

**THE EXPORTER AND PRODUCTIVITY  
DYNAMICS: THE EFFECT OF TRADE  
LIBERALIZATION**

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# The Exporter and Productivity Dynamics: The Effect of Trade Liberalization\*

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## Abstract

This paper studies how investment in R&D and export technology amplifies the welfare gains from trade liberalization. I develop a dynamic heterogeneous firm international trade model with investment in productivity-enhancing R&D and export technology. I find that R&D investment combined with a dynamic export technology enhances the welfare gains from trade liberalization. I quantitatively demonstrate that the welfare gain from trade liberalization in the dynamic trade model with elastic R&D is above 30% higher than that with inelastic R&D. By contrast, in a static trade model, the elasticity of R&D has a small impact on welfare gain. These findings suggest that static trade models may provide an even poorer approximation of dynamic trade models than we thought.

JEL Classification: F1, F4, O3

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# 1 Introduction

Recently, researchers have emphasized the importance of two firm-level dynamics in international trade: dynamic export decisions and endogenous productivity dynamics driven by R&D investment. A growing body of literature supports the positive effect of exports on a firm’s productivity. Another strand of literature argues the importance of firms’ dynamic export decisions to account for the gradual trade expansion. Although some paper examines the role of productivity dynamics in the canonical framework of a static trade model, such as [Melitz \(2003\)](#), the interaction between the firm’s dynamic export decision and productivity-enhancing R&D investment has not been explored<sup>1</sup>.

This paper quantifies the welfare gains from trade liberalization by incorporating two key margins: firms’ dynamic export decision (exporter dynamics) and productivity-enhancing R&D investment (productivity dynamics). To quantify the welfare gains, I develop a continuous-time general equilibrium international trade model with heterogeneous firms that incorporates both dynamics. The model builds on the canonical framework of heterogeneous firm international trade model, such as [Melitz \(2003\)](#), while I also integrate the exporter dynamics from [Alessandria and Choi \(2014\)](#) and productivity dynamics from [Atkeson and Burstein \(2010\)](#).

The model relies on three key assumptions. The first two are sunk export entry cost and lag before exporting, as in [Alessandria and Choi \(2014\)](#), which together generate the dynamic export decision. Firms invest in exporting by paying irreversible sunk costs to start exporting that are higher than the continuation costs. In addition to the sunk startup cost, there is a lag before exporting, which captures the gradual expansion of trade following trade liberalization. This assumption is typically embedded in the discrete-time model, but here it is newly incorporated into a continuous-time framework<sup>2</sup>. Due to these structures, firms initially enter the market as non-exporters. In the following discussion, I refer to the “Sunk-cost model” as the model with both sunk costs and a lag before exporting, while the “Fixed-cost model” refers to a static trade model without them.

The third assumption is that firms invest in R&D to enhance productivity. Firms face persis-

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<sup>1</sup>For example, [Costantini and Melitz \(2008\)](#), [Atkeson and Burstein \(2010\)](#), [Rubini \(2014\)](#), and [Impullitti and Licandro \(2018\)](#) examine the role of R&D investment in static export models.

<sup>2</sup>In a continuous-time model without a lag, firms begin exporting immediately after paying the startup cost.

tent productivity shocks that are endogenously determined by their R&D investment, generating productivity dynamics. R&D investment enhances the gain and intensifies competition from trade following trade liberalization. Exporters raise their R&D investment since higher productivity leads to higher profit under lower trade costs. In contrast, non-exporters face increased competition from both productive foreign and domestic firms. In the simulation, I compare the model with high R&D elasticity to one with low elasticity to clarify the importance of R&D investment.

I find that the elasticity of R&D investment affects the welfare gains from trade liberalization in the Sunk-cost model. By calibrating the model to fit the U.S. data, I show that the Sunk-cost model with high R&D elasticity generates about 55% larger long-run change in consumption and over 30% greater welfare gain than the Sunk-cost model with low R&D elasticity.

This result depends on the free entry conditions. In the Sunk-cost model, the entrants discount the potential gain from export since they cannot start exporting immediately after firm creation, and the sunk startup cost is required to access the gain from exporting. Conversely, entrants do not discount loss from the competition with productive firms and foreign exporters. This generates a substantial decline in the number of entrants, causing labor to be substituted away to production and R&D investment in the Sunk-cost model.

This finding contrasts with the result from Fixed-cost models. In the Fixed-cost model, differences in the elasticity of R&D have a small effect on the long-run change in consumption and welfare gain from trade liberalization as discussed in [Atkeson and Burstein \(2010\)](#). In the Fixed-cost model, entrants do not discount the gain from export. The increased gain from exporting provides a stronger incentive to start a business in the Fixed-cost model, and the labor is not substituted away from firm creation toward production and R&D investment.

In addition to labor substitution, the slow adjustment of trade and productivity, as well as the decline in investment in firm creation, are important to understand the welfare gains in the Sunk-cost models. Slow trade adjustment leads to a gain from trade liberalization in the long run, while it does not enhance short-run consumption. Slow productivity adjustment has a similar effect as slow trade adjustment. Moreover, the investment in R&D and export technology results in a short-run decline in consumption.

R&D investment has a compositional effect on trade dynamics in the Sunk-cost model. The elastic R&D investment raises the export participation rate but reduces the exporters' productivity

premium in the Sunk-cost model. Because of the difference between startup and continuation costs, the thresholds for starting and stopping exports differ. Exporters who are more likely to stop exporting increase their R&D investment to continue exporting and avoid paying the startup cost again. These less productive exporters tend to invest more in R&D than highly productive exporters, which increases the export participation rate while reducing the exporters' productivity premium.

This paper is related to several strands of literature on international trade. The first relevant body of work focuses on the aggregate effect of trade barriers, such as [Arkolakis et al. \(2012\)](#), based on [Krugman \(1980\)](#), [Eaton and Kortum \(2002\)](#), and [Melitz \(2003\)](#). Some studies (e.g., [Alessandria and Choi \(2014\)](#) and [Alessandria et al. \(2021\)](#)) highlight the importance of exporter dynamics to understand gain from trade<sup>3</sup>. [Impullitti et al. \(2013\)](#) develops the continuous-time model with sunk costs, based on [Dixit \(1989\)](#) and [Das et al. \(2007\)](#), but without R&D investment. I incorporate the assumption of R&D investment into this dynamic trade model and show that R&D investment amplifies the welfare gains from trade liberalization.

The second relevant strand of literature examines the relationship between trade and innovation. Research using micro-level data often indicates that international trade has a positive effect on firms' R&D investment and productivity. For instance, [Alvarez and López \(2005\)](#), [Aw et al. \(2007\)](#), [Aw et al. \(2011\)](#), [Loecker \(2013\)](#), and [Atkin et al. \(2017\)](#) find that firms' productivity or R&D investment increases when they engage in exporting. Also, [Lileeva and Trefler \(2010\)](#), [Bustos \(2011\)](#), and [Bloom et al. \(2016\)](#) show that the reduction in trade costs increases the firms' investment and productivity. Motivated by these micro-level findings, this paper develops the dynamic trade model with R&D investment. [Atkeson and Burstein \(2010\)](#) show that R&D investment does not significantly contribute to increasing the gains from trade cost reduction in a general equilibrium model with heterogeneous firms. In contrast, I demonstrate that R&D investment enhances the gains from trade liberalization not only at the firm level but also at the aggregate level, provided that the model includes exporter dynamics. Other studies, such as [Sampson \(2016\)](#); [Impullitti and Licandro \(2018\)](#); [Perla et al. \(2021\)](#), also find that trade cost reductions positively affect the gains when firms can invest in R&D within static trade models with knowledge diffusion within a country

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<sup>3</sup>Some studies argue that the trade volume gradually expands following the reduction in trade cost, and the short-run trade elasticity is lower than the long-run trade elasticity (e.g., [Baier and Bergstrand \(2007\)](#), [Yilmazkuday \(2019\)](#); [Boehm et al. \(2023\)](#)). This finding is consistent with the prediction of the Sunk-cost model.

or oligopolistic competition. In addition to these findings, this paper emphasizes the importance of exporter dynamics for the welfare gains. Another important perspective in trade and growth is technology diffusion across countries (e.g., [Grossman and Helpman \(1993\)](#); [Buera and Oberfield \(2020\)](#)). This paper does not incorporate technology diffusion and the welfare gains that arise from domestic reallocation.

