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A REINTERPRETATION OF THE KEYNESIAN CONSUMPTION FUNCTION AND MULTIPLIER EFFECT

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Abstract

We propose a microeconomic foundation of the multiplier effect and that of the consumption function using a dynamic optimization model that explains a shortage of aggregate demand and unemployment. We show that government purchases boost aggregate demand through a multiplier-like process but that the implication is quite different. It works through not an increase in disposable income but moderation of deflation, which makes money holding costly and stimulates consumption.

Keywords: Keynesian Consumption Function, Multiplier Effect, Keynesian Cross, Persistent Unemployment, Aggregate Demand

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1 Introduction

The United States, the European Union and Japan have suffered from serious economic depression and unemployment since the recent worldwide financial crisis of 2008. They expand fiscal spending so as to stimulate aggregate demand and reduce unemployment.[1] When doing so, they mostly have in mind the Keynesian multiplier theory. However, the theory is criticized for lack of a microfoundation of the Keynesian consumption function upon which the theory is founded. In particular, the intertemporal budget equation is ignored when the consumption function is assumed. New Keynesian economists use general equilibrium models with imperfect competition and present some microeconomic foundations for the multiplier effect. See Costa and Dixon (2009) for a recent survey on those models. However, they consider neither a shortage of aggregate demand nor involuntary unemployment. Full employment always obtains and an increase in labor supply that households determine by comparing utility of consumption and that of leisure plays a key role in creating the multiplier effect.

This paper reformulates the dynamic optimization model of Ono (1994, 2001) and proposes an alternative Keynesian cross model with demand shortage and involuntary unemployment. We obtain consumption as a function of aggregate demand and obtain the effect of government spending on aggregate demand that looks like the Keynesian multiplier effect. However, this consumption function implies not the Keynesian relationship between con-

[1]Feldstein (2009) states that the recent revival of interest in fiscal stimulus is due to differences between the current stagnation and previous recessions and to ineffectiveness of monetary policy under the current stagnation.

sumption and disposable income but the effect of an increase in aggregate demand on consumption that works through a change in the deflation rate. Since the intertemporal budget equation is taken into account in this model, Ricardian equivalence holds and the multiplier of a tax cut with bond issuance is zero. The multiplier effect works only when fiscal spending creates new employment and the deflationary gap shrinks.

2 The Model

Let us first summarize the model of Ono (1994, 2001). Since it uses the Sidrauski-type money-in-the-utility-function model, the first-order optimal condition is

$$\rho + \eta(c_t)\frac{\dot{c}_t}{c_t} + \pi_t = R_t = \frac{v'(m_t)}{u'(c_t)}, \tag{1}$$

where ρ (> 0) is the subjective discount rate, π_t is the inflation rate, R_t is the nominal interest rate of bonds, $u(c_t)$ is utility of consumption c_t, $v(m_t)$ is utility of real money balances m_t and $\eta(c_t) \equiv -[u''(c_t)c_t]/u'(c_t)$. $u(c_t)$ and $v(m_t)$ satisfy normal properties. The first equality implies the Ramsey equation and the second one shows portfolio choice between bonds and money.

The firm sector is assumed to have linear technology:

$$y_t = \theta n_t, \tag{2}$$

where y_t is output, θ (> 0) the labor productivity and n_t labor input. Given nominal wage W_t and nominal commodity price P_t, the firm sector chooses labor demand n_t^d so as to maximize profits. As far as n_t is finite and positive, profit maximizing behavior under the linear technology leads to

$$\theta = w_t \left(\equiv \frac{W_t}{P_t} \right) \text{ for all } t, \tag{3}$$

where w_t denotes the real wage.

The government finances government purchases g and interest payments $r_t b_t$, where r_t is the real interest rate on government bonds b_t, by imposing lump-sum tax-cum-subsidy τ_t and issuing new bonds $\dot{b}_t$. Thus,

$$g + r_t b_t = \tau_t + \dot{b}_t.$$

It adjusts b_t and τ_t so that the non-Ponzi game condition is satisfied. Nominal money supply M_t is kept constant at $\overline{M}$ and hence real money balances m_t $(= \overline{M}/P_t)$ follow

$$\frac{\dot{m}_t}{m_t} = -\pi_t, \tag{4}$$

where π_t $(\equiv \dot{P}_t/P_t)$ is the inflation rate.

