Firm Debt and Default over the Pandemic and Recovery *

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Abstract

We study the effects of a pandemic in an economy where firms differ in their productivity, capital, and debt. Firms, facing idiosyncratic shocks, finance investment using non-contingent debt and their default risk rises with leverage. Households share consumption risk given differences in their health and employment status. Healthy individuals may work but experience a higher risk of infection which increases with the number of ill individuals.

We show that a pandemic generates persistent aggregate dynamics. First, taking into account the distribution of health in the future, households reduce labor supply in an effort to restrain contagion. Decreases in consumption and employment in turn negatively affect firms' earnings in equilibrium. Highly leveraged borrowers become more likely to default, exit rises, and the number of producers falls. Continuing firms with lower earnings find it harder to finance investment, raising the dispersion of resources. Aggregate productivity falls endogenously. The interaction between households and firms propagates the impact of the pandemic through changes in their distributions.

We find that the recovery from a large shock that decreases household employment can be gradual and prolonged. As the pandemic ends, entry rises and the number of firms begins to return to its long-run level. However, entrants' growth is restricted by the loan rates associated with high leverage and their level of capital, relative to productivity. This implies that a pandemic is followed by a slow economic recovery characterized by a gradual improvement in aggregate productivity.

Keywords: heterogeneous firms, default risk, misallocation, pandemic, recovery

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1 Introduction

SARS-CoV-2 has created a pandemic that, beyond its enormous impact on health, has depressed economic activity around the world. In the US, households' attempts to reduce their risk of infection, alongside public health measures designed to curtail the spread of the disease, have brought a severe economic downturn. As restrictions on movement gradually suppress the contagion and vaccination programs are implemented, the health crisis is expected to end. Our attention is on the economic recession during a pandemic and the recovery that will follow.

In particular, we study the propagation of a pandemic shock in an economy with heterogeneity in both households and firms. Households differ in their health and employment status due to a contagious disease, and firms are subject to costly external borrowing that varies with individual default risk. This two-sided heterogeneity allows us to quantify the prolonged impact of a sudden rise in infection rates during and after the recession.

Firms in our model face persistent shocks to idiosyncratic productivity, given differences in their assets and leverage. Their investment is financed using one-period non-contingent debt which involves a risk of default. Large aggregate shocks that negatively affect firms' earnings lead to stagnant investment and excessive exit. The resulting changes in the distribution of firms lower aggregate total factor productivity (TFP), thereby amplifying an economic downturn. Thereafter, an economic recovery hinges on TFP growth which requires, in our setting, restoring the firm distribution. This is a gradual process as new firms' growth is restrained by loan rate schedules which rise with leverage. Such nontrivial changes in the distribution have an important role in determining the path of the aggregate economy following the resolution of a health crisis.

In our model economy, a large reduction in firms' earnings begins with households' responses to the pandemic. Individuals are different in their health and employment status, while pooling their consumption risk. Healthy individuals may work, in contrast to those with a disease, but they face a relatively higher risk of infection when working. In addition, the risk of infection increases with the share of the population that is infected. This leads to a *dynamic labor-leisure condition* that incorporates the change in the future value when an individual worker falls ill instead of staying healthy. Thus, the households' response to the pandemic critically depends on their expectation of the changes in infection rates and their impact on the distribution of health in the future. As a result, when the pandemic arrives, labor supply endogenously falls and hence aggregate production.

Following a pandemic shock in the model, we show that the responses of households and firms lead to a persistent recession with a slow recovery in the absence of policy interventions.

Specifically, when the model reproduces the observed patterns of infection in the early periods of the COVID pandemic in the US, aggregate output immediately decreases by about 4 percent and continues to fall further in the following periods until the infection rate reaches its peak. This u-shaped response of output slows down the economic recovery; its half-life rises to 8 quarters. Moreover, employment and consumption exhibit similar dynamics to those of output. At the same time, the gradual unraveling of the firm distribution delivers persistent decreases in aggregate productivity due to financial frictions and default risk. This further slows down the recovery later on. When a simple business support policy is implemented at the beginning of the pandemic recession, in addition, the model predicts a relatively larger recession while the decline in aggregate productivity less pronounced.

We argue that the households' intertemporal choices for consumption and labor supply shape the aggregate dynamics following the start of a pandemic. Households take into account expected changes in the distribution of health across individuals in the future periods; the expected evolution of infection rates affects current decisions. We highlight this point by contrasting the recession described above with another case of a pandemic shock. In the alternative case, the share of ill individuals immediately rises to 20 percent at impact and then monotonically declines over time. Given their anticipation of better economic and public health conditions, households raise the share of working individuals among its healthy members in the early stage of the recession. The economy-wide illness initially reduces the labor force, however, and hence aggregate output and employment fall and then gradually revert back to their pre-pandemic levels. In this case where the health crisis improves over time, the economic recovery is relatively rapid with a half-life of 3 quarters and consumption is countercyclical. This result is partly consistent with those in Buera et al. (2021) and Di Nola, Kaas, and Wang (2022). Our model, however, generates substantially different aggregate dynamics when the households' endogenous responses to a pandemic are considered and the patterns of infection are comparable to the data.

As employment falls with the start of the pandemic, the allocation of resources shifts across firms. The starting point for this is our assumption that a firm's capital serves as collateral for any loan it takes. Firms with more leverage face a greater probability of default, have less collateral, and find debt more costly. As earnings fall during the pandemic, highly leveraged firms become unable to refinance their debt. Default rates rise, increasing the costs of borrowing for continuing firms. These higher costs in turn reduce investment and further depress economic activities. In the cross-section, there is greater production in larger, less productive firms that operate with low levels of debt. Productive firms with little existing capital, in contrast, see a decline in their share of output or exit altogether, which worsens the allocative efficiency in the economy. Overall, aggregate productivity falls, amplifying the effects of household's reduction in labor supplied.

Financial frictions have an essential role in extending the downturn associated with a pandemic. First, they lead to exit and fewer firms in the economy. This lowers aggregate productivity in an economy where each firm has decreasing-returns-to-scale (DRS). Second, they increase resource misallocation across incumbents by raising the correlation between collateral and investment. This restricts borrowing and hence aggregate investment falls.

The reduction in the number of firms, in particular, implies a slow recovery after the health crisis. Increases in GDP and employment require both capital investment and a rise in aggregate TFP. The latter is driven by improvements in allocative efficiency across firms, which further relies on both increases in the number of producers and a higher correlation between firm-level productivity and investment. These changes, however, do not occur immediately in our model that reproduces the firm lifecycle patterns seen in the data. That is, entrants mature slowly given the rising marginal cost of borrowing. As a cohort of firms grows, their share of aggregate production gradually rises. Since risky lending slows growth in entrants, relative to a model with frictionless borrowing, the recovery of firm distribution is further delayed. Over this transition, as there are fewer firms, and the average firm operates at a smaller scale, aggregate productivity remains persistently below its long-run level. Hence, the economic recovery following the pandemic is sluggish because of the financial frictions that led to a rise in default during the recession and weak firm growth over the subsequent recovery.

Related Literature Since the outbreak of COVID-19 in early 2020, there are a number of papers studying the social and economic outcomes of the massive spread of a disease. In particular, studies including Eichenbaum, Rebelo, and Trabandt (2021) highlight the importance of the interaction between economic decisions and public health measures in quantifying the impacts of a pandemic and the effectiveness of policy responses.[1] Following the common approach in these studies, our model incorporates the key ingredients of the classic SIR model in the household side of an economy and reproduces the epidemic dynamics in the data.[2]

In particular, our paper contributes to the literature by looking at the propagation of a pandemic shock through the distribution of firms. While existing works, such as Glover et al. (2021), find the significant role of age and wealth heterogeneity across households in determining the distributional effects of a pandemic, we address questions about business investment and firm

[1] It is not possible to cite every paper in this rapidly growing research, so we restrict ourselves to highly influential works here. Alvarez, Argente, and Lippi (2021) consider a planning problem for analyzing the effects a lockdown policy, and Jones, Philippon, and Venkateswaran (2021) focus on the role of learning-by-doing when individuals are allowed to work from home.

[2] SIR means *Susceptible-Infectious-Recovered* status for an individual's health, originated from an epidemiological model. See Atkeson (2020) for a simple application of the model in the context of COVID-19.

default risk and focus on their role during an economic recovery.[3] This perspective is shared with Buera et al. (2021) and Di Nola, Kaas, and Wang (2022) by emphasizing the reallocation of resources across production units in the presence of financial frictions. To be specific, Buera et al. show that business shutdowns with a reallocative shock generate a persistent recession in an economy with rest unemployment and labor market frictions. Di Nola, Kaas, and Wang examine the effects of a rescue policy targeted for small businesses in a model with endogenous firm entry and exit. Our approach, on the other hand, additionally features households' endogenous responses to a pandemic shock, which leads to a prolonged recession with a time-varying distribution of firms.

Our paper is related to a large volume of works that use models with firm heterogeneity and financial frictions. Following the Great Recession, in particular, quantitative studies of production heterogeneity find that the real effects of financial shocks are substantial and persistent.[4] Most of these studies, however, abstract from the evidence on default risk, both in the cross-section and over time, by assuming that firms use risk-free financing only.[5] Notable exceptions are Khan, Senga, and Thomas (2016), Ottonello and Winberry (2020), and Gomes and Schmid (2021). These papers examine the aggregate dynamics of an economy in which firms are allowed to default on their external debt and the distribution of default risk varies over time.[6] Specifically, Khan, Senga, and Thomas look at the implications of a credit shock that raise firms' financing costs and default risk. Ottonello and Winberry, on the other hand, highlight the role of differences in default risk across firms in propagating monetary policy shocks. Lastly, Gomes and Schmid link the cyclical relationship of asset prices and leverage with aggregate volatility.

We complement this recently growing strand of research in three dimensions. First, our model features the endogenous margin of firm entry, which builds on a standard approach of studying firm dynamics in the literature.[7] This allows us to reproduce the cyclical changes in

[3]Our focus on firm default is motivated from the finding in Guerrieri et al. (2021) that firm exit amplifies the effect of a negative supply shock in a multi-sector model. Relatedly, Krueger, Uhlig, and Xie (2021) study the reallocation of consumption across sectors as a mitigation mechanism in equilibrium.

[4]Among others, see Khan and Thomas (2013) and Buera and Moll (2015) for the analysis of aggregate dynamics with resource misallocation arising from financial frictions. Buera and Shin (2013), on the other hand, study the persistent effects of financial and economic reforms in developing economies.

[5]For a systematic measurement of default risk and credit spreads among the US listed firms in corporate bonds markets, see Gilchrist and Zakrajsek (2012).

[6]At a stationary equilibrium, Corbae and D'Erasmo (2021) study the firm-level and aggregate implications of bankruptcy laws in the US. Gourio (2013) instead considers an environment with disaster risk and i.i.d. shocks across ex-ante identical firms.

[7]The production side of our model heavily builds on that in Khan, Senga, and Thomas (2016). On the other hand, incumbents in Ottonello and Winberry (2020) exhibit realistic firm-level dynamics, but new firms in ther model are exogenously born in each period. Gomes and Schmid (2021) introduce a free entry condition with stochastic investment costs, but the empirical patterns of firm entry are not targeted in their calibrated model.

firm entry rate and the characteristics of entrants in our model economy, consistent with the recent evidence documented by Sedlacek and Sterk (2017). Further, endogenous firm entry and exit lead to changes in the number of production units over transitional dynamics, which in turn affects the allocative efficiency of resources in an economy. This additional channel of resource misallocation responds differently to aggregate sources of fluctuations, as shown in Khan, Senga, and Thomas (2016). Second, and related to the previous point, we study the implications of a new aggregate shock in an otherwise standard heterogeneous-firm model, the spread of a disease. The pandemic shock affects the optimal decisions made by individuals that are differ in their health status. Such a negative shock endogenously lowers both consumption demand and labor supply in aggregate, which adversely affects firms' operation and decisions at the same time. The quantitative framework in this paper, therefore, offers an opportunity for us to assess the impact of the current pandemic on the production side of an economy, in comparison with those from conventional aggregate shocks. Lastly, we provide a new method of numerically solving a model with firm borrowing and default. In contrast to the grid-search method employed in previous studies, we utilize the first-order conditions (FOCs) derived from a firm's problem while accounting for potential kinks in its decision rules.[8]

This paper is also related to the recent empirical works that measure firm-level volatility and default risk. Among others, Baker, Bloom, and Terry (2022) look at the aggregate impact of uncertainty conditional on disasters, whereas Besley, Roland, and Van Reenen (2020) examine the loss in aggregate productivity arising from default risk at the firm level. In addition, Gourinchas et al. (2022) estimate a large increase in the number of business failures during a pandemic. Our quantitative analysis features the key elements of these studies while being consistent with the endogenous responses of households to a public health crisis.

