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THE SENSITIVITY ANALYSIS OF THE OPTIMAL LENGTH OF LIFE ～ THE NUMERICAL APPROACH ～

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Abstract

This paper solves the pure consumption Grossman model numerically following dynamic programming method, and undertakes sensitivity analysis of the optimal length of life. This is the first analysis that treats the determination of optimal length of life explicitly with numerical analysis. In numerical analysis, it is possible to treat the free terminal time problem directly, although previous analytical analyses have to consider a sequence of the modified fixed terminal time problem to treat the free terminal time problem. From the simulation results, decreases in the health capital depreciation rate or increases in the relative preference for health and the time discount rate make the individual live longer. Furthermore, longer lives significantly change the optimal allocation of consumption and the features of the optimal consumption path.

JEL classification: C61; C63; I10; I12

Keywords

Grossman Model, Optimal Endogenous Length of Life, The Sensitivity Analysis of the Optimal Length of Life

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Ⅰ Introduction

The number of the countries, which subsides the retired people or elderly through public sector, providing them with public pensions or public health care insurance[1)], has been increasing over the last few decades. Public pensions have grown, and now amount to one tenth of the gross domestic product of OECD countries. Moreover, in the United States, more than one third of total federal expenditures is devoted to income support and health care assistance for the old. As a result, these public assistance programs comprise a significant share of the income of retired people. The generosity of this system and the increase in subsidy per head is partly responsible for the growth of the longevity of retired people in OECD countries by between one and three months per cohort in the last decade (OECD health statistics in 1997)[2)]. However, at the same time, the increase in the longevity of the retired and of the subsidy per head has brought about a financial crisis in substantial public old age assistance programmes. In order to move to a more efficient system, it is essential to analyze the relationship between the public system and individual behavior affecting longevity, using the model that treats the longevity of the retired endogenously[3)].

Recently, Ehlrich and Chuma (1990), Ried (1998, 1999) and Grossman (1998) developed the Grossman model that was proposed by Grossman (1972a) in relation to endogenous length of life. Grossman (1972a) is the standard model that formalizes individual inter-temporal health investment behavior as an individual dynamic optimization problem. These studies use the analytical method and discuss the determination of the optimal length of life in a strictly delicate manner.

However, the studies suffer from certain problems or shortcomings concerning the sensitivity analysis of optimal longevity. Ehlrich and Chuma (1990) uses the continuous time model and shows how parameter changes affect individual optimal length of life, using the method of comparative dynamics proposed by Oniki (1972). However, Eisenring (1999) questions the plausibility of applying the Oniki's method to Ehlrich and Chuma (1990) model. Much more restrictive assumptions must be necessary, at least for the clear results presented in Ehlrich and Chuma (1990), although these are not explicitly mentioned in their paper.

While Ried (1998) uses the discrete time model and applies the comparative dynamic method, he focuses only on the case where marginal parameter changes are too small to alter the individual's optimal longevity. He can not show how changes in exogenous parameters affect the individual's optimal longevity, because the application of the comparative dynamic method to the discrete time model is impossible in the case where a parameter change causes the Lagrange multiplier to jump. As expounded in Ried (1999), sufficiently large non-marginal parameter changes alter individual longevity or terminal period and so causes this jump. However, as Grossman (1998) stresses, it is very important to consider the case where individual optimal loongevity responds elastically to change in exogenous parameters, i.e., the case of sufficiently large non-marginal change in parameters. The analytical method appears to suffer from certain limitations in dealing with this range of parameter changes in the discrete model.

Thus, the objective of this paper is to extend Ried(1999)'s contribution to the case of sufficiently large non-marginal parameter changes. To overcome the limitation of the analytical method, this paper solves the pure consumption Grossman model numerically, following the method of dynamic programming and undertakes the sensitivity analysis of optimal longevity.

The pure consumption Grossman model is a simplified version of the Grossman model. In the Grossman model, the marginal benefit of health capital consists of both monetary and psychic benefits. Monetary benefit are produced by the increase in working time stemming from longer healthy time. Psychic benefit are associated with these longer times of good health itself. The pure consumption Grossman model only focuses on psychic benefit and assumes that retired individuals are the targets for analysis[4)]. The pure consumption Grossman model is therefore appropriate for the analysis of the retired individual's health investment. Since the object of further analysis is mainly the longevity behavior of the retired old, we use the pure consumption model.

The merit of the numerical approach is as follows. First, it can explicitly show how exogenous parameter changes affect the optimal longevity. Second, although it is heavily related to the former, the algorithm for searching optimal longevity is different from the standard analytical approach.

It is well known that there are several methods for searching the optimal solution. Roughly there are two types of strategy. The simpler method computes the objective and constraint functions at several points and selects the feasible point that yields the greatest value. Judd (1998) calls this the 'comparison method'. The comparison method uses no derivative information. Moreover it is very powerful because it can establish a global maximum, supposing that grid search proceeds intensively and extensively over the feasible space. The second methods uses the information of the gradients or curvature of objective and constraints such as first order conditions (FOCs). Usually, the second method of establishing the optimum is more efficient in terms of calculation time than the first method if the optimization problem has a normal structure. However, the derivative information can establish only the local maximum. So, if the structure of the optimal problem is unusual, the former strategy is safer and better than the latter, as it excludes the local maximum.

Ried(1998,9) and Grossman(1998) propose the following method of establishing optimal longevity. They characterize the problem of individuals choosing their longevity as the 'free terminal time problem'. That is, the individual can actually choose the terminal period optimally. However, in the discrete time Grossman model, it seems impossible to treat the free terminal time problem directly with the analytical method. The authors thus use a step-by-step strategy. They consider the fixed terminal time problem with exogenous death constraints on the terminal health capital. This constraint forces the individual to die exogenously at terminal period, and individuals plan their optimal behavior in the right of this constraint. In addition, they consider the modified fixed terminal time problem. The modified fixed terminal time problem has no terminal health constraints. Thus individuals may die endogenously at the terminal period, or have sufficiently high health capital at terminal period. They then define that free terminal time problem as the modified fixed terminal time problem with endogenous death, which achieves the highest inter-temporal utility. Thus, they actually consider the sequence of the modified fixed terminal time problem with different terminal periods. They then solve the FOCs of each problem and evaluate each inter-temporal utility by reference to the solution satisfying each FOCs. Finally, they compare each utility level and choose the terminal period with the highest inter-temporal utility as the optimal length of life.

However, because the numerical approach adopted in this paper can treat the free terminal time problem directly, we do not have to follow the step by step strategy. Moreover, due to the numerical method, we do not have to use the information about FOC, curvature of the objectives and constraints, for solving free terminal time problem. In stead, we follow the comparison method. We directly evaluate the inter-temporal-utility of the free terminal time problem, and then choose the optimal path of health capital and financial assets and so does optimal length of life. Technically, in order to treat the free terminal time problem directly, the planning period must be sufficiently large. Intuitively, the planning period in the numerical dynamic programming method does not necessarily equal the individual optimal length of life, although in previous analytical analysis the terminal period become the individual optimal length of life.

It is impossible and pointless to determine which method is better for establishing the optimal length of life, as the latter approach seems analytically impossible. However, it may be safer and better to use our algorithm, which assures the global maximum, because the optimal problem has an unusual structure such that length of life is determined endogenously. Moreover, it enables us to under take sensitivity analysis of optimal longevity[5)].

To our knowledge, Picone et al.(1998) is the only analysis that solves the Grossman model analytically. It introduces uncertainty into the pure consumption Grossman model, and solves the model using the dynamic programming method[6)]. The authors analyze how the degree of uncertainty affects individual heath investment and saving behavior. Their analysis should be regarded highly, as there are few theoretical applied analyses of the Grossman model. However, because of their different concerns, they assume that the death rarely occurs endogenously. This paper, therefore, extends Piconne et al.(1998) to the study of endogenous optimal length of life. Eisenring (1999) points out that whether individual length of life is determined endogenously may influence the characteristics of an individual's optimal health investment path. Thus the endogenity of life may influence the optimal path[7)]. However, the objective of this paper is to demonstrate clearly the mechanism for determining optimal length of life. Thus it does not introduce any uncertainty into the model, and uses a simpler model than that of Picone et al.(1998). Hence it is impossible to directly compare our results with those of Picone et al.(1998). The introduction of uncertainty only induces earlier death only exogenously. Uncertainty is not, therefore, essential to determining optimal length of life.

This paper is organized as follows. Section II presents the model, and section III explain the dynamic programming procedure and the method of determining the optimal length of life. Section IV describes the parameter value setting, section Ⅴ and Ⅵ present and discuss the simulation results, and section Ⅶ summarizes the results and suggests further extensions.

Ⅱ The Model

This section presents the model that describes the optimal behavior of the retired individual. As individual has already retired, he/she has no labor income. In addition, uncertainty is not kept out of the model in order to demonstrate clearly the mechanism for determining endogenous death. The individual therefore chooses levels of consumption and health investment that maximize utility in retirement under several constraints, and builds up the optimal level of health capital and financial assets. More specifically, the model has a two-dimensional state space consisting of financial assets and health capital. The range of health capital H_t is defined from $\underline{H}$ to $\overline{H}$ and W_t is also defined from 0 to $\overline{W}$. In addition, individual is assumed to have no bequest motive: that is, it is optimal for the individual to exhaust all his or her financial assets during a lifetime. The individual therfore solves the following problem

$$\underset{\tau,\{C_t,M_t\}_{t=1}^{t=\tau}}{Max} \sum_{t=1}^{\tau} \beta^t u_t(C_t, H_t), \tag{1}$$

$$s.t. W_{t+1} = (1+r)W_t - C_t - M_t, \tag{2}$$

$$H_{t+1} = M_t + (1-\delta)H_t. \tag{3}$$

$$C_t > 0, M_t \geq 0 \;\; 1 \leq t < \tau \tag{4}$$

where β is the time discount rate, C_t is current consumption, M_t is the health investment and δ is the health capital depreciation rate. In addition, τ expresses the individual length of life (retirement periods). Thus from eq(1) the individual gains utility from consumption and health capital, and future utility is discounted by β. Eq(2) expresses the motion equation for financial assets and implies the individual's financial budget constraints. Eq (3) expresses the motion equation of health capital and implies the health investment function. This model concerns with the bang-bang control problem heavily discussed by Ehlrich and Chuma(1990) and thus uses the linear health investment function[8)] The depreciation of health capital δ is constant over time, and is different from the related studies. Suppose current health investment is 0, health capital decreases constantly through δ. Moreover, from eq(4), C_t must be positive and M_t must be non-negative during lifetime periods[9)]. In actual numerical analysis, the utility function is specified as

$u_t(H_t, C_t) = H_t^{\alpha} C_t^{1-\alpha}$ and $0 \le \alpha \le 1$ [10]. In this specification, α represents the preference for health capital. If α is higher, the preference for health capital is higher than for consumption. Finally, individual optimal length of life (retirement periods) in this model is defined as

$$\tau = \min\{t \mid H_t < \underline{H}\} \qquad (5)$$

Thus individual optimal length of life is determined when the optimal health capital becomes lower than the minimum level. For convenience, when the individual is dead, the set of health capital and financial asset is assumed as

$$D = \{H_t < \underline{\mathrm{H}}, \mathrm{W_t} \le 0 \mid \forall t > \tau\} \qquad (6)$$

In addition, the utility u_t in dead time and after become[11]

$$u_t = 0 \quad for\ t > \tau \qquad (7)$$

Ⅲ Solution procedure for the Model

This paper principally follows the method suggested by Bertsekas(1976). As is well known, dynamic programming consists of a backward process and a forward process. In this algorithm, the planning period, T, is set and the value function at T is defined as 0 by terminal condition. Note that T is different from τ, length of life. Given the value function at T, the value function at $T-1$ is calculated using the optimization problem at $T-1$. The value function for each proceeding period from $T-1$ to the initial period is calculated in a similar manner in $T-1$ steps. Then, in the forward process, the optimal problem is solved again from initial period to $T-1$, given initial value and the obtained value function in the backward process. As a result, the optimal values of state variables and control variables are determined sequentially. This model treats the free terminal time problem directly. Technically, in order to show the endogenous death in our algorithm, T must be large enough relative to τ for all the people with different initial endowments to die endogenouly. Thus, the appropriate size of T is determined according to the range of initial health capital and financial assets. Under a sufficiently large T, individual optimal length of life τ is determined through the optimal change in the feasibility set.

Ⅲ-1 Backward Solving

For the sake of clarity in the explanation below, we define the set of health capital and financial asset in t as

$$\psi = \lfloor \underline{H}, \overline{H} \rfloor \times \lfloor 0, \overline{W} \rfloor \qquad (8)$$

The backward process is expressed as follows. By terminal condition, the value function in

T , $V_T\left(H_T, W_T\right)$, is defined as

$$V_T\left(H_T, W_T\right) = 0 \tag{9}$$

Thus eq(9) indicates that all the values of $V_T\left(H_T, W_T\right)$ at any combination of H_T and W_T in ψ, are evaluated as 0 by terminal condition. Moreover, when the certain combination of H_t and W_t is the element of D or death status at t, the value of $V_t\left(H_t, W_t\right)$ of this H_t and W_t is also

$$V_t\left(H_t, W_t\right) = 0 \tag{10}$$

The optimal problem in T -1, using the value function in T, is expressed as follows

$$\max_{c_{T-1}, M_{T-1}} H_{T-1}{}^{\alpha} C_{T-1}{}^{1-\alpha} + \beta V_T(H_T, W_T) \tag{11}$$

$$s.t. C_{T-1} = (1+r)W_{T-1} - W_T - M_{T-1} \tag{12}$$

$$M_{T-1} = H_T - (1-\delta)H_{T-1} \tag{13}$$

$$M_{T-1} \geq 0, C_{T-1} > 0 \tag{14}$$

Thus the value function in T -1is evaluated as

$$V_{T-1}(H_{T-1}, W_{T-1}) = \max_{c_{T-1}, M_{T-1}} \left\{ H_{T-1}{}^{\alpha} C_{T-1}{}^{1-\alpha} + \beta V_T(H_T, W_T) \right\} \tag{15}$$

$$s.t C_{T-1} = (1+r)W_{T-1} - W_T - M_{T-1} \tag{16}$$

$$M_{T-1} = H_T - (1-\delta)H_{T-1} \tag{17}$$

$$M_{T-1} \geq 0, C_{T-1} > 0 \tag{18}$$

By substituting eq(16) and eq(17) into eq(15), we obtain

$$V_{T-1}(H_{T-1}, W_{T-1}) =$$
$$\max_{H_T, W_T} \left\{ H_{T-1}{}^{\alpha} ((1+r)W_{T-1} - W_T - H_T + (1-\delta)H_{T-1})^{1-\alpha} + \beta V_T(H_T, W_T) \right\} \quad (19).$$
$$\text{s.t.} H_T - (1-\delta)H_{T-1} \geq 0, (1+r)W_{T-1} - W_T - H_T + (1-\delta)H_{T-1} > 0$$

As a result $V_{T-1}\left(H_{T-1}, W_{T-1}\right)$ is defined over all combinations of H_{T-1} and W_{T-1} in ψ. Eq(19) expresses the evaluation of the optimal mapping from given H_{T-1} and W_{T-1} to optimal combination of H_T and W_T. Optimal H_T and W_T is chosen among the feasible combination of H_T and W_T, which satisfies eq(16) and eq(17), over the potential set of $\psi \cup D$. Note that D is also included in the feasible combination in T .

For example, the set of H_{T-1} and W_{T-1} over ψ, which satisfies the condition that $\underline{H} - (1-\delta)H_{T-1} \geq (1+r)W_{T-1}$, is defined as

$$B_{T-1} = \left\{ H_{T-1,} W_{T-1} \in \psi \mid \underline{H} - (1-\delta)H_{T-1} \geq (1+r)W_{T-1} \right\} \tag{20}$$

From eq(17), $\underline{H} - (1-\delta)H_{T-1}$ expresses health investment in T -1 which is necessary to maintain $\underline{H}$ in T . From the model's assumption, in case that $\underline{H}$ is not maintained, the individual dies and the utility level in t is 0. Thus, as a result of the optimal problem in T -1, when the combination of H_{T-1} and W_{T-1} satisfies B_{T-1}, optimal consumption and health investment in T -1 become

$$\begin{cases} C_{T-1} = (1+r)W_{T-1} \\ M_{T-1} = 0 \end{cases} \tag{21}$$

Thus, W_T become 0 and H_T become lower than $\underline{H}$. This means that the set of D is chosen through optimal behavior following the theoretical model in section Ⅱ .Therefore, $V_{T-1}(H_{T-1}, W_{T-1})$ is redefined as follows

$$V_{T-1}(H_{T-1}, W_{T-1}) = H_{T-1}{}^{\alpha}((1+r)W_{T-1})^{1-\alpha} \tag{22}$$

Intuitively, eq(22) expresses the value of $V_{T-1}(H_{T-1}, W_{T-1})$ at some combinations of H_{T-1} and W_{T-1} over ψ_{T-1} which choose the death status endogenously in T.

However, in T, all the values of $V_T(H_T, W_T)$ at any combination of H_T and W_T in ψ, are evaluated as 0 by terminal condition. Thus, the value function in T evaluates the element of both ψ and D as 0. The terminal condition also indicates that the utility at T cannot be obtained by the choice of any combination of H_T and W_T in ψ. This terminal condition is exogenous constraints, which is not shown in the theoretical model presented in sectionⅡ.This exogenous condition is added in order to apply the numerical dynamic programming method to solve the model. However this terminal condition actually influences the optimal choice process at T -1.

From eq(20), $\psi \setminus B_{T-1}$ is defined as

$$\psi \setminus B_{T-1} = \{(H_{T-1,} W_{T-1}) \in \psi_{T-1} \mid \underline{H} - (1-\delta)H_{T-1} < (1+r)W_{T-1}\} \tag{23}$$

Because of the terminal condition, given any combination of H_{T-1} and W_{T-1} in $\psi \setminus B_{T-1}$, consumption and health investment in T -1 become

$$\begin{cases} C_{T-1} = (1+r)W_{T-1} \\ M_{T-1} = 0 \end{cases} \tag{24}$$

Therefore, $V_{T-1}(H_{T-1}, W_{T-1})$ is also redefined as

$$V_{T-1}(H_{T-1}, W_{T-1}) = H_{T-1}{}^{\alpha}((1+r)W_{T-1})^{1-\alpha} \tag{25}$$

Due to eq(24), given any combination of H_{T-1} and W_{T-1} in $\psi \setminus B_{T-1}$, W_T become 0. On the other hand, from eq(17) and eq(24), H_T becomes

$$\begin{cases} \mathrm{H_T} = (1-\delta)H_{T-1} & \mathit{if}\ \ H_{T-1} \geq (1-\delta)^{-1}\underline{H} \\ \mathrm{H_T} \leq \underline{H} & \mathit{if}\ \ H_{T-1} < (1-\delta)^{-1}\underline{H} \end{cases} \tag{26}$$

Thus, supposing H_{T-1} is large enough, H_T can never fall below $\underline{H}$. In short, the value of $V_{T-1}(H_{T-1}, W_{T-1})$ at some combinations of H_{T-1} and W_{T-1} in $\psi \setminus B_{T-1}$ is potentially biased by the terminal condition, which is the exogenous constraint and is not assumed in the free terminal time problem presented in section Ⅱ. Because the terminal condition induces the choice as eq(24). Moreover, from eq(24), W_T become 0 even any given H_{T-1} and W_{T-1} in ψ, and thus cannot maintain positive consumption and non-negative health investment.. From the clarity of the expalnation, the choice that W_T=0 due to termianl condition through eq(24) is defined as exogenous death. Thus eq(25) express the value function of H_{T-1} and W_{T-1} which chooses exogenous death in T.

Similarly, the value function in T-2 is evaluated through the optimal problem in T-2.

$$V_{T-2}(H_{T-2},W_{T-2}) = \max_{c_{T-2},M_{T-2}} \left\{ H_{T-2}{}^{\alpha} C_{T-2}{}^{1-\alpha} + \beta V_{T-1}(H_{T-1},W_{T-1}) \right\} \tag{27}$$

$$s.t C_{T-2} = (1+r)W_{T-2} - W_{T-1} - M_{T-2} \tag{28}$$

$$M_{T-2} = H_{T-1} - (1-\delta)H_{T-2} \tag{29}$$

$$M_{T-2} \geq 0, C_{T-2} > 0 \tag{30}$$

By substituting eq (28) and eq(29) into eq(27)

$$V_{T-2}(H_{T-2},W_{T-2}) =$$
$$\max_{H_{T-1},W_{T-1}} \left\{ H_{T-2}{}^{\alpha}((1+r)W_{T-2} - W_{T-1} - H_{T-1} + (1-\delta)H_{T-2})^{1-\alpha} + \beta V_{T-1}(H_{T-1},W_{T-1}) \right\} \quad (31)$$
$$\text{s.t.} H_{T-1} - (1-\delta)H_{T-2} \geq 0, (1+r)W_{T-2} - W_{T-1} - H_{T-1} + (1-\delta)H_{T-2} > 0$$

Similar to the value function in T-1, eq(31) expresses the evaluation of the optimal mapping from given H_{T-2} and W_{T-2} in ψ to optimal combination of H_{T-1} and W_{T-1}. Especially, given some combination of H_{T-2} and W_{T-2} in ψ, the element of D is chosen in T-1 as the optimal choice, $V_{T-1}(H_{T-1},W_{T-1})$ become 0. Thus, as a result of optimal problem in T-2, following equations are satisfied.

$$\begin{cases} C_{T-2} = (1+r)W_{T-2} \\ M_{T-2} = 0 \end{cases} \tag{31}$$

Thus, eq(31) is redefined as

$$V_{T-2}(H_{T-2},W_{T-2}) = H_{T-2}{}^{\alpha}((1+r)W_{T-2})^{1-\alpha} \tag{32}$$

Intuitively, eq(32) expresses the value of $V_{T-2}(H_{T-2},W_{T-2})$ at H_{T-2} and W_{T-2} over ψ which chooses the death status endogenously in T-1. In addition, although exogenous death may not influence the optimal problem in T-2 directly, it influences it indirectly through $V_{T-1}(H_{T-1},W_{T-1})$. Eq(32) includes $V_{T-1}(H_{T-1},W_{T-1})$, which evaluates the optimal mapping from given H_{T-1} and W_{T-1} selected in the optimal problem in T-1 to optimal combination of H_T and W_T. Thus if the value of $V_{T-1}(H_{T-1},W_{T-1})$ is not biased by terminal condition, eq(32) evaluates the optimal path of all health capital and financial asset of all H_{T-2} and W_{T-2} in ψ

from T -2 to T. Conversely, if the value of $V_{T-1}(H_{T-1}, W_{T-1})$ is biased by terminal condition, the value of $V_{T-2}(H_{T-2}, W_{T-2})$ is also biased.

From the above discussion, supposing the value of $V_{T-1}(H_{T-1}, W_{T-1})$ is obtained without the bias of the terminal condition or exogenous death, $V_{T-2}(H_{T-2}, W_{T-2})$ can determine the optimal path of health capital and financial asset of all combination of H_{T-2} and W_{T-2} in ψ from T -2 to T. Of course, this expresses the optimal path in the free terminal time problem presented in sectionⅡ. By performing a similar calculation from T -2 backwards to the initial period, the value function in initial period, $V_1(H_1, W_1)$, is calculated. This, $V_1(H_1, W_1)$ can determine the optimal path of health capital and financial asset of all combination of H_1 and W_1 in ψ from initial to T.

In order to avoid the effect of the terminal condition on the value of $V_1(H_1, W_1)$ at any combination of H_1 and W_1 in ψ, the optimal path of health capital and financial asset, which $V_1(H_1, W_1)$ determines, have to be the element of D before T endogenously. More specifically, it is appropriate for the optimal health capital and financial assets path for any combination of H_1 and W_1 in ψ to be at least the element of B_{T-1} in T -1. In short, they choose the element of D in T. Supposing this condition is satisfied, the value of $V_1(H_1, W_1)$ at any combination of H_1 and W_1 in ψ can be evaluated without any bias from the terminal condition upon the sequence of the optimal choice. To satisfy this condition, T must be large enough not to affect the value of $V_1(H_1, W_1)$ at all combination of H_1 and W_1 in ψ. In practice, it is confirmed by performing the sensitivity analysis concerning T to evaluate the effect of $V_1(H_1, W_1)$, whether this condition is satisfied or not.

From the aspect of the optimal longevity, this backward process is loosely translated as follows[12)]. Let $V_t(H_t, W_t)$ as 0. This means individual dies at t. In this case, intuitively, the backward procedure from t to initial period means the process of establishing initial health capital and financial asset which achieve zero value function optimally in t. At the same time, it predicts the optimal sequence of health capital and financial asset. This indicates that backward process can establish the individual who dies optimally in t. Thus under the planning period of T, the backward process can establish not only the individual who dies optimally in T, but also those who dies optimally in t before T. This indicates the backward process can establish the optimal longevity of every one in initial period, so long as T is set at a sufficiently large value according to the range of initial endowments.