This paper is organized as follows. Section 2 presents the heterogeneous firm international trade model incorporating exporter and productivity dynamics. Section 3 discusses the calibration and the model fit. In section 4, I analyze the model dynamics following trade liberalization and assess the welfare gains. Section 5 concludes.

## 2 The model

I build a dynamic general equilibrium model with firm and export dynamics in the spirit of [Alessandria and Choi \(2014\)](#). I extend a simplified version of their model to continuous time and allow for the firms' R&D investment<sup>4</sup>. I have to introduce a lag before exporting when I extend their model to a continuous-time model in order to fill the discrepancy between the discrete-time and continuous-time Sunk-cost models. The lag before exporting is inherently embedded in discrete time, while the continuous-time model does not have this lag before exporting (e.g. [Impullitti et al. \(2013\)](#)). The lag before exporting is associated with the uncertainty of export investment and is important to generate a larger and more gradual export expansion.

There are two symmetric countries, *home* and *foreign*. Each country has identical, infinitely-lived consumers that supply  $L$  units of labor inelastically. Each country also has representative final goods firms, heterogeneous intermediate goods firms, and a government. All intermediate goods firms sell the product to their own country and some export. The intermediate goods firms are heterogeneous with respect to productivity,  $z_t$ , export states,  $m_t$ , and the state of waiting,  $w_t$ . The waiting state implies that intermediate goods firms cannot start exporting immediately after deciding to export, and there is a lag before they start exporting, leading to a gradual adjustment in trade, as [Alessandria and Choi \(2014\)](#). Here, firms in the waiting state,  $w_t = 1$  are those that have chosen to switch their export status, yet the state,  $m_t$ , remains unchanged. The measure of

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<sup>4</sup>I exclude the capital, intermediate inputs, and non-tradable sector to emphasize the importance of R&D investment. The benefit of using continuous-time models is discussed in [Achdou et al. \(2022\)](#).

home intermediate goods firms with productivity,  $z_t$ , export status,  $m_t = 0$  for non-exporters and  $m_t = 1$  for exporters, and waiting status,  $w_t = 0$  for non-waiting firms, and  $w_t = 1$  for waiting firms, is given by  $g_t(z, m, w)$ .

## 2.1 Consumers

Home consumers solve the following maximization problem:

$$\begin{aligned} \max_{C_t} \int_0^\infty e^{-\rho t} U(C_t) dt, \\ C_t \leq W_t L + \Pi_t + T_{g,t}, \end{aligned}$$

where  $\rho \in (0, 1)$  is the subjective discount factor. Here,  $C_t$  is the final goods consumption. In the budget constraint,  $W_t$  is the real wage rate,  $L$  is the inelastic labor supply,  $\Pi_t$  is the real transfer from home producers, and  $T_{g,t}$  is the real lump-sum transfer of local tariff revenue. Let home final goods price,  $P_t$ , be numeraire, which is omitted from the budget constraint. The foreign consumer's problem is analogous. Foreign prices and allocations are denoted with an asterisk.

## 2.2 Final goods firms

Final goods are produced by combining home and foreign intermediate goods. A home final goods firm purchases inputs from all home intermediate goods firms and some foreign intermediate goods firms exporting to the home market. The aggregation technology is a CES function,

$$Y_t = \left\{ \int \sum_{m=0}^1 \sum_{w=0}^1 y_{H,t}(z, m, w)^{\frac{\theta-1}{\theta}} g_t(z, m, w) dz + \int \sum_{w=0}^1 y_{F,t}(z, 1, w)^{\frac{\theta-1}{\theta}} g_t^*(z, 1, w) dz \right\}^{\frac{\theta}{\theta-1}} \quad (1)$$

where  $y_{H,t}$  and  $y_{F,t}$  are the inputs of intermediate goods purchased from home intermediate goods firms and foreign exporters, respectively. The elasticity of substitution between intermediate goods is  $\theta > 1$ .

The final goods market is competitive. A final goods firm maximizes the profit,

$$\begin{aligned} \max_{y_{H,t}, y_{F,t}} Y_t - \int \sum_{m=0}^1 \sum_{w=0}^1 p_{H,t}(z, m, w) y_{H,t}(z, m, w) g_t(z, m, w) dz \\ - \int \sum_{w=0}^1 (1 + \tau_t) p_{F,t}(z, 1, w) y_{F,t}(z, 1, w) g_t^*(z, 1, w) dz \end{aligned} \quad (2)$$

subject to the production technology equation (1). Here,  $p_{H,t}$  and  $p_{F,t}$  are the price of intermediate goods produced by home goods firms with  $(z, m, w)$  and foreign exporters with  $(z, 1, w)$ , respectively. Solving the problem in equation (2) yields the input demand functions,

$$y_{H,t}(z, m, w) = [p_{H,t}(z, m, w)]^{-\theta} Y_t, \quad (3)$$

$$y_{F,t}(z, 1, w) = [(1 + \tau_t) p_{F,t}(z, 1, w)]^{-\theta} Y_t, \quad (4)$$

where the price index is

$$P_t^{1-\theta} = \int \sum_{m=0}^1 \sum_{w=0}^1 p_{H,t}(z, m, w)^{1-\theta} g_t(z, m, w) dz + \int \sum_{w=0}^1 [(1 + \tau) p_{F,t}(z, 1, w)]^{1-\theta} g_t^*(z, 1, w) dz.$$

I assume that the home final good is the numeraire and  $P_t = 1$ .

### 2.3 Intermediate goods firms

The intermediate goods firms produce their differentiated goods using labor. Exporting incurs fixed and variable costs. There is an ad valorem tariff,  $\tau_t$ , and an ad valorem transportation cost,  $\xi$ . Exporters pay the flow cost,  $f_1$ , to continue to export. When non-exporters start to export, they pay the flow startup cost,  $f_0 > f_1$ , while they are waiting,  $w_t = 1$ . The decision to start waiting is irreversible<sup>5</sup>. To resume exporting, firms have to pay  $f_0$  again. These costs are valued in units of domestic labor. While firms are waiting, they can switch their export state,  $m_t$ , with an exogenous Poisson arrival rate,  $\gamma$ . I also suppose that the intermediate goods firms exit the market with an exogenous death rate,  $n_d(z)$ , conditional on the current productivity. Incumbents receive an exogenous death shock that depends on a firm's productivity,  $z_t$ ,  $0 \leq n_d(z) \leq 1$ . The

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<sup>5</sup>I assume that firms also wait when they stop exporting. This simplifies the computation, and it does not matter for the result qualitatively.



intermediate goods firms are described by  $s = (z, m, w)$ .

The incumbent's productivity,  $z_t$  follows,

$$dz_t = \mu_t(s)dt + \sigma d\mathcal{W}_t, \quad (5)$$

where  $\mu_t(s)$  is the endogenous drift,  $\sigma$  is the standard deviation, and  $\mathcal{W}_t$  denotes the Wiener process.

The endogenous drift,  $\mu_t(s)$  depends on R&D investment of intermediate goods firm,  $q_t(s)$ ,

$$\mu_t(s) = q_t(s) - \alpha z_t. \quad (6)$$

Here,  $\alpha$  denotes the exogenous drift of productivity. The incumbent firms invest  $q_t(s)$  in their productivity by hiring  $c_t(s)$  units of labor to increase future productivity<sup>6</sup>.

