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**THE SLOW DEMOGRAPHIC TRANSITION
IN REGIONS VULNERABLE
TO CLIMATE CHANGE**

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The slow demographic transition in regions vulnerable to climate change*

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Abstract

This paper considers the persistent effects of climate change on the speed of demographic transition, and hence on the size of the population in regions that are the least developed and the most vulnerable to climate change, such as Sub-Saharan Africa. These effects are transmitted through interactions between the education gender gap within families, fertility, and the local environment, through which the demographic transition is delayed. Environmental conditions affect intra-household labor allocation because of the impacts on local resources under the poor infrastructural system. Examples include the collection of essential resources, e.g. clean water and firewood, by women for their families' daily lives. Climate change causes damage to local resources, offsetting (partially) the role of technological progress and infrastructure investment in saving time that women spend on their housework duties. Hence, the gender inequality in education/income is upheld, delaying declines in fertility and creating population momentum. The bigger population, in turn, degrades local resources and the environment through expanded production. The interplay between local resources, gender inequality, and population, under the persistent effect of climate change, may thus generate a slow demographic transition and stagnation of the least developed regions. We provide empirical confirmation for our theoretical predictions using data from 44 African countries in the period from 1960 to 2017.

Keywords: Climate change, local resources, fertility, gender inequality in education, slow demographic transition.

JEL: J11, J16, Q01, Q20, Q54, Q56.

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1 Introduction

Our paper contributes to the literature by incorporating climate and local resource factors into a unified growth framework to study the persistent effects of climate change on fertility and the feedback loop between population growth, availability of local resources, gender inequality in education investment, and technological progress. We provide a new mechanism to explain the persistent stagnation of Sub-Saharan Africa along with the fast population growth and the depletion of its ecological system in the context of climate change. We also characterize the conditions and dynamic interactions between population and local resources through which an economy may fall into an environmental crisis within a finite time. The climate change economics literature for the most part considers the impact of climate change on economies through the adverse effects it has on the total factor productivity at aggregate production.

To gain a more accurate picture of the latter impact, we conduct an empirical investigation of the impact of climate on the basic local resources. Based on Dell et al. (2012), we proxy the climate variable with temperature. Our study uses a rich set of controls, including demographic and economic variables covering 44 African countries in the period from 1960 to 2017. Our results point to a negative link between the temperature and basic local resources. Moreover, we also explore the interplay between temperature, education gap, and fertility in our data. First of all, we find that there is a positive and significant effect exerted by temperature on fertility. Second, we highlight that the temperature has a negative and significant impact on education gap. However, the above results are contrary for other developing countries, which are more likely to see their fertility negatively affected by temperature. Our paper is the first to study the direct effects of climate change on the microstructure of the economy, and its subsequent effects on the macro level and long-run development of the economy.

The demographic transitions are differential across countries and regions. While developed countries have completed their demographic transitions and most developing countries have experienced fast demographic transition throughout the second half of the 20th century, the least developed ones in the regions most vulnerable to climate change, such as those in Sub-Saharan Africa, have been experiencing persistently high fertility rates and slow demographic transitions. Figure 1 illustrates such a distinctive difference in total fertility rates and speeds of decline in fertility between some developing countries and some Sub-Saharan African ones. Indeed, around the 1960s, the fertility rates between developing and least developed countries were almost the same. However, the fertility rates in developing countries in Asia and Latin America fell quickly from above six children per woman in the 1960s to below three children per woman after three decades, while the least developed countries in Sub-Saharan Africa have been exhibiting a slow decline in fertility rates. Their current fertility rates are more than five children per woman, and the figure is even higher in some countries.¹

¹The fertility rate in Niger is even at around seven children per woman, and the fertility dynamics of the country shows this persistent high level.

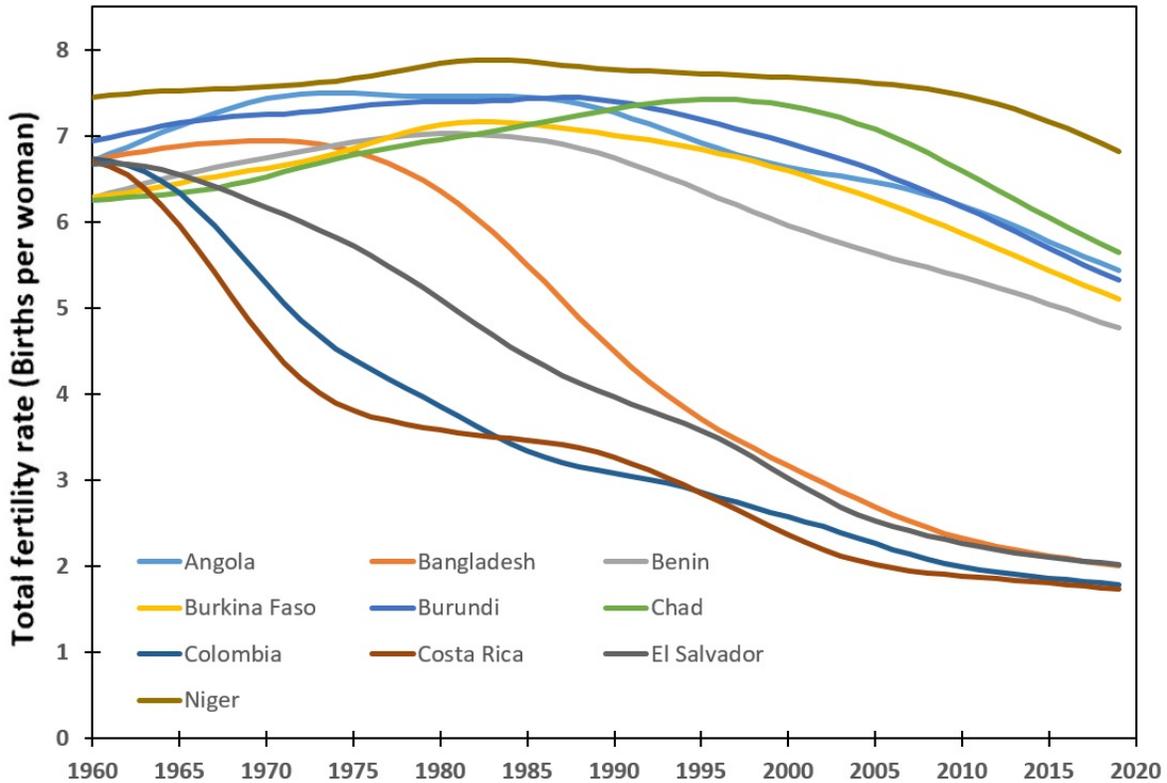


Figure 1: Total Fertility rate (births per woman, 1960–2020) among developing and least developed countries. Source: World Bank (2021).

Along with the difference in the speeds of demographic transition mentioned above, there are considerable differences in housework allocation across genders between Sub-Saharan countries and others. The figure 2 below shows the distribution of households by persons responsible for water collection, in which women in Sub-Saharan countries play a particularly predominant role. This figure also reflects the much poorer infrastructure of clean water systems in Sub-Saharan Africa, particularly in rural areas, compared with other developing regions. Indeed, the water infrastructure in Sub-Saharan Africa does not provide adequate freshwater to its inhabitants (Lutz 2011). As a result, there is an over reliance on freshwater resources such as local rivers and the local groundwater system, from which women need to spend an inordinate amount of time collecting water.

The key stylized facts depicted in Figures 1 and 2 indicate to us the merit of proposing a theoretical mechanism linking the persistent effects of climate change in terms of damaging local resources and the interplay between local resources, gender inequality, fertility, and population to explain the slow demographic transition and the stagnation of Sub-Saharan Africa. Our mechanism starts from the current scientific consensus that climate change has been likely resulting in widespread alterations to hydrologic conditions that very adversely affect the surface and groundwater systems, forest conservation, and biodiversity of Sub-Saharan Africa. These

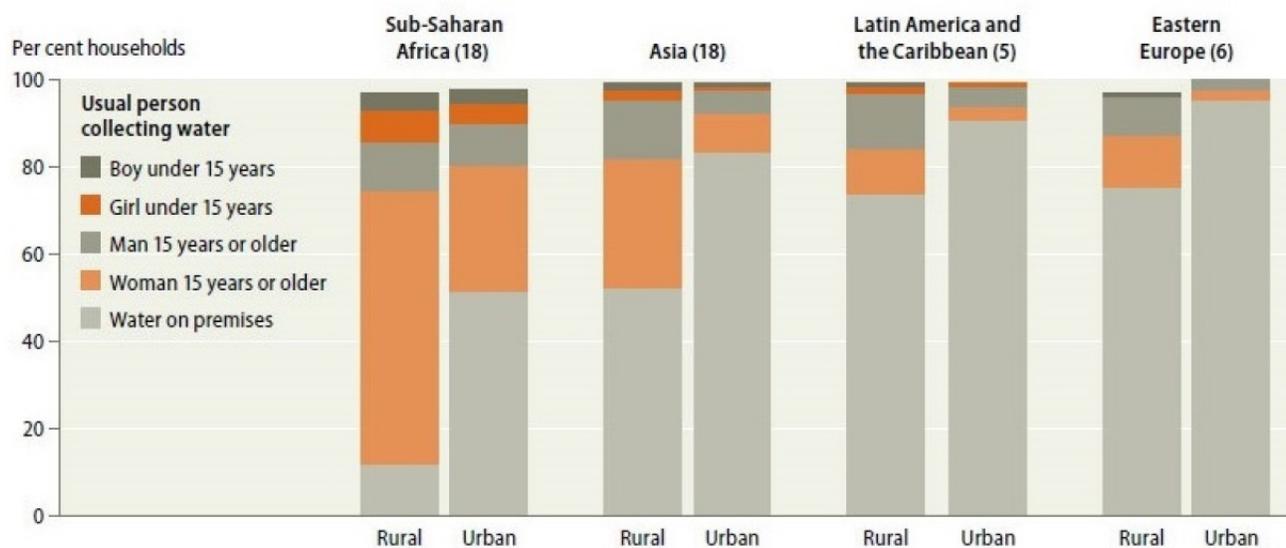


Figure 2: Distribution of households by gender responsible for water collection. Figure borrowed from World's Women, Trends and Statistics, UNDESA (2015)

adverse effects make it necessary for a woman in a household to spend more time collecting local resources, i.e. water from rivers and/or groundwater systems, firewood from forests, etc., for her family's daily life, instead of supplying labor to the formal labor markets. In a family, when the parents anticipate that their daughter(s) will spend more time on such housework, they rationally invest less in education for their daughter(s) and more for their son(s), creating a significant gap in education, and hence a significant gap in income, across genders. The lower income of women implies lower opportunity cost of child-rearing by which the household increases the fertility rate, generating a population momentum in the medium and long run. The bigger population, in turn, contributes to local resource depletion through the expansion of production. This reinforcing feedback loop has generated a slow demographic transition and stagnant economic growth in Sub-Saharan Africa.

Population growth is documented as one of key driving forces of global emissions (IPCC 2014) and is one of the main causes of natural resource depletion and environmental degradation (Ehrlich and Holden 1971, Meadow et al. 1972, Cleaver and Schreiber 1994, Kates 1996, Foster et al. 2002). The observation of differential fertility rates across regions suggests that several related research questions need to be answered. Why have the least developed regions, which are the most vulnerable to climate change and the most at risk of local environmental disasters, experienced persistently high fertility rates and a slow demographic transition? Does climate change have any effect on fertility decisions? If yes, through which mechanism? Answering these questions analytically and understanding the mechanisms in explaining the slow demographic transition, in our point of view, are key to combating global climate change through policies enabling management of population growth in relation to environmental protection in the least

developed regions vulnerable to climate change.

While the effects of population growth on climate change are included in many numerical works (O'Neill et al. 2010, Wheeler and Hammer 2010, Lutz and Striessnig 2015, Scovronick et al. 2017, among others), investigations into the effects of climate change and natural resource depletion on population growth are surprisingly rare in the related economic literature. There is a small empirical body of literature studying the effects of environmental and local resource conditions on fertility at the micro level in developing countries (e.g. Grace et al. 2016, Aggarwal et al. 2001, Filmer and Pritchett 2002, Nankhuni and Findeis 2004, Biddlecom et al. 2005). These studies show that environmental conditions affect households' fertility decisions through the channels of health and the intra-household division of labor. In addition, historians such as Diamond (2005) emphasize how overpopulation and excessive environmental degradation, e.g. due to deforestation, were responsible for the collapse of some past societies such as the Easter Island civilization and the Maya of Central America. While precise explanations for the collapse of the population of Easter Island remain controversial, environmental feedback mechanisms feature a prominent role in many studies (Brander and Taylor 1998, Reuveny and Decker 2000, de la Croix and Dottori 2008, among others). That is to say, fundamentally theoretical research on the interplay between population and environmental change should be developed in the context of climate change in the 21st century in the least developed regions. The present study attempts to develop such an analytical framework for this task.

