear difference equation

$$\triangle A_{t-1} = \delta A_{t-1}^{\phi} \left[N_t^H \left(1 - \frac{\xi}{\rho - \frac{\bar{e}-\varepsilon}{(1-\alpha)\left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}} \left[A_{t-1}+\triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}}\right]}} \right) - \frac{\left[A_{t-1} + \triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}}\right]}{\alpha^{\frac{1}{1-\alpha}}\delta A_{t-1}^{\phi}} \right]. \tag{42}$$

Using (35), we can rewrite (42) as an implicit function Ψ such that

$$\begin{aligned}\triangle A_{t-1} &= \delta A_{t-1}^{\phi} \left[N_t^H \left(1 - \frac{\xi}{\rho - \frac{(\bar{e}-\varepsilon)\alpha^{\frac{\alpha}{1-\alpha}}\left(A_{t-1}+\triangle A_{t-1}\alpha^{\frac{1}{1-\alpha}}\right)^{\alpha}}{(1-\alpha)K_t^{\alpha}A_{t-1}^{\alpha\phi}\left[A_{t-1}+\triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}}\right]^{1-\alpha}}} \right) - \frac{\left[A_{t-1} + \triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}}\right]}{\alpha^{\frac{1}{1-\alpha}}\delta A_{t-1}^{\phi}} \right] \\ &= \Psi(\triangle A_{t-1}, A_{t-1}, K_t, N_t^H),\end{aligned} \tag{43}$$

which depends only on the last available state of technology A_{t-1}, capital K_t, and the high-skilled population N_t^H. The values of these variables are all known at the beginning of each period.

Proposition 3. Given initial A_{t-1}, $K_t, N_t^H > 0$, and appropriate parameters, (43) admits 1 unique positive solution for $\triangle A_t$.

Proof. See Appendix B. ∎

Solving for $\triangle A_{t-1}$ will help us determine the values of other variables in the rest of the model at time t. We proceed to work out the dynamics for the subsequent period. Assuming full capital depreciation after one period, which is reasonable as one period in an OLG model is normally equivalent to 25 to 30 years, the capital accumulates according to the following

$$K_{t+1} = s_t^L N_t^L + s_t^H N_t^H. \tag{44}$$

Using the saving decision at (23), it can be written as

$$K_{t+1} = \frac{\beta}{1+\beta+\gamma}\left(w_t^L N_t^L + w_t^H N_t^H\right).$$

The wage for low-skilled is determined by (5) and wage for high-skilled labor is determined by (37), therefore we have

$$K_{t+1} = \frac{\beta}{1+\beta+\gamma}\left(A_{t-1}\theta N_t^L + (1-\alpha)\left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}}\left[A_{t-1} + \triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}}\right] N_t^H\right). \tag{45}$$

Given the necessary information available at the beginning of time t, such as population N_t^L, N_t^H, technology A_{t-1} and the capital stock K_t, we can solve the model at t and work out the capital available for the period $t+1$. The population for the next period shall be determined by the following

$$N_{t+1}^L = (1-\Pi_t^H)n_t^H N_t^H + (1-\Pi_t^L)n_t^L N_t^L, \tag{46}$$
$$N_{t+1}^H = \Pi_t^H n_t^H N_t^H + \Pi_t^L n_t^L N_t^L \tag{47}$$

where all variables on the RHS are solved and already determined at t. We are then entitled to state the dynamics of the intertemporal equilibrium of the model.

Definition 1 (Intertemporal Equilibrium)**.** Let time begin at $t = 1$ where A_0 is known. Given the initial stock of low-skilled population N_1^L, high-skilled population N_1^H, capital stock K_1, a competitive equilibrium is a sequence of $\{\triangle A_{t-1}\}_{t=1}^{\infty}$, factor prices $\{w_t^L, w_t^H, R_t\}_{t=1}^{\infty}$, household choices $\{c_t^L, c_t^H, s_t^L, s_t^H, d_{t+1}^L, d_{t+1}^H, n_t^H, n_t^L, e_t^H, e_t^L\}_{t=1}^{\infty}$, and inputs $\{K_t, L_t, H_t^Y, H_t^R\}_{t=1}^{\infty}$ such that

1. At each t, $\{\triangle A_{t-1}\}$ solves (42), which decides A_t and H_t^Y.
2. Then, the factor prices $\{w_t^L, w_t^H, R_t\}$ are decided following Eqs. (4), (5), and (35).
3. Given A_t and factor prices, $\{K_t, L_t, H_t^Y, H_t^R\}_{t=1}^{\infty}$ solve the problems for the final good sector and machine sector.
4. Given the factor prices, $\{c_t^L, c_t^H, s_t^L, s_t^H, d_{t+1}^L, d_{t+1}^H, n_t^H, n_t^L, e_t^H, e_t^L\}_{t=1}^{\infty}$ solve the households' problem according to (22), (23), (24), (25), (27).
5. Given the education inputs and A_t, whether a child becomes high-skilled or low-skilled will be determined by Π_t^L, Π_t^H at (19).
6. Given $\{s_t^L, s_t^H, n_t^H, n_t^L, Pi_t^L, \Pi_t^H\}$, the population and capital accumulation in the next period evolve according to (46), (47) and (45).

The following section presents some main results when we look at the economy in the long run.

3.2 Steady States

For reasons we will elaborate on later, the following assumption is necessary to guarantee a non-trivial steady-state equilibrium.

Assumption 2. *The long-run population growth is zero. In other words, $n_t^j \to 1$ for $j \in \{L, H\}$ as $t \to \infty$.*

3.2.1 Long run Education and Fertility

First, since fertility is endogenized, it is necessary to investigate the population dynamics. By definition, the young population evolves according to

$$N_{t+1} = n_t^L N_t^L + n_t^H N_t^H.$$

Assume that

$$w_t^H > w_t^L = A_{t-1}\theta > \frac{\bar{e} - \eta\varepsilon}{\eta\rho} \ \forall t.$$

so that the choice of education is an interior solution. We want to characterize the dynamics of the rest of the model.

Proposition 4. In the economy where technological advancement has full spillover effects $\nu = 1$, the long-run fertility rates of both high-skilled and low-skilled populations will converge to the same value where

$$n^H = n^L = \frac{(1-\eta)\gamma}{(1+\beta+\gamma)\rho}.$$

Proof. We first look at the low-skilled parents' choices of fertility.

$$\begin{aligned}
n_t^L &= \frac{(1-\eta)\gamma A_{t-1}\theta}{(1+\beta+\gamma)(\rho A_{t-1}\theta + \varepsilon - \bar{e})}, \\
e_t^L &= \frac{\eta(\rho A_{t-1}\theta + \varepsilon) - \bar{e}}{1-\eta} = \frac{\eta\rho A_{t-1}\theta}{1-\eta} + \frac{\eta\varepsilon - \bar{e}}{1-\eta}, \\
\Pi_t^L &= \mu^L \left(\frac{e^L + \bar{e}}{A_t}\right)^\eta.
\end{aligned}$$

One can see that when A_t increases, it will be more and more difficult for children of low-skilled parents to become high-skilled. Using the last two equations, we have

$$\begin{aligned}
\frac{e_t^L}{A_t} &= \frac{\eta\rho\theta}{1-\eta} \cdot \frac{A_{t-1}}{A_t} + \frac{\eta\varepsilon - \bar{e}}{(1-\eta)A_t} \\
&= \frac{\eta\rho\theta}{1-\eta} \cdot \left(\frac{1}{1 + \frac{\triangle A_{t-1}}{A_{t-1}}}\right) + \frac{\eta\varepsilon - \bar{e}}{(1-\eta)A_t}.
\end{aligned}$$

By letting technology $A_t \to \infty$, the benefits of default education vanish, and the possibility for a child from low-skilled parents to become high-skilled converges to

$$\Pi^L = \mu^L \left(\frac{\eta\rho\theta}{1-\eta} \cdot \left(\frac{1}{1+g^A}\right)\right)^\eta > 0, \tag{48}$$

where $g^A \equiv \frac{\triangle A_{t-1}}{A_{t-1}}$. The fertility decision converges to

$$n^L = \frac{(1-\eta)\gamma}{(1+\beta+\gamma)\rho} > 0. \tag{49}$$

Moving on to the high-skilled labor, which matters the most for growth, the high-skilled parents' choices are

$$\begin{aligned}
e_t^H &= \frac{\eta\rho}{1-\eta}(1-\alpha)\left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}}\left[A_{t-1}+\triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}}\right]+\frac{\eta\varepsilon-\bar{e}}{1-\eta},\\
n_t^H &= \frac{(1-\eta)\gamma}{(1+\beta+\gamma)(\rho+\frac{\varepsilon-\bar{e}}{w_t^H})},\\
\Pi_t^H &= \mu^L\left(\frac{e_t^H}{A_t}+\frac{\bar{e}}{A_t}\right)^{\eta}
\end{aligned}$$