2 Model

We first describe our model economy before the advent of the pandemic. In the *pre-pandemic economy*, identical households share their income risk as a large family of individuals. Heterogeneous firms, on the other hand, face idiosyncratic shocks and their decisions are subject to financial frictions and default risk. We then consider an outbreak of an infectious disease in the model by allowing persistent differences in health status among household members. Time is discrete, and markets are perfectly competitive.

[8] We generalize the idea of differentiability of a value function and first-order necessary conditions in Clausen and Strub (2020).

2.1 The Pre-pandemic Economy

2.1.1 Household

In the absence of an infectious disease, there is a unit measure of ex-ante identical individuals in each household. Each period, household members collectively decide how much to work, consume, and save. An individual household member's preference is represented by a constant-relative-risk-aversion (CRRA) utility function, $u(c, 1-h)$, where c is consumption and h is hours worked. Labor supply is indivisible, so all working individuals earn the same labor income at a given wage rate w. Markets are complete, and the household members share their consumption risk.[9] Let a be the household's total asset holdings at the beginning of a period.

In a given period, the household decides how many individuals will work and allocates consumption across its members. Let p be the fraction of household members randomly designated to work, and denote c^w and c^n respectively as the consumption allocated to each working and non-working individual. Then the household budget is given by,

$$pc^w + (1-p)c^n + a' \leq pwh + (1+r)a + \Pi_d, \tag{1}$$

where r is the real asset return and Π_d summarizes the lump-sum transfers between it and firms.[10] All relative prices and dividends vary over time, but we suppress time subscripts and use primes to denote variables in the future period.

Let $s \equiv \mu$ be the aggregate state of the economy, where μ represents the distribution of firms to be defined later. The household with $(a; s)$ solves the following recursive problem.

$$V^h(a; s) = \max_{c^w, c^n, p, a'} \left[pu(c^w, 1-h) + (1-p)u(c^n, 1) + \beta V^h(a'; s') \right] \tag{2}$$

subject to (1),

$c^w, c^n \geq 0$, $p \in (0, 1]$, and $s' = \Gamma(s)$,

where $\beta \in (0, 1)$ is the subjective discount factor and Γ is the mapping of the aggregate state over time.

We further assume that the utility function is logarithmic in consumption and leisure. An immediate result is that consumption is equalized between working and non-working individu-

[9] This is the indivisible-labor economy as in Rogerson (1988), which leads to a representative household with Arrow securities in a standard model of heterogeneous firms.

[10] We assume that the household owns all shares in firms. Hence, dividend payments from incumbent firms and initial capital costs for entrants are included in the aggregate term Π_d.

als, $c^w = c^n$. Moreover, the marginal value of consumption is linearly related with the wage rate in equilibrium. Let $C^h \equiv pc^w + (1-p)c^n$ and $N^h \equiv ph$ be the household's optimal consumption and labor supply in aggregate.

2.1.2 Firms

We now present the production side of the economy. A continuum of firms owned by the household face idiosyncratic shocks on their productivity.[11] Firms own their capital stock and hire labor to produce a homogeneous good. Conditional on their continuation to the next period, firms have two sources of funds for financing their investment: internal savings after production and external debt from competitive lenders. Debt contracts take the form of one-period discount loans, and due to the possibility of default, lenders offer a loan price schedule that varies with each firm's productivity and decisions. Defaulting firms immediately exit the economy with zero value, and those that do not default still face an exogenous risk of exit at the end of the period. Lastly, there is a fixed mass of potential entrants that decide whether to start operating by paying fixed costs. As exit is time-varying, the measure of firms engaged in production may change over time.

Incumbent Firms Each firm produces output using a DRS production technology, $y = z\epsilon f(k, n)$, where z is the exogenous TFP common across firms and ϵ is the idiosyncratic productivity. The firm's capital, $k \in \mathbf{K} \subset \mathbf{R}_+$, is predetermined and depreciates at the rate of $\delta \in (0, 1)$ in each period, while its labor input n can be flexibly adjusted. Idiosyncratic shocks on ϵ follow a Markov chain with $\epsilon \in \mathbf{E} \equiv \{\epsilon_1, \epsilon_2, \ldots, \epsilon_{N_\epsilon}\}$, $\pi_{ij} \equiv Pr(\epsilon' = \epsilon_j | \epsilon = \epsilon_j) \geq 0$, and $\sum_{j=1}^{N_\epsilon} \pi_{ij} = 1$ for $i = 1, 2, \ldots, N_\epsilon$.

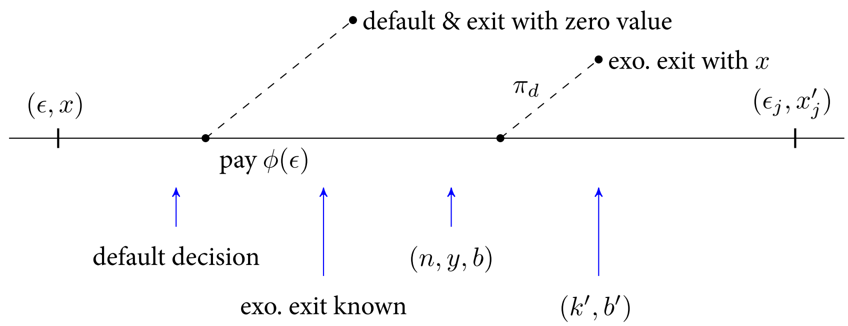
Prior to production in a given period, firms decide whether to default or not on their existing debt $b \in \mathbf{B} \subset \mathbf{R}$. When a firm defaults, it permanently exits the economy without repaying its debt, and lenders recover a fraction θ of the firm's capital.[12] Non-defaulting firms, on the other hand, must pay a fixed cost $\phi(\epsilon)$ in units of output to remain in the economy. Further, with probability π_d, these firms receive i.i.d. exit shocks before production takes place. As a result, a fixed measure of firms exogenously exit at the end of each period and their remaining assets are liquidated and transferred to the household.

Continuing firms then make intertemporal decisions of investment and borrowing, (k', b').

[11] The assumption of complete markets allows us to eliminate the terms for firm shares and dividend payments in the household budget (1).

[12] We assume a fixed recovery rate common across firms. For differences in business bankruptcy procedures and their recovery rates in the US, see Corbae and D'Erasmo (2021).

Figure 1 : Timing within a Period for Incumbents



Given these decisions, the discount loan price, $q(\epsilon, k', b'; s)$, determines the available loan size for these firms which may possibly default in the following period. As firms accumulate more physical and financial assets, the probability of default falls and thus $q(\epsilon, k', b'; s)$ approaches to the risk-free loan price $q_0(s) = (1 + r(s))^{-1}$. We further assume that firms are not allowed to issue equity for external financing.[13] Figure 1 illustrates the timing of exogenous shocks and firms' decisions in our model. Lastly, we summarize the distribution of incumbent firms by using a probability measure $\mu(\epsilon, k, b)$, which is defined on a Borel algebra generated by the open subsets of the product space, $\mathbb{S} \equiv \mathbf{E} \times \mathbf{K} \times \mathbf{B}$.

To formulate an incumbent firm's problem, we now define an individual state variable called *cash-on-hand*. Let $N^w(\epsilon, k; s)$ be the static labor demand of a firm with (ϵ, k, b).[14] Then the firm's cash-on-hand x, after production and debt repayment, is given by

$$x(\epsilon, k, b; s) = z\epsilon f(k, N^w) - w(s)N^w + (1 - \delta)k - b. \tag{3}$$

Given $(\epsilon_i, x; s)$ at the beginning of the current period, the firm solves the following recursive

[13] Equity financing can be simply introduced in our model by setting a negative bound for dividends, which does not significantly alter our quantitative results. Further, the assumption of non-negative dividends is common in recent macro studies of financial frictions, such as in Khan and Thomas (2013) and Ottonello and Winberry (2020).

[14] For the reminder of this paper, we simplify our functional notations by suppressing their arguments whenever necessary.

problem.

$$V_0(\epsilon_i, x; s) = \Big\{0, -\phi(\epsilon_i) + \pi_d x + (1-\pi_d)V_1(\epsilon_i, x; s)\Big\} \tag{4}$$

$$V_1(\epsilon_i, x; s) = \max_{k', b'} \Big[x - k' + q(\epsilon_i, k', b'; s)b' + \beta \sum_{j=1}^{N_\epsilon} \pi_{ij} V_0(\epsilon_j, x'_j; s')\Big] \tag{5}$$

subject to

$$(k', b') \in \Phi \equiv \{(k', b') : x - k' + q(\epsilon_i, k', b'; s)b' \geq 0\}$$
$$x'_j \equiv x(\epsilon_j, k', b'; s')$$
$$s' = \Gamma(s)$$

Equation (4) describes the firm's binary decision of default, in which the value of default is normalized to zero. We define the corresponding default threshold of cash-on-hand, $x_d(\epsilon_i)$ such that $\pi_d x_d + (1-\pi_d)V_1(\epsilon_i, x_d) = \phi(\epsilon_i)$. Firms with $x > x_d$ thus find it better to operate and produce in the current period by repaying their existing debt. A continuing firm then faces the exit shock with probability π_d, and conditional on its survival, it maximizes the sum of its current dividend and future expected discounted value in Equation (5).

Notice that numerically solving the above firm's problem requires a nonlinear method, since the value function V_1 may exhibit kinks. Instead of the grid-search method in existing studies, we make use of the FOCs derived from Equation (5). Our approach is based on the differentiability of value function at its interior optimum, as emphasized in Clausen and Strub (2020). In other words, FOCs are still necessary since it is not optimal for firms to choose a kinked point of V_1.[15] In particular, we exploit the fact that the marginal value of future cash-on-hand is binary, $D_2 V_0(\epsilon_j, x'_j; s') \in \{0, 1\}$.[16] We then robustly find the optimal choices of k' and b', considering all possible default decisions implied by the transition probability of ϵ. The details of our FOC approach are included in the online appendix.

Loan Price Schedule There is a competitive financial intermediary owned by the household. It lends resources to each borrowing firm at a specific loan price $q(\epsilon, k', b'; s)$. Suppose that $\tilde{k}'$ and $\tilde{b}'$ are chosen by a firm with (ϵ, x) in the current period, and let p_d be its conditional probability of default in the future. There is no private information, so the financial intermediary offers a

[15] Clausen and Strub (2020) provide the mathematical tools for solving a non-convex dynamic programming problem. They also show that kinked points of a value function can't be optimal in the sovereign default model of Arellano (2008). Given this result, we check the sufficiency of firm-level decision rules that satisfy the FOCs in our model.

[16] This observation is robust to changes in exit or default timing in our model, since we can re-define a firm's cash-on-hand accordingly.

firm-specific loan price that competitively evaluates the associated risk of default,

$$q(\epsilon, \tilde{k}', \tilde{b}'; s) = q_0(s)\Big((1-p_d) + p_d \cdot \min\big\{\frac{\theta\tilde{k}'}{\tilde{b}'}, 1\big\}\Big). \tag{6}$$

When the firm repays $\tilde{b}'$ with certainty, there is no default risk in this loan contract and the loan price equals to the risk-free price q_0. In case of default, on the other hand, the intermediary only recovers $\theta\tilde{k}'/\tilde{b}'$ for each unit of loan provided, implying $q < q_0$ from Equation (6).[17] Consequently, when there is non-zero probability of default, the firm's current decisions determine the available loan size qb' for financing its investment.

Potential Entrants A fixed measure M_e of potential entrants arrives in each period. They are ex-ante heterogeneous in their initial state (k_0, b_0). Specifically, we assume that k_0 and b_0 are jointly drawn from a bivariate uniform distribution $G(\bar{k}_0, \theta_0)$ with $k_0 \in (0, \bar{k}_0]$, $b_0 \in [0, \bar{b}_0]$, and $\bar{b}_0 \equiv \theta_e \bar{k}_0$.[18] In order to start its operation from the next period, a potential entrant has to pay ϕ_e units of output in the current period. This fixed cost of entry is sunk, and an entrant's initial productivity is realized before its first production. Let V_e be the value of a potential entrant such that

$$V_e(k_0, b_0; s) \equiv \max\Big\{0, -\phi_e + \beta\sum_{i=1}^{N_\epsilon}\pi_i V_0(\epsilon_i, x_{0,i}; s')\Big\}, \tag{7}$$

where $x_{0,i} \equiv x(\epsilon_i, k_0, b_0)$ is the initial cash-on-hand and π_i is the unconditional probability of having ϵ_i. From Equation (7), it is clear that only the potential entrants with $V_e > 0$ enter.

