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GOVERNMENT SPENDING IN A MODEL OF ENDOGENOUS GROWTH WITH PRIVATE AND PUBLIC CAPITAL

Dipankar Dasgupta

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The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

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Dipankar Dasgupta†
Institute of Social & Economic Research
Osaka University, 6-1 Mihogaoka, Ibaraki
Osaka 567-0047, Japan (Till March 31, 2001)
and
Economic Research Unit
Indian Statistical Institute
203 B.T. Road
Calcutta 700 035, India (From April 1, 2001)
e–mail : *ojaantrik@yahoo.com*

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ABSTRACT

The paper constructs a two-sector model of endogenous growth for a Mixed Economy characterized by two private inputs, labor and capital, and the services of an accumulable pure public input (of which an important example is infrastructure). Final goods are produced by a competitive private sector with the help of the three inputs, the public input being supplied free of charge by the Government. The Government accumulates infrastructural stocks noncompetitively with the same inputs, meeting the cost of accumulation from income tax revenues. The policy instruments for the Government are the tax rate and the revenue shares spent on the private inputs. The paper describes the equilibrium of the Mixed Economy and compares the optimum choices of the two instruments under alternative policy objectives. It also demonstrates the transitional stability of steady state paths. Inefficiencies of the Mixed Economy make its equilibria socially suboptimal. The best growth path of the economy is the unique solution to the Command Economy problem. However, despite the existence of suitably defined efficiency prices for a fictitious private economy that supports the Command Economy path, the latter is not decentralizable. The reason for this lies in the inability of the private economy to resolve the joint effect of labor and public capital into their individual effects. Paradoxically, the highest, though inefficient, growth rate for the Mixed Economy can be reasonably close to the socially best growth rate of the Command Economy. This gives rise to an identification problem, constituting thereby a dilemma for developing economies in search of market friendly systems.

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1 Introduction

The importance of the State in the process of economic development is hard to underestimate. This is particularly evident from the fact that an economy's growth path is often conditioned by the availability of inputs that qualify as pure public goods. The nonexcludability of such inputs implies that it will be incentive incompatible for private entrepreneurs to produce and supply a class of vital inputs necessary for economic growth. Consequently, there emerges a natural role for the Government to supplement the private sector, especially in developing economies.

Barro (1990) qualifies as the seminal paper in the field.[1] The novelty of this paper lay in endogenizing the rate of growth of an economy by linking it to the Government's fiscal exercises. The Government in Barro subsidized each household producer by providing a free infrastructural or public *input*, but covered the subsidy by levying an equivalent tax on the final *output* produced. This balanced budget procedure connected up the tax rate chosen by the Government to the post tax marginal productivity of capital, and hence, the rate of growth endogenously desired by the Representative Household in the economy. The link between fiscal policy and growth allowed for a comparison of the tax rate that maximized steady state welfare with the one that maximized steady state growth. In particular, for a Cobb-Douglas production structure (though not for a more general one), Barro found that the two rates were identical.

Barro's one-sector exercise was extended by Futagami *et al* (henceforth,

[1] Unlike the approach of the present paper, Barro is skeptical about the *pure* publicness of inputs in the real world. Hence, he concentrates only on rival public inputs. In spite of a continued skepticism regarding the issue, Barro & Sala-i-Martin (1999) present a model of nonrival public input also. Extensions of Barro's paper have appeared since its publication. Alesina and Rodrik (1992), , Dasgupta (1999, 2000), Futagami *et al* (1993) and Grenier & Semmler (2000) constitute a representative sample.

FMS) to a two-sector model, where infrastructural services were visualized as flowing out of an accumulable stock. FMS justified the necessity for the extension on the following grounds:

> First, many public infrastructures such as highways, airports, and electrical and gas facilities are stock variables in nature. $\cdots$ Second, and more importantly, there are several empirical studies supporting the importance of public capital in private production. For example, Aschauer (1988) finds empirically that public capital raises the marginal productivity of private capital. Estimating a version of the Cobb-Douglas production function with public capital for the Japanese economy, Iwamoto (1990) also finds a significant effect of public capital.

The FMS position is supported by the World Development Report, 1994, which found that a 1 % rise in the stock of infrastructure leads to a 1 % improvement in the GDP across countries. It also estimated that the annual economic costs of inadequate transport infrastructure in China was at least 1 % of its GNP. The India Infrastructure Report (1996) expressed a similar sentiment in discussing the fast-growing East and South East Asian countries.

The two-sector extension led FMS to consider the transitional dynamics for the model, which they proved to be stable. They also looked into the relationship between growth rate maximizing and welfare maximizing income tax rates in the presence of infrastructural stocks. As opposed to the Barro result in this context, they found that for a Cobb-Douglas production structure, the two tax rates were not equal. FMS argued that the implication of this result was that

> $\cdots$ the policy maker's work is more complex than simply maximizing the growth rate of the economy.

In spite of these significant differences between Barro and FMS, there were certain respects in which the economies they studied were similar. First, both were concerned with household producers with no explicit role for labor in the production function. Secondly, the public input was viewed as a rival commodity, though supplied freely by the Government. Thirdly, there being a single private input, capital, neither paper was concerned with market determined distribution of factor incomes. Finally, taxes were levied directly on the final good produced by the economy. In subsequent work, Barro & Sala-i-Martin (1999) (henceforth, BS) re-established the equality between between the growth and the welfare maximizing tax rates for an alternative one-sector version of Barro (1990), in which the household producer was replaced by a competitive entrepreneur, (b) the technology recognized labor as well as private capital and public infrastructure services, (c) the public input was allowed to be nonrival and (d) returns to labor and private capital were determined in competitive factor markets. To the extent, however, that the model had a single sector, the public input was a pure flow as in Barro (1990). Moreover, taxes continued to be raised on the final output.

In the present paper, I reinvestigate the Barro-BS-FMS issues by adding on to the BS framework infrastructural stocks as in FMS.[2] However, unlike the BS model, where taxation of output and incomes might well be equivalent instruments, I allow for taxes to be imposed directly on incomes, since there does not seem to be large-scale evidence of direct taxation of output in developing economies to create infrastructure necessary for growth and development. The model has a profit maximizing, competitive private sector producing a consumable and accumulable final output and a government sector producing changes in the public capital stock as well as supplying the service flows emanating from it. The service in question is assumed to be a *pure* public input, which prevents the Government sector from functioning on competitive principles. Consequently, there arises a divergence between marginal productivities and factor returns in the public sector, which is a major source of inefficiency in the Mixed Economy under consideration.

Private capital and labor employed in the public sector are assumed to be paid according to their opportunity costs, viz, the factor returns prevailing in the private sector, while the Government's tax revenue determines the budget available for purchasing these inputs in the factor markets. As in Barro-BS-FMS, the Government's budget is balanced. Unlike these papers, however, the budget has to be distributed over the two factors of production.

[2]An alternative interpretation of the endogenous growth model of the present work would be to view it as a generalization of the two sector, two factor Rebelo (1991) economy to a three factor world through the introduction of a (third) *pure* public (i.e. nonrival *and* nonexcludable) input into both production functions.

Thus, there are two choice parameters for the Government. The first, like Barro-FMS-BS, is a tax rate. The second is yet another *ratio* representing the allocation of the tax revenue amongst private inputs (capital and labor services) used for accumulating infrastructure. (See Section 2 below for the exact definition of the ratio.) The necessity of choosing the two parameters at their optimal levels simultaneously calls for a fresh consideration of the Barro-FMS-BS issues and justifies the exercise to follow.[3]

In the context of the Mixed economy, I demonstrate that a tax rate–expenditure ratio pair that maximizes the steady state growth rate is non-welfare maximizing. This confirms the robustness of the FMS conclusion stated above. However, I also show that growth and welfare maximization would be equivalent policies under the additional constraint that factor shares in the public sector, and not merely the factor returns, mimic the ones prevailing in the private sector. Consequently, a limited version of the Barro-BS result survives in the two-sector extension also. Quite apart from these issues, the paper proves, like FMS, that the economy is transitionally stable. Thus, given any choice of the two parameters and starting from any set of initial values of the private and public capital stocks, the dynamic path converges over time to the steady state value of the ratio of the two stocks. Without this result, policy choices of the Government with reference to steady state growth will amount to empty theorizing.

The inefficiencies in the Mixed Economy imply that its welfare maximizing equilibrium is not an overall social optimum. Hence, the paper goes on to prove the existence of a socially optimum (or, first best) steady growth path characterizing a centrally planned or Command Economy and suggests a scheme for decentralizing it. The scheme in question, however, treats the services of labor and public capital as a joint input. Since a market for such an input cannot function in the real world, the decentralization exercise is operationally vacuous, although the Command Economy optimum exhibits a higher growth rate than the highest achievable by the Mixed Economy. On the one hand, therefore, a decentralized economy that can support the first best growth path of the system is nonfunctional. On the other hand, a func-

[3]Both Dam (1997) and Dasgupta & Dam (2000) introduced the idea of the second choice parameter, but stopped short of the optimization exercise as well as the transitional dynamics of this paper.

tional market oriented system is grossly suboptimal. Interestingly enough, the highest growth rate achievable by the Mixed Economy for certain parameter values falls only marginally short of the Command Economy's growth rate, though the Mixed Economy can achieve its highest growth rate only at the cost of efficiency, and hence welfare. This poses a dilemma for developing economies engaged in the process of transition from planned to market economies.

I describe the model of the paper and set out the equilibrium solutions for a static and a dynamic steady state economy in Section 2. Sections 3 and 4 discuss respectively the problems of growth and welfare maximization with respect to the two policy parameters. I establish transitional stability of the economy in Section 5. Issues relating to the social optimum are discussed in Sections 6 and 7. Section 8 concludes the paper.

2 The Model

There are two productive sectors, referred to as the Y and the G-sector respectively. The Y-sector is privately controlled and produces Y, which is used for consumption as well as capital accumulation. Following Solow (1956), K denotes both the stock of capital and the flow of its services (i.e. the stock-flow ratio is assumed to be unity).

The G-sector is state owned and engaged in accumulating the infrastructural capital, denoted by G. The stock generates a flow that enters all production (including its own) as an essential input. As with K, the stock-flow ratio is a constant, so that G represents a flow also. The flow of G is a pure public good by assumption. Hence, both sectors use the same flow of G.[4] The Government supplies it free of user charge to all producers, including itself.

[4]This means of course that both are assumed to be nonsatiated in the use of the public input. This is particularly evident for infrastructure, which constitutes a bottleneck for developing economies.

As in BS, the technology for Y is given by[5]

$$Y = A\ K_y^{\alpha}\ L_y^{1-\alpha}\ G^{1-\alpha},\ 0 < \alpha < 1, \tag{2.1}$$

where K_y, L_y and G are the flows of capital, labor and the pure public input used to produce Y and A is parametrically specified. Thus, production exhibits constant returns to scale in private inputs, K_y and L_y. For fixed G, the economy faces diminishing returns to private capital. However, for fixed L_y, (2.1) implies constant returns to G and K_y. Commodity Y will be used as the numeraire in what follows.

