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COMPARING TESTS OF AUTOREGRESSIVE VERSUS MOVING AVERAGE ERRORS IN REGRESSION MODELS USING BAHADUR'S ASYMPTOTIC RELATIVE EFFICIENCY

C. R. McKenzie
and
Michael McAleer

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The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

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C. R. McKenzie
Osaka School of International Public Policy, Osaka University

Michael McAleer
Department of Economics, University of Western Australia

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Abstract
The purpose of this paper is to use Bahadur's asymptotic relative efficiency measure to compare the performance of various tests of autoregressive (AR) versus moving average (MA) error processes in regression models. Tests to be examined include non-nested procedures of the models against each other, and classical procedures based upon testing both the AR and MA error processes against the more general autoregressive-moving average model.

KEYWORDS: Autoregressive model; Bahadur efficiency; inappropriate alteratives; Lagrange multiplier test; moving average model; separate (non-nested) tests.

1. INTRODUCTION

In contrast to the numerous empirical studies which have reported estimates of regression models with autoregressive errors, there have been relatively few studies reporting estimates of regression models with moving average errors. This situation prevails in spite of the computational ease with which regression models with moving average errors can be estimated by maximum likelihood (ML) methods (see, for example, Pesaran and Pesaran [1] and Quantitative Micro Software EViews 4.0 [2]), or by asymptotically efficient two step methods (see Reinsel [3] and Hannan et al. [4]). The Lagrange multiplier (LM) test for serial correlation can detect serial correlation but provides no indication of whether the serial correlation in a regression model arises from an autoregressive process or a moving average process (see, for example, Godfrey [5]). The major aim of this paper is to compare some tests that will enable a choice to be made between the two error processes in a regression model using Bahadur's asymptotic relative efficiency measure.

Moving average (MA) errors are a viable alternative to autoregressive (AR) errors in a regression model since there are strong a priori reasons to expect the errors in certain models to have an MA form. For example, MA errors in a regression context may arise when: (i) there are random measurement errors associated with the dependent variable and lagged dependent variables appear as explanatory variables (Walker [6] and Pesaran [7]); (ii) the forecasting period exceeds the sampling period in forecasting equations (Hansen and Hodrick [8]); (iii) the equation is a solution of a rational expectations model (Broze et al. [9] and Evans and Honkapohja [10]); (iv) the model is a discrete time approximation to a continuous time model (Bergstrom [11]); (v) Koyck lag distributions are employed (Chow [12, pp. 102-103]); (vi) overlapping data on the dependent variable are used (Rowley and Wilton [13] and Kenward [14]); (vii) structural time series models are used (Harvey and Todd [15]); (viii) the data have been adjusted using filters such as X-11 (Wallis [16]); or (ix) an error correction model is estimated for series that are cointegrated (Engle and Granger [17]). Nicholls et al. [18] and Schwert [19, pp. 77-78] indicate a number of other instances where moving average errors can also be expected.

On the other hand, AR errors in a regression context may arise from common factor restrictions in dynamic models (see Hendry and Mizon [20] and Sargan [21]) and from stock adjustment models, or they may be indicative of general misspecification, especially in the form of the exclusion of important explanatory variables. If the excluded variables have the typical spectra of economic variables (Granger [22]), then the errors are likely to be AR. Alternatively, AR errors may be assumed because the consequences of doing so, even when the errors follow an MA error process, may not be too severe (see Griffiths and Beesley [23] for the case of a first-order MA, or MA(1), process).

Justifications for autoregressive-moving average (ARMA) errors rest on a combination of the reasons for AR and MA errors. The principle of parsimony, or Occam's razor (see, for example, Zellner et al. [24]), may lead to a simple AR or MA representation as an approximation to a higher-order ARMA process (see Box and Jenkins [25] and Hendry and Trivedi [26]).

The problem of testing between AR and MA models has been considered in the pure time series literature by Whittle [27], Walker [28], Pagan et al. [29], McAleer et al. [30], Hall and McAleer [31], Godfrey and Tremayne [32, 33], Franses [34], and Gourieroux and Monfont [35]. There have been a few attempts to examine the problem of testing between two regression models with different error processes; for example, King and McAleer [36], Godfrey and Tremayne [32] and Burke et al. [37] test AR(1) errors against MA(1) errors, Silvapulle and King [38] test MA(1) errors against AR(1) errors, Silvapulle and King [39] test joint AR(1)-AR(4) disturbances against joint MA(1)-MA(4) disturbances, and McKenzie et al. [40] test AR(p) disturbances against MA(q) disturbances. The available Monte Carlo evidence for both the pure time series and regression cases suggests that non-nested tests of these two models can have high power against each other, as well as against inappropriate alternatives (see McAleer et al. [30], Hall and McAleer [31] and McKenzie et al. [40]).

As a complement to existing research which tests these non-nested models against each other, this paper uses Bahadur's asymptotic relative efficiency to compare various tests of AR versus MA errors in regression models against each other.

The plan of the paper is as follows. Section 2 contains details of the model being considered as well as definitions of a number of variables that are used in later sections.

Section 3 briefly reviews tests of the null hypothesis that the errors follow either an AR or an MA process against the non-nested alternative of an MA or an AR process, respectively, as well as diagnostic tests for the respective null hypotheses against higher-order ARMA processes. Section 4 provides a comparison of the Bahadur approximate slopes of the diagnostic and non-nested tests against appropriate and inappropriate fixed alternatives. The final section contains some concluding comments.

2. MODEL AND NOTATION

Consider the linear regression model

$$y_t = x_t'\beta + u_t, \ t = 1,...,T \tag{1}$$

where x_t is presumed to be a k by 1 vector of non-stochastic variables and β is a k by 1 vector of unknown parameters. Two alternative non-nested hypotheses concerning the nature of u_t are the AR(p) and MA(q) processes, namely:

$$H_0: u_t = \rho_1 u_{t-1} + ... + \rho_p u_{t-p} + \varepsilon_t, \ \varepsilon_t \sim NID(0, \sigma^2) \tag{2}$$

$$H_1: u_t = \varepsilon_t - \gamma_1 \varepsilon_{t-1} - ... - \gamma_q \varepsilon_{t-q}, \ \varepsilon_t \sim NID(0, \sigma^2). \tag{3}$$

The AR(p) process in (2) is assumed to be finite and stationary, and the MA(q) process in (3) is assumed to be finite and invertible. Provided $\rho_i \neq 0$ for at least one i (0<i<p+1) and $\gamma_j \neq 0$ for at least one j (0<j<q+1), the hypotheses H_0 and H_1 can be shown to be globally non-nested. Assuming fixed initial values for u_t (t= -p+1,…,0) in H_0 and for ε_t (t= -q+1,…,0) in H_1, denote the ML estimates of (1) under H_0 as $(\hat{\beta}', \hat{\rho}_1,.., \hat{\rho}_p, \hat{\sigma}^2)$ with associated residuals $\hat{u}_t = y_t - x_t'\hat{\beta}$, and under H_1 as $(\tilde{\beta}', \tilde{\gamma}_1,.., \tilde{\gamma}_q, \tilde{\sigma}^2)$ with associated residuals $\tilde{u}_t = y_t - x_t'\tilde{\beta}$. The predictions and prediction errors from the two models are denoted by:

$$H_0: \hat{y}_t = x_t'\hat{\beta} + \hat{\rho}_1 \hat{u}_{t-1} + ... + \hat{\rho}_p \hat{u}_{t-p} \ \text{ and } \ \hat{\varepsilon}_t = y_t - \hat{y}_t,$$

and

$$H_1: \tilde{y}_t = x_t'\tilde{\beta} - \tilde{\gamma}_1 \tilde{\varepsilon}_{t-1} - \ldots - \tilde{\gamma}_q \tilde{\varepsilon}_{t-q} \quad \text{and} \quad \tilde{\varepsilon}_t = y_t - \tilde{y}_t.$$

For future reference, it will be useful to define the following polynomial lag functions:

$$\rho(L) = 1 - \rho_1 L - \ldots - \rho_p L^p, \quad \hat{\rho}(L) = 1 - \hat{\rho}_1 L - \ldots - \hat{\rho}_p L^p,$$

$$\gamma(L) = 1 - \gamma_1 L - \ldots - \gamma_q L^q, \quad \tilde{\gamma}(L) = 1 - \tilde{\gamma}_1 L - \ldots - \tilde{\gamma}_q L^q,$$

and the following transformed variables: $y_t^* = \hat{\rho}(L) y_t$, $x_t^* = \hat{\rho}(L) x_t$, $y_t^+ = \tilde{\gamma}(L)^{-1} y_t$, $x_t^+ = \tilde{\gamma}(L)^{-1} x_t$, $\tilde{\varepsilon}_t^+ = \tilde{\gamma}(L)^{-1} \tilde{\varepsilon}_t$, and $\tilde{\varepsilon}_t = \tilde{u}_t^+ = \tilde{\gamma}(L)^{-1} \tilde{u}_t$.