2.1.3 Equilibrium

We define stationary recursive competitive equilibrium (RCE) of the pre-pandemic economy.

An RCE is a set of functions: prices (w, q, q_0, r), quantities $(C^h, N^h, p, a', N^w, K, B)$, and values (V^h, V_0, V_1) that solve the optimization problems and clear each market, and the associated policy functions are consistent with the aggregate law of motion, as in the following conditions.

1. V^h solves the household's problem in Equation (2), and (C^h, N^h, p, a') are the associated policies.

2. V_0 and V_1 solve Equations (4) and (5), and (N^w, K, B) are the policy functions for firms.

[17] Since the default timing in our model is at the beginning of each period, we do not consider any depreciation of existing capital in (6). Modifying it to include δ would not change the main results of our paper.

[18] θ_e captures the maximum initial leverage b_0/k_0 of entrants. As a variation of Hopenhayn (1992), our setting features ex-ante heterogeneity in capital and debt across potential entrants. See Clementi and Palazzo (2016) and Jo and Senga (2019) for other applications.

3. The labor market clears,

$$N^h = \int_{\mathbb{S}} N^w \mu(d[\epsilon \times k]).$$

4. The law of motion for the firm distribution is consistent with individual decision rules.

2.1.4 Firm Types and Unconstrained Decisions

We follow the approach of distinguishing firm types in Khan and Thomas (2013). First, define a subset of firms that have accumulated sufficient wealth to become *unconstrained*. These firms face no default risk in any possible future state, and hence become indifferent between paying positive dividends and saving internally. The rest of firms are *constrained* with non-zero probability of default in the future, and they do not pay dividends to shareholders because the shadow value of their internal saving is higher.[19] In other words, these firms' constrained decisions imply that the zero-dividend policy holds, $x - k' + qb' = 0$. Depending on the level of cash-on-hand, some constrained firms are required to bear extra costs for discount loans due to their default risk, while others borrow or save at the risk-free rate.

For unconstrained firms, the optimal capital choice simply maximizes the future expected value in Equation (5), by definition. Let $K^w(\epsilon)$ be the corresponding policy for future capital. Next, we recursively define the optimal debt policy $B^w(\epsilon)$ for such firms as below.

$$\begin{aligned} B^w(\epsilon_i) &= \min_{\{\epsilon_j : \pi_{ij} > 0\}} \tilde{B}(\epsilon_j, K^w(\epsilon_i)) \\ \tilde{B}(\epsilon_i, K^w(\epsilon_j)) &= z\epsilon_i f(K^w(\epsilon_j), N^w) - wN^w + (1-\delta)K^w(\epsilon_j) \\ &\quad + \min\big\{ - K^w(\epsilon_i) + q_0 B^w(\epsilon_i), 0 \big\} \end{aligned}$$

The above *minimum-savings-policy* ensures that any unconstrained firm is able to choose K^w and B^w in the future period, by defining the threshold of debt holding $\tilde{B}$ for each possible future state implied by π_{ij}. We can further define the threshold level of cash-on-hand for being unconstrained, since unconstrained firms can pay positive dividends in the current period. Let $\tilde{x}(\epsilon) \equiv K^w - q_0 B^w$ be the threshold such that any firms with $x \geq \tilde{x}$ can be distinguished as unconstrained.

[19] Ottonello and Winberry (2020) prove this zero-dividend policy in a simplified model environment.

2.2 The Pandemic Economy with Differences in Health Status

We augment the pre-pandemic model by introducing an infectious disease that spreads across household members. Each individual will vary in their health status that affects both ability to work and marginal utility.

In the following, we assume that only healthy individuals may work, and activity in the labor market increases the probability of an individual's falling ill. Therefore, as it chooses the fraction of healthy individuals that work in a given period, the household takes into account the effect of its choice on the share of individuals that are infected next period. This leads to a dynamic condition for its labor supply.

Our approach extends the indivisible-labor economy by incorporating the key elements of an SIR model, while maintaining our original setup for the production side with heterogeneous firms. Since the aggregate state of the model now includes a time-varying distribution of health status across individuals, we formulate the household's sequence problem and solve for equilibrium decision rules along a transition path in perfect foresight.[20]

Health Status and Decisions During the pandemic, there can be two types of health status across household members, *healthy (Type-1)* and *ill (Type-2)*.[21] We assume that only Type-1 individuals are able to work, and further that the risk of infection is relatively higher for those who are designated for working. Due to their illness, the same level of consumption is more valuable to Type-2 individuals than Type-1. This leads to the assumption that the marginal utility of consumption for each Type-2, θ_2, is higher than that for Type-1, $\theta_1 = 1$.

In period t, the household decides the fraction p_t of Type-1 individuals that will work and allocates consumption by health and employment status. Let $c_t \equiv (c^w_{1,t}, c^n_{1,t}, c_{2,t})$ represent the consumption allocation within the household, where c^w_1 denotes the consumption for Type-1 working, c^n_1 for Type-1 non-working, and c_2 for Type-2 individuals. The optimal decisions of consumption, saving, and labor supply then depend on the distribution of health types which can potentially vary over time. This distribution is denoted by $m_t \equiv (m_{1,t}, m_{2,t})$, where m_1 is the number of Type-1 individuals and m_2 is that of Type-2. The budget constraint in period t is given by,

$$m_{1,t} \cdot \left(p_t c^w_{1,t} + (1-p_t) c^n_{1,t}\right) + m_{2,t} \cdot c_{2,t} + a_{t+1} \leq m_{1,t} \cdot p_t w_t h + (1+r_t) a_t + \Pi_{d,t}. \tag{8}$$

[20]It is still feasible to write down the household problem using a recursive approach. However, we find it more convenient to consider a sequence approach when the model economy experiences a pandemic.

[21]For simplicity, we do not distinguish healthy and recovered individuals. Nor do we allow for mortality. This simplification helps us focus on the persistent impact of a massive contagion that endogenously generates a large contraction in economic activity while abstracting from a time-varying population.

Modified Household Problem As in the standard SIR models, the probability of getting infected differs by an individual's health and employment status, as well as the number of individuals that are already infected. Hence, the household's decisions of consumption and labor may influence the contagion of a disease over time, which in turn affects its future economic decisions. To deal with this point previously emphasized by Eichenbaum, Rebelo, and Trabant (2021), we isolate the endogenous aggregate state $\bar{m}_t \equiv (\bar{m}_{1,t}, \bar{m}_{2,t})$ which will be equal to m_t in equilibrium.[22] The household takes such type distribution as given, understanding the transition probabilities between health types over time. That is, given $\bar{m}_t$, let $\hat{\pi}_{ij}$ be the transition probability of a type-i individual's becoming type-j in the next period for $i, j = 1, 2$.

$$\begin{aligned}
\hat{\pi}^w_{11}(\bar{m}_t) + \hat{\pi}^w_{12}(\bar{m}_t) &= 1 \qquad &&: \text{Type-1, working} \\
\hat{\pi}^n_{11}(\bar{m}_t) + \hat{\pi}^n_{12}(\bar{m}_t) &= 1 \qquad &&: \text{Type-1, non-working} \\
\hat{\pi}_{21}(\bar{m}_t) + \hat{\pi}_{22}(\bar{m}_t) &= 1 \qquad &&: \text{Type-2}
\end{aligned}$$

As mentioned earlier, working household members are more likely to be exposed to the disease, so we restrict that the probability of becoming infected is higher when working, $\hat{\pi}^w_{12} > \hat{\pi}^n_{12}$ for any given $\bar{m}_t$. Then the choice of p_t affects the law of motion of each health type as described in the following.

$$m_{1,t+1} = m_{1,t} \cdot \Big(p_t \hat{\pi}^w_{11}(\bar{m}_t) + (1 - p_t)\hat{\pi}^n_{11}(\bar{m}_t) \Big) + m_{2,t} \cdot \hat{\pi}_{21}(\bar{m}_t) \qquad : \text{Type-1} \tag{9}$$

$$m_{2,t+1} = m_{1,t} \cdot \Big(p_t \hat{\pi}^w_{12}(\bar{m}_t) + (1 - p_t)\hat{\pi}^n_{12}(\bar{m}_t) \Big) + m_{2,t} \cdot \hat{\pi}_{22}(\bar{m}_t) \qquad : \text{Type-2} \tag{10}$$

In the presence of a time-varying health distribution, the household maximizes the lifetime discounted utility under perfect foresight, by optimally choosing the allocations of consumption and labor supply in each period.

$$\max_{\{\mathbf{d}^h_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \bigg(m_{1,t} \Big[p_t u^1(c^w_{1,t}, 1-h) + (1-p_t) u^1(c^n_{1,t}, 1) \Big] + m_{2,t} u^2(c_{2,t}, 1) \bigg) \tag{11}$$

$$\begin{aligned}
&\text{subject to (8), (9), (10),} \\
&\qquad c^w_{1,t}, c^n_{1,t}, c_{2,t} \geq 0, p_t \in (0,1] \\
&\qquad (m_0, a_0) \text{ given, } \lim_{t\to\infty} \frac{a_{t+1}}{\Pi_{s=0}^t (1+r_s)} \geq 0,
\end{aligned}$$

where $\mathbf{d}^h_t \equiv (c^w_{1,t}, c^n_{1,t}, c_{2,t}, p_t, a_{t+1})$ is the vector of choices, and $u^i(\cdot)$ is the type-i individual's

[22] The aggregate state at the beginning of period t is then given by $s_t \equiv (\mu_t, \bar{m}_t)$ in the pandemic economy, where μ_t is the distribution of firms as defined in the previous subsection.

utility for $i = 1, 2$. Accordingly, we define the aggregate consumption and labor supply as below.

$$C_t^h \equiv m_{1,t}\big(p_t c_{1,t}^w + (1-p_t)c_{1,t}^n\big) + m_{2,t}c_{2,t}$$
$$N_t^h \equiv m_{1,t}p_t h$$

It is worth mentioning again that the labor-leisure condition from Equation (11) is dynamic, in contrast to the household problem in the pre-pandemic economy. As a result, the implied wage rate nonlinearly depends on the marginal values of consumption and health types in each period. In the online appendix, we derive the dynamic labor-leisure condition and the first-order nonlinear difference equation that characterizes the optimal sequence $\{\mathbf{d}_t^h\}_{t=0}^{\infty}$.

3 Quantitative Results

3.1 Calibration

The model period is one quarter, and we calibrate the parameter values of the pre-pandemic model to be consistent with the key annual moments of aggregates and firm dynamics in the US data. The production function is Cobb-Douglas, $y = z\epsilon k^{\alpha} n^{\nu}$ with $\alpha, \nu > 0$ and $\alpha + \nu < 1$. Our approach of incorporating health types is applicable to a general class of CRRA utility function, for both non-separable and separable cases of consumption and labor. For simplicity, we assume that the individual utility function of healthy household members is $u^1(c, 1-h) = \log c + \eta \log(1-h)$, and that of the infected is $u^2(c, 1) = \theta_2 u^1(c, 1) + \gamma$ with $\theta_2 > 1$ and $\gamma < 0$. The fixed cost of operation is given by $\phi(\epsilon) = \phi_0 \epsilon^{\phi_1 + 1 + \frac{1}{1-\nu}}$, where ϕ_0 is the scale parameter and ϕ_1 controls the curvature of the cost over ϵ.

We assume that there are two permanent types of idiosyncratic shock processes for ϵ. This allows us to closely reproduce the empirical firm size and age distributions in our model economy, in addition to capturing the relevant financial moments in the firm-level data. Specifically, let s_ϵ be the fixed share of firms with their individual productivity following a log-AR(1) process (*ln-type*),

$$\log \epsilon' = \rho_\epsilon \log \epsilon + \eta_\epsilon' \quad , \eta_\epsilon' \sim N(0, \sigma_{\eta_\epsilon}^2).$$

Other firms (*pr-type*) draw their productivity from a truncated Pareto distribution with its bounds $[\epsilon_m, \epsilon_M] \subset \mathbf{R}_+$ and the shape parameter ξ.[23] These firms retain their productivity with proba-

[23]This setting is common in quantitative studies of production heterogeneity such as Buera, Fattal-Jaef, and Shin (2015), Jo and Senga (2019), and Buera et al. (2021). These models generate a skewed distribution of producers in their employment which is comparable to the US data.