Similar to low-skilled parents, as A_t increases over time, the weight of $\bar{e}/A_t$ gradually disappears. However, for high-skilled, e_t^H increases with the current technology. To see the trajectory of e_t^H/A_t, we formulate it as

$$\frac{e_t^H}{A_t}=\frac{\eta\rho(1-\alpha)}{1-\eta}\left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}}\frac{\left[A_{t-1}+\triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}}\right]}{A_t}+\frac{\eta\varepsilon-\bar{e}}{(1-\eta)A_t} \tag{50}$$

By definition, $A_t = A_{t-1}+\triangle A_{t-1}$, therefore

$$A_{t-1}+\triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}} = A_t-(1-\alpha^{\frac{\alpha}{1-\alpha}})\triangle A_{t-1}$$

Thus, one can write (50) as

$$\frac{e_t^H}{A_t}=\frac{\eta\rho(1-\alpha)}{1-\eta}\left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}}\left[1-\frac{\triangle A_{t-1}}{A_{t-1}}(1-\alpha^{\frac{\alpha}{1-\alpha}})\right]+\frac{\eta\varepsilon-\bar{e}}{(1-\eta)A_t}$$

Since $0<\alpha<1$, the term in the square bracket tends to be constant once the economy reaches its balanced growth path. In fact, on the balanced growth path, we can derive

$$\Pi^H=\mu^H\left[\frac{\eta\rho(1-\alpha)}{1-\eta}\left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}}\left(1-g^A(1-\alpha^{\frac{\alpha}{1-\alpha}})\right)\right]^{\eta}>0, \tag{51}$$

where the interest rate reaches its steady state $R_t \to R^*$ as the capital-effective labor ratio reaches its steady state. Since w_t^H grows perpetually with A_t, the fertility of high-skilled workers also converges to

$$n^H=\frac{(1-\eta)\gamma}{(1+\beta+\gamma)\rho}=n^L. \tag{52}$$

Thus, it is implied that in the long run, the fertility rates of low-skilled and high-skilled parents will asymptotically be the same. ■

The implication is that since our income threshold is parameterized, the existence of a spillover effect can improve low-skilled individuals' income that will get higher than the threshold. A weaker degree of spillover only slows down the improvement in wages, which limits the speed of convergence in terms of fertility rates but does not affect the overall trend. Due to this feature, the population growth will stop at some point and reaches its steady-state value of 1. Without additional new research labor, R&D will stop, and the machine technology attains its maximum value $A_{\max} < \infty$, which also guarantees the steady state of Π^L, Π^H.

3.2.2 Long run population composition

Using the system (46) and (47), let us investigate the population ratio to check if the economy has a long-run equilibrium of population composition. Define

$$x_t = \frac{N_t^L}{N_t^H}. \tag{53}$$

then the population ratio respects the following dynamics

$$x_{t+1} = \frac{N_{t+1}^L}{N_{t+1}^H} = \frac{n_t^L(1-\Pi_t^L)x_t + n_t^H(1-\Pi_t^H)}{n_t^L\Pi_t^L x_t + n_t^H\Pi_t^H} = \Gamma(x_t).$$

By the assumption that $\Pi_t^H > \Pi_t^L \ \forall t$, we have the following results

$$\begin{aligned}
\Gamma'(0) &= \frac{1-\Pi_t^H}{\Pi_t^H} > 0, \\
\Gamma'(x_t) &= \frac{n_t^L n_t^H(\Pi_t^H - \Pi_t^L)}{(n_t^L\Pi_t^L x_t + n_t^H\Pi_t^H)^2} > 0, \\
\Gamma''(x_t) &= -2 \cdot \frac{n_t^L n_t^H(\Pi_t^H - \Pi_t^L)}{(n_t^L\Pi_t^L x_t + n_t^H\Pi_t^H)^3} \cdot n_t^L\Pi_t^L < 0.
\end{aligned}$$

The function Γ is concave and increasing in x_t. To find that steady state, we let $x_{t+1} = x_t = x^*$ and obtain a positive solution

$$x^* = \frac{(1-\Pi^L)n^L - \Pi^H n^H + \sqrt{(\Pi^H n^H - (1-\Pi^H)n^L)^2 + 4\Pi^L n^L n^H(1-\Pi^H)}}{2\Pi^L n^L}. \tag{54}$$

In the long run, when fertility and education decision reach their steady states, (54) reaches its steady state, which can be calculated by using (48), (49), (51), (52) so the time subscript can be omitted. An important implication from this result is its reliance on ν – the spillover effects for the low-skilled sector. The closer it is to 1, the more the low-skilled sector gain from the high-tech sector's advancements in technological development. However, in an extreme case where $\nu = 0$, as there are no spillover effects, earnings for low-skilled parents do not improve, which makes them unable to increase education expenses. Investment in children's education is discounted by A_t, which leads to a monotonic reduction in educational investment quality. As a result, if A_t is large enough, Π^L will asymptotically be zero, implying that there is zero chance for a child from a low-skilled family to become a high-skilled in the future. Eq. (54) thus indicates that the economy will have an asymptotically infinitesimal portion of high-skilled in the long run. Therefore, a positive and large enough ν must be guaranteed for the model to have a long-run stable population ratio. Furthermore, since innovation increases with population, fertility must reach its replacement level at some finite point A_t in time so that A stops growing and let Π^L reach its steady state.

3.2.3 Long run capital-effective labor ratio

From (35) and (38), we have

$$\left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}} = \left[\frac{K_t}{H_t^Y(A_{t-1} + \triangle A_{t-1}\alpha^{\frac{1}{1-\alpha}})}\right]^\alpha. \tag{55}$$

We can define the capital-effective labor ratio as

$$\tilde{k}_t = \frac{K_t}{A_t(L_t + H_t)}.$$

Using (30) and (31), knowing that in the long run, fertility rates for low-skilled and high-skilled are uniform, we can obtain

$$\frac{L_t}{H_t} = \frac{N_t^L(1-\rho n^*)}{N_t^H(1-\rho n^*)} = \frac{N_t^L}{N_t^H} = x_t.$$

Thus, we can express the capital-effective labor ratio as

$$\tilde{k}_t = \frac{K_t}{(x_t+1)A_t H_t}. \tag{56}$$

The law of motion for capital (45) can now be expressed as follows

$$K_{t+1} = \frac{\beta}{1+\beta+\gamma}\left[A_{t-1}\theta x_t N_t^H + (1-\alpha)\frac{K_t^\alpha(A_{t-1} + \triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}})N_t^H}{[H_t^Y(A_{t-1} + \triangle A_{t-1}\alpha^{\frac{1}{1-\alpha}})]^\alpha}\right].$$

Dividing both sides to $(x_{t+1}+1)A_{t+1}H_{t+1}$ and change $N_t^H = H_t/(1-\rho n^*)$ to obtain

$$\tilde{k}_{t+1} = \frac{\beta}{1+\beta+\gamma}\left[\frac{A_{t-1}\theta x_t}{(1+x_{t+1})n^* A_{t+1}(1-\rho n^*)}\right.$$
$$\left. + (1-\alpha)\tilde{k}_t^\alpha\left(\frac{H_t}{H_t^Y}\right)^\alpha \frac{(1+x_t)^\alpha A_t^\alpha(A_{t-1} + \triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}})}{(1+x_{t+1})n^* A_{t+1}(1-\rho n^*)(A_{t-1} + \triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}})}\right].$$

Grouping common terms yields

$$\tilde{k}_{t+1} = \frac{\beta}{(1+\beta+\gamma)(1+x_{t+1})n^*(1-\rho n^*)(1+g_t^A)}\left[\frac{\theta x_t}{(1+g_t^A)} + (1-\alpha)\tilde{k}_t^\alpha\left(\frac{H_t}{H_t^Y}\right)^\alpha\Theta_t\right]. \tag{57}$$

where

$$\Theta_t = \frac{(1+x_t)^\alpha A_t^\alpha(A_{t-1} + \triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}})}{A_t(A_{t-1} + \triangle A_{t-1}\alpha^{\frac{1}{1-\alpha}})^\alpha} \geq 0.$$

Note that for $0 < \nu < 1$, the dynamics of capital-effective labor ratio is

$$\tilde{k}_{t+1} = \frac{\beta}{(1+\beta+\gamma)(1+x_{t+1})n^*(1-\rho n^*)(1+g_t^A)} \left[\frac{\theta x_t A_{t-1}^{\nu-1}}{(1+g_t^A)} + (1-\alpha)\tilde{k}_t^{\alpha} \left(\frac{H_t}{H_t^Y} \right)^{\alpha} \Theta_t \right]. \tag{58}$$

Although it is impossible to pin down the closed-form solution for the steady-state capital-effective labor ratio, its existence can be studied. For that purpose, we use Proposition 5 from Galor and Ryder (1989).

