Discussion Paper No. 1248

ISSN (Print) 0473-453X
ISSN (Online) 2435-0982

INTERGENERATIONAL MOBILITY AND STUDENT LOANS

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July 2024

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Abstract

The decision of whether and how much to borrow from the credit market in order to finance education costs depends crucially on parental investment in education. This study constructs a simple two-period overlapping generations model incorporating both educational investment from parents and educational borrowing. The analysis shows that in the case where educational investment from parents and educational borrowing are substitutive, the relaxation of the borrowing constraint improves intergenerational mobility. In the complementary case, the relaxation of the borrowing constraint may impair intergenerational mobility. Implications differ depending on whether the relationship between parental investments and borrowings is substitutive or complementary.

Keywords: Educational borrowing, Intergenerational mobility, Education

JEL Classification Numbers: I22, I24, J62

*I would like to thank Tatsuro Iwaisako, Kohei Okada and the participants at the Japanese Economic Association 2024 Spring Meeting for helpful comments and suggestions. I gratefully acknowledges the financial support of the Joint Usage/Research Centers at the Institute of Social and Economic Research (ISER) and Grants-in-Aid for Scientific Research (20H5631). All remaining errors are my own.

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1 Introduction

As a measure of equality of opportunity, researches have paid attention to intergenerational mobility since the seminar paper by Becker and Tomes (1979). Intergenerational mobility relates to other fundamental concerns, such as income inequality, social welfare, and economic efficiency, for economists and policymakers. A large body of the empirical literature shows that the relationship between intergenerational mobility and income inequality is negatively correlated, which is referred to as the "Great Gatsby Curve" (Blanden, 2013; Corak, 2013, and references therein). Intergenerational mobility matters because if an individual's income is fully determined by her/his family background, income and wealth inequality will persist. If economic status were determined by birth, people would never make an effort to climb the social ladder. It leads to not only a lower competitiveness of society but also to a decline in economic efficiency.

Education is an important factor of intergenerational mobility, as it provides a viable pathway for climbing the social ladder. Recent empirical research emphasizes that educational inequality expands in developed countries. Blanden et al. (2023) has explored the "Educational Great Gatsby Curve," which shows the relationship between educational achievements and income inequality, and the relationship between educational attainments and income inequality.[1] Blanden et al. (2023) has reported that educational attainment does not seem to be positively correlated with income inequality. However, educational achievement is positively correlated with income inequality. These observations imply that expanding income inequality does not necessarily lead to an enlargement of educational attainment gap between the rich and the poor. Income inequality may lead to expansions of the gap of educational achievement between the rich and the poor. What causes the difference behind the observations might be affordability of higher education. Owing to the lack of money, children from low-income families cannot afford to undergo higher education, even if they are academically capable and eager to continue their studies. To fill in the gap regarding economic background, school loans and scholarships provided by government are prevalent in developed countries. Despite govern-

[1] In Blanden et al. (2023), educational achievements and attainments are measured by years of schooling and test scores, respectively.

mental interventions, the gap remains significant. Costs and credit constraints may partly drive these differences (Blanden et al., 2023).

Motivated by the aforementioned idea, the first aim of the present paper is to examine how individuals decide the amount of educational borrowing, considering both parental investment and their abilities. This study constructs a simple two-period overlapping generations (OLG) model incorporating both educational investment from parents and educational borrowing. In the first period of life, individuals make the decision of how much to borrow for educational purposes while facing the credit constraint, after observing parental investments and their abilities. In line with Fan and Stark (2008), Cremer et al. (2010), and Fan and Zhang (2013), we assume that whether an individual receives high wage or low wage depends on the probability. The second aim is to clarify the impact of the borrowing constraints on intergenerational mobility.

The analysis shows that in the case where educational investment from parents and educational borrowing are substitutive, the relaxation of the borrowing constraint improves intergenerational mobility. In the case where educational investment from parents and educational borrowing are complementary, however, the relaxation of the borrowing constraint may impair intergenerational mobility. Implications differ depending on whether the relationship between parental investments and borrowings is substitutive or complementary.