Each period, they choose current prices for each market,  $p_{H,t}(s)$  and  $p_{H,t}^*(s)$ , labor,  $l_t(s)$ , investment in R&D,  $q_t(s)$ , and timing to start waiting,  $T_t(z, m, 0)$ , to maximize the value function,  $V_t(s)$ , related to the profit,  $\pi_t(s)$ .

The firms have a linear production technology,  $y_t(s) = y_{H,t}(s) + m_t(1 + \xi)y_{H,t}^*(s) = e^{z_t}l_t(s)$ . Non-waiting firms solve the following stopping time problem subject to equation (5),

$$V_0(z, m, 0) = \max_{p_{H,t}, p_{H,t}^*, l_t, q_t, T_t} \left\{ \mathbf{E}_0 \int_0^{T_t(z, m, 0)} e^{-(\int_0^t r_v dv)t} (\pi_t(z, m, 0)) dt + e^{-(\int_0^{T_t} r_v dv)} \hat{V}_{T_t(z, m, 0)} \right\}.$$

where  $r_t = \rho + n_d(z)$  denotes the discount rate of firms and  $\hat{V}$  is the value of switching the state,  $w_t$ <sup>7</sup>. The first term on the right-hand side is value to continue the current state. The second term on the right-hand side is the value to switch status from non-waiting,  $w_t = 0$  to waiting,  $w_t = 1$ .

Waiting firms solve the following Hamilton-Jacobi-Bellman equation,

$$\begin{aligned} r_t V_t(z, m, 1) = & \max_{p_{H,t}, p_{H,t}^*, l_t, q_t} \pi_t(z, m, 1) + \mu_t(z, m, 1) \frac{\partial V_t(z, m, 1)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 V_t(z, m, 1)}{\partial z^2} \\ & + \gamma(V_t(z, -m, 0) - V_t(z, m, 1)) + \frac{\partial V_t(z, m, 1)}{\partial t}. \end{aligned}$$

Here,  $-m$  denotes the opposite export status of  $m$ . The second and third terms on the right-hand

<sup>6</sup> Atkeson and Burstein (2010) assume the jump productivity process. In this paper, I consider the continuous productivity process to examine the continuous change in export thresholds.

<sup>7</sup> This equation can be rewritten to Hamilton-Jacobi-Bellman variational inequality (HJBVI, henceforth). Noda and Teramoto (2024) use the combination of HJB and HJBVI for the analysis of panic buying.

side represent the value of the change in their productivity. The fourth term on the right-hand side represents the probability of switching the export state,  $m_t$ . With the Poisson arrival rate,  $\gamma$ , waiting firms switch their export state,  $m_t$  and they become non-waiting firms with  $w_t = 0$ . The last term denotes the value from a change in time,  $t$ .

The intermediate goods firms' profit,  $\pi_t(s)$  is given by

$$\pi_t(s) = p_{H,t}(s)y_{H,t}(s) + m_t p_{H,t}^*(s)y_{H,t}^*(s) - W_t l_t(s) - W_t c_t(s) - m_t W_t f_1 - w_t(1 - m_t)W_t f_0.$$

Both optimization problems are subject to the productivity process in equation (5), and the constraint that supplies to home and foreign goods markets,  $y_{H,t}(s)$  and  $y_{H,t}^*(s)$  with  $y_t(s) = y_{H,t}(s) + m(1 + \xi)y_{H,t}^*(s)$ , are equal to demands by final good firms from equation (3) and the foreign analogue of equation (4). The firms with  $m_t = 1$  pay the continuation cost to export,  $f_1$ , regardless of the state  $w_t$ . The firms with  $m_t = 0$  and  $w_t = 1$  pay the startup cost to export,  $f_0$ .

## 2.4 Entry

The new intermediate goods firms are created by hiring  $f_E$  workers. The entrants draw their productivity from the distribution,  $g_E(z)$ . The entry condition is

$$V_t^E = -W_t f_E + \int V_t(z, 0, 0)g_E(z)dz \leq 0. \quad (7)$$

Entrants cannot export immediately after they start their business; their export state is  $m = 0$ , and their waiting state is  $w = 0$ .

## 2.5 Government

The government collects tariffs and redistributes the revenue lump sum to domestic consumers. The government's budget constraint is

$$T_{g,t} = \tau_t \int \sum_{w=0}^1 p_{F,t}(z, 1, w)y_{F,t}(z, 1, w)g^*(z, 1, w)dz. \quad (8)$$

## 2.6 Evolution of intermediate goods firms' distribution

The mass of entrants in period  $t$  is  $N_{E,t}$ . The mass of exporters equals  $N_{1,t} = \int \sum_{w=0}^1 g_t(z, 1, w) dz$  and the mass of non-exporters equals  $N_{0,t} = \int \sum_{w=0}^1 g_t(z, 0, w) dz$ . The mass of intermediate goods firms equals  $N_t = N_{1,t} + N_{0,t}$ . The fixed costs of exporting imply that only a fraction  $n_{x,t} = N_{1,t}/N_t$  of home intermediate goods are available in the foreign country in period  $t$ . The starter ratio, the fraction of firms who start exporting among non-exporters, and the stopper ratio, the fraction of firms who stop exporting among exporters, are denoted by  $n_{0,t}$  and  $n_{1,t}$  respectively.

The dynamics of non-waiting and waiting firms' distribution are given by the Kolmogorov forward (KF) equations,

$$\begin{aligned}\partial_t g_t(z, m, 0) &= -\partial_z[\mu_t(z, m, 0)g_t(z, m, 0)] + \frac{1}{2}\partial_{zz}[\sigma^2 g_t(z, m, 0)] - n_d(z)g_t(z, m, 0) + \gamma g_t(z, -m, 1) - \hat{\gamma}g_t(z, m, 0), \\ \partial_t g_t(z, m, 1) &= -\partial_z[\mu_t(z, m, 1)g_t(z, m, 1)] + \frac{1}{2}\partial_{zz}[\sigma^2 g_t(z, m, 1)] - n_d(z)g_t(z, m, 1) - \gamma g_t(z, m, 1) + \hat{\gamma}g_t(z, m, 0),\end{aligned}$$

where  $\partial_z$  and  $\partial_{zz}$  denotes the first and second order derivatives with respect to  $z$ , respectively.  $\hat{\gamma}$  denotes the transition from  $w = 0$  to  $w = 1$ , which depends on  $s$ . Since entrants begin their business as non-exporters, the distribution of firms with  $m = 0$  and  $w = 0$  is

$$\begin{aligned}\partial_t g_t(z, 0, 0) &= -\partial_z[\mu_t(z, 0, 0)g_t(z, 0, 0)] + \frac{1}{2}\partial_{zz}[\sigma^2 g_t(z, 0, 0)] - n_d(z)g_t(z, 0, 0) \\ &\quad + \gamma g_t(z, 1, 1) - \hat{\gamma}g_t(z, 0, 0) + N_{E,t}g_E(z).\end{aligned}$$

## 2.7 Aggregate variables

Nominal exports and imports equal

$$\begin{aligned}EX_t^N &= \sum_{w=0}^1 \int p_{H,t}^*(z, 1, w) y_{H,t}^*(z, 1, w) g_t(z, 1, w) dz, \\ IM_t^N &= \sum_{w=0}^1 \int p_{F,t}(z, 1, w) y_{F,t}(z, 1, w) g_t^*(z, 1, w) dz,\end{aligned}$$

respectively.

Let  $IMY_t$  be the expenditure on imported goods relative to that on home goods,

$$IMY_t = \frac{(1 + \tau_t) \sum_{w=0}^1 \int p_{F,t}(z, 1, w) y_{F,t}(z, 1, w) g_t^*(z, 1, w) dz}{\sum_{m=0}^1 \sum_{w=0}^1 \int p_{H,t}(z, m, w) y_{H,t}(z, m, w) g_t(z, m, w) dz}.$$

I define the share of tradable expenditures on domestic goods and the trade elasticity as

$$\lambda_t = \frac{1}{1 + IMY_t}, \quad (9)$$

$$\varepsilon_{T,t} = -\frac{\ln(IMY_t/IMY_{-1})}{\ln((1 + \tau_t)/(1 + \tau_{-1}))}, \quad (10)$$

where variables with subscription  $-1$  denote the initial steady state values.