Modern demographic transition theories focus on non-environmental factors in explaining the demographic transitions that were mostly experienced in developed countries. The first prominent theory, which is linked to the rise in income per capita, was proposed by Becker (1960). Becker argues that the rise in per capita income leads to a decline in fertility because of the associated increase in the opportunity cost of raising children. In a refined model, Becker and Lewis (1973) suggest that when the income elasticity with respect to investments in child quality is high and the income elasticity for child quantity is low, then fertility declines when income increases. The second theory links fertility to the decline in infant and child mortality. Developed by demographers, this theory incorporates the empirical observation that in many countries, a decline in child mortality preceded a decline in fertility (Montgomery and Cohen 1998). O'Hara (1975) argues that the decline in infant and child mortality reduces gross fertility, but not necessarily net fertility (i.e. the demand for surviving children). Moreover, O'Hara points out that demographic transition only occurs if parents substitute child quality for child quantity. The third theory is the old-age security hypothesis (Neher 1971, Caldwell 1976, Boldrin and Jones 2002). This theory suggests that when capital markets or public safety nets are absent, children are treated as an asset that permits parents to transfer income to old age. The development of capital markets reduces the demand for children, which, in turn, leads to demographic transition. The fourth theory, proposed by Galor and Weil (2000) and Galor and Moav (2002), highlights the rise in demand for human capital. They argue that technological change increases the

demand for human capital and returns to educational investments. This, in turn, changes the trade-off between quantity and quality of children, thus generating demographic transition. The fifth theory is related to the decline in the gender gap with respect to income and employment during the development process. The decline in the gender gap is reflected by the rise in women's relative income and the associated increase in women's bargaining power. This theory, developed by Galor and Weil (1996), Doepke and Tertilt (2009), Diebolt and Perrin (2013), Prettnner and Strulik (2017), Bloom et al. (2020), and recently Dao et al. (2021), proposes that the rise in women's relative income increases the opportunity cost of child-rearing and, thereby reduces fertility.

While the modern demographic transition theories neglect environmental and resource factors, the natural environment and resources play a dominant role in early works on population dynamics. Particularly, in the influential treatise "*An Essay on the Principle of Population*", Thomas R. Malthus (1798) argued that as population growth is limited by natural resources, the population will eventually converge to a stable level. Despite Malthus's warning, the past two centuries experienced high rates of population growth along with climate change and environmental degradation. The fertility rates in the regions most vulnerable to climate change have been persistently high in spite of the climate-related damage on production and local natural resources. This stylized fact suggests the existence of an alternative pattern of demographic transition for these least developed regions in which climate change may play a crucial role. Unified growth theory (Galor and Weil 1996, Galor and Weil 2000, Galor and Moav 2002, and Galor 2011) generates a mechanism whereby economies escape Malthusian stagnation to enter the modern sustained growth regime along with the demographic transition. Differing from ours, this theory, however, so far does not aim at explaining the current stagnation in Africa in relation to climate change.

The rest of this paper is organized as follows. The next section introduces the model and derives the competitive equilibrium dynamics. Section 3 defines equilibria and the dynamic system of the economy. Section 4 analyzes the evolution of population and local resources, and the convergence to conditional steady states. Section 5 analyzes the impact of extreme climate change and the mechanisms that lead to environmental disasters. The global evolution of the economy is discussed in section 6. Section 7 tests empirically the hypotheses developed from the theoretical framework. Section 8 concludes the paper and discusses further works for this research agenda.

2 The model

We extend the model in Dao et al. (2021) by incorporating the role of local resources on women's time for doing housework and the dynamics of local resources under the persistent effects of climate change. By "*local resources*" we mean the entire geographical environment and

natural resources of the economy that support the daily lives of people, such as water resources, firewood, air quality, etc. We consider an economy consisting of L_t identical households at any period $t \in \mathbb{N}$. For simplicity, we assume that each household consists of two persons, as a couple, with gender male and female. Each member of a household lives for two periods—childhood and adulthood—and is endowed with one unit of time as an adult. Adult members of the household (i) allocate their total time between supplying labor, doing housework such as collecting essential resources for daily life (e.g. firewood and clean water), and child-rearing; (ii) decide how much to invest in their offspring’s human capital, and (iii) consume the remainder of their income. As in Becker (1985, p. 52), we assume that within a household the child-rearing and housework is taken care of by only the woman, which is in accordance with cross-cultural evidence. The time for doing housework in any period t , $\phi_t = \phi(R_t, a_t) \in [0, 1]$, depends on the availability of local resources, R_t , and the level of technology, a_t , of the economy. In particular, the following assumption describes the properties of function $\phi(R, a)$.

Assumption 1. $\phi_i(R, a) < 0$ with $i \in \{R, a\}$, $\phi_{RR}(R, a) > 0$, and $\lim_{R \rightarrow +\infty} \lim_{a \rightarrow +\infty} \phi(R, a) = 0$.

Assumption 1 implies the fact that the availability of local resources, such as water and firewood, saves time for women in the least developed regions in terms of collecting them for their family’s daily lives. In addition, a higher level of technology of the economy not only improves productivity but also saves women time on housework. Indeed, the level of technology of the economy could be reflected by infrastructure development, such as the diffusion of electricity and clean water supply, which strongly affect the time women allocate for housework. Greenwood et al. (2005) argue that technological progress plays a crucial role in a household’s allocation of time. Indeed, the appearance and the generalization of appliances such as washing machines, vacuum cleaners, and refrigerators (mid-20th century), as well as frozen foods and ready-made meals (mid- to late 20th century) has freed a considerable amount of time from housework, and which can thus be used otherwise.

Regarding child-rearing, the offspring of each household’s offspring—consisting of two-person, multi-gender households as well, for the sake of simplicity—requires devoting an amount of time $\rho_t > 0$, while the human capital of each member of the “child” household is assumed to be a power $\theta \in [\frac{1}{2}, 1)$ of the parent household’s educational investment in each of them, e_t for a male child and \tilde{e}_t for a female child respectively.² We assume that the time for raising children depends positively on the population density L_t/X , i.e. $\rho_t = \rho(L_t/X)$, where X is the size of land, which is fixed, of the economy. For simplicity, we normalize $X = 1$, hence $\rho_t = \rho(L_t)$. We introduce the following assumption on the functional form of $\rho(L)$.

Assumption 2. $\rho'(L) > 0$ and $\rho''(L) < 0$ for all $L \geq 0$.

This assumption implies that the cost in time of raising children is increasing concave in

²The condition $\theta \geq \frac{1}{2}$ implies plausibly that the returns on education do not decrease so quickly.

population density.³

2.1 Households

Adult households at t have preferences, and make choices regarding their consumption, number of children, and potential income that the children could earn as adults—as a function of the human capital they are endowed with by their parents’ educational investment. Specifically, households maximize their utility (1) under their budget constraint (2) and the time constraint of the women (3),

$$\max_{\substack{c_t, n_t > 0 \\ e_t, \tilde{e}_t \geq 0}} (1 - \gamma) \ln c_t + \gamma \ln (n_t w_{t+1} [e_t^\theta + (1 - \phi_{t+1}) \tilde{e}_t^\theta]) \quad (1)$$

subject to

$$c_t + n_t(e_t + \tilde{e}_t) \leq w_t [e_{t-1}^\theta + (1 - \phi_t - \rho_t n_t) \tilde{e}_{t-1}^\theta] \quad (2)$$

$$\phi_t + \rho_t n_t \leq 1 \quad (3)$$

given the wage rate w_{t+1} per efficient unit of labor at $t+1$; the fractions of household time ϕ_t , ϕ_{t+1} needed for housework at t , $t+1$,⁴ and past choices of educational investments e_{t-1} and \tilde{e}_{t-1} made by the household’s parents—where, at each period t , c_t is adult household consumption, n_t is the number of (household-)children per household.⁵ The parameter $\gamma \in (0, 1)$ captures the weight of the household’s offspring in its utility.

This paper focuses on the current least developed economies with fertility rates that have been falling slowly—i.e. those that are beyond a peak fertility rate—along with the increasing participation of female labor supply. Based on this, we introduce the following assumption guaranteeing that the time constraint of women is not binding.

Assumption 3. $\phi(0, 0) \leq 1 - \left[\frac{\gamma(1-\theta)}{1-\gamma(1-\theta)} \right]^{1-\theta}$.

The following lemma characterizes the optimal choice of a representative household.

Lemma 1. *Under assumption 3, the optimal choice of a representative household, which is the interior solution to (1) subject to (2) and (3), is*

$$c_t = (1 - \gamma) w_t [e_{t-1}^\theta + (1 - \phi_t) \tilde{e}_{t-1}^\theta], \quad (4)$$

$$e_t = \frac{\theta}{1 - \theta} \frac{\rho_t}{1 + (1 - \phi_{t+1})^{\frac{1}{1-\theta}}} w_t \tilde{e}_{t-1}^\theta, \quad (5)$$

³This idea is introduced in Goodsell (1937) and Thompson (1938), and was recently cited by De la Croix and Gosseries (2012) and Dao and D’Ávila (2013) to take into account that when households have small dwellings, child production is more costly and households have fewer children. More recently, De la Croix and Gobbi (2017) provide empirical evidence that confirms this idea.

⁴We assume that, when assessing the children’s potential income, the household perfectly foresees ϕ_{t+1} , which is a consequence of its own choices—because of the technological progress induced by its educational investments—but not what is the result of its children’s choices (hence the absence of $\rho_{t+1} n_{t+1}$ in the household’s objective).

⁵We assume that the gender ratio at birth (male to female) is 1 : 1, which is close to the natural gender ratio 1.05 : 1.

$$\tilde{e}_t = \frac{\theta}{1-\theta} \frac{\rho_t}{1 + (1 - \phi_{t+1})^{\frac{1}{\theta-1}}} w_t \tilde{e}_{t-1}^\theta, \quad (6)$$

$$n_t = \frac{\gamma(1-\theta)}{\rho_t} \left[\frac{e_{t-1}^\theta}{\tilde{e}_{t-1}^\theta} + 1 - \phi_t \right]. \quad (7)$$

Proof. The proof is available upon request. □

From (4) to (7), we have: (i) consumption is proportional to the household's potential income—*i.e.* net of housework costs; (ii) educational investments are proportional to the woman's potential income; and (iii) fertility is increasing in *the human capital gender gap*—resulting from differentiated educational investment—between the man and the woman of the household. Specifically, equations (5) and (6) show the different educational investment across genders due to the asymmetric time allocation for housework. These equations tell us that increasing the expected time for housework, which will be spent by daughter(s), leads parents to reduce education investment for their daughter(s). They instead increase the education investment for their son(s). Any exogenous effect on time for housework thus creates an impact on the education gender gap. Equation (7) suggests that the relative education gender gap among male and female parental members within a household determines the household's fertility rate. Indeed, the education gender gap generates an income gender gap that reflects the income effect and substitution effect on fertility decisions of the household such that the former raises fertility, whereas the latter, which reflects an increase in the opportunity cost of childrearing, reduces it. An increase in the relative education gender gap, e_{t-1}/\tilde{e}_{t-1} , makes the income effect dominant over the substitution effect, thereby fostering fertility.