Intergenerational mobility has been studied across various theoretical frameworks. In Owen and Weil (1998) and Garcia-Penalosa and Wälde (2000), children receive bequests from parents and decide whether to take education. They assume that due to the market imperfections, individuals cannot borrow to finance the education cost. In Hendel et al. (2005), individuals can borrow when they decide to undergo education if the fixed educational cost exceeds their received bequests. Educational borrowings in their setting can be interpreted as substitutes for educational investment from parents. However, dynamic complementarities for educational investments have been pointed out by Cunha et al. (2006), Cunha and Heckman (2008), and Cunha et al. (2010). In the present model, we can consider both the substitutability and complementarity of educational investments. In addition, individuals can choose the amount of borrowing unlike in the existing literature.

Student loans serve as a mechanism for mitigating educational inequality. However, due to credit market imperfections, such as a lack of appropriate collateral and income risks, the amount of borrowing tends to be smaller than the optimal level. Friedman (1962) suggests the income-contingent financing of students' investment in human capital. Several countries have already introduced income-contingent repayment programs. Australia started the Higher Education Contribution Scheme (HECS) in 1989, which was later followed by other countries, including Canada, Chile, New Zealand, the UK, and the US.[2] Several studies have analyzed income-contingent loan (hereafter, ICL) in theoretical frameworks. Eckwert and Zilcha (2010) examine how the precision of such screening information affects investment in education, human capital formation, and economic welfare. Del Rey and Racionero (2010) also analyze four types of educational schemes and show that an ICL with risk-pooling can induce the optimal level of participation. Eckwert and Zilcha (2012) consider separately three different forms of government intervention in the market for education loans: unrestricted access to credit markets, unrestricted insurance loans, and restricted insurance loans. The third regime stimulates investment in education and economic growth compared with the first regime. Eckwert and Zilcha (2017) consider the case in which income-contingent education finance coexists with competitive credit markets. They show that funding diversity leads to overinvestment due to adverse selection. These papers have explored the ideal system for educational funding in terms of the amount of investment, welfare, and income inequality. It is essential to examine the impact of the introduction of ICLs on intergenerational mobility. However, none of the literature on ICL examines its effect on intergenerational mobility. The present paper analyzes the impact of ICLs on intergenerational mobility in Section 4.

Some papers have analyzed educational borrowing constraint employing growth model with human capital accumulation. Kitaura (2012) constructs a three-period-lived OLG model in which individuals finance their educational expenditures by borrowing. He shows that if the elasticity of human capital to educational expenditure is great enough, the relationship between the tightness of the constraints and the growth rate is inverted U-shaped when the constraints are binding. Miyazaki (2016) examines how the burden of student loans affects young peo-

[2]See Lleras (2004), Chapman (2006), and Lochner and Monge-Naranjo (2016) for details.

ple's decisions. He shows a case in which the GDP growth rate decreases as the borrowing constraints are relaxed, whereas the growth rate of GDP per capita still increases. These papers especially focus on the impact of the tightness of the credit market for educational investment on economic growth, whereas the present paper aims to examine the effect on intergenerational mobility.

The remainder of this paper is organized as follows: Section 2 explains the model. Section 3 examines the impact on intergenerational mobility. Section 4 introduces the ICL regime in the baseline model and compares the results with those in the baseline regime. Finally, Section 5 concludes the paper.

2 Model

Consider a two-period OLG model. The population of each generation is constant and assumed to be one. Each individual lives for two periods. In the first period, an individual decides the amount of educational borrowing to fund their own education. Education increases the probability of acquiring a high level of human capital as we will comprehensively explain below. In the second period, an individual supplies effective labour and allocates earned wage to consumption, educational investment for their children, and repayment of educational borrowings. Individuals differ in their ability. Each generation consists of high and low abilities. The difference in ability affects the likelihood of acquiring a high level of human capital. Such attribution is random, and the fraction of individuals with high ability is $\lambda \in (0, 1)$.