Table 1 : Parameters Values, Pre-pandemic Economy

	Value	Description
β	0.99	subjective discount factor
h	0.40	indivisible working hours
ν	0.60	production function, labor coefficient
θ	0.45	debt recovery rate upon default
θ_e	0.40	maximum leverage for potential entrants
π_d	0.0135	exogenous exit rate
η	2.16	labor disutility
α	0.25	production function, capital coefficient
δ	0.0171	capital depreciation rate
ϕ_0	0.01	fixed operation cost, scale
ϕ_1	0.00	fixed operation cost, curvature
M_e	0.05	mass of potential entrants
ϕ_e	0.129	fixed entry cost
$\overline{k}_0$	0.114	maximum initial capital
N_ϵ	(15,15)	discrete values of idio. productivity, 2 types
s_ϵ	0.80	population share of ln-type
ρ_ϵ	0.90	persistence of idiosyncratic productivity
σ_{η_ϵ}	(0.04,0.10)	std. deviation of innovations, ln-type
ϵ_m, ϵ_M	(0.58,1.28)	bounds of Pareto distribution, pr-type
ξ	3.40	shape of Pareto distribution, pr-type

Note: Given the fixed parameters in the top panel, we calibrate other parameters in the middle and bottom panels at quarterly frequency.

bility ρ_ϵ in each period. We discretize the two processes respectively with N_ϵ^{ln} and N_ϵ^{pr} points, and target the average firm size and age moments in the Business Dynamics Statistics (BDS). Table 1 reports the calibrated parameter values, and Table 2 presents the corresponding data and model moments. Lastly, we compare the model-generated size and age distributions with their empirical counterparts in Table 3.

We set the subjective discount factor, β, to imply an annual real interest rate of 4 percent, and the value of labor disutility, η, is chosen to get the hours worked of 0.30 in equilibrium. The labor coefficient in the production function, ν, is set to have an average labor income share of 0.6, as targeted in Cooley and Prescott (1995). The quarterly depreciation rate, δ, is 0.0175 to be consistent with the average investment-capital ratio in the US. We choose the value of the coefficient for capital, α, to get the aggregate capital-output ratio close to that in the Bureau of Economic Analysis (BEA).

For the fixed cost parameters, their values are chosen to imply about 3 percent of firm defaults annually, as reported in Ottonello and Winberry (2020). We set the exogenous exit probability

Table 2 : Annual Moments, Pre-pandemic Economy

	Data	**Model**	**Description**
N	-	0.30	total hours worked
K/Y	2.30	2.30	capital-output ratio, BEA
I/K	0.07	0.07	investment-capital ratio, BEA
total exit (%)	8.41	8.46	average firm exit rate, BDS
default (%)	3.00	2.87	annual default rate, OW (2020)
$\mu(b_+)$	0.81	0.26	fraction of firms w/ debt, OW (2020)
b_+/k	0.34	0.24	mean firm leverage, OW (2020)
N_0/N (%)	2.25	1.16	relative employment of entrants, BDS
μ_0/μ (%)	9.02	7.39	relative population of entrants, BDS

Note: We target default and financial moments in Ottonello and Winberry (OW, 2020).

Table 3 : Size and Age Distributions, Pre-pandemic Economy

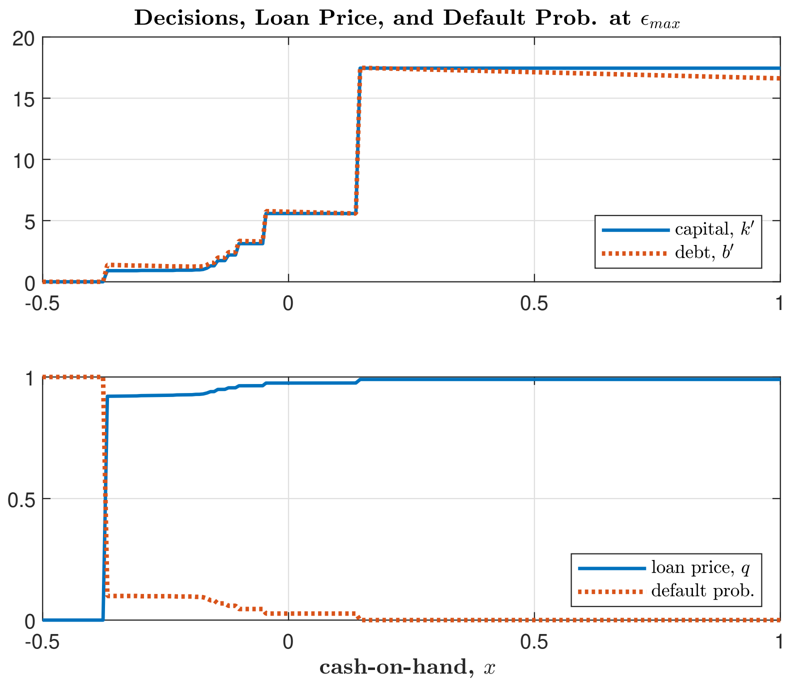
	Emp. Share (%)		**Pop. Share (%)**	
size/age bin	Data	Model	Data	Model
small (1-19)	19.98	19.98	88.60	86.01
medium (20-499)	31.56	31.56	11.01	11.65
large (500+)	48.46	48.46	0.39	2.34
young (0-4)	10.83	6.00	32.02	28.30
mature (5+)	89.17	94.00	67.98	71.70

Note: For firm size distribution, we exactly match the employment shares across 3 size bins following the method in Jo (2021). *Small* firms are defined as those with less than 20 employees, *medium*-sized firms are with 20-499, and *large* firms are with 500+ per year. *Young* firms are those with age 0 to 4. The empirical size distribution is the average from 1978 to 2019, and the age distribution is that from 1993 to 2019 in BDS.

at 0.0135 per quarter, so that the total annual exit rate in the model is aligned with its empirical counterpart. The loan recovery rate upon default, θ, is 0.45, reasonably close to those in earlier studies.[24] The remaining parameter values are jointly calibrated to match the relative population share and employment size of entrants, the mean leverage ratio, the fraction of firms with debt.

[24] Both Khan, Senga, and Thomas (2016) and Ottonello and Winberry (2020) set this value at 0.54. On the other hand, Corbae and D'Erasmo (2021) structurally estimate the relevant parameter values which imply the average recovery rate of 0.56 between Ch.7-liquidation and Ch.11-reorganization schemes for listed firms.

Figure 2 : Cash-on-hand and Decisions



Note: For visibility, the above figure is zoomed in at low levels of cash-on-hand x around the default threshold.

3.2 Firm Heterogeneity and Decisions

Investment and Borrowing Decisions Abstracting from the pandemic, we look first at firm-level investment and borrowing in the steady state equilibrium. This helps us understand how default risk affects these decisions and hence restricts the efficient allocation of resources across firms. Figure 2 plots the capital and debt choices, (k', b'), as functions of cash-on-hand x, at a given level of productivity ϵ_{max}.[25] In the upper panel of the figure, nonlinearities in decisions, across firms with different levels of cash-on-hand, are evident. Specifically, the figure shows that the threshold level of default, x_d, is located at around -0.4. Incumbent firms with $x \leq x_d$ find it better to default on their existing debt at the beginning of the current period, so their choices of k' and b' are zero. Since these firms are not in need of new loans, we assume, without loss of generality, that competitive lenders price q at zero, as seen in the lower panel of Figure 2.

When the level of x is slightly above x_d, on the other hand, firms decide to continue and repay their debt, regardless of the exit shock realization. Conditional on survival, however, such firms' financing is restricted by their non-zero probability of default in the future. This is again illus-

[25] We present the result at ϵ_{max} for visibility. Similar patterns are observed at other values of ϵ.

trated in the upper panel of Figure 2, in which the constrained capital choice for these marginal firms is far below that for unconstrained firms with $x \geq \tilde{x}$ which is around 20.7. Hence, firms with positive default probability in the future have to pay risk premia for their newly issued debt, and the implied discount rate is higher than the risk-free rate of $1/q_0$. The probability of default depends on the set of idiosyncratic productivity levels where cash-on-hand will fall below x_d in the future period. As the productivity process is finite, the discrete nature of default leads to kinks in capital and debt decisions when x is between x_d and $\tilde{x}$.[26]

Firms with higher cash-on-hand, but not enough to become unconstrained, are able to afford the efficient level of capital without high levels of debt. These relatively safe firms are offered risk-free loans, as shown in Figure 2, since their leverage is sufficiently low and their capital would be enough to cover any loss from default in the future. As x increases further, these firms gradually de-leverage by reducing their debt, and eventually become unconstrained at $\tilde{x}$.

Entry Decision We now discuss the entry decision made by potential entrants whose initial capital and debt, (k_0, b_0), are jointly drawn from a bivariate uniform distribution. Figure 3 presents the entry decision for each combination of (k_0, b_0), in which the blue-colored area corresponds to entry (=1). As their initial capital k_0 becomes larger, potential entrants are able to bear the entry cost and decide to start their operation in the following period. Further, at a given level of k_0, some potential firms may find it difficult to enter with higher levels of initial debt, which leads to the diagonal line of the entry margin in the figure. This margin of firm entry endogenously adjusts following an aggregate shock, so the number of producers in our model economy can vary over time. As will be shown in the next subsection, such changes affect the allocative efficiency of resources, propagating the effects of exogenous shocks.

3.3 Aggregate Dynamics

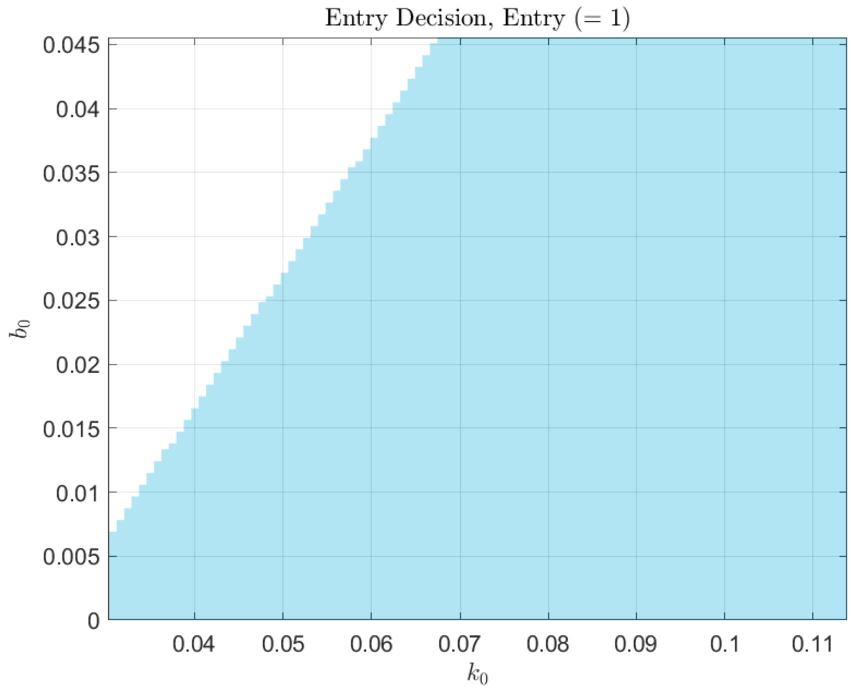
We present the equilibrium transitional dynamics of the model economy in response to aggregate shocks. The pre-pandemic economy is assumed to be at its steady state initially, and then hit by a persistent shock at date 1. We then report the corresponding aggregate dynamics under perfect foresight.

Since its outbreak at the beginning of 2020, the COVID-19 pandemic has seen a series of surges in infected cases.[27] It is also known that such multiple waves of infections are due to more

[26]The multiple kinks observed in Figure 2 tend to disappear when ϵ is low. That is, for low productive firms in the current period, the persistence in ϵ restricts their costly borrowing and the corresponding default probability collapses to a simple step function.

[27]As of January 2022, the US has experienced 5 waves of massive infections with the latest mainly driven by the

Figure 3 : Entry Decision



contagious variants of the virus and changes in restrictions and public health policies around the world, shaping different infection dynamics across countries and time periods. This makes it challenging to exactly replicate the observed patterns of the current pandemic in SIR-macro models. Instead, since the focus of this paper is on analyzing the aftermath of a pandemic shock in the presence of production heterogeneity, we assume an exogenous arrival of a disease and examine its propagation in our model economy.

In the following, we describe the pandemic across two different cases. In the first, which is used to illustrate the mechanics of the model, the pandemic is a large shock to health in the population and it declines monotonically. Thus, there is no rising infection rate over time. The second case, which is more empirically relevant, models a small initial infection that grows over time.