The technology for $\dot{G}$, the change in the *stock* of G, is similar to (2.1) except for the parameters of the production function. Thus,

$$\dot{G} = B\ K_g^{\beta}\ L_g^{1-\beta}\ G^{1-\beta},\ 0 < \beta < 1, \tag{2.2}$$

where K_g, L_g and G are the flows of capital, labor and the public input used to produce $\dot{G}$ and B is parametrically specified.[6] As with Alesina & Rodrik (1992), Barro and Sala-i-Martin (1999) and BS, I assume the aggregate labor force to be constant for the economy.[7]

Society's welfare is identical with that of the Representative Household and given by

[5] While all the variables appearing below are functions of time, the time index will often be dropped for notational simplicity.

[6] The form of both functions implies that G enters production in a labor augmenting, Harrod-neutral fashion. Harrod neutrality is of course unavoidable for steady state analysis. An alternative specification of the production functions might be (for example) $Y = K^{\alpha}\ L^{\beta}\ G^{1-\alpha-\beta}$ and $\dot{G} = K^{\delta}\ L^{\epsilon}\ G^{1-\delta-\epsilon}$. In per capita terms, these reduce to $(Y/L) = (K/L)^{\alpha}\ (G/L)^{1-\alpha-\beta}$ and $(\dot{G}/L) = (K/L)^{\delta}\ (G/L)^{1-\delta-\epsilon}$. Since steady state fixes the values of K/L and G/L in such a model, the endogeneity of the growth rate is lost in the process. The model then becomes inappropriate for analyzing the effect of policy on economic growth. See BS and Lucas (1988) on these issues.

[7] This is an innocuous assumption. Introducing an exponentially growing labor force is straightforward and does not affect the endogeneity of the growth rate under the paper's model specifications.

$$W = \int_0^\infty ln\ C(t)\ e^{-\rho t}\ dt, \tag{2.3}$$

where $C(t)$ represents consumption at point of time t and the constant ρ is a positive discount parameter.[8]

The Household owns all private capital and is the sole supplier of its services as well as labor. The final product Y is produced by a Representative Firm that maximizes instantaneous profit, assuming all prices to be parametrically specified. It is charged a price r and w respectively by the Household per unit flow of capital and labor services consumed. Further, there is a proportional tax τ on household income. The tax proceeds are used to pay for the competitive prices of capital and labor services used up in the production of $\dot{G}$.

Profit maximization gives rise to demands for K and L services by the Y-sector, but not for G services, because a demand function for G does not exist in this model.[9] The Y-sector demands for K and L are added to the corresponding requirements by the G-sector to yield the aggregate demands for K and L. Like Solow (1956), the available aggregates of the services are thrown inelastically on the factor market at each point of time, their prices being determined by equating the inelastic supplies to the aggregate demands.

To accumulate infrastructure, the G-sector pays K and L the competitive prices and, like the competitive Firm, has free use of G services. Since the tax proceeds are spent on the two factors capital and labor, it is necessary to specify a rule for distributing the expenditure over the factors. I assume that the expenditures are allocated to satisfy the condition that their ratio

[8]It is more common in Endogenous Growth Theory to deal with the welfare function

$$\int_0^\infty \frac{C(t)^{1-\sigma} - 1}{1-\sigma}\ e^{-\rho t}\ dt.$$

where $\sigma > 0$ is the constant elasticity of instantaneous marginal utility. As $\sigma \to 1$, the function reduces to (2.3). I choose the special case in the paper for notational simplicity. All results reported here carry through for the more general form.

[9]This analytical feature derives from Samuelson (1954).

equals a constant (to be chosen optimally) in steady state.

The mechanics of the model falls into two parts: Static Equilibrium at each t and the Dynamic Steady State Equilibrium. I describe these in turn.

2.1 Static Equilibrium

For the Y-sector to behave competitively, the factor returns must equal the marginal products. Thus,

$$\begin{aligned} r &= \alpha\ A\ K_y^{\alpha-1}\ L_y^{1-\alpha}\ G^{1-\alpha} & (2.4) \\ w &= (1-\alpha)\ A\ K_y^{\alpha}\ L_y^{-\alpha}\ G^{1-\alpha}, & (2.5) \end{aligned}$$

or, alternatively,

$$K_y = \frac{\alpha}{1-\alpha}\ L_y\ \frac{w}{r}. \tag{2.6}$$

Writing K and L respectively for the aggregate supplies of capital and labor, the G-sector employment of these factors is governed by a budget constraint,[10] viz,

$$r\ K_g + w\ L_g = \tau\ (r\ K + w\ L) \tag{2.7}$$

and the rule that

[10]I assume a uniform tax rate on profits and wages, partly to concentrate attention on the interplay between the two policy parameters mentioned earlier. Besides, there is no obvious theoretical reason that recommends differential taxation of the two factors in the Representative Household macro model I am concerned with. See, however, Alesina and Rodrik (1992) for the political economy of differential taxation in a model that recognizes agent heterogeneity in terms of factor endowments.

$$\frac{r\ K_g}{w\ L_g} = \text{ constant} \tag{2.8}$$

in steady state. The constancy of rK_g/wL_g in steady state implies that $rK_g/(r\ K_g + w\ L_g)$ as well as $wL_g/(r\ K_g + w\ L_g)$ are themselves constants. I denote the first of these by γ and the second by $1-\gamma$, so that (2.8) reduces to

$$\frac{r\ K_g}{w\ L_g} = \frac{\gamma}{1-\gamma}, \tag{2.9}$$

where $0 < \gamma < 1$.[11] The different sources of inefficiency characterizing the Mixed Economy may be noted at this stage. First, while capital and labor in the G-sector are paid the same returns that prevail in the private sector, these need not equal their marginal products as in equations (2.4) and (2.5). Secondly, the shares γ and $1-\gamma$ of capital and labor in the G-sector may not equal the competitive shares β and $1-\beta$. Thirdly, there is of course the usual distortion introduced by the tax rate. Lastly, while the marginal productivity of G is strictly positive in both sectors, the price paid for it is zero. These imply that the Mixed Economy can, at best, reach an inefficient solution of the dynamic resource allocation problem under consideration.

Equation (2.9) is written in a form similar to (2.6) as

$$K_g = \frac{\gamma}{1-\gamma}\ L_g\ \frac{w}{r}. \tag{2.10}$$

Equations (2.6), (2.7) and (2.9) together imply

[11]As in Dam (1997), one possible choice of the expenditure ratio is the ratio of the exponents of capital and labor in the G-sector production function. In other words, the expenditure allocation rule might mimic a necessary condition satisfied by factor returns under perfect competition. I shall show below (Remark 2 and Proposition 4.2) that $\gamma = \beta$ maximizes neither welfare nor the growth rate of the Mixed Economy.

$$L_y = \lambda(\tau, \gamma)\ L_g. \tag{2.11}$$

where $\lambda(\tau, \gamma) = ((1-\alpha)/(1-\gamma))\ ((1-\tau)/\tau)$. Assuming now that the total supply of labor is normalized to unity for all time, it follows from market clearing that

$$L = L_y + L_g = 1.$$

Substituting from (2.11), we see that

$$\begin{aligned} L_y &= \tfrac{\lambda(\tau,\gamma)}{1+\lambda(\tau,\gamma)} \\ L_g &= \tfrac{1}{1+\lambda(\tau,\gamma)}. \end{aligned} \tag{2.12}$$

Using (2.6), (2.10) and (2.12), the aggregate demand for K turns out to be

$$K_y + K_g = (\frac{\alpha}{1-\alpha}\ \frac{\lambda(\tau,\gamma)}{1+\lambda(\tau,\gamma)} + \frac{\gamma}{1-\gamma}\ \frac{1}{1+\lambda(\tau,\gamma)})\ \frac{w}{r}. \tag{2.13}$$

For every pair $(\tau,\ \gamma)$, the equilibrium value of w/r is determined by equating the aggregate demand for K to the aggregate supply, $K(t)$, at each t.[12] This is shown in Figure 1. The expression $(\alpha/(1-\alpha))\ (\lambda(\tau,\gamma)/(1+\lambda(\tau,\gamma))) + (\gamma/(1-\gamma))\ (1/(1+\lambda(\tau,\gamma)))$ will appear repeatedly in the paper. To make the notation simpler, I shall refer to it simply as χ. Note, however, that χ depends on α, τ and γ. Since χ lies between $\alpha/(1-\alpha)$ and $\gamma/(1-\gamma)$,

Figure 1 here.

[12]Note from the derivation of (2.12) that the labor market equilibrium condition is built into (2.13) through the requirement that $L_y + L_g = [\lambda(\tau,\gamma)/(1+\lambda(\tau,\gamma))] + [1/(1+\lambda(\tau,\gamma))] = 1$.

which are both positive and finite, there is a well-defined non-zero solution for w/r corresponding to each specification of τ and $K(t)$. Equations (2.6), (2.10) and (2.12) now give the equilibrium values of K_y and K_g. Equation (2.13) and the condition of market clearance at each point of time implies that (τ, γ) fixes the equilibrium value of w/K at

$$\frac{w}{K} = \frac{r}{\chi}. \tag{2.14}$$

Since r is a constant in steady state equilibrium, (2.14) shows that w and K change at the same rate in steady state.

2.2 Dynamic Steady State Equilibrium

A steady state equilibrium involves:

$$\begin{aligned} \frac{\dot{G}}{G} &= \frac{\dot{Y}}{Y} \\ &= \frac{\dot{K}}{K} \\ &= \frac{\dot{C}}{C} \\ &= \text{constant.} \end{aligned}$$

In what follows, $\dot{G}/G$ will be referred to as the supply rate of growth. On the other hand, $\dot{C}/C$ is the demand rate of growth chosen by the Household.

Using (2.2) and the derivations of the previous subsection, the supply rate of growth turns out to be

$$g^s = B\ \left((1-\alpha)\ \alpha^{\alpha/1-\alpha}\ A^{1/1-\alpha}\right)^{\beta}\ \frac{\gamma^{\beta}(1-\gamma)^{1-\beta}\ \tau}{(1-\gamma)\ \tau + (1-\alpha)(1-\tau)}\ \frac{1}{r^{\beta/1-\alpha}}. \tag{2.15}$$

Thus, given the size of the Government's budget and its distribution over the two factors, a rise in r lowers the employment of capital for producing $\dot{G}$. Consequently the supply rate of growth falls. The Representative Household is endowed with perfect foresight and is assumed to take the time paths of τ, r and w as parametrically given. Typically therefore, the Household's optimization problem over time is to maximize (2.3) subject to

$$\dot{K} = (1 - \tau)\ (r\ K + w) - C. \tag{2.16}$$

This is a standard Ramsey problem, whose solution is the demand rate of growth

$$g^d = (1 - \tau)\ r - \rho. \tag{2.17}$$

The intersection of (2.17) and (2.15) determines the corresponding equilibrium growth rate and interest rate. For any given $(\tau, \gamma) \in (0, 1)\ \times\ (0, 1)$,

$$g^s \to \infty \text{ as } r \to 0 \text{ and } g^s \to 0 \text{ as } r \to \infty.$$

Similarly,

$$g^d \to -\rho \text{ as } r \to 0 \text{ and } g^d \to \infty \text{ as } r \to \infty.$$

This guarantees an intersection between the two curves, and hence the existence of an equilibrium pair (r, g) for each $(\tau, \gamma) \in (0, 1) \times (0, 1)$.