We assume a small open economy with a constant returns to scale technology. The gross rate of return on capital R_t is exogenously given. The capital-efficiency unit of labour ratio is fully determined by R_t, so the wage rate per efficiency unit of labour $\bar{w}_t$ is also given. We assume that both rates are constant along time, $R_t = R > 0$ and $\bar{w}_t = \bar{w}$ for all t. The number of efficiency units of labour supplied by a worker born in period $t-1$ depends on her level of human capital h_i, which is determined by the probability. If an individual succeeds in education, she becomes a skilled worker and supplies h_s efficiency units of labour. The level

of human capital is given by the following function:

$$h_i = \begin{cases} h_s \text{ if she succeeds in education} \\ h_u \text{ if she fails in education} \end{cases} \tag{1}$$

where $h_s > h_u$. As the wage per efficiency unit is constant, we define the total wage as follows:

$$w_i \equiv \bar{w} h_i. \tag{2}$$

Whether each individual acquires a high level of human capital or not is determined by the probability $P(a_j, b_i^j, e_i^j) = a_j \pi(b_i^j, e_i^j)$. The probability depends on innate ability a_j, educational expenditure by borrowing b_i^j, and parental investment e_i^j. The function $\pi(b_i^j, e_i^j)$ satisfies $\pi_b > 0, \pi_e > 0, \pi_{bb} < 0$, and $\pi_{ee} < 0$, where π_k denotes the partial derivative with respect to k. The probability is an increasing function of educational borrowings, educational investments from parents, and innate ability. This is also a concave function with respect to educational borrowing and educational investment from parents.

2.1 Individuals

An individual draws utility from consumption and educational investments for their children. For tractability, an individual of generation t with ability j and type i parent maximizes the following quasi-linear utility function:

$$U_{i,t}^j = c_{i,t}^j + \gamma_i \ln e_{i,t}^j, \tag{3}$$

where γ_i denotes the preference over educational investment for their children and is assumed that $\gamma_s > \gamma_u$. The assumption reflects that wealthier families tend to place greater emphasis on

their children's education than poorer families. Individuals face three constraints:

$$c_{i,t}^{j} + Rb_{i,t-1}^{j} + e_{i,t}^{j} \leq w_{i}^{j}, \tag{4}$$

$$Rb_{i,t-1}^{j} \leq \theta w_{u}, \tag{5}$$

$$0 \leq b_{i,t-1}^{j}. \tag{6}$$

The first is budget constraint in the second period of life. The second is the borrowing constraint of educational borrowing, where $\theta \in (0, 1)$ denotes the tightness of credit constraint. An individual has access to the credit market and can borrow for educational purposes. We consider a situation where an individual can borrow up to a certain proportion of the minimum wage in the next period. This ensures that the lender never incurs a loss. The third is non-negative constraint for educational borrowing. The amount of educational borrowing cannot be negative. In other words, individual cannot lend money for others.

We solve the maximization problem backwards. First, we derive the optimal allocation in adulthood given the amount of educational borrowing. Then, we solve the maximization problem in childhood, which determines the amount of educational borrowing. From (3) and (4), first-order conditions yield optimal consumption and educational investment:

$$c_{i,t}^{j} = w_{i} - Rb_{i,t-1}^{j} - \gamma_{i}, \tag{7}$$

$$e_{i,t}^{j} = \gamma_{i}. \tag{8}$$

Since we assume that $\gamma_{s} > \gamma_{u}$, it follows that skilled parents invest more than unskilled parents, but the amount of educational investment neither depend on their own abilities or those of their children.

In the first period of life, individuals choose the amount of educational borrowing to maximize the expected indirect utility. Substituting (7) and (8) into (3), we obtain the expected

indirect utility function:

$$\begin{aligned} V_{t,i}^{j} &= P(a_j, b_{i,t-1}^{j}, e_{i,t}) U_{s,t}(b_{i,t-1}^{j}) + [1 - P(a_j, b_{i,t-1}^{j}, e_{i,t})] U_{u,t}(b_{i,t-1}^{j}) \\ &= a_j \pi(b_{i,t-1}^{j}, e_{i,t}) \Omega + (w_u - \gamma_u - R b_{i,t-1}^{j} + \gamma_u \ln \gamma_u), \end{aligned} \tag{9}$$

where $\Omega \equiv (w_s - w_u) - (\gamma_s - \gamma_u) + (\gamma_s \ln \gamma_s - \gamma_u \ln \gamma_u)$. The optimal educational borrowing satisfies the following equation:

$$a_j \pi_b(b_{i,t-1}^{j}, e_{i,t}) \Omega \geq R. \tag{10}$$

When the credit constraint is binding or the optimal educational borrowing is zero, inequality holds. It is clear that the optimal educational borrowing is time independent. In the following, we omit the time subscript.