In particular, we contrast the case of rising infection rates with the monotonically declining public health crisis. The latter is an economy where households and firms expect the pandemic to gradually disappear over time. The large initial number of infected people leads to a rise in spending and employment rates among individuals that are able to work. Despite the fall in the number of idle workers, economy-wide illness reduces the labor force. Overall, aggregate output

Omicron variant. The resulting cumulative cases of infection amount to about 22 percent of the US population.

falls then recovers monotonically, as does employment. That is, output and employment dynamics are similar to those following a persistent, negative aggregate TFP shock, while consumption behaves countercylically. Most importantly, the recession is severe but relatively short, with a half-life of output of roughly 3 quarters. These highlight the mechanics of a pandemic where infections rates fall over time.

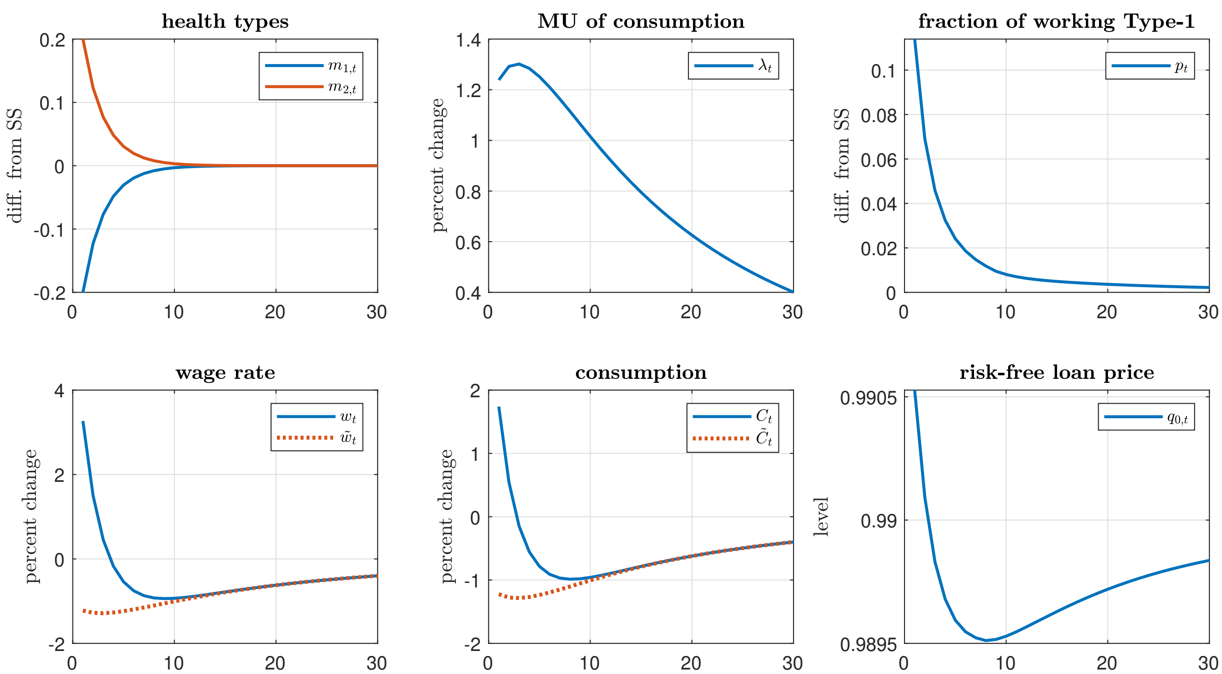
The more realistic case of a pandemic where the fraction of infected people initially rises is a sharp contrast to the falling infection case just described. We calibrate the transition probabilities between health status to reproduce the timing and magnitude of peak infection rate in the US from January 2020.[28] As households expect a rise in infection rates, they reduce employment rates in an effort to contain the spread of disease. Since the share of infected individuals is hump-shaped, initially rising for a year, aggregate output and employment both exhibit non-monotone dynamics. After the initial fall, they deteriorate further over time before starting to recover just before the peak of the pandemic. It follows that the recession persists for far longer with a half-life of 8 quarters. Households efforts to reduce the risk of infection, therefore, has a far larger role in this scenario. The initial choice to reduce employment rates leads to a rapid downturn in output that propagates the recession.

Outbreak of a Pandemic Beginning with the first case, we consider an unanticipated arrival of a number of ill individuals in the model. Specifically, at impact, we assume that the number of Type-2 individuals becomes 20 percent. The resulting fall in labor supply is about 8 percent in equilibrium.[29] Thereafter, as determined by Equations (9) and (10), the evolution of the health distribution is jointly governed by the household's labor supply decision, the relative share of each type, and the exogenous probabilities $(\pi_I^w, \pi_I^n, \pi_R)$. These probabilities reflect the micro-level infection and recovery rates for each individual in the model, where π_I^w is the probability of becoming ill for a working Type-1 individual and π_I^n is that of a non-working Type-2, and π_R as the recovery rate from the disease within a period. We set the values of these parameters to generate a persistent infection dynamics for about 12 quarters following the one-time pandemic shock. Later, we also consider the second case of a pandemic shock, with time-varying values of π_I^w and π_I^n. This generates hump-shaped infection dynamics over time. In both cases, further, we assume that firms' fixed operation cost rises by 20 percent for the initial 8 periods. This captures the additional costs and losses for businesses during a pandemic such as remote working, loss of intangible knowledge, or supply chain disruptions.

[28]The massive infection from delta-variant of COVID reaches its peak in about 13 months after the outbreak. We calibrated a simplified version of our model at monthly frequency.

[29]The Bureau of Labor Statistics (BLS) reports a 11.9 percent drop in aggregate non-farm employment between 2020Q1 and 2020Q2. The employment level in 2021 Q4 still remains about 2 percent below its pre-pandemic level.

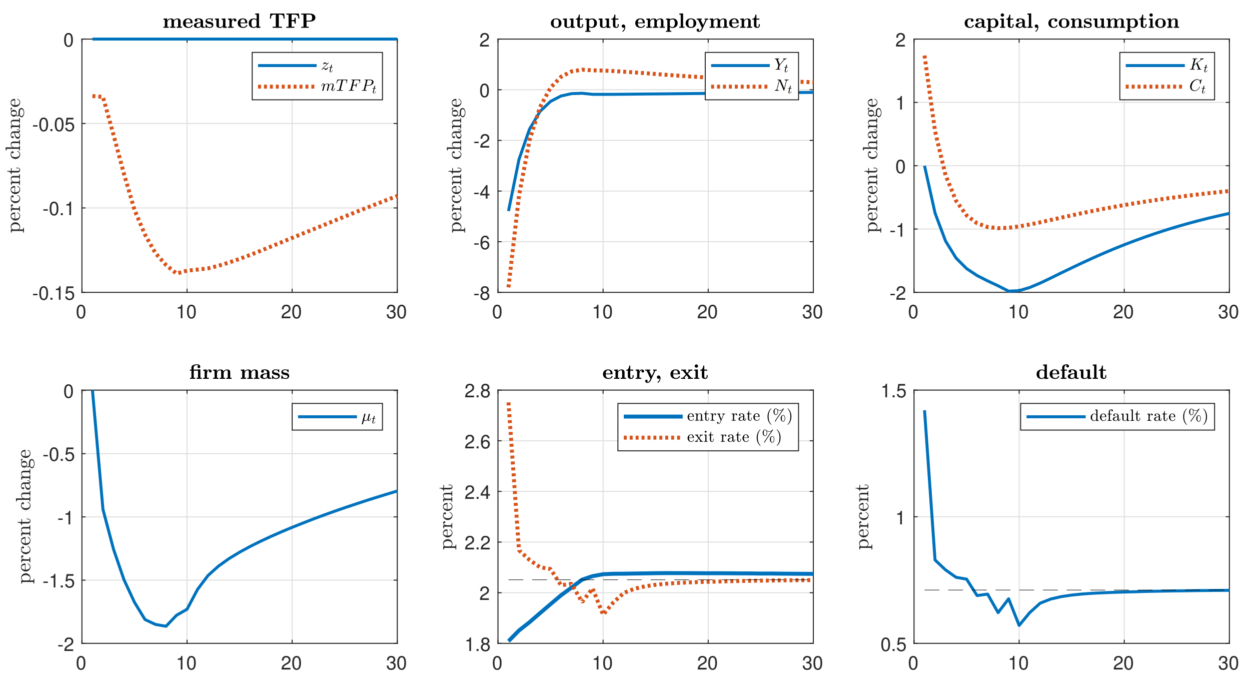
Figure 4 : Aggregate Dynamics, Declining Pandemic 1



Responses to a Declining Pandemic Shock Figure 4 presents the impulse responses to the pandemic shock that introduces a 20 percent share of Type-2 individuals at impact. Accordingly, the relative share of Type-1 individuals decreases by the same magnitude, as shown in the top-left panel of the figure. As the number of people able to work falls unexpectedly, the household's budget constraint becomes tighter for a given level of assets. This raises the marginal value of consumable output, λ_t, as illustrated in the top-center panel of Figure 4. In an effort to smooth consumption over time and also to provide more consumption for Type-2 individuals with higher marginal utility, the household initially puts more healthy individuals to work. The probability of employment for Type-1 individuals rises by more than 10 percentage points (top-right panel). Although such a decision further risks its members' health and thus lowers their welfare, the household also understands that the pandemic will lead to a persistent loss of future earnings. The return on savings falls (bottom-right panel), enhancing the incentive to consume beyond providing the necessary rise in consumption for Type-2 individuals (bottom-center panel).[30] As the infection rate falls to zero, however, aggregate consumption remains relatively low for a long period of time. This is because the production side of the economy still suffers from losses in

[30] As the household shares consumption risk across individual members, consumption is equalized across individuals with the same health status. Nonetheless, even with the initial rise in C_t, the increase in p_t is large enough to make the household welfare fall after the shock.

Figure 5 : Aggregate Dynamics, Declining Pandemic 2



capital and productivity, as will be discussed below.

The pattern of consumption changes is absent in standard business cycle models where the labor supply decision is atemporal. We emphasize this property of our model by looking at the artificial series of wages and consumption, $\{\tilde{w}_t, \tilde{C}_t\}_{t=1}^{T}$, assuming that the the wage rate is inversely related with λ_t as in the indivisible-labor economy.[31] Together with the changes in the marginal value of output, the dotted-red lines at the bottom panels of Figure 4 indicate that the recession following a pandemic shock would be otherwise similar to that from a standard model with a TFP shock.[32]

Figure 5 reports the dynamics of additional variables describing production. First, total employment falls significantly, by about 8 percent at the impact (top-center panel). This is because the drop in the number of Type-1 individuals is large enough to offset the increase in the share of working Type-1, given $N_t^s = p_t m_{1,t} h$. In addition, the change in aggregate employment is also affected by the the distribution of firms in equilibrium. As the bottom-center and bottom-right panels of the figure show, the total exit rate of firms increases due to the rise in firm default

[31] Our utility function implies that, in the absence of a pandemic, $w_t = W/\lambda_t$ where $W \equiv \frac{-\eta \log(1-h)}{h} > 0$.

[32] The responses of consumption can be larger when the pandemic shock is accompanied by an economy-wide business shutdown as in Buera et al. (2021) and Di Nola, Kaas, and Wang (2022).

rate at the impact, whereas the entry rate decreases immediately after the shock. This lowers the number of producing firms persistently (bottom-left panel).[33]

More importantly, both entry and exit rates are persistently different from their respective levels at the steady state during the pandemic. This implies that the economy experiences a prolonged churning of firms at the entry and exit margins, keeping the number of firms at lower levels for a long period of time. Further, such changes involve continued replacement of relatively productive incumbents with small and financially constrained entrants, which raises the extent of resource misallocation arising from financial frictions and firm default risk. Borrowing limits and default risk prevent entrants from growing rapidly. This pattern of gradual firm growth upon entry becomes more prolonged during the pandemic, worsening the misallocation. As a result, aggregate capital remains substantially low for more than 30 periods (top-right panel), and therefore, aggregate productivity in terms of measured TFP falls and then gradually recovers.[34] Overall, the endogenous firm entry and exit margins in our model affect both the number of firms and their composition in the distribution over the pandemic. This persistently lowers the allocative efficiency of productive factors in the economy.

