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LUXURY AND WEALTH ACCUMULATION

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Abstract

This paper develops a model of luxury goods by incorporating weakly non-separable, recursive preferences. In a two-good framework, a quasi-luxury is defined as a good whose marginal rate of substitution is increasing in wealth. Under certain conditions, it is identical to a luxury good. Consumers wait for quasi-luxuries more (less) patiently than for quasi-necessities when they expect to be happier (unhappier) in the future. The preference for quasi-luxuries promotes optimal wealth accumulation and hence growth. In a two-country economy, the less patient country with stronger quasi-luxury preferences can be wealthier than the more patient country.

Keywords: Quasi-luxury goods; luxury; wealth; weakly non-separable preferences; time preference; growth; wealth distribution.

JEL classification: D91, E21, F34.

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Countries which have sumptuary laws, are generally poor. (Sir Dudley North, *Discourses Upon Trade*, 1691, p. 14)

1 Introduction

By definition, the wealthier allocate higher proportions of their expenditures to luxuries than the poorer do. The standard price theory (see, e.g., Deaton and Muellbauer (1983)) would describe this by saying that the richer consume more luxuries because they have more wealth. In dynamic consumer theory, however, the statement is not perfect: how much wealth consumers accumulate is a part of their lifetime utility-maximization problem, as is how much of each good they consume in each period. Comprehensive understanding of luxury expenditures entails dynamic analysis.

This paper develops a dynamic theory of luxury consumption, focusing on the bilateral relationship between luxury and wealth accumulation. Particularly emphasized is the causal effect that pursuit of luxury goods has on wealth accumulation. The topic of how preferences for luxury goods affect the total amount of national wealth goes back to at least the era of David Hume and Adam Smith.[1] The paper is a tentative response to their question using modern consumption theory. Based on the procedures used by Shi (1994), weakly non-separable preferences are specified in a simple two-good model of recursive preferences, such that the intratemporal marginal rate of substitution (MRS) between two goods depends on the future consumption streams through current welfare.[2] Relative preferences for the two goods, measured by the MRS, then depend on current welfare and hence current wealth holdings. With the resultant non-homothetic preference structure, a good whose MRS is increasing (decreasing) in wealth is called a *quasi-luxury* (*quasi-necessity*). Under a certain condition, quasi-luxuries are identical to luxury goods. The purposes of the present paper are (i) to characterize luxury goods from the viewpoint of intertemporal resource allocation; and (ii)

[1] See Brewer (1998) and Mason (1998).

[2] Weakly non-separable preferences under recursive preferences are analyzed by several authors (e.g., Lucas and Stokey (1984), Judd (1985), Epstein, Ham, and Zin (1988), Shi (1994), Ikeda (2001)). Shi conducts the most systematic analysis to discuss the intertemporal leisure-consumption choice under distortionary taxation on capital and labor. By applying his analysis to a small-country model with two traded goods, Ikeda (2001) shows that, in contrast to the literature, a terms-of-trade deterioration can worsen the current account, i.e., the Harberger-Laursen-Metzler effect can take place.

to show that wealth accumulation and distribution depend on consumers' *intratemporal* preferences for these luxury goods.

Quasi-luxuries are characterized by two properties. First, consumers wait for quasi-luxury consumption more (less) patiently than for quasi-necessity when they expect to be happier (unhappier) in the future. This property helps to understand consistently two contrasting patterns of luxury consumption that are commonly observed: Some people save luxuries today to enjoy tomorrow, while other people enjoy luxurious lives today by loans and live on necessity goods tomorrow to pay back their debts. Which pattern takes place depends on the future course of the welfare of consumers.

Second, given constant market prices, strong preferences for quasi-luxury goods are shown to promote consumers' optimal wealth accumulation. Alternatively stated, quasi-luxury induces a preference for wealth. This saving-promoting property of quasi-luxury preferences is then applied to the models of neoclassical growth and a two-country world economy, showing two main results: (i) the stronger the quasi-luxury preference, the more capital is accumulated in steady state; and (ii) even if production flows and utility-discounting functions are both internationally (interpersonally) identical, a country with a stronger quasi-luxury preference holds more wealth in the long run than a country with a weaker preference. As a corollary of (i), luxury taxes harm growth, as Sir Dudley North pointed out more than three centuries ago (see the epigraph). From (ii) and continuity, a less-patient country with a stronger quasi-luxury preference can be wealthier than a more-patient one.

Although there are strong empirical evidences against preference homotheticity (e.g., Blundell, Browning, and Meghir (1994), Attanasio and Weber (1995), and Parker (1999)), homothetic preferences have usually been assumed for simplicity in dynamic consumer theory, and few attempts have been made to analyze luxury consumption from the viewpoint of intertemporal utility maximization.[3, 4] Browning and Crossley (2000), in an important

[3]Besley (1989) proposes a new definition for luxury which is useful in "dynamic" applications. However, dynamic optimization is not discussed there. Baland and Ray (1991) examine the effect of capital accumulation on unemployment by using a model in which luxury and basic goods compete for the use of the scarce resources. However, the analysis is essentially static, assuming that capital accumulation is exogenous.

[4]There are several important contributions to dynamic macroeconomics that can be *reinterpreted* in the luxury-necessity terms. For example, in the status (wealth)-seeking literature (e.g., Cole, Mailath, and Postlewaite (1992), Corneo and Jeanne (1999)), a high

exception using a two-good, two-period model of the time-additive utility function, show that luxury goods have higher elasticities of intertemporal substitution and hence are easier to postpone than necessity goods. This paper re-examines this issue in the recursive preference framework by relating quasi-luxury goods to luxury goods. My contribution in this regard is the finding that luxury goods can be featured by the relative magnitudes of good-specific time preferences as well as of good-specific intertemporal substitution elasticities. Even when good x is not easier to postpone than good c, it turns out that x can be a luxury good if consumers are more patient with respect to x than with respect to c when they are getting happier, i.e., if it is a quasi-luxury.

In the dynamic macro literature, international (interpersonal) wealth distribution has been explained by referring to differences in four determinants: (i) the subjective discount rate (e.g., Ramsey (1928), Devereux and Shi (1991)); (ii) productivity growth (e.g., Obstfeld and Rogoff (1996), Frenkel and Razin (1992)); (iii) age structures (e.g., Buiter (1981), Obstfeld and Rogoff (1996), Frenkel and Razin (1992)); and (iv) random income fluctuation (e.g., Clarida (1990), Becker and Zilcha (1997)). To focus on implications of the luxury preference, these factors are not considered. The punch line of my result lies in the finding that wealth accumulation is still affected by intratemporal preferences for luxury.

The rest of the paper is organized as follows. Section 2 presents the basic framework to define (quasi-) luxury goods, and analyses the relation between preferences for them and consumers' optimal wealth accumulation. Section 3 considers implications of the luxury preference for economic growth by using a simple neoclassical model. In section 4, the analysis is extended to a two-country economy model, where a box diagram is introduced to aid the discussions. Section 5 concludes the paper.

"status" can be regarded as a luxury good, so that the preference for luxury is growth promoting. Our result differs from theirs in that the wealth preference is induced by preferences for usual-marketed commodities, instead of for non-marketed goods such as status and marriage.

2 Consumption behavior with quasi-luxury goods

2.1 Consumer preferences and quasi-luxury goods

Consider infinitely lived consumers. There are two distinct consumption goods c and x. Let $u(c,x)$ be the instantaneous utility function of a representative consumer. The $u(c,x)$ is assumed to satisfy the regularity conditions such as concavity, monotonicity, and the Inada condition. Preferences of consumers are given by the following lifetime utility function:

$$U(0) \equiv \int_0^\infty u(c(t), x(t)) \exp(-\Delta(t))\, \mathrm{d}t, \tag{1}$$

where

$$\Delta(t) = \int_0^t \delta(c(s), x(s))\, \mathrm{d}s, \tag{2}$$

with $\delta(\cdot,\cdot)\,(>0)$ representing the instantaneous subjective discount rate. As in the literature (e.g., Uzawa (1968)), $\delta_c > 0$ and $\delta_x > 0$ are assumed, where $\delta_c = \partial\delta/\partial c$, etc. Letting $\phi(t)$ denote the time-t lifetime utility $U(t)$, the corresponding generating function g is given by

$$g(c,x,\phi) = u(c,x) - \phi\delta(c,x),$$

with which utility evolution is expressed as

$$\dot{\phi} = -g(c,x,\phi) \ \text{ s.t. } \lim_{t\to\infty} \phi(t) \exp(-\Delta(t)) = 0, \tag{3}$$

where a dot represents the time derivative, i.e., $\dot{\phi}(t) = \mathrm{d}\phi(t)/\mathrm{d}t$. The first-order partial derivatives g_c and g_x equal the current-value marginal utilities of $c(t)$ and $x(t)$ defined in terms of the Voltera derivative. It is assumed that g_c and g_x are positive. The intratemporal marginal rate of substitution (MRS) between c and x is given by $\mathrm{d}c/\mathrm{d}x|_{U(0)=\text{const.}} = g_x/g_c(c,x,\phi)$.