Proposition **2.1** *For any admissible tax rate and ratio of input expenditure on infrastructural accumulation, there exists a positive equilibrium steady growth rate and a corresponding positive equilibrium rate of interest.*

Figure 2, where the equilibrium g and r are represented by $g(\tau, \gamma)$ and $r(\tau, \gamma)$ respectively, illustrates the proposition. The corresponding value of w is denoted by $w(\tau, \gamma)$. I will refer to $g(\tau, \gamma)$ as the equilibrium steady state growth rate ($ESSGR$).

Figure 2 here.

The aggregate welfare for the economy in dynamic steady state equilibrium may be found by integrating (2.3) and substituting from (2.16) and (2.17). Thus,

$$W(\tau, \gamma) = ln\ [K(0)\{\rho + \frac{(1-\tau)\ w(\tau, \gamma)}{K}\}]\ \frac{1}{\rho} + \frac{g(\tau, \gamma)}{\rho^2}, \tag{2.18}$$

where $K(0)$ is the value of the initial capital stock for the economy. Using in succession equation (2.14), equation (2.17) and the fact that $g^d = g(\tau, \gamma)$ in dynamic steady state equilibrium, the value of equilibrium welfare reduces to

$$W(\tau, \gamma) = ln\ [K(0)\{\rho + \frac{g(\tau, \gamma) + \rho}{\chi}\}]\ \frac{1}{\rho} + \frac{g(\tau, \gamma)}{\rho^2}. \tag{2.19}$$

While welfare maximization is an obvious objective for the Government to pursue, it is of interest to compare this policy with the alternative policy of growth rate maximization. In particular, it is important to know if growth rate maximization is a proxy for welfare maximization. I shall begin with the problem of growth maximization.

3 Growth Maximization

Using (2.4) and inverting (2.17) for $\tau \neq 1$,

$$\frac{G\ L_y}{K_y} = (\frac{\rho + g^d}{(1-\tau)\ \alpha\ A})^{1/(1-\alpha)}. \tag{3.1}$$

Similarly, (2.2) yields

$$\frac{K_g}{L_g\ G} = (\frac{g^s}{B\ L_g})^{1/\beta}. \tag{3.2}$$

Multiplying equations (3.1) and (3.2), using (2.6), (2.10), (2.12) and substituting $g^d = g^s = g$,

$$(\frac{g}{B})^{1/\beta}\ (\frac{\rho + g}{\alpha\ A})^{1/(1-\alpha)} = \frac{1-\alpha}{\alpha}\ \frac{\gamma}{1-\gamma}\ (\frac{1}{1+\lambda(\tau,\gamma)})^{1/\beta}\ (1-\tau)^{1/1-\alpha}. \tag{3.3}$$

Equation (3.3) gives a necessary condition that is satisfied when $g^d = g^s = g$. It is easily checked that the LHS of (3.3) is strictly convex and increasing in g. Hence, a unique solution to (3.3) exists for any $(\tau, \gamma) \in (0,1) \times (0,1)$. This is identically the same as the $ESSGR$ $g(\tau, \gamma)$ established in Proposition 2.1.

Maximizing the growth rate amounts to maximizing the solution of (3.3) with respect to τ and γ. However, (3.3) being a polynomial equation in g, I solve the maximization exercise in two steps. The first involves maximization with respect to τ for any given $\gamma \in (0,1)$.

3.1 Growth Maximizing Tax Rate

It is not hard to show that for each $\gamma \in (0,1)$, $\exists$ a unique $\tau \in (0,1)$ for which the expression

$$\frac{1-\alpha}{\alpha}\ \frac{\gamma}{1-\gamma}\ (\frac{1}{1+\lambda(\tau,\gamma)})^{1/\beta}\ (1-\tau)^{1/1-\alpha} \tag{3.4}$$

on the *RHS* of (3.3) reaches an interior maximum with respect to τ. This fact, coupled with the behavior of the *LHS* of (3.3), implies that for each $\gamma \in (0,1)$, $\exists\ \tau(\gamma) \in (0,1)$ maximizing the *ESSGR*. Expression (3.4) monotonically rises with γ, thereby implying that $\tau(\gamma)$ rises as γ rises. Moreover, $\tau(\gamma) \rightarrow\ \underline{\tau} > 0$ as $\gamma \rightarrow\ 0$. The following proposition summarizes the conclusions:

PROPOSITION **3.1** *For each proper fraction representing the ratio of expenditures on infrastructural inputs, there exists a unique fraction representing the ESSGR maximizing tax rate. The growth maximizing tax rate is a strictly increasing function of the expenditure ratio and approaches a positive limit as the latter approaches zero.*

Remark 1: The intuitive reasoning underlying the behavior of $\tau(\gamma)$ is as follows. First, given τ, r and w, a rise in γ raises K_g and lowers L_g. These are two opposing effects on g^s. The joint effect is to raise the latter unambiguously in the region $\gamma \leq \beta$, but possibly beyond also, as the term $1-\gamma$ in the denominator of equation (2.15) indicates. Moreover, g^s rises unambiguously with τ, since more revenue is spent on the accumulation of G. On the other hand, g^d is unaffected by γ and falls with τ (as (2.17) shows), because the Household demands a smaller rate of growth as its post tax income from capital falls. The joint positive effect of γ and τ on g^s turns out to be larger than the isolated negative effect of τ on g^d. Consequently, a simultaneous rise in γ and τ is likely to raise $g(\tau,\gamma)$ and $\tau(\gamma)$. The behavior of the function $\tau(\gamma)$ is shown in Figure 3.

Figure 3 here.

The strict monotonicity of $\tau(\gamma)$ allows the function to be inverted. This is done by solving the first order condition for a maximum of (3.4) for fixed γ. Denote the resulting expression by $\tilde{\gamma}(\tau)$, where

$$\tilde{\gamma}(\tau) = 1 + \frac{(1-\alpha)(1-\tau)}{\tau} - \frac{(1-\alpha)^2(1-\tau)}{\beta\tau^2}. \tag{3.5}$$

This expression will be useful in locating the joint maximum with respect to τ and γ. I proceed now to a maximum with respect to γ given τ.

3.2 Growth Maximizing Expenditure Ratio

Once again, the existence of a growth rate maximizing γ is governed by equation (3.3). For $\tau \in (0,1)$, the behavior of the RHS depends on the behavior of $(\gamma/(1-\gamma))\ (1/(1+\lambda(\tau,\gamma)))^{1/\beta}$, or, on its positive monotonic transformation $(\gamma/(1-\gamma))^{\beta}\ (1/(1+\lambda(\tau,\gamma)))$. Denoting the latter by $\phi(\gamma)$, the following properties are easily derived:

$$\begin{aligned}
\phi'(\gamma) &> 0 \text{ for } \gamma \text{ small and positive ;} \\
\phi'(\gamma) &\to 0 \text{ as } \gamma \to 0; \\
\phi'(\gamma) &\cong -\infty \text{ for } \gamma \cong 1.
\end{aligned}$$

These observations imply that for any $\tau \in (0,1)$, $\exists\, \gamma \in (0,1)$ such that the $ESSGR$ is maximized. Differentiating $\phi(\gamma)$, equating to zero and solving, it follows that

$$\gamma = \frac{\beta\tau + \beta(1-\alpha)(1-\tau)}{\beta\tau + (1-\alpha)(1-\tau)}. \tag{3.6}$$

Thus, the $ESSGR$ maximizing γ is unique for each $\tau \in (0,1)$. I denote the relationship by $\gamma(\tau)$. Differentiating $\gamma(\tau)$, it follows that $\gamma'(\tau) > 0$. Also, $\gamma(\tau) \to \beta$ as $\tau \to 0$. It is possible now to state a result parallel to Proposition 3.1:

Proposition **3.2** *For each proper fraction representing the tax rate, there exists a unique fraction representing the ESSGR maximizing expenditure ratio. The growth-maximizing ratio is an increasing function of the tax rate and approaches the exponent of capital in the infrastructural sector as the tax rate approaches zero.*

Remark 2: For the Cobb-Douglas technology under consideration, efficient allocation of resources calls for the ratio of capital and labor incomes in each sector to equal the ratio of factor exponents in the corresponding production function. The condition is always satisfied by the competitive Y-sector. The last proposition shows this to be the case for the G-sector also as the tax rate approaches zero, i.e. when there is no tax induced distortion in the system. The noncompetitive behavior of the G-sector along with the distortion brought about by the positive tax rate makes the Mixed Economy's optimal choice of γ larger than β. On the other hand, $\gamma = \beta$ is not realizable since $\tau = 0$ rules out production in the G-sector and therefore positive steady state growth.

3.3 Growth Maximizing τ and γ

I will state the conclusions of this subsection in the form of a Remark.

Remark 3: A necessary condition for the existence of an *interior* growth rate maximizing pair (τ^*, γ^*) is that the two functions $\tau(\gamma)$ and $\gamma(\tau)$ intersect in the region $(0,1) \times (0,1)$. Looking for an intersection of $\tau(\gamma)$ and $\gamma(\tau)$ is equivalent to looking for an intersection of $\tilde{\gamma}(\tau)$ and $\gamma(\tau)$. The functions $\tilde{\gamma}(\tau)$ and $\gamma(\tau)$ are described completely by the two parameters α and β, the exponents of capital in the two production functions (equations (3.5) and (3.6)). In order to get a feel for the figures involved, it is interesting to test for the possibility of existence numerically. I do this for two different choices of the parameters (α, β).

Case 1: Using a narrow concept of physical capital (structures and equipment), α and β should be around 0.33. (See, for example, Denison (1962), Maddison (1982), Jorgenson, Gollop and Fraumeni (1987)). For $\alpha = 0.3$ and $\beta = 0.35$, Figure 4 (drawn by *Mathematica*) shows that the two curves do intersect in $(0,1) \times (0,1)$. However, the coordinates of the point of intersection exceed 0.8 for both parameters. Thus, the tax rate exceeds 80%. Similarly, $\gamma^* > 0.8$ is large compared to $\beta = 0.35$, the share of capital income in the G-sector that would emerge under competitive conditions. This raises the possibility that capital earns substantially more than its marginal product in the G-sector. The exceptionally high values of τ^* and γ^* suggest severe social costs. Hence, (τ^*, γ^*) is not expected to maximize welfare in a Mixed Economy. This will be confirmed by Proposition 4.2 below.

Case 2: Following Barro and Sala-i-Martin (1999), however, and using a broader definition of capital (to include, for example, human capital), the two parameters should have values close to 0.75. Using this value, the tax rate falls to a more reasonable figure of 28%, while γ^* still hovers around a value slightly exceeding 0.8. This is shown in Figure 4 also. Once again, $\gamma^* > \beta$ indicates a source of inefficieny.