Since we take into account the possibility of unemployment, employment n_t is determined by the short side of labor demand n_t^d and inelastic labor supply $\overline{n}$:

$$n_t = \min\left\{n_t^d, \overline{n}\right\}.$$

Nominal wage W_t is assumed to adjust in a sluggish manner:

$$\frac{\dot{W}_t}{W_t} = \alpha\left(\frac{n_t^d}{\overline{n}} - 1\right),$$

where α (> 0) is exogenous and constant.[2] Meanwhile, commodity price P_t instantaneously adjusts so as to satisfy (3) and realize the commodity market

[2] Although how nominal or real wages adjust is an important issue that many economists have long addressed, we assume a simple adjustment process because our purpose is not to examine why wages are rigid but to analyze how government spending affects GDP under stagnation. Using the idea of the fair wage, Ono and Ishida (2009) give a microfoundation to such an adjustment process, and show that under the microfounded adjustment process an economy also reaches a steady state of which properties are similar to those of the unemployment steady state of the present paper.

equilibrium:

$$c_t + g = y_t. \tag{5}$$

Since $\pi_t = \dot{W}_t/W_t$ from (3) and $y_t = \theta n_t$ from (2), we have

$$\pi_t = \alpha\left(\frac{n_t^d}{\overline{n}} - 1\right) = \alpha\left(\frac{y_t}{\overline{y}} - 1\right), \tag{6}$$

where $\overline{y}$ denotes full-employment output:

$$\overline{y} \equiv \theta\overline{n}.$$

3 The Consumption Function and the Multiplier Effect

This section derives a relationship between aggregate demand y and consumption c that looks like the Keynesian consumption function. Using it we propose an analytical framework similar to the Keynesian cross. A multiplier-like effect of government purchases on aggregate demand arises but the economic implication is quite different.

In the full employment steady state ($n = n^d = \overline{n}$), if it exists, $\dot{c} = 0$ and $y = \overline{y}$. Thus, from (1), (4), (5) and (6) we find

$$\pi = 0, \quad \dot{m} = 0, \quad c + g = \overline{y}, \quad \rho = \frac{v'(m)}{u'(c)}. \tag{7}$$

However, if the marginal utility of money has a positive lower bound β:[3]

$$\lim_{m\to\infty} v'(m) = \beta > 0$$

[3]Ono (1994, chapter 1) mentions the validity of the assumption of insatiable liquidity preference, quoting the statements by Karl Marx and Georg Simmel. Based on recent findings in neuroscience, Ono and Ishida (2009) also discuss the validity. Using both parametric and non-parametric methods, Ono, Ogawa and Yoshida (2004) empirically support the assumption. Murota and Ono (2008) show that the marginal utility of money stays positive in the presence of status preference, and Murota and Ono (2009) demonstrate that it reaches a positive lower bound under zero nominal interest rates if liquidity of deposits is considered.

and β is high enough to satisfy

$$\rho < \frac{\beta}{u'(\overline{y} - g)}, \tag{8}$$

then there is no value of m that satisfies (7) and hence the full employment steady state does not exist.[4] The second equality of (1), $R = v'(m)/u'(c)$, implies that this is the case of the Keynesian liquidity trap, where money demand m is infinite when the nominal interest rate R takes a positive lower bound $\beta/u'(c)$.

Under (8) the marginal benefit of money (the liquidity premium) exceeds that of consumption (the time preference rate ρ) if consumption c takes the full-employment level $\overline{y} - g$. In order for the first order condition (1) to be satisfied, c is set to be lower than $\overline{y} - g$ and then unemployment ($n < \overline{n}$) and deflation ($\pi < 0$) occur. Hence, the economy reaches an unemployment steady state where $\dot{c} = 0$, $y < \overline{y}$, and $\dot{m}/m > 0$, which causes $v'(m) = \beta$. Then, (1) and (6) lead to

$$\rho + \alpha \left(\frac{y}{\overline{y}} - 1 \right) = \frac{\beta}{u'(c)}, \tag{9}$$

which gives the consumption function in the present model:

$$c = u'^{-1} \left(\frac{\beta}{\rho - \alpha + \dfrac{\alpha}{\overline{y}} y} \right) \equiv c(y). \tag{10}$$

From (10) we obtain[5]

$$c'(y) = -\frac{\left[u'(c)\right]^2 \alpha}{u''(c)\beta\overline{y}} > 0. \tag{11}$$

[4]This condition for the non-existence of the full-employment steady state is the same as in Ono (1994, 2001).

[5]If $u(c)$ is a logarithmic function, $c'(y)$ is constant.

As is proven by Ono (2001), the unemployment steady state uniquely exists and satisfies saddle-path stability under the following conditions:

$$\rho > \alpha, \ \ -\frac{\beta u''(c)}{[u'(c)]^2} - \frac{\alpha}{\overline{y}} > 0,$$

and then from (10) and (11) $c(y)$ satisfies

$$c(0) = u'^{-1}\left(\frac{\beta}{\rho - \alpha}\right) > 0, \ \ 1 > c'(y) > 0. \tag{12}$$

These properties look the same as those of the Keynesian consumption function but the implications are quite different. Neither does $c(0)$ mean autonomous consumption nor does $c'(y)$ mean the marginal propensity to consume. It means an effect that works through a change in deflation, and hence consumption c depends on not disposable income $y-\tau$ but aggregate demand y.