2.2 Production, technology, and locally natural resources

We assume that the final good production in each period t is linear in the effective units of labor supplied in-elastically by the household adults

$$Y_t = A(a_t, E_t)H_t = A(a_t, E_t)L_t h_t,$$

where $H_t = L_t h_t$ is aggregate human capital in final good production, and $h_t = e_{t-1}^\theta + (1 - \phi_t - \rho_t n_t) \tilde{e}_t^\theta$ is the average human capital that the representative household devotes to the production.⁶ The marginal return to human capital is

$$w_t = A(a_t, E_t),$$

⁶For sake of simplicity, we abstract from physical capital. The physical capital, indeed, can be introduced without changing the qualitative results of the model if we consider a small and open economy with perfect physical capital mobility. In this case, we define the constant returns scale technology-augmented labor production function as $Y_t = F(K_t, A(a_t, E_t)H_t)$, and we extend the households to live for three periods with saving motives.

where $A(a_t, E_t) > 0$ is the total factor productivity of the economy in period t that depends on the contemporary level of technology a_t and exogenous global climate index $E_t \in [0, +\infty)$. We can read E_t as the index of global carbon concentration. An increase in E_t reflects the more climate change to be. We assume plausibly

$$A_a(a_t, E_t) > 0 \quad \text{and} \quad A_E(a_t, E_t) < 0,$$

which tell us that technological progress enhances total factor productivity, whereas climate change damages it.

The technology evolves according to

$$a_{t+1} = (1 + g_t)a_t,$$

where $g_t = g\left(\frac{e_{t-1}^\theta + \bar{e}_{t-1}^\theta}{a_t}\right)$ is the rate of technological progress between t and $t + 1$. We assume that

$$g(x), g'(x) \begin{cases} > 0 & \text{if } x > \bar{x} \\ = 0 & \text{if } x \in [0, \bar{x}] \end{cases} .$$

That is to say, the rate of technological progress is non-decreasing in average human capital and non-increasing in level of technology; and the higher level of technology a_t requires a sufficiently high average human capital h_t in production to guarantee a strictly positive rate of technological progress.

The locally natural resources or locally environmental quality index, R_{t+1} , evolves according to the following dynamic equation

$$R_{t+1} = R_t + \Omega(R_t, E_t) - \xi_t Y_t,$$

where $\xi_t Y_t$ is the damage to local natural resources coming from the production activity, and $\xi_t > 0$ measures the damage level of production in period t . We assume that $\xi_t = \frac{\xi}{a_t h_t}$, which captures the idea that higher level of technology and average human capital go hand in hand with cleaner production.⁷ Without loss of generality, we normalize $\xi = 1$. The function $\Omega(R_t, E_t)$ is the regeneration capacity of local resources, which depends on the quality of the local resources themselves, R_t , and on the exogenous global climate index E_t . An increase in E_t has an impact on the regeneration capacity of the local natural environment and resources.⁸

⁷Swinton and Quiroz (2003) pointed out, in a study on Peru, that higher human capital favors the choice of more sustainable practices.

⁸We follow the laws of thermodynamics as mentioned in Georgescu-Roegen (1971) in order to set up the general form of function $\Omega(R, E)$. Indeed, the state of the environment is constrained by biophysical principles, specifically through two processes: (i) the *entropic process* and (ii) the *preservation process*. The economy is basically a closed system with respect to material. According to the law of material or energy conservation, material is neither lost nor created in any transformation process. The entropic process transforms the availability of material or energy in a closed system in the sense that the available energy is continuously

Assumption 4. The function $\Omega(R, E)$ is continuous in its variables and satisfies the following properties

- (i) $\Omega_E(R, E) < 0$, $\Omega(0, E) = \Omega(\bar{R}(E), E) = 0$ for all $E \geq 0$
- (ii) $\Omega_R(R, E) > (<)(=) 0$ if $R < (>)(=) \hat{R}(E) \in (0, \bar{R}(E))$
- (iii) $\Omega_R(R, E) > -1$ and $\Omega(R, +\infty) = 0$ for all R .

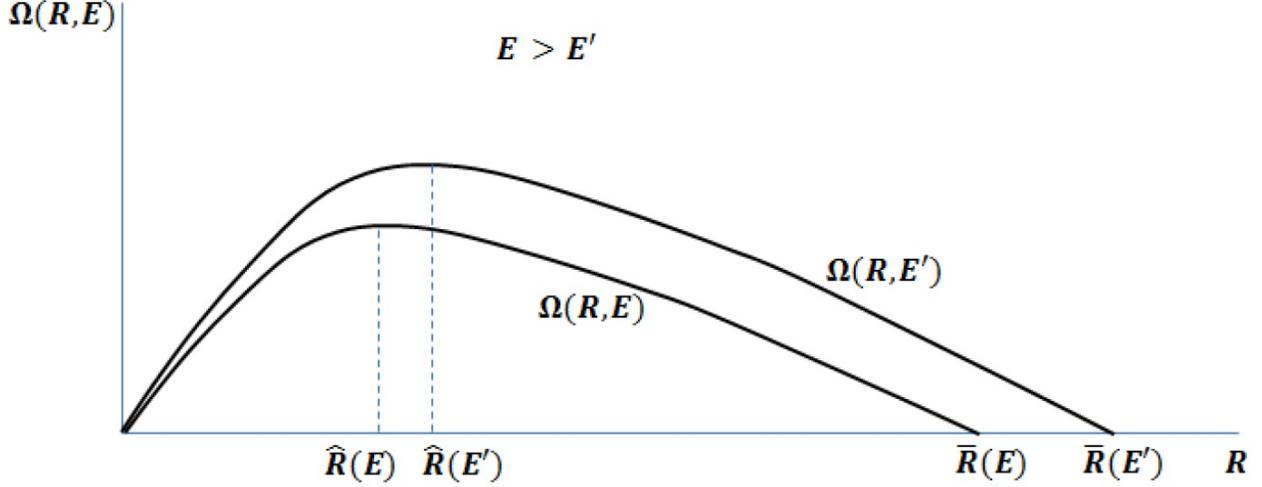


Figure 3: The regeneration capacity of local resources

2.3 Some static analyses

In this section, we carry out some important static analyses that help us understand the effects of technological progress and availability of local resources on education gender gap and fertility.

Definition 1. (*Education gender gap*) The education gender gap in any t is the relative gap in education investments between the man and the woman in a representative household. That is,

$$\zeta_t = \frac{e_{t-1}}{\tilde{e}_{t-1}}.$$

By definition 1, from equations (5) and (6), with one period lagged, we have

$$\zeta_t = [1 - \phi(R_t, a_t)]^{\frac{1}{\theta-1}} \equiv \zeta(R_t, a_t) \quad (8)$$

and hence, the fertility rate is determined by

transformed into unavailable energy until it disappears completely. Fortunately, the economy is an open system with respect to energy. The preservation process refers to the constant receiving of solar radiation, which provides energy to compensate for the entropic process, thus making resources renewable. That is to say, while natural and human transformation processes destroy the availability of material, new energy inflows provide energy to recollect material and energy and to offset such destruction. This explains the equilibrium in our ecology systems and the renewable nature of natural resources (Smulders 1995a,b).

$$n_t = \frac{\gamma(1-\theta)}{\rho(L_t)} \left\{ [1 - \phi(R_t, a_t)]^{\frac{1}{\theta-1}} + 1 - \phi(R_t, a_t) \right\} \equiv n(L_t, R_t, a_t) \quad (9)$$

Proposition 1 states some important effects of local resources and technology on gender equality in education and fertility.

Proposition 1. *The following properties hold:*

- (i) $\zeta(R_t, a_t) > 1 \forall (R_t, a_t) \geq (0, 0)$, and $\zeta_i(R_t, a_t) < 0$ with $i \in \{R, a\}$;
- (ii) $n_j(L_t, R_t, a_t) < 0$ with $j \in \{L, R, a\}$.

Proof. The proofs for these statements are obtained from equations (8), (9), and properties of functions $\rho(L)$ and $\phi(R, a)$. □

The statement (i) in Proposition 1 tells us that males always receive more education investment than females. This is because of the asymmetric time allocation across gender within households, as reflected in equation (8). As time on housework is unpaid and taken by women while the potential income of a child-household also constitutes the utility for their parents, a parent-household chooses optimally to invest more in education for their son(s) than for their daughter(s). The relative education gender gap ζ_t , however, decreases in both the availability of local resources R_t and level of technology a_t . That is because both the increase in availability of local resources and technological progress save women time on housework. When the time women spend on housework, $\phi(R_t, a_t)$, is expected to be reduced, a parent-household will have incentive to invest more in education for their daughter(s)—because the initial imbalance makes the marginal return to female education higher—while the total education investment for one couple of children (one son and one daughter), derived from equations (5) and (6) with one period lagged, is

$$e_{t-1} + \tilde{e}_{t-1} = \frac{\theta}{1-\theta} \rho_{t-1} w_{t-1} \tilde{e}_{t-2}^\theta,$$

which is given, independent on a_t and R_t , and determined by past (predetermined) variables. Therefore, to increase education investment for the daughter(s), a parent-household has to reduce education investment for the son(s). As a result, the relative education gender gap is reduced.

Statement (ii) says that the fertility rate n_t decreases in the size of population L_t , the availability of local resources R_t , and the level of technology a_t . That is rather intuitive. A larger population L_t , i.e. higher population density, increases the cost in time of raising children, resulting in a decline in demand for children. Higher availability of local resources R_t and/or higher level of technology a_t save the women time on housework, resulting in women being endowed with more education investment, which increases the opportunity cost of childrearing, making the substitution effect dominate the income effect. Hence, the fertility rate is reduced.

3 Equilibria and dynamics

The competitive equilibria of the economy are characterized by: (i) the household's utility maximization under its constraints, (ii) the aggregate output equating the total return to human capital, (iii) the dynamics of population size, (iv) the dynamics of technology, and (v) the dynamics of the local natural environment and resources. That is to say a competitive equilibrium $(c_t, e_t, \tilde{e}_t, n_t, a_{t+1}, L_{t+1}, R_{t+1}, w_t)$ is determined by the following system of equations

$$c_t = (1 - \gamma)w_t(e_{t-1}^\theta + [1 - \phi(R_t, a_t)]\tilde{e}_{t-1}^\theta), \quad (10)$$

$$e_t = \frac{\theta}{1 - \theta} \frac{\rho(L_t)}{1 + [1 - \phi(R_{t+1}, a_{t+1})]^{\frac{1}{1-\theta}}} w_t \tilde{e}_{t-1}^\theta, \quad (11)$$

$$\tilde{e}_t = \frac{\theta}{1 - \theta} \frac{\rho(L_t)}{1 + [1 - \phi(R_{t+1}, a_{t+1})]^{\frac{1}{\theta-1}}} w_t \tilde{e}_{t-1}^\theta, \quad (12)$$

$$n_t = \frac{\gamma(1 - \theta)}{\rho(L_t)} \left\{ [1 - \phi(R_t, a_t)]^{\frac{\theta}{\theta-1}} + 1 - \phi(R_t, a_t) \right\}, \quad (13)$$

$$a_{t+1} = \left[1 + g \left(\frac{e_{t-1}^\theta + \tilde{e}_{t-1}^\theta}{a_t} \right) \right] a_t, \quad (14)$$

$$L_{t+1} = n_t L_t, \quad (15)$$

$$R_{t+1} = R_t + \Omega(R_t, E_t) - \xi_t Y_t, \quad (16)$$

$$w_t = A(a_t, E_t), \quad (17)$$

given an exogenous global climate change index $E_t \geq 0$.

The competitive equilibria are fully characterized by the following reduced system describing the equilibrium dynamics of the educational investments for male and female children e_t and \tilde{e}_t , the level of technology a_{t+1} , the population size L_{t+1} , and the local natural environment and resources R_{t+1} .

$$e_t = \frac{\theta}{1 - \theta} \frac{\rho(L_t)}{1 + [1 - \phi(R_{t+1}, a_{t+1})]^{\frac{1}{1-\theta}}} A(a_t, E_t) \tilde{e}_{t-1}^\theta$$

$$\tilde{e}_t = \frac{\theta}{1 - \theta} \frac{\rho(L_t)}{1 + [1 - \phi(R_{t+1}, a_{t+1})]^{\frac{1}{\theta-1}}} A(a_t, E_t) \tilde{e}_{t-1}^\theta$$

$$a_{t+1} = \left[1 + g \left(\frac{e_{t-1}^\theta + \tilde{e}_{t-1}^\theta}{a_t} \right) \right] a_t$$

$$L_{t+1} = \frac{\gamma(1-\theta)}{\rho(L_t)} \left\{ [1 - \phi(R_t, a_t)]^{\frac{\theta}{\theta-1}} + 1 - \phi(R_t, a_t) \right\} L_t$$

$$R_{t+1} = R_t + \Omega(R_t, E_t) - L_t$$

given initial conditions e_{-1} , \tilde{e}_{-1} , a_0 , L_0 , $R_0 > 0$, and a sequence of exogenous climate change indexes $\{(E_t)_{t=0}^{+\infty}\}$.