3. TESTS OF AR(p) VERSUS MA(q) ERRORS

3.1 Testing $H_0: AR(p)$ by Variable Addition

The tests of the AR(p) null against the MA(q) alternative developed in McKenzie et al. [40] are based on the following auxiliary regression equation:

$$y_t^* = x_t^{*}{}'\beta + \phi_1 \hat{u}_{t-1} + \ldots + \phi_p \hat{u}_{t-p} + Z_t'\theta + w_t \tag{4}$$

where $w_t = \varepsilon_t + (\hat{\rho}(L) - \rho(L)) x_t'(\beta - \hat{\beta})$, $\phi_i = \rho_i - \hat{\rho}_i$, Z_t is an r by 1 vector of added variables that are asymptotically uncorrelated with ε_t, and θ is an r by 1 vector of unknown parameters. Estimating (4) by OLS and denoting the F test of $H_0: \theta = 0$ by F, then $rF \xrightarrow{d} \chi^2_{(r)}$ under H_0 as $T \to \infty$.

The test associated with the hypothesis $\theta = 0$ in (4) can be computed as

$$rF = (ESS_R - ESS_U)/(ESS_U / T) \tag{5}$$

where ESS_R and ESS_U are the restricted and unrestricted error sums of squares from estimating (4) subject to $\theta = 0$ and $\theta \neq 0$, respectively. An identical procedure can be obtained by subtracting $x_t^{*}{}'\hat{\beta}$ from both sides of (4) to give

$$\hat{\varepsilon}_t = x_t^{*}{}'(\beta - \hat{\beta}) + \hat{\phi}_1 u_{t-1} + \ldots + \hat{\phi}_p u_{t-p} + Z_t{}'\theta + w_t. \quad (6)$$

Equation (6) is particularly useful for calculating the asymptotic relative efficiencies of the various tests. For testing the AR(p) null against the MA(q), AR(p+r) or ARMA(p, r) alternatives, the choices of Z_t are given in Table 1.

3.2 Testing $H_1 : MA(q)$ by Variable Addition

The tests of the MA(q) null against the AR(p) alternative developed in McKenzie et al. [40] are based on the following auxiliary regression equation:

$$y_t^{+} = x_t^{+}{}'\beta + \psi_1 \tilde{\varepsilon}_{t-1}^{+} + \ldots + \psi_q \tilde{\varepsilon}_{t-q}^{+} + Z_t{}'\theta + \eta_t \quad (7)$$

where $\eta_t = \varepsilon_t + [\gamma(L) - \tilde{\gamma}(L)]\tilde{\gamma}(L)^{-1}(\varepsilon_t - \tilde{\varepsilon}_t)$ and $\psi_i = \tilde{\gamma}_i - \gamma_i$. Estimating (7) by OLS, and denoting the F test of $H_0 : \theta = 0$ by F, then $rF \xrightarrow{d} \chi^2_{(r)}$ under H_0 as $T \rightarrow \infty$.

The test associated with the hypothesis $\theta = 0$ in (7) can be computed as rF in (5), where ESS_R and ESS_U are the restricted and unrestricted sums of squares from estimating (7) subject to $\theta = 0$ and $\theta \neq 0$, respectively. An identical procedure can be obtained by subtracting $x_t^{+}{}'\tilde{\beta}$ from both sides of (7) to give

$$\tilde{\varepsilon}_t = x_t^{+}{}'(\beta - \tilde{\beta}) + \psi_1 \tilde{\varepsilon}_{t-1}^{+} + \ldots + \psi_q \tilde{\varepsilon}_{t-q}^{+} + Z_t{}'\theta + \eta_t. \quad (8)$$

Equation (8) is particularly useful for calculating the asymptotic relative efficiencies of the various tests. For testing the MA(q) null against the AR(p), MA(q+r) or ARMA(p, q) alternatives, the choices of Z_t are given in Table 2.

4. BAHADUR'S ASYMPTOTIC RELATIVE EFFICIENCY

Given the number of consistent tests in section 3, some criterion must be chosen to compare their performance. Since the AR(p) and MA(q) error processes are non-nested, non-local power comparisons are used to evaluate the tests. Specifically, Bahadur's [41, 42] asymptotic relative efficiency criterion is used for purposes of comparison (see Geweke [43, 44, 45], Pesaran [46], Wascher [47], Pesaran and Smith [48], and Zabel [49] for some econometric applications). In this context, since only the asymptotic properties of the estimators under both the null and alternative hypotheses are known, it is necessary to work with the approximate rather than the exact slopes of the tests, despite the difficulties noted in Geweke [43]. The difference between the tests labelled 1 and 2 in Tables 1 and 2 is whether the added variable is not modified before being included (method 1), or is modified by the AR transformation in Table 1 (method 2) or the MA transformation in Table 2 (method 2).

There are three important questions relating to the various tests: (i) For any given added variable, is any one of the testing methods clearly superior? (ii) For any given testing method, is any one of the added variables dominant? (iii) Is there an optimal test?

Bahadur's approach keeps the alternative hypothesis fixed and allows the probability of type I error (size) to tend to zero as the sample size increases. The slopes of the power functions can be approximated by evaluating the various statistics for testing the null hypothesis under the fixed alternative. In this section, the tests are compared using Bahadur's asymptotic relative efficiency for testing, in the context of the linear regression model, an AR(1) null hypothesis against an MA(1) alternative (section 4.1) and an MA(1) null hypothesis against an AR(1) alternative (section 4.2).

4.1 Approximate Slopes of Tests of the AR(1) Model

A particularly useful property of the ML estimator $\hat{\beta}$ is that $\text{plim}\,\hat{\beta} = \beta$ regardless of whether the true disturbances follow an AR or an MA process. As x_t^* is asymptotically orthogonal to the remaining regressors in the equation for the LM, E and DOP tests, the term $x_t^*{}'(\beta - \hat{\beta})$ in (6) can be omitted without affecting the asymptotic properties of the test under either the null or the alternative hypothesis. Therefore, the LM, E and DOP statistics can be calculated from the auxiliary regression

$$\hat{\varepsilon}_t = \phi_1 \hat{u}_{t-1} + \ldots + \phi_p \hat{u}_{t-p} + Z_t{}'\theta + w_{1t} \qquad (9)$$

where $w_{1t} = w_t + x_t^*{}'(\beta - \hat{\beta})$. The Bahadur criterion requires an evaluation of the probability limit of the test statistic under the fixed alternative. When the linear regression model has been estimated assuming an AR(1) disturbance, the pseudo-true value of $\hat{\rho}$, ρ^*, is given, in general, as $\rho^* = p\lim_1 \hat{\rho} = -\gamma/(1+\gamma^2)$, where γ is the true value of the MA(1) parameter and $p\lim_1 \hat{\rho}$ denotes the probability limit of $\hat{\rho}$ under H_1.