When the MRS indeed depends on current utility level ϕ, consumer preferences are not weakly separable since the choice of the time-t consumption basket depends on future consumption plans. With the preferences, which are called by Shi (1994) weakly non-separable preferences, I define *quasi-luxury goods* as follows:

Definition: *A good is a **quasi-luxury (-necessity) good** or simply a **quasi-luxury (-necessity)** to a consumer if and only if (iff) his or her* *relative preferences for it, measured by the MRS, are increasing (decreasing) in wealth.*

It is convenient if the non-separability index ξ is defined as

$$\xi\left(c, x, \phi\right) \equiv \frac{1}{g_x/g_c}\frac{\partial\left(g_x/g_c\right)}{\partial\phi} = \frac{\delta_c}{g_c} - \frac{\delta_x}{g_x},$$

where the following relations are assumed:

Assumption 1:

$$g_{cx} = 0, g_{cc} < 0, g_{xx} < 0, \frac{\delta g_{cc}}{g_c^2} < \xi < -\frac{\delta g_{xx}}{g_x^2}.$$

The last three inequalities ensure the local concavity of the preference.[5,6] The index ξ captures how relative preferences for the two goods depend on current welfare and hence on current wealth. Without loss of generality, let us assume the following:

Assumption 2: Good x is a quasi-luxury good:

$$\xi\left(c, x, \phi\right) > 0.$$

From the definition of ξ, the MRS g_x/g_c for the quasi-luxury x along the indifference curve is larger than that along the discount rate, δ_x/δ_c, implying that consumers are more reluctant to sacrifice x for c to keep lifetime utility constant than to keep the discount rate constant:

$$\left.\frac{\mathrm{d}c}{\mathrm{d}x}\right|_{U(0)=\text{const.}} > \left.\frac{\mathrm{d}c}{\mathrm{d}x}\right|_{\delta=\text{const.}}.$$

Put otherwise, to increase quasi-luxury x in exchange for quasi-necessity c, keeping the discount rate constant, improves lifetime utility and underlying wealth positions. This property plays an important role for consumers' quasi-luxury preferences to affect wealth accumulation.

[5] See Shi (1994, appendix).

[6] The index ξ would be zero if the discount rate δ were a function of felicity u, as in Uzawa (1968) and Obstfeld (1982), or if δ were constant, as in the case of time-additive preference, or if u were constant, as in Epstein and Hynes (1983).

Taking the good c as numeraire, let p be the relative price of the quasi-luxury, r the interest rate, and a the consumer's *total* wealth, which may be composed of financial wealth and human capital. Consumers maximize lifetime utility (1) subject to four constraints: (i) the flow budget constraint,

$$\dot{a}(t) = r(t)\,a(t) - c(t) - p(t)\,x(t)\,; \tag{4}$$

(ii) the law of motion (2) for the discount factor; (iii) the initial condition, a_0 =given; and (iv) the positivity condition on a. Letting λ denote the current-value shadow price of savings, the optimal conditions are given by:

$$\begin{aligned} &g_c(c,\phi)\,(\equiv u_c(c,x) - \phi\delta_c(c,x)) = \lambda, \\ &g_x(x,\phi)\,/g_c(c,\phi) = p, \\ &\dot{\lambda} = (\delta(c,x) - r)\,\lambda, \end{aligned} \tag{5}$$

and the transversality conditions for a and ϕ.

The optimal dynamics for (a, c, x, ϕ, λ) are generated by five equations (3), (4), and (5) under the initial and transversality conditions. To reduce the system, define the rate of time preference ρ^c with respect to c as

$$\rho^c \equiv -\left.\frac{\mathrm{d}\ln g_c(c(t),\phi(t))\exp(-\Delta(t))}{\mathrm{d}t}\right|_{\dot{c}=0},$$

where $g_c \exp(-\Delta)$ represents the present-value marginal utility of c. Then, the first and third equations in (5) can be reduced to

$$\dot{c} = -\frac{g_c}{g_{cc}}\,(r - \rho^c(c,x,\phi))\,, \tag{6}$$

where

$$\rho^c(c,x,\phi) = \delta(c,x) - \frac{\delta_c(c,x)}{g_c(c,\phi)} g(c,x,\phi)\,. \tag{7}$$

In exactly the same way, the optimal dynamics of quasi-luxury consumption can be obtained by defining the rate of time preference ρ^x with respect to x. From (5), the dynamics are not independent of (6).

By comparing (7) with the corresponding equation for good x, quasi-luxury goods can be characterized from the viewpoint of impatience: from

(3) and the definition of ξ, the difference between the two good-specific time preferences satisfies

$$\rho^c - \rho^x = \xi\dot{\phi},$$

which, under Assumption 3, implies $\rho^x \lesseqgtr \rho^c$ as $\dot{\phi} \gtreqless 0$. This characterizes quasi-luxuries as follows:

Proposition 1: *When welfare is improving (deteriorating) over time, consumers are more (less) patient with respect to quasi-luxury consumption than with respect to quasi-necessity consumption.*

Alternatively stated, consumers can wait for consuming quasi-luxuries more patiently than for consuming quasi-necessities when they expect to be happier or wealthier in the future, whereas they cannot when getting unhappier or poorer. This describes well our daily consumption behavior towards luxuries. For example, many young people with poor future prospects often enjoy outrageous luxurious consumption (luxurious suits, brand-new cars, expensive restaurant dinners with friends, etc.) by using credit cards and, at the same time, save daily necessity consumption (living in cheap apartment rooms, having junk food for daily dinners, etc.). The "Last Supper" might well have been luxurious since the sufferings on the cross were expected next day. People tend to spend large portions of unlabored lucky income, e.g., lottery prizes, poker, etc., on luxuries.[7] In contrast, many entrepreneurs and professional sports players likely save luxury consumption when young, instead of enjoying luxuries by loans, to accumulate human and nonhuman capital, until they can afford to enjoy luxurious lives in the future.[8] Upon an unexpected unlucky expense, consumers usually decrease more luxuries than necessities. These two contrasting consumption patterns, i.e., consuming more luxuries today and less tomorrow than necessities, or consuming less luxuries today and more tomorrow, can be rationalized consistently by Proposition 1: either pattern can take place, depending on whether the consumer's welfare state is deteriorating or improving, or whether he or she is happier or unhappier than expected to be in the future.

[7] In this sense, "ill got, ill spent" could be regarded as rational behavior.

[8] Likewise, during World War II, Japanese people saved luxury consumption under the slogan: "Luxury is our enemy!".

2.2 Optimal consumption

Given the market prices (p, r) and the initial total wealth a_0, the optimal consumption plan $\{a(t), c(t), x(t), \phi(t)\}_{t=0}^{\infty}$ for the consumer is generated by (3), (4), and (5). The steady-state consumption basket $(\bar{c}, \bar{x})$ is determined by

$$\delta(\bar{c}, \bar{x}) = r, \tag{8}$$

and

$$\frac{g_x(\bar{x}, u(\bar{c}, \bar{x})/\delta(\bar{c}, \bar{x}))}{g_c(\bar{c}, u(\bar{c}, \bar{x})/\delta(\bar{c}, \bar{x}))} = p. \tag{9}$$

Steady-state wealth holding and welfare are then given by

$$r\bar{a} = \bar{c} + p\bar{x}, \tag{10}$$

and

$$\bar{\phi} = u(\bar{c}, \bar{x})/r,$$

respectively.

Figure 1 depicts the determination of the steady-state consumption plan. Schedule RR' represents (8), depicting the locus of $(\bar{c}, \bar{x})$ that equalizes the steady-state rate of time preference to a given interest rate. It could be referred to as the steady-state time preference curve. Schedule FF' is a locus along which the MRS between the two goods is equal to the corresponding relative price. The steady-state consumption basket $(\bar{c}, \bar{x})$ is determined at the intersection point E of the two schedules. With the consumption basket, the no-savings condition (10) is depicted as schedule AA' which goes through point E with slope $-1/p$. Its horizontal intercept gives the steady-state total wealth holding $\bar{a}$.