Responses to a Worsening Pandemic Shock Notice that aggregate employment dynamics following the above pandemic shock are largely driven by the household's optimal labor supply and the relative share of infected people over time. The highest infection rate occurs at the impact date, as opposed to the persistent aspects of the current COVID-19 pandemic. We now turn to our second case and consider a different pandemic shock that gradually raises the infection rate in the economy. Instead of directly raising the number of Type-2 individuals, we exogenously increase the probabilities of becoming ill for each individual, π_I^w and π_I^n, for the initial periods of a pandemic.[35] The transition probabilities of the type distribution, ($\hat{\pi}_{1j}^w$, $\hat{\pi}_{1j}^n$, $\hat{\pi}_{2j}$) for $j = 1, 2$, continue to depend on the share of infected people in the population, but they now have additional exogenous variations for several initial periods. Specifically, we increase the values of π_I^w and π_I^n at the start of the pandemic, by 0.016 and 0.012, respectively.[36] This exogenous rise gradually disappears in 8 periods. Then the infection rate rises gradually, reaching its peak of 6 percent at date 5, as shown in the top-left panel of Figure 6.[37] Thus, this *worsening* pandemic

[33] According to BDS, there are more than 5.3 mil. firms and an average firm hires about 24 workers in 2019. Given that our model closely reproduces the empirical firm size, the above result roughly amounts to a job loss of 1.2 mil.

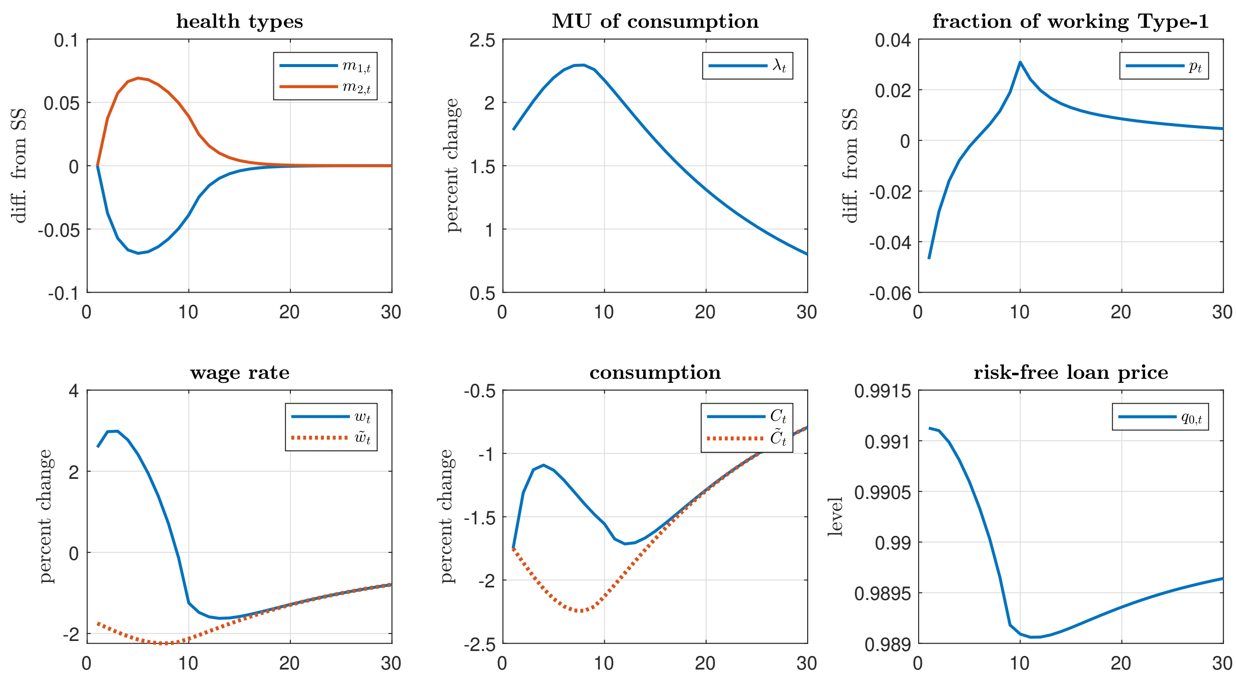
[34] As in Khan and Thomas (2013), we calculate the measured TFP level in each period as $mTFP_t \equiv \frac{Y_t}{K_t^{\alpha} N_t^{\nu}}$.

[35] This captures the potential externality in infection rates when there is a massive contagion across individuals during the early periods of a pandemic.

[36] At impact, the share of Type-2, $m_{2,t}$, remains at zero in this setting, and it increases to the weighted average of π_I^w and π_I^n over p_t in the next period.

[37] This magnitude is comparable to the infection rate observed from 2020Q4 to 2021Q1 in the US, mainly due to the delta-variant of COVID-19.

Figure 6 : Aggregate Dynamics, Worsening Pandemic 1



shock generates an initially small but growing pandemic with more persistent changes in the distribution of health in our model.

In Figure 6, both the fraction of working individuals and aggregate consumption initially fall (top-right and bottom-center panels). This stark contrast to the first case is the result of expectations of increases in the share of infected people. Given a small number of initially infected, the household tries to slow down the rise in the family infection rate by reducing employment sharply. This supply-driven reduction in employment drives up the equilibrium real wage as shown in the bottom-left panel of the figure.

The household reduces its consumption beyond the loss in income as it smooths consumption over time. There is a tension here. An initially low number of ill family members, together with higher probability of infection from work in the future, leads the household to reduce its labor supply at date 1 (top-right panel). Over time as infections grow, consumption spending on those unable to work rises. This drives up the employment rate and brings further increases in the share of infected individuals. Consumption exhibits a u-shaped response ordinarily seen following a persistent exogenous shock to aggregate productivity. However, output and employment are also u-shaped as seen in the top-center panel of Figure 7. This is the result of differences in a pandemic driven recession compared to that from a TFP shock. That is, the pandemic leads to a fall in the labor supply, part of which is endogenous as the household tries to reduce the growth in infections across its members.

As illness and efforts to slow further infection reduce labor supply, the rise in the real wage reduces equilibrium employment by firms and so does production. Aggregate output falls by 4 percent, then continues to fall further in the following periods. The initial fall in output is purely the result of the household choosing to reduce its labor supply. Further reductions in employment arise since the number of individuals in the labor force falls as infection rates rise. This leads to the u-shaped response in output.

As in our first example above, moreover, adjustments in firm entry and exit margins lead to persistent decreases in the number of producing firms and a larger number of smaller firms that are more constrained in their investment choice. Higher wages reduce cash-on-hand across firms. This disproportionately affects entrants who, as a result, face tougher borrowing conditions. The result is a rise in misallocation of resources across firms. This is illustrated by the drop in aggregate productivity as in the top-left panel of Figure 7. These adverse impacts further reduce the value of entry, which drives a persistent decrease in entry rates seen in the bottom-center panel of the figure. The number of firms thus fall, worsening aggregate TFP. Beyond the persistently weak demand for consumption and investment, this fall in aggregate productivity slows down the economy's recovery from the recession.

Effectiveness of Business Subsidy In Figure 7, the overall responses of aggregate variables to a pandemic shock are relatively moderate when compared to their empirical counterparts in the US data. It is also clear that our model economy abstracts from various policy measures for preventing the spread of a disease. The results in Eichenbaum, Rebelo, and Trabant (2021), however, indicate that an implementation of containment measures may lead to substantially larger decreases in aggregate consumption and employment in equilibrium.[38] Such a tension between macroeconomic outcomes and policy interventions further motivates a quantitative investigation in our model economy.

We conduct a simple policy counterfactual when the economy is hit by the worsening pandemic shock. Specifically, we consider a business support scheme of subsidizing firms for the first 8 periods of the pandemic recession. The subsidy reduces firms' fixed operation costs to the level in the pre-pandemic economy, alleviating their losses from increased factor prices. This further mitigates the rise in the number of firm defaults so that the employment by vulnerable firms can be sustained temporarily .[39]

[38]For instance, their simple containment policy only reduces the peak infection rate by 2 percent, whereas consumption drops more than 2.5 times than that without the policy.

[39]In this regard, the policy mainly targets financially constrained firms in our model economy. We assume that

Figure 7 : Aggregate Dynamics, Worsening Pandemic 2

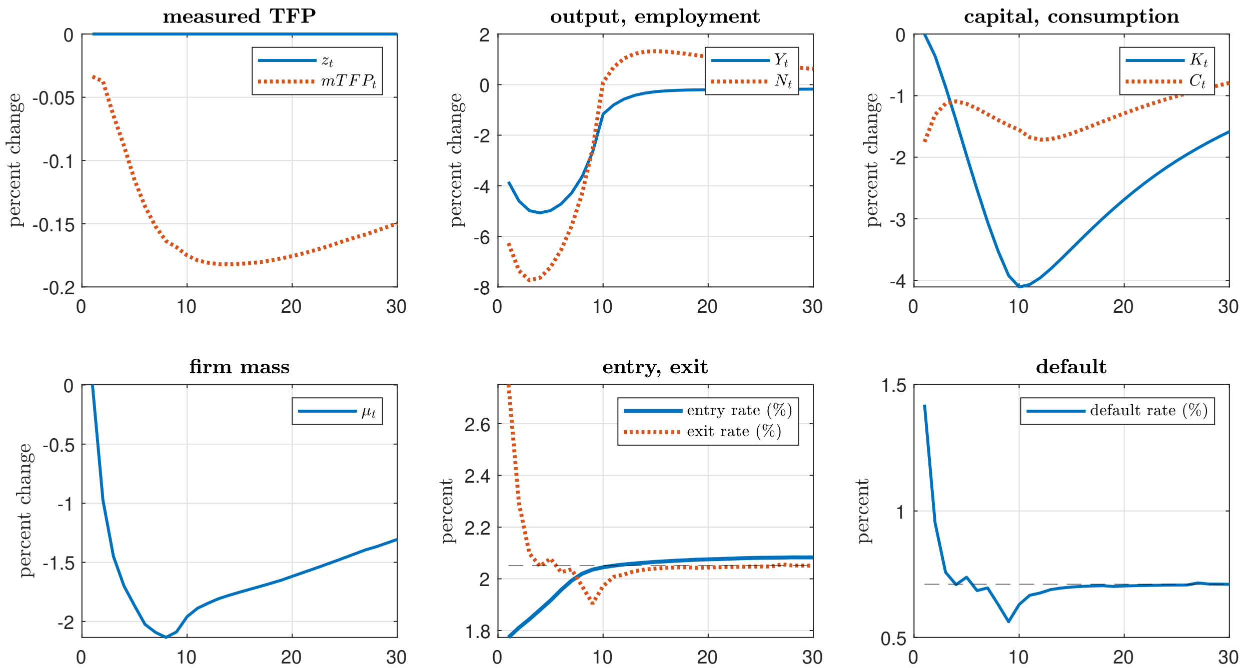
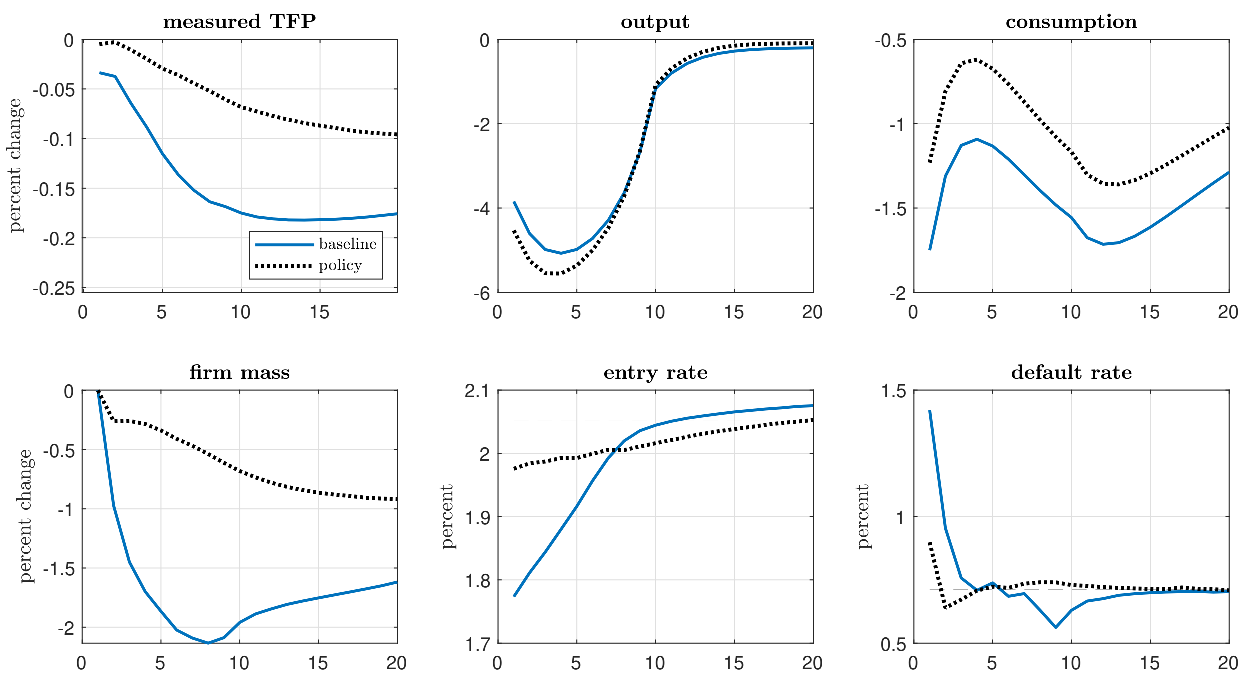


Figure 8 compares the aggregate dynamics during the worsening pandemic, with or without the business support policy. As expected, the subsidy substantially lowers the rise in firm defaults at the impact of the shock (*black-dotted line*). Due to relatively lower cost of operating with the subsidy, firm entry margin adjusts less. This moderates the declines in the number of firms significantly, and the measured TFP only decreases slightly. However, the economy experiences a relatively larger recession when the policy is implemented. The top-middle panel of the figure shows that aggregate output falls more by 0.5 percent in the first 7 periods of the recession. This is mainly because the subsidy substantially reduces the average productivity of incumbents by deterring the marginal firms with low productivity from exiting the economy. Our results therefore suggest that the unintended effects of business subsidies can be quantitatively significant when the policy targets relatively unproductive firms during a pandemic.

the subsidy is financed from households in lumpsum.