Figure 4 here.

The stage is now set for a discussion of welfare maximization.

4 Welfare Maximization

As with the analysis of a growth maximizing pair, I approach the question of welfare maximization in two stages, by considering in succession the partial problems of with respect to γ and τ.

4.1 Effect of the Expenditure Ratio on Welfare

Differentiating the RHS of (2.19) with respect to γ,

$$
\begin{aligned}
\frac{\partial W(\tau,\gamma)}{\partial \gamma} &= \frac{1}{[K(0)(\rho + \frac{(1-\tau)\ w}{K})]\ \frac{1}{\rho}} \times \\
&\quad \frac{\frac{\partial g(\tau,\gamma)}{\partial \gamma}\ (\chi) - (g(\tau,\gamma)+\rho)\ z}{(\chi)^2} \\
&\quad + \frac{1}{\rho^2}\ \frac{\partial g(\tau,\gamma)}{\partial \gamma},
\end{aligned} \tag{4.1}
$$

where

$$
\begin{aligned}
z &= \frac{\partial \chi}{\partial \gamma} \\
&= \frac{\tau^2 + \alpha\ \tau(1-\tau) + (1-\alpha)(1-\tau)\tau}{((1-\gamma)\tau + (1-\alpha)(1-\tau))^2} \\
&> 0.
\end{aligned}
$$

Since $\partial g(\tau,\gamma)/\partial\gamma$ is nonpositive for $\gamma \geq \gamma(\tau)$ (by virtue of Proposition 3.2), the strict positivity of z implies that $\partial W/\partial\gamma < 0$ whenever $\gamma \geq \gamma(\tau)$. The next proposition states this fact.

Proposition **4.1** *For each proper fraction representing the tax rate, lowering the expenditure ratio below its growth maximizing value will increase welfare.*

It follows from Proposition 4.1 that a welfare maximizing pair cannot lie on the curve $\gamma(\tau)$. Consequently, it cannot be identical with a growth maximizing pair. The next proposition states this fact.

Proposition **4.2** *A pair of policy parameters maximizing the growth rate is distinct from a pair that maximizes welfare.*

Remark 4: In this context, it is worth considering Barro's observation (Barro (1990), p S107):

> As long as the government and the private sector have the same production functions, the results would be the same if the government buys private inputs and does its own production, instead of purchasing only final output from the private sector, as I assume.

Proposition 4.2 of the present paper shows that Barro's (as well as the BS) finding on the equivalence of growth and welfare maximization is untrue for *all* choices of the parameters of the production functions, *including the case where the production functions are identical.* As noted in the Introduction, FMS reach a similar conclusion for a two-sector model. However, unlike the present paper, the FMS model does not fit the economy described by the quote from Barro. The two policies are equivalent in my two-sector extension under the additional constraint that income distribution in the sectors match. (See Remark 6.) In BS, of course, the public sector is absent and income distribution automatically follows the pattern of the private sector. Hence, in that model, the policies for growth and welfare maximization merge.

4.2 Effect of the Tax Rate on Welfare

In order to narrow down the search for a welfare maximizing policy, differentiate the RHS of (2.19) with respect to τ and use (2.18) to get

$$\begin{aligned}\frac{\partial W(\tau,\gamma)}{\partial \tau} &= \frac{1}{\rho\ [K(0)(\rho + \frac{(1-\tau)\ w(\tau,\gamma)}{K})]} \times \\ & \frac{\frac{\partial g(\tau,\gamma)}{\partial \tau}\ (\chi) - \frac{(g(\tau,\gamma)+\rho)\ (\gamma-\alpha)}{((1-\gamma)\tau+(1-\alpha)(1-\tau))^2}}{(\chi)^2} \\ & + \frac{1}{\rho^2}\ \frac{\partial g(\tau,\gamma)}{\partial \tau}. \end{aligned} \tag{4.2}$$

The behavior of the RHS of (4.2) depends on the relative values of γ and α. Consider first the case $\gamma = \alpha$. Then $\chi = \alpha/(1-\alpha)$, a constant. Consequently, the sign of the RHS of (4.2) depends on the sign of $\partial g(\tau,\gamma)/\partial\tau$, which is zero at $\tau = \tau(\gamma)$, positive for $\tau < \tau(\gamma)$ and negative for $\tau > \tau(\gamma)$. Hence, welfare is maximized at $\tau = \tau(\gamma)$.

For $\gamma > \alpha$, sgn $\partial W/\partial\tau = -ve$ whenever $\tau \geq \tau(\gamma)$, since $\partial g(\tau,\gamma)/\partial\tau < 0$. Hence, welfare improves with a marginal fall in τ below $\tau(\gamma)$ in this case. Similarly, when $\gamma < \alpha$, sgn $\partial W/\partial\tau = +ve$ for $\tau \leq \tau(\gamma)$. Consequently, welfare rises with a mariginal rise in τ above $\tau(\gamma)$.

The following result is accordingly established:

Proposition **4.3** *Welfare can be increased by reducing (raising) the tax rate below (above) the growth maximizing tax rate when the expenditure ratio is greater (less) than the share of capital in the private sector. When the expenditure ratio equals the share of capital in the private sector, the growth maximizing tax rate equals the welfare maximizing tax rate.*

Remark 5: It might appear that it is the value of γ relative to α alone that matters in determining the behavior of welfare as τ varies relative to $\tau(\gamma)$, with β playing no role in the matter at all. This, however, is not the case. To appreciate the fact, consider expression (2.19) for the equilibrium value of steady state welfare. The numerators of both terms on the RHS of (2.19) involve $g(\tau,\gamma)$, which, being the equilibrium growth rate, depends on both α and β. On the other hand, the denominator of the first term on the RHS of (2.19) derives from equation (2.14), which depends on the static equilibrium at each t. Since the static equilibrium ignores the marginal productivities of both private factors in the G-sector, (2.14) involves α and γ, but not β. Going over now to (4.2), the sign of $\partial W/\partial\tau$ is determined by (i) the sign of $\partial g(\tau,\gamma)/\partial\tau$ and (ii) the sign of $\partial\chi/\partial\tau$. The first of these signs depends on the value of τ relative to $\tau(\gamma)$ and the latter depends on α *as well as* β. The second sign, however, depends on the value of γ relative to α only. Thus, conclusions such as "For $\gamma > \alpha$, sgn $\partial W/\partial\tau = -ve$ whenever $\tau \geq \tau(\gamma)$" that appear above involve both α and β.

4.3 Joint Effcet of $\mathcal{T}$ and γ on Welfare

Figure 5 shows the plausible relative positions of growth and welfare maximizing pairs. Following Figure 4, the growth maximizing pair is shown by the coordinates (τ^*, γ^*) of the intersection of $\tau(\gamma)$ and $\gamma(\tau)$. The horizontally striped areas represent the possible locations of welfare maximizing τ's. Similarly, welfare maximizing γ's must lie in the vertically striped zone. A (τ, γ)-pair jointly maximizing welfare can belong then only to the crosshatched area, i.e. the set theoretic union of the areas marked P and Q. It should be noted that $P \bigcup Q$ has *no point in common* with the segment of $\gamma(\tau)$ connecting the point $(0, \beta)$ and the point of intersection (τ^*, γ^*) (including (τ^*, γ^*) itself). It also *excludes* all points on the segment of $\tau(\gamma)$ joining $(\underline{\tau}, 0)$ to the point (τ^*, γ^*), *except the point* ω.

Figure 5 here.

Remark 6: The point $\omega = (\alpha, \tau(\alpha))$ is of special interest. First of all, it falls in the region to which a welfare maximizing pair (τ_w^*, γ_w^*) would belong. Hence, $\omega = (\tau_w^*, \gamma_w^*)$ is not ruled out. Even if this is not the case, ω has a property worth noting. It says that a Government that is committed to a similar pattern of income distribution in the public and private sectors, viz, $\gamma = \alpha$, will choose $(\alpha, \tau(\alpha))$ whether it wishes to maximize the growth rate or welfare. However, even when $\omega = (\tau_w^*, \gamma_w^*)$, this is a second best *growth rate* maximization policy, since it fixes γ arbitrarily at the level α. Once γ is allowed to be chosen optimally also, Proposition 4.2 will apply, making the growth maximizing parameters distinct from the welfare maximizing pair. When $\omega \neq (\tau_w^*, \gamma_w^*)$, the point ω is second best from the points of view of both maximization problems.

Remark 7: Assuming $A = 0.2$, $B = 0.2$, $\rho = 0.2$, the steady state growth rates associated with ω and (τ^*, γ^*) are as follows. For Case 1 of Remark 3, the growth rate at ω is 6.4%, while that at (τ^*, γ^*) is 8.1%. For Case 2 of the same Remark, the respective growth rates turn out to be 7.7% and 7.9%. Remark 8 in Section 6 reports on the growth rates for the Command Economy.

5 Transitional Dynamics

As FMS point out, the two sector exercise remains incomplete without a discussion of the transitional dynamics properties.[13] Towards this end, I define two new variables, $k = K/G$ and $c = C/K$. Using (2.4), (2.5), (2.9), (2.14), (2.15) and (2.16), it follows that

$$\frac{\dot{k}}{k} = R_1\ k^{\alpha-1} - R_2\ k^{\beta} - c \tag{5.1}$$

$$\frac{\dot{c}}{c} = -R_3\ k^{\alpha-1} + c - \rho, \tag{5.2}$$

where

$$R_1 = \alpha\ A\ (\frac{\alpha}{1-\alpha}\ \frac{1}{\chi})^{\alpha-1} \times (1-\tau)\ (1+\frac{1}{\chi});$$

$$R_2 = B(\frac{\gamma}{1-\gamma}\ \frac{1}{\chi})^{\beta}\ \frac{1}{1+\lambda(\tau,\gamma)};$$

$$R_3 = \alpha\ A\ (\frac{\alpha}{1-\alpha}\ \frac{1}{\chi})^{\alpha-1} \times (1-\tau)\frac{1}{\chi}.$$

This system of differential equations describes the transitional dynamics of the economic system under consideration. The stationary state for the system corresponds to the dynamic equilibrium described earlier in terms of the demand and supply rates of growth for each possible choice of (τ, γ). Such an equilibrium must have associated with it admissible stationary values of k and c. These stationary values will simultaneously solve

$$c = R_1\ k^{\alpha-1} - R_2\ k^{\beta} = \Psi(k) \tag{5.3}$$

[13]The exercise considers out of steady state dynamics of k and c for each *fixed* pair (τ, γ).

and

$$c = R_3\ k^{\alpha-1} + \rho = \Phi(k). \tag{5.4}$$

It is easily checked that both $\Psi(k)$ and $\Phi(k)$ are monotone decreasing, strictly convex functions. Again, the function $\Phi(k) \to +\infty$ as $k \to 0$ and $\Phi(k) \to \rho$ as $k \to \infty$. On the other hand, $\Psi(k) \to +\infty$ as $k \to 0$ and $\Psi(k) \to -\infty$ as $k \to \infty$. For meaningful solutions to the equations, eliminate c to get

$$(R_1 - R_3)\ k^{\alpha-1} = R_2 + \rho.$$

Taking account of the definitions of R_1, R_2 and R_3, the equation admits a unique, positive solution, say k^*. The uniqueness of k^* implies a corresponding unique solution c^* for c. To see that $c^* > 0$, consider $\Psi(k) - \Phi(k)$, which approaches ∞ as $k \to 0$. Thus, $\Psi(k) > \Phi(k)$ for small k. Since $\Psi(k) \to -\infty$ as $k \to \infty$ and $\Phi(k) > \rho\ \forall\ k$, it follows that $\Psi(k) < \Phi(k)$ for large enough k. Hence, $0 < c^* = \Psi(k^*) = \Phi(k^*)$, as shown in the phase diagram of Figure 6 below.