Ⅲ-2 Forward Simulation

Given the value of $V_t(H_t, W_t)$ for all t from the above procedure, the individual whose initial state variables are H_1 and W_1 solves the following problem to show explicitly the optimal path of health capital and financial assets:

$$V_1(H_1,W_1)=\max_{H_2,W_2}\left\{H_1{}^{\alpha}((1+r)W_1-W_2-H_2+(1-\delta)H_1)^{1-\alpha}+\beta V_2(H_2,W_2)\right\}$$
$$s.t.\{H_2,W_2\}\in\Lambda_2 \tag{34}$$

where Λ_2 shows the feasible set of health capital and financial assets in the second period. Λ_2 consists of survival, Ω_2 and death, D. Thus Λ_2 is defined as

$$\Lambda_2=\Omega_2\cup D \tag{35}$$

The individual chooses optimal health capital and financial assets in the second period to maximize the utility in retirement periods. The set of survival in the second period is expressed as

$$\Omega_2=\left\{\max\left[\underline{H},(1-\delta)H_1\right]\le H_2<(1+r)W_1+(1-\delta)H_1, 0\le W_2<(1+r)W_1\right\} \tag{36}$$

where $(1-\delta)H_1$ expresses health capital in the second period with no health investment at the first period. Supposing $(1-\delta)\mathrm{H}_1$ is below $\underline{H}$, it is not included in Ω_2. $(1+r)W_1+(1-\delta)H_1$ expresses the health capital in the second period, if all the interest and principle is spent initially. However, it is also not included in Ω_2, because consumption has to be positive for the individual to survive. In addition, financial assets in the second period are 0, if the individual exhausts all initial financial assets and interest through initial consumption and health investment. On the other hand, the financial assets in the second period are $(1+r)W_1$, with no initial health investment and consumption. However, it is also not included in Ω_2 because of the strictly positive constraints for consumption. If it is infeasible to choice an element of Ω_2, the individual dies and choices an element of D. In the case of D, the value function in the second period is expressed as

$$V_2(H_2,W_2)=0 \tag{37}$$

Therefore, the distinction between death and survival is expressed as the differences in the size of the feasibility set.

Under Λ_2, optimal health capital and financial asset in the second period are determined simultaneously as follows

$$\{H_2(H_1,W_1),\mathrm{W}_2(H_1,W_1)\}$$
$$=\begin{cases}\arg\max\limits_{H_2,W_2}(H_1{}^{\alpha}((1+r)W_1-W_2-H_2+(1-\delta)H_1)^{1-\alpha}+\beta V_2(H_2,W_2)) \\ \qquad\qquad\qquad\qquad if\ \Lambda_2\setminus D\neq\phi \\ D \\ \qquad\qquad\qquad\qquad if\ \Lambda_2\setminus D=\phi\end{cases} \tag{38}$$

In the case that $\Lambda_2\setminus D=\phi$, individual cannot choose an element of Ω_2 and dies before or at 2. Thus health capital and financial assets are defined as an element of D.

Moreover, in the case of survival, $\Lambda_2\setminus D\neq\phi$, consumption and health investment at the first period are expressed as

$$\begin{cases} C_1(H_1,W_1) = (1+\mathrm{r})W_1 - W_2(H_1,W_1) - H_2(H_1,W_1) + (1-\delta)H_1 \\ M_1(H_1,W_1) = \mathrm{H}_2(H_1,W_1) - (1\text{-}\delta)H_1 \\ \qquad\qquad\qquad\qquad if\ \ \Lambda_2 \setminus D \neq \phi \end{cases} \tag{39}$$

Contrary when individual dies just at 2nd period, these are expressed as

$$\begin{cases} C_1(H_1,W_1) = (1+\mathrm{r})W_1 \\ M_1(H_1,W_1) = 0 \\ \qquad\qquad\qquad\qquad if\ \ \Lambda_2 \setminus D = \phi \end{cases} \tag{40}$$

In this case, individual consumes all the initial financial assets and interest income and never undertakes health investment.

Suppose an individual with certain initial endowments satisfies the condition that $\Lambda_2 \setminus D = \phi$, and T is defined appropriately to satisfy $2 < T$. In this case, the individual's optimal length of life, $\tau = 2$, is determined endogenously. Moreover, in the actual calculation process, this individual technically continues to choose an element of D and gains 0 utility during the period from 2 to T, as if he or she lived after period 2.

A similar process is performed from the second to T-1 period, and the sequence of optimal health capital and financial assets can be calculated. In addition, consumption and health investment are also determined optimally. The development of Λ_{t} depends heavily upon initial endowments. It means that the optimal length of life and the optimal path differs by the difference in initial endowments. Therefore, technically T must be large enough to satisfy the condition that all individual with different initial health capital and financial assets die endogenously and satisfy that $\tau < T$.

Generally, the optimal problem in t is presented as

$$\begin{aligned} &V_t(H_t,W_t) = \max_{H_{t+1},W_{t+1}} \left\{ H_t{}^{\alpha}((1+r)W_t - W_{t+1} - H_{t+1} + (1-\delta)H_t)^{1-\alpha} + \beta V_{t+1}(H_{t+1},W_{t+1}) \right\} \\ &s.t. \{H_{t+1},W_{t+1}\} \in \Lambda_{t+1} \end{aligned} \tag{41}$$

where $\Lambda_{\mathrm{t+1}}$ consists of the case of survival $\Omega_{\mathrm{t+1}}$ and death D, and is thus expressed as

$$\Lambda_{\mathrm{t+1}} = \Omega_{\mathrm{t+1}} \cup D. \tag{42}$$

The set of survival $\Omega_{\mathrm{t+1}}$ is defined as

$$\Omega_{\mathrm{t+1}} = \left\{ \max\left[\underline{H}, (1-\delta)H_{\mathrm{t}}\right] \leq H_{\mathrm{t+1}} < (1+r)W_{\mathrm{t}} + (1-\delta)H_{\mathrm{t}}, 0 \leq W_{\mathrm{t+1}} < (1+r)W_{\mathrm{t}} \right\} \tag{43}$$

Then, suppose $\Lambda_{t+1} \setminus D = \phi$, the value function in $t+1$ is defined as

$$V_{t+1}(H_{t+1},W_{t+1}) = 0 \tag{44}$$

Under $\Lambda_{\mathrm{t+1}}$, optimal health capital and financial assets are determined simultaneously as follows

$$\{H_{t+1}(H_t, W_t), W_{t+1}(H_t, W_t)\}$$

$$= \begin{cases} \underset{H_{t+1}, W_{t+1}}{\arg\max}(H_t{}^{\alpha}((1+r)W_t - W_{t+1} - H_{t+1} + (1-\delta)H_t)^{1-\alpha} + \beta V_{t+1}(H_{t+1}, W_{t+1})) \\ \qquad\qquad\qquad\qquad\qquad if\ \Lambda_{t+1} \setminus D \neq \phi \\ D \\ \qquad\qquad\qquad\qquad\qquad if\ \Lambda_{t+1} \setminus D = \phi \end{cases} \tag{45}$$

When $\Lambda_{t+1} \setminus D = \phi$, the individual dies before or at $t+1$, and health capital and financial assets are defined as D. Moreover, $\{H_t, W_t\} \in D, \{H_s, W_s\}_{s \geq \tau} \in D$ technically means that the individual continues to choose the element of D after death. We can, therefore, treat dead people's behavior as well as that of survivors following eq.(44). In this sense, death and survival are continual phenomena.

Moreover, when the individual survives in $t+1$, consumption and health investment in t are expressed as

$$\begin{cases} C_t(H_t, W_t) = (1+r)W_t - W_{t+1}(H_t, W_t) - H_{t+1}(H_t, W_t) + (1-\delta)H_t \\ M_t(H_t, W_t) = H_{t+1}(H_t, W_t) - (1-\delta)H_t \\ \qquad\qquad\qquad\qquad if\ \Lambda_{t+1} \setminus D \neq \phi \end{cases} \tag{46}$$

Conversely, when individual dies in just $t+1$, consumption and health investment in t are expressed as

$$\begin{cases} C_t(H_t, W_t) = (1+r)W_t \\ M_t(H_t, W_t) = 0 \\ \qquad\qquad if\ \Lambda_{t+1} \setminus D \neq \phi\ and\ B_t \end{cases} \tag{47}$$

where B_t is defined in eq(20). When $\underline{H} - (1-\delta)H_t$ is more than $(1+r)W_t$, the individual cannot maintain minimal health due to budget constraints. In this case, the individual consumes his or her entire financial assets and interest in t, and never undertakes health investment. Finally, when the individual dies before the $t+1$ period, both consumption and health investment in t should be 0, That is

$$\begin{cases} C_t(H_t, W_t) = 0 \\ M_t(H_t, W_t) = 0 \\ \qquad\qquad if\ \Lambda_s \setminus D \neq \phi \quad (s < t+1) \end{cases} \tag{48}$$

Therefore, the dead individual chooses the element of D during the period of $\{t \mid \tau \leq t \leq T\}$ from eq(44), and gains 0 utility. Since the development in Λ_{t+1} depends heavily upon initial endowments, T must be set up appropriately according to the range of initial endowments so that all initial individuals can satisfy the condition that $\Lambda_t \setminus D_t = \phi$ before T. If T is not large enough for all people to die endogenously, some individuals who survive in T die exogenously by the terminal condition that $V_T(H_T, W_T) = 0$. In the case of exogenous death[13)], the individual plans his or her optimal behavior by regarding the 0 value function as given. Thus, the terminal condition induces this individual to exhaust his or her financial assets more rapidly so that he or she has no financial asset at T as shown in eq(24). This path may not coincide with the optimal path when T is sufficient large and the individual experiences endogenous death.

Ⅳ Parameter Values and Numerical Evaluation

This model may be relatively simpler than that of Picone et al.(1998) for the following reasons. First, as one of the objectives of this model is to show the mechanism of the determination of optimal length of life clearly and perform the sensitivity analysis for some parameters over reasonable range, thus the simpler model is appropriate for this purpose. Secondly, there are no adequate empirical studies for the parameter used in the Grossman model. The model consists of the following parameters defined above: health capital depreciation rate (δ); the preference parameter for health capital (α); time discount rate (β); and interest rate (r). However, there exist no adequate empirical studies of the possible values of α and δ. Grossman (1972a) is the only investigation with estimates, but the estimation method is not so appropriate. Picone et al. (1998) set α as 0.6[14)] and made δ a function of age[15)] in their benchmark case following Grossman (1972a). However, the plausibility of these values is not well established, and much more precise empirical studies are necessary.

In the benchmark case, α is 0.7, δ is 0.3, β is 0.9 and r is 0. Moreover, $\underline{H}$ is 0.5, $\underline{W}$ is 0, $\overline{H}$ is 99.5 and $\overline{W}$ is 99 and 100 equal-scaled grid[16)]. Thus the health capital of entire periods (from initial to T) are divided into 100 different equally-levels from 0.5 to 99.5. Financial asset are also divided into 100 equally spaced from 0 to 99. This implies that there exist 10000 combinations over every period. Especially initial 10000 combination is interpreted as 10 000 people with different initial combinations who plans their optimal life after the retirement. In other words, we assume the society that retired-people distributes uniformly over heath capital and financial asset. In addition, this paper does not consider the range that the financial assets become negative. It is impossible to return resources, even if the retired people borrow from financial market. Because the they only gain the interest income from positive financial asset

The assumption that δ is the increasing function of age is unnecessary complexity for the finite length of life in the pure consumption Grossman model. Thus we set δ as constant value of 0.3 and relatively high for the clarity of endogenous optimal length of life. Moreover, we set r as 0, since. the value of r heavily affects the characteristics of optimal health investment path under a linear health investment function. As the purpose of this paper is to demonstrate the determination of optimal length of life, we do not treat this matter in depth. We then set r as 0, and discuss the optimal length of life under a linear health investment function. While the assumption that retired-people distributed uniformly over heath capital and financial asset is unrealistic, it is unnecessary to use a more realistic distribution for our purposes as a benchmark case. Nevertheless, it is easy to obtain the result from a realistic distribution only by weighting the results from a uniform distribution[17)].

Finally, this paper undertakes sensitivity analysis in order to assure everyone's endogenous death and to confirm the robustness of the model. First, we discuss the value of T. If T is of sufficiently large value, everyone in initial period dies endogenously and satisfies the condition that $\Lambda_t \setminus D = \phi$ before T. Thus optimal length of life is determined endogenously. Theoretically, it is preferable to set T at a sufficiently large and nearly infinite value, since an individual with any level of initial health capital and financial assets can die endogenously. However, too large value for T takes too much time and is difficult in practice. Thus, we attempt sensitivity analysis concerning T from 5 to 30 for every period, given the parameters held constant at the benchmark case, and then establish the appropriate value of T[18)]. The appropriate value of T is determined when the value of $V_1(H_1, W_1)$ over ψ is not influenced by the length of T. Second, after fixing T at 25, we undertake sensitivity analysis concerning δ, α and β respectively. The range of δ is from 0.5 to 0.1, of α from 0.5 to 0.9, and of β from 0.5 to 0.9. The size of the steps is 0.1.

Ⅴ Sensitivity analysis concerning T and endogenous death

This section tries to find the appropriate value of T under which the value of $V_1(H_1, W_1)$ at any combination of H_1 and W_1 in ψ is not influenced by the terminal condition. This indicates that all initial people die endogenously before T. The endogenous death of all initial people is assured by sensitivity analysis of $V_1(H_1, W_1)$ at all elements in ψ concerning T. If the value of $V_1(H_1, W_1)$ over ψ is not influenced by the length of T and converges to constant, all $V_1(H_1, W_1)$ over ψ are not biased by terminal condition at all. Under such $V_1(H_1, W_1)$, the optimal path, determined by $V_1(H_1, W_1)$, follows the free terminal time problem presented in section Ⅱ.

In order to demonstrate the characteristics of the model, we first look at the typical individual with a certain combination of H_1 and W_1 in ψ and find the T under which the value of $V_1(H_1, W_1)$ at given H_1 and W_1 converge to the constant. In other words, we try to find the T that is sufficiently large for this individual to die endogenously. Then we move to the discussion of the value of T under which the value of $V_1(H_1, W_1)$ at any combination of H_1 and W_1 over ψ becomes constant, or all initial people die endogenously.

Figure 1 presents the relationship between the value of $V_1(H_1, W_1)$ of the typical individual and T. This typical individual has 60th level of H_1 and W_1, so the value of each is 59.5 and 59 respectively. When T is under 11, the value of $V_1(H_1, W_1)$ increases as T increases. However, when T is over 11, it converges to a constant. Therefore, if T is over 11, the value of $V_1(H_1, W_1)$ of this individual is not influenced by the terminal condition. Figure 2 shows the relationship between this individual's length of life and T to explain the intuitive meanings of Figure1. When T is less than 11, this individual length of life τ is equal to T. This indicates that the individual dies exogenously by the terminal condition. However when T is more than 11, the individual's length of life τ become less than T by choosing an element of D endogenously before T. This indicates that the individual dies endogenously.

Figure 3 to Figure 6 show the relationship between the financial assets, consumption, health investment , and health capital path of this individual and T. The solid line represents the path when T is 5. Similarly, dash, dash-dot, and dot lines present the case of 7, 9,11 and 15 respectively. As shown explicitly in these figures, dash-dot and dot lines have exactly the same lengths and values at each period. Therefore, if T is more than 11, individual optimal paths of financial assets, consumption and health capital converge on the same line. Moreover, the path of these variables is co0nsistent with the predicted results in the section Ⅲ. For example, as T gets shorter, health investment and especially consumption in the initial period increase, as shown in Figure 4 and 5. As a result, from Figure 3, financial asset decrease more rapidly as T gets shorter, and fall to nearly 0 at each T. It is consistent with the predicted result in eq(24) because of the inducement effect of the terminal condition of exogenous death. The differences in the paths of the solid line and the dot line apparently express the effect of the terminal condition on the optimal path. Moreover, the obtained health capital path is also consistent with the predicted result in eq (26). From Figure 7, supposing H_{T-1} is sufficiently large, H_T remains to be positive.

The results presented above show that the value of $V_1(H_1, W_1)$ of this typical individual becomes constant under a sufficiently large T. This means that optimal length of life is determined endogenously under a sufficiently large T. In addition, the value of $V_1(H_1, W_1)$ of this typical individual indicates the welfare level of his or her life in retirement [19)]. Thus, when the individual dies exogenously by the terminal condition, individual welfare falls below the level in the case of endogenous death. This is because the terminal condition induces the individual to consume the financial assets more rapidly in order to exhaust them completely at T. However, neither does health capital, since individual gains utility from health capital, and thus has an incentive to maintain health through the initial health investment. Moreover, the results of converged welfare under a large T can be translated as follows. As the individual can choose the longevity he or she wants, the individual determines it so as to maximize lifetime welfare. Therefore, even if T gets larger, individual optimal length of life does not alter, and neither does individual welfare.

Next, we move to the discussion of the value of T under which the value of $V_1(H_1, W_1)$ at any combination of H_1 and W_1 in ψ becomes constant or all initial people die endogenously. Figure 7 presents the relationship between the average value of $V_1(H_1, W_1)$ over all the combination of H_1 and W_1 in ψ and T. As T becomes larger, the average value of $V_1(H_1, W_1)$ increases monotonically. Then, when T is over 13, it converges on the constant Therefore, supposing T is over 13, the value of $V_1(H_1, W_1)$ at any combination of H_1 and W_1 in ψ is not influenced by the terminal condition. As a result, as explained abpve, all initial people die endogenously if T is more than 13. To confirm this results, Figure 8 shows the relationship between the social average longevity and T. Social average longevity represents average retirement periods of 10000 retired-people at the initial period. From Figure 8, as T lengthens from 5 to 30, the numbers of retired people who die exogenously by the terminal condition decrease, and thus social average longevity also lengthens linearly. Then, when T is more than 13, social average longevity converges on the constant. Therefore, all retired people die endogenously, when T is more than 13.

From these results, if T is more than 13, all retired people die endogenously in the whole society, and thus everyone determines his or her longevity so as to maximize welfare during retirement periods. Therefore, the difference in T does not alter either social average longevity or welfare. As a result, when T is 25, we can confirm that all retired-people die endogenously in any parameter value of the following sensitivity analysis.

Ⅵ The sensitivity analysis of the optimal path

This section undertakes sesitivity analysis for various parameters. Since T is large enough, length of life is determined endogenously. First, the optimal length of life and the optimal path at the benchmark case are presented. Then we undertake sensitivity analysis for δ, α and β sequentially.

Ⅵ-1 The benchmark case

This sub-section describes the cumurative death rate in the whole society and the optimal path in the benchmark case. At the benchmark, α is 0.7, δ is 0.3, β is 0.9 and r is 0. We also set T at 25. For the clarity of description, this paper presents figures for the individuals whose initial levels of health capital and financial asset are 20th, 40th, 60th, 80th, and 100th, respectively. Moreover the initial 60th case is mainly referred to in words.

The cumurative death rate at t in the society is defined as the ratio of the cumulative number of those who die until t period to the retired people in initial period. In the benchmark case, as shown in Figure 9, everyone has died by the 13th period, and thus the average length of life becomes about 9.4 periods. Figure 10 shows the individual optimal health investment path. Almost all health investment is undertaken at the initial period. In the case of 60th initial stocks, initial health investment occupies the 26.9% of initial financial assets, becoming nearly 0 during the remainder of retirement. As a result, as shown in Figure 11, optimal health capital decreases exponentially through δ after the second period. Similarly, from Figure 12, optimal finantial assets decrease exponentially. This means that people use up financial assets to finance health investment and consumption. Optimal consumption also decreases, although more gradually than financial assets, as shown in Figure 13.

In the above figures, endogenity of longevity is confirmed by the fact that the optimal path of health capital becomes 0 before the 25th period. Principally, the time discount rate and the linear health investment function explain the features in the dynamics of the endogenous variables. Owing to the time discount rate, an individual prefers to maintain higher health capital and to consume much in earlier periods. Since the health investment function is linear, the current health investment additively increases the health capital of the next period if the constant health capital depreciation effect is ignored. This means that the marginal effect of health investment on health capital does not change due to the level of the current health capital. Therefore health investment is mostly undertaken in the initial period; thereafter it becomes nearly 0, and thus health capital decreases exponentially through δ after second period. This feature of health investment, which becomes 0 in certain period, is consistent with the Ehlrich and Chuma (1990) which emphasized the point of the bang-bang control problem under the linear health investment function. Consumption also decreases because of the time discount rate, but more gradually, partly because of the specification of the utility function. As a result, financial assets also decrease exponentially. These simulation results are almost consistent with the results at Picone et al. (1998). However, as Picone et al. (1998) assume a high negative value of disutility for death so as to avoid endogenous death- moreover, their specification of health capital depreciation and utility functions also differs- there are some differences in the characteristics of optimal health capital and consumption path[20].

Ⅵ-2 Changes in the health capital depreciation rate (δ)

This subsection first analyses how the depreciation for health capital, δ, affect the optimal path of the typical individual when changing from 0.5 to 0.1 by the step of 0.1. Other parameters are the same as in the benchmark case. This individual has the 60th level of initial health capital and finantial assets. This subsection then discusses how the death rate and average longevity in the whole society alter due to the same change in δ.

The relationship between optimal longevity and δ is shown in Figure 14. As δ falls, optimal length of life increases, and thus the longevity when δ = 0.1 is twice as long as when δ =0.5. This tendency is consistent with the analytical results of Ehlrich and Chuma (1990). Health investment at the initial period also declines, as in Figure 15. Although health investment decreases, health capital increases at every period due to the direct effect of δ, as shown in Figure 16. Similarly, financial assets also increase, as shown in Figure 17. This means that the individual runs down financial asset more slowly. This affects the level of consumption at earlier periods, and so the consumption-age profile becomes flatter as shown in Figure 18. For example, when δ is 0.5, consumption decreases by 83.3 % from the initial to period 4, but when δ is 0.1, it changes only by 36.6 % over the same duration.

Figure 19 and 20 show how the death rate and the average longevity in the whole society respond to the same change in δ. The death rate decreases over all retirement periods due to the decrease in δ. Thus, although all retired people die by period 10 when δ =0.5, they survive until period 20 when δ =0.1. Therefore, average longevity increases as δ decreases, and thus average longevity when δ =0.1 is nearly twice as long as when δ =0.5. These results show explicitly that longevity responds elastically to the change in parameters. The decrease in δ increases individual health investment in previous analysis, such as Ehlrich and Chuma (1990) and Eisenring (1999). However, our results suggest that health investment in the initial period declines. One of the reasons of this somewhat counter-intuitive result is the use of the pure consumption model, whereas Ehlrich and Chuma (1990) use the full Grossman model. Indeed, the analysis of Grossman (1972b) also indicates that the sign of the response of health investment is not clear in the pure consumption model. This result is explained intuitively as follows. Under the linear health investment function such that $H_{t+1} = M_t + (1-\delta)H_t$, a lower value of δ can affect health capital in the next period, not only through health investment in the present period but also through the smaller direct depreciation effect of the health capital H_t. If this direct effect sufficiently increases health capital at the next period, the individual may be able to achieve more utility through less current health investment. The decrease in δ also affects the consumption pattern through longevity. Due to the direct effect of δ, health capital is maintained more than $\underline{H}$. Therefore, it is optimal for the individual to save financial assets for the longer remaining period of life, and so consumption decreases in earlier periods. In addition, longer optimal life changes the optimal inter-temporal allocation of consumption, and so the features of the optimal consumption path. Since endowment stays constant and investment is not affected significantly by changes in δ, a longer life results in the smaller financial assets per lifetime period.

VI-3 Changes in the Relative Preference for Health Capital (α)

This subsection analyses how the parameter of the preference for H_t, α, affect the optimal path of the same individual when changing from 0.5 to 0.9. As shown in Figure 21, optimal longevity lengthens as α become larger. However, it is not affected when α is more than 0.7[21)]. Figure 22 shows that health investment at initial period increases. For example, lifetime health investment when α =0.9 is 21 times as large as when α =0.5. Thus, health capital also increases, as shown in Figure 23. On the other hand, financial assets are run down more rapidly at earlier periods and more slowly at later periods, as shown in Figure 24. For example, when α is 0.5, financial assets decrease by 87 % from period 2 to period 6, while when α is 0.9, financial assets change only by 66.7 % over the same duration. Consumption also decreases over a lifetime, as shown in Figure 25. In particular, consumption stays at a low level for almost every period if α=0.9. In addition, the consumption-age profile becomes flatter when α is higher.