2.2 Optimal educational borrowing

As we do not specify the functional form of the probability of acquiring a high level of human capital, we cannot derive the solutions for optimal educational borrowings in an explicit form. Based on some assumptions regarding the probability function, we can compare the amount of educational borrowings among individuals using (10). The introduction of parental investment in eduction influences the decision that their children make in the first period of life. The relationship between educational investments from parents and educational borrowing plays a crucial role in determining the extent to which individuals borrow in childhood.

Educational investments from parents and educational borrowings are relatively substitutive if the cross partial derivative with respect to educational investments from parents and educational borrowing is negative, that is, $\pi_{be} < 0$. If educational investments from parents are assumed to be the expenditure for university or college tuition, educational borrowings are viewed as potential substitutes for parental financial contributions. Ultimately, children are not required to borrow money if their parents fully cover tuition. In this case, a larger parental investment decreases the amount of educational borrowing. Educational borrowings of wealth-

ier students are less than those of poorer students. There are two candidates of order in the substitutive case:

$$b_s^l < b_s^h < b_u^l < b_u^h \quad \text{or} \quad b_s^l < b_u^l < b_s^h < b_u^h.$$

The magnitudes of b_s^h and b_u^l depend on the functional form of probability of acquiring a high level of human capital. Figure 1 shows how the amount of borrowing is determined. The downward-sloping curves represent the left-hand side of (10).

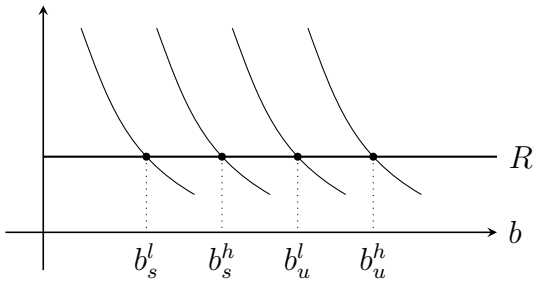

Figure 1: Optimal educational borrowing. In the substitutive case, there are two candidates regarding the order of the amount borrowing for education. We show the first case in this figure. As we have seen (10), the marginal cost of borrowing remains constant. The marginal benefit of borrowing is decreasing in the amount of borrowing. If the interior solution exists, it is depicted in figures.

Parental investment does not affect the amount of educational borrowing if $\pi_{be} = 0$. The amount of borrowing for education is irrelevant to the amount of educational investment from parents. Children who have same ability borrow the same amount in this case, so we obtain

$$b_s^l = b_u^l < b_s^h = b_u^h.$$

Educational investment from parents and educational borrowing are relatively complementary if $\pi_{be} > 0$. The dynamic complementarity of educational investments has been reported by Cunha et al. (2006), Cunha and Heckman (2007), and Cunha et al. (2010). In these papers, the introduction of dynamic complementarity explains a variety of findings established in the child development and child intervention literature. If educational investments from parents are assumed to be expenditures for education at early stages, educational borrowings for higher education can be considered as complementary inputs. A larger $e_{i,t-1}$ increases the amount of

educational borrowing. There are two candidates of order in this case:

$$b_u^l < b_u^h < b_s^l < b_s^h, \quad \text{or} \quad b_u^l < b_s^l < b_u^h < b_s^h.$$

The order of b_u^h and b_s^l is indeterminate unless the functional form of the probability is specified.

3 Intergenerational mobility

Following Iyigun (1999) and Fan and Zhang (2013), we define intergenerational mobility as the odds ratio by

$$M \equiv \frac{P_u^l + P_u^h}{P_s^l + P_s^h} \in [0, 1]. \tag{11}$$

If M is equal to one, equality of opportunity is achieved. Children of any parents have an equal opportunity to acquire a high level of human capital. If M is equal to zero, the economy is unfair in terms of equality of opportunity. Only children who have skilled parents have a chance to acquire a high level of human capital.