Production labor,  $L_{P,t}$ , and Research labor,  $L_{R,t}$ , are, respectively

$$\begin{aligned} L_{P,t} &= \sum_{m=0}^1 \sum_{w=0}^1 \int l_t(z, m, w) g_t(z, m, w) dz, \\ L_{R,t} &= \sum_{m=0}^1 \sum_{w=0}^1 \int c_t(z, m, w) g_t(z, m, w) dz. \end{aligned} \quad (11)$$

The domestic labor hired by exporters to cover the fixed costs of exporting,  $L_{X,t}$ , and the domestic labor hired to create new firms,  $L_{C,t}$ , are, respectively,

$$\begin{aligned} L_{X,t} &= f_0 \int g_t(z, 0, 1) dz + f_1 \int \sum_{w=0}^1 g_t(z, 1, w) dz, \\ L_{C,t} &= f_E N_{E,t}. \end{aligned}$$

Aggregate profits equal profits minus entry costs,

$$\Pi_t = \int \sum_{m=0}^1 \sum_{w=0}^1 \pi_t(z, m, w) g_t(z, m, w) dz - W_t L_{C,t},$$

which is transferred to home consumers. Since the final goods market is competitive, the profit of the final goods firm is zero<sup>8</sup>.

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<sup>8</sup>If I assume that consumers own firms, it reduces the entry in the short run following trade liberalization, while the result does not change in the long run.

## 2.8 Equilibrium

In an equilibrium, variables satisfy several resource constraints. The market clearing conditions are:

1. Final goods market  $Y_t = C_t$ .
2. Labor market  $L = L_{p,t} + L_{r,t} + L_{X,t} + L_{C,t}$ .
3. Government budget constraint given by equation (8).

The foreign analogues also hold. The firm's profits are distributed to the consumers,  $\Pi_t$ , and foreign analogue. Writing the budget constraints in local currency units allows us to normalize the consumption price in each country as  $P_t = P_t^* = 1$ .

An equilibrium is a collection of allocations for home and foreign consumers,  $C_t, C_t^*$ ; allocation for home and foreign final good firms; allocations, prices, R&D investment, and export decisions for home and foreign intermediate good firms; labor used for exporting costs at home and foreign; labor used for entry costs at home and foreign; transfers,  $T_{g,t}, T_{g,t}^*$  by home and foreign governments; real wages,  $W_t, W_t^*$  that satisfy the following conditions: (i) the consumer allocations solve the consumer's problem; (ii) the final good firms' allocation solves their profit maximization problem; (iii) the intermediate good firms' allocations, prices, R&D investment, and export decisions solve their profit maximization problems; (iv) the entry conditions for intermediate good firms hold; (v) the market clearing conditions hold; and (vi) the transfers satisfy the government budget constraint.

## 3 Calibration

This section describes the functional forms, parameter values, and calibration strategy. The instantaneous utility function equals  $U(C) = \log(C)$ . The cost function of R&D investment is  $c_t = e^{(\theta-1)z_t + \phi q_t} / \phi$ , where  $1/\phi$  is the elasticity of R&D investment. With this functional form, the drift of productivity  $\mu_t(s)$  becomes mean reversion, and the distribution of productivity does not diverge. Since the cost depends on the firms' productivity, the investment cost gets larger as firms are larger. An entrant draws productivity from the unconditional distribution,  $z = \mu_E + \varepsilon_E$ ,  $\varepsilon_E \stackrel{iid}{\sim} N\left(0, \frac{\sigma^2}{2\hat{\alpha}}\right)$ , where  $\mu_E < 0$  is the entrants' disadvantage and set to match the fact that entrants are smaller than incumbents.  $\hat{\alpha}$  denotes the parameter of entrant distribution.

The subjective discount factor is 4% in annual frequency,  $\rho = 0.04$ , and labor supply,  $L$ , is set to 1. The elasticity of substitution,  $\theta$ , is set to 5, which yields a producer markup of 25% and is consistent with [Broda and Weinstein \(2006\)](#). The entry cost,  $f_E$ , is chosen to set the mass of the establishments of the initial steady state to be 1. The Poisson arrival rate,  $\gamma$ , is set to 1, which implies that the average waiting time to start and stop exporting is one year<sup>9</sup>. The baseline tariff rate,  $\tau$ , is set to 8%.

The R&D elasticity,  $\phi$ , is the important parameter of the model, which determines how firms change their R&D investment after trade liberalization. In this paper, instead of estimating the value of it, I compare two different values of  $\phi$  to clarify the importance of R&D investment. I set  $\phi = 20$  for the high R&D elasticity and  $\phi = 100$  for the low R&D elasticity. I refer to the model with  $\phi = 20$  as the model with high R&D elasticity and the model with  $\phi = 100$  as the model with low R&D elasticity. To make the initial steady state of all variants match the firm distributions of data, I calibrate  $\alpha$  to match the data firm distribution. Hence, the models may have different R&D investments,  $q_t$ , but in the initial steady state, all models have similar firm distributions and similar endogenous drift,  $\mu_t(s)$ . I also calibrate the models without R&D investment, which is described in the appendix. The parameters of entrant distribution,  $\hat{\alpha}$ , are set to 0.29 in the Sunk-cost model and 0.72 in the Fixed-cost model, which corresponds to the value of the models without R&D for simplicity.

I internally calibrate the remaining eight parameters,  $f_0$ ,  $f_1$ ,  $\xi$ ,  $\lambda_D$ ,  $n_{d0}$ ,  $\mu_E$ ,  $\alpha$ , and  $\sigma$  to match the six moments and two distributions for the United States:

1. An exporter rate of 22.3% (1992 Census of Manufactures (CM)).
2. A stopper rate of 17% as in [Bernard and Jensen \(1999\)](#) based on the Annual Survey of Manufactures (ASM) of the Bureau of the Census 1984-1992.
3. Exporters' export sales to the total sale of 13.3% from 1992 CM.
4. Five-year exit rate of entrants of 37% ([Dunne et al. \(1989\)](#)).
5. Entrants' labor share of 1.5% reported in [Davis et al. \(1998\)](#) based on the ASM.

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<sup>9</sup>The empirical studies of exporter dynamics use the firm-level data in annual frequency (e.g., [Das et al. \(2007\)](#)).  $\gamma = 1$  is consistent with their findings.

6. Shut down establishments' labor share of 2.3% (Davis et al. (1998)).
7. Employment size distribution as in the 1992 CM.
8. Establishment size distribution as in the 1992 CM.

Since there are more moments than parameters, the model fit is imperfect. The iceberg cost,  $\xi$ , is set to match exporters' export sales to total sales. I choose the rest of the seven parameters to minimize the sum of squared residuals between the data and model. The parameters and target moments are reported in table 1. I also calibrate the Fixed-cost model, in which firms decide their current export and  $f_0 = f_1$ . This model is the dynamic variation of Melitz (2003). I examine the role of R&D investment for the Fixed-cost model as well as the Sunk-cost model.

The size of the startup cost is about 6 times larger than the continuation cost across the Sunk-cost models. The elasticity of R&D investment does not have a large effect on the gap between the continuation cost,  $f_1$ , and the startup cost,  $f_0$ . In this model, R&D investment itself has a small effect on the export decision.

The top and middle panel of figure 1 plots the distribution of establishment and employment by employment size in the data and the models. The bar represents the data. Each line represents the Sunk-cost model with high R&D elasticity (circle), the Sunk-cost model with low R&D elasticity (plus), the Fixed-cost model with high R&D elasticity (asterisk), and the Fixed-cost model with low R&D elasticity (times), respectively.