Figure 6 here.

In order to study the local stability of the dynamic system, I rewrite equations (5.1) and (5.2) as

$$\begin{aligned}
\dot{k} &= (R_1\ k^{\alpha-1} - R_2\ k^{\beta} - c)\ k \\
\dot{c} &= (-R_3\ k^{\alpha-1} + c - \rho)\ c.
\end{aligned}$$

Linear approximation around (k^*, c^*) gives

$$\begin{aligned}
\dot{k} &= (\alpha\ R_1\ k^{*\alpha-1} - (1+\beta)\ R_2\ k^{*\beta} - c^*)(k - k^*) \\
&\quad -k^*(c - c^*) \\
\dot{c} &= (1-\alpha)\ R_3\ k^{*\alpha-2}\ c^*(k - k^*) \\
&\quad +(2c^* - R_3\ k^{*\alpha-1} - \rho)(c - c^*).
\end{aligned} \tag{5.5}$$

Using (2.15) and (5.4), the relevant determinant simplifies to

$$(\alpha - (1+\beta))\ g(\tau,\gamma)\ c^* + (\alpha - 1)\ \rho\ c^* < 0,$$

where $g^d = g^s = g(\tau,\gamma)$. The sign of the determinant establishes that the stationary equilibrium is a saddle point. In other words, for each possible choice of the initial value of $k(0)$ in a neighborhood of k^*, $\exists$ a choice of c in a corresponding neighborhood of c^*, such that the system (5.5) converges to (k^*, c^*). The thick arrowheads in Figure 6 indicate the stable path more clearly. The results are summarized by the next proposition, which parallels Proposition 2 of FMS.

PROPOSITION **5.1** *For each specification of the values of the two policy instruments, the steady state equilibrium values of the private–public capital ratio and consumption–private capital ratio are unique. Further, there exists a unique stable path of these ratios converging to the steady growth equilibrium.*

6 Command Economy

So far, I have considered the case of a Mixed Economy only. Quite apart from the tax rate induced distortion and the non-competitive behavior of the G-sector, the growth rate of such an economy was socially suboptimal because the agents were not concerned with optimality conditions surrounding the state variable G. The socially optimum path on the other hand may be found by solving the corresponding Command Economy's optimization exercise. I proceed now to prove the existence of a social optimum and construct prices that may support it as the competitive equilibrium of a fictitious two-sector economy.[14]

[14]The existence of a unique steady state growth path for the Command Economy was proved by Dam (1997) and Dasgupta & Dam (2000) also. However, these papers were not concerned with the decentralization exercise to follow.

Resource allocation in the Command Economy is carried out by solving a grand optimization exercise by an altruistic social planner organizing production in both sectors and allocating resources between them. The welfare function of the planner is identically the same as that of the Representative Household. The planner maximizes (2.3) subject to[15]

$$\dot{K} = A(\phi K)^{\alpha}\ (\theta G)^{1-\alpha} - C \tag{6.1}$$

and

$$\dot{G} = B((1-\phi)K)^{\beta}\ ((1-\theta))G^{1-\beta}, \tag{6.2}$$

where ϕ and $1-\phi$ are respectively the shares of K and θ and $1-\theta$ are the shares (as well as the absolute amounts) of labor employed in the Y and G sectors respectively.[16] The problem is solved by maximizing the current value Hamiltonian

$$\begin{aligned} H \ &= \ ln\ C \ + \ \eta(A(\phi K)^{\alpha}(\theta\ G)^{1-\alpha} - C) \\ &\quad + \xi(B((1-\phi)K)^{\beta}\ ((1-\theta)\ G)^{1-\beta}), \end{aligned}$$

where η and ξ are the costate variables associated with $\dot{K}$ and $\dot{G}$. The steady state solution for this problem may be called the Command Equilibrium and the resulting rate of steady growth denoted g^*.

The first order optimality conditions for the maximization of H are

$$\frac{\partial H}{\partial C} \ = \ 0, \tag{6.3}$$

[15]The calculations for the more general utility function mentioned in footnote 9 may be found in Dasgupta & Dam.

[16]Mino (1996) considers a two sector production structure similar to, though more general than, (6.1) and (6.2). However, the thrust of Mino's paper is totally different from the present exercise.

$$\frac{\partial H}{\partial \phi} = 0, \tag{6.4}$$

$$\frac{\partial H}{\partial \theta} = 0, \tag{6.5}$$

$$\dot{\eta} = -\frac{\partial H}{\partial K} + \eta\rho, \tag{6.6}$$

$$\dot{\xi} = -\frac{\partial H}{\partial G} + \xi\rho. \tag{6.7}$$

Since both the instantaneous utility function and the two production functions are strictly concave, it follows from Cass (1965) that (6.3) through (6.7), along with the transversality conditions

$$\eta(t)\ K(t)\ e^{-\rho t} \to 0 \text{ as } t \to \infty$$

and

$$\xi(t)\ G(t)\ e^{-\rho t} \to 0 \text{ as } t \to \infty,$$

are a sufficient characterization of the *unique* optimum path solving the planner's problem.

To facilitate the arguments of the next section, we note that conditions (6.3) through (6.7) lead respectively to the following equations:

$$C^{-1} = \eta \tag{6.8}$$

$$\eta\alpha A(\phi K)^{\alpha-1}(\theta G)^{1-\alpha} = \xi\beta B((1-\phi)K)^{\beta-1}((1-\theta)G)^{1-\beta} \tag{6.9}$$

$$\eta(1-\alpha)A(\phi K)^{\alpha}(\theta G)^{-\alpha} = \xi(1-\beta)B((1-\phi)K)^{\beta}((1-\theta)G)^{-\beta} \tag{6.10}$$

$$\dot{\eta} = -A\eta\alpha(\phi K)^{\alpha-1}(\theta G)^{1-\alpha} + \eta\rho \tag{6.11}$$

$$\dot{\xi} = -B(1-\beta)\xi((1-\phi)K)^{\beta}((1-\theta)G)^{-\beta} + \xi\rho. \tag{6.12}$$

The steady state optimum values of all variables associated with equations (6.8), (6.9), (6.10), (6.11) and (6.12) will be denoted by asterisks.

Using (6.8) and (6.11),

$$g^d = \frac{\dot{C}}{C} = A\alpha(\phi K)^{\alpha-1}(\theta\ G)^{1-\alpha} - \rho. \tag{6.13}$$

Analogous to a Market Equilibrium,

$$\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{G}}{G} = \frac{\dot{Y}}{Y} \tag{6.14}$$

in steady state. Denote the common growth rate by g. Consistency between the demand and the supply rate requires (using (2.2)) that

$$g = B(\frac{(1-\phi)K}{(1-\theta)\ G})^{\beta}(1-\theta). \tag{6.15}$$

Differentiating (6.5) and using (6.3)

$$\frac{\dot{\eta}}{\eta} = \frac{\dot{\xi}}{\xi} = -g. \tag{6.16}$$

It can be shown that equation (6.16) implies that the transversality conditions are satisfied. Using (6.16) in (6.12),

$$(1-\beta)\ B\ ((1-\phi)\ K)^{\beta}\ ((1-\theta)\ G)^{-\ \beta} = g + \rho \tag{6.17}$$

Equations (6.17) and (6.15) lead to

$$(1-\theta)^{1/\beta} = (\frac{(1-\beta)\ g}{g+\rho})^{1/\beta}. \tag{6.18}$$

Inverting (6.13) and (6.15) and multiplying out,

$$(\frac{g}{B})^{1/\beta}(\frac{g+\rho}{\alpha A})^{1/(1-\alpha)} = \frac{\theta}{1-\theta}\ \frac{1-\phi}{\phi}\ (1-\theta)^{1/\beta}. \tag{6.19}$$

Finally, manipulating (6.3), (6.5), (6.19) and (6.18) in succession, we get

$$(\frac{g}{B})^{1/\beta}(\frac{g+\rho}{\alpha A})^{1/(1-\alpha)} = \frac{1-\alpha}{\alpha}\ \frac{\beta}{1-\beta}\ (\frac{(1-\beta)\ g}{g+\rho})^{1/\beta}. \tag{6.20}$$

The behavior of the curves in (6.20) is shown in Figure 7. The LHS is the convex, increasing curve $\Lambda(g)$, while the RHS is the concave, increasing curve $\Gamma(g)$. The curve Γ is bounded above by $((1-\alpha)/\alpha)\ \beta\ (1-\beta)^{(1-\beta)/\beta}$, thereby guaranteeing a unique intersection in the positive orthant. The solution g^* to (6.20) represents the command optimum steady state growth rate for the economy. This gives rise to

Proposition **6.1** *A unique, strictly positive welfare maximizing growth rate exists for the Command Economy.*

Figure 7 here.

Remark 8: For Case 1 of Remark 3, the Command Economy growth rate is $g^* = 8.5\%$. This is only slightly higher than the maximal growth rate of 8.1% associated with (τ^*, γ^*). Clearly, the welfare associated with the Command Economy growth rate is higher than that associated with a welfare maximizing (τ_w^*, γ_w^*) in the Mixed Economy. Moreover, Proposition 4.2 has established that (τ_w^*, γ_w^*) itself dominates (τ^*, γ^*) welfare wise. Quite obviously therefore, it is not merely the best possible growth rate that matters. The instruments that are used to achieve it are equally important. As the present example demonstrates, a Mixed Economy might be capable of growing at a rate close to the best possible growth rate for the system (i.e. the Command solution) and yet enjoy a low level of welfare. Thus, the proximity of the highest growth rate of the Mixed Economy and the Command Economy may give rise to an identification problem. Going over to Case 2 of Remark 3, the Command Economy growth rate turns out to be 9.3%, which is significantly higher than that achievable by the Mixed Economy. In this case the Command Economy dominates the Mixed Economy *by a wide margin*, both in terms of the growth rate as well as welfare. Nonetheless, the degree of centralization called for by the Command Economy has its well-established social costs and cannot constitute a viable choice in a free society. It is worth considering, therefore, the possibility of decentralizing the Command solution.