4 Evolution of local resources and population

In this section, we study the evolution of the economy conditionally on a given level of technology a and a given climate change index E .

4.1 Conditional steady states

The conditional dynamics of the economy is characterised by the following dynamic system of equations

$$L_{t+1} = \frac{\gamma(1-\theta)}{\rho(L_t)} \left\{ [1 - \phi(R_t, a)]^{\frac{\theta}{\theta-1}} + 1 - \phi(R_t, a) \right\} L_t \quad (18)$$

$$R_{t+1} = R_t + \Omega(R_t, E) - L_t \quad (19)$$

Given any level of technology a and global climate change index E , we define the loci $LL(a)$ and $RR(E)$ as follows

$$LL(a) \equiv \{(L_t, R_t) \in \mathfrak{R}_+^2 : L_{t+1} = L_t\}$$

$$\text{i.e. } L_t = \rho^{-1} \left(\gamma(1-\theta) \left[[1 - \phi(R_t, a)]^{\frac{\theta}{\theta-1}} + 1 - \phi(R_t, a) \right] \right) \equiv \psi(R_t; a) \quad (20)$$

and

$$RR(E) \equiv \{(L_t, R_t) \in \mathfrak{R}_+^2 : R_{t+1} = R_t\}$$

$$\text{i.e. } L_t = \Omega(R_t; E) \quad (21)$$

Lemma 2. $\psi_a(R; a) < 0$, and $\psi_R(R; a) < 0$, $\psi_{RR}(R; a) > 0$.

Proof. By applying the implicit function theorem for equation (20) with respects to L_t , R_t , a , and noting that $\theta \in [\frac{1}{2}, 1)$, we have

$$\psi_i(R; a) = \frac{\gamma}{\rho'(L)} \left(\theta [1 - \phi(R, a)]^{\frac{1}{\theta-1}} - 1 + \theta \right) \phi_i(R, a) < 0, \quad i \in \{R, a\}$$

and

$$\psi_{RR}(R; a) = \frac{\gamma}{\rho'(L)} \left\{ \frac{\theta}{1-\theta} [1 - \phi]^{\frac{2-\theta}{\theta-1}} \phi_R^2 + \left(\theta [1 - \phi]^{\frac{1}{\theta-1}} - 1 + \theta \right) \phi_{RR} \right\} - \frac{\rho''(L)}{\rho'(L)} \psi_R^2 > 0$$

□

The properties of the function $\psi(R; a)$, as stated in lemma 2, and those of the function $\Omega(R; E)$ are very important in determining the number of conditional steady states as pointed out in proposition 2. In addition, as will be apparent in the subsequent subsections, they allow us to study the impacts of technological progress and climate change on the size of the population and the availability of local resources at the conditional steady states.

Lemma 3. *For the dynamic system characterized by equations (18) and (19), then*

$$(i) \ L_{t+1} - L_t \begin{cases} > 0 & \text{if } L_t < \psi(R_t; a) \\ = 0 & \text{if } L_t = \psi(R_t; a) \\ < 0 & \text{if } L_t > \psi(R_t; a) \end{cases} \quad \text{and} \quad (ii) \ R_{t+1} - R_t \begin{cases} > 0 & \text{if } L_t < \Omega(R_t; E) \\ = 0 & \text{if } L_t = \Omega(R_t; E) \\ < 0 & \text{if } L_t > \Omega(R_t; E) \end{cases}$$

Proof. The proofs are fairly straightforward.

□

Given a level of technology, a , and a global climate change index, E , the conditional steady states of the economy are determined by the intersections between the loci $LL(a)$ and $RR(E)$. Analytically, they are the solutions, R , to the following equation

$$\Omega(R; E) = \psi(R; a) \tag{22}$$

The following proposition states the existence of conditional steady states and their properties.

Proposition 2. *Given a level of technology a and a global environmental quality index E ,*

(i) *There exists at most two distinct conditional steady states;*

(ii) *If there exists a steady state(s), then there is at least a steady state at which it holds that $R > \hat{R}(E)$;*

(iii) *If there exists two distinct conditional steady states characterized by $R > \tilde{R}$, then:*

(iii.a) *the one with higher availability of local resource R is a locally stable node;*

(iii.b) *if $\psi_R(\tilde{R}; a) \geq -1$, the steady state with $R^* = \tilde{R}$ is a saddle node.*

Proof. See Appendix □

Proposition 2 characterizes the number of conditional steady states and their properties, particularly their stability. In the case that two distinct conditional steady states prevails, the condition $\psi_R(\tilde{R}; a) \geq -1$ is just a sufficient condition under which the steady state (\tilde{R}, \tilde{L}) is a stable node.⁹ We will focus on this most interesting case (the case of two distinct conditional steady states). The diagram in figure 4 depicts the existence of and the convergences to conditional steady states. In this case, there is a unique saddle path (stable arm) leading to the conditional steady state (\tilde{R}, \tilde{L}) . This saddle path can be represented by a monotonically increasing function $\mathcal{L}(R; E, a)$, and there exists an availability level of local resources, which depends on the level of technology and the global environmental index, $\underline{R}(a, E) \in [0, \tilde{R})$.

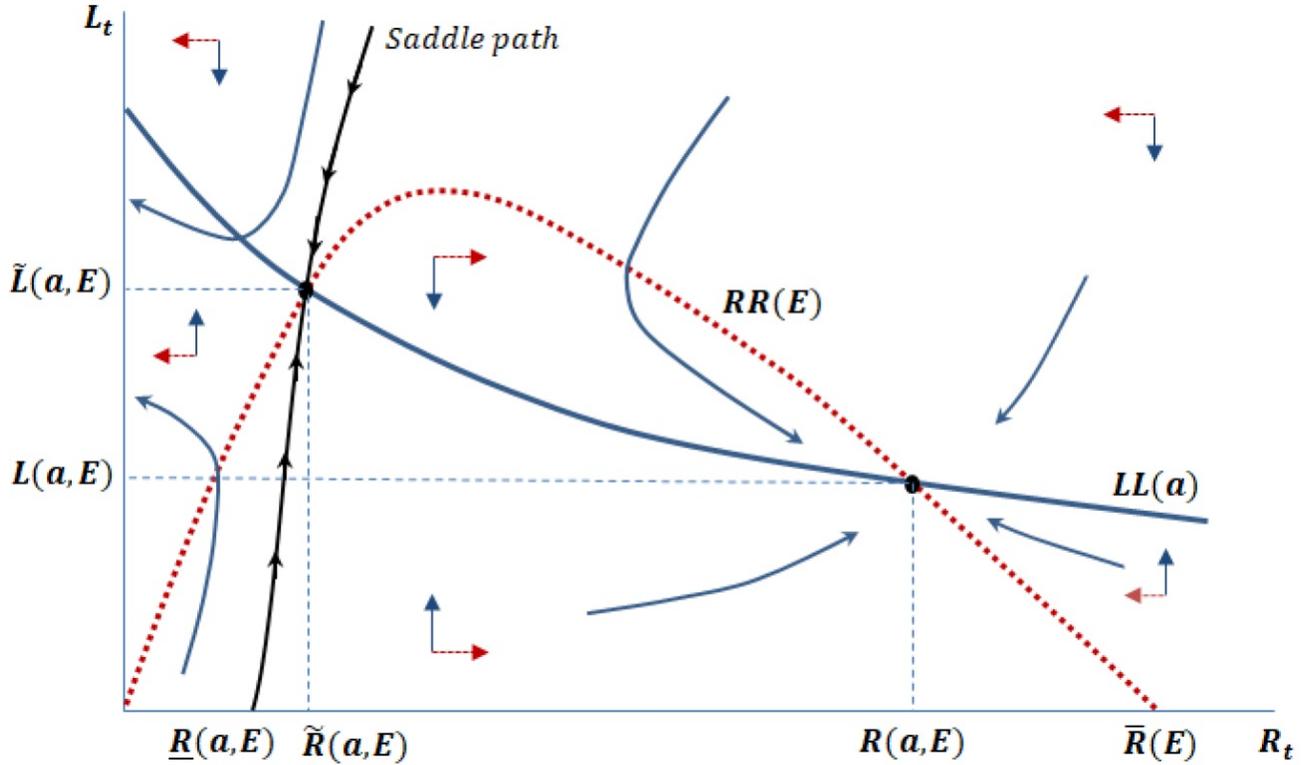


Figure 4: Dynamics and convergence to conditional steady states

Definition 2. (*Environmental disaster*) The economy is called falling in an *environmental disaster* in some period t if $R_t \leq 0$.

⁹Indeed, we have room to introduce the following condition on parameters and functional forms under which $\psi_R(R; a) \geq -1$ always holds. It is

$$\gamma \left(\theta [1 - \phi(0, 0)]^{\frac{1}{\theta-1}} - 1 + \theta \right) \phi_R(0, 0) + \rho'(\sup \Omega(R; E)) \geq 0$$

Proposition 3. *If, in some period t , the state of economy is characterized by (R_t, L_t) such that*

$$L_t > \mathcal{L}(R_t; a, E) \quad \text{or} \quad R_t < \underline{R}(a, E)$$

then an environmental disaster will occur.

Proposition 3 characterizes a dangerous area that is located on the left side of the saddle path, as depicted graphically by the diagram in Figure 4. Proposition 3 tells us that, if the economy falls into this dangerous area, then in the long run an environmental disaster will occur. This result implies an important implication that, as will be more apparent in the later analyses, if there is a sufficiently strong environmental shock that puts the economy into the dangerous area, without sufficient technological progress, then, in the long run, the economy will fall into an environmental disaster.

4.2 Impact of technological progress

In this subsection, we consider the impact of technological progress on the availability of local resources and the size of population over the long term. Let us focus on the case where the economy converges to a conditional stable steady state. We know from lemma 2 that $\psi_a(R_t; a) < 0$. That is to say, the increase in the level of technology, a , makes the locus $LL(a)$ move downward as depicted in figure 5. The downward shift of LL locus from $LL(a)$ to $LL(a')$ makes the economy tend to converge to a conditional steady state with higher availability of local resources, $R(a', E) > R(a, E)$, and smaller population size, $L(a', E) < L(a, E)$.

The economic intuitions for the convergence to the conditional steady state with higher availability of local resources and smaller population size, under the impact of increasing technology, is as follows. As analyzed in section 2.3, technological progress reduces the fertility rate because it makes the substitution effect dominate the income effect through increasing education investment for daughters relatively more compared with sons. The lower fertility rate leads to the smaller population size, which will be associated with better human capital and higher female labor force participation, in the future. The smaller population size, in turn, as reflected in equation (19), results in less damages to local resources, increasing the availability of local resources. The higher availability of local resources, in turn, saves women time on housework for their families' daily lives, thereby reinforcing the convergence to the conditional steady state $(R(a', E), L(a', E))$.