To analyze the impact of changes in the tightness of the borrowing constraint on intergenerational mobility, we consider the case in which the borrowing constraint is initially binding for all individuals. In this case, individuals choose the same constant amount of borrowing $\bar{b}$ regardless of the substitutability of investments discussed above. As $P_s^h > P_u^h$ and $P_s^l > P_u^l$, it follows that $M < 1$. In the following subsections, we examine the impact on intergenerational mobility where the tightness θ decreases.

3.1 Substitutive case

Proposition 1. *In the case where educational investment from parents and educational borrowing are substitutive, the relaxation of the borrowing constraint increases intergenerational mobility.*

Proof. From (5), it follows that $\bar{b} = \theta w_u / R$. We can consider intergenerational mobility as a

function of θ, that is, $M(\theta)$. Differentiating $M(\theta)$ with respect to θ, we obtain

$$\frac{\partial M(\theta)}{\partial \theta} = \frac{1}{(P_s^l + P_s^h)^2}\left[\left(\frac{\partial P_u^l}{\partial \theta} + \frac{\partial P_u^h}{\partial \theta}\right)(P_s^l + P_s^h) - (P_u^l + P_u^h)\left(\frac{\partial P_s^l}{\partial \theta} + \frac{\partial P_s^h}{\partial \theta}\right)\right]. \quad (12)$$

When $\pi_{be} < 0$ holds, then $\partial P_u^j/\partial\theta > \partial P_s^j/\partial\theta$, so we obtain

$$\left(\frac{\partial P_u^l}{\partial \theta} + \frac{\partial P_u^h}{\partial \theta}\right)(P_s^l + P_s^h) > (P_u^l + P_u^h)\left(\frac{\partial P_s^l}{\partial \theta} + \frac{\partial P_s^h}{\partial \theta}\right). \quad (13)$$

When (13) holds, $\partial M(\theta)/\partial\theta > 0$. Once the interior solutions achieve, individuals do not change the amount of borrowing any longer. It means that $\partial P_i^j/\partial\theta$ will be zero at some point. In either order of educational borrowings, (13) holds even after some individuals achieve their optimal amount. Thus, (13) holds for all θ. □

The intuition is as follows. Children with wealthy parents require smaller amount of educational borrowing than those with poor parents due to the substitutability. As we have seen in Section 2.2, the interior solutions of poor children are larger than those of wealthy children. An increase in the tightness of the borrowing constraint from a sufficiently small value increases the amount of educational borrowing for both wealthy and poor children. The marginal benefit of increasing educational borrowing for poor children is higher than that for wealthy children. It leads to increase intergenerational mobility. Once the interior solutions are achieved, children no longer change their decisions, even if the borrowing constraint continues to be relaxed. As the tightness of the borrowing constraint relaxes, wealthy children achieve their optimal level of educational borrowing faster than poor children. Even after wealthy children achieve their optimal level, poor children can increase their possibilities until they achieve their optimal amount. Overall, the benefit from a continued increase in the tightness for poor children is greater than that for wealthy children.

3.2 No correlation case

Proposition 2. *In the case where there is no correlation between educational investment from parents and educational borrowings, the relaxation of the borrowing constraint increases in-*

tergenerational mobility.

Proof. When $\pi_{be} = 0$ holds, $\partial P_u^j/\partial\theta = \partial P_s^j/\partial\theta$ always holds. Therefore, (13) can be rewritten by $(P_s^l + P_s^h) \geq (P_u^l + P_u^h)$. It holds for all θ because $M \leq 1$. □

Children with same abilities have the same amount of educational borrowing because the left-hand side of (10) coincides for both types of children. The marginal benefit of borrowing for both types of children coincides as well. In turn, changes in the tightness of the credit market affect the decisions of both children in a same manner. The remaining mechanism of increasing intergenerational mobility is the same as the substitutive case.