The property of quasi-luxury goods is illustrated by noting from (5) that the slope of schedule AA' should equal the gradient g_d/g_f at point E of the steady-state indifference curve $I(E)$:

$$I(E) = \left\{ (\bar{c}, \bar{x}) \mid \frac{u(\bar{c}, \bar{x})}{\delta(\bar{c}, \bar{x})} = \text{utility at } E \right\}. \tag{11}$$

Therefore, for good x to be a quasi-luxury good, as in Assumption 2, the slope of the AA' schedule, $-1/p\,(= -g_c/g_x)$ should be smaller than the gradient at point E of schedule RR', i.e., $-\delta_c/\delta_x$. This property plays a crucial role for the quasi-luxury preference to promote wealth accumulation. Figure 1 is similar to the usual map for static consumption choice, except that the location of the "budget schedule" AA' is endogenously determined to finance the consumption basket $(\bar{c}, \bar{x})$ that is determined by schedules RR' and FF'. The resultant steady-state wealth holdings reflect preferences for the two goods as well as time preference.

An autonomous local dynamic system can be obtained with respect to $n \equiv (c, \phi, b)$ by substituting (5) into (3), (4), and (6) and linearizing the resulting system around steady state as $\dot{n}(t) = A\hat{n}(t)$, where the coefficient matrix A is given in Appendix A; and the hat above variable n represents deviations from the steady-state value of the variable. As is also shown in the appendix, the linear system has two positive roots and one negative root ω:

$$\omega \equiv \frac{r - \sqrt{r^2 + \frac{4g_c^2 g_x^2}{g_{cc}g_{xx}}\Psi}}{2};$$

where

$$\Psi \equiv -\left\{\xi^2 + \frac{r\delta_c\delta_x}{g_c g_x}\left(\frac{g_{cc}}{\delta_c g_c} + \frac{g_{xx}}{\delta_x g_x}\right)\right\},$$

which can be shown to be positive under Assumption 1. Any other paths than the saddle path governed by ω cannot satisfy the transversality conditions. The optimal consumption plan is uniquely determined on the saddle arm, which can be derived from the eigen vector associated with ω as:

$$\hat{c}(t) = \frac{g_x^2}{(g_{xx} + p^2 g_{cc})}\left\{\frac{g_{xx}(r-\omega)}{g_x^2} + \xi\right\}\hat{a}(t),$$

$$\hat{x}(t) = \frac{g_x g_c}{(g_{xx} + p^2 g_{cc})}\left\{\frac{g_{cc}(r-\omega)}{g_c^2} - \xi\right\}\hat{a}(t), \tag{12}$$

and

$$\hat{\phi}(t) = g_c \hat{a}(t),$$

where the state variable $a(t)$ evolves by $\dot{a}(t) = \omega\hat{a}(t)$ subject to $\hat{a}(0) = a_0 - \bar{a}$.

As total wealth holding monotonically approaches its steady-state quantity from a given a_0, the transitional paths for consumptions c and x and welfare ϕ are determined on stable arms (12). Under Assumption 1, the stable arms are all positively-sloping: consumptions c and x as well as lifetime utility ϕ co-move positively with net foreign assets b. That is, the two goods are normal.

2.3 The Engel curve and luxury goods

Based on the above analysis, quasi-luxury goods are related to luxury goods. In considering luxury goods in a dynamic setting, two points should be noted: first, relevant income in considering consumption baskets is not usual current income but permanent income, i.e., income from total wealth; second, however, total wealth is an endogenous variable in the intertemporal consumption choice setting. It is proposed that luxury goods be defined on the saddle arm (12).

By eliminating $\hat{a}$ from the first two equations in (12), a positively-sloping saddle trajectory in the (c, x)-space is obtained as

$$\hat{x}(t) = \frac{g_c}{g_x}\left\{\frac{g_{cc}(r-\omega)}{g_c^2} - \xi\right\}\left\{\frac{g_{xx}(r-\omega)}{g_x^2} + \xi\right\}^{-1}\hat{c}(t). \tag{13}$$

This schedule depicts positive co-movements of c and x generated by (endogenous) wealth variation. This can be regarded as the wealth-consumption curve or the *Engel curve* defined with respect to total wealth or permanent income. The Engel curve is also illustrated in figure 1. Luxury goods are defined along this schedule:

Definition: *A good is a **luxury (necessity) good** or simply a **luxury (necessity)** to a consumer iff, for given constant market prices* (p, r) and initial total wealth a_0*, the consumption share of the good in the consumer's total expenditure is increasing (decreasing) in total wealth along the Engel curve (13).*[9, 10]

[9]To be precise, it should be called a (p, r)-luxury good, for example, because how optimal consumption baskets depend on wealth depends on (p, r).

[10]Hamermesh (1982) estimates the *permanent*-income elasticities of various consumptions.

Along the Engel curve (13), $\frac{\mathrm{dln}px}{\mathrm{dln}c} = \frac{\gamma^c + c\xi/(r-\omega)}{p\gamma^x - px\xi/(r-\omega)}$, where $\gamma^c \equiv -cg_{cc}/g_c$ and $\gamma^x \equiv -xg_{xx}/g_x$ represent the measures of the desire to smooth consumptions c and x, respectively.[11] The identity $\frac{c}{z}\frac{\mathrm{dln}c}{\mathrm{dln}z} + \frac{px}{z}\frac{\mathrm{dln}px}{\mathrm{dln}z} = 1$ implies

$$\frac{\mathrm{dln}px}{\mathrm{dln}z} = \left(\frac{px}{c} + \frac{p\gamma^x - px\xi/(r-\omega)}{\gamma^c + c\xi/(r-\omega)}\right)^{-1}\left(\frac{px}{c} + 1\right).$$

It thus follows:

$$\frac{\mathrm{d}(px/z)}{\mathrm{d}a} \gtreqless 0 \Leftrightarrow \frac{\xi z}{r-\omega} + (\gamma^c - p\gamma^x) \gtreqless 0,$$

which can be summarized by:

Proposition 2: *The quasi-luxury good x is a luxury good iff*

$$\frac{\xi z}{r-\omega} + (\gamma^c - p\gamma^x) > 0,$$

implying that quasi-luxuries are likely to be luxuries in the sense that a quasi-luxury is a luxury good unless the degree γ^x of the desire to smooth the x consumption times p is much larger than that γ^c to smooth c.

Remark 1: By using a time-additive utility function over two-period consumption, Browning and Crossley (2000) proved that luxury goods have a higher intertemporal substitution elasticity (ISE) and hence are easier to postpone than necessities. Since the reciprocals of γ^c and γ^x equal the ISEs with respect to goods c and x, respectively, Proposition 2 could be regarded as an extension of their analysis to the recursive preference framework. In the setting, a good can be a luxury, even if it has a lower ISE, provided it is quasi-luxurious, that is so long as consumers are more patient with respect to the good with the lower ISE than with respect to the other.

Proposition 2 can be interpreted intuitively by recalling the Euler equation (6). From the definition (7) of time preference, the optimal consumption ratio of the two goods evolve according to:

$$\begin{aligned}\dot{\ln}(px/c) &= \frac{1}{p\gamma^x}(r-\rho^x) - \frac{1}{\gamma^c}(r-\rho^c) \\ &= r\left(\frac{1}{p\gamma^x} - \frac{1}{\gamma^c}\right) + \left(\frac{1}{\gamma^c}\rho^c - \frac{1}{p\gamma^x}\rho^x\right).\end{aligned}$$

[11] Epstein (1987, p. 76) uses a similar terminology.

From Proposition 1, when the utility index ϕ increases over time, consumers are more patient with respect to quasi-luxury x than with respect to quasi-necessity c (i.e., $\rho^x < \rho^c$). Proposition 2 thus implies that, if the ISEs are the same between the two goods, the x-c consumption ratio increases as total wealth a increases over time. On the other hand, if x and c were neither quasi-luxury nor quasi-necessity, i.e., when $\xi = 0$, the x-c consumption ratio is increasing in a if and only if x is easier to postpone than c, i.e., iff $1/p\gamma^x > 1/\gamma^c$, as is shown by Browning and Crossley.