4.3 Impact of climate change

We study in this subsection the impact of climate change on the availability of local resources and the population size over the long term. As in the previous subsection, we also focus on the case where the economy converges to a conditional stable steady state. We have from

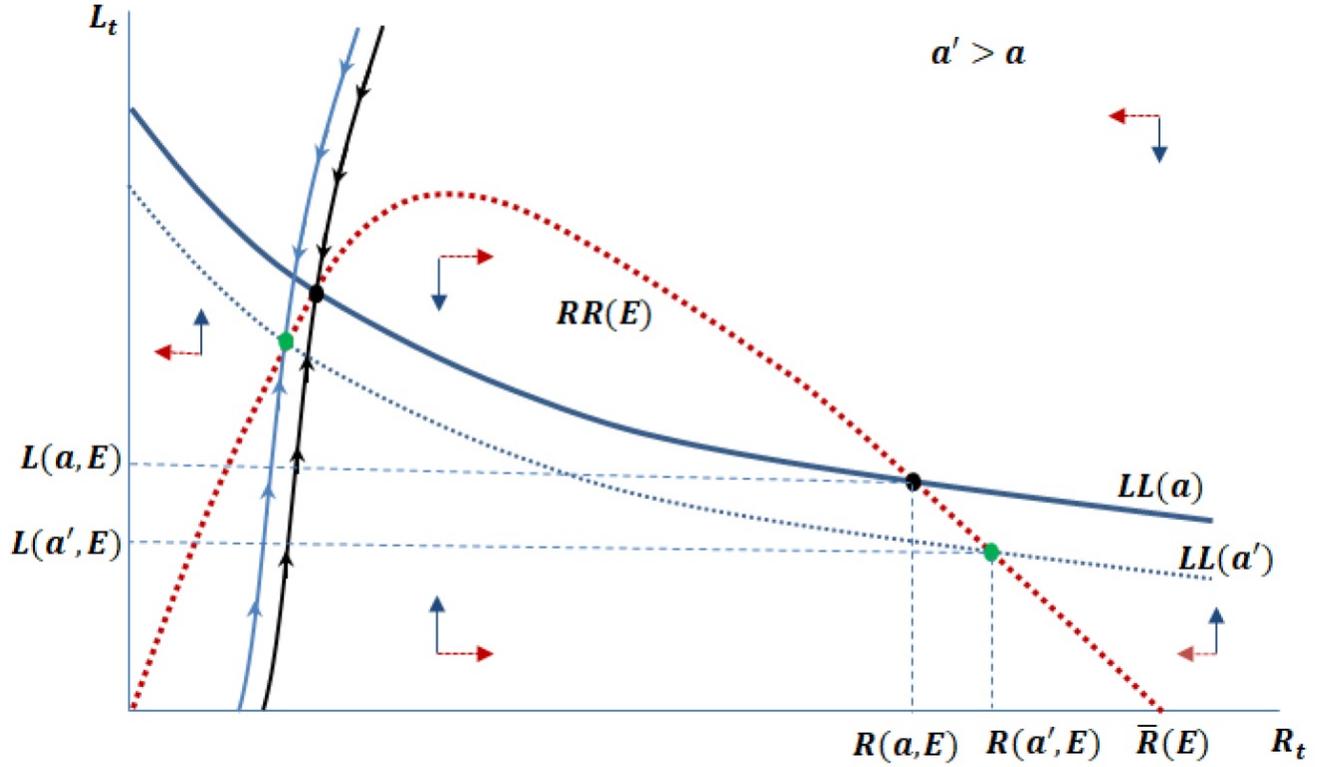


Figure 5: Dynamics and convergence to conditional steady states

assumption 4 that $\Omega_E(R, E) < 0$. That is to say, the increase in climate change makes the RR locus shift downward from $RR(E)$ to $RR(E')$, as illustrated in Figure 6. This downward shift of the locus RR makes the conditional stable steady state move from $(R(a, E), L(a, E))$ to $(R(a, E'), L(a, E'))$ with $R(a, E') < R(a, E)$ and $L(a, E) > L(a, E')$.

The mechanism associated with the persistent impact of climate change is as follows. The increase in climate change reduces the regeneration capacity of the local resources, thereby making the local resources scarcer, requiring women to spend more time collecting essential resources for their families' daily lives. Through this channel, the education gender gap is enlarged because households tend to reduce education investment for their daughter(s) and instead invest more in education for their son(s). As a result, the fertility will be increased, creating a bigger population in the future associated with low average human capital. The bigger population and lower average human capital, in turn, damage local resources through (more polluted) production activity. The feedback loop between local resources, population, and gender inequality in education, under the persistent effects of increased climate change, leads to the convergence to the conditional stable steady state $(R(a, E'), L(a, E'))$.

Compared with the effects of technological progress, increased climate change has the opposite effect over the long term on local resources and population through the opposite mechanism linking women's time in doing housework, education investment across genders, and fertility.

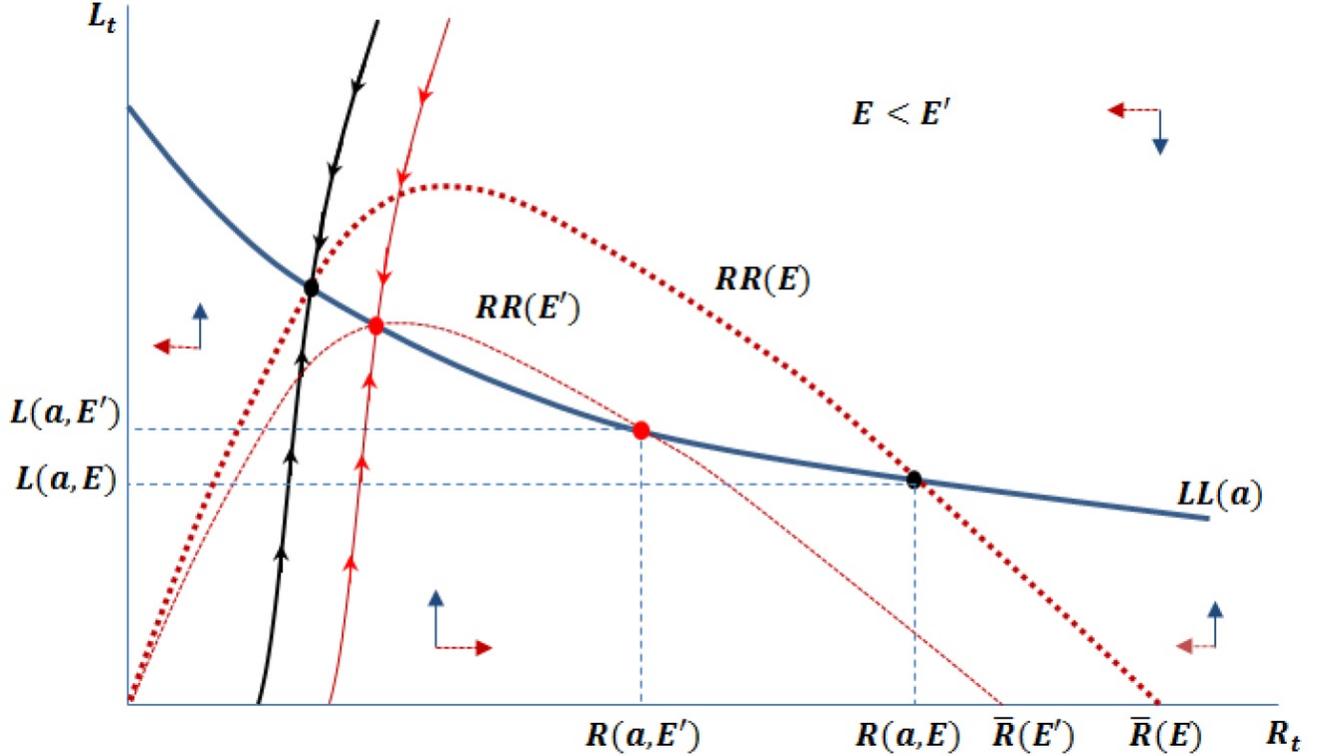


Figure 6: Dynamics and convergence to conditional steady states

5 Climate change and disaster

The analyses and phase diagrams in the previous section suggest that when climate change is too extreme such that the $RR(E)$ locus always lies below the $LL(a)$ locus—i.e. $RR(E) \cap LL(a) = \emptyset$ —as depicted in figure 7, then an environmental disaster will occur. In this section, we will characterize the precise conditions regarding climate change and level of technology under which the interactions between population and local resources will lead to an environmental disaster regardless of the starting state $(R_0, L_0) \in \mathfrak{R}_{++}^2$ of the economy.

In order to characterize the condition under which climate change always leads to an environmental disaster, we study the economy under the following assumption.

Assumption 5. $LL(0) \cap RR(E) \neq \emptyset$ for some $E < +\infty$.

To facilitate the analyses, in subsection 5.1, we consider the conditional convergence to an environmental disaster in the sense that we fix the level of technology, a , when we study the dynamics. In subsection 5.2 we instead study unconditional convergence by allowing the level of technology to change over time.

5.1 Conditional convergence to an environmental disaster

Proposition 4 provides us the properties of conditional dynamics of population L_t and local

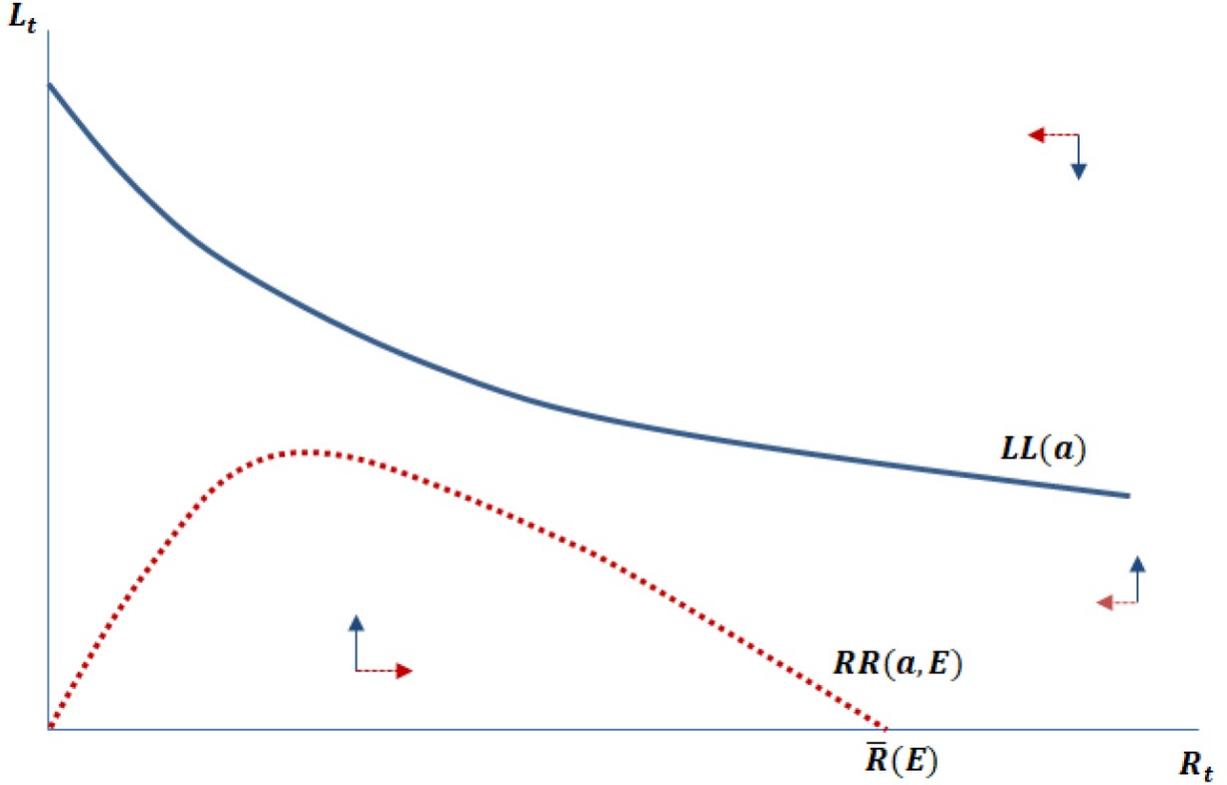


Figure 7: Extreme climate change

resources R_t , which, under certain conditions, generate a monotonic convergence to an environmental disaster.

Proposition 4. *If $LL(a) \cap RR(E) = \emptyset$ and in some T , the state of the economy $(R_T, L_T) \in \mathfrak{R}_{++}^2$ is characterized by*

$$L_T \in (\Omega(R_T, E), \psi(R_T, a)) \quad (23)$$

then the economy converges monotonically in an environmental disaster, in which for all $t \geq T$ until the environmental disaster occurs, the following properties hold

$$R_{t+1} < R_t \quad \text{and} \quad L_t < L_{t+1} \in (\Omega(R_{t+1}, E), \psi(R_{t+1}, a)).$$

Proof. See Appendix. □

Proposition 4 implies that, in the absence of technological progress¹⁰, when the economy falls into a certain area characterized by condition (23), the size of population will increase monotonically, whereas the availability of local resources will decrease monotonically, making an environmental disaster occur within a finite time. The economic intuition for these dynamics

¹⁰or if technology progresses too slowly

is rather obvious. $L_t \in (\Omega(R_t, E), \psi(R_t, a))$ implies two aspects of the population size L_t in relation to the availability of local resources, R_t . First, $L_t > \Omega(R_t, E)$ means that the size of population L_t , and hence labor force, is relatively high compared with the regeneration capacity $\Omega(R_t, E)$ of contemporary local resources. Hence, the environmental damage from aggregate production exceeds the regeneration of local resources, making the availability of local resources decline. The decline in the availability of local resources, as pointed out in an earlier section, enhances the fertility rate by enlarging the gender gap in education—and hence the gender gap in income—because it requires women to spend more time on housework duties, resulting in parents investing less in education on their daughter(s). Second, $L_t < \psi(R_t, a)$ indicates that the size of population L_t , and hence population density, is not sufficiently high to make the cost in time of raising children physically expensive enough such that the demand for children of the representative household is $n_t \leq 1$. Thus, the size of the population becomes bigger over time. The mechanism of interactions between population and local resources creates a monotonic convergence to an environmental disaster.