3.3 Complementary case

Figure 2 illustrates how individuals decide the amount of borrowing in the complementary case. First, we consider the case in which $b_u^l < b_u^h < b_s^l < b_s^h$ holds, as shown in the left graph in Figure 2. In the region where θ is larger than θ', intergenerational mobility is a decreasing function of θ. In the shaded region, an increase in the tightness of the borrowing constraint only changes the educational borrowing of children with skilled parents. The educational borrowing of children with skilled parents continues to increase until the amount reaches the interior solution. The numerator of (11) remains constant, but that of the denominator still increases, and thus, intergenerational mobility declines. Both educational borrowings change in the remaining region as the tightness of the borrowing constraint increases. Both the numerator and denominator of (11) increase, and therefore, the effect on mobility is ambiguous. Next, we consider the case in which $b_u^l < b_s^l < b_u^h < b_s^h$ holds, as shown in the right graph in Figure 2. In the region where θ is larger than θ'', mobility is a decreasing function of θ. In the rest of the region, the effect on mobility is ambiguous.

3.4 The effect of the expansion of the wage gap

An expansion of the wage gap plays a crucial role when children decide the amount of borrowing. A natural question is how expansions of the wage gap influence children's decisions of educational borrowing. Intuitively, an increase in the wage gap motivates children to borrow

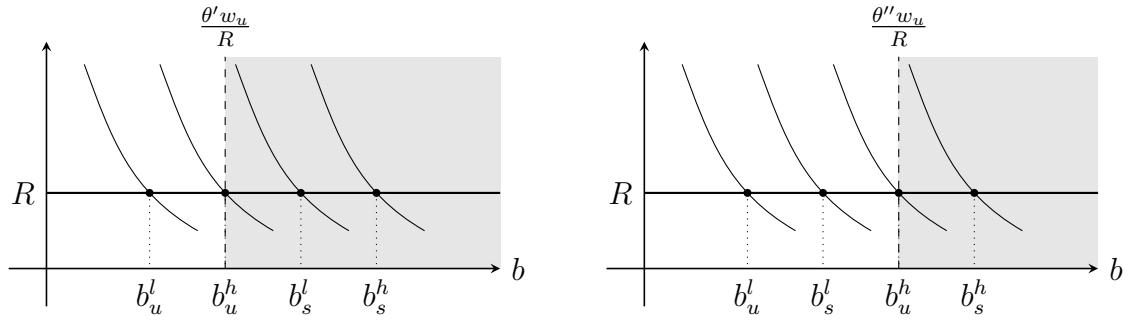

Figure 2: Complementary case. In the shaded region, intergenerational mobility is a decreasing function of θ. The impact on intergenerational mobility in the remaining region is ambiguous.

more, as the marginal benefit of borrowing rises. However, both wealthy and poor children are motivated to increase their borrowing. Thus, the overall effect on intergenerational mobility is unclear.

This subsection examines the effect of changes in the wage gap between skilled and unskilled wages, defined as $\hat{w} \equiv w_s - w_u$.[3][4] Assume that all agents achieve their interior solutions. Differentiating M with respect to $\hat{w}$, we obtain

$$\frac{\partial M(\hat{w})}{\partial \hat{w}} = \frac{1}{(P_s^l + P_s^h)^2}\left[\left(\frac{\partial P_u^l}{\partial \hat{w}} + \frac{\partial P_u^h}{\partial \hat{w}}\right)(P_s^l + P_s^h) - (P_u^l + P_u^h)\left(\frac{\partial P_s^l}{\partial \hat{w}} + \frac{\partial P_s^h}{\partial \hat{w}}\right)\right]. \quad (14)$$

We compare $\left(\partial P_u^l/\partial \hat{w} + \partial P_u^h/\partial \hat{w}\right)$ with $\left(\partial P_s^l/\partial \hat{w} + \partial P_s^h/\partial \hat{w}\right)$ to evaluate whether (14) is positive. Because the interior solutions satisfy (10) with equality, we can obtain the condition where (14) is positive as follows:

$$[a_l \pi_{bb}(b_u^{l*}, e_u)]^{-1} + [a_h \pi_{bb}(b_u^{h*}, e_u)]^{-1} > [a_h \pi_{bb}(b_s^{l*}, e_s)]^{-1} + [a_h \pi_{bb}(b_s^{h*}, e_s)]^{-1} \quad (15)$$

$[a_j \pi_{bb}(b_i^{j*}, e_i)]$ represents the slope of the left-hand side of (10) evaluated at each optimal value. It can be interpreted as the sensitivity in response to changes in the wage gap. If (15) holds, the marginal increase in optimal borrowing for children who have unskilled parents is larger than

[3] An increase in wage gap ensures from three reasons: an increase in w_s, a decrease in w_u, or both. The source of the wage gap does not affect the result because individuals decide the amount of borrowing based on the wage gap itself, not the source of the wage gap.