In sum, there are two factors that determine whether a good is a luxury or a necessity: inter-commodity differences in ISE, and time preference. By considering quasi-luxury goods, the role of the second factor is focused on. As shown later, it is luxury preferences induced by this factor that play an important role in promoting wealth accumulation.

2.4 The preference for quasi-luxuries and optimal wealth accumulation

By using the optimal intertemporal consumption plan derived, let us examine the implication of consumers' preferences for quasi-luxury goods x for steady-state wealth holding and hence wealth accumulation. The relative preferences for quasi-luxury goods are parameterized by α (> 0), with which the instantaneous utility function is re-specified as

$$u = u\left(c, \alpha x\right),$$

where an increase in α increases the marginal felicity αu_x for given u_x.

The intratemporal marginal rate of substitution g_x/g_c in the steady state is given by $\mathrm{MRS}(\bar{c}, \bar{x}; \alpha)$:

$$\mathrm{MRS}\left(\bar{c}, \bar{x}; \alpha\right) = \frac{\alpha u_x\left(\bar{c}, \alpha\bar{x}\right) - \left(u\left(\bar{c}, \alpha\bar{x}\right)/\delta\left(\bar{c}, \bar{x}\right)\right)\delta_x\left(\bar{c}, \bar{x}\right)}{u_c\left(\bar{c}, \alpha\bar{x}\right) - \left(u\left(\bar{c}, \alpha\bar{x}\right)/\delta\left(\bar{c}, \bar{x}\right)\right)\delta_c\left(\bar{c}, \bar{x}\right)}. \tag{14}$$

Parameter α affects the MRS through three channels: (i) by raising the marginal felicity of x, αu_x for given u_x; (ii) by lowering $u_x\left(\bar{c}, \alpha\bar{x}\right)$; and (iii) by raising the utility level $u\left(\bar{c}, \alpha\bar{x}\right)/\delta\left(\bar{c}, \bar{x}\right)$. The effects (i) and (iii) enhance the MRS for x, whereas (ii) reduces it. Formally

$$\frac{\partial \mathrm{MRS}\left(\bar{c}, \bar{x}; \alpha\right)}{\partial \alpha} \gtreqless 0 \Leftrightarrow (\Lambda \equiv)\, 1 + \frac{\alpha x u_{xx}}{u_x} + \frac{x g_x}{r}\xi \gtreqless 0,$$

where the three terms on the right-hand side represent the effects (i) through (iii), respectively. To consider upward shifts in quasi-luxury preferences, it is assumed that the sum of (i) and (iii) dominates (ii):[12]

Assumption 3: An increase in α enhances the MRS for x: $\Lambda > 0$.

With this assumption, parameter α is referred to as the degree of consumers' preferences for quasi-luxury goods x, or simply quasi-luxury preference. By differentiating (8) through (10) with respect to α, the effect of an increase in the quasi-luxury preference on steady-state wealth holding can be computed as

$$\frac{\mathrm{d}\bar{a}}{\mathrm{d}\alpha} = \frac{\xi u_x \Lambda}{g_c g_x \Psi},$$

which is positive under Assumptions 2 and 3. The result can be summarized as follows:

Proposition 3: *An increase in the preference for quasi-luxury increases optimal steady-state wealth holding and hence promotes optimal wealth accumulation.*

As explained in section 2.1, increasing quasi-luxury consumption in exchange for quasi-necessity consumption, maintaining the discount rate at r, increases steady-state welfare and wealth holding, which results in Proposition 2. Intuitively, the preference for quasi-luxury goods induces a preference for wealth.

Figure 2 illustrates the property. An increase in α shifts the FF' schedule upward, bringing the steady-state point from point E_0 to E_1. Since the steady-state time preference curve RR' is steeper than the budget lines E_0A_0 and E_1A_1, this shift increases steady-state total wealth from OA_0 to OA_1.

Transition dynamics are monotonic, as depicted by figure 2. An increase in quasi-luxury preference shifts the Engel curve counter-clockwise. Note that consumers may instantly *reduce* quasi-luxuries $x(0)$ as well as quasi-necessities $c(0)$ to enjoy more luxuries in the future steady state, as illustrated by the discrete jump from point E_0 to E_{01} in figure 2. Consumers

[12] However, if the felicity function u is specified in the quadratic form: $u = -\frac{1}{2}\gamma_{cc}c^2 + \gamma_c c - \frac{1}{2}\gamma_{xx}x^2 + \alpha x$, where γ_c, γ_{cc}, and γ_{xx} are all positive, then $\mathrm{MRS}(\bar{c}, \bar{x}; \alpha)$ is definitely increasing in α, so that an increase in α can be regarded as an upward shift of the preference for x without any additional assumption.

become more patient instantly with respect to either good and increase savings since (7) and (12) imply

$$\mathrm{d}\rho^x(0)/\mathrm{d}\alpha = -\frac{g_c g_{xx}\omega}{(g_{xx}+p^2 g_{cc})}\left\{\frac{g_{cc}(r-\omega)}{g_c^2} - \xi\right\}\frac{\mathrm{d}\bar{a}}{\mathrm{d}\alpha} < 0,$$

and

$$\mathrm{d}\rho^c(0)/\mathrm{d}\alpha = -\frac{g_x^2 g_{cc}\omega}{g_c(g_{xx}+p^2 g_{cc})}\left\{\frac{g_{xx}(r-\omega)}{g_x^2} + \xi\right\}\frac{\mathrm{d}\bar{a}}{\mathrm{d}\alpha} < 0.$$

As a result, for example, the life of a consumer with stronger preferences for luxuries may be *less* luxurious in the short run than that of another with weaker luxury preferences even when they hold the same amount of wealth. In the interim run, both consumptions gradually increase up to the level at point E_1 as wealth is monotonically accumulated. Quasi-luxury consumption at point E_1 is larger than that at E_0, whereas quasi-necessities consumption at E_1 is less than before. In the long run, a consumer with stronger luxury preferences enjoys more wealth and a more luxurious life than another with weaker preferences for luxury. These possibilities are consistent with the fact that the lives of the wealthier and more luxurious are likely to have been less luxurious than those of the poorer when they were young.

3 Growth

From Proposition 1, it could be conjectured that, in the context of economic growth, the preference for quasi-luxuries promotes capital accumulation. Let us next examine this problem by recasting the previous consumer model in the neoclassical growth context.

Suppose that two goods are produced using capital and labor. The production functions in the two sectors are given by the usual linearly-homogeneous, concave functions $F^i(K^i, L^i)$ $(i = c, x)$, where K^i and L^i are the capital and labor employed by sector i, respectively. Labor is supplied inelastically. The total amount of labor L is constant. The total capital stock K accumulates from savings. To avoid complexities due to inter-sectoral differences in factor intensity, it is assumed that the two production functions $F^i(K^i, L^i)$ are similar in that the production functions satisfy: $F^c(K^c, L^c) = BF(K^c, L^c)$ and $F^x(K^x, L^x) = F(K^x, L^x)$, where F

is a linearly homogeneous concave function. Letting $f(k^i)$ be $F(K^i, L^i)/L^i$ with k^i denoting K^i/L^i, profit maximization yields:

$$\frac{w}{r} = \frac{f(k^c) - k^c f'(k^c)}{f'(k^c)} = \frac{f(k^x) - k^x f'(k^x)}{f'(k^x)},$$

and

$$w = B\{f(k^c) - k^c f'(k^c)\} = p\{f(k^x) - k^x f'(k^x)\},$$

where w and r are the wage rate and the capital rent, respectively. The first equation implies

$$k^c = k^x = k,$$

where $k \equiv K/L$ represents the aggregate capital-labor ratio. The above maximum profit condition thus implies

$$p = B \text{ and } r = Bf'(k). \tag{15}$$

The relative price p is fixed by the productivity factor B.

The demand side is the same as in the previous section except that total wealth a for the representative agent is specified explicitly as the sum of the per capita capital stock k as nonhuman wealth and the present value of the wage income flow as human wealth. The resultant optimal conditions are essentially the same.

The market-clearing conditions depend on which good is accumulated as capital goods. In either case, however, capital accumulation is generated by the same aggregate equation,

$$\dot{k} = Bf(k) - c - Bx, \tag{16}$$

where (15) is substituted. To see this, consider the case in which good c is used for both investment and consumption whereas good x is consumed for the pure consumption purpose. Market equilibria are then expressed as

$$Bl^c f(k) = c + \dot{k},$$

and

$$(1 - l^c) f(k) = x,$$

where $l^c \equiv L^c/L$ represents the proportion of labor employed in sector c. By multiplying the second equation by B, these equations can be aggregated into (16). When good x is used for investment purposes, the $\dot{k}$ term moves to the right-hand side of the second equation, so that the same aggregation again yields (16).