Substituting (10) into (5) yields

$$c(y) + g = y. \tag{13}$$

Note that (13) gives a unique value of y satisfying $0 < y < \overline{y}$ because (8) implies $c(\overline{y}) + g < \overline{y}$ and (12) is valid. This leads to seemingly the same multiplier effect as the conventional Keynesian one but the present effect works very differently. An increase in g by the magnitude of $\mathrm{d}g$ initially expands y by $\mathrm{d}g$, which moderates deflation and causes the household to increase c by $c'(y)\mathrm{d}g$. It further expands y by $c'(y)\mathrm{d}g$, which again moderates deflation and increases c and y by $[c'(y)]^2\mathrm{d}g$. This process continues and eventually[6]

$$\frac{\mathrm{d}c}{\mathrm{d}g} = \frac{c'(y)}{1 - c'(y)}, \ \ \frac{\mathrm{d}y}{\mathrm{d}g} = \frac{1}{1 - c'(y)}.$$

[6]Blanchard and Perotti (2002) find that an increase in government purchases leads to an increase in consumption. See Galí et al. (2007) for a similar finding.

Since the intertemporal budget equation is taken into account, Ricardian equivalence holds in the present model.[7] Therefore, there is no difference in the multiplier effect between under a balanced budget and a deficit budget. Government purchases g increase consumption c even under a balanced budget. Lump-sum tax-cum-subsidy τ has any effect on neither consumption nor aggregate demand even under a deficit budget.[8]

If wage adjustment speed α is low, the deflation rate is less sensitive to the output gap and therefore the effect on c of an increase in g is small. Moreover, if $\alpha = 0$ and hence prices and wages are fixed (i.e., $\pi = 0$), from (10) and (13) c is independent of y and is not affected by g. Thus, in the typical Keynesian case with fixed prices and wages, fiscal expansion does not affect consumption. This property holds true whether a liquidity trap occurs or not since from (1) where $\pi = 0$ and P is constant at $\bar{P}$ one finds

$$\rho = \frac{v'(M/\bar{P})}{u'(c)},$$

implying that c is constant. Although the stimulative effect of government purchases g decreases as α declines, a lower α leads to larger consumption and aggregate demand for given g. This property is derived from (10) and

[7] Our multiplier effect is also very different from those discussed by Bénassy (2007a, 2007b) and Galí et al. (2007). They assume non-Ricardian frameworks with price rigidities. Bénassy uses overlapping generations models, and Galí et al. assume rule-of-thumb consumers, who do not take intertemporal decisions.

[8] This may be consistent with the consequence of the tax rebate implemented by the US government in 2008. Feldstein (2009) mentions that consumption in the second quarter of 2008 increased by only 12 billion dollar although 80 billion dollar went back to taxpayers in May and June 2008, and mentions that the estimated marginal propensity to consume from the corresponding rebate variable is only 0.13 whereas that out of real per capita disposable income is 0.70. Similarly, Shapiro and Slemrod (2009) report that only one-fifth of the recipients of the tax rebate planed to mostly spend the rebate and the rest planed to mostly save it or to mostly pay off debt with it.

(13):

$$\frac{\mathrm{d}c}{\mathrm{d}\alpha} = \frac{\mathrm{d}y}{\mathrm{d}\alpha} = \frac{\left[u'(c)\right]^2 (\overline{y} - y)}{u''(c)\beta\overline{y}[1 - c'(y)]} < 0.$$

Note that the present multiplier analysis holds only in the unemployment steady state given by (9). If (8) is not valid and the economy is not in the liquidity trap, the second equality of (1) yields consumption c as a function of m and R:

$$c = \phi(m, R),$$

and the real balance effect works. A decrease in P expands m and hence increases c until full employment is reached and (7) is satisfied.

4 Conclusion

From a dynamic optimization monetary model with a liquidity trap we derive a consumption function and a multiplier effect of fiscal expansion on aggregate demand. In the steady state with demand shortage and deflation consumption is a function of aggregate demand and aggregate demand is determined in a way similar to the Keynesian cross. However, working of the effect is quite different since the consumption function does not represent the relationship between disposable income and consumption, as in the conventional Keynesian model. An increase in government purchases expands consumption by mitigating deflation. This effect arises whether under a deficit budget or a balanced budget since households take into account the intertemporal budget equation.

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