The directions of motion of the population size and the availability of local resources, as pointed out in lemma 3, tell us that when the economy starts from a state such that $L_0 \notin (\Omega(R_0, E), \psi(R_0, a))$ then it will enter the area $L_T \in (\Omega(R_T, E), \psi(R_T, a))$ within some finite time T , and, eventually, the economy will converge monotonically in an environmental disaster.

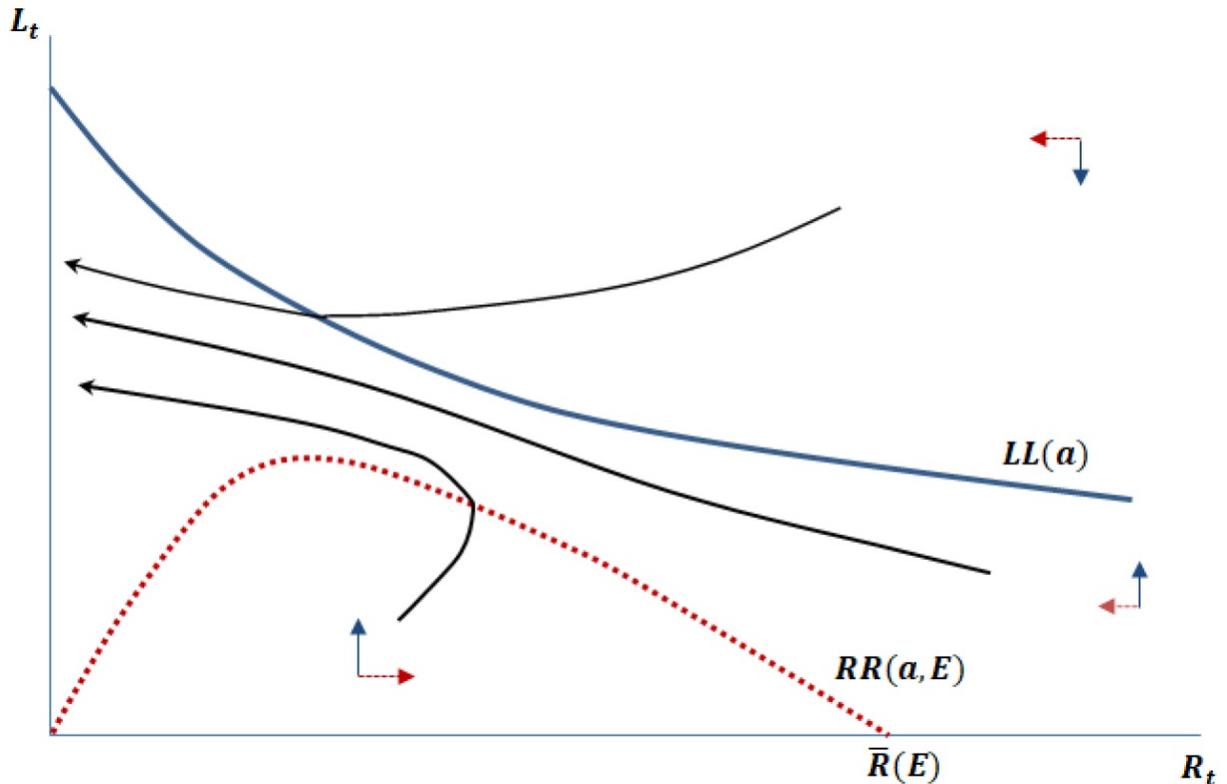


Figure 8: Extreme climate change

The next proposition characterizes the level of climate change under which an environmental disaster will occur.

Proposition 5. *Given any level of technology a unchanged (more strictly, non-increasing) over time, there exists a unique threshold $E_*(a) > 0$ such that for all $E > E_*(a)$, an environmental disaster will occur, regardless of the starting state of the economy $(R_0, L_0) \in \mathfrak{R}_{++}^2$. Moreover, $E'_*(a) > 0$.*

Proof. See Appendix. □

Proposition 5 provides an important condition under which an economy will fall into an environmental disaster. It implies that if the level of climate change is too strong relatively compared with the level of technology of the economy—i.e. when it holds that $E > E_*(a)$ —then in the long run the availability of local resources is zero, $R = 0$, i.e. an environmental disaster will occur, even if the economy starts from an initial state characterized by small population size L_0 and high availability of local resources R_0 . In the absence of technological progress, the interplay between population and local resources, under the persistent effects of climate change, makes the population size increase along with the decrease in the availability of local resources until the latter reaches zero.

5.2 Unconditional convergence to an environmental disaster

In this section, we relax the conditions whereby technological levels and global environmental indexes are fixed. We allow these variables to change over time. Let us define $\mathcal{A}_T = \{(a_\tau)_{\tau=T}^{+\infty}\}$ and $\mathcal{E}_T = \{(E_\tau)_{\tau=T}^{+\infty}\}$ to be respectively the set of levels of technology and the set of global climate change indexes from some period $T \geq 0$ onward.

Proposition 6. *If $LL(\sup \mathcal{A}_T) \cap RR(\inf \mathcal{E}_T) = \emptyset$ and in some period T , the state of the economy $(R_T, L_T) \in \mathfrak{R}_{++}^2$ is characterized by*

$$L_T \in (\Omega(R_T, \inf \mathcal{E}_T), \psi(R_T, \sup \mathcal{A}_T))$$

then the economy converges monotonically in an environmental disaster, in which for all $t \geq T$ until an environmental disaster occurs, the following properties hold

$$R_{t+1} < R_t \quad \text{and} \quad L_t < L_{t+1} \in (\Omega(R_{t+1}, \inf \mathcal{E}_{t+1}), \psi(R_{t+1}, \sup \mathcal{A}_{t+1})).$$

Proof. The proof for this proposition is similar to that for proposition 4 □

The following theorem extends the result stated in proposition 5.

Proposition 7. *If in some period T ,*

$$\inf \mathcal{E}_T > E_*(\sup \mathcal{A}_T) \quad \text{or equivalently} \quad \sup \mathcal{A}_T < E_*^{-1}(\inf \mathcal{E}_T),$$

then an environmental disaster occurs within some finite time $t \geq T$.

Proof. Under the condition stated in proposition 7, in any period $t \geq T$, it holds that $LL(a_t) \cap RR(E_t) = \emptyset$ —i.e. there is no conditional steady state. Hence, from the directions of motions of R_t and L_t , as pointed out in lemma 3, the economy eventually falls into an environmental disaster. □

6 Global dynamics and discussion

The global dynamics of the economy is governed by the following system of equations

$$e_t = \frac{\theta}{1-\theta} \frac{\rho(L_t)}{1 + [1 - \phi(R_{t+1}, a_{t+1})]^{\frac{1}{1-\theta}}} A(a_t, E_t) \tilde{e}_{t-1}^\theta \quad (24)$$

$$\tilde{e}_t = \frac{\theta}{1-\theta} \frac{\rho(L_t)}{1 + [1 - \phi(R_{t+1}, a_{t+1})]^{\frac{1}{\theta-1}}} A(a_t, E_t) \tilde{e}_{t-1}^\theta \quad (25)$$

$$a_{t+1} = \left[1 + g \left(\frac{e_{t-1}^\theta + \tilde{e}_{t-1}^\theta}{a_t} \right) \right] a_t \quad (26)$$

$$L_{t+1} = \frac{\gamma(1-\theta)}{\rho(L_t)} \left\{ [1 - \phi(R_t, a_t)]^{\frac{\theta}{\theta-1}} + 1 - \phi(R_t, a_t) \right\} L_t \quad (27)$$

$$R_{t+1} = R_t + \Omega(R_t, E_t) - L_t \quad (28)$$

given initial conditions $e_{-1} > 0$, $\tilde{e}_{-1} > 0$, $a_0 > 0$, $L_0 > 0$, $R_0 > 0$, and given an exogenous sequence of global carbon concentration index $(E_t)_{t=0}^{+\infty}$.

In addition to the previous section, we study in this section the impact of climate change on technological progress. We learn from the previous section that the level of technology has a negative impact on fertility by narrowing the gender gap in education investment. Hence, we show in this section that by affecting technological progress, climate change has other reinforcing effects on gender inequality in education and fertility in addition to the effects studied in the previous section. The following proposition states the impact of climate change on technological progress

Proposition 8. *An increase in global climate change index E_t in period t slows the rate of technological progress g_{t+1} between periods $t+1$ and $t+2$.*

Proof. In effect, we come back with the education gender gap as defined in definition 1,

$$\zeta_{t+1} = \frac{e_t}{\tilde{e}_t} = [1 - \phi(R_{t+1}, a_{t+1})]^{\frac{1}{\theta-1}} \equiv \zeta(R_{t+1}, a_{t+1}),$$

where in any period $t + 1$, variables a_{t+1} and R_{t+1} are predetermined by equations (26) and (28) respectively.

We have

$$\frac{\partial \zeta(R_{t+1}, a_{t+1})}{\partial E_t} = \frac{1}{1 - \theta} [1 - \phi(R_{t+1}, a_{t+1})]^{\frac{2-\theta}{\theta-1}} \phi_R(R_{t+1}, a_{t+1}) \Omega_E(R_t, E_t) > 0,$$

which implies that the climate change enlarges the gender inequality in education investment, while the total education investment for a couple of children is

$$e_t + \tilde{e}_t = \frac{\theta}{1 - \theta} \rho(L_t) A(a_t, E_t) \tilde{e}_{t-1}^\theta$$

which decreases in the climate change index E_t .

The decrease in total education investment for a couple of children, the reallocation in education investment being more biased toward the son, and the concavity of human capital formation function together induce the decline in the rate of technological progress. □

The theoretical results so far may suggest a mechanism to explain the slow demographic transition in the least developed regions that are the most vulnerable to climate change, accompanied by gender inequality in education. Climate change persistently plays a role in maintaining high rates of fertility in these regions through several channels. First, it causes damage to the local resources that are essential for life, thereby making it necessary for women to spend more time collecting resources for their families. When households anticipate that their daughters will have less labor supply because of spending more time doing housework, they will invest less in education for their daughters and more in education for their sons. Through this channel, the gender inequality in education, which is biased toward the sons, is maintained. Hence, the high fertility rate is also upheld because the opportunity cost of child-rearing becomes relatively inexpensive, making the substitution effect dominated by the income effect on fertility. Second, climate change directly decreases the return to human capital, reducing household income, and thus indirectly makes households invest less in education for their offspring. That is to say, climate change slows the rate of technological progress because it decreases the average human capital of the economy. The low level of technology, in turn, inhibits the development of infrastructure, such as electricity and clean water supply, as well as the diffusion of appliances, which save women time on housework. Therefore, through this channel, climate change also upholds gender inequality in education and thus maintains a high fertility rate.

7 Empirical Analysis

In this section, we test our theoretical predictions derived from our model for African countries. In particular, our model suggests that climate change and population negatively affect the local resources, which cause a reduction in the female labor force and female incomes given that the women need to spend significant time at home. Because the women need to spend significant time at home they will receive less investment in education. As a result, the gender gap in education will increase and fertility will increase. More specifically, our model-based hypotheses that will be tested in this section can be summarized as follows:

- H1: Climate change positively affects fertility.
- H2: Climate change has a negative relationship with local resources.
- H3: Climate change increases the gender gap in education investment and reduces the female labor force participation and female income.