Figure 26 and Figure 27 present the influence of a change in α on the death rate and the average longevity in the whole society. As shown in Figure 26, an increase in α makes the death rate lower over entire lifetimes. However, the marginal effect on the death rate is smaller than δ. When α is 0.5, all retired old people die by period 12, whereas when α is 0.9, they survive until period 14. The marginal effect on average longevity is also smaller than δ, and thus average longevity grows only 20 % due to the increase in α from 0.5 to 0.9. The increase in α means that the individual has a relative preference for health capital over his or her entire retirement periods. Thus, the individual undertakes larger initial health investment in order to increase health capital over his or her entire lifetime. Hence, the death rate at each period declines and longer average longevity is achieved. In order to finance the increase in health investment, the individual not only decreases consumption but also exhausts the financial assets more rapidly. However, at the same time individuals plan a longer life due to the increase in health capital, and so it is not optimal to exhaust the financial asset completely over their remaining lifetimes. The speed of the decrease in financial assets therefore falls in later periods. Since the longer optimal length of life changes the optimal inter-temporal allocation of consumption, the slope of the consumption-age profile becomes flatter.

Ⅵ-4 Changes in the time discount rate (β)

Finally, we show how change in the time discount rate, β, affects the optimal path of the same typical individual when changing from 0.5 to 0.9. Surely other paprameters are held constant at the benchmark. Figure 28 shows how the optimal length of life differs due to the change in β. The optimal length of life lengthens as β increases. As a result, lonvevity when β = 0.9 is about 2.2 times as long as when β=0.5. Moreover, health investment continues for slightly longer periods and so does health capital (Figure 29 and 30) as β become larger[22)]. Similarly, financial assets increase at every period, as shown in Figure 31. This means that individuals exhaust financial assets more slowly. Therefore, Figure 32 shows how the consumption-age profile becomes flat. When β is 0.75, for example, consumption changes by 86.9 % from the first period to the forth. On the other hand, when β is 0.95, it changes by only 60 % over the same duration.

Figure 33 and 34 show the effect of β on the death rate and average longevity in the whole society. As shown in Figure 33, the death rate during retirement decreases monotonically as β

increases. For example, all retired people live until period 7 when β is 0.5, whereas they survive until 13 periods when β is 0.9. As a result, average longevity lengthens as β increases, and so average longevity when β =0.9 is about 1.9 times as long as when β =0.5. The increase in the time discount rate means that the individual places more value on a longer retirement priods. Therefore, health capital and health investment increases so as to maintain the health capital above $\underline{H}$ for longer periods; conversely, individuals try to save their assets by reducing consumption at earlier periods. As a result, death rate during retirement decreases and average longevity increases.

Ⅶ Concluding Remarks

This paper solves the pure consumption Grossman model numerically by using the dynamic programming method, and undertakes sensitivity analysis of the optimal length of life. Grossman (1998) and Ried (1999) have discussed the determination of optimal length of life analytically. However, in the discrete time model, these authors cannot determine how the parameters affect the optimal length of life. Although Picone et al.(1998) is the rare analyses try to extend the Grossman model numerically, it does not discuss the determination of optimal length of life. This paper, hterefore, focuses on the issue of the optimal length of life and extends the Picone et al.(1998) in this direction. To our knowledge, this is the first analysis that treats the optimal length of life with numerical analysis.

This paper shows the numerical algorithm that individual length of life is determined endogenously under a sufficiently large planning period, T . To confirm the availability of this method, this paper undertakes sensitivity analysis concerning T to search appropriate value of T . From the simulation results, it is proved that all people's value of the value function of initial period converge under sufficiently large T . In short, the longevity of all retired people in the whole society is determined endogenously. After fixing T at 25, we undertake sensitivity analysis with several parameters, health capital depreciation, relative preference for health capital, and the time discount rate. Simulation results show that a longer optimal length of life results from decreases in the health capital depreciation rate or increases in the relative preference for health and in time discount rate. Furthermore, longer lives significantly change the optimal allocation of consumption and the features of the optimal consumption path.

By making the individual length of life endogenous in the numerical method, applied analysis of longevity becomes easier in the Grossman model than the analytical method. For example, Philipson and Becker (1998) analyzes how pension affects the retired people's behavior for longevity. It claims that public pension will have the effect of inducing more health investment and longer lives than private insurance, although it deteriorates the welfare level due to the distortion of the lifetime resource allocation. The author calls it moral hazard effect in annuities. Moreover, they discuss how this longevity inducement affects the premiums of annuities and health care insurance. However, their model is inappropriate for rigorous analysis for longevity. Thus our model enables us to investigate this longevity effect more carefully supposing annuity and health care insurance are introduced. Also, it is also important to check how the endogeneity of the length of life itself affect the optimal path of health capital and health investment. In addition, there are very few analyses which try to extend the Grossman model by introducing insurance and annuities[23)]. Therefore, It is also essential to show how the existence or the characteristic of health insurance and annuity affects the individual health investment behavior. Nevertheless, we may be able to consider the optimal insurance or annuity under the Grossman model. Finally in order to extend numerical applied analysis of Grossman model, the reliability of parameter value is of great importance. Therefore, more reliable empirical analysis for Grossman model is also significant theme for the future research.

Footnote

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1)According to Philipson and Becker (1998), the number of the countries that provide the retired people income transfer or health care assistance program through public sector has increased fivefold. (from 33 to 155) since 1940.

2)Zweifel and Breyer (1997) emphasize the close relationship between longer average longevity and the amount of subsidy per retired person. As the rate of the retired increase in the population due to the improvement of average longevity, retired people become more influential in the political decision-making process. The public sector is therefore induced to expend excess resources on the medical technology preferred by the retired, or on higher subsidies through old-age assistance programs. As a result, average longevity improves further and reinforces the political power of the retired. Zweifel and Breyer (1997) call this phenomenon Sisyphus syndrome.

3)Philipson and Becker(1998)is the first analysis which analyzes this issue formally. However the method and the model used seems to be inappropriate for rigorous analysis.

4)On the contrary, the model which focuses on the wage through healthy time is called ‘the pure investment Grossman model’. Thus it mostly targets young individuals.

5)We have tried to use the terminology of the existing literature. However, as our method of establishing optimal length of life is totally different from the analytical studies, it is sometimes difficult to describe our ideas with the terminology of the existing literature. Thus, to avoid misunderstanding, we mainly use terminology that is familiar in the literature of dynamic programming.

6)The introduction of uncertainty into the Grossman model is discussed in Chang (1996), Dardanoni and Wagstaff (1987), Liljas (1998) and Picone et al (1998)

7)Eisenring (1999) applies the comparative dynamic method to the fixed terminal time model and compares the results with the analysis of Ehlrich and Chuma (1990), which applies the same method to the model with endogenous longevity. In Ehlrich and Chuma (1998), an increase in initial health does not induce lower health investment, as the demand for longevity also increases. However in Eisenring (1999) health investment decreases due to the increase in the initial health stock.

8)The bang -bang control problem means that health investment in each period becomes the corner solution on the feasible set in the optimization problem due to the linear health investment function. In this case, the maximum level of health investment is undertaken under constraints, when the marginal value of health capital happens to become larger than the marginal value of financial assets. Ehlrich and Chuma (1999) discusses the possibility of the bang-bang control problem in Grossman (1972a), as it assumes that the health production function is constant return to scale.

9)If financial assets are completely exhausted by the one-period before, the individual cannot consume a positive amount in current period and thus dies in this period. Therefore, endogenous death occur even if the health capital at one-period before is high enough to keep the current health capital above the minimum without health investment in one-period before. Needless to say, in almost all cases, it may not be optimal.

10) Certainly, the specification of the utility function influences longevity and the optimal path. Ried (1999) argues the possibility of the non-existence of the solution, if H_t is near $\underline{H}$ and the utility function satisfies the Inada condition. We can also employ the utility function such as. $u(H_t, C_t) = (H_t - \underline{H})^{\alpha} C_t^{1-\alpha}$ alternatively. However, the obtained results do not change significantly. Thus, this paper presents only the results obtained from $H_t^{\alpha} C_t^{1-\alpha}$. The non-existence issue proposed by Ried (1999) may occur only under the fixed terminal time problem, because non-existence means that individuals cannot survive, even though they have to survive until the terminal period under the fixed terminal time problem. In other words, it is obviously not optimal to survive for such long fixed periods. As our simulation method treats the free terminal time problem directly, this problem does not matter, though it is one of the important issues in the existing analytical literature.

11) As referred in Bleinchrodt and Quiggin (1999), 0 utility assumption in dead time is normal in health economics or health evaluation. However, there is no positive reason for it. If utility in dead time has different values, the difference in the utility level in death and survival may have a different jump. As a result, it may significantly alter the optimal path and optimal longevity.

12) This intuitive explanation may be incorrect mathematically. As we explained earlier, the value function in t just evaluates the optimal mapping from all grid points of t to the feasible set in $t+1$, using the information of the value function from $t+1$ to T.

13) This does not imply that the optimal problem with a shorter T is equal to the fixed terminal time problem in the existing literature. Some persons die by terminal condition that $V_T(H_T, W_T) = 0$ or dies exogenously at T . However, there is the case that $H_T \geq \underline{H}$ as a result of optimal choice, as shown in eq (26). This apparently contradicts the definition of the fixed terminal time problem. Needless to say, this may be the modified fixed terminal time problem

14) α is set as 0.7 in our paper, because the grid of state variables used in this paper is coarse. Supposing we use the lower value, the response of health investment to the change in the parameter cannot be shown clearly in the figure. This is mostly due to the insufficient precision of our grid. However, the usage of more precise grid setting consumes too much time . Therefore, this paper uses α, which is 0.1 point higher than Picone et al.(1998).

15) Picone et al.(1998) specify health capital depreciation as the increasing function of age such as $\delta = 1 - 0.012\exp(0.021(32+t))$, following Grossman (1972b). As a result, δ changes from 0.124 to 0.152. Moreover, in order to investigate how individual attitudes against uncertainty affect individual optimal behavior, the utility function is specified as $\frac{\left[C_t^{\gamma} H_t^{1-\gamma}\right]^{1-\sigma} - 1}{1-\sigma}$. Then $\gamma - \sigma(\gamma - 1)$ express the Arrow-Pratt measure of individual attitudes for relative risk aversion.

16) As it is plausible to suppose that some retired people have no financial assets, this paper sets the lowest financial assets level at 0. For the finite length of life, $\underline{H}$ must be positive. If an individual does not make any health investment in his or her life, health capital decreases exponentially through eq(3), but health capital never reaches 0 within finite periods.

17)As there is no interaction between people in the initial period, the optimal path of the certain value of the initial endowment does not change, even if the initial distribution is different from the uniform distribution.

18) This sensitivity analysis is undertaken to find the appropriate value of T. Therefore, this is different from the sequence of the modified-fixed terminal time problem for searching optimal longevity in the existing literature. Our method can evaluate the free terminal time problem directly under appropriate T and thus there is no need of the sequence of the problem with different

planning periods to find optimal longevity.

19) This individual welfare exactly equals to the inter-temporal utility in the existing literature due to the well-known characteristics of the value function.

20) In the footnote5, Picone et al.(1998) assume utility in death ($H_t \leq 0.001$) as $5 * \frac{[(0.0001)]^{1-\sigma} - 1}{1-\sigma}$. As σ is 0.9 at the benchmark case, it indicates that death is assumed have a large negative value.

21)This result shows the one of shortcomings of our numerical analysis. If the grid of the state variables is not so fine, the subtle difference in individual length of life is approximated to a same number. However, the usage of the more precise grid takes too much time for computation. Therefore, we cannot help using the rather rough grid.

22)This figure also shows the one of the shortcomings of our analysis. As referred before, the grid of the state variables in our analysis is rough and thus the subtle difference in health investment and health capital in each period is approximated a same number. Thus the image of the figure may change slightly when we use the more precise grid but it does not change the essence of our results.

23) Liljas(1998) is probably the only analysis which introduces the insurance into the Grossman model

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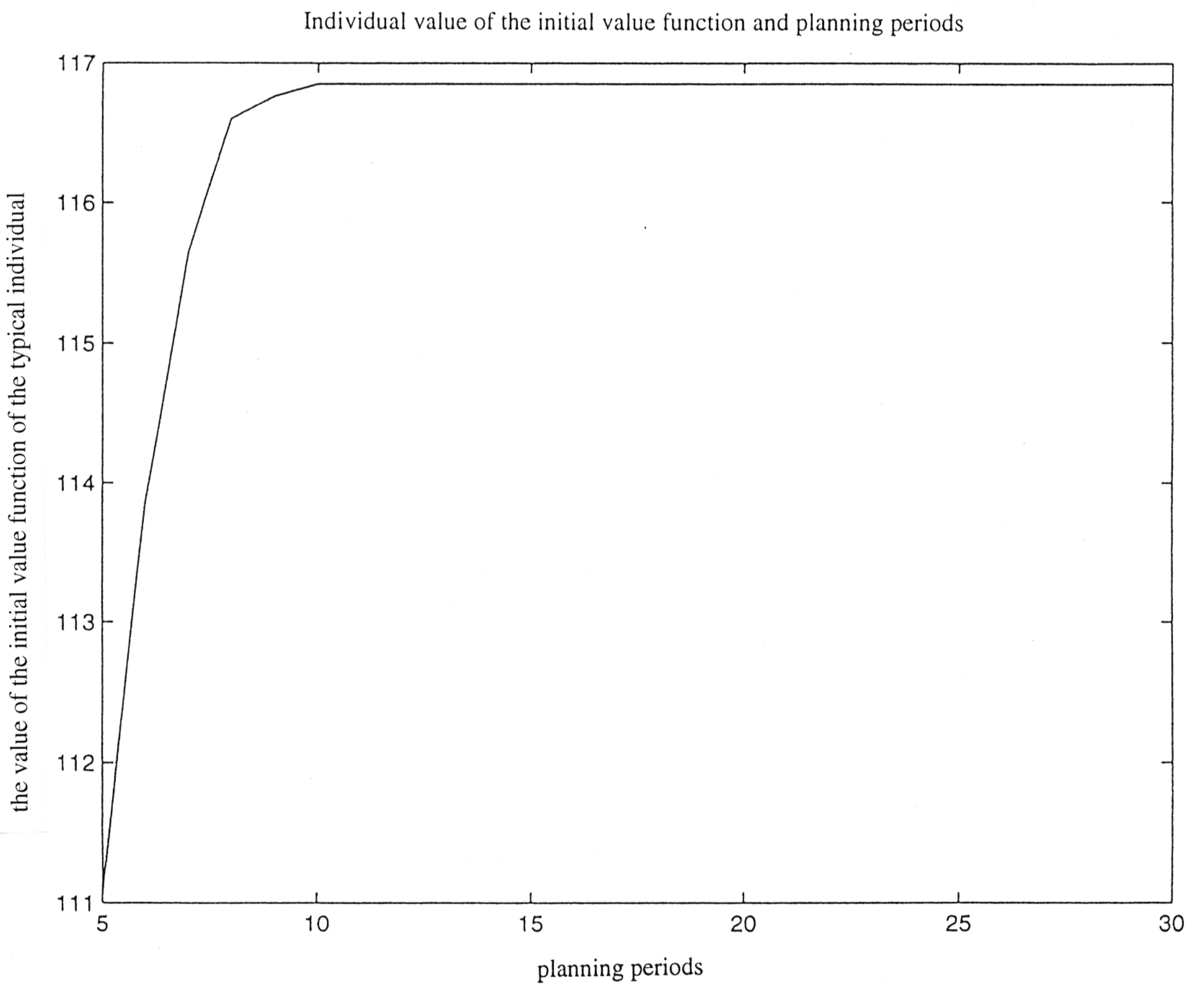
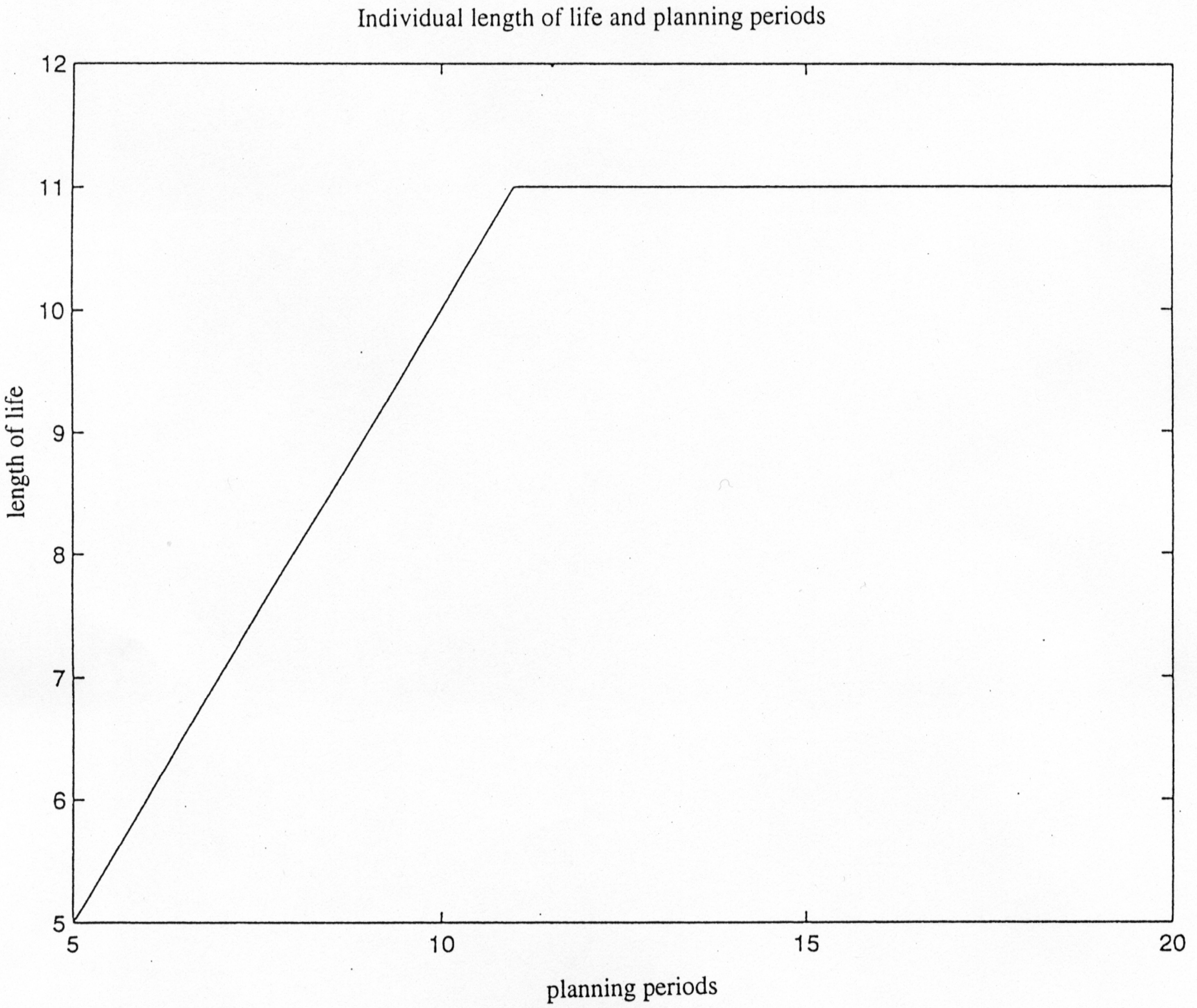
Figure 1



Note

This figure shows the relationship between the value of the initial value function of the typical individual and planning periods. This individual has 60th levels of initial health capital and financial asset.

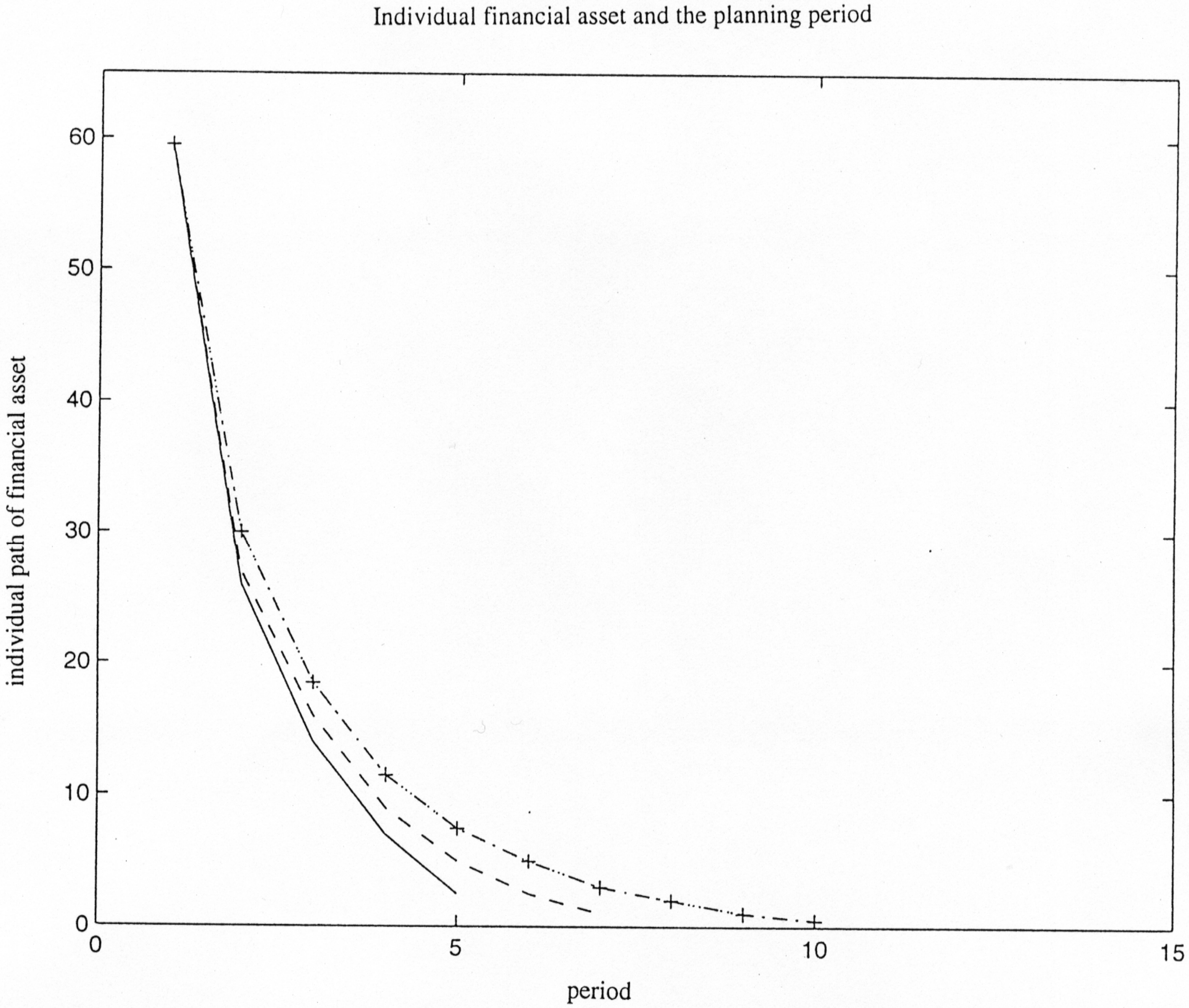
Figure 2



Note

This figure shows the relationship between length of life of the typical individual and planning periods. This individual has 60th levels of initial health capital and financial asset.