7 Command Solution through Markets?

From a purely formal point of view, the current value Hamiltonian H for the Command Economy treats G as a private rather than a public input. This is evident from the fact that while θ and $1-\theta$ are defined to be flows of labor into the two sectors, the Hamiltonian is unable to distinguish this from an alternative scenario, where G is a private input with allocations $\theta\ G$ and $(1-\theta)\ G$ into the two sectors. Strictly speaking therefore, the Hamiltonian, recognizes only two factors, viz, K and G, though the corresponding Mixed economy model involves *three*. As a result, the equilibrium of the Command Economy resembles two sector, two factor models of endogenous growth (such as Rebelo (1991) or Mino (1996)). Any attempt to decentralize this equilibrium must therefore establish a correspondence between the Command Economy and an artificially constructed two-sector economy whose equilibrium imitates the Command Economy path.

Accordingly, the Decentralized Economy visualized below does not distinguish between L and G. Instead, it deals with a surrogate for the two, denoted by the fictitious factor of production N. As with G, the factor N is accumulable, but unlike G, it generates a *rival* and *excludable* flow of services entering the production functions of the two sectors. Thus, the economy produces Y and $\dot{N}$ by means of the two rival inputs K and N. Hence the two production functions reduce to

$$Y = A\ K_y^{\alpha}\ N_y^{1-\alpha},\ 0 < \alpha < 1 \tag{7.1}$$

and

$$\dot{N} = B\ K_g^{\beta}\ N_g^{1-\beta},\ 0 < \beta < 1, \tag{7.2}$$

where N_y and N_g are the respective flows of N into the two sectors.

The Decentralized Economy is assumed to be made up of four agents, viz, the representative Household, two aggregative firms and the Government. The first three agents have well-defined objective functions which they maximize at parametrically specified prices. The government provides the initial stock of N and finances its acquisition of $\dot{N}$ from the revenue generated by the sale of N-services to the firms. The Household has a dynastic structure and maximizes (2.3) subject to an instantaneous budget constraint. Its budgetary resources fall into two parts. First, it has an income, $rK(t)$ from private capital holdings, where r stands for the constant rate of interest on K. Secondly, it receives a subsidy equal to $\Delta(t)$ from the Government at each t as a compensation for the labor it supplies to the two firms. The amount of the subsidy, which is beyond the control of the Household, will be specified later. The level of $\Delta(t)$ for each t may also be viewed as a wage rate, since $L(t) = 1\ \forall\ t$. The firms, however, *do not* pay wages directly to the Household.[17] Hence, the Household's budget constraint is

$$C(t) + \dot{K}(t) = rK(t) + \Delta(t), \tag{7.3}$$

[17] Alternatively, $\Delta(t)$ could be distributed over the two firms, in a manner that will be obvious from below, to purchase labor from the Household.

where $K(0) = K_0$ has the same value as the initial capital stock for the Command Economy. Firm I produces Y and is assumed to maximize the instantaneous profit function

$$\Pi^1 = AK_y^\alpha N_y^{1-\alpha} - rK_y - \nu N_y, \tag{7.4}$$

where ν represents the price charged by the Government per unit use of N-services. Commodity Y is treated as the numéraire. Firm II produces $\dot{N}$ by maximizing instantaneous profits given by

$$\Pi^2 = \mu BK_g^\beta N_g^{1-\beta} - rK_g - \nu N_g, \tag{7.5}$$

where μ is the price per unit charged by Firm II for its product, viz, $\dot{N}$.[18]

The Government accumulates N by purchasing $\dot{N}$ from Firm II.[19] The net revenue accruing to the Government on the N-account is the difference between its sales revenue from N services and its investment cost for creating additional N. By definition of ν and μ, the Government's revenue and expenditure at each t are

$$\nu N \tag{7.6}$$

and

$$\mu BK_g^\beta N_g^{1-\beta}. \tag{7.7}$$

Thus, the Government has a net revenue from infrastructure accumulation and sale equal to

[18]It may be noted that while ν is the price charged for the flow of N-services, the price μ applies to the *stock* of N.

[19]Without loss of generality, Firm II could be operated by the Government itself, in which case the relevant prices are used for bookkeeping.

$$D = \nu N - \mu B K_g^{\beta} N_g^{1-\beta}. \tag{7.8}$$

This completes the description of the decentralized economy.

The next and last proposition of the paper demonstrates the existence of a price-subsidy scheme for the Decentralized Economy that supports the first best steady growth path of the Command Economy. By the very construction of the subsidy, the Government's budget will be seen to be balanced.

PROPOSITION **7.1** *There exist time dependent values of the Household subsidy and time invariant values of the rate of interest, the price of the fictitious factor of production and the price of the output of Firm II such that*
(a) the Decentralized Economy chooses identically the same growth rate as the Command Economy;
(b) the fraction of the fictitious input employed by the first (second) firm is identically the same as the fraction of labor force employed by the Command Economy in producing the final (incremental stocks of the public) good;
(c) the growth path of the fictitious commodity is identical with the growth path of the stock of the public good in the Command Economy; and
(d) the Government maintains a balanced budget along the chosen growth path.

Proof: Choose

$$r = r^* = \alpha A(\phi^* K^*)^{\alpha-1}(G^*)^{1-\alpha}, \tag{7.9}$$

where $\alpha A(\phi^* K^*)^{\alpha-1}(G^*)^{1-\alpha}$ is the marginal product of K along the optimal steady state path of the Command Economy. Since both K and G grow at the same rate, r^* is a constant for all t. Similarly, let

$$\mu = \frac{\xi^*(t)}{\eta^*(t)} \;\; \forall\, t \tag{7.10}$$

and

$$\nu = (g^* + \rho)\mu. \tag{7.11}$$

Given the uniqueness of the optimal path chosen by the Command Economy, it follows that μ and ν are well-defined. Further, μ is a constant from (6.16). Hence, ν is a constant also.

The Household's optimization exercise leads to the maximization of the current value Hamiltonian

$$H^h = ln\ C + \eta_h(r^* K + \Delta - C), \tag{7.12}$$

where η_h is the relevant costate variable associated with $\dot{K}$. A set of necessary conditions for the optimum solution to the problem is

$$C^{-1} = \eta_h \tag{7.13}$$

and

$$\dot{\eta_h} = -\eta_h r^* + \eta_h \rho = -\eta_h \alpha A (\phi^* K^*)^{\alpha-1} (G^*)^{1-\alpha} + \eta_h \rho. \tag{7.14}$$

The equation on the extreme right of (7.14) follows from (7.9).

The instantaneous profit maximization problems of Firms I and II lead to the following two conditions each:

$$\alpha A K_y^{\alpha-1} N_y^{1-\alpha} = r^* \tag{7.15}$$
$$(1-\alpha) A K_y^{\alpha} N_y^{-\alpha} = \nu \tag{7.16}$$

and

$$\mu\beta BK_g^{\beta-1}N_g^{1-\beta} = r^* \tag{7.17}$$
$$\mu(1-\beta)BK_g^{\beta}N_g^{-\beta} = \nu. \tag{7.18}$$

Equations (7.15) and (7.17) together imply

$$\alpha AK_y^{\alpha-1}N_y^{1-\alpha} = \mu\beta BK_g^{\beta-1}N_g^{1-\beta} = r^*. \tag{7.19}$$

Similarly, (7.16) and (7.18) imply

$$(1-\alpha)AK_y^{\alpha}N_y^{-\alpha} = \mu(1-\beta)BK_g^{\beta}N_g^{-\beta} = \nu \tag{7.20}$$

Using (6.16) and the definitions of μ and ν, (7.19) and (7.20) reduce to

$$\eta^*\alpha AK_y^{\alpha-1}N_y^{1-\alpha} = \xi^*\beta BK_g^{\beta-1}N_g^{1-\beta}, \tag{7.21}$$

$$\eta^*(1-\alpha)AK_y^{\alpha}N_y^{-\alpha} = \xi^*(1-\beta)BK_g^{\beta}N_g^{-\beta} \tag{7.22}$$

and

$$\dot{\xi}^* = -\xi^*\ (1-\beta)\ BK_g^{\beta}N_g^{-\beta} + \xi^*\rho. \tag{7.23}$$

Equations (6.9), (6.12) and the definitions of ϕ^*, K^* and G^* imply that $K_y = \phi^* K^*$, $K_g = (1-\phi^*)K^*$, $N_y = \theta^*\ G^*$ and $N_g = (1-\theta^*)\ G^*$ satisfy (7.21), (7.22) and (7.23).

Going over now to the Household's problem, we may differentiate (7.13) and substitute from (7.14) to obtain

$$\frac{\dot{C}}{C} = r^* - \rho.$$

From (6.13) and (7.9), the Household chooses the same growth rate as that of the Command Economy. However, this still does not determine the *level* of $C(t)$. To fix the latter, choose $\Delta(t) = D(t)$ for each t. That is, the subsidy provided to the Household is exactly equal to the net revenue of the Government as defined in (7.8). Hence, the Government's Budget is balanced. Moreover, $D(t) > 0 \; \forall \; t$. For, using (7.10) and (7.11),

$$\begin{aligned} \nu G^* - \mu B((1-\phi^*)K^*)^\beta((1-\theta^*)G^*)^{1-\beta} &= \mu((g^*+\rho) \\ &\quad -B(\frac{(1-\phi^*)K^*}{(1-\theta^*)G^*})^\beta(1-\theta^*))G^* \\ &= \mu(\theta^* g^* + \rho)G^* \\ &\quad (\text{ using } (6.2)\ ) \\ &> 0. \end{aligned} \tag{7.24}$$

The positivity of $D(t) > 0$ allows it to be treated as compensation for labor, or its imputed wage rate.

Equations (7.15) through (7.18) imply then that the RHS of (7.3) reduces to $Y(t)$. Rewriting (7.3) as

$$\frac{C}{K} + \frac{\dot{K}}{K} = \phi^* A(\frac{\phi^* K^*}{G^*})^{\alpha-1},$$

balanced growth requires C and K to grow at the same rate. In other words, $\dot{K}/K = g^*$. This in turn means that

$$C(t) = K(t)(\phi^* A(\frac{\phi^* K^*(t)}{G^*(t)})^{\alpha-1} - g^*) \;\; \forall \; t.$$

Hence, (6.1) implies that the choice of $C(t)$ by the Household is the same as that for the Command Economy. Therefore, comparing (6.8) and (7.13), we see that $\eta_h(t) = \eta^*(t)\ \forall\ t$.

Collecting the results, it follows that equations (7.13), (7.14), (7.21), (7.22)and (7.23) describing the evolution of the Decentralized Economy are the same as (6.8), (6.9), (6.10), (6.11) and (6.12) characterizing the Command Economy path. The solution to the Command Economy problem being unique, the Decentralized Economy chooses identically the same path.

Q.E.D.

Remark 9: Even though the above proposition establishes support prices for the Command Economy, it does so in terms of an economy which behaves very differently from the economy I started out with. First, the Household supplying labor freely to the firms in exchange for a Government subsidy may not be an operationally feasible idea. Secondly, it is difficult to imagine profit maximizing entrepreneurs treating the two factors of production, labor and infrastructure, as a single factor and coming out with a market demand for it. A private economy requires prices of inputs separately to define the corresponding demand and supply curves. This calls for a separation of labor from the public input, which the decentralized economy prevents.[20] Hence, the Mixed Economy's inefficient equilibrium may be the only feasible growth path for the system. As pointed out in the Introduction, this poses a dilemma for developing economies trying to adapt their economies to free market structures.