Figure 8 : Aggregate Dynamics, Business Subsidy with Worsening Pandemic



4 Concluding Remarks

We have developed a model of the pandemic where households differ in their health status, the risk of illness is higher for workers and rises in the number of infected people, and firm vary in their productivity, debt and capital. Loan rate schedules that rise with the risk of default restraints firm growth. Changes in the distribution of firms, in the economic downturn that begins with the pandemic, increase the misallocation of resources and depresses aggregate TFP. As default risk increases with leverage, and thus loan rates, new firms find it costly to finance efficient investment. On average, they operate with levels of capital that are low relative to their productivity. This propagates over time since capital serves as collateral, and insufficient levels of collateral lead to higher loan rates. Thus our model, through financial frictions and without capital adjustment costs, reproduces the slow growth of entrants observed in the data. The recovery that follows the end of the health crisis, in our environment, is slow given the gradual growth of new firms. Thus costly borrowing with default, in a model with production heterogeneity, predicts that changes in the aggregate state over the pandemic propagate through the economy by slowing the recovery.

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Online Appendix

A Household Decisions in the Pandemic Economy

We derive the first-order necessary conditions (FOCs) from the household problem described in the main text, when the economy experiences a pandemic. In particular, we characterize the optimal labor supply condition and then derive a first-order difference equation for the marginal value of adjusting the health-type distribution $m_t = (m_{1,t}, m_{2,t})$ along the transition path. Note that our approach of introducing simple heterogeneity among households is generally applicable to a standard heterogeneous-firm model, when studying equilibrium aggregate dynamics in perfect foresight.

First, we re-write the household's sequence problem as below.

$$\max_{\{\mathbf{d}_t^h\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \Big(m_{1,t} \Big[p_t u^1(c_{1,t}^w, 1-h) + (1-p_t) u^1(c_{1,t}^n, 1) \Big] + m_{2,t} u^2(c_{2,t}, 1) \Big) \tag{11}$$

subject to

$$m_{1,t} \cdot \Big(p_t c_{1,t}^w + (1-p_t) c_{1,t}^n \Big) + m_{2,t} \cdot c_{2,t} + a_{t+1} \leq m_{1,t} \cdot p_t w_t h + (1+r_t) a_t + \Pi_{d,t} \tag{8}$$

$$m_{1,t+1} = m_{1,t} \cdot \Big(p_t \hat{\pi}_{11}^w(\bar{m}_t) + (1-p_t) \hat{\pi}_{11}^n(\bar{m}_t) \Big) + m_{2,t} \cdot \hat{\pi}_{21}(\bar{m}_t) \qquad \text{: Type-1} \tag{9}$$

$$m_{2,t+1} = m_{1,t} \cdot \Big(p_t \hat{\pi}_{12}^w(\bar{m}_t) + (1-p_t) \hat{\pi}_{12}^n(\bar{m}_t) \Big) + m_{2,t} \cdot \hat{\pi}_{22}(\bar{m}_t) \qquad \text{: Type-2} \tag{10}$$

$$c_{1,t}^w, c_{1,t}^n, c_{2,t} \geq 0, p_t \in (0, 1]$$

$$a_0 \text{ given}, \lim_{t\to\infty} \frac{a_{t+1}}{\hat{\pi}_{s=0}^t (1+r_s)} \geq 0,$$

where $\mathbf{d}_t^h \equiv (c_{1,t}^w, c_{1,t}^n, c_{2,t}, p_t, a_{t+1})$ be the vector of choices, and $u^i(\cdot)$ is the type-i individual's utility for $i = 1, 2$. Let λ_t, $\lambda_{1,t}$, $\lambda_{2,t}$ be the multipliers respectively for (8), (9), and (10). The first derivatives with respect to $c_{1,t}^w$, $c_{1,t}^n$, $c_{2,t}$, and p_t are

$$\begin{aligned}
&[c_{1,t}^1] \qquad m_{1,t} p_t D_1 u^1(c_{1,t}^w, 1-h) - m_{1,t} p_t \lambda_t = 0 \\
&[c_{1,t}^0] \qquad m_{1,t}(1-p_t) D_1 u^1(c_{1,t}^n, 1) - m_{1,t}(1-p_t)\lambda_t = 0 \\
&[c_{2,t}] \qquad m_{2,t} D_1 u^2(c_{2,t}, 1) - m_{2,t}\lambda_t = 0 \\
&[p_t] \qquad m_{1,t}\big[u^1(c_{1,t}^w, 1-h) - u^1(c_{1,t}^n, 1)\big] + \lambda_t m_{1,t}\big[w_t h - (c_{1,t}^w - c_{1,t}^n)\big] \\
&\qquad\qquad + \lambda_{1,t} m_{1,t}\big(\hat{\pi}_{11}^w(\bar{m}_t) - \hat{\pi}_{11}^n(\bar{m}_t)\big) + \lambda_{2,t} m_{1,t}\big(\hat{\pi}_{12}^w(\bar{m}_t) - \hat{\pi}_{12}^n(\bar{m}_t)\big) = 0.
\end{aligned} \tag{A.1}$$

The first two conditions in the above imply that $c_{1,t}^w = c_{1,t}^n$ for all t, when the CRRA utility function is logarithm in consumption and leisure. The multiplier $\lambda_{1,t}$ ($\lambda_{2,t}$), in addition, represents the marginal value of a healthy (ill) worker within the household in period $t+1$. Next, the FOCs with respect to a_{t+1}, $m_{1,t+1}$, and $m_{2,t+1}$ are

$$[a_{t+1}] \qquad -\lambda_t + \beta\lambda_{t+1}(1+r_{t+1}) = 0 \tag{A.2}$$

$$\begin{aligned}[m_{1,t+1}] \qquad \lambda_{1,t} = \beta\Big[&p_{t+1}u^1(c^w_{1,t+1}, 1-h) + (1-p_{t+1})u^1(c^n_{1,t+1}, 1)\Big] \\ &+ \beta\lambda_{t+1}\Big[p_{t+1}w_{t+1}h - \big(p_{t+1}c^w_{1,t+1} + (1-p_{t+1})c^n_{1,t+1}\big)\Big] \\ &+ \beta\lambda_{1,t+1}\Big[p_{t+1}\hat{\pi}^w_{11}(\bar{m}_{t+1}) + (1-p_{t+1})\hat{\pi}^n_{11}(\bar{m}_{t+1})\Big] \\ &+ \beta\lambda_{2,t+1}\Big[p_{t+1}\hat{\pi}^w_{12}(\bar{m}_{t+1}) + (1-p_{t+1})\hat{\pi}^n_{12}(\bar{m}_{t+1})\Big] \end{aligned} \tag{A.3}$$

$$\begin{aligned}[m_{2,t+1}] \qquad \lambda_{2,t} = &\beta u^2(c_{2,t+1}, 1) - \beta\lambda_{t+1}c_{2,t+1} \\ &+ \beta\lambda_{1,t+1}\hat{\pi}_{21}(\bar{m}_{t+1}) + \beta\lambda_{2,t+1}\hat{\pi}_{22}(\bar{m}_{t+1}), \end{aligned} \tag{A.4}$$

where the transition probabilities in (A.3) and (A.4) are functions of the aggregate state $\bar{m}_{t+1}$.

Our goal is to derive an equation for $(\lambda_{1,t} - \lambda_{2,t})$ by using the previous FOCs, which eventually allows us to solve the optimal sequence of consumption and labor supply over transitional dynamics. To this end, it is convenient to define the average utility and consumption among Type-1 individuals in period $t+1$.

$$\begin{aligned} U^1_{t+1} &\equiv p_{t+1}u^1(c^w_{1,t+1}, 1-h) + (1-p_{t+1})u^1(c^n_{1,t+1}, 1) \\ C^1_{t+1} &\equiv p_{t+1}c^w_{1,t+1} + (1-p_{t+1})c^n_{1,t+1} \end{aligned}$$

Using the above expressions, we can re-write (A.3).

$$\begin{aligned} \lambda_{1,t} =& \beta U^1_{t+1} + \beta\lambda_{t+1}\Big[p_{t+1}w_{t+1}h - C^1_{t+1}\Big] \\ &+ \beta\lambda_{1,t+1}\Big[p_{t+1}\hat{\pi}^w_{11}(\bar{m}_{t+1}) + (1-p_{t+1})\hat{\pi}^n_{11}(\bar{m}_{t+1})\Big] \\ &+ \beta\lambda_{2,t+1}\Big[p_{t+1}\hat{\pi}^w_{12}(\bar{m}_{t+1}) + (1-p_{t+1})\hat{\pi}^n_{12}(\bar{m}_{t+1})\Big] \end{aligned} \tag{A.3'}$$

Since the transition probabilities for each working status sum to 1, we can replace $\hat{\pi}^w_{12}(\bar{m}_{t+1})$ and $\hat{\pi}^n_{12}(\bar{m}_{t+1})$ in the last term of (A.3').

$$\begin{aligned} \lambda_{1,t} =& \beta U^1_{t+1} + \beta\lambda_{t+1}\Big[p_{t+1}w_{t+1}h - C^1_{t+1}\Big] \\ &+ \beta(\lambda_{1,t+1} - \lambda_{2,t+1})\Big[p_{t+1}\hat{\pi}^w_{11}(\bar{m}_{t+1}) + (1-p_{t+1})\hat{\pi}^n_{11}(\bar{m}_{t+1})\Big] + \beta\lambda_{2,t+1} \end{aligned} \tag{A.5}$$

Combining (A.4) with the above, we have

$$\begin{aligned}\lambda_{1,t} - \lambda_{2,t} = \beta\Big[U^1_{t+1} - u^2(c_{2,t+1}, 1)\Big] + \beta\lambda_{t+1}\Big[p_{t+1}w_{t+1}h - (C^1_{t+1} - c_{2,t+1})\Big] \\ + \beta(\lambda_{1,t+1} - \lambda_{2,t+1})\Big[p_{t+1}\hat{\pi}^w_{11}(\bar{m}_{t+1}) + (1 - p_{t+1})\hat{\pi}^n_{11}(\bar{m}_{t+1})\Big] \\ - \beta\lambda_{1,t+1}\hat{\pi}_{21}(\bar{m}_{t+1}) - \beta\lambda_{2,t+1}\hat{\pi}_{22}(\bar{m}_{t+1}) + \beta\lambda_{2,t+1}\end{aligned} \tag{A.6}$$

Since $\hat{\pi}_{22}(\bar{m}_{t+1}) = 1 - \hat{\pi}_{21}(\bar{m}_{t+1})$, the last two terms in (A.6) collapse to $\beta\lambda_{2,t+1}\hat{\pi}_{21}(\bar{m}_{t+1})$, and we have the following first-order difference equation for $(\lambda_{1,t} - \lambda_{2,t})$.

$$\begin{aligned}\lambda_{1,t} - \lambda_{2,t} = \beta\Big[U^1_{t+1} - u^2(c_{2,t+1}, 1)\Big] + \beta\lambda_{t+1}\Big[p_{t+1}w_{t+1}h - (C^1_{t+1} - c_{2,t+1})\Big] \\ + \beta(\lambda_{1,t+1} - \lambda_{2,t+1})\Big[p_{t+1}\hat{\pi}^w_{11}(\bar{m}_{t+1}) + (1 - p_{t+1})\hat{\pi}^n_{11}(\bar{m}_{t+1}) - \hat{\pi}_{21}(\bar{m}_{t+1})\Big]\end{aligned} \tag{A.7}$$

Note that $(\lambda_{1,t} - \lambda_{2,t})$ summarizes the marginal value of a healthy individual by reducing the number of ill individuals in the health-type distribution in the future period. At the steady state equilibrium without pandemic, we have $\lambda_{1,t} - \lambda_{2,t} = \lambda_{1,t+1} - \lambda_{2,t+1}$ for all t. Suppose that a pandemic occurs at $t = 0$ and the economy reverts back to its steady state at $t = T$ with T large enough. Then we can solve for $\{\lambda_{1,t} - \lambda_{2,t}\}_{t=0}^{T-1}$ backward from T, given $(c^w_{1,t+1}, c^n_{1,t+1}, c_{2,t+1}, p_{t+1})$ and w_{t+1} at each $t = 1, 2, \ldots, T-1$.