Proposition 5. The dynamics converge to a non-trivial steady-state equilibrium if and only if population growth stops at some finite $A < \infty$. In other words, Assumption 2 must hold. As a result, $N_t^H \to N^H < \infty, H_t \to H < \infty$.

Proof. This is directly seen by taking the limits of Θ_t as $A_t \to \infty$

$$\lim_{A_t \to \infty} \Theta_t = \lim_{A_t \to \infty} (1+x_t)^{\alpha} \frac{(A_{t-1} + \triangle A_{t-1} \alpha^{\frac{\alpha}{1-\alpha}})}{A_t (A_{t-1} + \triangle A_{t-1} \alpha^{\frac{1}{1-\alpha}})^{\alpha}} A_t^{\alpha-1} = 0.$$

And since g_t^A grows with population growth, unless the population stops growing at a finite A (in other words, $n = 1$ in the long run), the dynamics will converge to $\tilde{k}^* = 0$. ■

As a result, to obtain a non-trivial steady state, it is necessary to impose the condition that the population stays stable (zero growth) in the long run. This assumption is natural, considering the fertility rates will be just as large as the reproduction level (de la Croix, 2012). For this reason, the model can be classified as a semi-endogenous type. After this phase, the dynamics follow the traditional Diamond-type OLG.

Proposition 6. The model admits a unique and globally stable non-trivial steady-state equilibrium so long as Assumption 2 holds.

Proof. Eq.(57) can be rewritten as $\tilde{k}_{t+1} = \phi(\tilde{k}_t)$. Plugging (36) and (55) into the production function (3), then using the intensive form (56), the output-effective labor ratio can be written as

$$\tilde{y}_t = \frac{Y_t}{(1+x_t)A_t H_t} = \frac{\theta x_t}{(1+x_t)(1+g_t^A)} + \frac{\Theta_t}{H_t} \left(\frac{H_t}{H_t^Y} \right)^{\alpha} \tilde{k}_t^{\alpha} = f(\tilde{k}_t).$$

We can easily see that

$$\lim_{\tilde{k}_t \to \infty} f'(\tilde{k}_t) = 0.$$

and we can impose parameters to restrict $n_{\max}$ such that the slope of $\phi(\tilde{k}_t)$ is steeper than the 45-degree line at the origin when $\tilde{k}_t \to 0$. Notice the presence of a negative scale effect in $f(\tilde{k}_t)$ (the term Θ_t/H_t), which necessitates the condition of zero population growth. Otherwise, $\tilde{y}$ will decrease to zero as H_t gets higher with N_t^H in the case of positive long-run population growth. Furthermore, assuming that $\tilde{k}_0 > 0$, since $0 < \alpha < 1$, it is easy to see that

$$
\phi'(\tilde{k}_t) = (1-\alpha)\left(\frac{H_t}{H_t^Y}\right)^{\alpha} \Theta_t \alpha \tilde{k}_t^{\alpha-1} > 0 \; \forall k_t > 0,
$$
$$
\phi''(\tilde{k}_t) = (1-\alpha)\left(\frac{H_t}{H_t^Y}\right)^{\alpha} \Theta_t \alpha (1-\alpha) \tilde{k}_t^{\alpha-2} < 0 \; \forall k_t > 0.
$$

so that the dynamics of $\tilde{k}$ exhibits concavity and thus guarantees the existence of a fixed point $\tilde{k}^* = \tilde{k}_t = \tilde{k}_{t+1}$ and is globally stable. Nevertheless, the trajectory can be non-monotonic due to our assumptions of a spillover effect of technology, endogenous fertility, and R&D-based endogenous growth. In fact, as Galor and Ryder (1989) have pointed out, multiple equilibria may exist despite the concavity of both the production and utility function. Our model falls into this category. ■

Proposition 7. The steady-state $\tilde{k}^*$ is increasing in the strength of the technological spillover effect ν.

Proof. To show this, it is sufficient to show that (i) equation (58) is higher than (57) for all values of k_t, which is obvious since $A_{t-1}^{\nu-1} > 0 \; \forall t$; and (ii) k_{t+1} is an increasing function of ν, which is true since

$$
\frac{\partial \tilde{k}_{t+1}}{\partial \nu} = \Omega A_{t-1}^{\nu-1} \ln(A_{t-1}) > 0
$$

where $\Omega \equiv \frac{\beta \theta x_t}{(1+\beta+\gamma)(1+x_{t+1})n^*(1-\rho n^*)(1+g_t^A)^2} > 0$. ■

It is necessary to note that as income rises with the increase in technology, population growth will approach zero (that is, one child per person). Furthermore, as is well known in the endogenous growth literature where $\phi < 1$, which represents the "standing-on-shoulders" effect, is strong but not enough to guarantee sustained growth, and therefore the variety growth rate, in the long run, should be zero (Growiec, 2006). Nevertheless, convergence to zero growth is very slow. Hence, the economy still exhibits an asymptotic balanced growth path where g_t^A grows with the rate of the population. At the same time, the ratio of researcher labor service over total high-skilled labor approaches a constant. We provide a numerical illustration to show the convergence of the capital-effective labor ratio in Fig. 4.

Initially, fertility rates are high thanks to improvement in earnings of the low-skill cohorts, who enjoy the technological spillovers. Furthermore, as the stock of knowledge is sufficiently low, education spending is productive enough to allow children to become high-skilled. As a result, we see a surge in the high-skilled population ratio. The increase in the high-skilled population means that the R&D sector also has more research labor, which enhances the machine variety growth mechanism. Technological growth and capital accumulation (savings) are strong enough to overcome population growth.

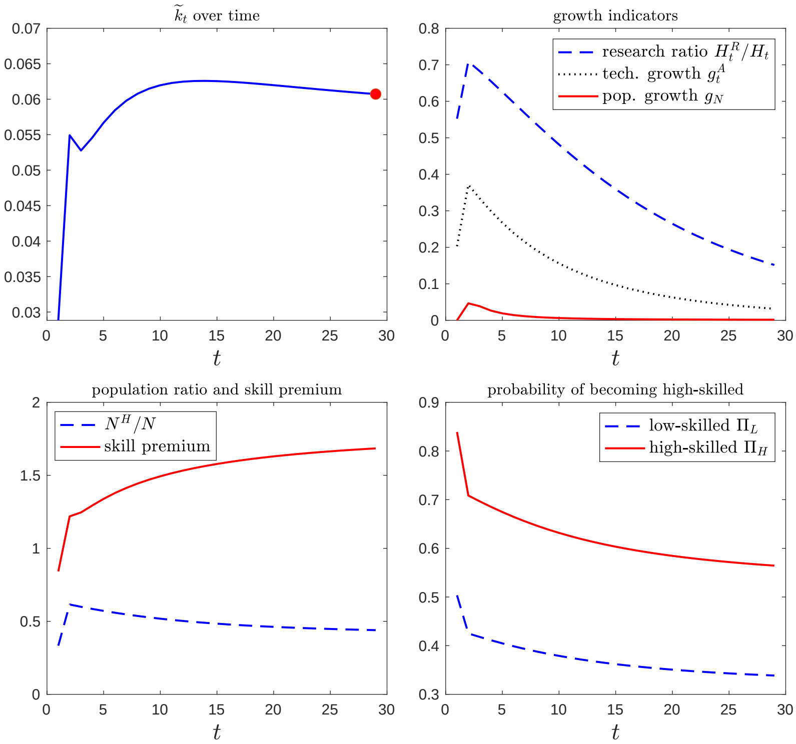

Figure 4: Numerical Simulation with $\tilde{k}_0 < \tilde{k}^*$.