Figure 3



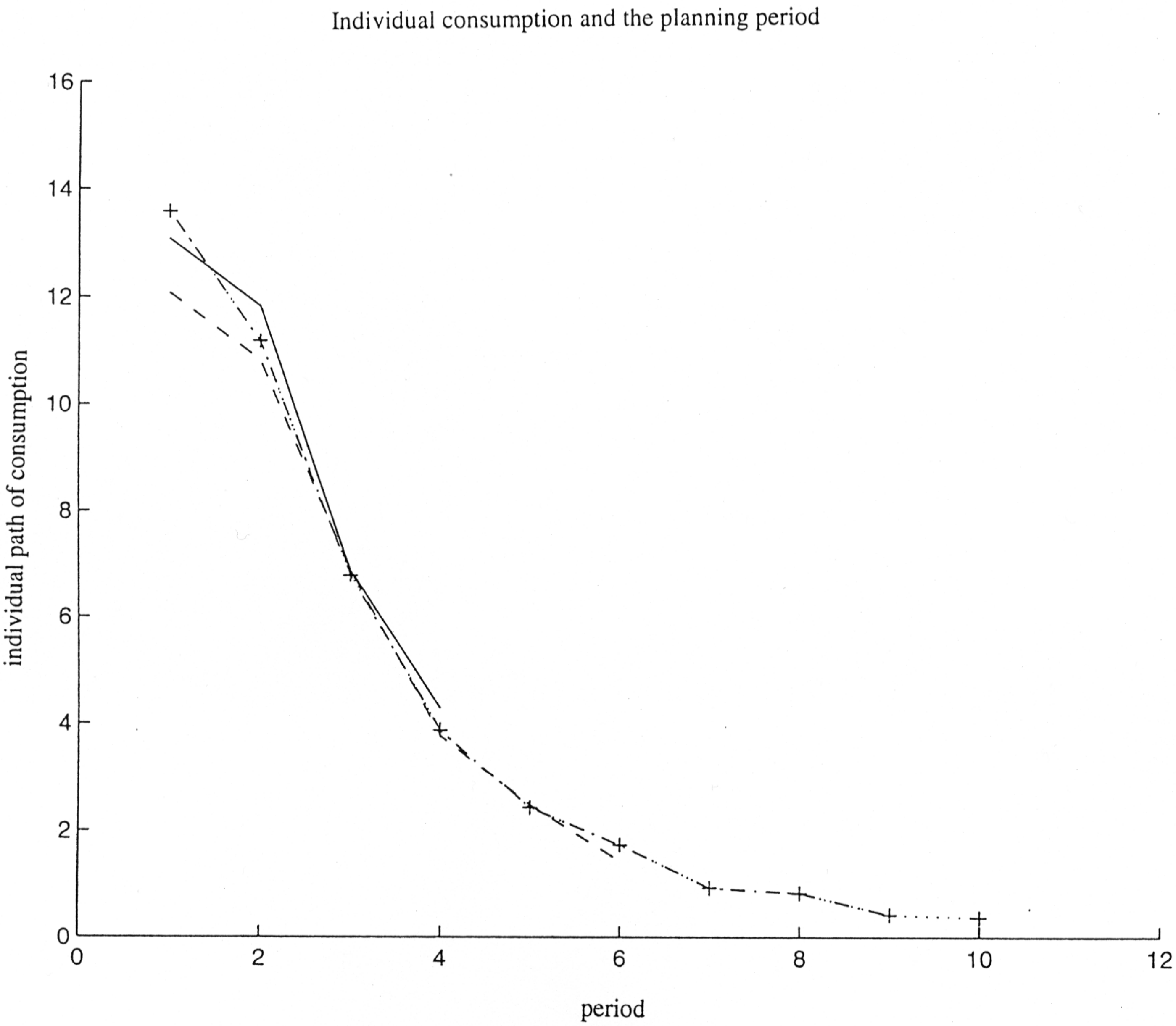
Description

Solid: 5, Dash: 7, Dash dot: 10, dot: 11,plus: over13

Note

This figure shows the typical individual financial asset path under different planning period. This individual has 60th levels of initial health capital and financial asset. The planning period is 5,7,9,11 and over13 periods.

Figure 4



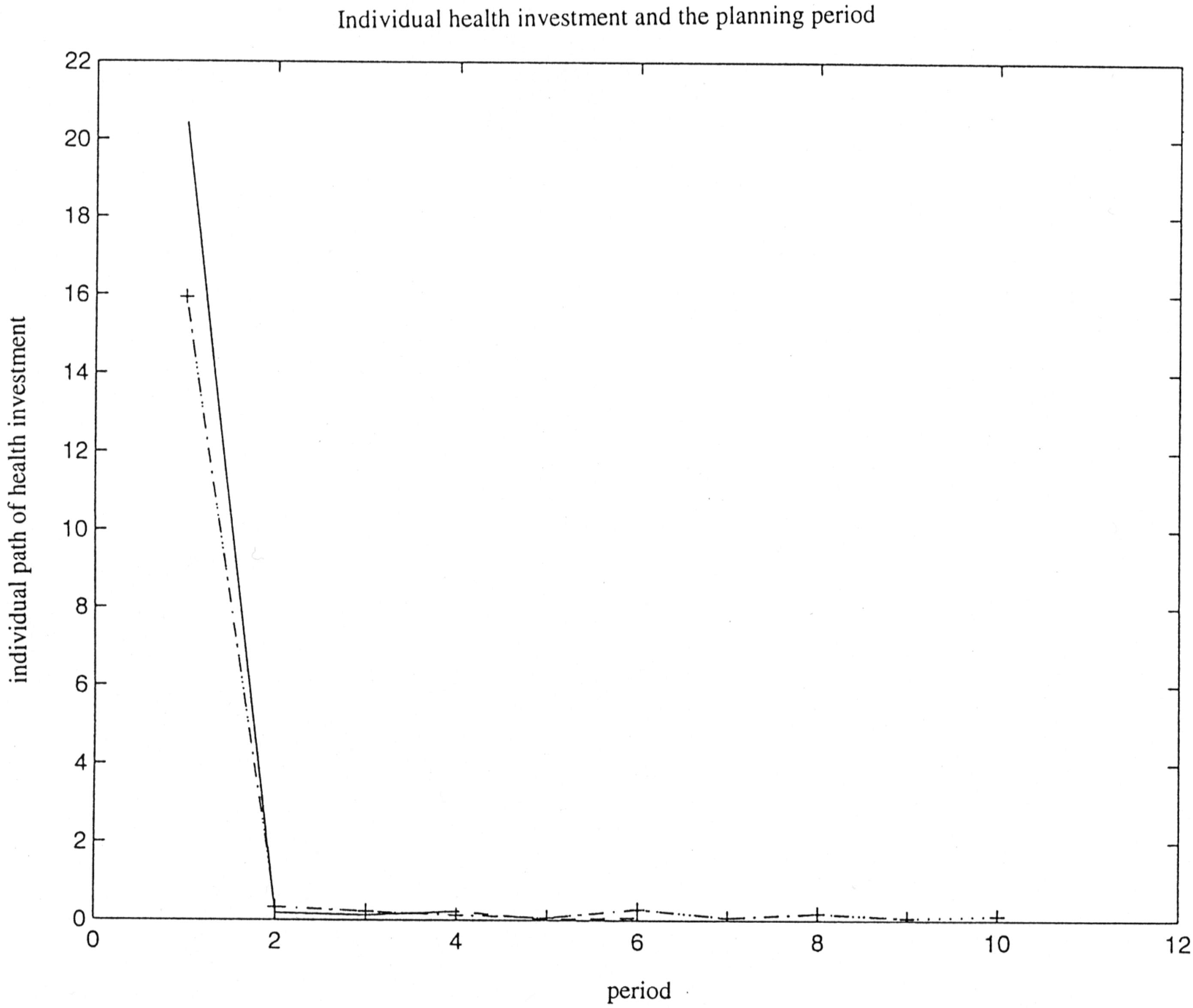
Description

Solid: 5, Dash: 7, Dash dot: 9, dot: 11,plus: over13

Note

This figure shows the typical individual consumption path under different planning period. This individual has 60th levels of initial health capital and financial asset. The planning period is 5,7,9,11 and over13 periods.

Figure 5



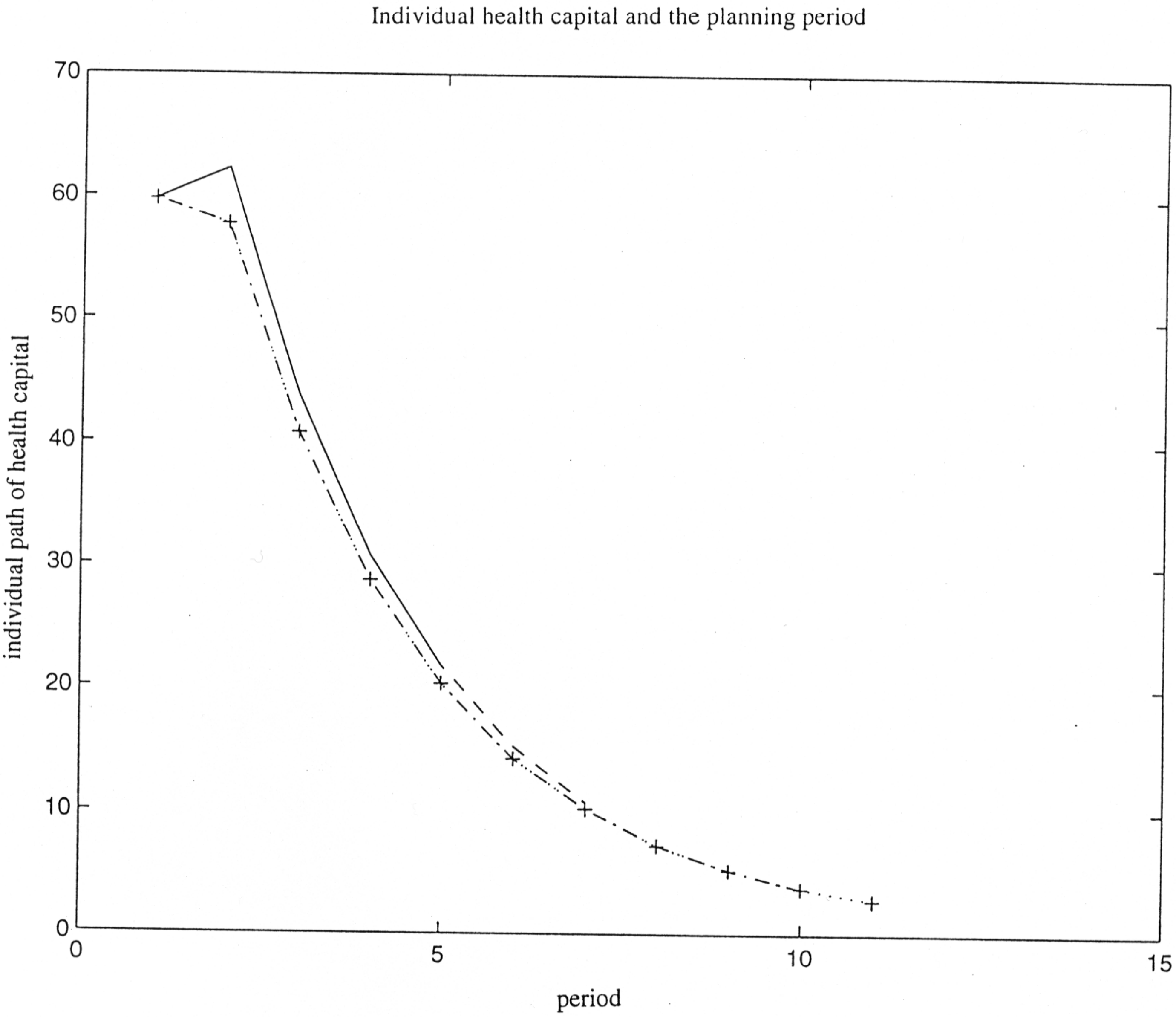
Description

Solid: 5, Dash: 7, Dash dot: 9, dot: 11,plus: over13

Note

This figure shows the typical individual health investment path under different planning period. This individual has 60th levels of initial health capital and financial asset. The planning period is 5,7,9,11 and over13 periods.

Figure 6



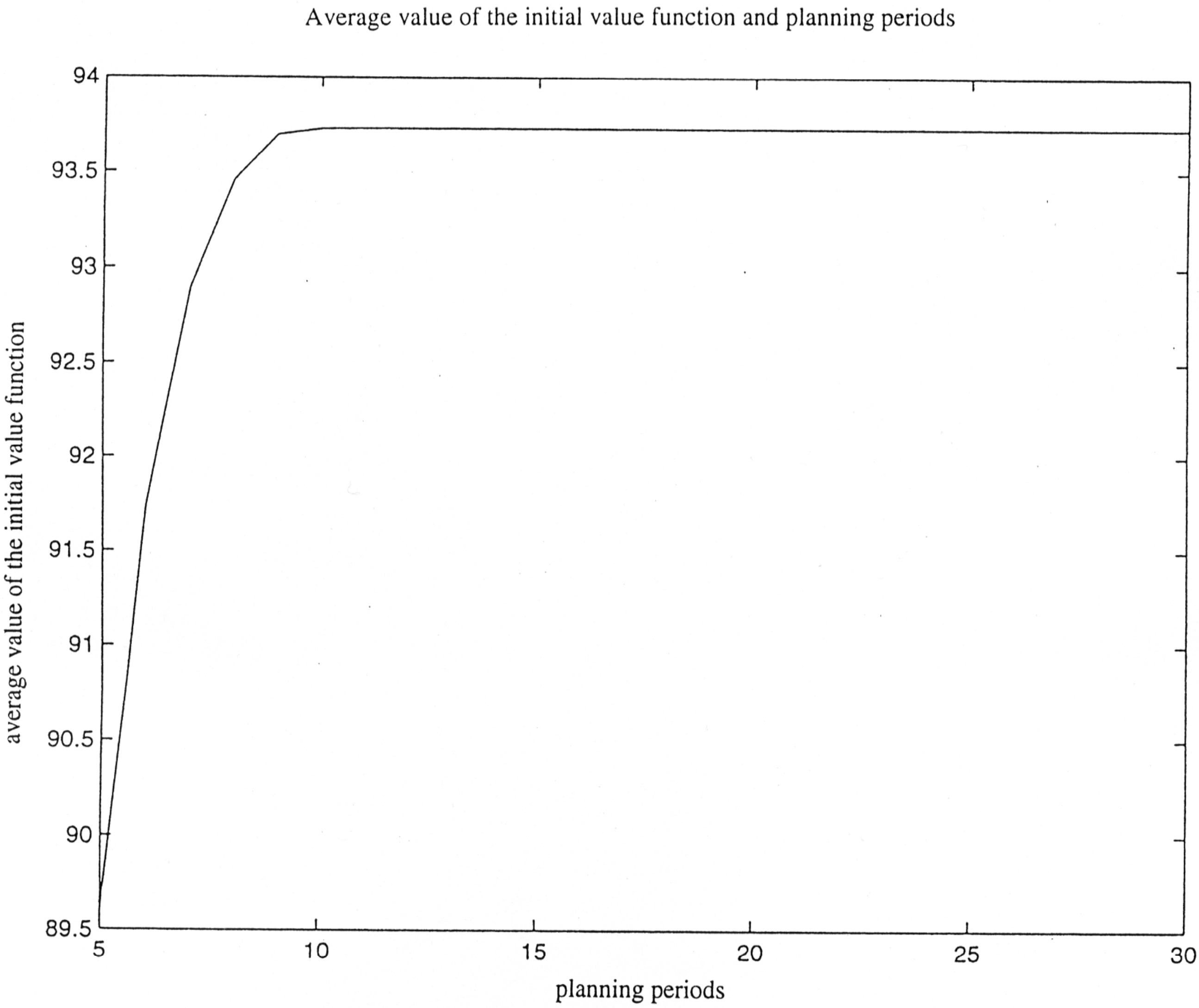
Description

Solid: 5, Dash: 7, Dash dot: 9, dot: 11,plus: over13

Note

This figure shows the typical individual health capital path under different planning period. This individual has 60th levels of initial health capital and financial asset. The planning period is 5,7,9,11 and over13 periods.

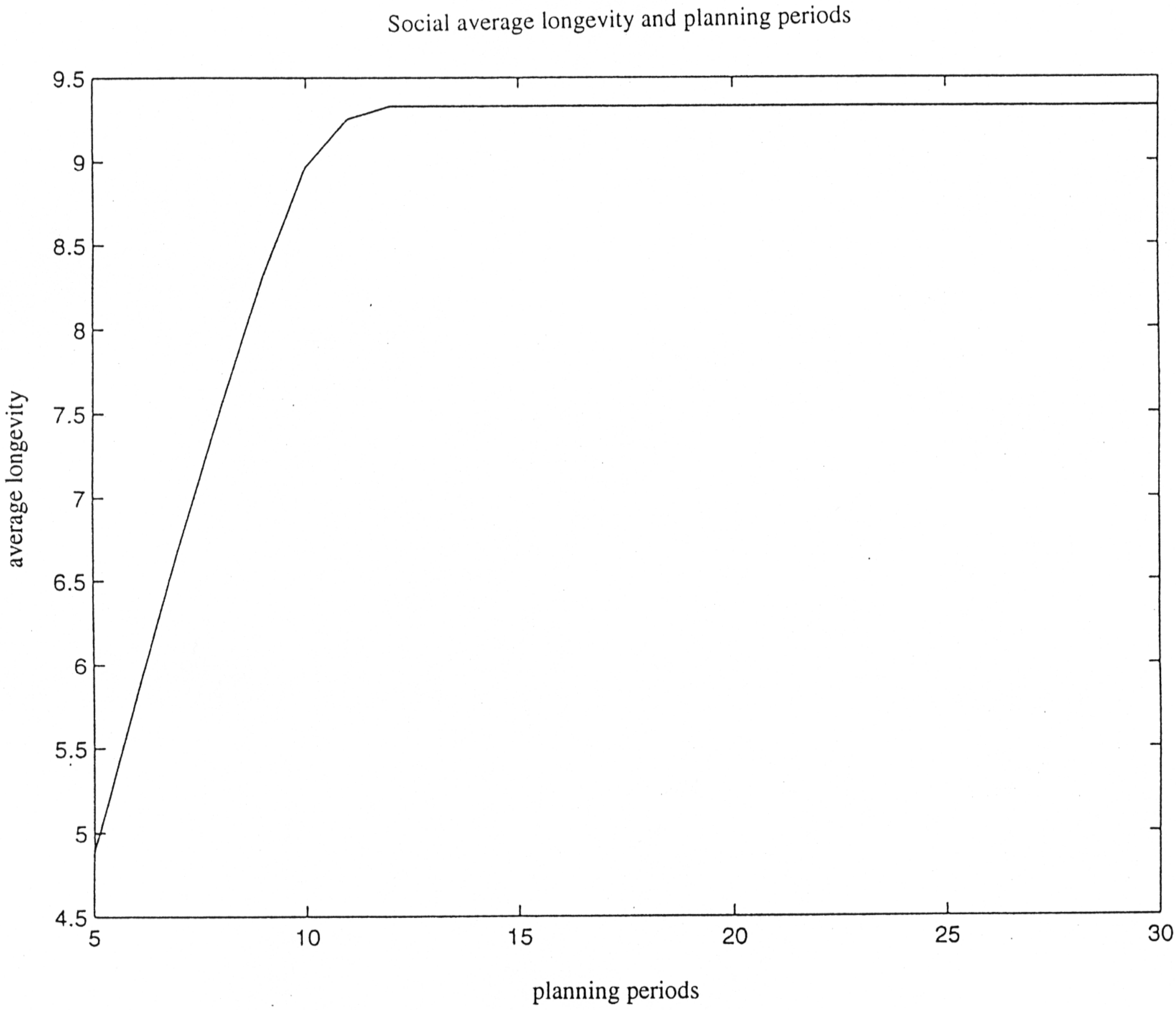
Figure 7



Note

This figure shows the relationship between the social welfare and planning periods. Social welfare is defined as the average initial value of 10000 retired individual's value function.

Figure 8

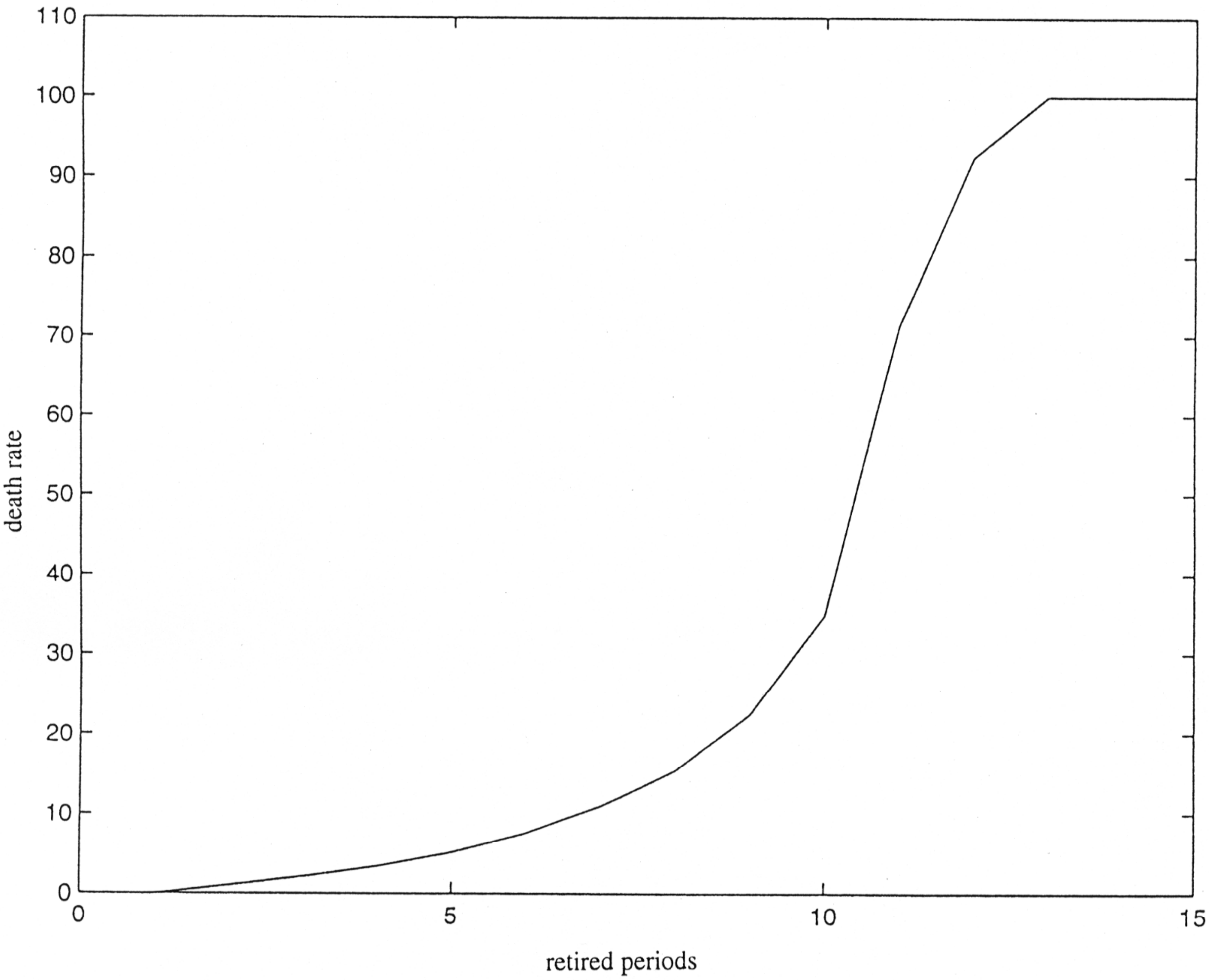


Note

This figure shows the relationship between the average longevity and planning periods. Average longevity is defined as simple average longevity of 10000 retired-individuals in initial period.

Figure 9

The death rate for the benchmark case.

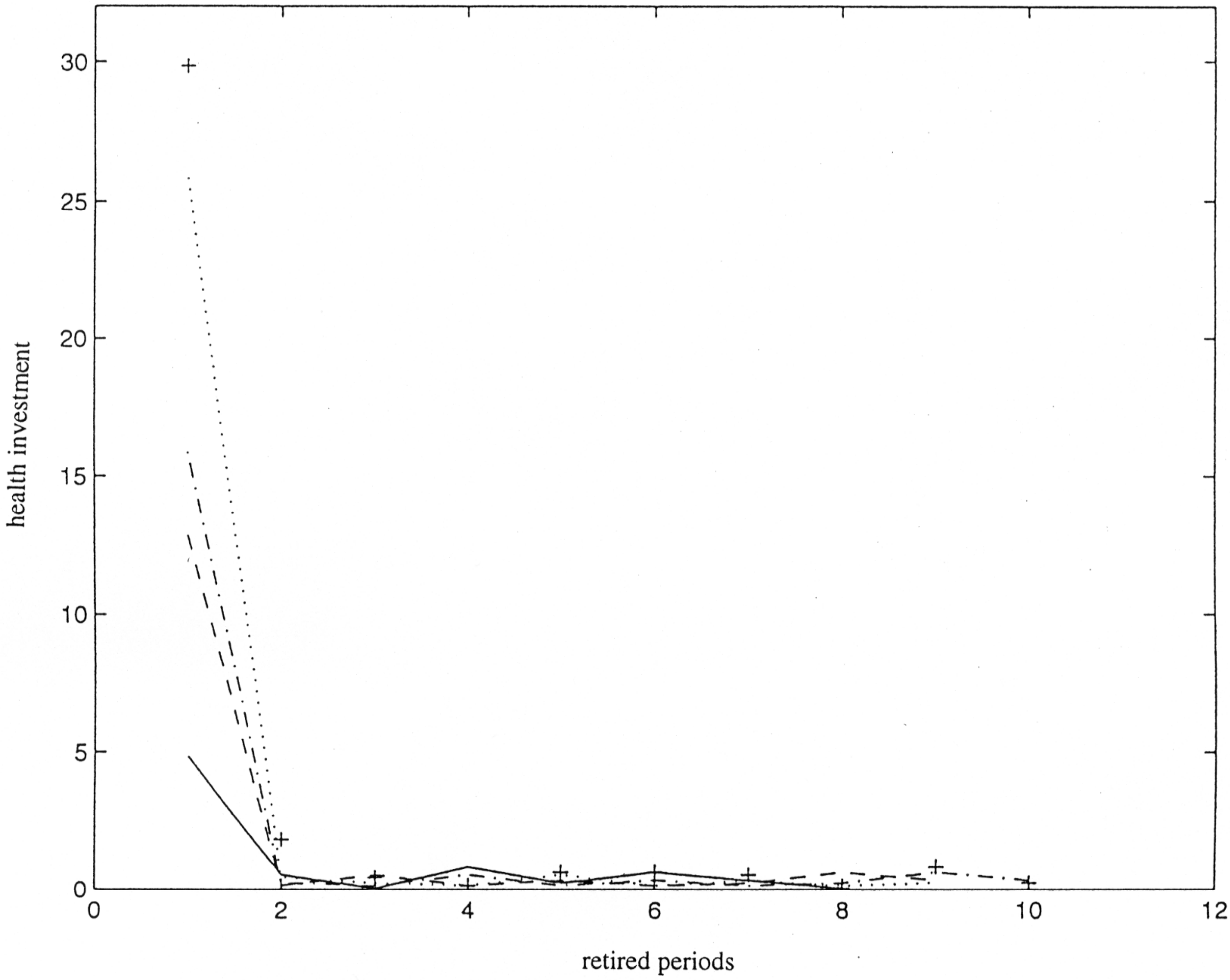


Note

The death rate for the benchmark case: $\alpha = 0.7$, $\beta = 0.9$, $\delta = 0.3, r = 0$. The death rate at t is defined as the ratio of the cumulative number of those who die before t to 10 000 retired individuals in initial period with different initial endowments.