Using the test statistics calculated from (9), the Bahadur approximate slopes are computed as $p\lim_1 rF/T$ for the various forms of the LM, E and DOP tests of the regression model with AR(1) errors, assuming a fixed alternative of a regression model with MA(1) errors. The probability limits of the approximate slopes of the tests will depend only on γ, given the consistency of $\hat{\beta}$ under both hypotheses.

Using the results in the Appendix, it can be shown that the approximate slope, AS, of each test defined using (9) can be computed as

$$AS = (ESS_R - ESS_U)/ESS_U, \qquad (10)$$

where ESS_U is the unrestricted error sum of squares from one of the regressions defined in Table 3, and $ESS_R = s_1{}'s_1$ (with s_1 defined in Table 3). Let $\tilde{s}_1{}'\tilde{s}_1$ be the unrestricted error sum of squares from a particular auxiliary regression arising from Table 3, so that the approximate slope of the test statistic is given by

$AS = (s_1's_1 - \tilde{s}_1'\tilde{s}_1) / \tilde{s}_1'\tilde{s}_1$.

An interesting point to note from Table 3 is that the approximate slopes of E1, E2 and DOP1 are identical since the regressors used, a_2, f_2 and r_2, respectively, are linearly dependent.

Another test considered here is the adaptation by Burke et al. [37] of the test developed in Godfrey and Tremayne [32] for pure time series models for testing the null hypothesis of AR(1) errors against the alternative of MA(1) errors in a regression model. The test statistic is given by

$$\tau = T^{1/2}(r_2 - r_1^2)/(1 - r_2) \qquad (11)$$

where $\tau \xrightarrow{d} N(0,1)$ under the null hypothesis of AR(1) errors, and r_i is an estimate of the ith-order autocorrelation coefficient (i=1,2) calculated using OLS residuals. As the test has a negative mean under the MA(1) alternative, the AR(1) null is not rejected if $\tau > c_\alpha$, where c_α is such that $\Pr(N(0,1) < c_\alpha) = \alpha$ and α is a given critical level. Since the theoretical autocorrelation coefficient for an MA(1) process cannot exceed 0.5, Burke et al. [37, p. 138] also suggest not rejecting the AR(1) model if $r_1 > (0.5 + T^{-1/2})$. However, the interpretation of this outcome should be that the MA(1) model is rejected, not that the AR(1) model is accepted. The reason for this interpretation is that there are many other stationary processes for which the theoretical autocorrelation coefficient at lag one can exceed 0.5, such as an MA(2) process. Under the MA(1) alternative, $\text{plim}_1 T^{-1}\tau^2 = \gamma^4/(1+\gamma^2)^4$, where γ is the true value of the MA(1) parameter (see Bera and Newbold [50]).

Figures 1 and 2 graph the approximate slopes of the tests for values of the true MA parameter between 0 and 0.9. Burke et al.'s [37] τ test is labelled TAU in Figure 1. In order that all these tests are distributed as $\chi^2_{(1)}$ under the null hypothesis, the rule that the MA(1) model should be rejected when $r_1 > (0.5 + T^{-1/2})$ is ignored in this analysis. One important result is that E1=E2=DOP1, so that no single testing method or added variable is preferred. It is clear from these figures that the asymptotic relative efficiencies of the tests that use information from the alternative (E and DOP) are generally higher than the corresponding tests based on the LM principle (LMM and LMA). Moreover, in this simple case, of all the tests in the comparison, the E1 (=E2=DOP1) test would appear to be the best. The E1 test dominates Burke et al.'s [37] τ test.

4.2 Approximate Slopes of Tests of the MA(1) Model

Since under H_0 and H_1, $\text{plim}\,\tilde{\beta} = \beta$ and x_t^+ is asymptotically orthogonal to all the other regressors in the equation for the LM, E and DOP tests, $x_t^{+\prime}(\beta - \tilde{\beta})$ in (8) can be omitted without affecting the asymptotic properties of the test under either the null or alternative hypotheses. Thus, the LM, E and DOP test statistics can be calculated from the auxiliary regression

$$\tilde{\varepsilon}_t = \psi_1 \tilde{\varepsilon}_{t-1}^+ + \ldots + \psi_q \tilde{\varepsilon}_{t-q}^+ + Z_t'\theta + \eta_{1t} \qquad (12)$$

where $\eta_{1t} = \eta_t + x_t^{+\prime}(\beta - \tilde{\beta})$. When the linear regression model has been estimated assuming an MA(1) error, the pseudo-true values of $\tilde{\beta}$ and $\tilde{\gamma}$ under H_0:AR(1) are $\text{plim}\,\tilde{\beta} = \beta$ and $\text{plim}_0\,\tilde{\gamma} = \gamma^*$, respectively, where γ^* is the solution to

$$\gamma^{*3}\rho^2 + \gamma^{*2}\rho - \gamma^* - \rho = 0 \qquad (13)$$

that lies in the interval (-1,1) (Whittle [27, p. 91]), ρ is the true value of the AR(1) parameter, and $\text{plim}_0\,\tilde{\gamma}$ denotes the probability limit of $\tilde{\gamma}$ under H_0. Thus, the probability limits for calculating the approximate slopes of the tests of H_1 will depend only on ρ.

Using the test statistics calculated from (12), the Bahadur approximate slopes are computed as $\text{p}\lim_0 rF/T$ for the various forms of the LM, E and DOP tests of the regression model with MA(1) errors, assuming a fixed alternative of a regression model with AR(1) errors. The probability limits of the approximate slopes of the tests will depend only on ρ, given the consistency of $\tilde{\beta}$ under both hypotheses.

Based on (12) and using the results in the Appendix, it can be shown that the approximate slope of each test defined using (12) can be computed as

$$AS = (\lim ESS_R - \lim ESS_U)/\lim ESS_U, \qquad (14)$$

where ESS_U is the unrestricted error sum of squares from one of the regressions defined in Table 4, ESS_R is the error sum of squares from the regression of $\underset{\sim}{q}$ on $\underset{\sim}{nq_1}$ (with $\underset{\sim}{q}$ and $\underset{\sim}{nq_1}$ defined in the Appendix), and lim denotes the limit as $T \rightarrow \infty$.

Figures 3-4 graph the approximate slopes of the tests for values of the true AR parameter between 0 and 0.9. It should be noted that the approximate slopes of LMA1, LMA2, LMM1, E2 and DOP2 are the same so that no single testing method or added variable is preferred. When they differ, the tests that use information from the alternative generally have higher approximate slopes than the corresponding tests based on the LM principle. In this case, the DOP1 test would appear to be the best of the tests in the comparison.

5. CONCLUSION

This paper has used Bahadur's asymptotic relative efficiency (ARE) to compare several simple procedures for the purpose of testing AR versus MA error processes in regression models. For the case of comparing a regression model with AR(1) errors as the null and MA(1) errors as the alternative, it was found on the basis of Bahadur's ARE criterion that the preferred test uses information from the alternative model through the prediction errors or the difference in prediction errors of the two models. When the roles of the null and alternative hypotheses are reversed, so that the MA(1) model is the null and the AR(1) model is the alternative, the preferred test on the basis of Bahadur's ARE criterion uses information from the alternative model through the difference in the prediction errors.

APPENDIX

Let A and B be T by T band matrices with elements $a_{i,i} = \alpha_1$, $a_{i+1,i} = \alpha_2, \ldots, a_{i+k-1,i} = \alpha_k$ and $a_{i,j} = 0$ otherwise, and $b_{i,i} = \beta_1$, $b_{i+1,i} = \beta_2, \ldots, b_{i+k-1,i} = \beta_k$ and $b_{i,j} = 0$ otherwise, respectively. Denote the ith columns of A and B as $\underset{\sim}{a}_i$ and $\underset{\sim}{b}_i$, respectively. In addition, the vectors $\underset{\sim}{a}_i^j$ and $\underset{\sim}{b}_i^j$ are j by 1 vectors with elements identical to the first j elements of $\underset{\sim}{a}_i$ and $\underset{\sim}{b}_i$, respectively. Other vectors with a superscript are defined in a similar manner.