The equilibrium dynamics are obtained by combining the supply side, represented by (15) and (16), and the demand side, described by (3), (5), (6), and the transversality condition. As proven in Appendix B, the equilibrium dynamics are uniquely given by a saddle time-path governed by a negative root. Since the transition dynamics are monotonic, I focus on the steady-state effect of an increase in quasi-luxury preference α. The steady-state equilibrium $\left(\bar{c}, \bar{x}, \bar{k}\right)$ is determined by

$$\delta\left(\bar{c}, \bar{x}\right) = Bf'(\bar{k}), \tag{17}$$

$$\frac{g_x\left(\bar{x}, u\left(\bar{c}, \alpha\bar{x}\right)/\delta\left(\bar{c}, \bar{x}\right)\right)}{g_c\left(\bar{c}, u\left(\bar{c}, \alpha\bar{x}\right)/\delta\left(\bar{c}, \bar{x}\right)\right)} = B, \tag{18}$$

and

$$Bf\left(\bar{k}\right) = \bar{c} + B\bar{x}. \tag{19}$$

Substituting (19) into (17) yields

$$\delta\left(\bar{c}, \bar{x}\right) = Bf'(f^{-1}\left(\bar{c} + B\bar{x}\right)). \tag{20}$$

The steady-state consumption basket $(\bar{c}, \bar{x})$ is jointly determined by (18) and (20). Capital stock $\bar{k}$ is then given by (19). These relations can be depicted precisely by reinterpreting figure 1: the long-run time preference schedule RR' can be read as representing (20); the contract curve FF' as (18); and the long-run budget schedule AA' as (19). In particular, since $\delta_c/\delta_x > g_c/g_x = 1/B$ by Assumption 2, from (19) and (20), the slopes of schedules RR' and AA' satisfy:

$$\begin{aligned}
\left|\left.\frac{\mathrm{d}x}{\mathrm{d}c}\right|_{RR'}\right| &= \frac{\delta_c - f''/\left(Bf'\right)}{\delta_x - f''/\left(f'\right)} \\
&> \frac{1}{B} \\
&= \left|\left.\frac{\mathrm{d}x}{\mathrm{d}c}\right|_{AA'}\right|,
\end{aligned}$$

just as in section 2. Therefore, from the same discussion as for figure 2, an increase in quasi-luxury preference α increases the steady-state capital stock $\bar{k}$: it shifts the FF' to the left, thereby increasing the total expenditures $\bar{c} + B\bar{x}$ from A_0 to A_1 and raising the steady-state capital stock.

Proposition 4: *The greater is the preference α for quasi-luxury, the more steady-state capital is accumulated.*

Proof. Differentiating equations (18)-(20) with respect to α yields

$$\frac{\mathrm{d}\bar{k}}{\mathrm{d}\alpha} = \frac{u_x \Lambda \xi}{\Delta \left(f' \delta_x - f''\right)} > 0,$$

where

$$\Delta \equiv \frac{1}{f'\delta_x - f''} \left\{ g_c g_x \Psi + f'' \left(\frac{g_{xx}}{g_x} + \frac{B g_{cc}}{g_c} \right) \right\} > 0.$$

■

Remark 2: From the continuity property, even if consumer i is less patient than consumer j, in that $\delta^i(c,x) > \delta^j(c,x)\,\forall(c,x)$, the less-patient consumer i will accumulate *more* steady-state capital than the more-patient consumer j if consumer i's quasi-luxury preference is sufficiently larger than j's.

Remark 3: When a one-factor production economy is considered, in which goods are produced using only capital with constant-to-scale (i.e., linear in this case) production functions, the interest rate r is also fixed by the productivity factor, say B, so that consumers have no interaction through markets. As a result, Proposition 4 can be extended to the case of the N heterogeneous agent economy: Suppose that there are N heterogeneous consumers, indexed by $n = 1, \cdots, N$ with an identical utility-discounting function $\delta(c,x)$ and felicity functions $u^n(c,x) = u(c, \alpha^n x)$. Then, a consumer with a stronger luxury preference accumulates more steady-state capital in that:

$$\alpha^i > \alpha^j \Leftrightarrow \bar{K}^i > \bar{K}^j \;\forall i, j = 1, \cdots, N, i \neq j.$$

As can easily be conjectured from the above analysis, taxation on quasi-luxury goods harms capital accumulation. To show this, consider a tax τ on quasi-luxury consumption x, assuming that the tax revenue τx is paid back to households in a lump-sum manner. Then, the first-order condition between the two-good consumptions becomes

$$\frac{g_x(x,\phi)}{g_c(c,\phi)} = (1+\tau)\,p.$$

Since any other equilibrium conditions are unchanged, this implies that an increase in the quasi-luxury tax τ has qualitatively the same effect as a decrease in quasi-luxury preference α: it decreases the steady-state capital stock $\bar{K}$. Figure 2 can be reinterpreted as illustrating this. A quasi-luxury tax increase shifts the FF' schedule to the right, bringing the steady-state point from E_1 to E_0.

4 Wealth distribution in a two-country world economy

Let us finally examine the implication of quasi-luxury preferences for wealth distribution. Consider a two-country world economy composed of home and foreign countries, H and F. Foreign country variables are denoted by asterisks. Any production activities are neglected for simplicity. The representative agents in both countries are equally endowed with $Y/2$ and $X/2$ of the two goods, quasi-necessity good c and quasi-luxury good x. Countries H and F can freely trade these two goods at the world price p, and bonds b at the world interest rate r. The household sector here is exactly the same as in section 2 if total asset holding a in (4) is specified by

$$a(t) = b(t) + \int_t^{\infty} \frac{Y + p(s)\,X}{2} \exp\left(-\int_t^s r(\tau)\right) \mathrm{d}\tau,$$

implying

$$\dot{b}(t) = r(t)\,b(t) + Y/2 + p(t)\,X/2 - c(t) - p(t)\,x(t)\,. \tag{21}$$

The market equilibrium conditions for the two goods and bonds are expressed as:

$$c(t) + c^*(t) = Y,$$

$$x(t) + x^*(t) = X,$$

and

$$b(t) + b^*(t) = 0.$$

By the Walrus law, when the two-good markets are in equilibrium for all t, the international bond market is also in equilibrium. Given the time paths of $p(t)$ and $r(t)$, the optimal consumption path $\{b(t), c(t), x(t), \phi(t)\}_{t=0}^{\infty}$ for the representative agent in country H is determined by (3), (5), (6), and (21), and that for country F by the corresponding equations with respect to $\{b^*(t), c^*(t), x^*(t), \phi^*(t)\}$. The equilibrium price path $\{p(t), r(t)\}_{t=0}^{\infty}$ is determined such that these optimal consumption paths satisfy the above market-equilibrium conditions.

For simplicity, it is assumed that the two countries are initially identical and hence that the initial equilibrium is autarchic. Using this framework, let us examine the effects of a permanent increase in country F's preference for quasi-luxury goods. Our main interest is to show that due to the resultant stronger preference for quasi-luxury country F becomes wealthier than H in the steady state. The initial autarchy equilibrium is constructed by assuming the following:

Assumption 4: Both the felicity and subjective discount functions are initially identical between the two countries:

$$u^*(c^*, x^*) = u(c^*, \alpha x^*), \delta^*(c^*, x^*) = \delta(c^*, x^*),$$

where country F's quasi-luxury preference α initially equals one.