Figure 10

Health investment of the individual with different initial endowments for the benchmark case.



Description

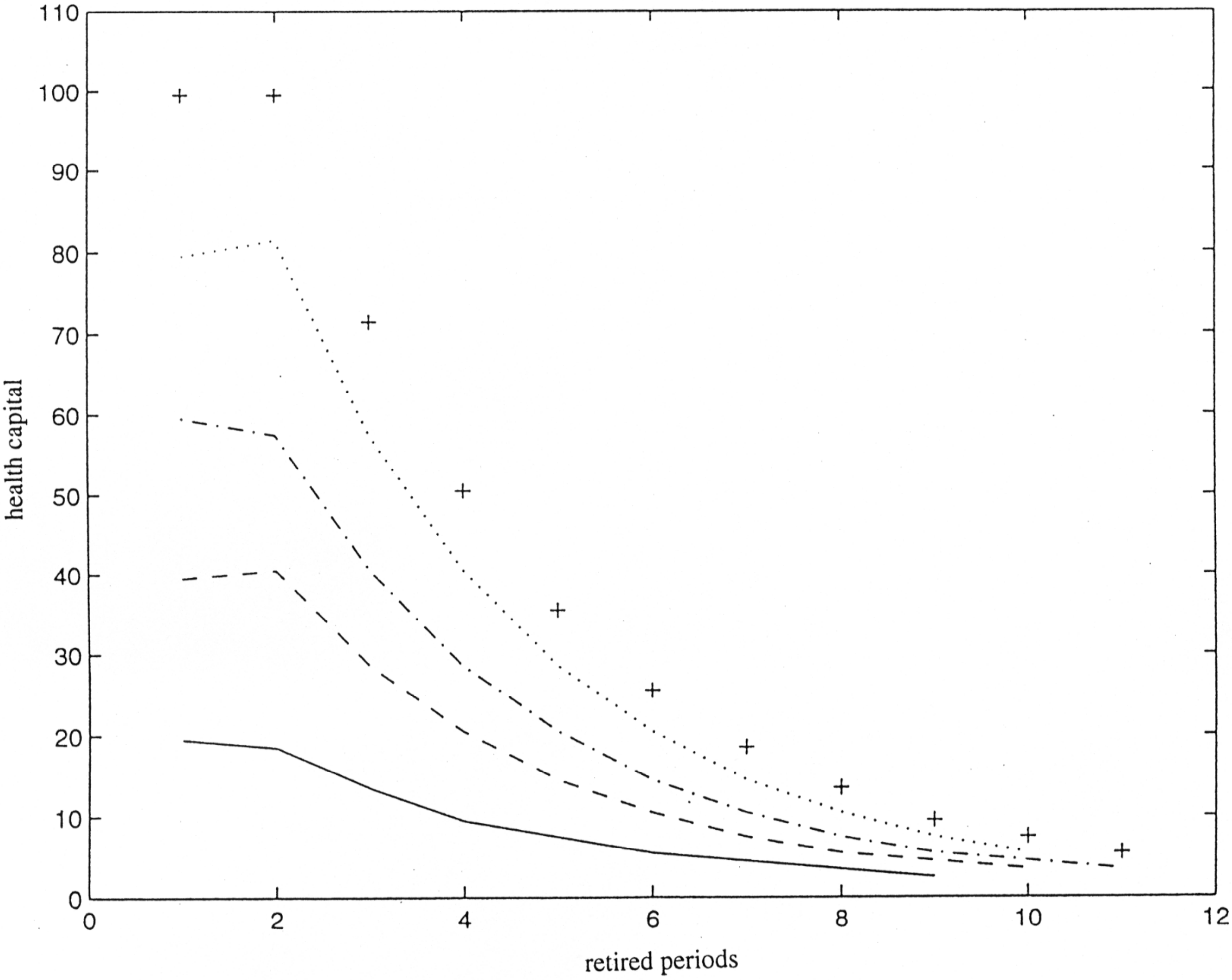
Solid: 20, Dashed: 40, Dashdot: 60, Dotted: 80, Plus: 100

Note

Health investment of the individual with different initial endowments for the benchmark values: α = 0.7, β = 0.9, δ = 0.3, r = 0. Individuals have 20th, 40th, 60th, 80th, 100th levels of initial health capital and financial asset.

Figure 11

Health capital of the individual with different initial endowments for the benchmark case.



Description

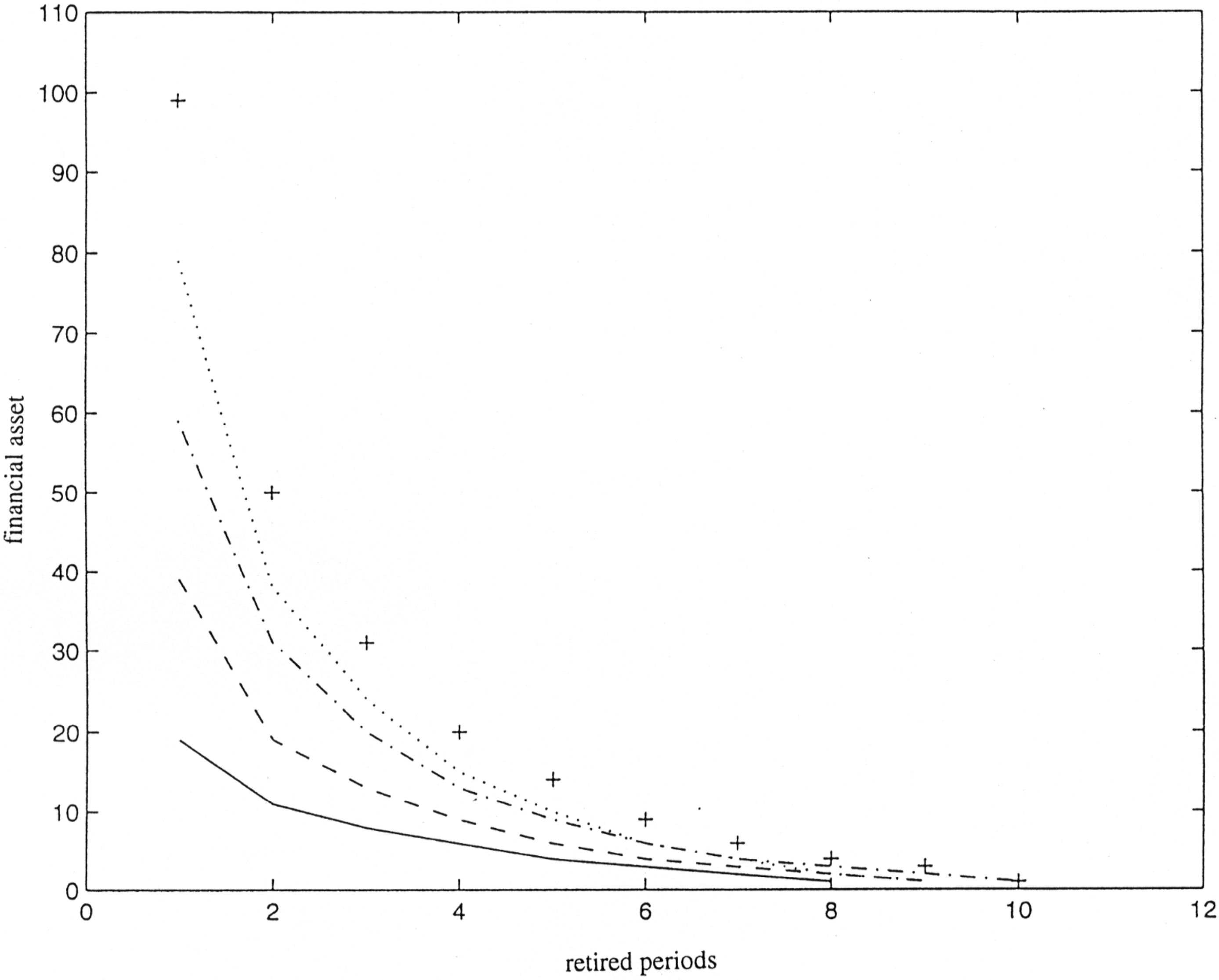
Solid: 20, Dashed: 40, Dashdot: 60, Dotted: 80, Plus: 100

Note

Health capital of the individual with different initial endowments for the benchmark values: α = 0.7, β = 0.9, δ = 0.3, r = 0. Individuals have 20th, 40th, 60th, 80th, 100th levels of initial health capital and financial asset.

Figure 12

Financial asset of the individual with different initial endowments for the benchmark case.



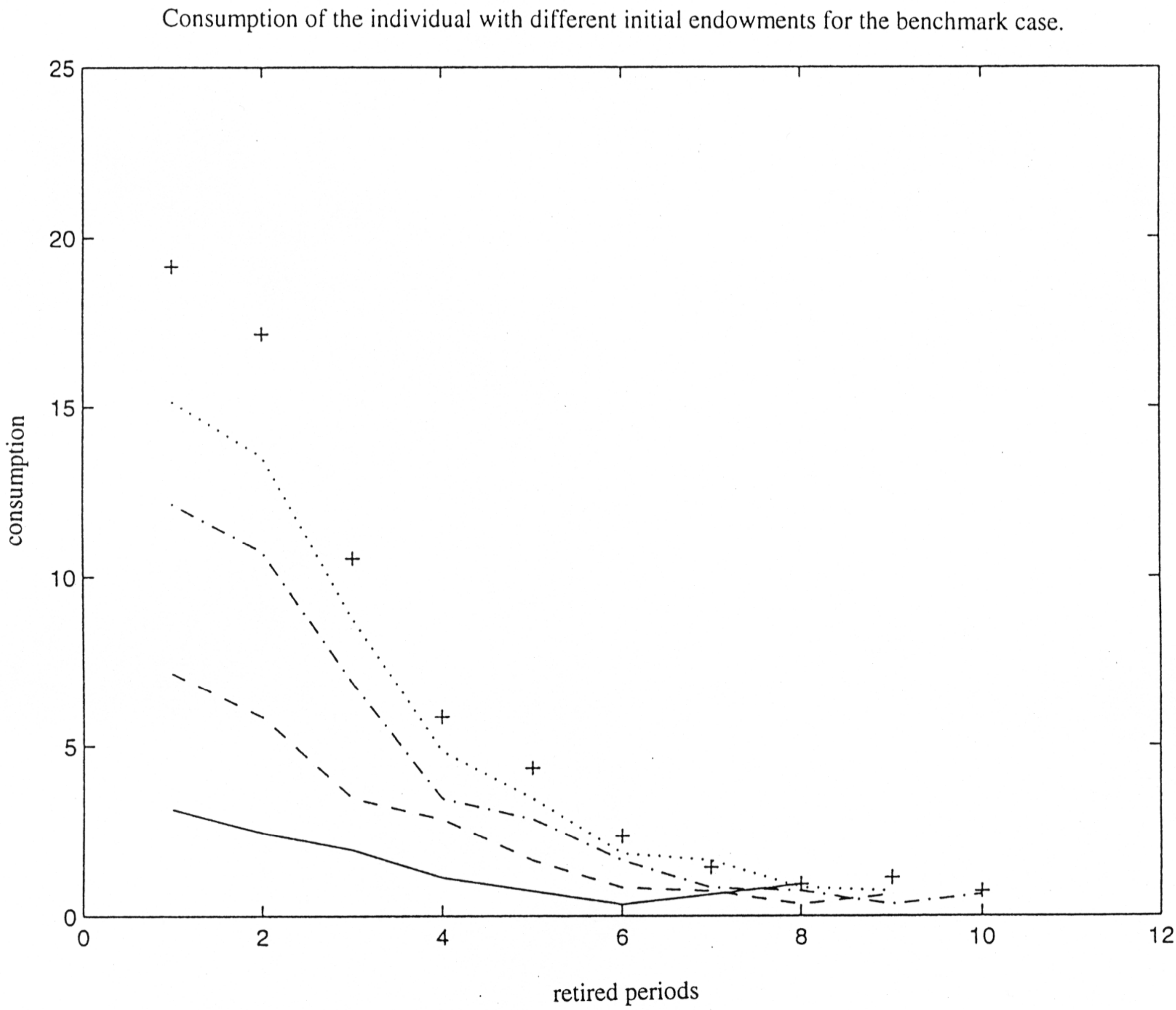
Description

Solid: 20, Dashed: 40, Dashdot: 60, Dotted: 80, Plus: 100

Note

Financial asset of the individual with different initial endowments for the benchmark values: α = 0.7, β = 0.9, δ = 0.3, r = 0. Individuals have 20th, 40th, 60th, 80th, 100th levels of initial health capital and financial asset.

Figure 13



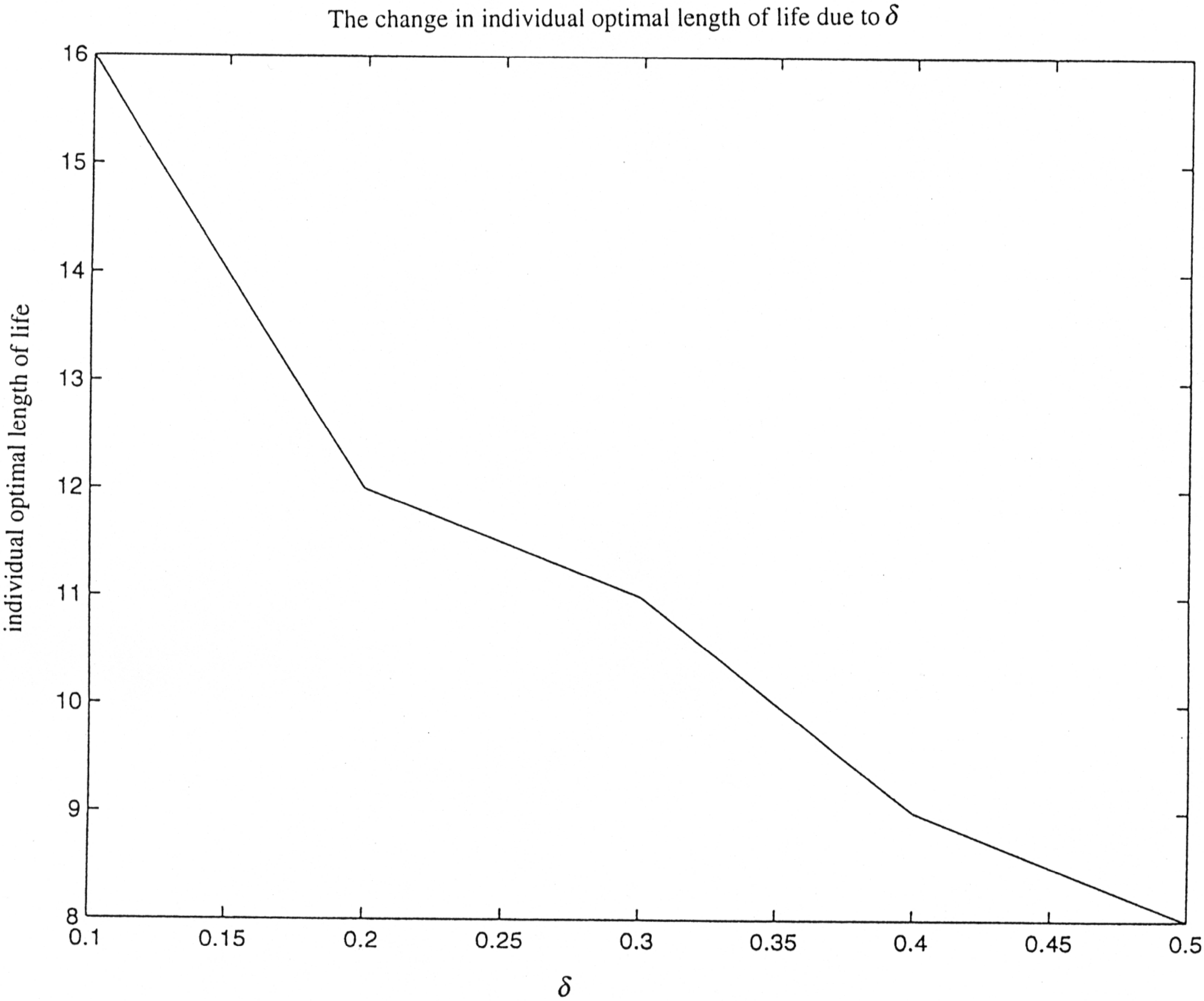
Description

Solid: 20, Dashed: 40, Dashdot: 60, Dotted: 80, Plus: 100

Note

Consumption of the individual with different initial endowments for the benchmark values: $\alpha = 0.7$, $\beta = 0.9$, $\delta = 0.3$, $r = 0$. Individuals have 20th, 40th, 60th, 80th, 100th levels of initial health capital and financial asset.

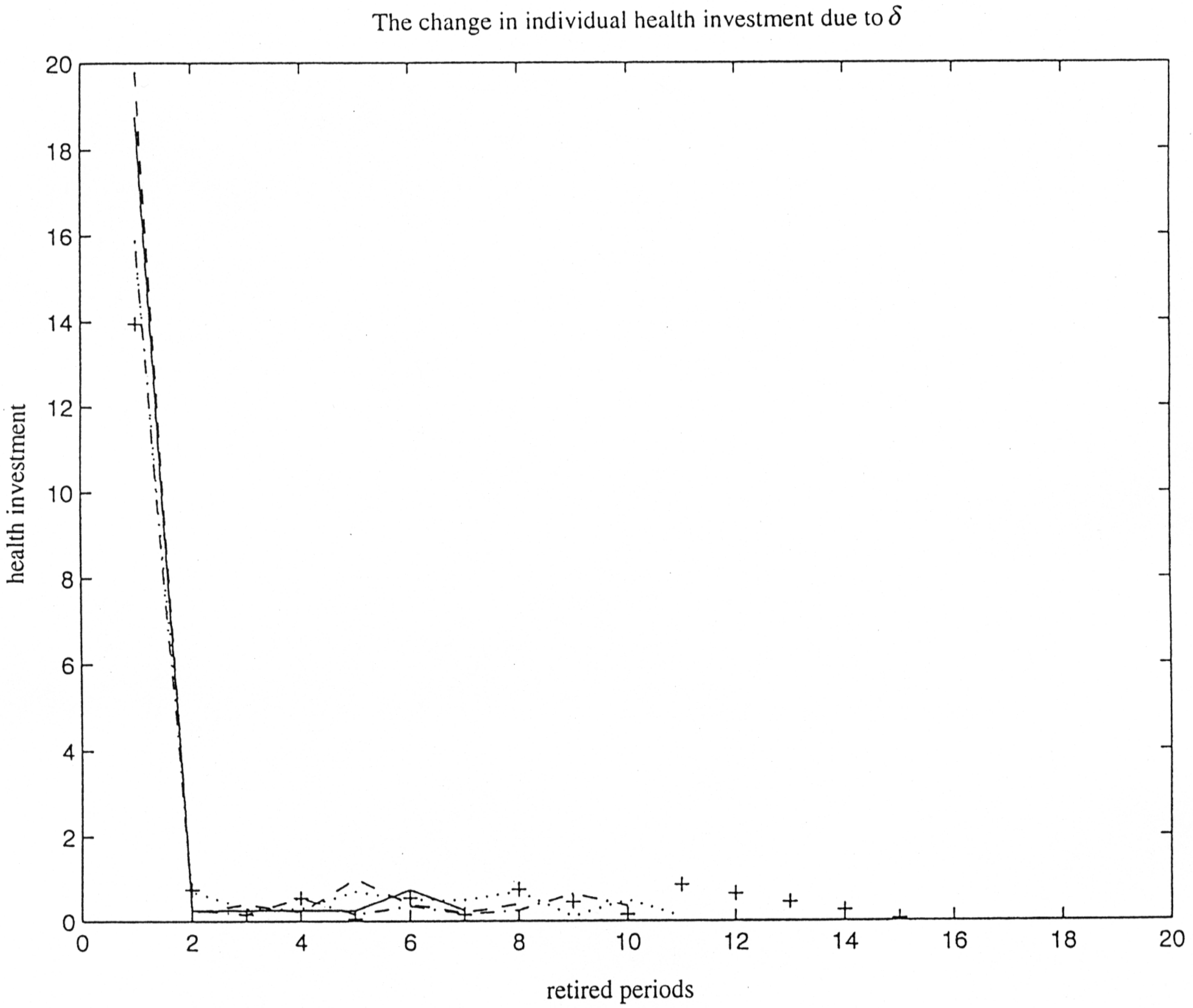
Figure 14



Note:

The change in individual optimal length of life due to δ. Other parameters are held at the benchmark values: $\alpha = 0.7$, $\beta = 0.9$, $r = 0$. This individual has 60th levels of initial health capital and financial asset.