Lemma 1: (a) If k=T and $\lim \underset{\sim}{a}_1{}'\underset{\sim}{b}_1$ exists, then $\lim \mathrm{tr}(A'B)/T = \lim \underset{\sim}{a}_1{}'\underset{\sim}{b}_1$.

(b) If k is fixed as $T \to \infty$, $\lim \mathrm{tr}(A'B)/T = \lim \underset{\sim}{a}_1^k{}'\underset{\sim}{b}_1^k$.

Proof: (a) If k=T, using the definitions of A and B, tr(A'B) is given by

$$\mathrm{tr}(A'B) = \sum_{j=1}^{T} (T+1-j)\,\alpha_j \beta_j.$$

Taking limits as $T \to \infty$ gives

$$\lim \mathrm{tr}(A'B)/T = \lim \sum_{j=1}^{T} [1-(j-1)/T]\,\alpha_j \beta_j.$$

The result follows from the properties of Cesaro sums (see Hatanaka [51]).
(b) Given (a), the case of fixed k is obvious.

QED

For the purpose of computing the Bahadur asymptotic relative efficiencies associated with tests of the AR(1) null under a fixed MA(1) alternative, and the MA(1) null under a fixed AR(1) alternative, it is useful to rewrite the models given in equations (1)-(3) in matrix form for p=q=1. The regression model with AR(1) errors is rewritten as

$$y = X\beta + U, \quad R(\rho)U = e, \; E(e) = 0, \quad V(e) = \sigma^2 I_T,$$

where y, U and e are T by 1 vectors, X is a T by k matrix of non-stochastic variables, β is a k by 1 vector of unknown parameters, and $R(\rho)$ is a T by T band matrix with $r_{i,i} = 1$, $r_{i,i-1} = -\rho$ and $r_{i,j} = 0$ otherwise. Denote the columns of $R(\rho)$ by $\underset{\sim}{r}_1(\rho)$, $\underset{\sim}{r}_2(\rho)$,…, $\underset{\sim}{r}_T(\rho)$. The regression model with MA(1) errors is rewritten as

$$y = X\beta + U, \quad U = N(\gamma)e, \; E(e) = 0, \quad V(e) = \sigma^2 I_T,$$

where $N(\gamma)$ is a TxT band matrix with $a_{i,i} = 1$, $a_{i,i-1} = -\gamma$ and $a_{i,j} = 0$ otherwise. Denote the columns of $N(\gamma)$ by $\underset{\sim}{a}_1(\gamma)$, $\underset{\sim}{a}_2(\gamma)$,…, $\underset{\sim}{a}_T(\gamma)$.

For the analysis that follows, define the T by 1 vectors U_j, $\hat{e}$, $\hat{e}_j$, $\hat{U}$, $\hat{U}_j$, $\tilde{U}$, $\tilde{U}_j$, $\tilde{e}$, $\tilde{e}_j$, $\tilde{e}^+$, $\tilde{e}_j^+$, $\tilde{e}_j^{++}$ with typical elements u_{i-j}, $\hat{\varepsilon}_i$, $\hat{\varepsilon}_{i-j}$, $\hat{u}_i$, $\hat{u}_{i-j}$, $\tilde{u}_i$, $\tilde{u}_{i-j}$, $\tilde{\varepsilon}_i$, $\tilde{\varepsilon}_{i-j}$, $\tilde{\varepsilon}_i^+$, $\tilde{\varepsilon}_{i-j}^+$ and $\tilde{\varepsilon}_{i-j}^{++}$ (j>0). Now $\hat{e} = R(\hat{\rho})\hat{U}$ where $\hat{U} = y - X\hat{\beta}$, so that

$$\hat{e} = R(\hat{\rho})(y - X\hat{\beta}) = R(\hat{\rho})[U + X(\beta - \hat{\beta})] \tag{A1}$$

and $\hat{U}_i$ can be written as

$$\hat{U}_i = y_i - X_i\hat{\beta} = U_i + X_i(\beta - \hat{\beta}) \tag{A2}$$

where $y_i = L^i y$, $X_i = L^i X$ and L is the lag operator. Similarly, $\tilde{U} = N(\tilde{\gamma})\tilde{e}$ where $\tilde{U} = y - X\tilde{\beta}$, so that

$$\tilde{e} = N(\tilde{\gamma})^{-1}(y - X\tilde{\beta}) = N(\tilde{\gamma})^{-1}[U + X(\beta - \tilde{\beta})] \quad \text{(A3)}$$

and $\tilde{e}_i^+$ can be written as

$$\tilde{e}_i^+ = N(\tilde{\gamma})^{-1}\tilde{e}_i = N(\tilde{\gamma})^{-2}[U_i + X_i(\beta - \tilde{\beta})]. \quad \text{(A4)}$$

A. Tests of the AR(1) Null Under a Fixed MA(1) Alternative

It is useful to define the matrix $S(\gamma, \rho) = R(\rho)N(\gamma)$, which is a T by T band matrix with elements $s_{i,i} = 1$, $s_{i,i-1} = -(\rho + \gamma)$, $s_{i,i-2} = \rho\gamma$ and $s_{i,j} = 0$ otherwise. Denote the columns of S by $\underset{\sim}{s_1}(\gamma, \rho)$, $\underset{\sim}{s_2}(\gamma, \rho), \ldots,$ $\underset{\sim}{s_T}(\gamma, \rho)$. If the columns of the T by T identity matrix are defined as $\underset{\sim}{f_1}$, $\underset{\sim}{f_2}$, $\underset{\sim}{f_3}, \ldots,$ $\underset{\sim}{f_T}$, then define the T by T matrix L_l with the first (T-1) columns being $\underset{\sim}{f_2}$, $\underset{\sim}{f_3}, \ldots,$ $\underset{\sim}{f_T}$ and the last column being a column of zeros. It should be noted that $\underset{\sim}{s_i}(\gamma, \rho) = \underset{\sim}{r_i}(\rho) - \gamma \underset{\sim}{r_{i+1}}(\rho) = \underset{\sim}{a_i}(\gamma) - \rho \underset{\sim}{a_{i+1}}(\gamma)$ and $\underset{\sim}{r_i}(\rho) = \underset{\sim}{f_i} - \rho \underset{\sim}{f_{i+1}}$. Define the T by T matrix $N_1(\gamma)$ with columns $\underset{\sim}{a_2}$, $\underset{\sim}{a_3}, \ldots,$ $\underset{\sim}{a_T}$, with the last being a column of zeros.

Using (9), for the tests of the AR(1) model, ESS_R is given by

$$ESS_R = \hat{e}'\hat{e} - \hat{e}'\hat{U}_l(\hat{U}_l'\hat{U}_l)^{-1}\hat{U}_l'\hat{e}.$$

As indicated in section 4.1, $\rho^* = \mathrm{plim}_l\, \hat{\rho} = -\gamma(1 + \gamma^2)$, where γ is the true value of the MA(1) parameter. Let $E_1 Z$ denote expectation of Z evaluated when H_1, the regression model with MA(1) errors, is true.

Proposition 1: $\mathrm{plim}_l\, T^{-1}\hat{e}'\hat{e} = \sigma^2 \underset{\sim}{s_1^3}(\gamma, \rho^*)' \underset{\sim}{s_1^3}(\gamma, \rho^*)$.