The distributions of all models are fitted to the data. The share of establishments is decreasing in size in the data and the model. The distribution of the share of manufacturing employment is hump-shaped, and the establishments with 100-249 employees account for 20% of total employment in the data and model. The mean squared errors between the model and data of two distributions are reported in panel C of table 1.

The bottom panel of figure 1 plots the stationary distribution of intermediate goods firms of the Sunk-cost model with high R&D elasticity as a representative example. I plot the distributions of exporters (right red distribution), non-exporters (left blue distribution), and newborn firms (yellow dashed line) by productivity level of the Sunk-cost model with high R&D elasticity. I also plot the exogenous shutdown probability of producers and the probability of starting and stopping exporting. To match the low employment share and high shutdown rate of entrants, the average

newborn productivity is about 25% lower than that of incumbents. The exogenous death rate depends on productivity, and the unproductive firms are more likely to exit. Since the startup cost is higher than the continuation cost, the threshold to start export is larger than that to stop export. Hence, the productive non-exporters have a higher probability of starting exports, and the unproductive exporters have a higher probability of stopping exports. Also, the distribution shows that there are unproductive exporters and productive non-exporters since the export decision is persistent.

[Table 1 about here.]

[Figure 1 about here.]

## 4 The effect of trade liberalization

In this section, I examine the short-run and long-run effects of an unanticipated permanent global cut in tariffs,  $\tau$  from 8% to 0%. This estimates the expected change in the U.S. and the rest of the world from eliminating tariffs. I examine how R&D investment enhances the welfare gains from trade liberalization in the Sunk-cost and Fixed-cost models. I decompose the consumption change in the equilibrium and show that the labor is substituted toward production and R&D investment from firm creation in the Sunk-cost model with high R&D elasticity.

### 4.1 The change in the firms' R&D investment

Firstly, I describe how firms change their R&D investment and export decisions after trade liberalization in the long run. Figure 2 plots the difference in the R&D investment between the initial and new steady state. It also plots the threshold to start and stop exporting. The dot and dash lines denote the initial threshold and the new threshold, respectively. In the upper figures plotting the Sunk-cost model, the two vertical lines on the left-hand side represent the thresholds to stop, and those on the right-hand side represent the thresholds to start. The top two panels plot the change in R&D investment of the Sunk-cost model with high and low R&D elasticity. The red lines on the right-hand side plot the change in investment of non-waiting exporters, and the blue lines on the left-hand side plot the change of non-waiting non-exporters. The bottom two panels plot those of



Fixed-cost models. In the Sunk-cost models, the difference in R&D investment discontinues since firms change their state of waiting immediately and firms do not exist in some regions.

In the new steady state, exporters raise their R&D investment compared to the initial steady state. The changes are the largest around the threshold of export decisions. Exporters who are more likely to stop exporting increase their R&D investment more than other exporters to continue exporting. Also, non-exporters who are more likely to start exporting increase their investment more than other non-exporters to start exporting. In the Fixed-cost model, there is one spike since the startup cost is equal to the continuation cost.

The threshold to export moves to the left-hand side in all models. Because of the decline in trade costs, unproductive firms can earn profit from exporting, and thus, those firms continue and start exporting. In the Sunk-cost models, the threshold to stop exporting moves larger than that to start exporting. This means that trade cost reduction makes export decisions more persistent.

[Figure 2 about here.]

## 4.2 Welfare gain and decomposition

I decompose the general equilibrium effect of the tariff reduction on consumption:

$$\hat{C}_t = \hat{L}_{P,t} + \frac{1}{\theta - 1} \left( \hat{N}_t + \hat{\Psi}_t - \hat{\lambda}_t \right) + \hat{S}_t, \quad (12)$$

where  $\hat{S}_t = -\hat{\lambda}_t + \hat{A}_t$ ,  $A_t^{-1} = 1 + (1 + \xi)^{1-\theta}(1 + \tau_t)^{-\theta} \frac{\Psi_{X,t}}{\Psi_t}$ , and  $\lambda_t^{-1} = 1 + (1 + \xi)^{1-\theta}(1 + \tau_t)^{1-\theta} \frac{\Psi_{X,t}}{\Psi_t}$ .  $\lambda_t$  is derived from equation (9). If  $\tau = 0$ ,  $A_t = \lambda_t$ . Here,  $\Psi_t = \int \sum_{m=0}^1 \sum_{w=0}^1 e^{(\theta-1)z} g_t(z, m, w) dz$  and  $\bar{\Psi}_t = \Psi_t / N_t$  denote the aggregate productivity and the average productivity, respectively.  $\Psi_{X,t} = \int \sum_{w=0}^1 e^{(\theta-1)z} g_t(z, 1, w) dz$  denotes the aggregate productivity of exporters.  $\hat{x}_t$  denotes the log change of  $x_t$  from the initial steady state. This equation is derived from the aggregate production labor, equation (11) and the aggregate technology of the final goods firm, equation (1),

$$L_{P,t} = \left( \frac{\theta}{\theta - 1} W_t \right)^{-\theta} C_t \left( \Psi_t + (1 + \xi)^{1-\theta}(1 + \tau_t)^{-\theta} \Psi_{X,t} \right), \quad (13)$$

$$W_t = \frac{\theta - 1}{\theta} \left( \Psi_t + (1 + \xi)^{1-\theta}(1 + \tau_t)^{1-\theta} \Psi_{X,t} \right)^{\frac{1}{\theta-1}}. \quad (14)$$

By eliminating  $W_t$  in equation (13) with equation (14), the consumption,  $C_t$  is decomposed into changes attributed to variations in labor,  $\hat{L}_{p,t}$ , the mass of producers,  $\hat{N}_t$ , average productivity,  $\hat{\Psi}_t$ ,  $\hat{\lambda}_t$ ,  $\hat{S}_t$ .

Then, I define the manufacturing labor productivity,  $Z_{p,t} = \frac{Y_t}{L_{p,t}}$  following [Atkeson and Burstein \(2010\)](#). Since  $Y_t = C_t$  from the market clearing condition, by equation (12),

$$\begin{aligned}\hat{Z}_{p,t} &= \hat{C}_t - \hat{L}_{p,t}, \\ &= \frac{1}{\theta - 1} \left( \hat{N}_t + \hat{\Psi}_t - \hat{\lambda}_t \right) + \hat{S}_t.\end{aligned}\tag{15}$$

This equation implies that manufacturing productivity depends on the firms' distribution.

Table 2 reports the welfare gain and long-run change in consumption. This table also reports the decomposition according to equation (12) and trade elasticity. The first row of panels A and B in the table shows the welfare gain and long-run consumption change from the trade liberalization of each model, respectively. I also report the manufacturing productivity in the sixth row of each panel. Here, the discount value of  $x_t$  is defined as  $\rho \int_0^\infty e^{-\rho t} \log(x_t) dt$ , which is shown in percent.

Trade liberalization in the Sunk-cost model with high R&D elasticity leads to larger welfare gain than that with low R&D elasticity. The welfare gain in the Sunk-cost model with high R&D elasticity is about 30% larger than that with low R&D elasticity. I also examine the long-run changes in consumption. The long-run change in consumption in the Sunk-cost model with high R&D elasticity is about 55% larger than that with low R&D elasticity.

The decomposition clarifies the key factors that increase the welfare gain in the Sunk-cost model. By comparing the Sunk-cost model with high R&D elasticity to that with low R&D elasticity, I find that the production labor contributes to raising the welfare gain and long-run change in consumption. Additionally, the average productivity accounts for some of the differences. On the other hand, the decline in the mass of establishment is amplified by R&D investment. R&D investment has a small effect on the contribution of trade to the welfare gains from trade liberalization.