The steady-state equilibrium of the world economy is determined from the following ten equations:

$$\delta(\bar{c}, \bar{x}) = \delta(\bar{c}^*, \bar{x}^*) = \bar{r}, \tag{22}$$

$$\frac{g_x(\bar{x}, u(\bar{c}, \bar{x})/\delta(\bar{c}, \bar{x}))}{g_c(\bar{c}, u(\bar{c}, \bar{x})/\delta(\bar{c}, \bar{x}))} = \frac{g_x^*(\bar{x}^*, u^*(\bar{c}^*, \bar{x}^*)/\delta(\bar{c}^*, \bar{x}^*))}{g_c^*(\bar{c}^*, u^*(\bar{c}^*, \bar{x}^*)/\delta(\bar{c}^*, \bar{x}^*))} = \bar{p}, \tag{23}$$

$$\bar{r}\bar{b}^{(*)} + Y/2 + \bar{p}X/2 = \bar{c}^{(*)} + \bar{p}\bar{x}^{(*)}, \tag{24}$$

$$\bar{\phi}^{(*)} = \frac{u^{(*)}\left(\bar{c}^{(*)}, \bar{x}^{(*)}\right)}{\delta\left(\bar{c}^{(*)}, \bar{x}^{(*)}\right)},$$

and the market equilibrium conditions:

$$\bar{c} + \bar{c}^* = Y, \bar{x} + \bar{x}^* = X.$$

Due to Assumption 4 and the assumption of the symmetric supply sides, the initial steady-state equilibrium $(\bar{r}_0, \bar{p}_0, \bar{b}_0, \bar{c}_0, \bar{x}_0, \bar{\phi}_0, \bar{b}_0^*, \bar{c}_0^*, \bar{x}_0^*, \bar{\phi}_0^*)$ is autarchic:

$$\bar{c}_0 = \bar{c}_0^* = Y/2; \bar{x}_0 = \bar{x}_0^* = X/2; \bar{b}_0 = \bar{b}_0^* = 0;$$

$$\bar{r}_0 = \delta\left(Y/2, X/2\right); \bar{p}_0 = \frac{g_x\left(X/2, u\left(Y/2, X/2\right)/\delta\left(Y/2, X/2\right)\right)}{g_c\left(Y/2, u\left(Y/2, X/2\right)/\delta\left(Y/2, X/2\right)\right)},$$

and

$$\bar{\phi}_0 = \bar{\phi}_0^* = u\left(Y/2, X/2\right)/\delta\left(Y/2, X/2\right).$$

It is useful to construct a box diagram, as in figure 3, to depict the determination of the steady-state equilibrium. Schedule RR' represents the downward-sloping curve that is obtained by eliminating $\bar{c}^*$ and $\bar{x}^*$ from (22) using the market-clearing conditions, and schedule FF' the upward-sloping curve obtained from (23) in the same way, respectively:

$$RR' : \delta\left(\bar{c}, \bar{x}\right) = \delta\left(Y - \bar{c}, X - \bar{x}\right), \tag{25}$$

and

$$FF' : \mathrm{MRS}\left(\bar{c}, \bar{x}\right) = \mathrm{MRS}^*\left(Y - \bar{c}, X - \bar{x}; \alpha\right), \tag{26}$$

where the intratemporal MRS function MRS $(\bar{c}, \bar{x}; \alpha)$ is defined by (14). Schedules FF' and RR', the two-country versions of FF' and RR' in figure 1, could be regarded as the *intratemporal* and *intertemporal* steady-state contract curves, respectively. The equilibrium consumption allocation $(\bar{c}, \bar{x}, \bar{c}^*, \bar{x}^*)$ is determined at the intersection E of the two contract curves. Given point E, consider the steady-state indifference curve $I(E)$, defined by (11). $\bar{p}$ is then determined as the gradient of the curve at E. As discussed in section 2, by Assumption 2, the intertemporal contract curve RR' is steeper than the indifference curve $I(E)$ at E. Recall that this is the key property of quasi-luxuries that have produced Propositions 3 and 4. Furthermore, it can be shown that the equilibrium time-path of the economy around the steady state is uniquely given by a saddle path governed by a negative root (see Appendix C for the proof). The same discussion as in the previous sections can thus be applied to obtain the implication of the quasi-luxury preference.

Suppose that country F's preference α for quasi-luxury goods increases. As depicted by figure 4, it shifts the intratemporal contract FF' to the right, bringing the steady-state point from point E_0 to E_1. For country H, the new consumption basket is below the initial indifference curve $I(E_0)$, implying that its steady-state welfare deteriorates. Accordingly, the steady-state wealth holding of country H decreases, whereas that of country F increases, as depicted by the change from point A_0 to A_1. This can be summarized as follows:

Proposition 5: *The country with a stronger preference for quasi-luxury holds more wealth in the steady state than the other country with a weaker preference.*

Proof. Equations (22)-(24) yield:

$$\frac{\mathrm{d}\bar{b}}{\mathrm{d}\alpha} = -\frac{\mathrm{d}\bar{b}^*}{\mathrm{d}\alpha} = -\frac{u_x \xi}{2\Psi g_c g_x}\Lambda < 0.$$

■

An increase in country F's preferences for quasi-luxury raises the relative price p of the quasi-luxury good. In country H, to raise the marginal rate of substitution $g_x\left(\bar{x}, \bar{\phi}\right)/g_c\left(\bar{c}, \bar{\phi}\right)$ of the quasi-luxury up to the higher p, the representative agent shifts consumption away from the quasi-luxury good to the quasi-necessity. It should take place along the intertemporal contract curve, i.e., the quasi-luxury consumption should be reduced in exchange for the quasi-necessity at the rate of $\mathrm{d}x/\mathrm{d}c = \delta_c/\delta_x$ (see (8)), which, for given levels of $\bar{b}$ and, hence, $\bar{\phi}$, is larger than required to equalize $g_x\left(\bar{x}, \bar{\phi}\right)/g_c\left(\bar{c}, \bar{\phi}\right)$ to p. To reduce the MRS for quasi-luxury down to p, steady-state welfare $\bar{\phi}$and, hence, the underlying wealth holdings $\bar{b}$ should decrease. From the market clearing conditions, country F's consumption of the quasi-luxury good increases and that of the quasi-necessity decreases. Its wealth holdings $\bar{b}^*$ should increase to finance more quasi-luxuries than before.

As in the previous section, the following result can also be obtained from the continuity property of the steady-state solution with respect to preference parameters.

Corollary 1: *Even when there is an international difference in the steady-state time preference (utility-discounting functions), the less-patient country can have more wealth in the steady state than the more-patient one if the less-patient one has sufficiently stronger preference for quasi-luxury.*

5 Conclusions

There is an old idea that certain goods are related to wealth accumulation. I have provided a model to formalize the idea by incorporating weakly non-separable preferences. The model helps to understand luxury consumption from a dynamic viewpoint. The phenomenon that wealthier agents consume more luxuries can be described by stating that the consumers are wealthier because they prefer luxuries (quasi-luxuries). A new insight is that wealth accumulation should reflect consumers' preferences for various kinds of goods as well as for time.

There are several ways in which the above analysis can be extended. First, it should be extended to the case of more than two goods. Quasi-luxury goods are defined here in terms of the MRS between two goods. With more than two goods, some other devices would be required. Second, empirical testing of our model should be conducted. The model could be hypothesized by the property that differences in commodity-specific time preferences between luxury and necessity goods depend on wealth accumulation. This could be done by estimating the commodity-specific Euler equations.

Appendices

A Coefficient matrix A in section 2.2

$$A \equiv \begin{pmatrix} -\frac{\xi g_x^2}{g_{xx}} & \frac{g_c}{g_{cc}}\left(\frac{g_x^2 \xi^2}{g_{xx}} + \frac{r\delta_c}{g_c}\right) & 0 \\ -g_c\left(1 + p^2 \frac{g_{cc}}{g_{xx}}\right) & r + \frac{\xi g_x^2}{g_{xx}} & 0 \\ -\left(1 + p^2 \frac{g_{cc}}{g_{xx}}\right) & p\frac{\xi g_x}{g_{xx}} & r \end{pmatrix}. \quad (27)$$

B Dynamics of the growth model in section 3

This appendix proves that the equilibrium dynamics in the growth model in section 3 can be obtained as a saddle time-path around steady state. The equilibrium dynamics for (c, k, p, r, x, ϕ) are generated by (3), (5), (6), (15), (16) and the transversality condition. The system can be reduced to:

$$\dot{c} = -\frac{g_c}{g_{cc}}(Bf'(k) - \rho^c(c, x, \phi)),$$

$$\dot{\phi} = u(c, \alpha x) - \phi\delta(c, x),$$

$$\dot{k} = Bf(k) - c - Bx.$$

Substituting (5) for x in the above system yields an autonomous differential equation system with respect to $h = (c, \phi, k)$. The resulting system can be linearized as $\dot{\hat{h}} = G\hat{h}$, where

$$G = \begin{pmatrix} -\frac{\xi g_x^2}{g_{xx}}, & \frac{g_c}{g_{cc}}\left(\frac{g_x^2 \xi^2}{g_{xx}} + \frac{r\delta_c}{g_c}\right), & -\frac{g_c}{g_{cc}}Bf'' \\ -g_c\left(1 + B^2 \frac{g_{cc}}{g_{xx}}\right), & r + \frac{\xi g_x^2}{g_{xx}}, & 0 \\ -\left(1 + B^2 \frac{g_{cc}}{g_{xx}}\right), & B\frac{\xi g_x}{g_{xx}}, & r \end{pmatrix}.$$

The trace of G equals $2r$. Its determinant can be computed as

$$\det G = -\frac{g_c^2 g_x^2}{g_{cc} g_{xx}}\Psi - \frac{rBg_c f''}{g_{cc}}\left(1 + B^2 \frac{g_{cc}}{g_{xx}}\right) < 0,$$

implying that the equilibrium dynamics are saddle-point stable. It can be shown that the two positive roots are inconsistent with the transversality condition.