Proof: Given the non-stochastic nature of x_t, and using (A1) and $\mathrm{plim}(\beta - \hat{\beta}) = 0$ under both H_0 and H_1, gives

$$\text{p}\lim T^{-1}\hat{e}'\hat{e} = \text{p}\lim T^{-1}[U'R'(\hat{\rho})R(\hat{\rho})U].$$

Evaluation under H_1 gives

$$\begin{aligned}
\text{p}\lim_1 T^{-1}\hat{e}'\hat{e} &= \text{p}\lim_1 T^{-1}[e'N'(\gamma)R'(\rho^*)R(\rho^*)N(\gamma)e] \\
&= \text{p}\lim_1 T^{-1}[e'S'(\gamma,\rho^*)S(\gamma,\rho^*)e] \\
&= \sigma^2 \lim T^{-1}\,\text{tr}[S'(\gamma,\rho^*)S(\gamma,\rho^*)] \\
&= \sigma^2 \underset{\sim}{s}{}_1^3(\gamma,\rho^*)'\underset{\sim}{s}{}_1^3(\gamma,\rho^*)
\end{aligned}$$

where Lemma 1 is used to obtain the last result.

QED

Proposition 2: $\text{p}\lim_1 T^{-1}\hat{e}'\hat{U}_1 = 0.$

Proof: Using (A1) and (A2), and by similar reasoning to Proposition 1, gives

$$\text{p}\lim T^{-1}\hat{e}'\hat{U}_1 = \text{p}\lim T^{-1}[U'R'(\hat{\rho})\,U_1].$$

Evaluation under H_1 yields

$$\begin{aligned}
\text{p}\lim_1 T^{-1}\hat{e}'\hat{U}_1 &= \text{p}\lim_1 T^{-1}[e'N'(\gamma)R'(\rho^*)\,L_1\,N(\gamma)e] \\
&= \text{p}\lim_1 T^{-1}[e'S'(\gamma,\rho^*)\,N_1(\gamma)e] \\
&= \sigma^2 \lim T^{-1}\,\text{tr}[S'(\gamma,\rho^*)\,N_1(\gamma)] \\
&= \sigma^2 \underset{\sim}{s}{}_1^3(\gamma,\rho^*)'\underset{\sim}{a}{}_2^3(\gamma) \\
&= -\sigma^2[(\gamma+\rho^*)+\gamma^2\rho^*].
\end{aligned}$$

Since $\rho^* = -\gamma/(1+\gamma^2)$, $\text{p}\lim_1 T^{-1}\hat{e}'\hat{U}_1 = 0.$

QED

Proposition 3: $\text{p}\lim_1 T^{-1}\hat{U}_1'\hat{U}_1 = \sigma^2 \underset{\sim}{a}{}_2^3(\gamma)'\underset{\sim}{a}{}_2^3(\gamma).$

Proof: Using (A2), and by similar reasoning to Proposition 1, gives

$$\text{p}\lim T^{-1}\hat{U}_1'\hat{U}_1 = \text{p}\lim T^{-1}U_1'U_1.$$

Evaluation under H_1 gives

$$\text{plim}_1 T^{-1} \hat{U}_1' \hat{U}_1 = \text{plim}_1 T^{-1} [e' N'(\gamma) L_1' L_1 N(\gamma) e]$$
$$= \text{plim}_1 T^{-1} [e' N_1'(\gamma) N_1(\gamma) e]$$
$$= \sigma^2 \lim T^{-1} \text{tr}[N_1'(\gamma) N_1(\gamma)]$$
$$= \sigma^2 \underset{\sim}{a}_2^3(\gamma)' \underset{\sim}{a}_2^3(\gamma).$$

QED

Combining the results in Propositions 1-3 implies $\text{plim}_1 ESS_R / T = \sigma^2 \underset{\sim}{s}_1^3(\gamma, \rho^*)' \underset{\sim}{s}_1^3(\gamma, \rho^*)$.

The other terms that appear in the various tests can be calculated in the same way using the following results for i, j > 0:

$$\text{plim}_1 \hat{e}' \hat{U}_j / T = \sigma^2 \underset{\sim}{s}_1^k(\gamma, \rho^*)' \underset{\sim}{a}_{j+1}^k(\gamma), \quad \text{where } k=j+2$$

$$\text{plim}_1 \hat{U}_i' \hat{U}_j / T = \sigma^2 \underset{\sim}{a}_{i+1}^k(\gamma)' \underset{\sim}{a}_{j+1}^k(\gamma), \quad \text{where } k=2+\max(i,j)$$

$$\text{plim}_1 \hat{e}' \tilde{e}_j / T = \sigma^2 \underset{\sim}{s}_1^k(\gamma, \rho^*)' \underset{\sim}{f}_{j+1}^k, \text{ where } k=1+\max(2,j)$$

$$\text{plim}_1 \hat{U}_i' \tilde{e}_j / T = \sigma^2 \underset{\sim}{a}_{i+1}^k(\gamma)' \underset{\sim}{f}_{j+1}^k, \quad \text{where } k=1+\max(i+1,j)$$

$$\text{plim}_1 \hat{e}_i' \hat{e}_j / T = \sigma^2 \underset{\sim}{s}_{i+1}^k(\gamma, \rho^*)' \underset{\sim}{s}_{j+1}^k(\gamma, \rho^*), \quad \text{where } k=3+\max(i,j)$$

$$\text{plim}_1 \hat{U}_i' \hat{e}_j / T = \sigma^2 \underset{\sim}{a}_{i+1}^k(\gamma)' \underset{\sim}{s}_{j+1}^k(\gamma, \rho^*), \quad \text{where } k=2+\max(i,j+1)$$

$$\text{plim}_1 \hat{e} \hat{e}_i / T = \sigma^2 \underset{\sim}{s}_1^k(\gamma, \rho^*)' \underset{\sim}{s}_{i+1}^k(\gamma, \rho^*), \quad \text{where } k=i+3.$$

B. Tests of the MA(1) Null Under a Fixed AR(1) Alternative

It is useful to define the matrix $Q(\gamma, \rho) = N(\gamma)^{-1} R(\rho)^{-1} = S(\gamma, \rho)^{-1}$, which is a T by T band matrix with elements $q_{i,i} = 1$, $q_{i,i-1} = \rho + \gamma$, $q_{i,i-j} = (\rho + \gamma) q_{i,i-j+1} - \rho\gamma q_{i,i-j+2}$ (j>1) and

$q_{i,j} = 0$ otherwise, and the matrix $NQ(\gamma,\rho) = N(\gamma)^{-1}Q(\gamma,\rho)$, which is also a T by T band matrix with elements $nq_{i,i} = 1$, $nq_{i,i-1} = \rho + 2\gamma$, $nq_{i,i-2} = (\rho + 2\gamma)\, nq_{i,i-1} - (2\rho\gamma + \gamma^2)$, $nq_{i,i-j} = (\rho + 2\gamma)\, nq_{i,i-j+1} - (2\rho\gamma + \gamma^2)\, nq_{i,i-j+2} + \rho\gamma^2 nq_{i,i-j+3}$ (j>2) and $nq_{i,j} = 0$ otherwise. Denote the columns of $Q(\gamma,\rho)$ by $\underset{\sim}{q}_1(\gamma,\rho), \underset{\sim}{q}_2(\gamma,\rho), .., \underset{\sim}{q}_T(\gamma,\rho)$ and the columns of $NQ(\gamma,\rho)$ by $\underset{\sim}{nq}_1(\gamma,\rho), \underset{\sim}{nq}_2(\gamma,\rho), .., \underset{\sim}{nq}_T(\gamma,\rho)$. Define a T by T matrix $NQ_1(\gamma,\rho)$ with columns $\underset{\sim}{nq}_2(\gamma,\rho), \underset{\sim}{nq}_3(\gamma,\rho), .., \underset{\sim}{nq}_T(\gamma,\rho)$, and the last is a column of zeros. Denote the columns of $R^{-1}(\rho)$ by $\underset{\sim}{rr}_1(\rho), \underset{\sim}{rr}_2(\rho), .., \underset{\sim}{rr}_T(\rho)$. $R^{-1}(\rho)$ is a T by T band matrix with typical element $rr_{i,i-j} = \rho^j$ (j=0,1,2,3,..) and $rr_{i,j} = 0$ otherwise. Define the T by T matrix $NNQ(\gamma,\rho) = N(\gamma)^{-2}Q(\gamma,\rho)$ which is a band matrix with typical elements $nnq_{i,i} = 1$, $nnq_{i,i-1} = \rho + 3\gamma$, $nnq_{i,i-2} = (\rho + 3\gamma)\, nnq_{i,i-1} - (3\rho\gamma + 3\gamma^2)$,

$nnq_{i,i-3} = (\rho + 3\gamma)\, nnq_{i,i-2} - (3\rho\gamma + 3\gamma^2)\, nnq_{i,i-1} + (3\rho\gamma^2 + \gamma^3)$,

$nnq_{i,i-j} = (\rho + 3\gamma)\, nnq_{i,i-j+1} - (3\rho\gamma + 3\gamma^2)\, nnq_{i,i-j+2} + (3\rho\gamma^2 + \gamma^3)\, nnq_{i,i-j+3} - \rho\gamma^3\, nnq_{i,i-j+4}$ (j>3)

and $nnq_{i,j} = 0$ otherwise. Denote the columns of $NNQ(\gamma,\rho)$ by $\underset{\sim}{nnq}_1(\gamma,\rho), \underset{\sim}{nnq}_2(\gamma,\rho), .., \underset{\sim}{nnq}_T(\gamma,\rho)$.