8 Conclusions

In this paper I have attempted to take a fresh look into the issues raised by Barro-FMS-BS in the context of optimal taxation for infrastructure devel-

[20] As should be obvious, the cause underlying market failure in this economy is somewhat different from the standard externality issues associated with public goods (Samuelson (1954)).

opment and growth by constructing a full-fledged two-sector model producing a final good and changes in the infrastructural stock respectively. The final good is privately produced. The flow of infrastructure being a pure public input in the model, the sector producing infrastructure cannot function competitively and is viewed as being under the Government's control. Factor shares are not determined by marginal productivities in this sector. This called for the introduction of a new variable into the model, the share of expenditure out of tax revenues on the different private inputs used for infrastructure accumulation. Thus, the optimal taxation problem had to be considered in tandem with the problem of optimal choice of the share. I established that growth maximization is not equivalent to welfare maximization in the economy considered. The equivalence can hold, however, in a second best sense, under the further constraint that factor shares be the same in the public and the private sectors.

The two-sector structure of the model made it imperative to study the transitional stability of the equilibrium growth path. Parallel to FMS therefore, I demonstrated the dynamic steady state path of the Mixed Economy to be transitionally stable.

Finally, the paper considered the nature of the first best steady state path for the system. This is the solution to the Command Economy exercise. Given the parameters of the technology and preferences, the Command Economy simply calculates the economy's best growth potential. I established that the highest growth rate of a Mixed Economy (for certain parameter values) might not fall too short of the Command Economy growth rate. The fact that a partially marketized economy may be capable of growing at a rate close to that of the Command Economy is no cause for rejoice; for the high growth rate may be sustained at the cost of efficiency and welfare. Thus, performance wise, a Mixed Economy growing at an impressively high rate might give rise to an identification problem. Nonetheless, the centralization implied by a Command Economy being an unpractical choice, I looked into the possibility of decentralizing the first best growth path. Though apparently decentralizable, I argued that it is actually not so, because the private economy in question (that sustains the Command Economy path) needs to treat the public service and labor as a joint input. Markets for such inputs cannot exist in the real world. The conclusion that emerges is that the existence of accumulable pure public inputs imposes severe constraints on the growth paths available to a developing economy, especially those engaged in the process of liberalization.

I have adhered all through to the Barro-FMS-BS specification that the Government's budget is balanced at each point of time. An extension of the paper might consider the possibility that the Government has recourse to revenue raising instruments other than income taxes. In particular, it is of some interest to consider the possibility of deficit financing through borrowing from the public under the constraint that the Government's budget is balanced *over time*. A second extension would be to follow BS and introduce public services with congestion. Yet another possibility is to look into Alesina and Rodrik (1994) type political economy issues by introducing agent specific endowments and nonlinear taxation. Some of these will be taken up in future work.

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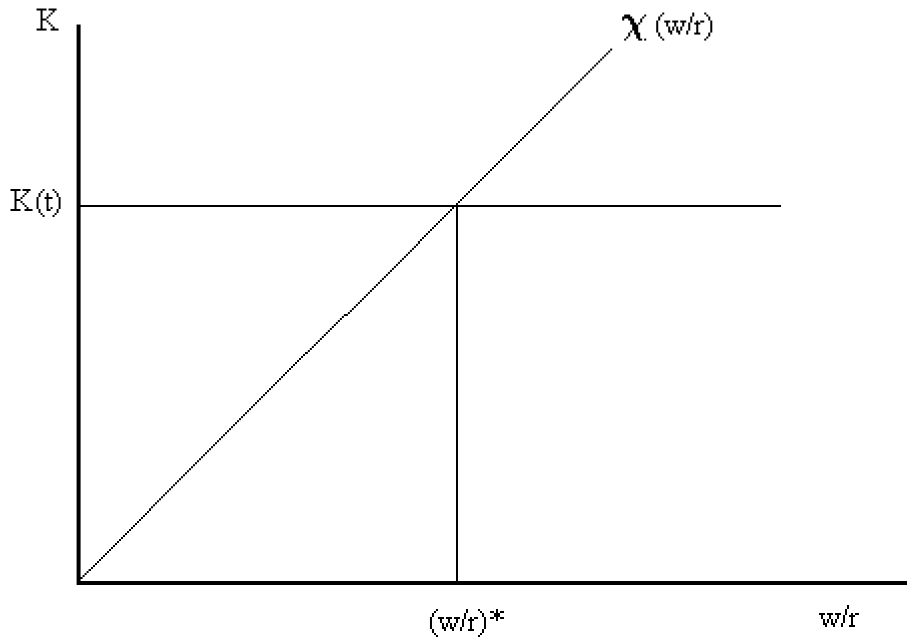