C Dynamics of the two-country model in section 4

With the steady-state equilibrium being given above, the local dynamic system for $m \equiv (c, \phi, \phi^*, b)$ can be obtained as

$$\dot{m}(t) = D\hat{m}(t); \tag{28}$$

$$D = \begin{pmatrix} -\frac{\xi g_x^2}{g_{xx}} & \frac{g_c}{2g_{cc}}\left(\frac{g_x^2\xi^2}{g_{xx}} + \frac{r\delta_c}{g_c}\right) & -\frac{g_c}{2g_{cc}}\left(\frac{g_x^2\xi^2}{g_{xx}} + \frac{r\delta_c}{g_c}\right) & 0 \\ -g_c\left(1 + p^2\frac{g_{cc}}{g_{xx}}\right) & r + \frac{\xi g_x^2}{2g_{xx}} & -\frac{\xi g_x^2}{2g_{xx}} & 0 \\ g_c\left(1 + p^2\frac{g_{cc}}{g_{xx}}\right) & -\frac{\xi g_x^2}{2g_{xx}} & r + \frac{\xi g_x^2}{2g_{xx}} & 0 \\ -\left(1 + p^2\frac{g_{cc}}{g_{xx}}\right) & p\frac{\xi g_x}{2g_{xx}} & -p\frac{\xi g_x}{2g_{xx}} & r \end{pmatrix}.$$

The linear system can be reduced in the form of a block-structure by using the difference ϕ^- of the two countries' welfare levels and their average ϕ^+:[13]

$$\phi^- = \frac{\phi - \phi^*}{2}, \phi^+ = \frac{\phi + \phi^*}{2}, \tag{29}$$

as, for $h \equiv (c, \phi^-, b, \phi^+)$,

$$\dot{h}(t) = H\hat{h}(t): \tag{30}$$

$$H = \begin{pmatrix} A & 0 \\ 0 & r \end{pmatrix},$$

where A is matrix (27) describing the local dynamics for individuals' optimal consumption in section 2. As can easily be seen, H has three positive roots and one negative root and the negative root is the same as ω in section 2.[14] The resultant equilibrium path is also very similar to that obtained in (12):

$$\dot{b}(t) = \omega\hat{b}(t), \ \hat{b}(0) = b_0 - \bar{b},$$

$$\hat{c}(t) = -\hat{c}^*(t) = \frac{g_x^2}{(g_{xx} + p^2 g_{cc})}\left\{\frac{g_{xx}(r - \omega)}{g_x^2} + \xi\right\}\hat{b}(t),$$

$$\hat{x}(t) = -\hat{x}^*(t) = \frac{g_x g_c}{(g_{xx} + p^2 g_{cc})}\left\{\frac{g_{cc}(r - \omega)}{g_c^2} - \xi\right\}\hat{b}(t),$$

$$\hat{\phi}(t) = -\hat{\phi}^* = g_c\hat{b}(t).$$

[13]For this procedure, see Aoki (1981).

[14]The magnitude of ω is different from that in section 2 because the points at which ω is evaluated are different.

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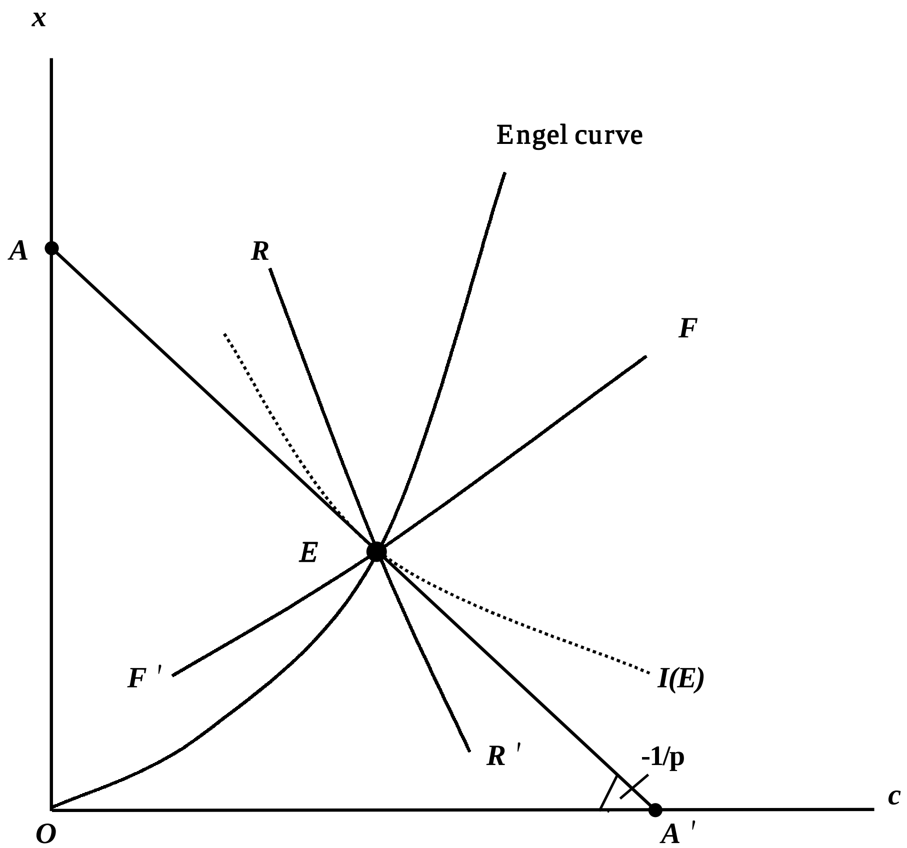