7.1 Data

Our analysis is restricted to African countries (44 countries) from the period 1960 to 2017. We corroborate the strategy of Dell et al. (2012) for constructing the climate variable, which is the the temperature. More precisely, we collect the historical weather data from the Terrestrial Air Temperature: 1900 - 2017 Gridded Monthly Time Series, Version 5.01 (Matsuura and Willmott 2007). The dataset includes worldwide monthly mean temperature data at 0.5 Å 0.5 degrees resolution (approximately 56km Å 56km at the equator). The values are interpolated for each grid node from an average of 20 different weather stations, with corrections for elevation. We use the ArcGIS software to aggregate the weather data to the country-year level. In our specification, we use population-weighted average temperature where the weights are constructed from 1990 population data at Gridded Population Count Data (GRUMP). We also consider averaging based on geographic area, which produces broadly similar temperatures for most countries.

To proxy the local resources, we use two variables. The first one is the percentage of the population that has access to basic hand washing facilities including soap and water. The other variable is the percentage of the population with access to basic drinking water. Both variables are collected from the World Bank’s World Development Indicators. Data on female formal labor force participation and female incomes in agriculture also come from the World Bank World Development Indicators, covering the period 1990-2015 (yearly observations). Data on education comes from the Barro and Lee Database, and observations are available from 1950 on, on a five-year basis (Barro and Lee 2013). The population, fertility rate, exports (expressed as percentage of GDP), urban, income per capita, excess mortality, and age dependency ratio are collected from the World Bank Indicators.

We logarithm the variables population, income per capita, temperature, local resources and age dependency ratio in order to make them normal. For the robustness of our analysis, we replicate our empirical specification using data for the rest of developing countries and the same time span.

Table 1 illustrates the summary of statistics for all the variables for African countries in Panel A and the summary of statistics of the rest of the developing countries in Panel B.

Table 1: Summary statistics

Panel A:African Countries	Mean	Std. Dev.	N
Water sources (in logs)	4.448	0.287	2534
Temperature (in logs)	2.522	0.149	2552
Gender Education Gap	0.727	0.5	342
Basic Water Sources (in logs)	4.102	0.298	791
Labour female participation	55.741	21.652	1233
Exports (% GDP)	28.134	17.414	1973
Wage female in Agriculture	1.458	0.198	2303
Urbanization (% GDP)	30.456	17.874	2460
Population(in logs)	15.402	1.444	2460
Total fertility rate	5.897	1.405	2552
Under 5-mortality rate	154.43	78.261	2232
Income per capita (in logs)	6.878	0.982	2040
Panel B:Other Developing Countries	Mean	Std. Dev.	N
Total fertility rate	3.718	1.831	6100
Temperature (in logs)	2.504	0.144	4634
Exports (% GDP)	32.613	20.291	3286
Urbanization (% GDP)	45.691	21.195	4700
Under 5-mortality rate	71.608	67.005	3967
Income per capita (in logs)	7.836	0.992	3384
Population(in logs)	15.126	2.294	4718
Gender Education Gap	0.816	0.233	660
Female labor force participation	47.147	16.005	2919

7.2 Model

Following Dell et al. (2012) and Dao et al. (2021), we use OLS fixed-effect estimation to exploit the within-country variation of temperature to total fertility rates, female labor force participation, local resources and the female wages.

$$Y_{it} = a_i + a_t + \beta_1 \ln(\text{temperature}_{it}) + \beta_2 X_{it} + \epsilon_{it} \quad (29)$$

The key coefficient is β_1 , which shows the impact of temperature on the outcome variable Y_{it} . We include country fixed effects (a_i) and year fixed effects (a_t) to capture the unobserved heterogeneity and eliminate all the co-founding factors so that they are not captured by the other control variables (X_{it}), such as income per capita (in logs), population (in logs), excess child mortality, urbanization (% GDP), and exports (% GDP) included in the regression. ϵ_{it} is error term. We cluster heteroskedasticity adjusted standard errors at the bank-level to account

for serial correlation and small standard errors (Bertrand, Mullainathan, and Duflo 2004).

7.3 Results

Table 1 illustrates the results related to the effects of climate change on local resources. As mentioned in the data section, access to local resources is proxied by two variables: the percentage of the population that has access to drinking water and the percentage that has access to handwashing water. Column 1 shows the estimates of Equation (29) having the percentage of the population with access to drinking water as a dependent variable, with the only independent variable the temperature in logs. Column 2 presents similar estimates, with the percentage of population having access to handwashing water as a dependent variable. Both columns show that the temperature has a negative and significant impact on basic water sources.

More specifically, a percentage increase in the temperature decreases the local resources by 0.5 (see Column 1) and 0.1 (see Column 2) percentage. The results remain robust when we include control variables for both of the different measures of basic water resources in columns 3 and 4. Notably, the only variables that are significant are income per capita in logs and the population in logs. The population has a negative and significant coefficient at 5%, showing that the larger the population, the less access to the local resources will be. On the other hand, we observe that the estimated coefficient for income per capita is quite intuitive given that the wealthier the country is, the greater the population's access to drinking water. To the end, we note that the adjusted R-square is around 0.92 on average, indicating that the model has a good fit with the independent variables. All the estimated models include country and year fixed effects and the standard errors are clustered at country level.

Table 2 illustrates the impact of climate on fertility. Column 1 presents the effect of temperature on fertility without including any control variables. In column 2, we introduce only the economic variables as controls in addition to the temperature variable and in column 3, we add only the demographic variables such as population and child mortality. To the end, column 4 includes all the variables (economic and demographic variables). All the columns show a similar result, which is that the temperature has a positive and significant effect on fertility. In particular, we find that a rise in temperature increases fertility by 0.5 children per woman on average. This result is in line with Case et al. (2019). By contrast, studies by Chang et al. (1963), Mathew (1941), and Barreca et al. (2018) show that temperature has a negative effect on the birth rate or the health of infants. Consequently, we need to further examine our results to highlight the mechanism of our current results.

Table 3 examines the effects of climate change on education and labor outcomes. Column 1 shows the impact of temperature in logs on education gap. Following Dao et al. (2021), the gender gap in education is measured by the female-to-male ratio of average years of total

Table 2: Local basic resources and Climate

VARIABLES	(1)	(2)	(3)	(4)
	Model	Model	Model	Model
	Basic water sources	Basic water sources	Basic water sources	Basic water sources
Temperature (in logs)	-0.469*** (0.092)	-0.113** (0.043)	-0.316*** (0.076)	-0.051** (0.020)
Population(in logs)			-0.595*** (0.161)	-0.409*** (0.157)
Exports (% GDP)			0.000 (0.001)	-0.000 (0.001)
Urbanization (% GDP)			0.005** (0.002)	0.001 (0.006)
Under 5-mortality rate			0.000 (0.000)	-0.000 (0.001)
Income per capita (in logs)			0.402* (0.228)	0.785* (0.420)
Income per capita squ. (in logs)			0.028* (0.015)	-0.039 (0.025)
Constant	5.630*** (0.233)	4.396*** (0.111)	15.745*** (2.450)	-5.873 (4.378)
Observations	2,534	791	1,784	627
Adjusted R-squared	0.815	0.931	0.878	0.963
Year fixed effects	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes

Notes: ***, **, and * indicate statistical significance at the 1%, 5% and 10% levels respectively.

Table 3: Fertility and Climate

VARIABLES	(1)	(2)	(3)	(4)
	Model	Model	Model	Model
	Total fertility rate	Total fertility rate	Total fertility rate	Total fertility rate
Temperature (in logs)	0.683** (0.279)	0.509** (0.187)	0.566** (0.256)	0.444** (0.220)
Exports (% GDP)		0.007 (0.005)		0.004 (0.005)
Urbanization (% GDP)		-0.013 (0.019)		-0.035** (0.016)
Income per capita (in logs)		-1.021* (0.623)		-0.353* (0.530)
Income per capita sq. (in logs)		0.052 (0.088)		0.010 (0.095)
Population (in logs)			0.424 (0.776)	0.475 (1.025)
Under 5-mortality rate			-0.000 (0.003)	0.000 (0.003)
Constant	4.175*** (0.704)	9.286* (4.702)	-2.035 (12.480)	0.124 (16.481)
Observations	2,552	1,886	2,228	1,796
R-squared	0.806	0.854	0.828	0.880
Year fixed effects	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes
widstat

Notes: ***, **, and * indicate statistical significance at the 1%, 5% and 10% levels respectively.

schooling for ages 20â24. Column 1 shows the impact of temperature in logs on education gap. We find that the temperature (in logs) has a negative and significant impact on education gap. This result shows that climate change decreases the number of females that attend school, instead remaining at home. The latter result is supported by the other two columns in Table 3. Columns 2 and 3 show the effects of climate change on the percentage of the women that are in the labor force and female wages in the agriculture (in logs).

Both regression models show that there is a significant negative effect of temperature on the female labor participation and female wages in agriculture. In particular, we find that one percentage point increase in temperature results in a decrease of 2.1 percentage points in female wages. The adjusted R-square of all the models is above 80%, which shows that there is a good fit of the model with the included variables. In all the models, we include control variables such as income per capita, income per capita squared, child mortality, population (in logs), exports, urbanization, and country and year fixed effects.

Table 4: Education, labor, and climate

VARIABLES	(1)	(2)	(3)
	Model Gender Education Gap	Model Female labor force participation	Model Female wages in Agriculture
Temperature (in logs)	-0.612** (0.290)	-9.403** (4.121)	-2.281*** (0.479)
Exports (% GDP)	0.000 (0.007)	-0.111 (0.087)	-0.002 (0.001)
Urbanization (% GDP)	-0.002 (0.011)	0.381 (0.399)	0.009** (0.004)
Under 5-mortality rate	0.000 (0.001)	0.076 (0.054)	0.001** (0.000)
Income per capita (in logs)	0.476 (0.720)	2.080 (17.512)	-0.082 (0.277)
Income per capita sq. (in logs)	-0.040 (0.054)	-0.072 (1.109)	0.011 (0.019)
Population(in logs)	0.303 (0.484)	16.641 (26.963)	0.004 (0.117)
Constant	-3.815 (7.107)	-214.615 (459.158)	9.596*** (2.005)
Observations	278	1,004	1,591
R-squared	0.542	0.810	0.908
Year fixed effects	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes

Notes: ***, **, and * indicate statistical significance at the 1%, 5% and 10% levels respectively.

So far, our analysis has been focused on African countries. Our stylized facts indicate that our hypotheses hold only for the African countries but not for the rest of the developing countries. In Table 4, we estimate Equation (29) with the variables of interest as dependent variables for the rest of the developing countries (120 countries). We define developing countries based on the World Bank income classification. Columns 1, 2, 3, and 4 show the impact of climate on basic access to water resources, total fertility rate, female labor force participation and gender education gap, respectively. All the columns include control variables such as the income per capita, income per capita squared, child mortality, population (in logs), exports, urbanization, and country and year fixed effects.

We find that temperature as a proxy of climate does not have any impact on local resources and female labor force participation. This result could be explained by the fact that other developing countries could have better infrastructure for access to the basic local resources compared to African countries. Consequently, there is no need for women to remain at home. We reach this outcome based on the fact that we find that the temperature has a negative and significant impact at 10% on the total fertility rate. This outcome is consistent with empirical literature that shows that the climate is responsible for the drop in all pregnancy outcomes in several countries (Cachon et al. 2012). The mechanism of the latter result could be that the hot temperatures might reduce sexual activity or does not affect certain sectors.

Table 5: Other developing countries and climate

VARIABLES	(1)	(2)	(3)	(4)
	Model Basic Water Sources	Model Total fertility rate	Model Labour	Model Gender Education Gap
Temperature (in logs)	0.003 (0.015)	-0.138* (0.075)	0.236 (0.596)	-0.047* (0.024)
Exports (% GDP)	-0.000 (0.000)	-0.004* (0.002)	-0.012 (0.017)	0.001 (0.000)
Urbanization (% GDP)	0.001 (0.001)	-0.024** (0.010)	0.094 (0.106)	-0.001 (0.002)
Under 5-mortality rate	0.001* (0.000)	0.004 (0.003)	0.021 (0.039)	-0.001** (0.000)
Income per capita (in logs)	-0.065 (0.159)	-2.530*** (0.849)	-27.061* (14.667)	0.397** (0.156)
Income per capita sq. (in logs)	0.008 (0.010)	0.176*** (0.054)	1.574 (0.951)	-0.027** (0.011)
Population(in logs)	-0.018 (0.030)	-1.401*** (0.377)	8.964** (3.775)	0.155** (0.073)
Constant	4.746*** (0.806)	36.636*** (7.066)	9.947 (80.184)	-2.916** (1.165)
Observations	2,283	2,362	1,436	399
R-squared	0.764	0.950	0.965	0.941
Day fixed effects	Yes	Yes	Yes	Yes
Sector fixed effects	Yes	Yes	Yes	Yes

Notes: *** p<0.01, ** p<0.05, * p<0.1.