However, after an intense growth period, the economy enters a phase where it becomes more difficult for the general population to keep up with technological progress. First, the high-skilled population ratio reduction implies that the labor market gets more competitive. As machines become more intelligent and capable of doing more tasks, parents' education spending and intergenerational transmission become more critical. Consequently, households (whether rich or poor) must sacrifice their children's quantity to spend on their education, which leads to a gradual decrease in fertility rates, which means that future generations continue to have fewer high-skilled individuals (especially in R& D) and therefore, growth slows down. Second, since machine variety (or the state of technology) augments labor productivity, individuals earn a higher income, which raises the opportunity cost of childbearing, causing them to give birth to fewer babies. Third, it is essential to note that the wage rate paid to researchers resembles what is paid to the final-good manufacturing high-skilled workers, so it increases with technology. As a result, when the supply of high-skilled labor decreases, researchers are getting more expensive. In contrast, their productivity decreases over time due to the "standing-on-shoulders" effect, which forces innovators to hire fewer researchers, thus inducing lower machine variety growth. On the demand side, the R&D sector's profit also relies on the market scale (in this case, the population), so when the population decreases, the profit from new inventions is lowered. Hence firms have fewer incentives to innovate.

As the growth rate of machines variety and population growth approach their respective steady states, capital per effective labor also slows down. We can see a sharp increase in the capital-effective labor ratio at the beginning when population and variety growth rates are high. After that, however, growth becomes slower as population growth approaches zero. Interestingly, there is a prolonged period of overshooting where the capital-effective labor ratio stays at a higher equilibrium state. Here, innovators' profit is still marginally positive, incentivizing them to hire research labor, thus making the ratio $H_t/H_t^Y > 1$. When variety growth g_t^A and innovation profit π_t^A tend to zero at the asymptote, the H_t/H_t^Y ratio tends to 1, and the capital-effective labor ratio gradually converges to the lower (but more stable) steady state. Intuitively, since the variety growth function exhibits increasing return to scale, so long as there is a proportion of the population hired as researchers in R&D, the economy generates a large enough labor augmenting productivity to stay at a high steady state. However, as this "extra growth" phases down when all growth variables reach zero growth asymptotes, the capital-effective labor ratio returns to the zero-growth equilibrium state. Furthermore, other distortable dynamics, such as those related to the skill-based population ratio (x_t), can contribute to the non-monotonic convergence of the capital-effective labor ratio towards its steady state.

4 Numerical Simulations for Japan

4.1 Empirical Evidence

Before performing simulations, as a way to concretize our theoretical framework, let us look at the empirical evidence for Japan. We focus mainly on fertility rates, skill premiums, and the increasing stock of robots. There are many reasons why we chose Japan as a case study. First, it is one of the leading pioneer countries in robotic development, and the data cover a relatively long period from 1980 until today. Second, Japanese society is experiencing rapid aging and a reduction in the birth rate, putting it at the forefront of other developed countries. Therefore, studying and understanding the changes in the past years in Japanese society during the wake of robots and artificial intelligence can bring about a lot of valuable lessons. Finally, we found that the movement of the skill premium in Japan has been peculiar (Hara et al., 2014). Noticeably, the gap between high-skilled and low-skilled labor has been on a narrowing trend from the 1980s to 2005, only to pick up a rising trend in recent years. For comparison, in the US, robotic production and birth rate declines resemble Japan's trends (albeit to a lesser magnitude). Still, the skill premium has kept widening during the same time frame. It implies that the spillover effects of technological advances in the US may not be that strong.

Figure 5 plots the relationship among our four main variables of interest: the birth rates (per 1000 people), log of the stock of industrial robots per worker (called robot density), log of annual income for a household, and the university enrollment rate (which is a proxy to measure the high/low-skilled labor ratio), using annual data from 1980 to 2015. Table 3 reports the detailed regression results.

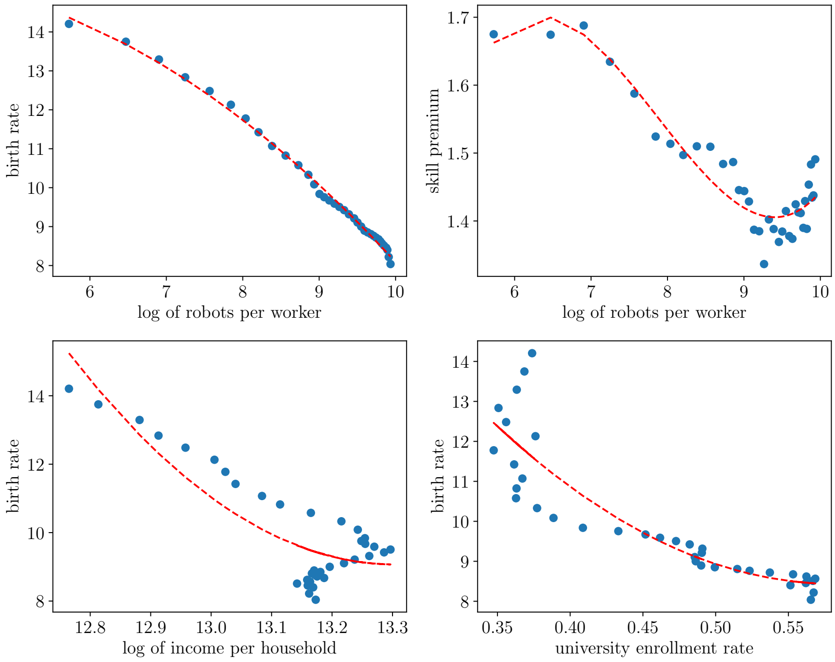

Figure 5: Birth rates, robot stock, income & skill-labor ratio for Japan. Dashed lines are fitted trends.

Although the observations are not so large, we can confirm significant correlations among these variables. Specifically, the birth rate tends to decrease with a rise in robots per worker, household income, and the high-skilled population ratio. As laid out in the theoretical analysis, the correlation between reducing the birth rate and robot stock can be reciprocal. The economy produces more (and smarter) machines to encounter the labor shortage and take advantage of capital stock. At the same time, a rise in robot stock increases productivity and labor income, which results in a further decline in fertility. As we discussed, this effect results from the children's quantity-quality trade-off, which is confirmed by a negative correlation between birth rate and income per household. Furthermore, during this period, Japanese society also observed a transition in terms of human capital, where more and more children progressed to university and pursued higher education. The increased portion of skilled workers also contributed immensely to the rise of industrial robot design and production. Nevertheless, the more interesting character lies in the dynamics of skill premium with respect to robot density, as it appears to exhibit a U shape.

We know that industrial robot production and density in Japan have been increasing yearly, similar to many advanced economies. However, unlike many other countries (such as the US or UK), skill premiums in Japan fell during the 80s and 90s, only to rise in recent years. This is perhaps due to substantial spillover and absorption effects. As documented by Hollanders and Ter Weel (2002), during this period, a positive change in R&D employment leads to positive changes in both high-skilled and low-skilled workers, representing a spillover effect. Furthermore, among other OECD countries at the time, Japan observed the second largest percentage increase in labor skills (Table 1 in Colecchia and Papaconstantinou (1996)), which highlights a strong absorption effect where labor employed in the low-skilled sector can learn and utilize a significant portion of the technology spillover. A rise in the supply of high-skilled workers also contributes to the decline in skill premiums during this period. That was when the number of machines competing with human labor was still relatively small, and education was able to ensure quality. As time progresses, when the stock of machine and their capability increases, and the availability of (higher) education opportunity, the competition among children get tighter, while the returns to higher education start to decrease. This can result in a situation where there are lower savings to fuel capital growth Horii et al. (2008), and it is more difficult to become a high-skilled worker. In other words, the truly high-skilled individuals become scarcer. At the same time, more university graduates might end up working in the low-skilled sector, which results in a gradual increase in the skill premium.

4.2 Parameters

In this section, we calibrate the parameters to match Japan's labor and demographic development from 1975 forward, assuming that one period of the model is equivalent to 30 years in real life. The choice of the starting time is due to the availability of data and the fact that the application of industrial robots marked a boom in the 1980s, both in the US (Acemoglu and Restrepo, 2018) and Japan (Kumaresan and Miyazaki, 1999). Since the focus of this paper is to investigate the relations among fertility, skill premium, and the progression of automation technologies, we calibrate the parameters in a way to match the developments of the following key variables: the graduates share N_t^H/N_t (used as a proxy to measure skilled population), the machine sector growth g_t^A, population growth, and skill premium.