Panel A of table 3 reports the discounted change in the labor of production, research, firm creation, and exports. The change is in level. When R&D elasticity is high, labor for new firm creation is substituted away toward production. In contrast to the average productivity, the decline in the research labor is enhanced by R&D investment. This is because individual firms increase their

R&D investment, while the aggregate R&D investment declines due to the reduction in the mass of establishments. Similarly, the increase in production labor and average productivity accounts for the difference in the long-run change in consumption in the models.

The amplification effect of R&D investment follows from the free entry condition. The equation (7) implies that, in the Sunk-cost model, the entrants begin their business as non-exporters and must pay startup costs to start exporting. As a result, the entrants discount gain from exports compared to current exporters. After trade liberalization, exporters increase their R&D investment, which raises the gain from exporting and intensifies competition. The entrants discount the gains from exporting, while they do not discount the loss from competition. Consequently, the mass of entrants declines largely, and the labor is substituted away from firm creation toward production and R&D investment.

In the Fixed-cost models, R&D elasticity weakly affects the size of welfare gains. In the Fixed-cost model, the difference in R&D elasticity does not lead to the difference in the long-run change in consumption either. The entrants do not discount the gain from export in the Fixed-cost models, and the labor substitution does not occur. In Appendix A, I show that the welfare gain in the Sunk-cost model without R&D is similar to the Sunk-cost model with low R&D and that in the Fixed-cost model without R&D is similar to the Fixed-cost models with high and low R&D elasticity.

The difference in manufacturing productivity is weak relative to that in consumption. The increased R&D investment raises the average productivity, while the change in the average productivity is offset by the decline in the mass of establishment in the equilibrium. In contrast, there is no difference between the fixed-cost models.

Table 2 also reports trade elasticity. Here, the discounted trade elasticity is given by  $\rho \int_0^\infty e^{-\rho t} \varepsilon_{T,t}$ . The long-run trade elasticity in the Sunk-cost models is larger than that in the Fixed-cost models. In the Sunk-cost models, trade liberalization enhances the gain from continuing exports, and it makes the firms' export decisions persistent. The long-run trade elasticity is larger than the discounted trade elasticity in the Sunk-cost models. This is because the trade elasticity gradually increases and converges to the long-run level in the Sunk-cost models in contrast to the Fixed-cost models.

Arkolakis et al. (2012) show that the gain from trade cost reduction is explained by the trade elasticity,  $\hat{C} = \hat{\lambda}/\varepsilon_T$  (ACR formula). In the static trade model, trade adjustment is completed

immediately, and the consumption jumps to the new steady state level. Hence, the comparison of long-run trade elasticity is a good approximation to evaluate the gain from trade. On the other hand, [Alessandria and Choi \(2014\)](#) and [Alessandria et al. \(2021\)](#) argue that the long-run trade elasticity is a bad approximation of the welfare gains in the dynamic trade model and there is a gap between the long-run change in consumption and welfare gain. They also show that the ACR formula overstates the gain from trade liberalization in the model with firm dynamics. Here, I discuss how R&D investment affects the welfare gains from trade liberalization.

The ACR formula predicts the gain from trade liberalization is 0.91% in the Sunk-cost model with high R&D elasticity and 0.89% in that with low R&D elasticity. Hence, the ACR formula overstates the welfare gain when R&D investment is inelastic, while it approximates well when R&D investment is elastic. In the Sunk-cost model with high R&D elasticity, the R&D investment enhances an increase in consumption while it has a small effect on the trade elasticity. This fills the gap between the prediction of the ACR formula and the actual welfare gain. In the Fixed-cost model, the R&D elasticity weakly affects the welfare gains, and the ACR formula does not approximate the gain from trade liberalization.

R&D investment has an effect on the difference between the welfare gain and the long-run change in consumption. The labor substitution occurs the whole time following trade liberalization, and it contributes to the increase in the welfare gain and the long-run change in consumption. However, the short-run difference in consumption is smaller than the long-run difference, and R&D investment makes the larger gap between the long-run consumptions than that between the welfare gains. This is because firms invest in R&D investment and export technology intensively in the short run. In the Fixed-cost models, R&D also does not affect the gap between them, and the welfare gain is larger than the long-run change due to the entrant disadvantage.

[Table 2 about here.]

[Table 3 about here.]

### 4.3 Transition dynamics

Figures [3](#) and [4](#) plot the transitions after trade liberalization of the models for the first 50 years. The blue line and red dashed line denote the Sunk-cost model with high R&D elasticity and with

low R&D elasticity, respectively. The yellow dot and purple chain lines denote the Fixed-cost model with high R&D elasticity and with low R&D elasticity, respectively. Except for trade elasticity, I plot the change relative to the initial steady state.

Figure 3a plots the transition dynamics of the consumption of each model. The change in consumption of the Sunk-cost model with high R&D elasticity is larger than that with low R&D elasticity the whole time by about 0.4 points. In the Sunk-cost model, the short-run and long-run consumption contribute to the increase in the welfare gain. Also, the Sunk-cost models show the delayed overshoot of consumption dynamics. The consumption gradually increases, reaching a peak after 5 periods of trade liberalization, which is about 0.4 points higher than the long-run change.

In the Fixed-cost model, the elasticity of R&D investment has a small effect on the long-run consumption, as pointed out in Atkeson and Burstein (2010). The transition dynamics of the Fixed-cost model are flatter than the Sunk-cost model. Across the Fixed-cost models, the consumption at  $t = 0$  is higher than the long-run consumption, and it gradually converges to the long-run level. Due to the entrant disadvantage,  $\mu$ , the entrants are smaller than the incumbents, which raises the consumption in the short run.

Manufacturing productivity and production labor are plotted in figure 3b and 3c, respectively. The shape of the transition dynamics is similar to the consumption both in the Sunk-cost and Fixed-cost models. On the other hand, the size of the change is different from the consumption, and the elasticity of R&D investment has a small effect on manufacturing productivity over time across models compared to the consumption. The manufacturing productivity of the Sunk-cost model also shows the delayed overshoot, and it achieves the peak after 5 periods of trade liberalization. In contrast to the consumption, the manufacturing productivity does not increase at  $t = 0$  since it is the state variable. The production labor increases in the initial period and gradually converges to the new steady-state level.

The average productivity also differs across the models. Figures 3e, 3f illustrate the transition dynamics of the average productivity, and mass of establishment, respectively. In the Sunk-cost model with high R&D elasticity, the changes in average productivity and the mass of the establishments are larger than in the other models. Discounting the gain from exporting leads to a large decline in entry and the mass of establishment. This leads to the substitution toward production in figure 3c and R&D investment in figure 3d.

Figure 4a plots the trade elasticity. The assumption of sunk cost and delayed exporting generates the gradual trade adjustment after trade liberalization. The trade elasticity of the Sunk-cost models gradually grows, which is not observed in the Fixed-cost model as shown in Alessandria and Choi (2014). It takes about 20 years to reach the long-run level after trade liberalization. The short-run and long-run trade elasticity of the Fixed-cost models is almost constant over time. In the Sunk-cost model, the long-run trade elasticity is about 7, while the short-run trade elasticity is about 4.5. Figure 4d plots the mass of waiting firms, which is the proxy of starter and stopper. The gradual adjustment of the starter and stopper leads to the gradual adjustment of the extensive margin of export.

To understand the role of R&D investment in trade, I decompose the trade elasticity. Trade elasticity (10) is given by the change in imported goods share,  $IMY_t$ . Imported goods share is decomposed into the trade cost change, productivity premium,  $\frac{\bar{\Psi}_{X,t}}{\bar{\Psi}_t}$ , and exporter ratio,  $n_{x,t}$ ,

$$IMY_t = (1 + \xi)^{1-\theta} (1 + \tau_t)^{1-\theta} \frac{\bar{\Psi}_{X,t}}{\bar{\Psi}_t} n_{x,t}. \quad (16)$$

The change in the trade costs  $\tau$  and  $\xi$  are the same across the models, while the change in the exporter ratio and productivity premium are different across the models.