Using (12) for the tests of the MA(1) model, ESS_R is given by

$$ESS_R = \tilde{e}'\tilde{e} - \tilde{e}'\tilde{e}_1^+(\tilde{e}_1^{+\prime}\tilde{e}_1^+)^{-1}\tilde{e}_1^{+\prime}\tilde{e}.$$

As indicated in section 4.2, $\gamma^* = p\lim_0 \tilde{\gamma}$ and γ^* is given by the solution to (13) that lies in the interval (-1, 1). Let $E_0 Z$ denote the expectation of Z evaluated when H_0, the regression model with AR(1) errors, is true.

Proposition 4: $p\lim_0 T^{-1}\tilde{e}'\tilde{e} = \sigma^2 \lim \underset{\sim}{q_1}(\gamma^*, \rho)' \underset{\sim}{q_1}(\gamma^*, \rho)$.

Proof: Given the non-stochastic nature of x_t, and using (A3) and $p\lim(\beta - \tilde{\beta}) = 0$ under both H_0 and H_1, gives

$$p\lim T^{-1}\tilde{e}'\tilde{e} = p\lim T^{-1}[U'N'(\tilde{\gamma})^{-1}N(\tilde{\gamma})^{-1}U].$$

Evaluation under H_0 gives

$$p\lim_0 T^{-1}\tilde{e}'\tilde{e} = p\lim_0 T^{-1}[e'R'(\rho)^{-1}N'(\tilde{\gamma})^{-1}N(\tilde{\gamma})^{-1}R(\rho)^{-1}e]$$

$$= p\lim_0 T^{-1}[e'Q'(\gamma^*, \rho)Q(\gamma^*, \rho)e]$$

$$= \sigma^2 \lim \underset{\sim}{q_1}(\gamma^*, \rho)' \underset{\sim}{q_1}(\gamma^*, \rho).$$

QED

Proposition 5: $p\lim_0 \tilde{e}'\tilde{e}_1^+ = \sigma^2 \lim \underset{\sim}{q_1}(\gamma^*, \rho)' \underset{\sim}{nq_2}(\gamma^*, \rho)$.

Proof: Using (A3) and (A4), and by similar reasoning to Proposition 4, gives

$$p\lim T^{-1}\tilde{e}'\tilde{e}_1^+ = p\lim T^{-1}[U_1'N'(\tilde{\gamma})^{-1}N(\tilde{\gamma})^{-2}U_1].$$

Evaluation under H_0 gives

$$p\lim_0 T^{-1}\tilde{e}'\tilde{e}_1^+ = p\lim_0 T^{-1}[e'R'(\rho)^{-1}N'(\gamma^*)^{-1}N'(\gamma^*)^{-2}L_1 R(\rho)^{-1}e]$$

$$= p\lim_0 T^{-1}[e'Q'(\gamma^*, \rho)\, NQ_1(\gamma^*, \rho)e]$$

$$= \sigma^2 \lim \underset{\sim}{q_1}(\gamma^*, \rho)' \underset{\sim}{nq_2}(\gamma^*, \rho).$$

QED

Proposition 6: $p\lim_0 T^{-1}\tilde{e}_1^{+\prime}\tilde{e}_1^+ = \sigma^2 \lim \underset{\sim}{nq_2}(\gamma^*, \rho)' \underset{\sim}{nq_2}(\gamma^*, \rho)$.

Proof: Using (A4), and by similar reasoning to Proposition 4, gives

$$\text{p lim}\, T^{-1} \tilde{e}_1^{+}{}' \tilde{e}_1^{+} = \text{p lim}\, T^{-1}[U_1{}' N'(\tilde{\gamma})^{-2} N(\tilde{\gamma})^{-2} U_1].$$

Evaluation under H_0 gives

$$\text{p lim}_0\, T^{-1} \tilde{e}_1^{+}{}' \tilde{e}_1^{+} = \text{p lim}_0\, T^{-1}[e' R'(\rho) L_1{}' N'(\tilde{\gamma})^{-2} N(\tilde{\gamma})^{-2} L_1 R'(\rho) e]$$

$$= \text{p lim}_0\, T^{-1}[e' N Q_1{}'(\gamma^*, \rho) N Q_1(\gamma^*, \rho) e]$$

$$= \sigma^2 \lim \underset{\sim}{nq}_2(\gamma^*, \rho)' \underset{\sim}{nq}_2(\gamma^*, \rho).$$

QED

The other terms that appear in the various tests can be calculated in the same way using the following results for i,j>0:

$$\text{p lim}_0\, \tilde{e}_i^{+}{}' \hat{U}_j / T = \sigma^2 \lim \underset{\sim}{nq}_{i+1}(\gamma^*, \rho)' \underset{\sim}{rr}_{j+1}(\rho)$$

$$\text{p lim}_0\, \tilde{e}_i^{+}{}' \tilde{e}_j^{+} / T = \sigma^2 \lim \underset{\sim}{nq}_{i+1}(\gamma^*, \rho)' \underset{\sim}{nq}_{j+1}(\gamma^*, \rho)$$

$$\text{p lim}_0\, \hat{U}_i{}' \hat{U}_j / T = \sigma^2 \lim \underset{\sim}{rr}_{i+1}(\rho)' \underset{\sim}{rr}_{j+1}(\rho)$$

$$\text{p lim}_0\, \hat{U}_i{}' \tilde{e}_j / T = \sigma^2 \lim \underset{\sim}{rr}_{i+1}(\rho)' \underset{\sim}{q}_{j+1}(\gamma^*, \rho)$$

$$\text{p lim}_0\, \tilde{e}_i^{+}{}' \tilde{e}_j^{++} / T = \sigma^2 \lim \underset{\sim}{nq}_{i+1}(\gamma^*, \rho)' \underset{\sim}{nnq}_{j+1}(\gamma^*, \rho)$$

$$\text{p lim}_0\, \tilde{e}_i^{++}{}' \tilde{e}_j^{++} / T = \sigma^2 \lim \underset{\sim}{nnq}_{i+1}(\gamma^*, \rho)' \underset{\sim}{nnq}_{j+1}(\gamma^*, \rho)$$

$$\text{p lim}_0\, \tilde{e}' \tilde{e}_j / T = \sigma^2 \lim \underset{\sim}{q}_1(\gamma^*, \rho)' \underset{\sim}{q}_{j+1}(\gamma^*, \rho).$$

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Table 1: Added Variables for Testing the AR(p) Null Against Three Alternatives

Alternative Hypothesis	Added Variables	Degrees of Freedom	Test
MA(q)	$\tilde{\varepsilon}_{t-1},\ldots,\tilde{\varepsilon}_{t-q}$	q	E1
MA(q)	$\tilde{\varepsilon}^*_{t-1},\ldots,\tilde{\varepsilon}^*_{t-q}$	q	E2
MA(q)	$\hat{\varepsilon}_t - \tilde{\varepsilon}_t$	1	DOP1
MA(q)	$\hat{\varepsilon}^*_t - \tilde{\varepsilon}^*_t$	1	DOP2
AR(p+r)	$\hat{u}_{t-p-1},..,\hat{u}_{t-p-r}$	r	LMA1
AR(p+r)	$\hat{u}^*_{t-p-1},..,\hat{u}^*_{t-p-r}$	r	LMA2
ARMA(p,r)	$\hat{\varepsilon}_{t-1},..,\hat{\varepsilon}_{t-r}$	r	LMM1
ARMA(p,r)	$\hat{\varepsilon}^*_{t-1},..,\hat{\varepsilon}^*_{t-r}$	r	LMM2

Notes:

1. An asterisk is used to denote the transformation of the relevant variable by $\hat{\rho}(L)$; for example, $\tilde{\varepsilon}^*_t = \hat{\rho}(L)\,\tilde{\varepsilon}_t$.