[4] An introduction of the subsidy for repayment can be considered in the model. However, its effect on intergenerational mobility is the same as an increase in wage gap. The decrease in the RHS of (10) due to the subsidy is equal to the increase in the LHS of (10). Individuals recognize the increase in the wage gap as being equivalent to the subsidy for repayment.

that for wealthy children when the wage gap rises. In other words, children who have unskilled parents are more responsive to the change in the wage gap.

Proposition 3. *If poor children are more responsive to changes in the wage gap than wealthy children, increases in the wage gap improve intergenerational mobility.*

4 Income-contingent loan

Several countries have adopted the income-contingent repayment schemes for educational loans. Switching from the conventional credit market regime to ICL regime, the incentive for borrowers may have changed. Borrowers know that regardless of how much they borrow in childhood, they are not obligated to repay as long as they fail in education and earn a lower wage in adulthood.[5] Thus, the introduction of ICLs may have different implications of intergenerational mobility.

This section examines the impact of the ICL on intergenerational mobility. Consider the following simple ICL scheme for tractability. If an individual succeeds and becomes a skilled worker, her repayment will be the sum of borrowing and interest, Rb_{t-1}. If an individual fails and becomes a unskilled worker, she will repay nothing.[6] To balance total borrowing and repayment in the economy, the government imposes a lump-sum tax T for every agent. Only successful individuals repay their borrowing in the second period.

The budget constraints of borrowers in the second period are rewritten as follows:

$$c_u^j + e_u^j + T \leq w_u, \tag{16}$$

$$c_s^j + Rb_s^j + e_s^j + T \leq w_s. \tag{17}$$

[5]One may concern about moral hazard and adverse selection. In the present setting, moral hazard and adverse selection problems do not arise. Gary-Bobo and Trannoy (2015) examine optimal school loan contract under moral hazard and adverse selection.

[6]This simplification does not seem to be realistic. However, students who use the HECS in Australia do not have to repay if their income is below a certain level.

The optimal condition for educational borrowing is[7]

$$a_j \pi_b(b_i^j, e_i)(\Omega - R b_i^j) = a_j \pi(b_i^j, e_i) R. \tag{18}$$

Let $\tilde{b}_i^j$ and $\hat{b}_i^j$ denote the interior solution in market loan regime and in the ICL regime, respectively. To compare the amount of borrowing in market loan regime with that of the ICL regime, we define the following function:

$$G(b_i^j) \equiv \frac{\pi_b(b_i^j, e_i)}{\pi(b_i^j, e_i)} \left(\Omega - R b_i^j\right) - R. \tag{19}$$

Substituting the optimal amount of borrowing in the market loan regime into (19), we obtain

$$G(\tilde{b}_i^j) = \frac{R}{a_j \Omega \pi(\tilde{b}_i^j, e_i)} \left\{ \Omega[1 - \pi(\tilde{b}_i^j, e_i)] - R\tilde{b}_i^j \right\}. \tag{20}$$

When (20) is positive, we confirm that $\tilde{b}_i^j < \hat{b}_i^j$ because $G(\hat{b}_i^j) = 0$. We can rewrite condition (20) as follows:

$$[1 - a_j \pi(\tilde{b}_i^j, e_i)](\Omega - R\tilde{b}_i^j) > a_j \pi(\tilde{b}_i^j, e_i) R\tilde{b}_i^j. \tag{21}$$

The left-(right-)hand side of (21) means the expected loss (gain) in the ICL regime, evaluated at the optimal amount of borrowing in the market loan regime, if an individual fails to become a skilled worker. An individual needs to repay the loan in exchange for becoming a skilled worker. It can be interpreted as a gain from the failure to become a skilled worker because she does not have to repay the loan. When the loss is larger than the gain, valued at optimal borrowing in the market loan regime, the shift therefrom to the ICL regime increases the amount of borrowing. When (21) holds, the expected loss of utility is larger than the gain. Individuals can reduce their expected loss and increase the gain by increasing the borrowing in the ICL regime.