Figure 1. Optimum Consumption Plan

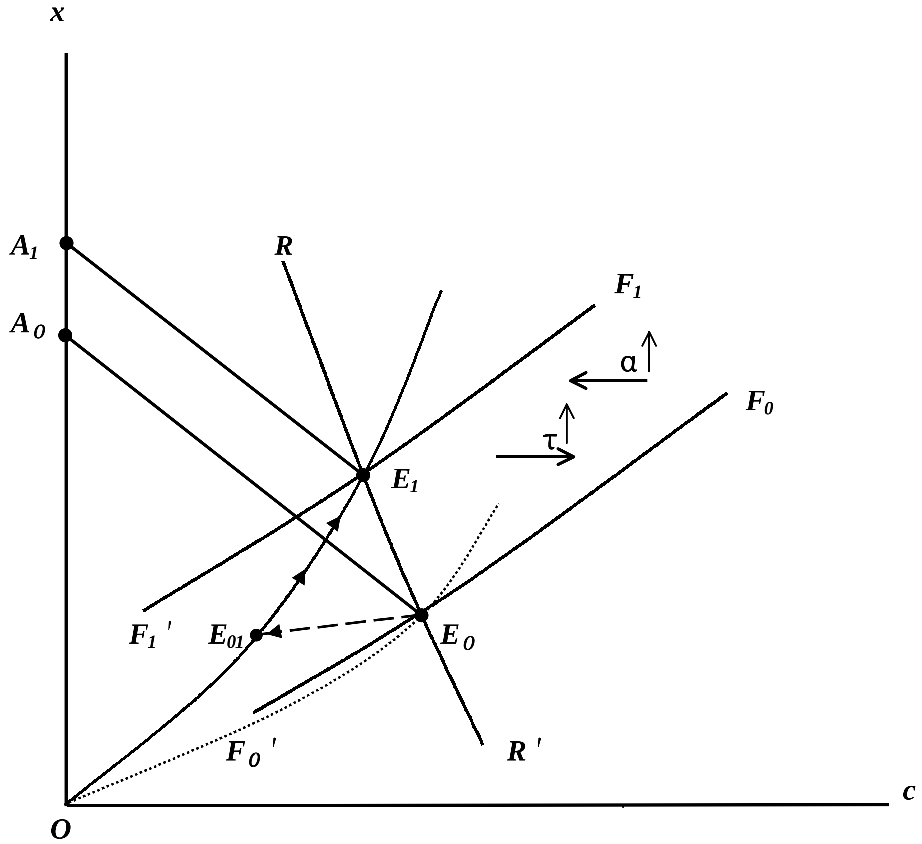

Figure 2. Quasi-Luxury Preference and Optimum Wealth Accumulation

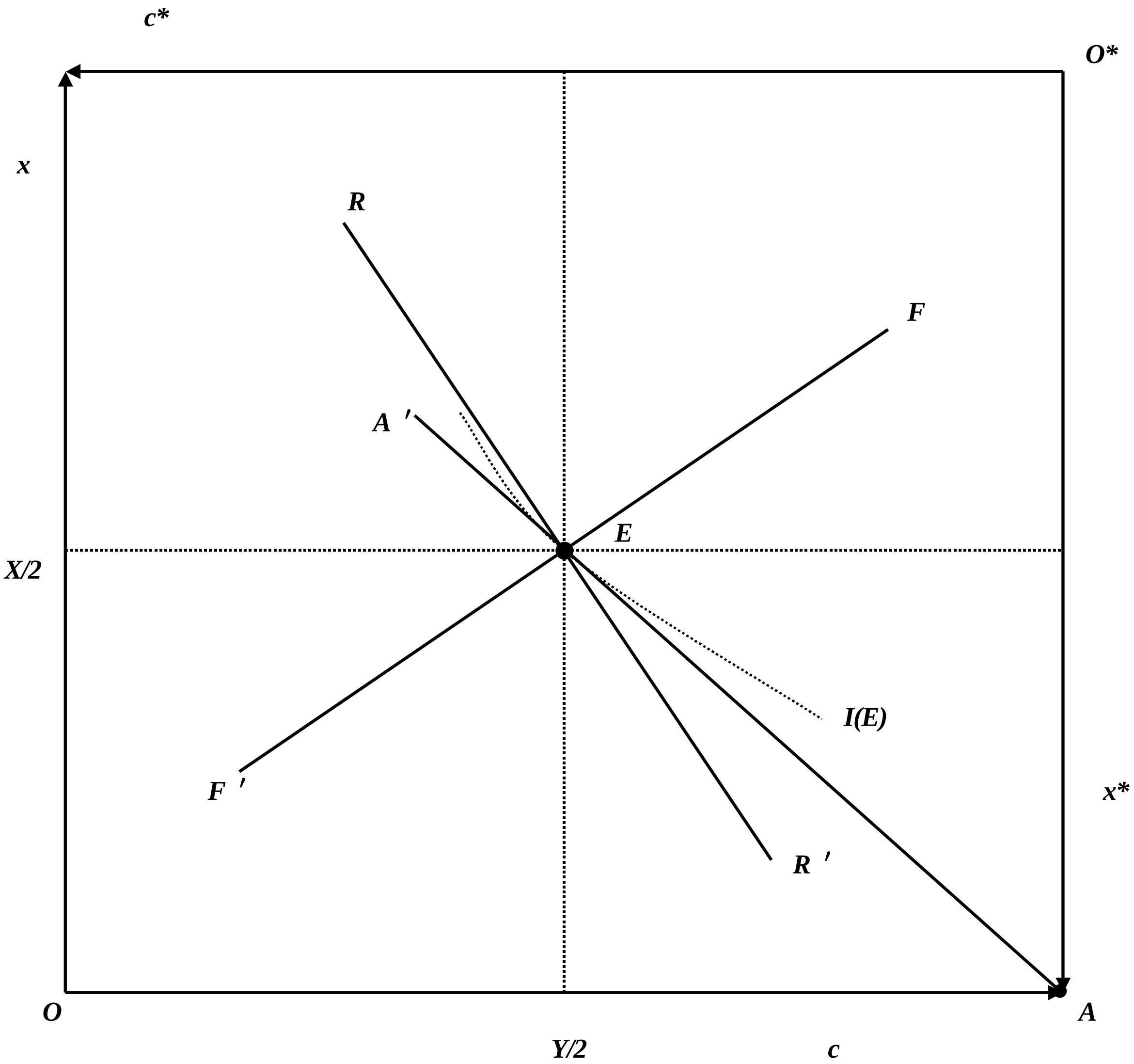

Figure 3. Two-Country Equilibrium

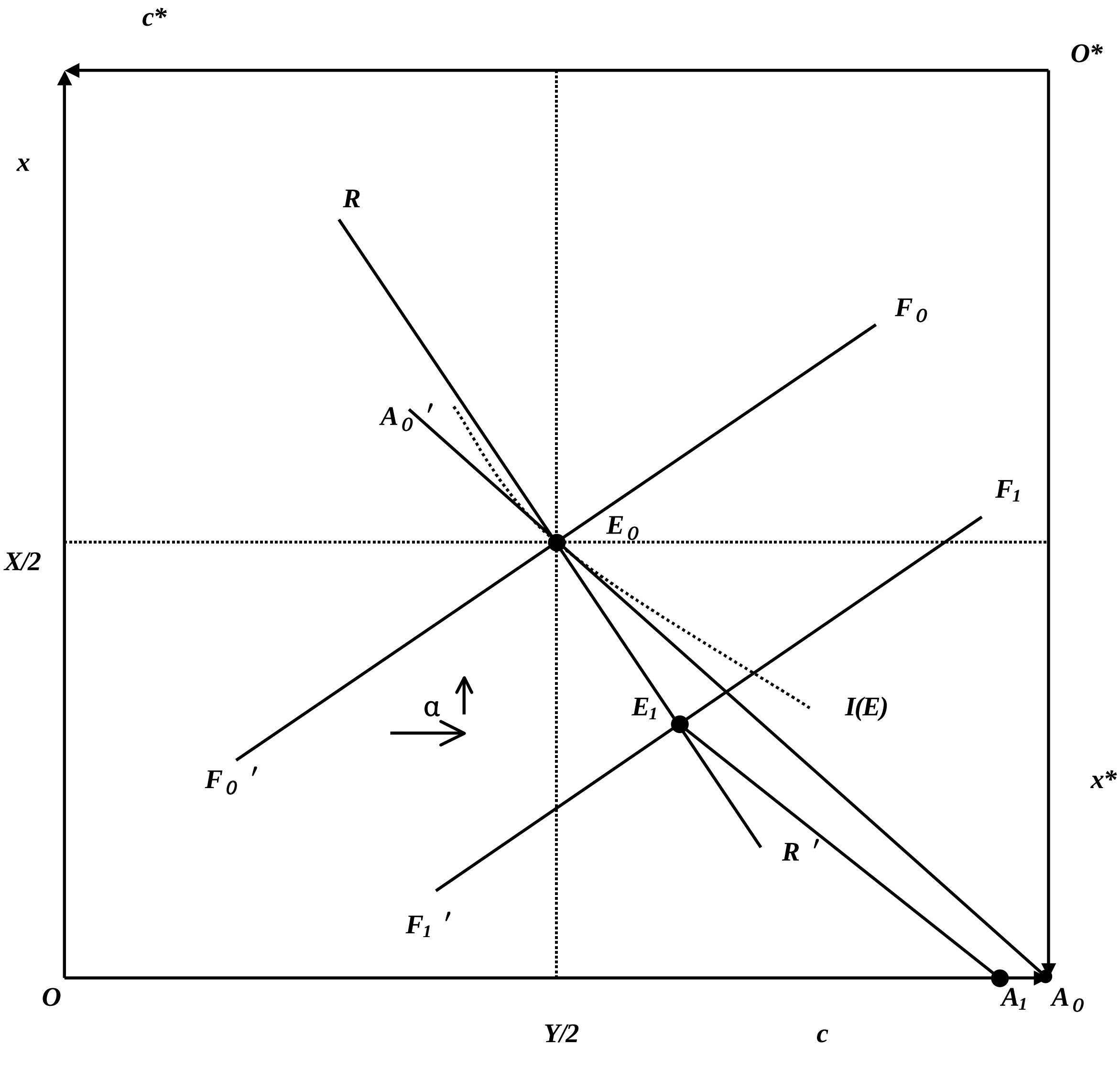

Figure 4. Quasi-Luxury Preference and Wealth Distribution