2. LMA1=LMM1 (see Godfrey [5]).

3. E denotes the prediction error test, and DOP denotes the difference of prediction errors test.

Table 2: Added Variables for Testing the MA(q) Null Against Three Alternatives

Alternative Hypothesis	Added Variables	Degrees of Freedom	Test
AR(p)	$\hat{u}_{t-1},\dots,\hat{u}_{t-p}$	p	E1
AR(p)	$\hat{u}^{+}_{t-1},\dots,\hat{u}^{+}_{t-p}$	p	E2
AR(p)	$\hat{\varepsilon}_t - \tilde{\varepsilon}_t$	1	DOP1
AR(p)	$\hat{\varepsilon}^{+}_t - \tilde{\varepsilon}^{+}_t$	1	DOP2
MA(q+r)	$\tilde{\varepsilon}^{+}_{t-q-1},..,\tilde{\varepsilon}^{+}_{t-q-r}$	r	LMM1
MA(q+r)	$\tilde{\varepsilon}^{++}_{t-q-1},..,\tilde{\varepsilon}^{++}_{t-q-r}$	r	LMM2
ARMA(r,q)	$\tilde{u}^{+}_{t-1},..,\tilde{u}^{+}_{t-r}$	r	LMA1
ARMA(r,q)	$\tilde{u}^{++}_{t-q-1},..,\tilde{u}^{++}_{t-r}$	r-q [r>q]	LMA2

Notes:

1. A plus sign is used to denote the transformation of the relevant variable by $\tilde{\gamma}(L)^{-1}$; for example, $\hat{u}^{+}_t = \tilde{\gamma}(L)^{-1}\hat{u}_t$.

2. LMM1=LMA1 (see Godfrey [5]).

3. E denotes the prediction error test, and DOP denotes the difference of prediction errors test.

4. Following the convention for the other tests, the added variables for LMA2 should be $\tilde{u}^{++}_{t-1},..,\tilde{u}^{++}_{t-r}$. Since $\tilde{\varepsilon}^{+}_t = \tilde{u}^{++}_t$, it follows that $\tilde{u}^{++}_{t-1},..,\tilde{u}^{++}_{t-q}$ are perfectly correlated with the existing regressors in (7) and so are excluded from the above Table.

Table 3: Auxiliary Regressions to Calculate Bahadur Approximate Slopes for Tests of the AR(1) Null Evaluated Under the MA(1) Alternative

Test	Unrestricted Regression
E1	s_1 on a_2 and f_2
E2	s_1 on a_2 and r_2
DOP1	s_1 on a_2 and $-(\gamma f_2 + \rho^* a_2)$
DOP2	s_1 on a_2 and $-(\gamma r_2 + \rho^*(a_2 - \rho^* a_3))$
LMA1	s_1 on a_2 and a_3
LMA2	s_1 on a_2 and $a_3 - \rho^* a_4$
LMM1	s_1 on a_2 and s_2
LMM2	s_1 on a_2 and $s_2 - \rho^* s_3$

where $s_1 = \underset{\sim}{s}_1^5(\gamma, \rho^*) = (1,-(\gamma+\rho^*), \gamma\rho^*,0,0)'$, $s_2 = \underset{\sim}{s}_2^5(\gamma, \rho^*) = (0,1,-(\gamma+\rho^*), \gamma\rho^*,0)'$, $s_3 = \underset{\sim}{s}_3^5(\gamma, \rho^*) = (0,0,1,-(\gamma+\rho^*), \gamma\rho^*)'$, $a_2 = \underset{\sim}{a}_2^5(\gamma) = (0,1,-\gamma,0,0)'$, $a_3 = \underset{\sim}{a}_3^5(\gamma) = (0,0,1,-\gamma,0)'$, $a_4 = \underset{\sim}{a}_4^5(\gamma) = (0,0,0,1,-\gamma)'$, $f_2 = \underset{\sim}{f}_2^5 = (0,1,0,0,0)'$, and $r_2 = \underset{\sim}{r}_2^5(\rho^*) = (0,1,-\rho^*,0,0)'$. The vectors $\underset{\sim}{s}_i^5(\gamma, \rho^*)$ [i=1,2,3], $\underset{\sim}{a}_j^5(\gamma)$ [j=2,3,4], $\underset{\sim}{f}_2^5$ and $\underset{\sim}{r}_2^5(\rho^*)$ are defined in the Appendix.

Table 4: Auxiliary Regressions to Calculate Bahadur Approximate Slopes for Tests of the MA(1) Null Evaluated Under the AR(1) Alternative

Test	Unrestricted Regression
E1	$\underset{\sim}{q}_1(\gamma^*, \rho)$ on $\underset{\sim}{nq}_2(\gamma^*, \rho)$ and $\underset{\sim}{rr}_2(\rho)$
E2	$\underset{\sim}{q}_1(\gamma^*, \rho)$ on $\underset{\sim}{nq}_2(\gamma^*, \rho)$ and $\underset{\sim}{q}_2(\gamma^*, \rho)$
DOP1	$\underset{\sim}{q}_1(\gamma^*, \rho)$ on $\underset{\sim}{nq}_2(\gamma^*, \rho)$ and $-(\gamma^* \underset{\sim}{q}_2(\gamma^*, \rho) + \rho \underset{\sim}{rr}_2(\rho))$
DOP2	$\underset{\sim}{q}_1(\gamma^*, \rho)$ on $\underset{\sim}{nq}_2(\gamma^*, \rho)$ and $-(\gamma^* \underset{\sim}{nq}_2(\gamma^*, \rho) + \rho \underset{\sim}{q}_2(\gamma^*, \rho))$
LMM1	$\underset{\sim}{q}_1(\gamma^*, \rho)$ on $\underset{\sim}{nq}_2(\gamma^*, \rho)$ and $\underset{\sim}{nq}_3(\gamma^*, \rho)$
LMM2	$\underset{\sim}{q}_1(\gamma^*, \rho)$ on $\underset{\sim}{nq}_2(\gamma^*, \rho)$ and $\underset{\sim}{nnq}_3(\gamma^*, \rho)$
LMA1	$\underset{\sim}{q}_1(\gamma^*, \rho)$ on $\underset{\sim}{nq}_2(\gamma^*, \rho)$ and $\underset{\sim}{q}_2(\gamma^*, \rho)$
LMA2	Not defined due to perfect collinearity among regressors.

where $\underset{\sim}{q}_i(\gamma^*, \rho)$, $\underset{\sim}{nq}_i(\gamma^*, \rho)$, $\underset{\sim}{nnq}_i(\gamma^*, \rho)$ and $\underset{\sim}{rr}_i(\rho)$ are defined in the Appendix.