[7] In addition to the condition, $a_l \pi(b^*, e_u)(\Omega_l - R b^*) \geq T$ is required because when an individual borrows nothing, she does not have to pay the lump-sum tax. It means that the expected gain of borrowing is larger than the expected amount of the lump-sum tax even if the individual has a lower innate ability. If this condition holds, the individual will choose a positive amount of borrowing in equilibrium.

In contrast, if (21) violates, individuals will reduce the educational borrowing in response to the change of regime. Intuitively, the shift from the market loan regime to the ICL regime seems to increase the amount of educational borrowing. However, in the ICL regime, an increase in the amount of educational borrowing reduces the benefit of becoming a skilled worker because only skilled workers have the obligation to repay the borrowing. In contrast to the ICL regime, both skilled and unskilled workers have the obligation to repay the borrowing in the market loan regime. Therefore, the shift from the market loan regime to the ICL regime might decrease the amount of educational borrowing under some conditions.

Without the specification of the probability function, the impact of intergenerational mobility is unclear. To evaluate the impact on intergenerational mobility, we consider two functional forms. In the first case, we assume that $P_i^j(a_j, b_i^j, e_i) = a_j[1 - (b_i^j + e_i)^{-1}]$. From (10), we can derive the probability where individuals borrow the optimal amount in the market loan regime as follows:

$$\tilde{P}_i^j = 1 - \left(\frac{R}{a_j \Omega}\right)^{\frac{1}{2}}. \tag{22}$$

The probability does not depend on the amount of parental investments. Thus, intergenerational mobility in the market loan regime is 1. From (18), we derive the probability in the ICL regime,

$$\hat{P}_i^j = 1 - \left(\frac{R}{\Omega + Re_i}\right)^{\frac{1}{2}}. \tag{23}$$

In contrast to the market loan regime, the probability does not depend on their abilities. Thus, intergenerational mobility in the ICL regime is less than 1. In this case, the shift from the market loan regime to the ICL regime reduces intergenerational mobility. Second, we assume that $P_i^j(a_j, b_i^j, e_i) = a_j (b_i^j)^{\alpha} e_i^{1-\alpha}$. To ensure the probability $P_i^j < 1$, we assume that $b_i^j < a_j^{-1/\alpha} \gamma_i^{(\alpha-1)/\alpha}$. From (10) and (18), we derive the optimal amount of borrowings, $\tilde{b}_i^j = (a_j \alpha / R)^{1/(1-\alpha)}$ and $\hat{b}_i^j = \alpha\Omega/[(1+\alpha)R]$. Substituting optimal borrowings into (11), we obtain $M_{ICL} = (\gamma_u/\gamma_s)^{1-\alpha}$ and $M_{ML} = \gamma_u/\gamma_s$. Because $\gamma_u < \gamma_s$, it follows that $M_{ICL} > M_{ML}$. Thus, the shift from the market loan regime to the ICL regime leads to improved intergenerational mobility in the Cobb-Douglas case.

5 Conclusion

The decision of whether and how much to borrow for educational costs from the credit market crucially depends on parental investment in education. In this study, taking parental investment into consideration, we examine how changes in the tightness of the credit market for educational investment affect the decision of educational borrowing for children. We construct a simple two-period OLG model with both parental investment in education and educational borrowing. The analyses show that in the case where educational investment from parents and educational borrowing are substitutive, the relaxation of the borrowing constraint increases intergenerational mobility. In the case where educational investment from parents and educational borrowings are complementary, however, the relaxation of the borrowing constraint may decrease intergenerational mobility. In reality, a significant number of students borrow due to the costs associated with higher education. In this context, both forms of educational investment appear to be substitutive. Thus, the relaxation of the borrowing constraint improves intergenerational mobility.

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