Figures 4b and 4c illustrate the exporter ratio and productivity premium, respectively. The change in the export participation rate of the Sunk-cost model with high R&D elasticity is larger than that with low R&D elasticity. The increase in R&D investment of exporters who are likely to stop exporting has a compositional effect. The increase in R&D investment of those exporters raises their survival rate, leading to an increase in the export participation rate. On the other hand, it has a negative effect on productivity premiums since those firms are less productive, and the productivity premium gets smaller.

Both consumption and manufacturing productivity grow gradually and show the overshoot in the Sunk-cost model. The short-run investment in export technology and R&D generates this shape of transition. The gradual adjustment in trade implies that the economy enjoys the gain from trade in the long run while the gain is small in the short run. In the short-run, firms intensively start exporting, as shown in 4d, and labor is used to invest in export technology. Also, in the short run, research labor in 3d increases and gradually converges to a new steady state. Hence, introducing

R&D investment generates a new channel that reduces the short-run increase in production labor and consumption.

The change in the mass of establishments is important to understand the transition of consumption. In all models, the mass of establishments declines following trade liberalization. The gradual decline in the mass of establishments implies the economy can utilize the over-accumulated mass of establishments soon after the trade liberalization. This also contributed to several initial periods of increase in consumption relative to long-run consumption.

[Figure 3 about here.]

[Figure 4 about here.]

#### 4.4 The flow startup cost

The continuous-time model with flow startup cost and lag generates a non-monotonic increase in consumption after trade liberalization. In figure 3a, the consumption increases at  $t = 0$  when the economy realizes the trade liberalization, which is higher than the consumption for several periods after  $t = 0$ . Also, the production labor increases in the initial period, as shown in figure 3c. This feature is not observed if I assume the stock startup cost as [Impullitti et al. \(2013\)](#). Also, it is not observed in the discrete-time model as [Alessandria and Choi \(2014\)](#). With the flow startup cost and lag, the firms do not pay the startup cost when they decide to start exporting, while the firms pay the cost in the following periods until they start exporting. Thus, in period 0, when trade cost is reduced, the firms do not use the labor for startup costs. Instead, the economy utilizes labor to produce goods, raising consumption in period 0. Although the temporary increase in consumption at  $t = 0$  surely contributes to an increase in the welfare gains, the difference in consumption between models is persistent the whole time, and I can observe the difference in the welfare gains without it in the Sunk-cost models.

## 5 Conclusion

This paper develops a dynamic heterogeneous firm international trade model with two firm-level investments in export and productivity. I introduce three assumptions highlighted in the literature:

sunk export cost, lag before exporting, and endogenous R&D investment. I calibrate the model to fit the U.S. data and examine the aggregate effect of an unanticipated tariff reduction. I find that the welfare gains depend on the elasticity of R&D investment in the Sunk-cost models. In the Sunk-cost models, elastic R&D raises the welfare gain from trade liberalization by 30%, compared to the model with low R&D elasticity. On the other hand, the elasticity of R&D investment has a small effect on the welfare gains in the Fixed-cost model. I decompose the welfare gains from trade liberalization and show that the labor substitution toward production and research from firm creation accounts for the difference between the Sunk-cost model and the Fixed-cost model. While R&D investment raises the gain from export following trade liberalization in the Sunk-cost models, entrants discount the gain from export, leading to a decline in the mass of entrants. This result suggests that the lack of exporter dynamics may underestimate the importance of R&D investment for the welfare gain from trade liberalization.

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## A The model without R&D investment

In the model without R&D investment,  $\alpha$ , governs the whole productivity persistence and R&D investment,  $q_t$ , is eliminated from equation (6). The calibrated parameters and targets are reported in table 4. Here, the asterisk means that the variable is not the target. The common parameters are the same as the main part.

Figures 5 and 6 show the transition dynamics of models without R&D investment. The line with a circle shows the Sunk-cost model without R&D investment, and the line with a plus shows the Fixed-cost model without R&D investment. I also plot the transition dynamics of the models with low R&D elasticity as a reference. The welfare gain from trade liberalization is 0.63%, and the long-run change in consumption is 0.33% in the Sunk-cost model without R&D investment. These values are close to the value of the Sunk-cost model with low R&D elasticity. The welfare gain from trade liberalization is 0.20%, and the long-run change in consumption is 0.13% in the Fixed-cost model without R&D investment. These values are close to the value of the Fixed-cost models with high and low R&D elasticity.

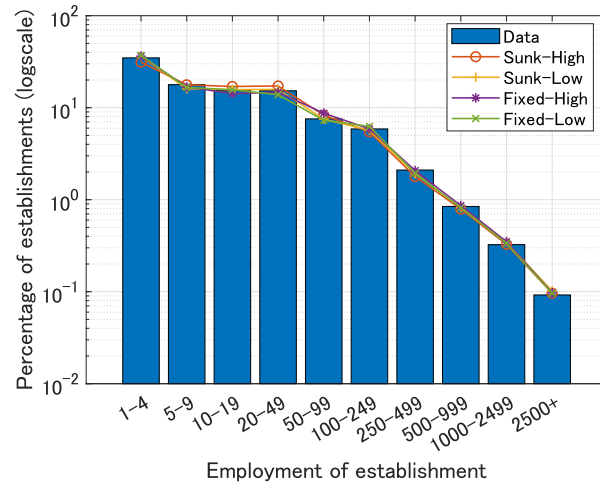
[Table 4 about here.]

[Figure 5 about here.]

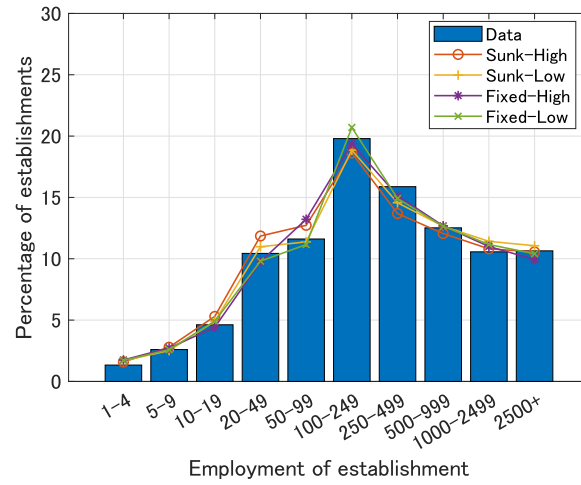
[Figure 6 about here.]