Figure 15



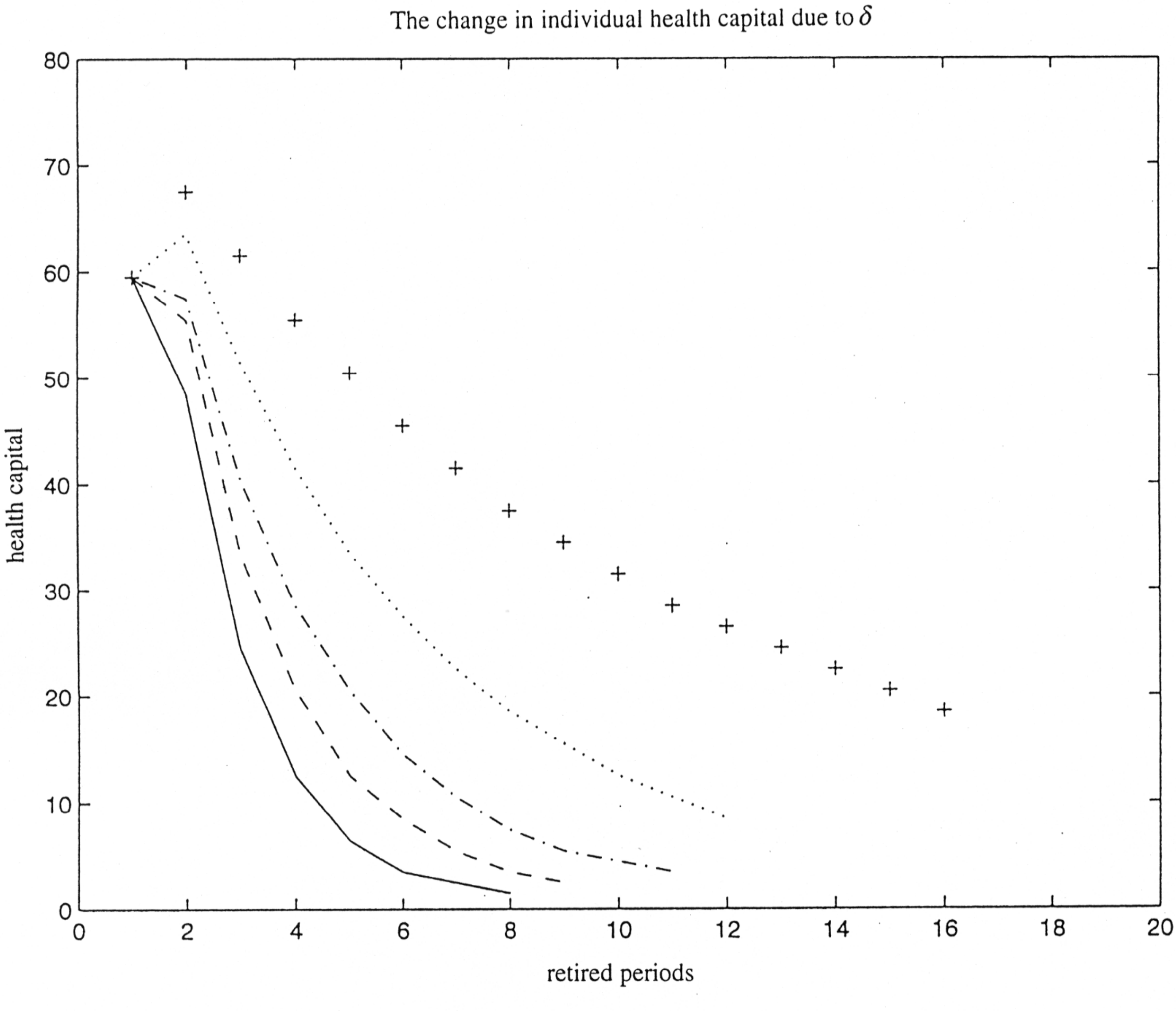
Description:

Solid: δ =0.5, Dashed: δ =0.4, Dashdot: δ =0.3,Dotted: δ =0.2, Plus: δ =0.1

Note:

The change in individual health investment due to δ . Other parameters are held at the benchmark values: α = 0.7, β = 0.9, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 16



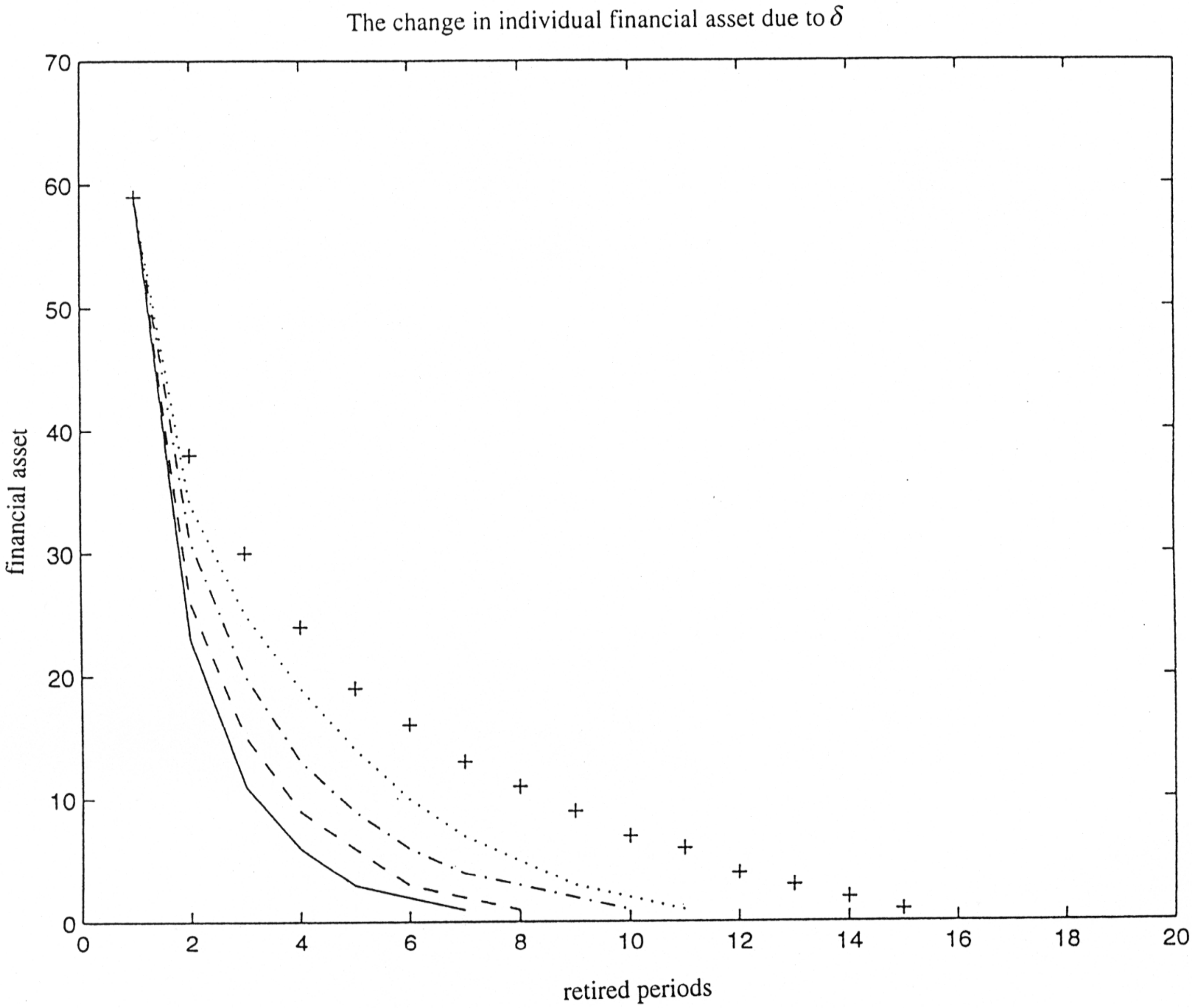
Description:

Solid: δ =0.5, Dashed: δ =0.4, Dashdot: δ =0.3, Dotted: δ =0.2, Plus: δ =0.1

Note:

The change in individual health capital due to δ. Other parameters are held at the benchmark values: α = 0.7, β = 0.9, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 17



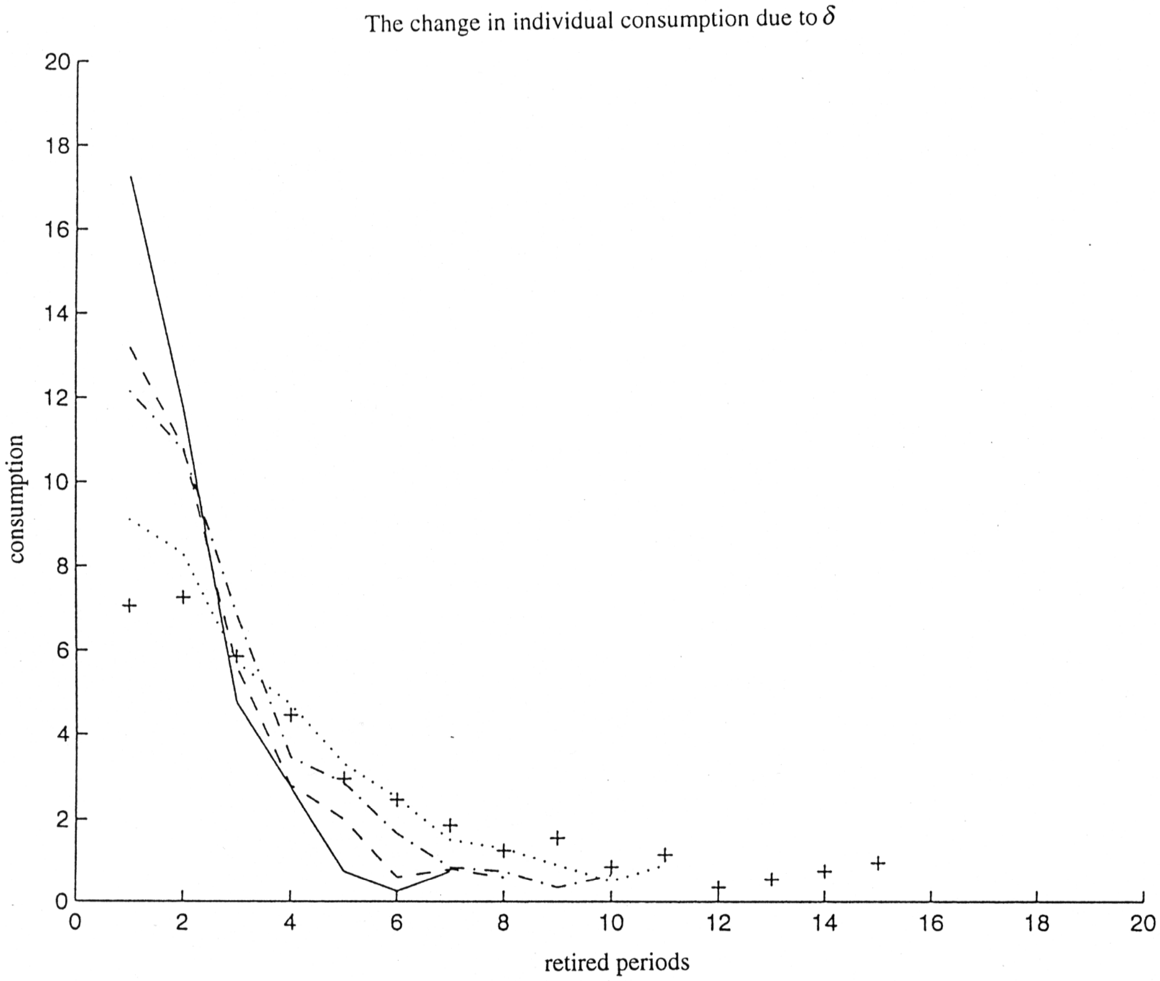
Description:

Solid: δ =0.5, Dashed: δ =0.4, Dashdot: δ =0.3,Dotted: δ =0.2, Plus: δ =0.1

Note:

The change in financial asset due to δ . Other parameters are held at the benchmark values: α = 0.7, β = 0.9, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 18



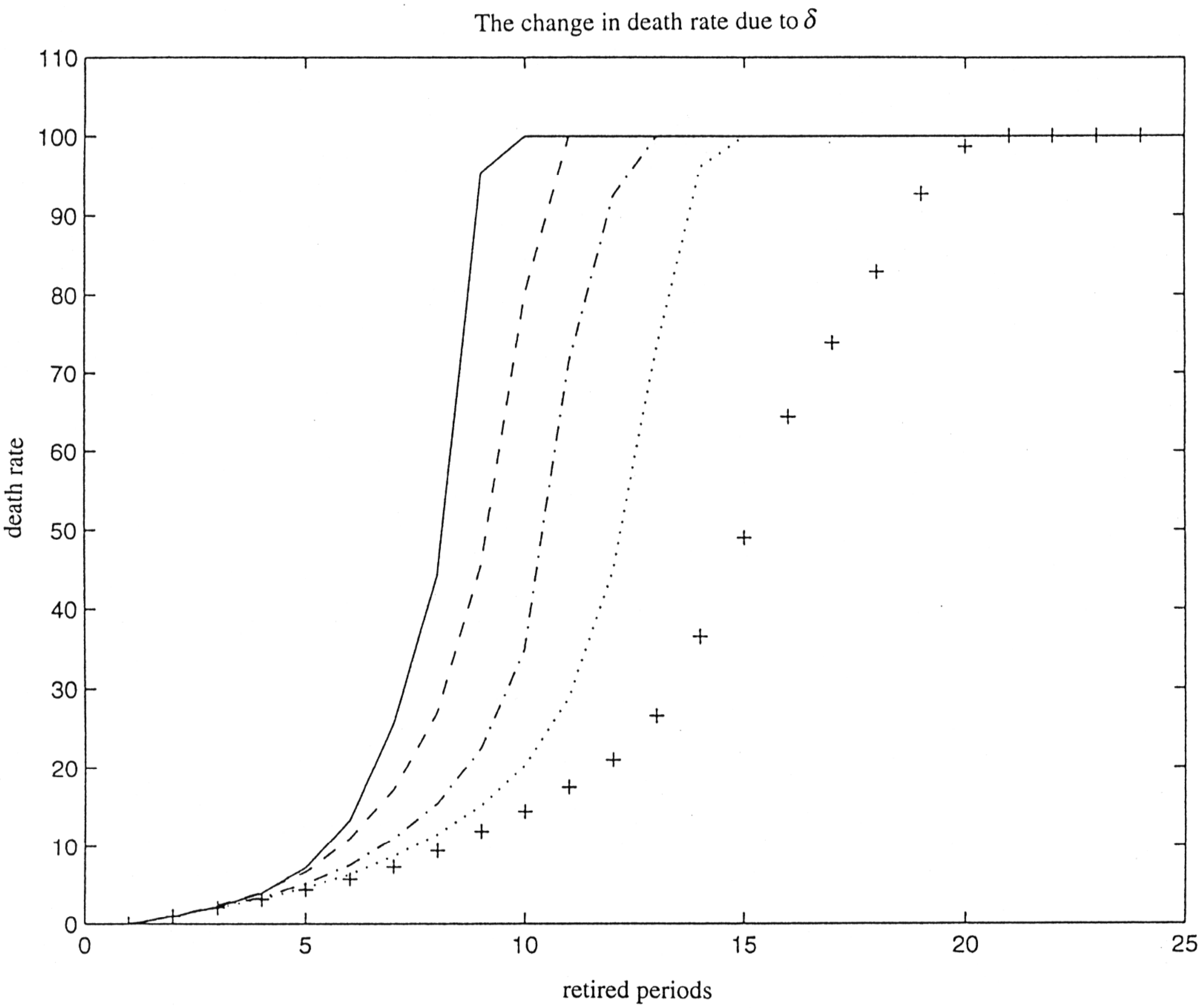
Description:

Solid: δ =0.5, Dashed: δ =0.4, Dashdot: δ =0.3,Dotted: δ =0.2, Plus: δ =0.1

Note:

The change in individual consumption due to δ . Other parameters are held at the benchmark values: α = 0.7, β = 0.9, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 19



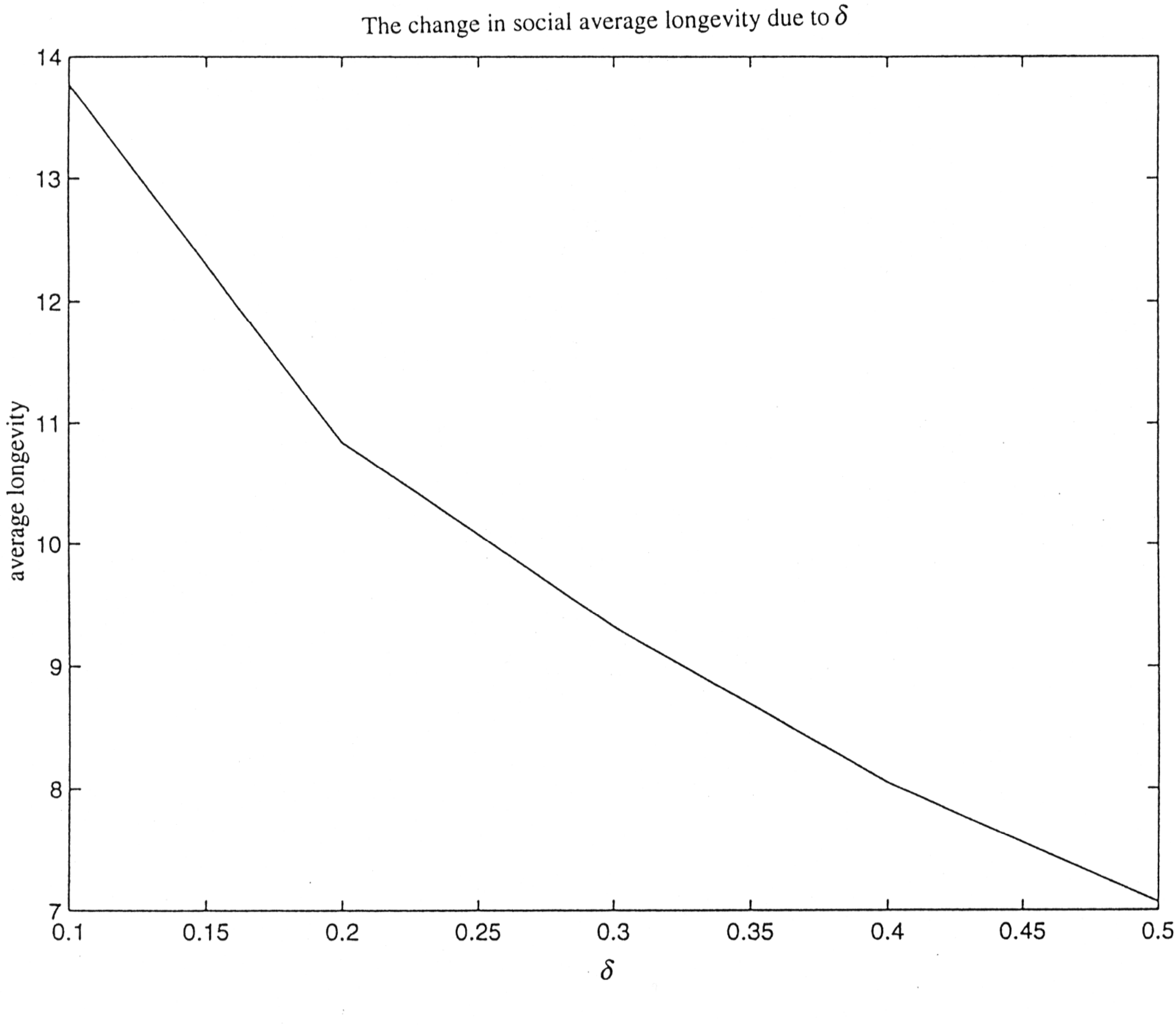
Description:

Solid: δ =0.5, Dashed: δ =0.4, Dashdot: δ =0.3,Dotted: δ =0.2, Plus: δ =0.1

Note:

The change in the death rate due to δ. Other parameters are held at the benchmark values: α = 0.7, β = 0.9, r = 0. The death rate at t is defined as the ratio of the cumulative number of those who die before t to 10 000 retired individuals in initial period with different initial endowments.

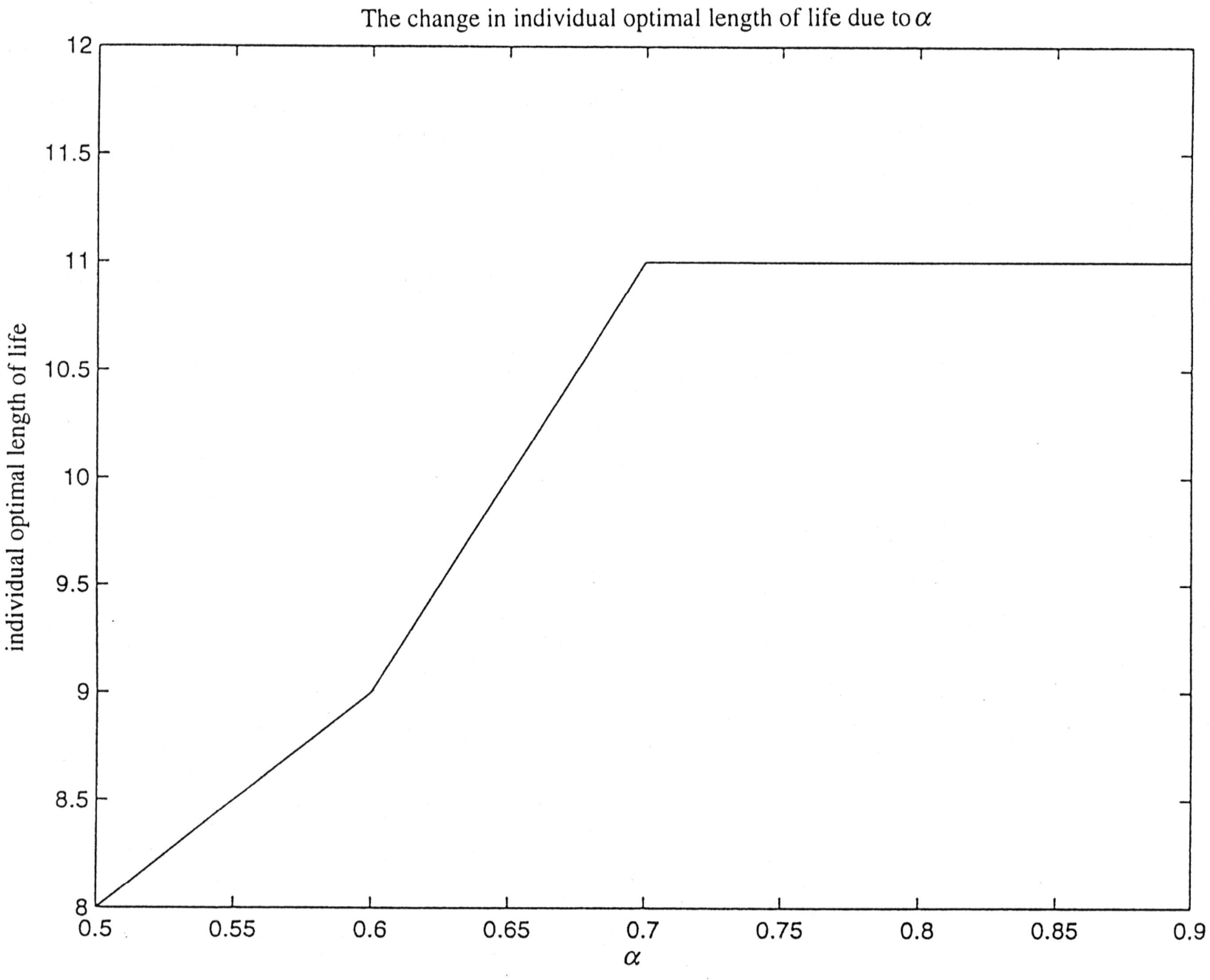
Figure 20



Note:

The change in average longevity due to δ . Other parameters are held at the benchmark values: α = 0.7, β = 0.9, r = 0. Average longevity is defined as simple average longevity of 10000 retired-individuals in initial period.