First, let us focus on some specific data on Japan. For skill premium, we use the Annual Earnings by Educational Background published by MEXT in the Survey on Wage Structure. The skill premium is calculated as the ratio of the annual wage rate earned by males with a university degree or higher versus people with only a high school diploma or less, average for different age cohorts. The initial ratio of high-skilled over the population is calibrated based on the Percentage of high-school graduates who proceed to university or college, published by the Statistic Bureau of Japan. The data on robot production is taken from the "domestic use (units) of industrial robots", reported by Japan Robot Association's Annual Report (JARA) dating back to 1989. Furthermore, since A_t is regarded as machines' variety, we consider the most relevant data for calibration is the Industrial Design Patent Applications for Japan (both Residents and Non-Residents), published by World Intellectual Property Organization (WIPO). Since the model assumes a spillover affects ϕ on past inventions, we calculate that the number of new patents applied yearly only adds $(1 - \phi)$ of its value to the stock of existing patents.

We now turn to the calibration of demographic factors. To get a reasonable fertility rate, we rely on the maximum fertility differential induced by the model without the child-rearing cost, which can be calculated to be $(n_t^L/n_t^H)_{max} = 1/(1-\eta)$. This indication for Japan is calibrated based on data provided by Uchikoshi (2018), where the mean fertility was 1.96 with a standard deviation of 0.86. Thus, we assume that the maximum differential is around two standard deviations, equivalent to around 2.5636. Using this number, the value for η is 0.6099. In addition, the parameter γ governs the decision of fertility. We assume zero population growth in the long run, so the long-run fertility rate is set at 1. This is also necessary for the model's stability. As a result, the parameter γ is calibrated to be 0.3093.

Finally, we select some standard parameters. The child-rearing time cost is calibrated based on Guryan et al. (2008) to set it at 0.075. The subjective discount is set at $.99^{120}$, giving us a saving rate of around 20%. The default education $\bar{e}$ is based on de la Croix (2012) and is set at 0.012. Since the good cost of child-rearing ε needs to be smaller than $\bar{e}$ for the model to be meaningful, we set it arbitrarily at 0.01. These parameters matter most initially and do not affect the long-run fertility decision. The capital share α is set at 0.3, as in the literature. The data for other standard variables such as GDP per capita, fertility rates, and population follow internationally recognized statistics.

To start the economy, we need to set the initial values. We normalize the high-skilled population to 1 and accordingly select the low-skilled individuals to around 2, making the high-skilled population represent approximately 1/3 of the population. This is set to match the proportion of high-school graduates advancing to college or university in 1975. For technology, the initial stock is set to 1 (to avoid complications if $\nu < 1$), the knowledge spillover from past research ϕ is 0.7 (Prettner and Strulik, 2020), whereas the productivity of research sector δ is set to 0.5. The spillover effects of technologies on the low-skilled sector ν is set at 0.7, while the degree of absorption of such technologies is set to be 0.55. Calibration for the initial stock of capital $\tilde{k}_0$ is more tricky as skill premium is sensitive to this value. To reproduce the declining skill premium trend in Japan following the late 1980s, we have to assume that the initial capital-effective labor ratio $\tilde{k}_0$ is higher than the long-run steady state. The detailed calibration of parameters and initial conditions is summarized in Table. 2.

4.3 Benchmark Simulations

The main results are shown in Fig. 6. Based on the first movement of the model, its realization fits the data very well. Overall, the model predicts that technology (represented by the number of patents filed) and the stock of industrial robots will rise and accelerate in the latter half of the century. The proportion of the high-skilled population is expected to rise, which follows the data, where more than 50% of high school graduates will proceed to the university. However, as the robot stock increases, the proportion of the skilled population is expected to decrease and finally settle at 35.67% in the long run. This trend is because the probability of becoming high-skilled is decreasing in both

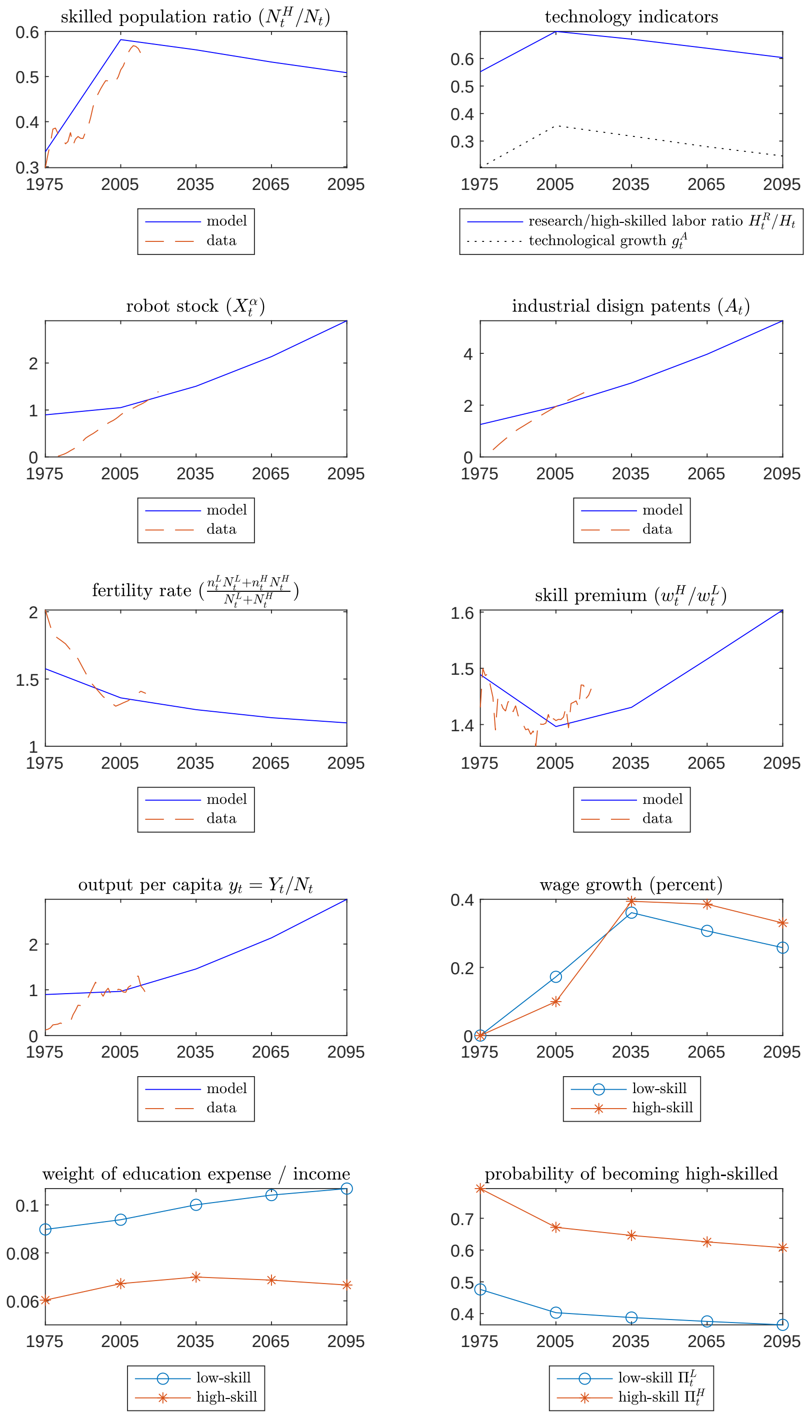

Figure 6: Benchmark Simulation Results for Japan.

cohorts. This reflects the race between education and machine. The high-skilled wage equation has the term $A_{t-1}+\triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}} < A_{t-1}+\triangle A_{t-1} = A_t$, implying that the wage paid for high-skilled increases with technology but the amount gained is smaller. Our formulation asserts that the quality of education spending tends to lag behind technological progress. Putting differently, even though the ratio of university graduates may increase in the future, as households keep education expenses high, the number of those who truly win the race against the machines will be lower.