Notes: ***, **, and * indicate statistical significance at the 1%, 5% and 10% levels respectively.

8 Conclusion and further research

This paper advances a theory integrating two emerging and important topics in the literature, demography and climate change, in order to explain the slow demographic transition in the least developed regions of Africa. It provides a novel mechanism to explain the slow demographic transition and stagnation of the least developed regions. It also characterizes the conditions on population size, availability of local resources, and extreme of climate change through which an environmental disaster will occur within some finite time.

On empirical grounds, we use a database covering 44 African countries over the period 1960 - 2017 to identify the role of temperature on basic local resources and fertility. We find that as the temperature increases, basic water resources go down. By contrast, we show that there is a positive relationship between temperature and fertility. However, the latter result does not hold for the other developing countries.

The global dynamics of the model needs numerical exercises to illustrate the mechanism that leads to slow demographic transition along with the slow technological progress and the persistent gender inequality in education, as well as the mechanism that leads to an environmental disaster. Several hypotheses, further research questions, and policy implications should be addressed based on the theoretical results of the model. These include, but are not limited to, the following: (i) Damage caused by climate change to local resources increases gender inequality in education/income (and hence fertility) through its asymmetric effects on the time allocations between genders; (ii) Is only public investment in education sufficient for triggering development in the least developed regions?; (iii) Using carbon tax revenue to invest in essential infrastructure (e.g. clean water supply and electricity) may trigger development in the least developed regions. These further directions of research are ongoing in our research agenda.

9 Appendix

Proof of Proposition 2

Proof. (i) From equation (22), the strict concavity of $\Omega(R; E)$ in R and the strict convexity of $\psi(R; a)$ in R guarantees that at most two distinct conditional steady states exist.

(ii) We consider first the case where there is only one conditional steady state. Since $\psi(0; a) > \Omega(0; E) = 0$, $\psi(\bar{R}(E); a) > \Omega(\bar{R}(E); E) = 0$, and the strict concavity of $\Omega(R; E)$ in R and the strict convexity of $\psi(R; a)$ in R , this case occurs when the curves $\Omega(R; E)$ and $\psi(R; a)$ are tangent to each other at such a steady state (R, L) . That is to say, at a steady state it holds that $\Omega_R(R; E) = \psi_R(R; a)$. From lemma 2 we have $\Omega_R(R; E) < 0$, and hence from assumption 4 and equation (21) we have $R > \hat{R}(E)$.

In the case that there are two distinct conditional steady states characterized by $0 < \tilde{R} < R < \bar{R}(E)$, let us suppose a negation that $R \leq \hat{R}(E)$. Hence, $\Omega(R; E) > \Omega(\tilde{R}; E)$ while

$\psi(R; a) < \psi(\tilde{R}; a)$. In addition, we have $\Omega(\tilde{R}; E) = \psi(\tilde{R}; a)$. Therefore, $\Omega(R; E) > \psi(R; a)$, which contradicts the property that $\Omega(R; E) = \psi(R; a)$. That is to say, $R > \hat{R}(E)$.

(iii) In order to examine the local stability of steady states, we linearize the dynamic system, which is characterized by two equations (18) and (19), around the steady states. The associated Jacobian matrix appears as

$$J = \begin{pmatrix} 1 - \eta_L^* & \eta^* \psi_R(R^*; a) \\ -1 & 1 + \Omega_R(R^*; E) \end{pmatrix}$$

where $\eta^* = \frac{L^* \rho'(L^*)}{\rho(L^*)}$ is the elasticity of cost of raising children with respect to the population density evaluated at a steady state (R^*, L^*) . By assumption 2 we have $\eta^* \in (0, 1)$.

The trace and determinant of the Jacobian matrix J are respectively

$$\text{Tr}(J) = 1 - \eta^* + 1 + \Omega_R(R^*; E) > 0$$

$$\det(J) = (1 - \eta^*) [1 + \Omega_R(R^*; E)] + \eta_L^* \psi_R(R^*; a)$$

Since $\psi_R(R^*; a) < 0$, it is straightforward that $\text{Tr}(J)^2 > 4 \det(J)$. That is to say the characteristic polynomial

$$C(\lambda) \equiv \lambda^2 - \text{Tr}(J)\lambda + \det(J) = 0$$

has two real distinct eigenvalues $\lambda_1 < \lambda_2$, which satisfy the following properties

$$\lambda_1 + \lambda_2 = \text{Tr}(J) \quad \text{and} \quad \lambda_1 \lambda_2 = \det(J)$$

It is fairly straightforward to prove that $\psi_R(R; a) > \Omega_R(R; E)$ and $\psi_R(\tilde{R}; a) < \Omega_R(\tilde{R}; E)$. So, we have

$$C(1) = \eta_L^* [\psi_R(R^*; a) - \Omega_R(R^*; E)] \begin{cases} > 0 & \text{if } R^* = R \\ < 0 & \text{if } R^* = \tilde{R} \end{cases} \quad (30)$$

(iii.a) **For the steady state with $R^* = R$** : We have $-1 < \Omega_R(R; E) < \psi_R(R; a) < 0$, then $\lambda_1 + \lambda_2 = \text{Tr}(J) \in (0, 2)$ and $\lambda_1 \lambda_2 = \det(J) \in (-1, 1)$. We know that $-1 < \lambda_1 < \lambda_2 < 1$. Indeed, since $\lambda_1 + \lambda_2 < 2$ then $\lambda_1 < 1$. Let us now suppose a negation that $\lambda_1 \leq -1$. In this case $\lambda_2 > 1$ because $\lambda_1 + \lambda_2 > 0$. Hence, $\lambda_1 \lambda_2 < -1$ which contradicts the property $\det(J) > -1$. Therefore, $-1 < \lambda_1 < 1$. Since, at $R^* = R$, $C(1) > 0$, then $\lambda_2 < 1$. In summary, it holds $-1 < \lambda_1 < \lambda_2 < 1$. That is to say, the steady state with $R^* = R$ is a stable node.

(iii.b) **For the steady state with $R^* = \tilde{R}$** : From (30) we have $C(1) < 0$ then $\lambda_1 < 1 < \lambda_2$.

Since $\psi_R(\tilde{R}; a) \geq -1$ then $\det(J) > -1$, therefore

$$C(-1) = 4 + 2\Omega_R(\tilde{R}; E) - 2\tilde{\eta} - \left[\Omega_R(\tilde{R}; E) - \psi_R(\tilde{R}; a) \right] \tilde{\eta} > 2 + \Omega_R(\tilde{R}; E) + \psi_R(\tilde{R}; a) > 0$$

where $\tilde{\eta}_L \in (0, 1)$ is the elasticity of cost raising children with respect to the population density evaluated at the steady state (\tilde{R}, \tilde{L}) . Hence, $\lambda_1 \in (-1, 1)$ while $\lambda_2 > 1$. That is to say, the steady state (\tilde{R}, \tilde{L}) is a stable node. \square

Proof of Proposition 4

Proof. We have for

$$R_{T+1} = R_T + \Omega(R_T, E) - L_T < R_T \quad (31)$$

and, as in (18),

$$L_{T+1} = \gamma(1 - \theta) \left\{ [1 - \phi(R_T, a)]^{\frac{\theta}{\theta-1}} + 1 - \phi(R_T, a) \right\} \frac{L_T}{\rho(L_T)} = \rho(\psi(R_T, a)) \frac{L_T}{\rho(L_T)}$$

because, as defined in (20), $\psi(R_T, a) = \rho^{-1} \left(\gamma(1 - \theta) \left\{ [1 - \phi(R_T, a)]^{\frac{\theta}{\theta-1}} + 1 - \phi(R_T, a) \right\} \right)$.

Hence, by assumption 2, equation (31), and $\psi_R(R, a) < 0$, we obtain

$$L_T = \rho(L_T) \frac{L_T}{\rho(L_T)} < L_{T+1} < \rho(\psi(R_T, a)) \frac{\psi(R_T, a)}{\rho(\psi(R_T, a))} = \psi(R_T, a) < \psi(R_{T+1}, a) \quad (32)$$

We prove next that $L_{T+1} > \Omega(R_{T+1}, E)$. Indeed, it is trivially true for $R_T, R_{T+1} \in (0, \hat{R}(E)]$. For the case $R_T, R_{T+1} \in (\hat{R}(E), \bar{R}(E))$, since $\Omega_R(R, E) < 0$ for $R_{T+1} \in (\hat{R}(E), \bar{R}(E))$ and $L_{T+1} > L_T$, we have

$$\Omega(R_{T+1}, E) = \Omega(R_T + \Omega(R_T) - L_T, E) < \Omega(R_T + \Omega(R_T) - L_{T+1}, E) \quad (33)$$

Let us define the following function

$$G(L_{T+1}, R_T) = L_{T+1} - \Omega(R_T + \Omega(R_T) - L_{T+1}, E) \quad (34)$$

We have

$$G_L(L_{T+1}, R_T) = 1 + \Omega_R(R_T + \Omega(R_T) - L_{T+1}, E) > 0$$

since, by assumption 4, $\Omega_R(R_T, E) > -1$.

Note from (32) that $L_{T+1} > L_T > \Omega(R_T, E)$, hence we have

$$G(L_{T+1}, R_T) > G(\Omega(R_T, E), R_T) = 0 \quad (35)$$

Combining (33), (34), (35), (32), and (31), we have

$$L_T < L_{T+1} \in (\Omega(R_{T+1}, E), \psi(R_{T+1}, a)) \quad \text{and} \quad R_{T+1} < R_T.$$

Thus, by inducing for all $t > T$, we obtain the results stated in proposition 4. \square

Proof of Proposition 5

Proof. We need to prove that for all given a , there exists a unique pair $(R_*, E_*) \in \mathfrak{R}_{++}^2$ such that the following system of equations (36) and (37) holds

$$\Omega(R_*, E_*) - \psi(R_*, a) = 0 \tag{36}$$

$$\Omega_R(R_*, E_*) - \psi_R(R_*, a) = 0 \tag{37}$$

This system of equations tells us that the loci $LL(a)$ and $RR(E)$ are tangent to each other at (R_*, E_*) . Thus, by the strict concavity of $\Omega(R, E)$ in R and the strict convexity of $\psi(R, a)$ in R when $R \in (0, \bar{R}(E))$, the system above implies that

$$LL(a) \cap RR(E_*) = \{(R_*, \psi(R_*, a))\}$$

By assumptions 4 and 5, there always exists $E_* > 0$ such that the equation (36) has at least a solution R_* . By applying the implicit function theorem for the equation (36) with respect to E_* and a , we have E_* to be a function of a ,

$$E_* = E_*(a) \quad \text{with} \quad E'_*(a) = \frac{\psi_a(R_*, a)}{\Omega_E(R_*, E_*)} > 0$$

We now complete the proof by showing that, for all a , there exists a unique R_* making equation (37) holds. Indeed, let us define

$$\Delta(R; a) = \Omega_R(R, E_*(a)) - \psi_R(R, a)$$

We have, for all $R \in (0, \bar{R}(E_*(a)))$,

$$\Delta_R(R; a) = \Omega_{RR}(R, E_*(a)) - \psi_{RR}(R, a) < 0; \quad \Delta(\hat{R}(E_*(a)); a) > 0 \quad \text{and} \quad \Delta(\bar{R}(E_*(a)); a) < 0.$$

Thus, there exists a unique $R_* \in (\hat{R}(E_*(a)), \bar{R}(E_*(a)))$ to be the solution to $\Delta(R; a) = 0$. Hence, there exists a unique pair $(R_*, E_*) \in (0, \bar{R}(E_*)) \times \mathfrak{R}_{++}$ solving the system of equations (36) and (37).

That is to say, for a given a there exists a unique $E_* = E_*(a)$ such that the loci $LL(a)$ and $RR(E)$ are tangent to each other. For any $E > E_*(a)$, $LL(0) \cap RR(E) = \emptyset$. Hence, the evolutionary processes of R_t and L_t , as depicted by the directions of motions in figure 7, will lead the economy to fall into an environmental disaster. □

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