Given the value of $(\lambda_{1,t} - \lambda_{2,t})$, we can further determine w_t in each period. Recall the dynamic labor-leisure condition in (A.1), and replace substitute out the transition probabilities, $\hat{\pi}^w_{12}$ and $\hat{\pi}^n_{12}$, in its last two terms. Then (A.1) becomes

$$\begin{aligned}& m_{1,t}\big[u^1(c^w_{1,t}, 1-h) - u^1(c^n_{1,t}, 1)\big] \\ & \quad + \lambda_t m_{1,t}\big[w_t h - (c^w_{1,t} - c^n_{1,t})\big] + (\lambda_{1,t} - \lambda_{2,t})m_{1,t}\big(\hat{\pi}^w_{11}(\bar{m}_t) - \hat{\pi}^n_{11}(\bar{m}_t)\big) = 0 \qquad \text{(A.8)} \\ \Leftrightarrow \quad & \big[u^1(c^1_{1,t}, 1-h) - u^1(c^0_{1,t}, 1)\big] \\ & \quad + \lambda_t\big[w_t h - (c^1_{1,t} - c^0_{1,t})\big] + (\lambda_{1,t} - \lambda_{2,t})\big(\hat{\pi}^w_{11}(\bar{m}_t) - \hat{\pi}^n_{11}(\bar{m}_t)\big) = 0.\end{aligned}$$

Once $\{\lambda_t\}_{t=0}^T$ is solved in perfect foresight, we have the values of $c^w_{1,t}$ and $c^n_{1,t}$ for each period. This allows for determining w_t in (A.8) by knowing the value of $\lambda_{1,t} - \lambda_{2,t}$ solved in (A.7).

Lastly, we can also isolate $\lambda_{2,t}$ from $\lambda_{1,t} - \lambda_{2,t}$. In the last term of (A.4), replace $\hat{\pi}_{22}(\bar{m}_{t+1})$ with $1 - \hat{\pi}_{21}(\bar{m}_{t+1})$. Then (A.4) becomes

$$\lambda_{2,t} = \beta u^2(c_{2,t+1}, 1) - \beta\lambda_{t+1}c_{2,t+1} + \beta(\lambda_{1,t+1} - \lambda_{2,t+1})\hat{\pi}_{21}(\bar{m}_{t+1}) + \beta\lambda_{2,t+1}. \tag{A.9}$$

Given the value of $\lambda_{2,t}$ from (A.9), we can determine $\lambda_{1,t}$ by using the known value of $\lambda_{1,t} - \lambda_{2,t}$ in each t.

B Constrained Firm Decisions with Default Risk

In this section, we provide the details of our approach of using the FOCs derived from the incumbent firm's problem. As mentioned earlier, the key insight is that the marginal value of future cash-on-hand is binary, provided that the firm's value function is differentiable at its interior optimum. This allows us to consider all possible default state in the future period and then find the optimal decisions of capital and borrowing (k', b').

First, recall the firm's problem in the main text.

$$V_0(\epsilon_i, x; s) = \Big\{0, -\phi(\epsilon_i) + \pi_d x + (1-\pi_d) V_1(\epsilon_i, x; s)\Big\} \tag{4}$$

$$V_1(\epsilon_i, x; s) = \max_{k', b'} \Big[x - k' + q(\epsilon_i, k', b'; s) b' + \beta \sum_{j=1}^{N_\epsilon} \pi_{ij} V_0(\epsilon_j, x'_j; s')\Big] \tag{5}$$

subject to

$$(k', b') \in \Phi \equiv \{(k', b') : x - k' + q(\epsilon_i, k', b'; s) b' \geq 0\}$$
$$x'_j \equiv x(\epsilon_j, k', b'; s')$$
$$s' = \Gamma(s),$$

where the cash-on-hand is defined as $x(\epsilon, k, b; s) = z\epsilon f(k, N^w) - w(s)N^w + (1-\delta)k - b$. Suppose that V_1 in (4) and (5) are differentiable except at kinked points arising from discrete choices. Then the optimal choices of k' and b' solve the following necessary conditions.

$$[k'] \quad -1 + \frac{\partial q(\epsilon, k', b')}{\partial k'} b' + \beta \sum_{j=1}^{N_\epsilon} \pi_{ij} D_2 V_0(\epsilon_j, x'_j) \frac{\partial x'_j}{\partial k'} \geq 0 \tag{A.10}$$

$$[b'] \quad \frac{\partial q(\epsilon, k', b')}{\partial b'} b' + q(\epsilon, k', b') + \beta \sum_{j=1}^{N_\epsilon} \pi_{ij} D_2 V_0(\epsilon_j, x'_j) \frac{\partial x'_j}{\partial b'} \leq 0. \tag{A.11}$$

Given the default threshold $x_d(\epsilon)$ defined by (4), the differentiability of the value function implies that $D_2V_0(\epsilon, x) = 0$ for firms with $x \leq x_d(\epsilon)$ and $D_2V_0(\epsilon, x) = 1$ otherwise. Lastly, recall the discount loan price for a firm's chosen $\tilde{k}'$ and $\tilde{b}'$

$$q(\epsilon, \tilde{k}', \tilde{b}') = q_0\Big((1-p_d) + p_d \cdot \min\Big\{\frac{\theta \tilde{k}'}{\tilde{b}'}, 1\Big\}\Big), \tag{6}$$

where the risk-free loan price q_0 equals to β in the pre-pandmic economy.

We first analytically derive the *candidate* constrained choice of capital by defining the set of possible default state for a constrained firm with (ϵ, x). Given the set of constrained choices over default state, we check their sufficiency by choosing the best feasible one that maximizes the RHS of (5). This particularly requires the assumption that a set of default state is finite and discrete, which is naturally satisfied by assuming a discrete idiosyncratic shock process. Further, unlike the grid-search methods used in Khan, Senga, and Thomas (2016) and Ottonello and Winberry (2020), we additionally check the *consistency* of the constrained decisions with the assumed default state.

Consider a constrained firm continuing to the next period by choosing (k', b'). Let $s \in \{0, 1, 2, \ldots, N_\epsilon - 1\}$ be an index for a *default set* each with default probability $p_d \in [0, 1)$.[40] At each s, define $\Delta_s \equiv \{1, 2, 3, \ldots, s\}$ as the ordered set of indices for ϵ_j that induce the firm to default with $x'_j \leq x_d(\epsilon_j)$. The corresponding default probability, $p_d(s) \equiv \sum_{j \in \Delta_s} \pi_{ij}$, increases in s. Consider a specific $s \geq 1$ and suppose that a firm engages in *risky borrowing* such that $b' > \theta(1-\delta)k'$. Let $k'(s)$ and $b'(s)$ denote the corresponding decisions given s. Given $p_d(s) > 0$, the loan price schedule in (6) implies

$$q(\epsilon, k'(s), b'(s); s) = \beta(1 - p_d(s)) + \beta p_d(s)\theta \frac{k'(s)}{b'(s)} \tag{6'}$$

$$\Rightarrow \quad \frac{\partial q(\epsilon, k'(s), b'(s); s)}{\partial k'(s)} b'(s) = \beta p_d(s)\theta. \tag{A.12}$$

Using $D_2 V_0(\epsilon, x) \in \{0, 1\}$ and (A.12), the FOC for k' becomes

$$-1 + \beta p_d(s)\theta + \beta \sum_{j \notin \Delta_s} \pi_{ij} \frac{\partial x'_j(s)}{\partial k'(s)} \geq 0. \tag{A.10'}$$

Rearranging terms in the above, we can find the upper bound of the constrained capital at s.

$$k'(s) \leq \bar{K}(s). \tag{A.13}$$

As the value of a firm increases with k', it is clear that $\bar{K}(s)$ is chosen at s. Note that $k'(s)$ equals to the unconstrained capital choice K^w if $s = 0$ and hence $p_d(s) = 0$, and that $k'(s)$ is decreasing in s. Given $q(s)$ and $k'(s)$ at each (ϵ, x), the zero-dividend policy implies the constrained borrowing

[40] We abuse our notation here by denoting s as an index, not the aggregate state of our model described in the main text.

decision at s.

$$b'(s) = \frac{\big(1 - \beta\theta p_d(s)\big)k'(s) - x}{\beta(1 - p_d(s))} \tag{A.14}$$

We now have a set of $(k'(s), b'(s))$ for all possible s, which allows for computing the implied cash-on-hand in the future period $x'_j(s)$. Given the set of $x'_j(s)$ and current state (ϵ, x), we then check whether the constrained decisions are consistent with the assumed default set s. Let $\mathbf{d} \equiv \#\{j : x'_j(s) \leq x_d(\epsilon_j)\}$ be the number of default indices implied by $x'_j(s)$. When $\mathbf{d} = s$, the constrained decisions $k'(s)$ and $d'(s)$ are *consistent* with the assumed s.[41] Only among consistent cases over s, we find the optimal decisions that maximizes the firm value in (5). This indicates that all constrained decision rules and loan prices are functions of (ϵ, x).

C Algorithm for Solving the Transitional Dynamics

We numerically solve for the equilibrium transition path of the economy in perfect foresight. This requires solving the steady state of the Pre-pandemic economy and then finding the sequence of $\{\lambda_t, p_t\}_{t=1}^T$ that clears goods and labor markets in each period. The following describes the corresponding numerical algorithm.

1. Solve the pre-pandemic economy by bisecting $\lambda \in [\lambda_l, \lambda_r]$.
   (a) At each λ, get the steady-state wage rate $w = \frac{-\eta \log(1-h)}{h} \cdot \frac{1}{\lambda}$. Solve firm-level decisions and then find the stationary distribution μ.
   (b) Compute aggregate labor demand $N^d \equiv \int N^w d\mu$, and set p such that $N^d = ph$ given $h > 0$.
   (c) Given p, get the values of consumption and utility by health type and check the goods market clearing, $Y = C + I$ where $C = \frac{1}{\lambda}$.
2. Given guessed $\{\tilde{\lambda}_t, \tilde{p}_t\}_{t=1}^T$, solve the transitional dynamics in perfect foresight.
   (a) Set the initial and final values at $t = 1$ and $t = T$.
   (b) *Forward-Pass 1* for households from $t = 1$ to $T - 1$: from Equations (9) and (10), compute the evolution of health distribution $(m_{1,t}, m_{2,t})$ given $\{\tilde{p}_t\}_{t=1}^T$.
   (c) *Backward-Pass 1* for households from $t = T - 1$ to 1: at each t, get the difference of the marginal values $(\lambda_{1,t} - \lambda_{2,t})$ from Equation (A.7) and then calculate $\lambda_{2,t}$ by using (A.9). Compute the implied wage rate $\tilde{w}_t$ using (A.8), and get aggregate labor supply $N_t^s = \tilde{p}_t m_{1,t} h$.

[41] In practice, we impose a weaker version of the consistency requirement which robustly delivers the monotonicity of k' and V_1 in x.

(d) *Backward-Pass 2* for firms from $t = T - 1$ to 1: solve decisions at each t given $(z_t, \tilde{\lambda}_t, \tilde{w}_t)$ while updating value function $V_{1,t}$.

(e) *Forward-Pass 2* for firms from $t = 1$ to $T-1$: update firm distribution μ_t along the transition, and compute aggregates $\{K_t, Y_t, N_t^d\}_{t=1}^{T-1}$.

3. Check excess supply (ES) in each market for $t = 1, 2, \ldots, T - 1$, and then update the guessed sequence $\{\tilde{\lambda}_t, \tilde{p}_t\}_{t=1}^{T}$.