We now turn to skill premiums. As documented in Japanese data, the model verifies that skill premium will decrease initially. This is explained by a surge in the high-skilled population, which dragged the wage for high-skilled down due to congestion. On the other hand, low-skilled workers enjoy technological spillover and see their productivity increase. With that picture, the wage gap improved. However, after the first period, the capital-skill complementary effect dominates once the high-skilled population settles down to the balanced growth path. The model predicts that the skill premium may rise again, with a steady-state value of around 1.87. Our simulation shows that at the beginning, during the transition period, and when the robot stock (as well as technology) was sufficiently low, the wage growth in the low-skilled sector can be higher than that of the high-skilled sector. However, over time, the spillover effects weaken, driving down the wage improvement in the intensive margin for the low-skilled sector. Meanwhile, the high-skilled population becomes scarcer, getting more rewards from robotic development, so the skill premium is more likely to widen.

We now turn to the individuals' side of the story. To compensate for the education expenses, we can expect a monotonic decrease in the fertility rate. The last graph shows that as society races against machines, the burden of education is heavier for households with lower skills, where educational expense takes up a higher proportion of their income. The absolute number required for education is the same across cohorts. Still, due to the income disparity, education is more expensive for poorer households, and one alleviation is to give birth to fewer children. The overall birth rate per woman in Japan from 1975–2005 was 1.58 children and will keep on decreasing, which was also predicted by the model. The declining population also shrinks the demand for new products, making technological advances less explosive than before.

Overall, the model's dynamics so far are consistent with the data in Japan. The simulation result implies that the spillover effect from the high to the low-skilled sector in Japan is considerably high, perhaps due to the culture of on-the-job training and lifetime employment (Hashimoto, 1979), even in labor-intensive sectors. However, technological advancement may tighten the strength of spillover effects as the number of necessary skills accumulates. Furthermore, in the future, the demand for high-skilled individuals will increase in order to manage more sophisticated technologies. There will be tighter competition between university graduates (reflected by the decreasing Π^H) so that being a college/university graduate will see fewer chances of becoming a high-skilled. Although the skill premium has remained the same since the beginning of the twenty-first century, it may increase again as companies need to pay more for the genuinely high-skilled and the spillover weakens. According to a MEXT dataset, we have observed that such a trend is being materialized in Japan. Fertility is expected to continue declining, which necessitates robot production. On the other hand, due to lower population growth, both the demand and supply for scientific research may see lower growth in the future. At least, thanks to robotic development, we expect the Japanese economy will grow considerably.

4.4 Counterfactual Simulations: Spillover Effects

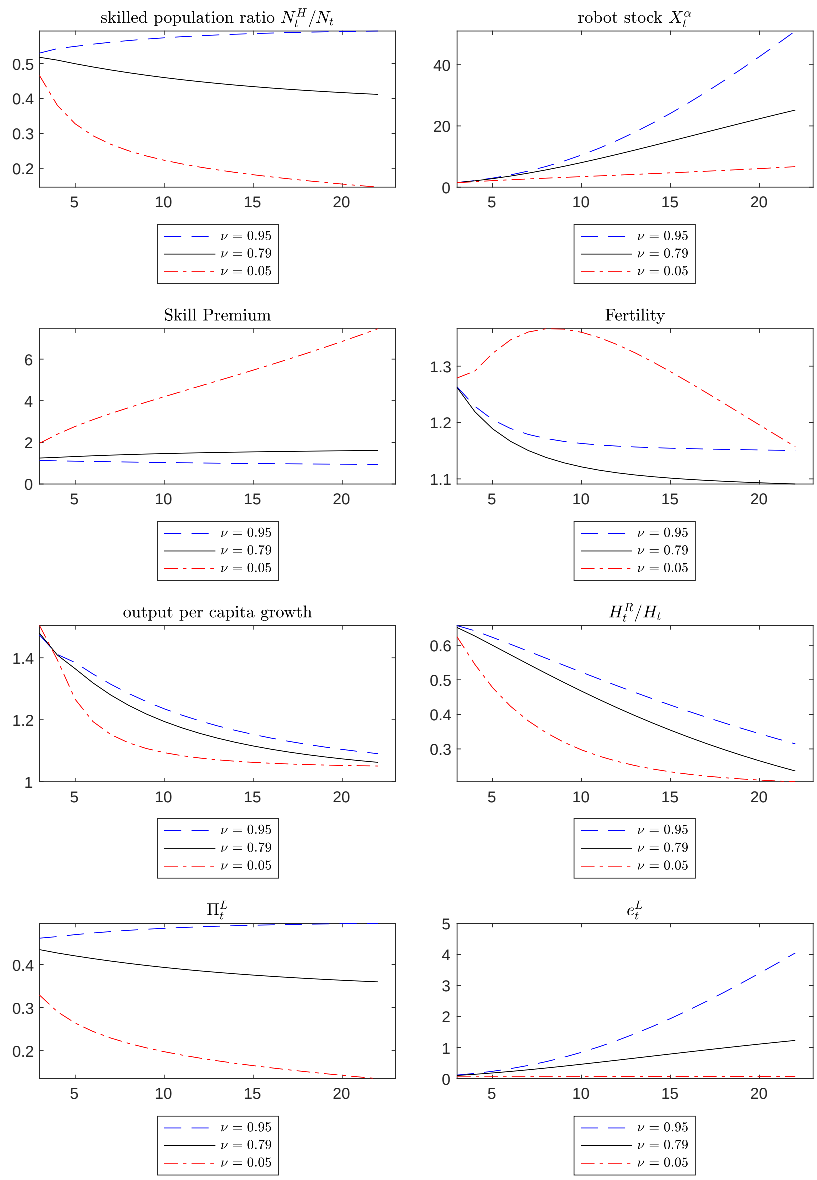

Figure 7: Counterfactual Simulations

The novel contribution of this paper lies in the assumption of spillover effects on the low-skilled sector, which is different from other papers such as Prettner and Strulik

(2020), where there is no trickle-down effect from the high-skilled sector to the lower-skilled sector and skill premium is predicted to rise indefinitely. As a result, we simulate two more scenarios in addition to the benchmark case, a high spillover ($\nu = 0.95$) and a low (almost no) spillover $\nu = 0.05$, then compare their dynamics.

First, we examine the case of low ν. We observe that, in a low spillover economy, without substantial improvement in wage or productivity for the low-skilled sector, these workers are left behind significantly, and it is impossible for them to increase educational investment for their children. With technology advancing, the chance for the generations of these low-skilled parents to become high-skilled and compete with machines is getting slimmer. Since educational expense cannot be improved, the low-skilled family compensate for the altruistic taste by having more children, further driving down the skilled population of the whole society. In this economy, high-skilled people benefit the most from technological progress. The fact that skill premium rises steadily in this economy shows that the high-skilled population captures most of the wealth from the economy. It is also essential to pay attention to the stock of robots. Since machines are produced using only capital, their availability determines how many robots can be made. With K accumulated from savings of both low-skilled and high-skilled cohorts, the low-spillover economy fails to utilize savings from low-skilled parents and therefore exhibits very slow growth in the robot sector. Furthermore, since children from low-skilled families are likely to be stuck at that level due to inadequate education investment from the parents, this economy sees a sharp decline in the high-skilled population ratio. As a result, the research service ratio declines, further contributing to this stagnancy.

On the other hand, the economy with a high spillover effect shows a brighter picture. Thanks to the spillover of technology from the high-skilled sector, the low-skilled sector sees significant wage improvement, enabling them to spend more on children's education than in other cases. As a result, the probability of their children becoming high-skilled also gradually increases, contributing significantly to the high-skilled population counts. Skill premium is also kept low since the difference in skills between high-skilled and low-skilled is closing down thanks to high spillovers of technology. We can see a positive cycle where the rise in robots and patents fosters growth and innovation, which can be transferred even to the low-skilled sector, albeit one period slower. In turn, children from low-skilled parents are more likely to proceed to become high-skilled workers. Increased savings generated by low-skilled parents help fuel more robotic development and so on.

In general, an economy with high spillover effects benefits social welfare and economic growth in many aspects more than those with limited spillovers. It is also essential for the survival of the economy since the source of savings comes from both low-skilled and high-skilled cohorts, and if the high-skilled population captures more and more wealth in the economy, savings from the low-skilled cannot contribute to sustainable growth. In other words, wage inequality due to a sufficiently low spillover effect can constrain economic growth. Although we did not discuss government interventions in this model, it is natural to suggest that governments should deliver policies that can strengthen the technological spillover effects. This support or subsidies can be in the form of skills upgrading (learning or training). In such a way, low-skilled workers have the opportunity to take advantage of the inexpensively available spillover technologies to improve not only their own lives but also the lives of their future generations.