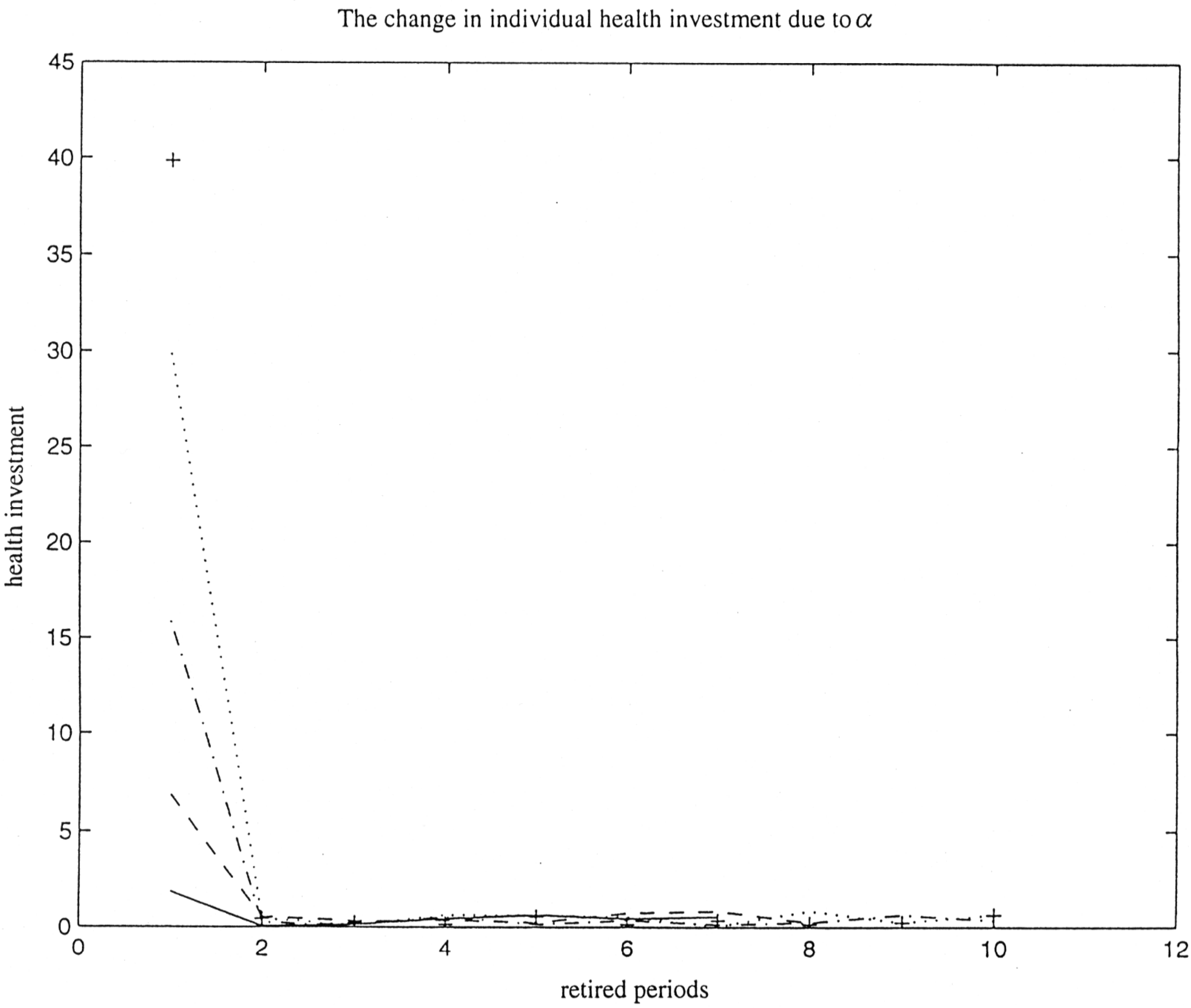
Figure 21



Note

The change in individual optimal length of life due to α. Other parameters are held at the benchmark values: δ = 0.3, β = 0.9, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 22



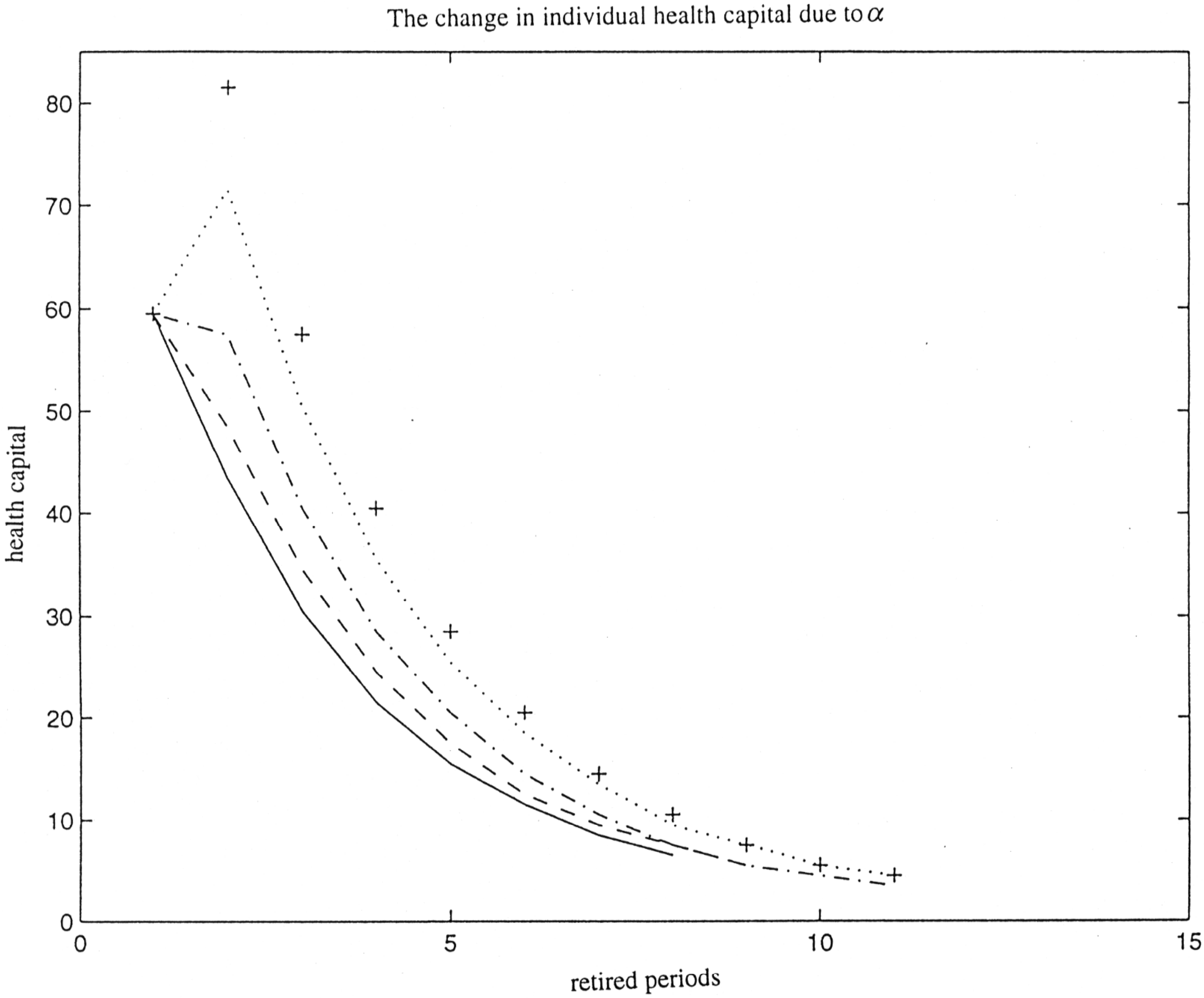
Description:

Solid: α =0.5, Dashed: α =0.6, Dashdot: α =0.7,Dotted: α =0.8, Plus: α =0.9

Note

The change in individual health investment due to α . Other parameters are held at the benchmark values: δ = 0.3, β = 0.9, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 23



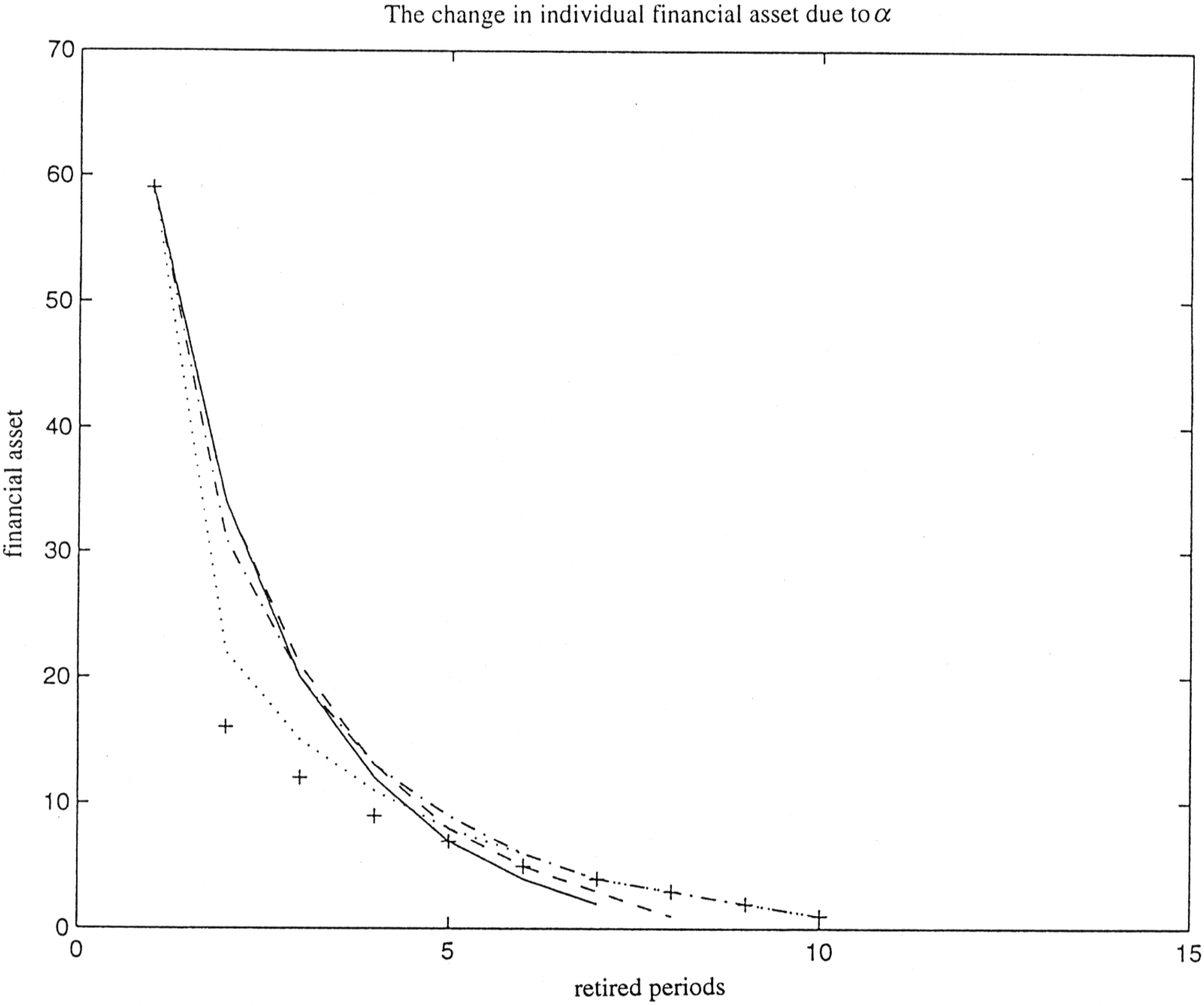
Description:

Solid: α =0.5, Dashed: α =0.6, Dashdot: α =0.7,Dotted: α =0.8, Plus: α =0.9

Note

The change in individual health capital due to α . Other parameters are held at the benchmark values: δ = 0.3, β = 0.9, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 24



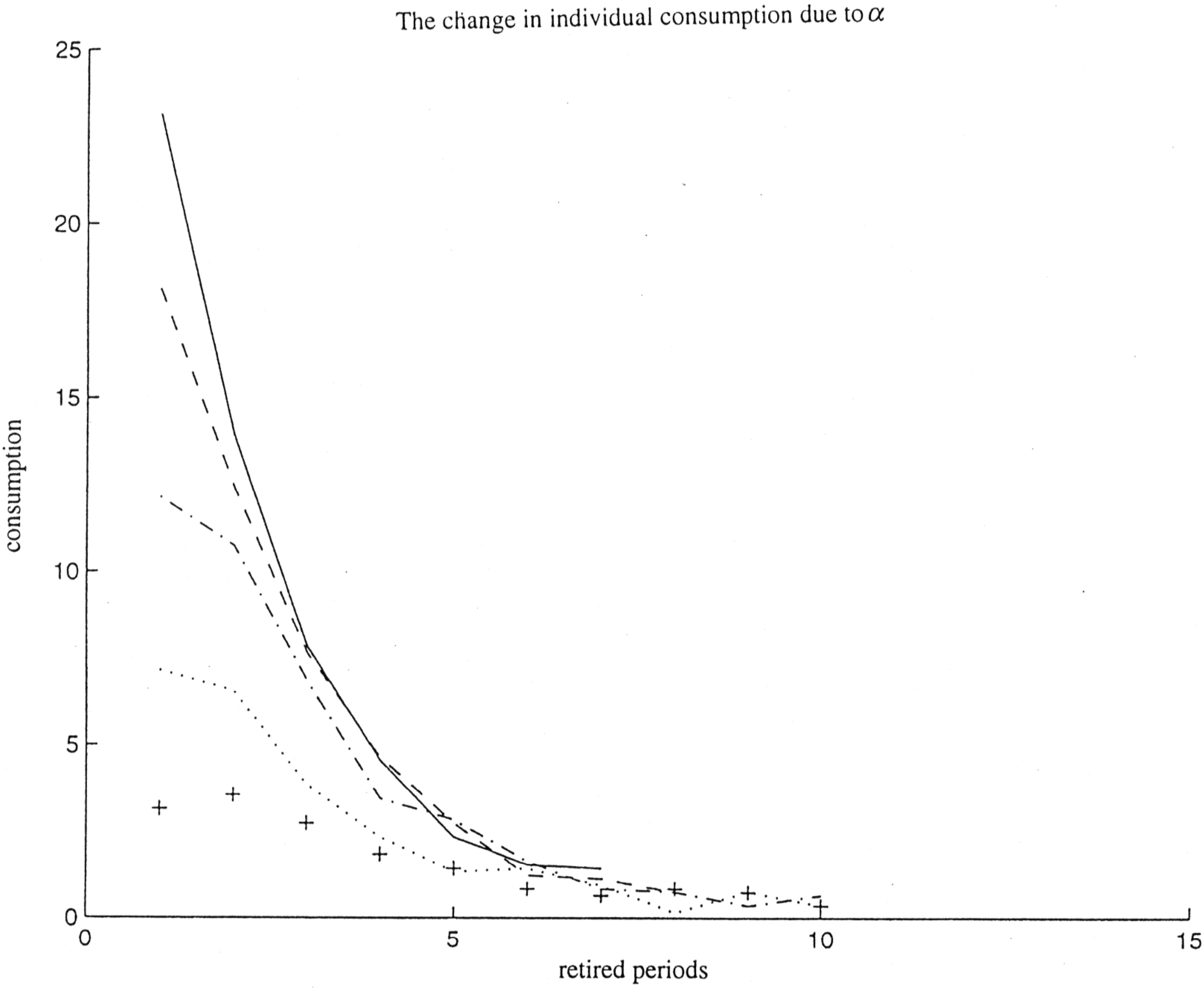
Description:

Solid: α =0.5, Dashed: α =0.6, Dashdot: α =0.7,Dotted: α =0.8, Plus: α =0.9

Note

The change in individual financial asset due to α. Other parameters are held at the benchmark values: δ = 0.3, β = 0.9, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 25



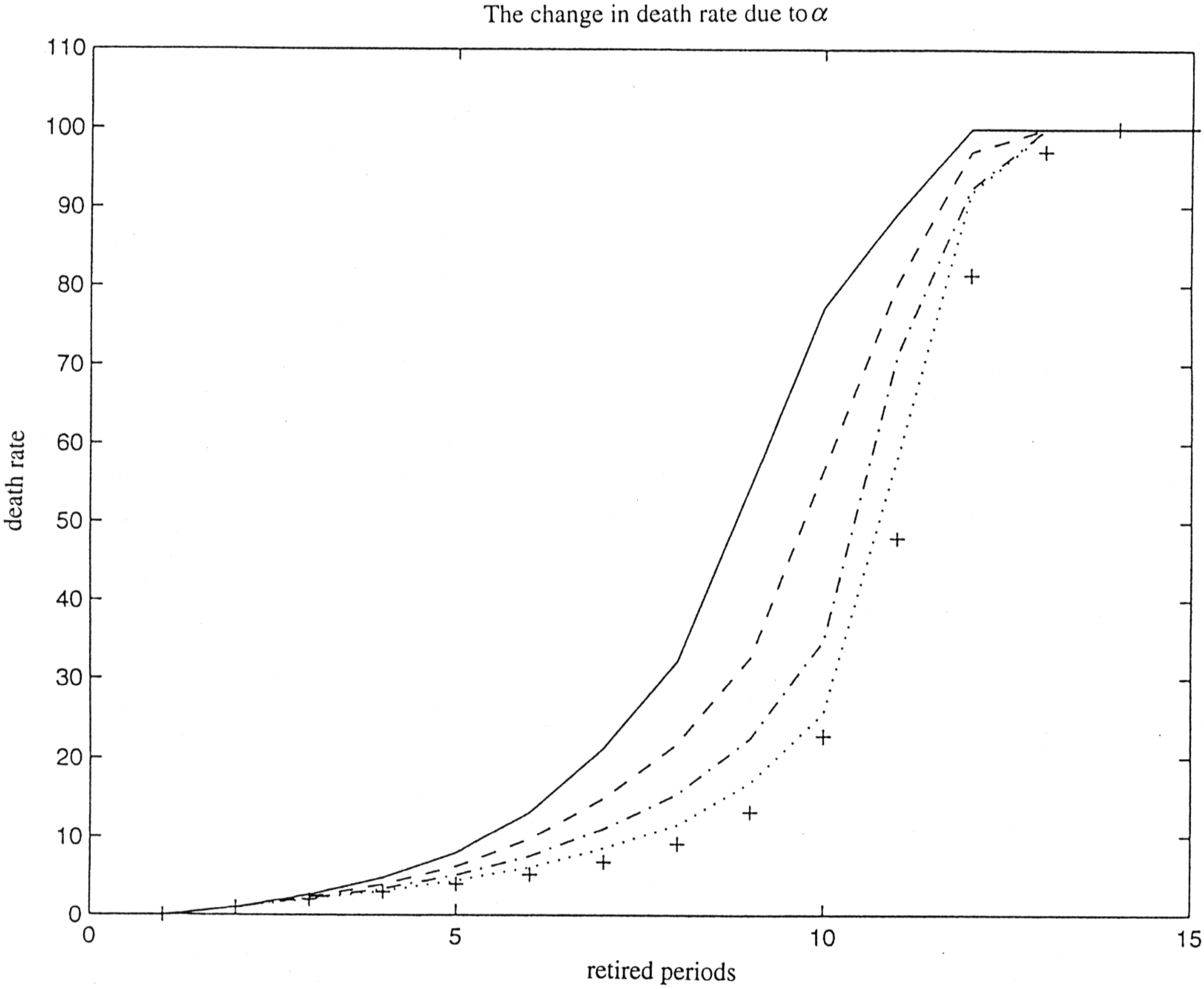
Description:

Solid: α =0.5, Dashed: α =0.6, Dashdot: α =0.7,Dotted: α =0.8, Plus: α =0.9

Note

The change in individual consumption due to α . Other parameters are held at the benchmark values: δ = 0.3, β = 0.9, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 26



Description:

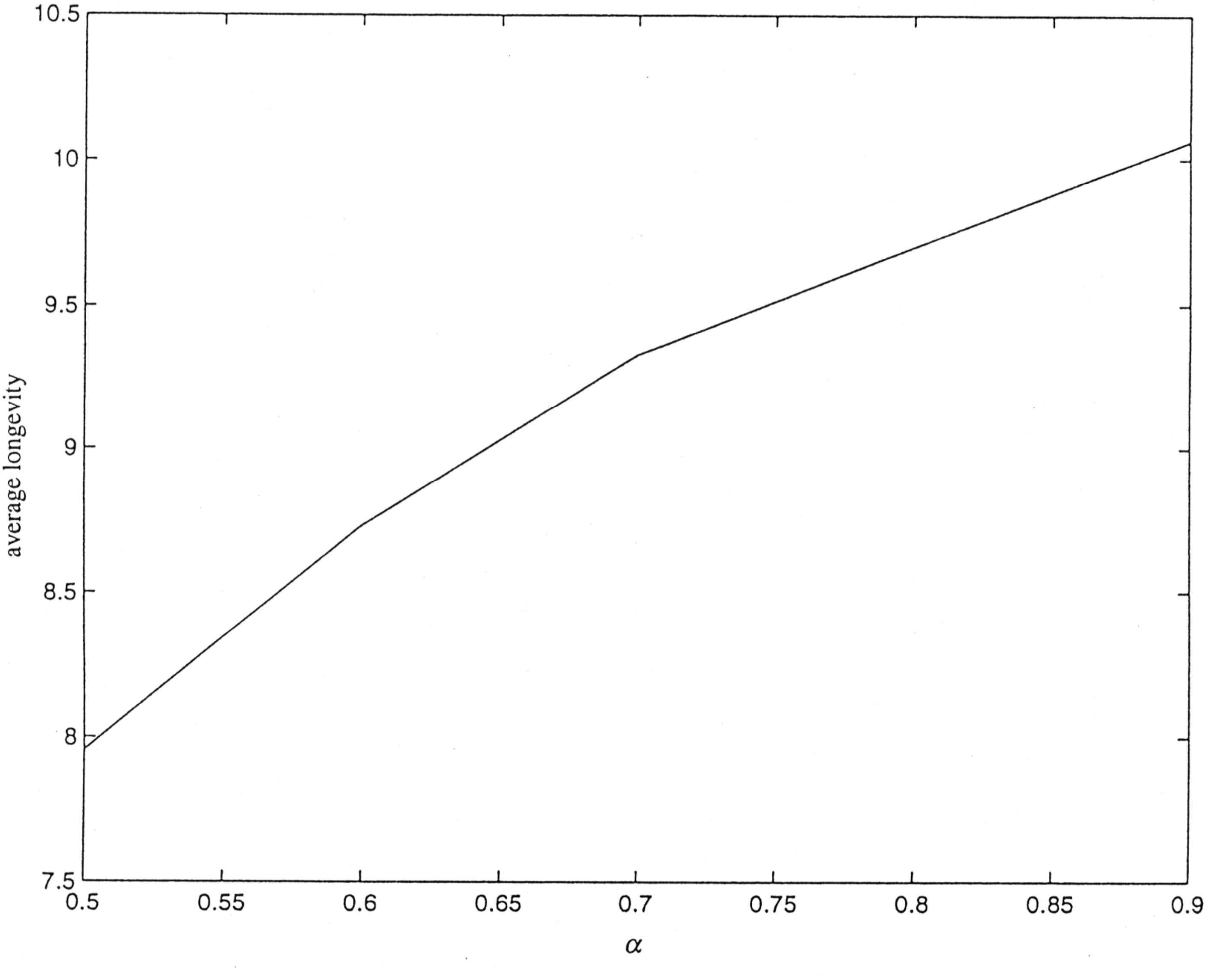
Solid: α =0.5, Dashed: α =0.6, Dashdot: α =0.7,Dotted: α =0.8, Plus: α =0.9

Note

The change in death rate due to α . Other parameters are held at the benchmark values: δ = 0.3, β = 0.9, r = 0. The death rate at t is defined as the ratio of the cumulative number of those who die before t to 10 000 retired individuals in initial period with different initial endowments.

Figure 27

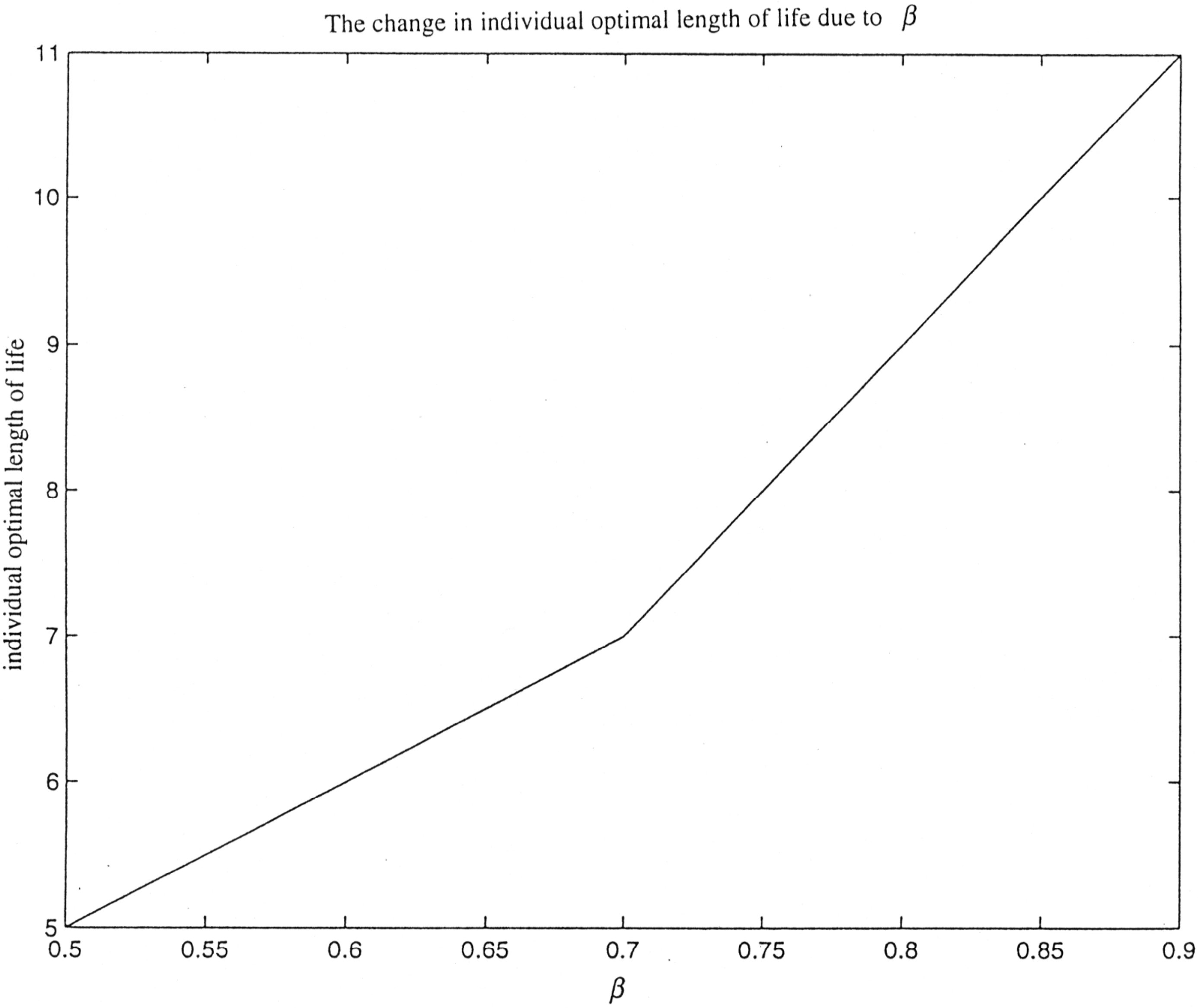
The change in social average longevity due to α



Note

The change in social average longevity due to α. Other parameters are held at the benchmark values: $\delta = 0.3$, $\beta = 0.9$, $r = 0$. Average longevity is defined as simple average longevity of 10000 retired-individuals in initial period.