Figure 1: Determination of Equilibrium w/r

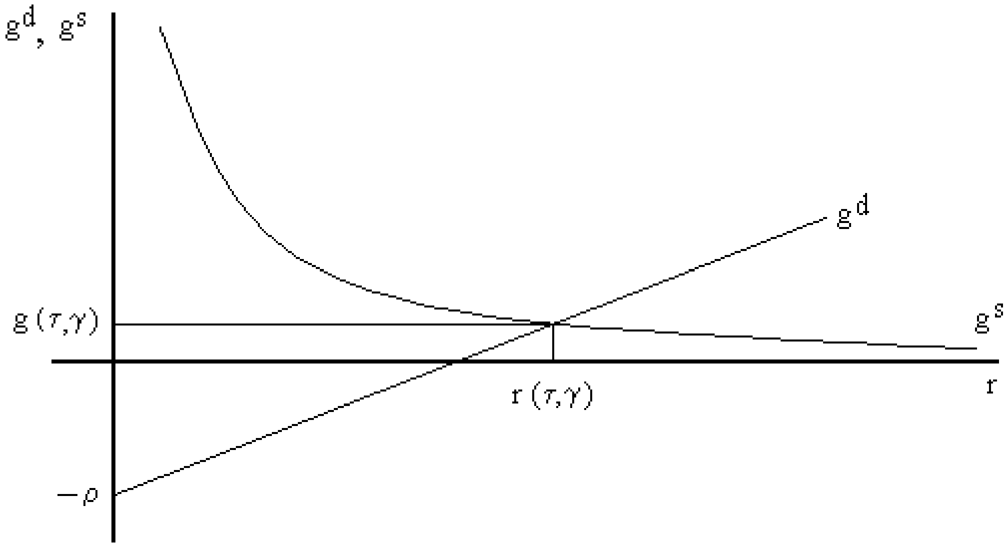

Figure 2: Determination of Equilibrium Growth and Interest Rates

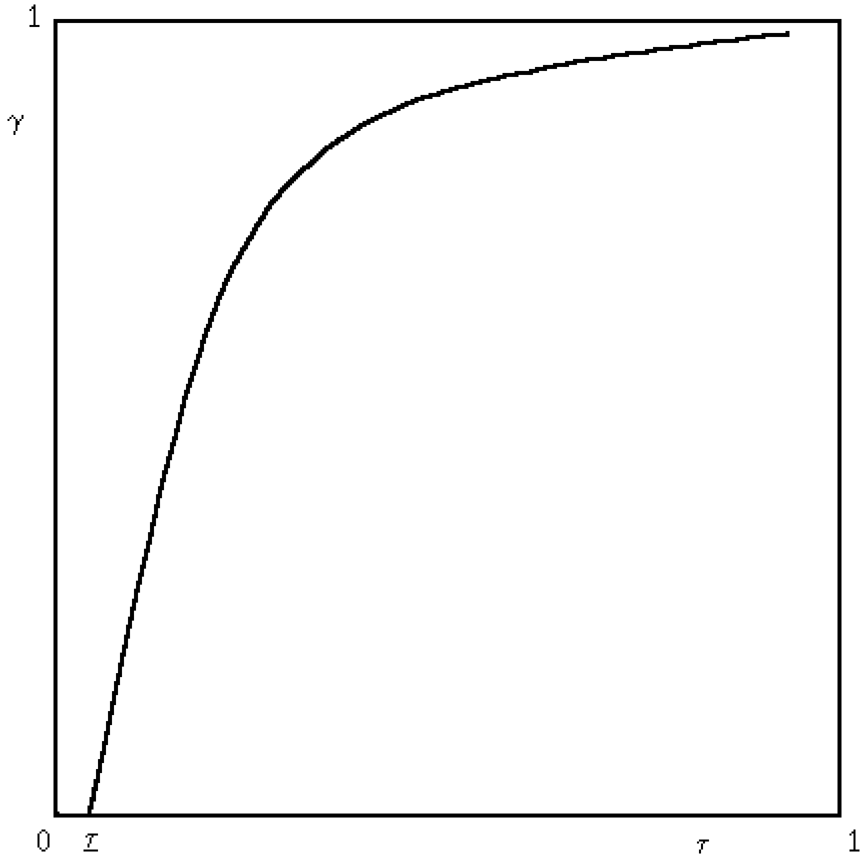

Figure 3: Behavior of $\tau(\gamma)$

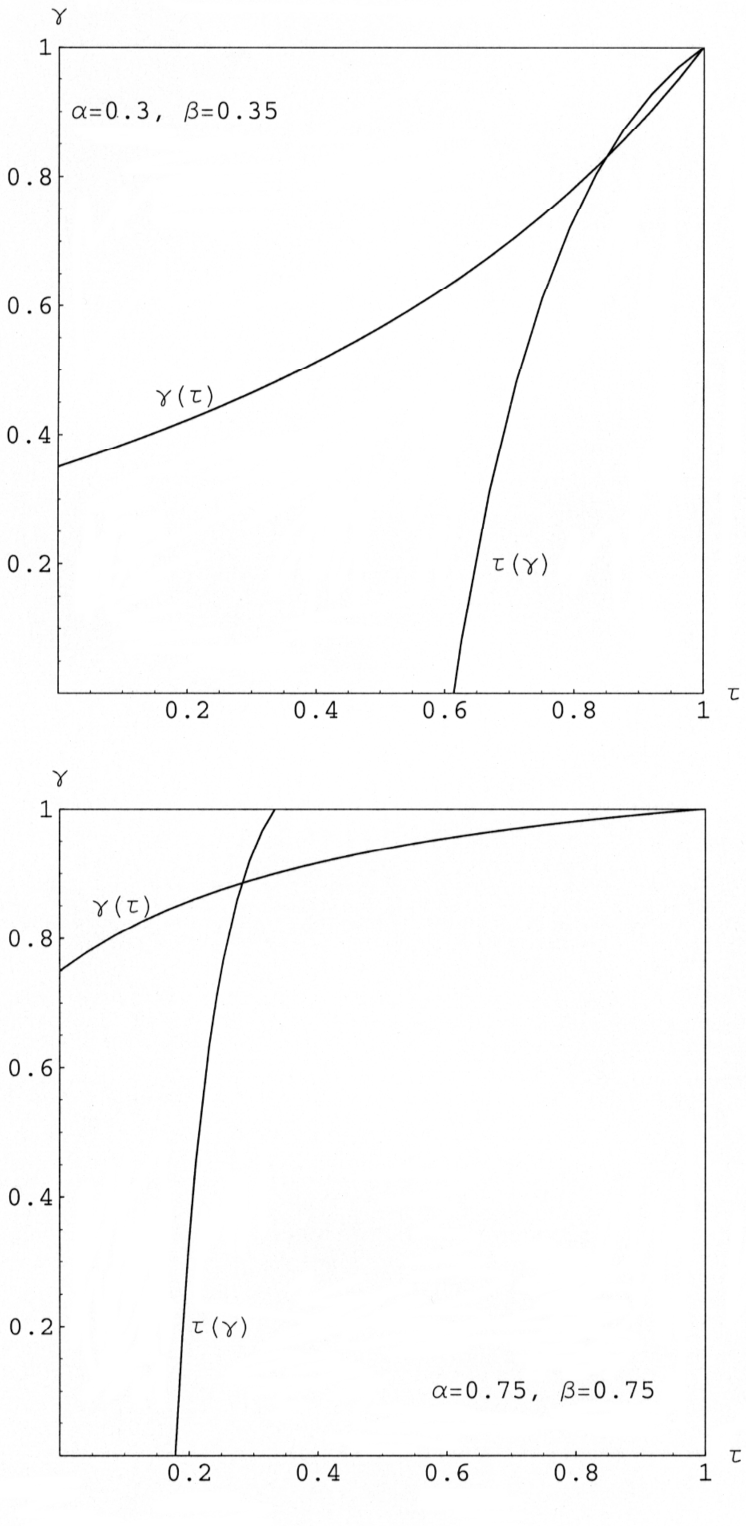

Figure 4: Growth Maximizing τ and γ for a Mixed Economy under Narrow and Broad Definitions of Capital

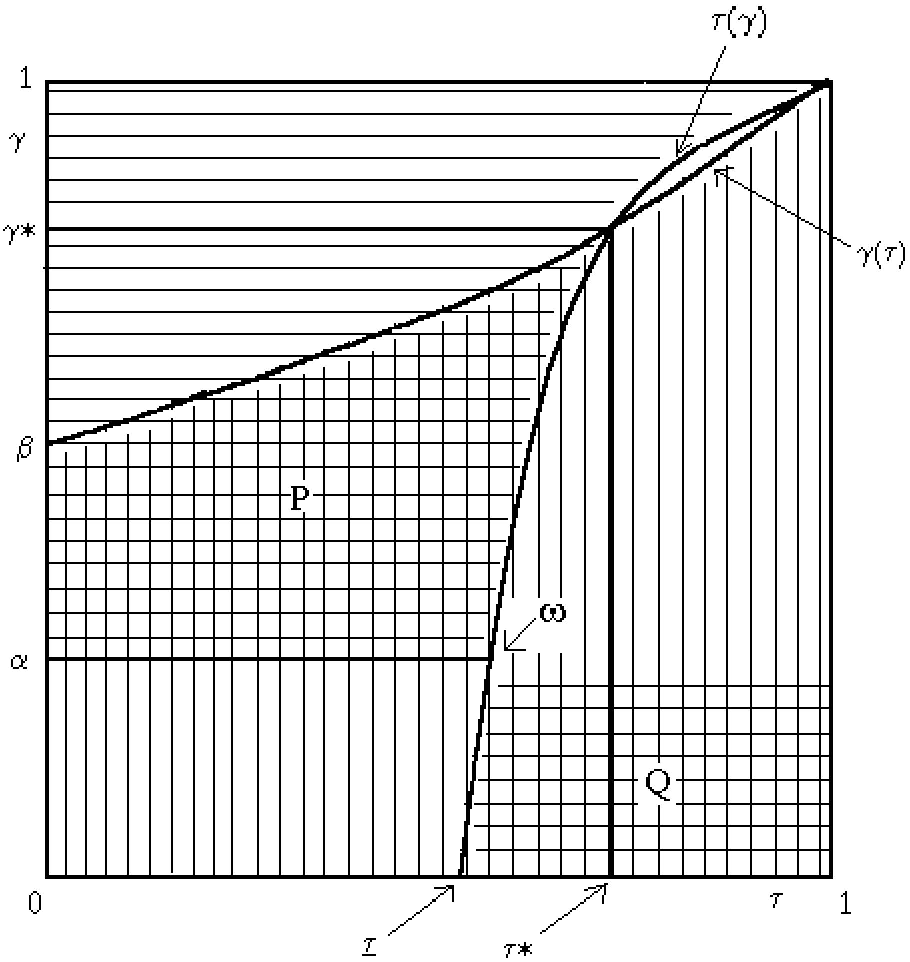

Figure 5: Comparison of Growth Maximizing and Welfare Maximizing τ and γ

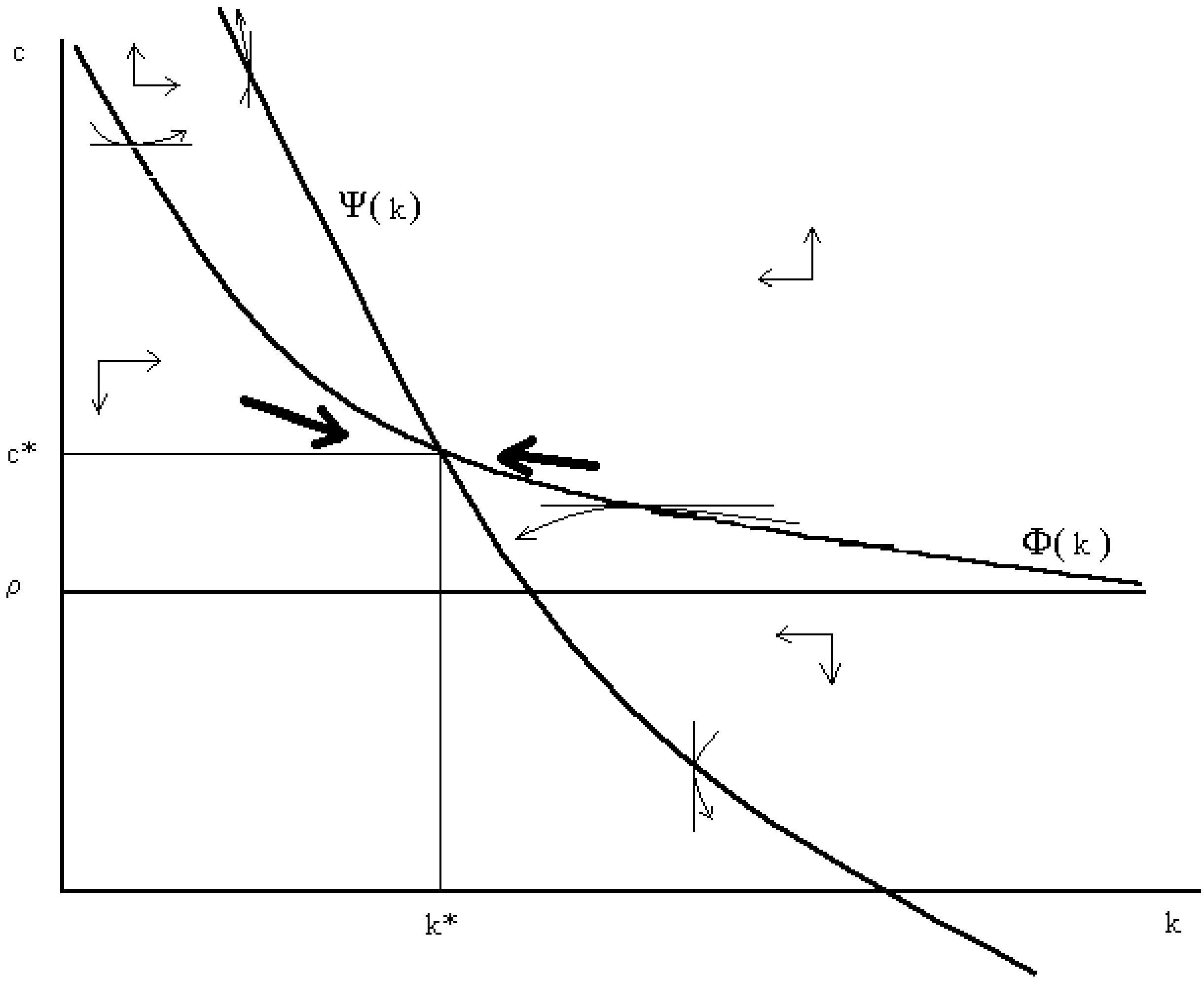

Figure 6 : Transitional Dynamics

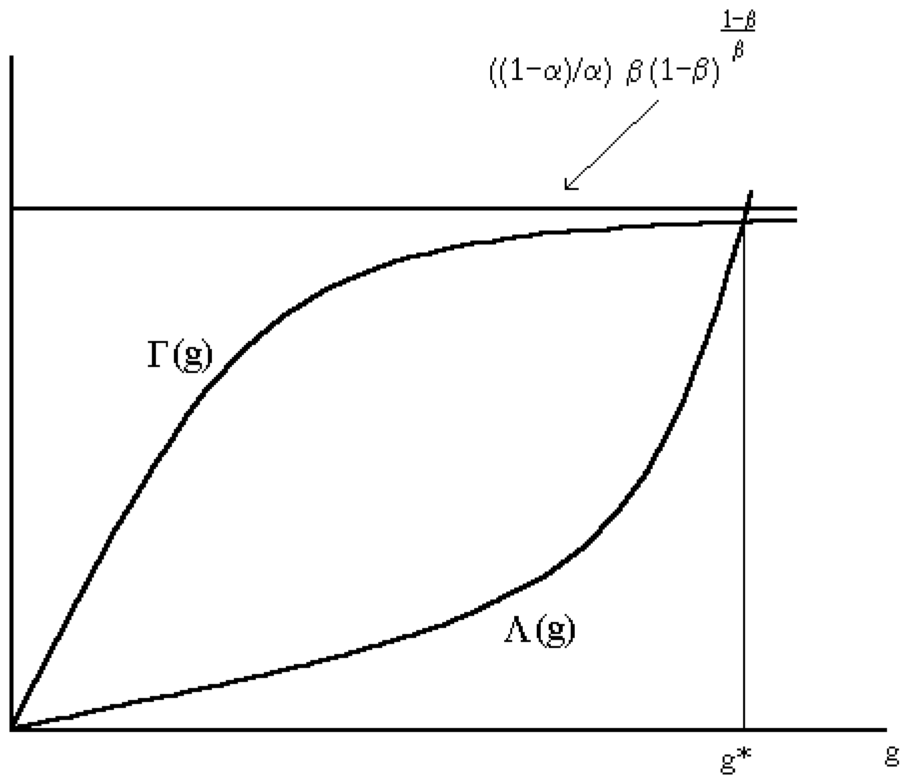

Figure 7 : Equilibrium of the Command Economy