5 Conclusion

In this paper, we use a Diamond-type OLG model with endogenous R&D growth to analyze the relationship between fertility, machines development, and their implications on skill premium. Unlike other papers where the low-skilled sector is assumed to be idle – that is, they gain nothing from the technological process – we consider a model with spillover effects from the high-skilled sector. This assumption is reasonable in that most technological advances are concentrated in the semiconductive industries, which fuels both robotic development and smart devices, of which prices have decreased dramatically and become more assessable and affordable to all persons, regardless of skill type. Given that most software can be used and learned freely online, we think allowing technological spillovers holds substantial merits. Such effects, however, are discounted and governed by parameters controlling the degree of absorption and the strength of spillover. Interesting results emerge from this setup. Our framework is also rich enough to include endogenous fertility and education expenses. We deem these features essential because individuals react to substantial changes in robot development. Parents tend to have fewer babies to invest more human capital in their children, who must compete with machines. On the other hand, fertility rates decide the progression of technological growth by determining the skilled population ratio and how much research labor service can be put into robot development.

To demonstrate these interactions, we calibrate appropriate parameters tailored to the Japanese economy. The simulations' dynamics are consistent with the development of Japanese demographics and other important indicators. It predicts that with the rise of robotic and smart machines, the labor market will get more competitive, and it will be more challenging for an individual to become a high-skilled worker. With decreasing population, a large stock of capital, and patents, the model agrees that Japan's supply of industrial robots and machines will continue to rise rapidly. As a result, the skill premium tends to rise again after a fall due to congestion from a large intake of high-skilled when the stock of knowledge is still sufficiently low. In the race between machines and education, fertility rates are expected to continue falling so parents can spend more on their children's education. Furthermore, we also emphasize the importance of considering spillover effects. A high spillover effect – or high knowledge transfer – helps close the gap between high-skilled and low-skilled, improving low-skilled workers' productivity, which can generate more savings and high-skilled workers to fuel economic growth. On the other hand, a low spillover effect forbids the knowledge generated in research to be tricked down to the low-skilled sector. Without any productivity improvements, the low-skilled sector falls behind, skill premium increases, and the issue of "low-hanging fruits" exacerbates. In this economy, machine production is also hampered due to a shortage of high-skilled population and capital stock, and we end up with substantially slower growth.

Several extensions are possible. Since fertility and spillover effects play important roles in our model, a natural extension is to incorporate some government policies that can affect fertility decisions (Fanti and Gori, 2014) or subsidize innovation in the case of high spillover while taxing innovation in the case of low spillover (Jones and Williams, 2000). Another potential extension often seen in an OLG framework is to incorporate the model in the context of population aging. Tran (2022) shows that education is essential in determining old workers' productivity. Since education is endogenized in our paper, it will be interesting to see how education and fertility affect workers' retirement decisions once we allow individuals to work in their second period of life, which may affect growth in the long run. This possibility can be well incorporated with Irmen (2021), where the author explores the relationship between aging and automation. Another extension concerns the innovation process. We can consider a gestation lags akin to Kitagawa and Shibata (2005) where innovation takes time (more than one period) to incubate and reach maturity, or a longer duration of patent-holding (Chou and Shy, 1993). In our model, in exchange for ease in analysis, innovation mechanically takes only one period to realize, and monopolistic power from such an invention also mechanically vanishes after one period, thus may exhibit some restrictions. Finally, in our model, parents are assumed to have decisive power over children's education. One can relax this assumption by incorporating endogenous school or occupation decisions, such as Kimura and Yasui (2007) or Prettner and Strulik (2020) where the authors consider college choice (to be a high-skilled worker) of either 1 or 0 where the pursuit of higher education causes a disutility for individuals. It is also reasonable to consider a skill acquiring opportunity costs for individuals who want to pursue higher education as in Morimoto and Tabata (2020).

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A Household's Problem

Rewrite the problem as

$$\max_{s_t^j, n_t^j, e_t^j} U_t^j = \ln(w_t^j(1-\rho n_t^j) - s_t^j - e_t^j n_t^j - \varepsilon n_t^j) + \beta \ln(R_{t+1} s_t^j) + \gamma \ln \left(\mu^j \left(\frac{e_t^j + \bar{e}}{A_t} \right)^\eta n_t^j \right).$$

The First order conditions (FOCs) are

$$\begin{aligned}
(s_t^j) &: -\frac{1}{c_t^j} + \frac{\beta}{s_t^j} = 0 && \Leftrightarrow s_t^j = \beta c_t^j, \\
(n_t^j) &: -\frac{\rho w_t^j + e_t^j + \varepsilon}{c_t^j} + \frac{\gamma}{n_t^j} = 0 && \Leftrightarrow n_t^j = \frac{\gamma c_t^j}{\rho w_t^j + e_t^j + \varepsilon}, \\
(e_t^j) &: -\frac{n_t^j}{c_t^j} + \frac{\gamma\eta}{e_t^j + \bar{e}} = 0 && \Leftrightarrow e_t^j = \frac{\gamma\eta c_t^j}{n_t^j} - \bar{e}.
\end{aligned}$$

From the last 2

$$\begin{aligned}
\frac{n_t^j}{c_t^j} &= \frac{\gamma}{\rho w_t^j + e_t^j + \varepsilon} = \frac{\gamma\eta}{e_t^j + \bar{e}}, \\
e_t^j &= \frac{\eta(\rho w_t^j + \varepsilon) - \bar{e}}{1-\eta}.
\end{aligned}$$

Observe that

$$e_t^j = 0 \Leftrightarrow w_t^j \leq \frac{\bar{e} - \eta\varepsilon}{\eta\rho}.$$

Plugging back to the FOC of n_t^j yields

$$n_t^j = \begin{cases} \dfrac{\gamma(1-\eta)}{\rho w_t^j + \varepsilon - \bar{e}} c_t^j & \text{for } e_t^j > 0 \ , \\ \dfrac{\gamma}{\rho w_t^j + \varepsilon} c_t^j & \text{for } e_t^j = 0 \end{cases}$$

The optimal decisions for an agent of type j are

$$\begin{aligned}
c_t^j &= \frac{1}{1+\beta+\gamma} w_t^j, \\
s_t^j &= \frac{\beta}{1+\beta+\gamma} w_t^j, \\
d_{t+1}^j &= \frac{\beta}{1+\beta+\gamma} R_{t+1} w_t^j.
\end{aligned}$$

If $w_t^j \leq \dfrac{\bar{e} - \eta\varepsilon}{\eta\rho}$,

$$\begin{aligned}
e_t^j &= 0, \\
n_t^j &= \frac{\gamma w_t^j}{(1+\beta+\gamma)(\rho w_t^j + \varepsilon)}.
\end{aligned}$$

If $w_t^j > \dfrac{\bar{e} - \eta\varepsilon}{\eta\rho}$,

$$\begin{aligned}
e_t^j &= \frac{\eta(\rho w_t^j + \varepsilon) - \bar{e}}{1-\eta}, \\
n_t^j &= \frac{(1-\eta)\gamma w_t^j}{(1+\beta+\gamma)(\rho w_t^j + \varepsilon - \bar{e})}.
\end{aligned}$$

Since the constraint is nonlinear (due to the existence of the $n_t^j e_t^j$ term), we need to check the definiteness of the second-order Hessian matrix for sufficient conditions.