FIGURE 1: APPROXIMATE SLOPES (AS) OF METHOD 1 TESTS WITH AR(1) NULL

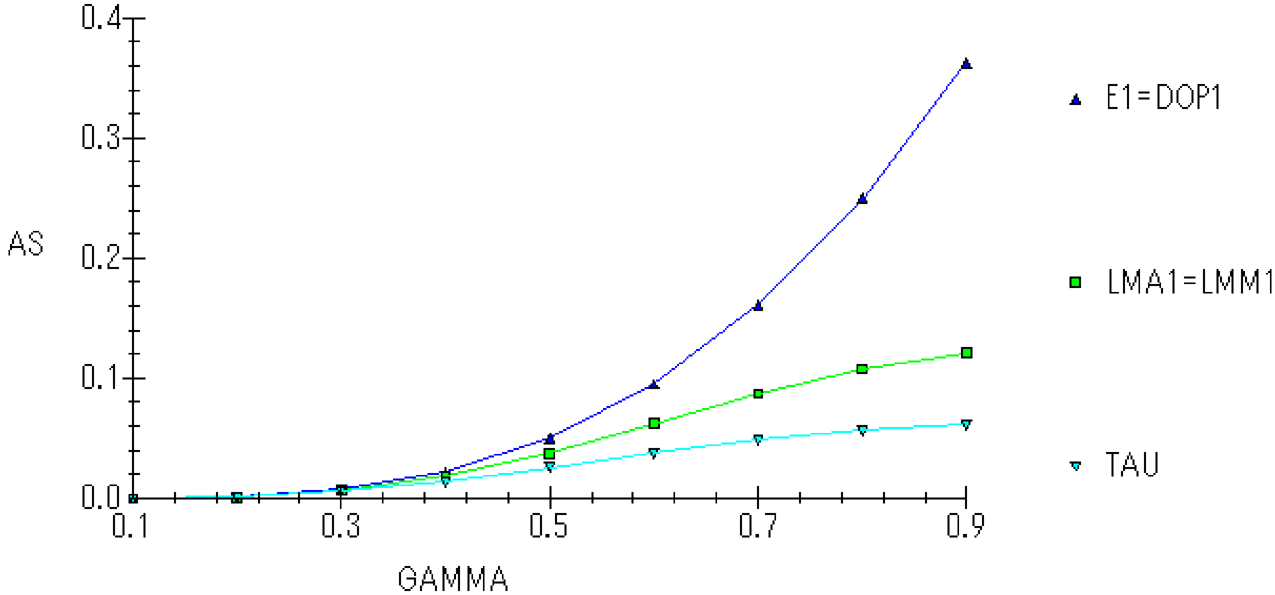

FIGURE 2: APPROXIMATE SLOPES (AS) OF METHOD 2 TESTS WITH AR(1) NULL

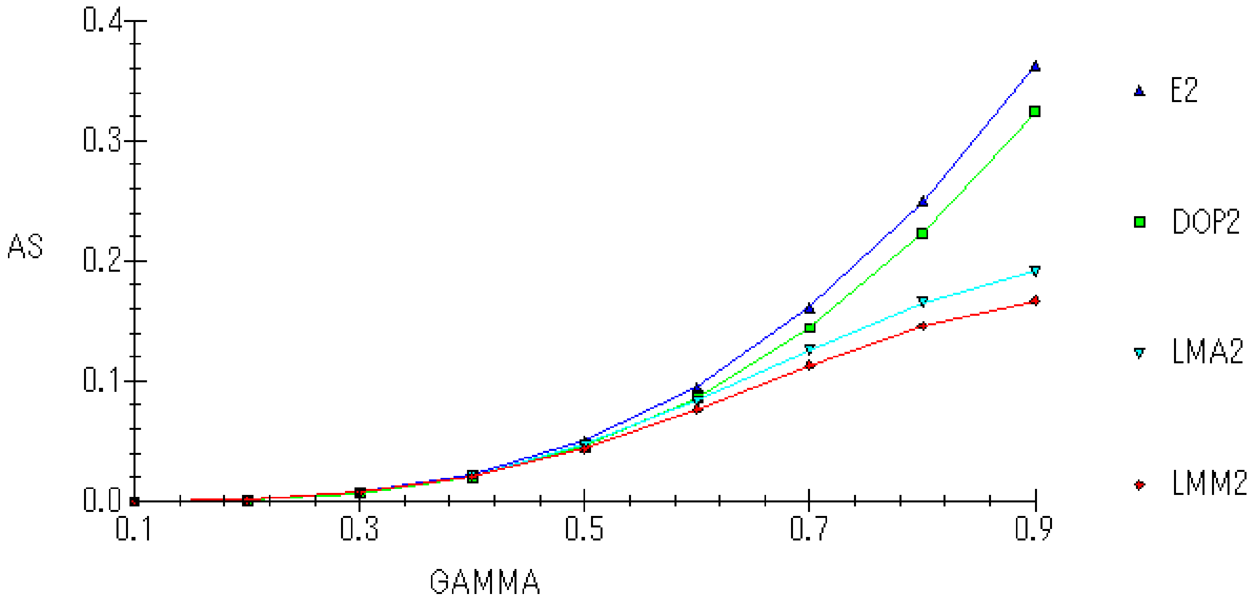

FIGURE 3: APPROXIMATE SLOPES (AS) OF METHOD 1 TESTS WITH MA(1) NULL

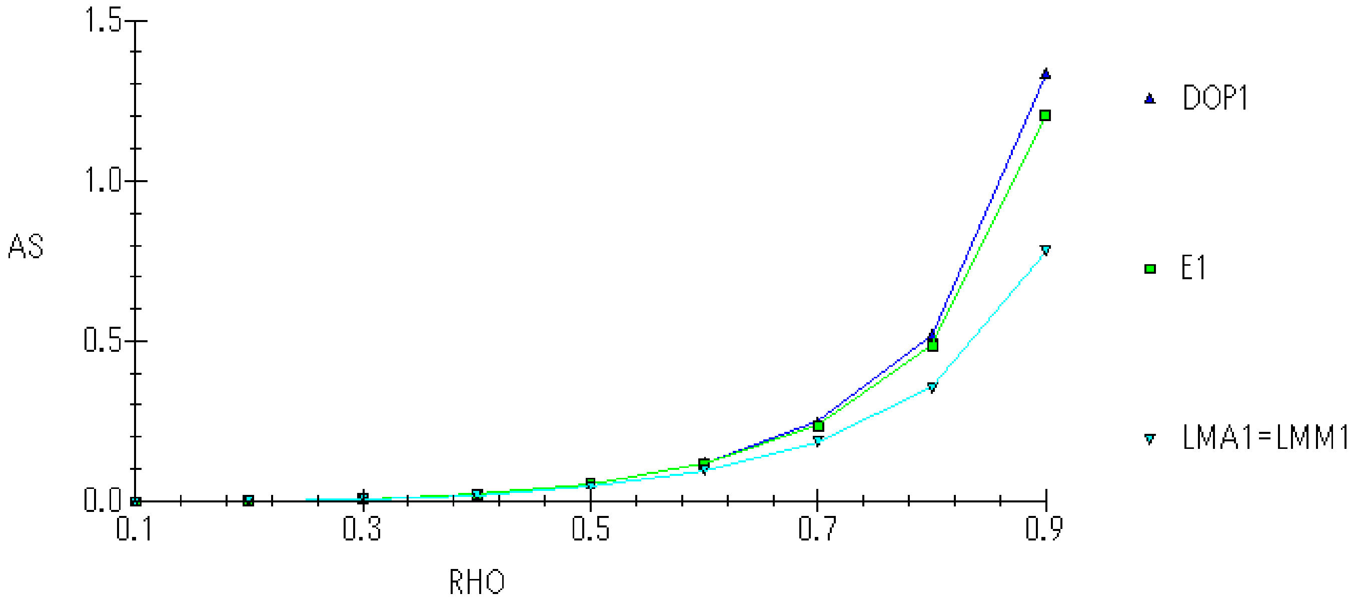

FIGURE 4: APPROXIMATE SLOPES (AS) OF METHOD 2 TESTS WITH MA(1) NULL