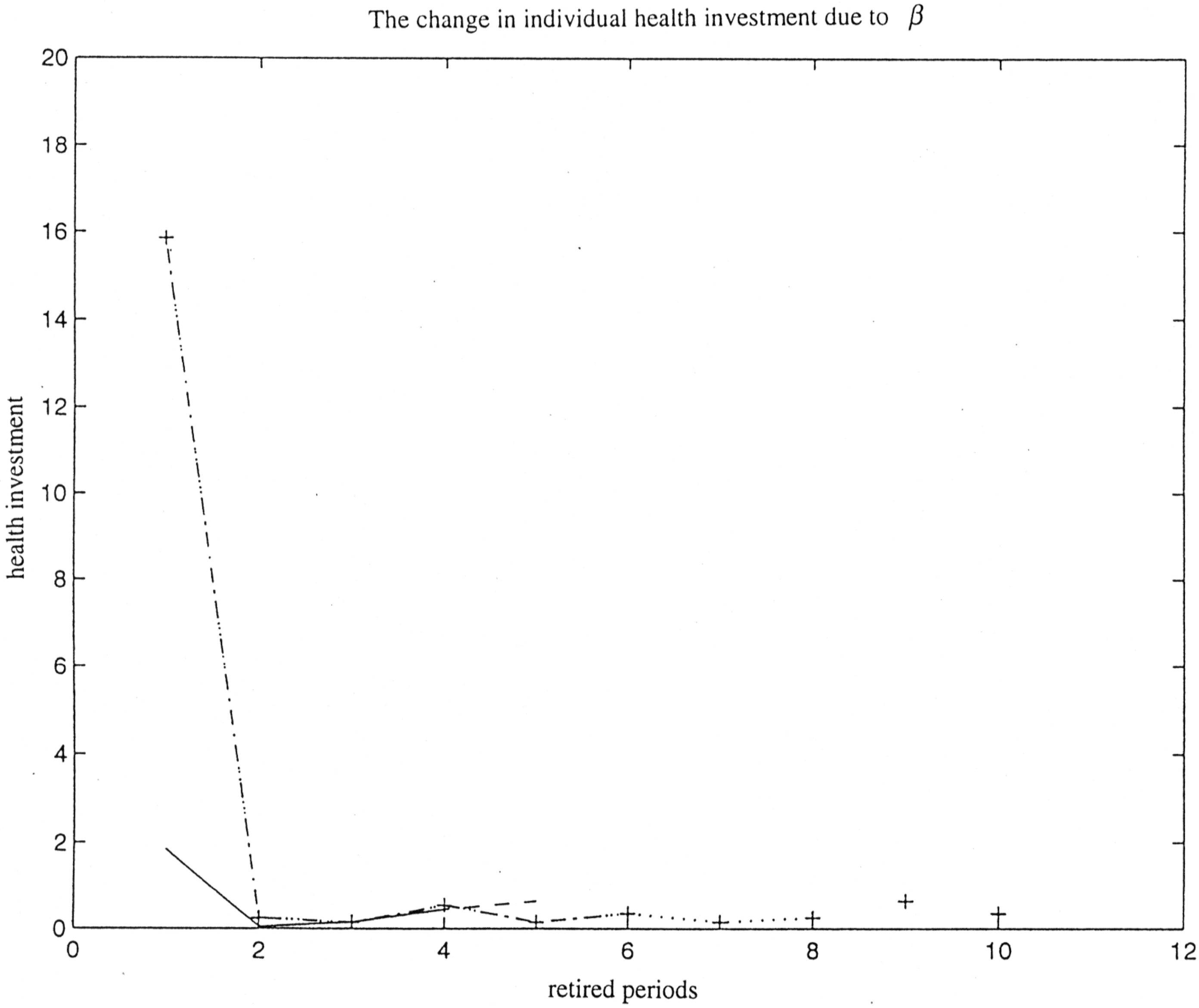
Figurc 28



Note

The change in individual optimal length of life due to β . Other parameters are held constant at the benchmark values: α = 0.7, δ = 0.3, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 29



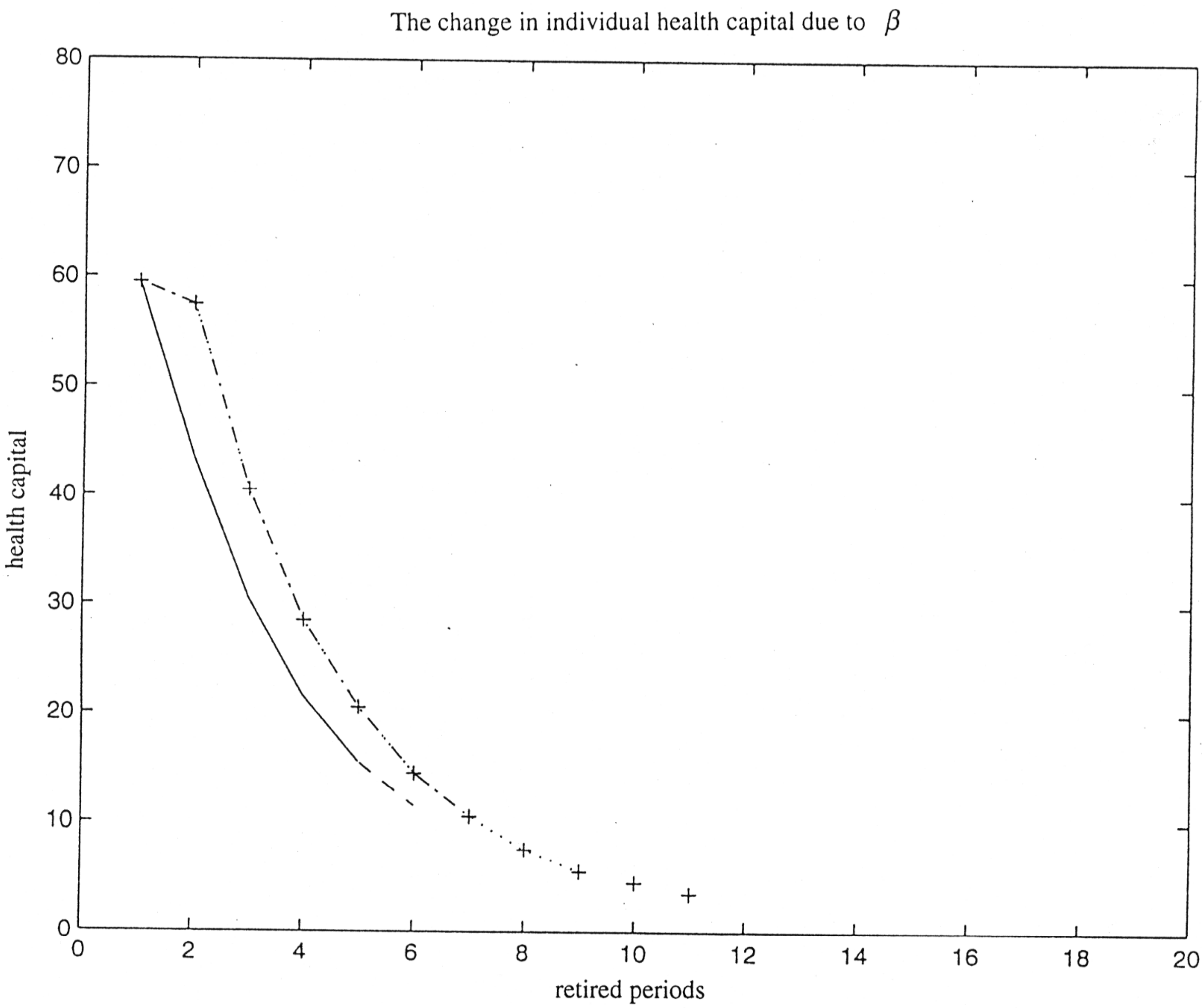
Description:

Solid: β =0.5, Dashed: β =0.6, Dashdot: β =0.7, Dotted: β =0.8, plus β =0.9

Note

The change in individual health investment due to β. Other parameters are held constant at the benchmark values: α = 0.7, δ = 0.3, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 30



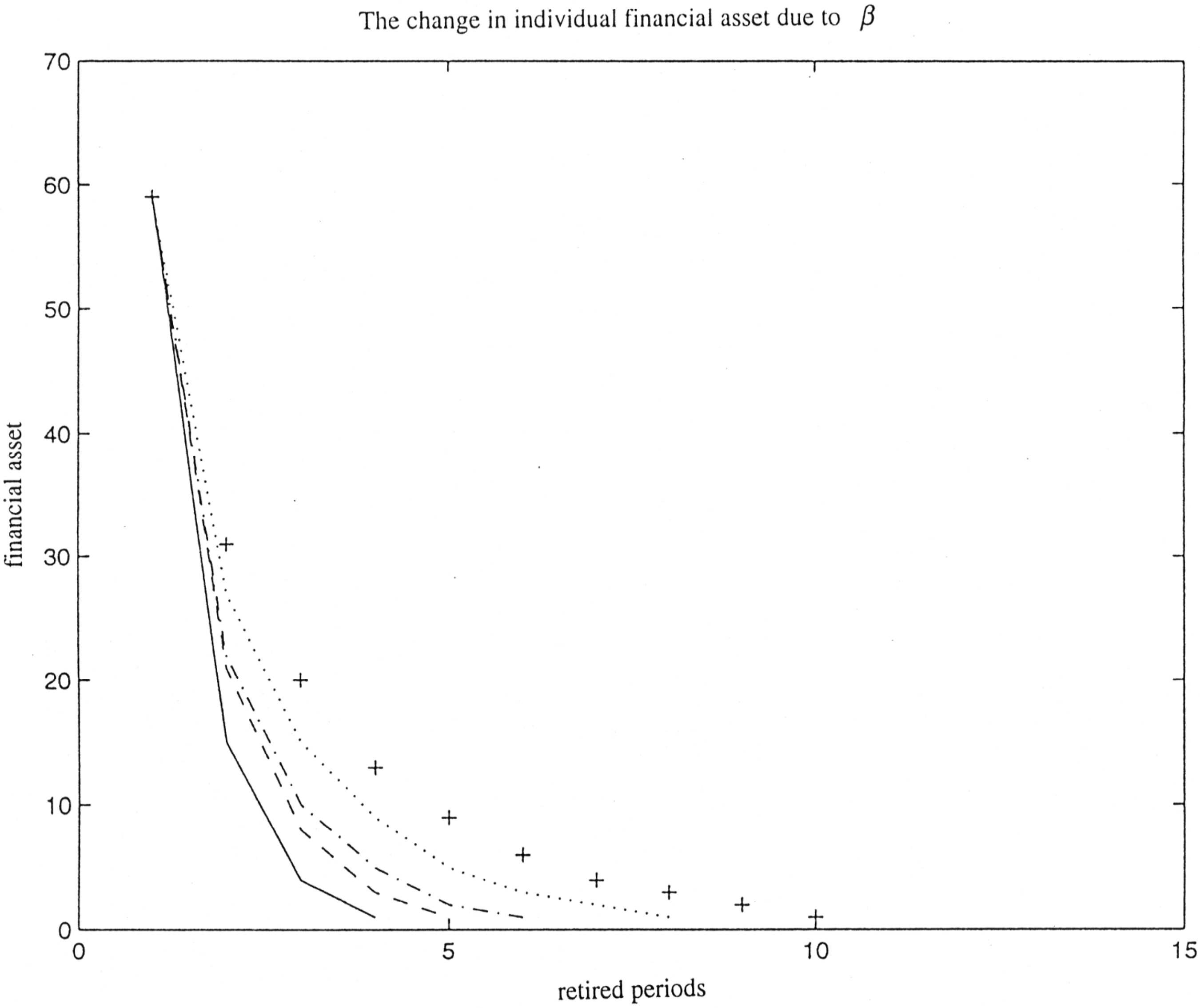
Description:

Solid: β =0.5, Dashed: β =0.6, Dashdot: β =0.7,Dotted: β =0.8,plus β =0.9

Note

The change in individual health capital due to β. Other parameters are held constant at the benchmark values: α = 0.7, δ = 0.3, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 31



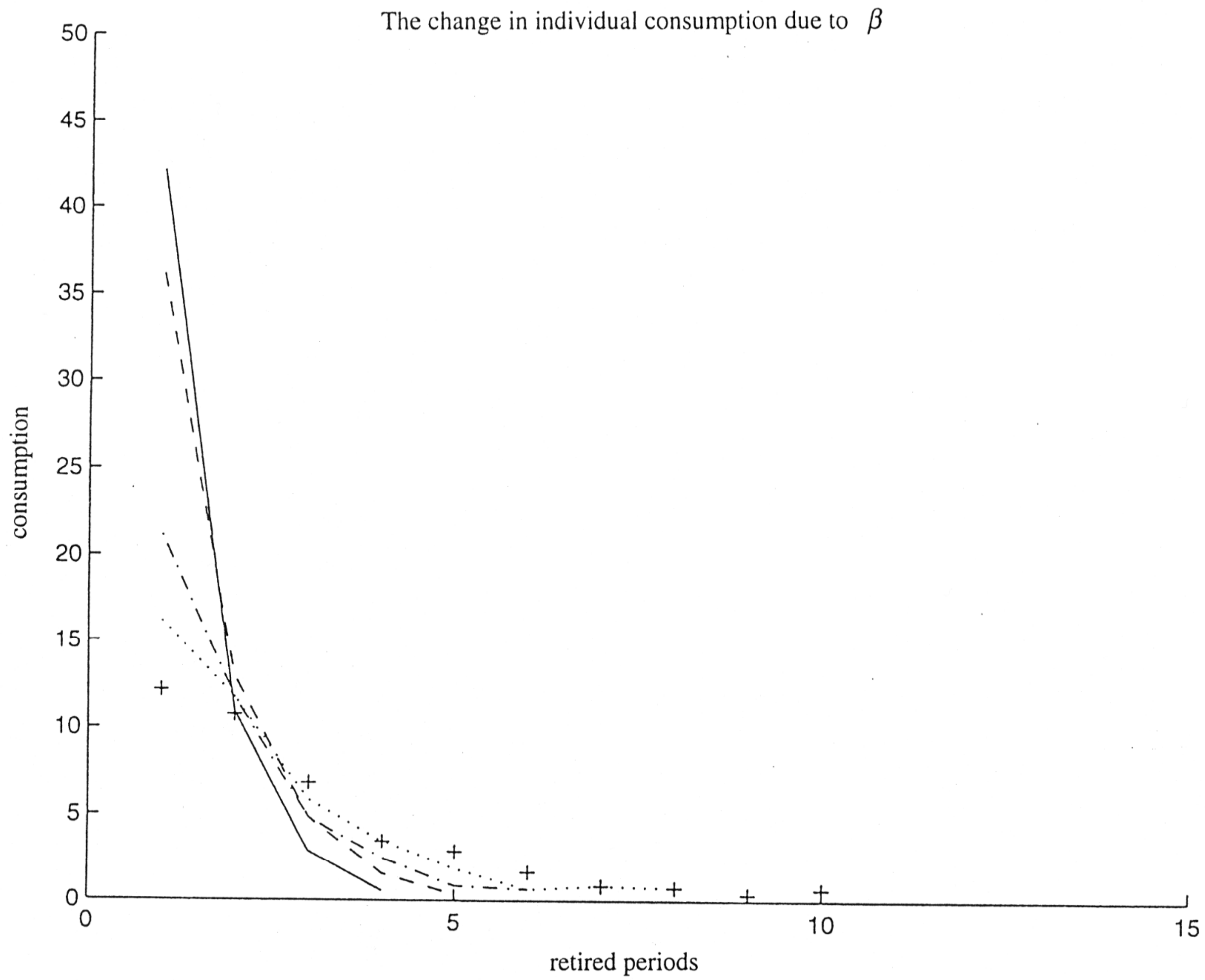
Description:

Solid: β =0.5, Dashed: β =0.6, Dashdot: β =0.7,Dotted: β =0.8,plus β =0.9

Note

The change in individual financial asset due to β. Other parameters are held constant at the benchmark values: α = 0.7, δ = 0.3, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 32



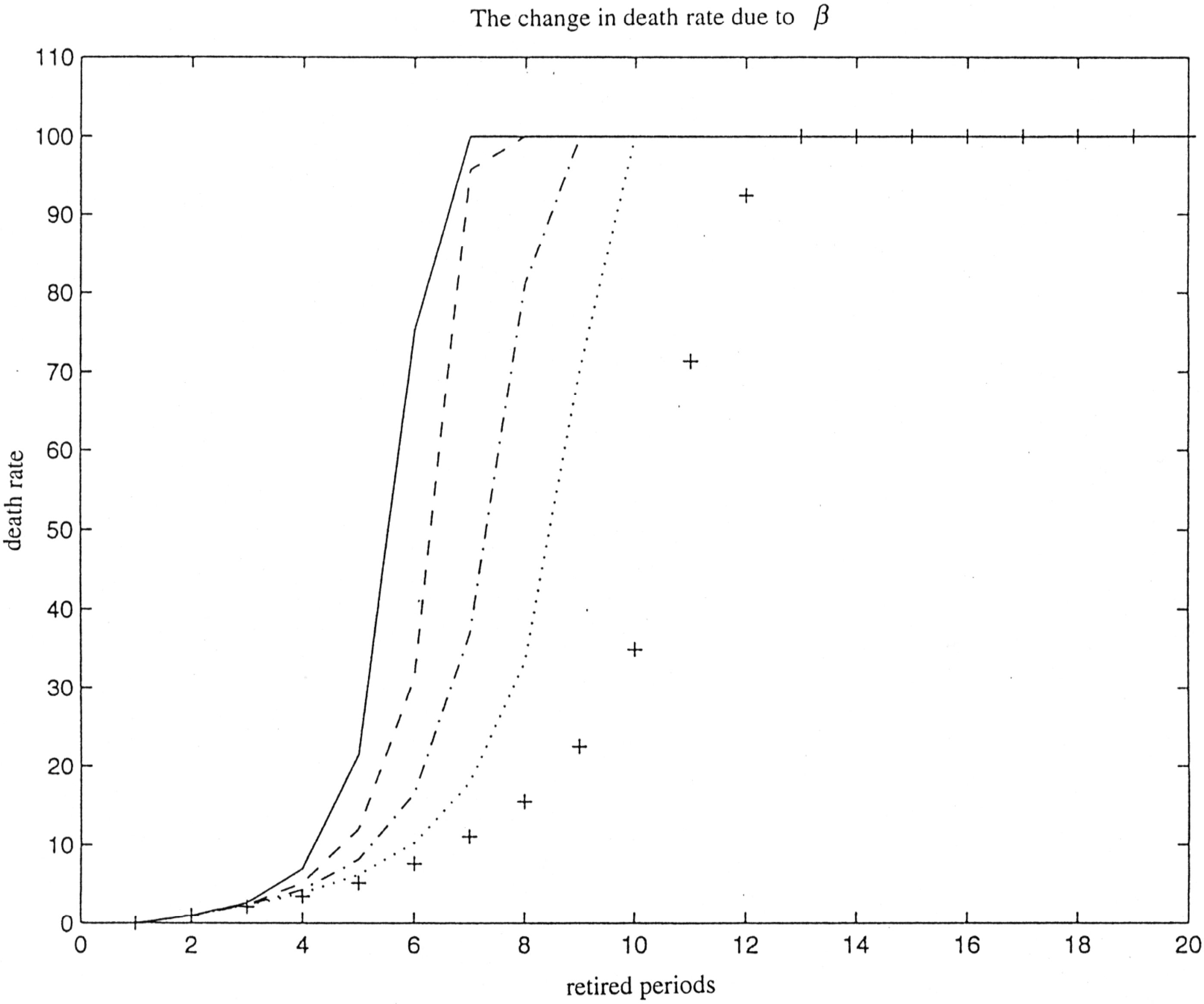
Description:

Solid: β =0.5, Dashed: β =0.6, Dashdot: β =0.7,Dotted: β =0.8,plus β =0.9

Note

The change in individual consumption due to β. Other parameters are held constant at the benchmark values: α = 0.7, δ = 0.3, r = 0. This individual has 60th levels of initial health capital and financial asset.

Figure 33



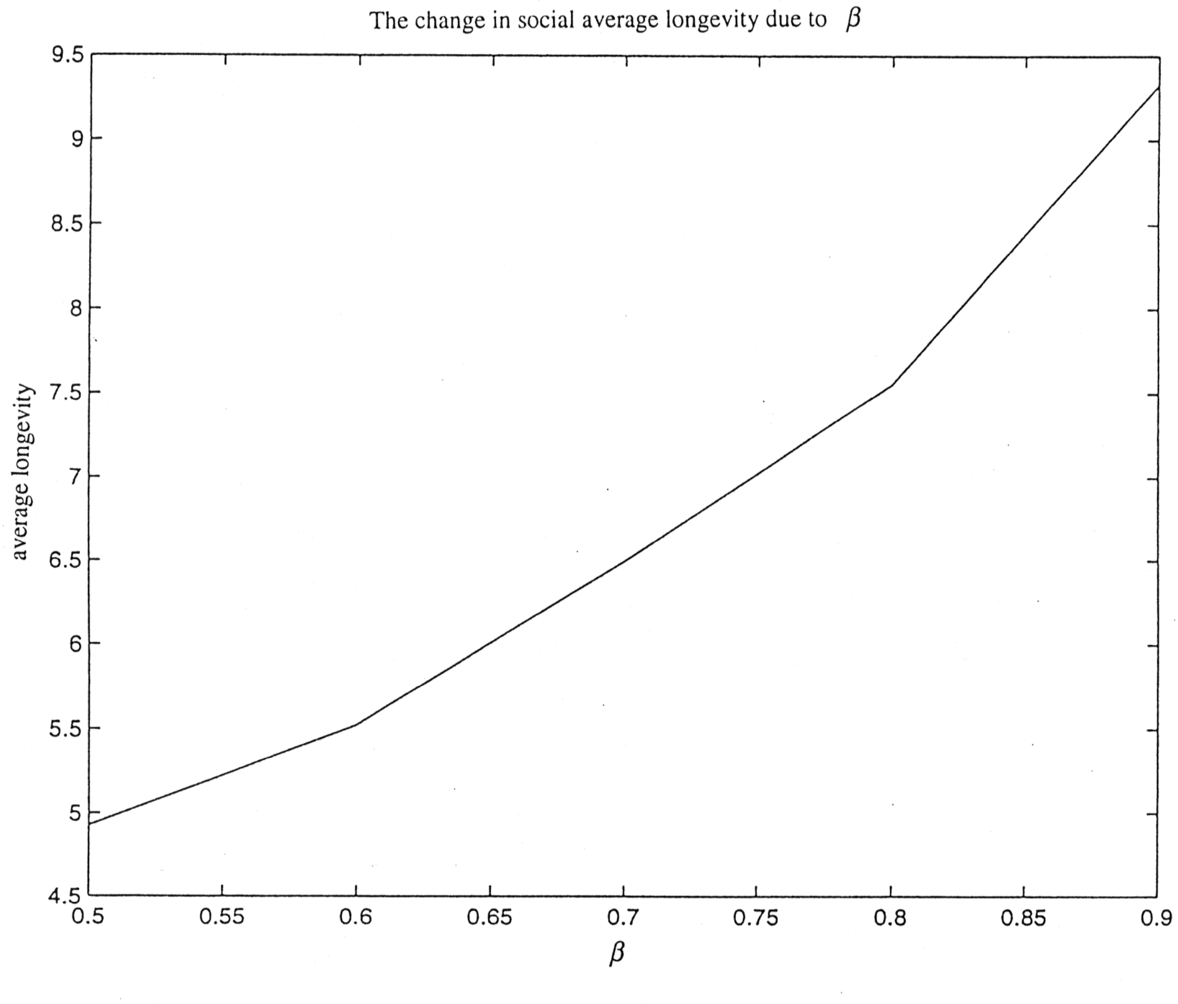
Description:

Solid: β =0.5, Dashed: β =0.6, Dashdot: β =0.7,Dotted: β =0.8,plus β =0.9

Note

The change in death rate due to β . Other parameters are held constant at the benchmark values: α = 0.7, δ = 0.3, r = 0. The death rate at t is defined as the ratio of the cumulative number of those who die before t to 10 000 retired individuals in initial period with different initial endowments.

Figure 34



Note

The change in social average longevity due to β. Other parameters are held constant at the benchmark values: α = 0.7, δ = 0.3, r = 0. Average longevity is defined as simple average longevity of 10000 retired-individuals in initial period.