Sufficient Conditions

The Jacobian of the First-order derivatives of U_t^j w.r.t s_t^j, n_t^j, e_t^j is

$$\mathbf{J} = \begin{pmatrix} \dfrac{\partial U_t^j}{\partial s_t^j} \\ \dfrac{\partial U_t^j}{\partial n_t^j} \\ \dfrac{\partial U_t^j}{\partial e_t^j} \end{pmatrix} = \begin{pmatrix} -\dfrac{1}{w_t^j(1-\rho n_t^j) - s_t^j - e_t^j n_t^j - \varepsilon n_t^j} + \dfrac{\beta}{s_t^j} \\ -\dfrac{\rho w_t^j + e_t^j + \varepsilon}{w_t^j(1-\rho n_t^j) - s_t^j - e_t^j n_t^j - \varepsilon n_t^j} + \dfrac{\gamma}{n_t^j} \\ -\dfrac{n_t^j}{w_t^j(1-\rho n_t^j) - s_t^j - e_t^j n_t^j - \varepsilon n_t^j} + \dfrac{\gamma\eta}{e_t^j + \bar{e}} \end{pmatrix}.$$

We can construct a second-order Hessian matrix as follows

$$\mathbf{H} = \begin{pmatrix} -\dfrac{1}{(c_t^j)^2} - \dfrac{\beta}{(s_t^j)^2} & -\dfrac{\rho w_t^j + e_t^j + \varepsilon}{(c_t^j)^2} & -\dfrac{n_t^j}{(c_t^j)^2} \\ -\dfrac{\rho w_t^j + e_t^j + \varepsilon}{(c_t^j)^2} & -\dfrac{(\rho w_t^j + e_t^j + \varepsilon)^2}{(c_t^j)^2} - \dfrac{\gamma}{(n_t^j)^2} & -\dfrac{w_t^j - s_t^j}{(c_t^j)^2} \\ -\dfrac{n_t^j}{(c_t^j)^2} & -\dfrac{w_t^j - s_t^j}{(c_t^j)^2} & -\dfrac{(n_t^j)^2}{(c_t^j)^2} - \dfrac{\gamma\eta}{(e_t^j + \bar{e})^2} \end{pmatrix},$$

where

$$c_t^j = w_t^j(1-\rho n_t^j) - s_t^j - e_t^j n_t^j - \varepsilon n_t^j.$$

Exploiting the FOCs, the Hessian can be reduced to

$$\mathbf{H} = \begin{pmatrix} -\dfrac{(1+\beta)\Phi^2}{\beta(w_t^j)^2} & -\dfrac{\gamma\Phi}{w_t^j n_t^j} & -\dfrac{n_t^j\Phi^2}{(w_t^j)^2} \\ -\dfrac{\gamma\Phi}{w_t^j n_t^j} & -\dfrac{\gamma(\gamma+1)}{(n_t^j)^2} & -\dfrac{(1+\gamma)\Phi}{w_t^j} \\ -\dfrac{n_t^j\Phi^2}{(w_t^j)^2} & -\dfrac{(1+\gamma)\Phi}{w_t^j} & -\dfrac{\gamma\eta(\gamma\eta+1)}{(e_t^j + \bar{e})^2} \end{pmatrix},$$

where

$$\Phi \equiv 1 + \beta + \gamma.$$

Evaluating the three leading principal minors leads to

$$|\mathbf{H}_1| = -\frac{(1+\beta)\Phi^2}{\beta(w_t^j)^2} < 0,$$

$$|\mathbf{H}_2| = \begin{vmatrix} -\dfrac{(1+\beta)\Phi^2}{\beta(w_t^j)^2} & -\dfrac{\gamma\Phi}{w_t^j n_t^j} \\ -\dfrac{\gamma\Phi}{w_t^j n_t^j} & -\dfrac{\gamma(\gamma+1)}{(n_t^j)^2} \end{vmatrix} = \frac{\gamma\Phi^3}{\beta(w_t^j n_t^j)^2} > 0,$$

$$|\mathbf{H}_3| = \begin{vmatrix} -\dfrac{(1+\beta)\Phi^2}{\beta(w_t^j)^2} & -\dfrac{\gamma\Phi}{w_t^j n_t^j} & -\dfrac{n_t^j\Phi^2}{(w_t^j)^2} \\ -\dfrac{\gamma\Phi}{w_t^j n_t^j} & -\dfrac{\gamma(\gamma+1)}{(n_t^j)^2} & -\dfrac{(1+\gamma)\Phi}{w_t^j} \\ -\dfrac{n_t^j\Phi^2}{(w_t^j)^2} & -\dfrac{(1+\gamma)\Phi}{w_t^j} & -\dfrac{\gamma\eta(\gamma\eta+1)}{(e_t^j+\bar{e})^2} \end{vmatrix} = -\frac{\Phi^5}{\beta(w_t^j)^4} < 0.$$

Since the three leading principal minors of $\mathbf{H}$ alternate in signs, starting with $\mathbf{H}_1 < 0$, the second-order Hessian matrix $\mathbf{H}$ is negative definite. The set of solutions obtained at the FOCs is indeed a maximizer.

B Proof of Proposition 3

Proof. Eq. (43) can also be written simply as

$$\triangle A_{t-1} = \delta A_{t-1}^{\phi}\left[N_t^H(1-\rho n_t^H)\right] - \frac{\left[A_{t-1} + \triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}}\right]}{\alpha^{\frac{1}{1-\alpha}}\delta A_{t-1}^{\phi}}. \tag{59}$$

Let $J_t = A_{t-1} + \triangle A_{t-1}\alpha^{\frac{\alpha}{1-\alpha}}$, we can rewrite (59) as a continuous function of J_t

$$\begin{aligned}\frac{J_t - A_{t-1}}{\alpha^{\alpha/(1-\alpha)}} &= \delta A_{t-1}^{\phi}\left[N_t^H(1-\rho n_t^H)\right] - \frac{J_t}{\alpha^{\frac{1}{1-\alpha}}} \\ \Leftrightarrow J_t &= \frac{\alpha^{\frac{\alpha}{1-\alpha}}\delta A_{t-1}^{\phi}\left[N_t^H(1-\rho n_t^H)\right] + A_{t-1}}{(1+\frac{1}{\alpha})}.\end{aligned} \tag{60}$$

By (37), $w_t^H = (1-\alpha)\left(\frac{\alpha}{R_t}\right)^{\frac{\alpha}{1-\alpha}} J_t$, and so from Lemma 1, $n_t^H \to n_{\max}^H$ if $J_t \to \frac{\bar{e}-\eta\varepsilon}{\eta\rho(1-\alpha)}\left(\frac{R_t}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}$ and $n_t^H \to n_{\min}^H$ if $J_t \to \infty$. In other words, for any value of $J_t \in \left(\frac{\bar{e}-\eta\varepsilon}{\eta\rho(1-\alpha)}\left(\frac{R_t}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}, \infty\right)$, the RHS of (60) has n_t^H determined, and it can be solved explicitly for J_t given A_{t-1}, thus yields a unique value of $\triangle A_{t-1}$.

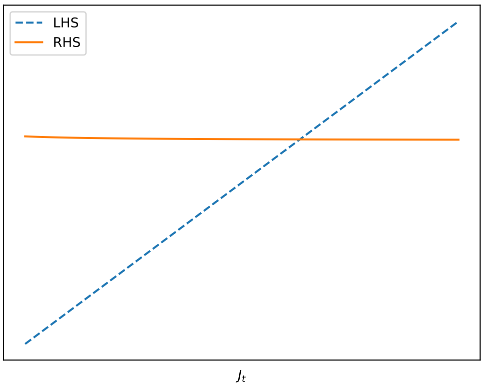

Figure 8: Illustration for Appendix B.

■

C Parameter Calibration

Table 2: Calibration for the Japanese economy.

Parameters	Value	Meaning
T	30	Period's length
α	0.3	Capital share
δ	0.5	Research productivity shifter
ϕ	0.7	Past inventions' spillover
β	0.99^{120}	Consumption weight
γ	0.3093	Altruistic weight
ρ	0.075	Child-rearing time cost
ε	0.01	Child-rearing good cost
$\bar{e}$	0.012	Default education
η	0.6099	Return to education
μ^L	3	Low-skilled's human capital transmission
μ^H	5	High-skilled's human capital transmission
ν	0.7	Spillover effect to low-skilled sector
θ	0.55	Degree of absorption
Initial Conditions		
A_0	1	Initial stock of machines' variety
K_0	0.5	Initial stock of capital
N_0^L	2	Initial low-skilled population
N_0^H	1	Initial high-skilled population

D Regression Results

Table 3: Correlation Testing for Figure 5

	Dependent variable:	
	birth_rate	skill_premium
	(1)	(2)
log(robots_per_worker)	-1.165^{***}	
	(0.085)	
log income	-1.228^{**}	
	(0.461)	
enrollment_rate	-4.580^{***}	
	(0.678)	
std_robots_per_worker^2		0.075^{***}
		(0.006)
std_robots_per_worker		-0.295^{***}
		(0.018)
Constant	38.653^{***}	1.670^{***}
	(5.580)	(0.012)
Observations	36	36
R^2	0.993	0.917
Adjusted R^2	0.993	0.912
Residual Std. Error	0.145 (df = 32)	0.027 (df = 33)
F Statistic	$1,567.013^{***}$ (df = 3; 32)	181.539^{***} (df = 2; 33)
Note:		*p<0.1; **p<0.05; ***p<0.01
		std_robots_per_worker is the positive standardization of robot density.