Figure 1: Steady state distribution and policy function

(a) Distribution of establishment



(b) Distribution of employment



(c) Stationary distribution of the Sunk-cost model

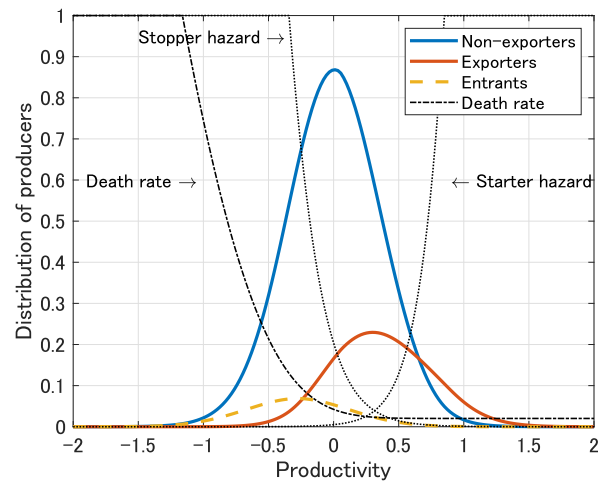
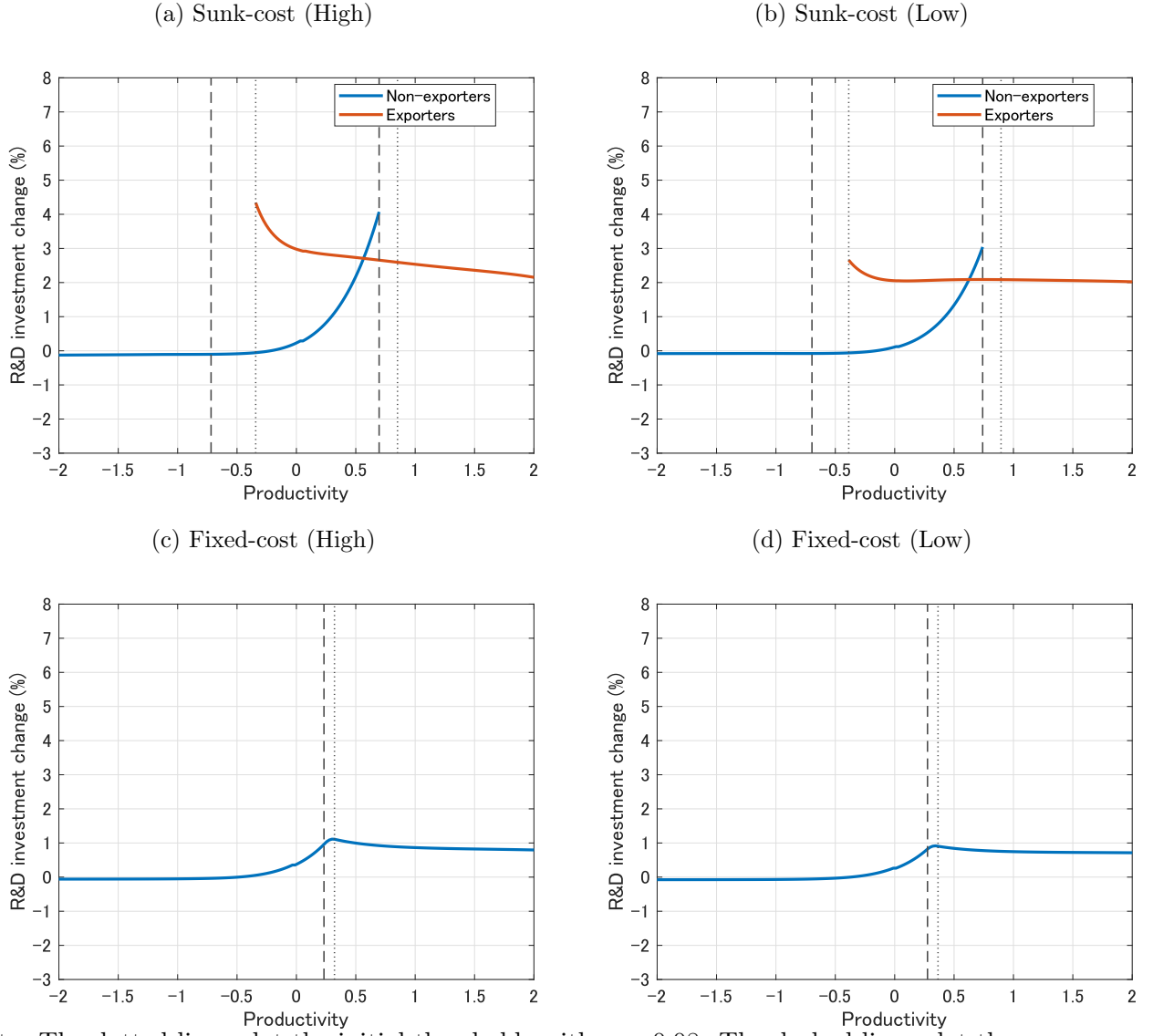


Figure 2: Change in the R&D investment



Note: The dotted lines plot the initial thresholds with  $\tau = 0.08$ . The dashed lines plot the new thresholds with  $\tau = 0$ . The Sunk-cost models have two thresholds. The right dashed and dotted lines in the top figures show the thresholds to start exporting. The left lines show the thresholds to continue exporting.

Figure 3: Transition after 8% tariff reduction

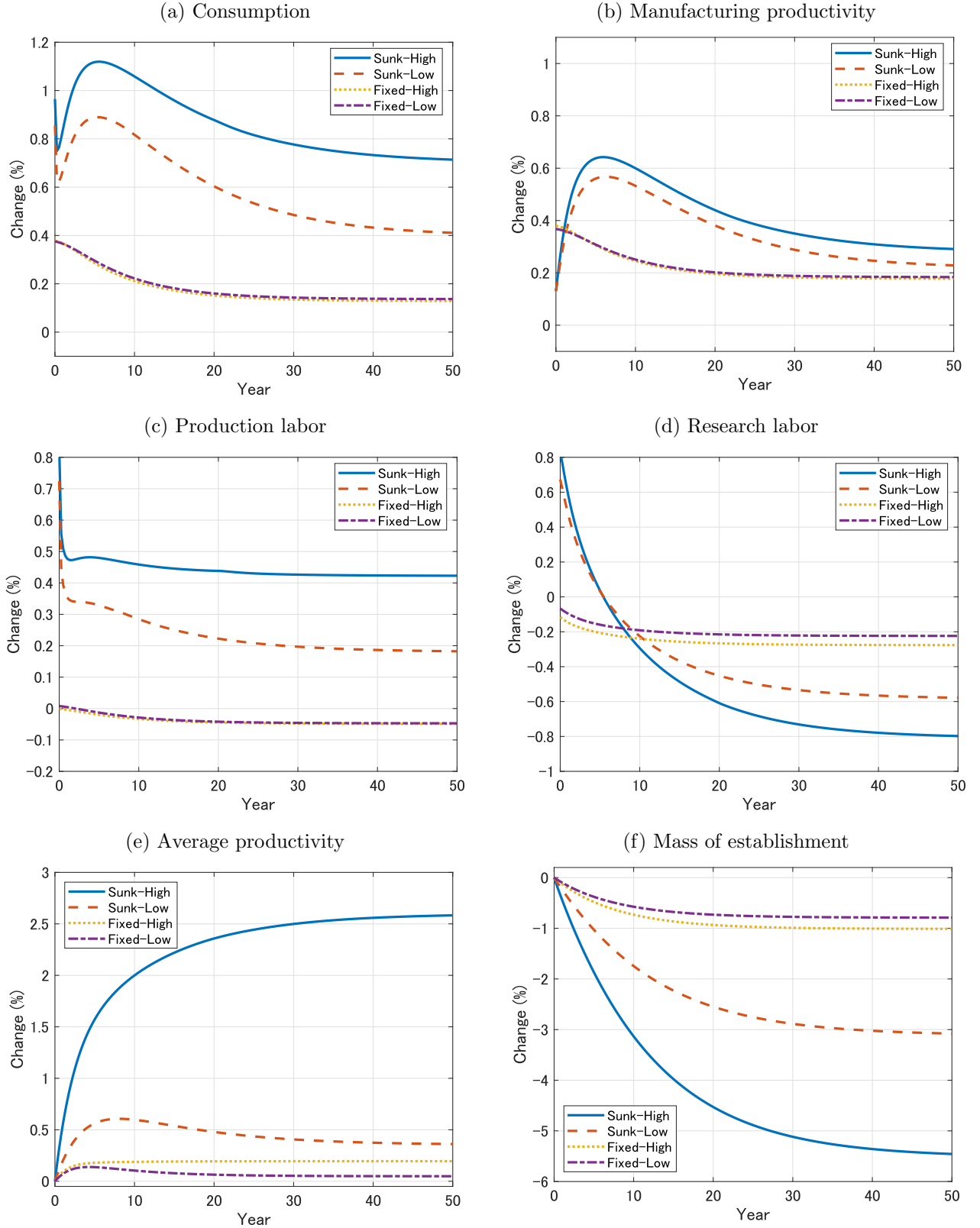


Figure 4: Transition dynamics of trade after 8% tariff reduction

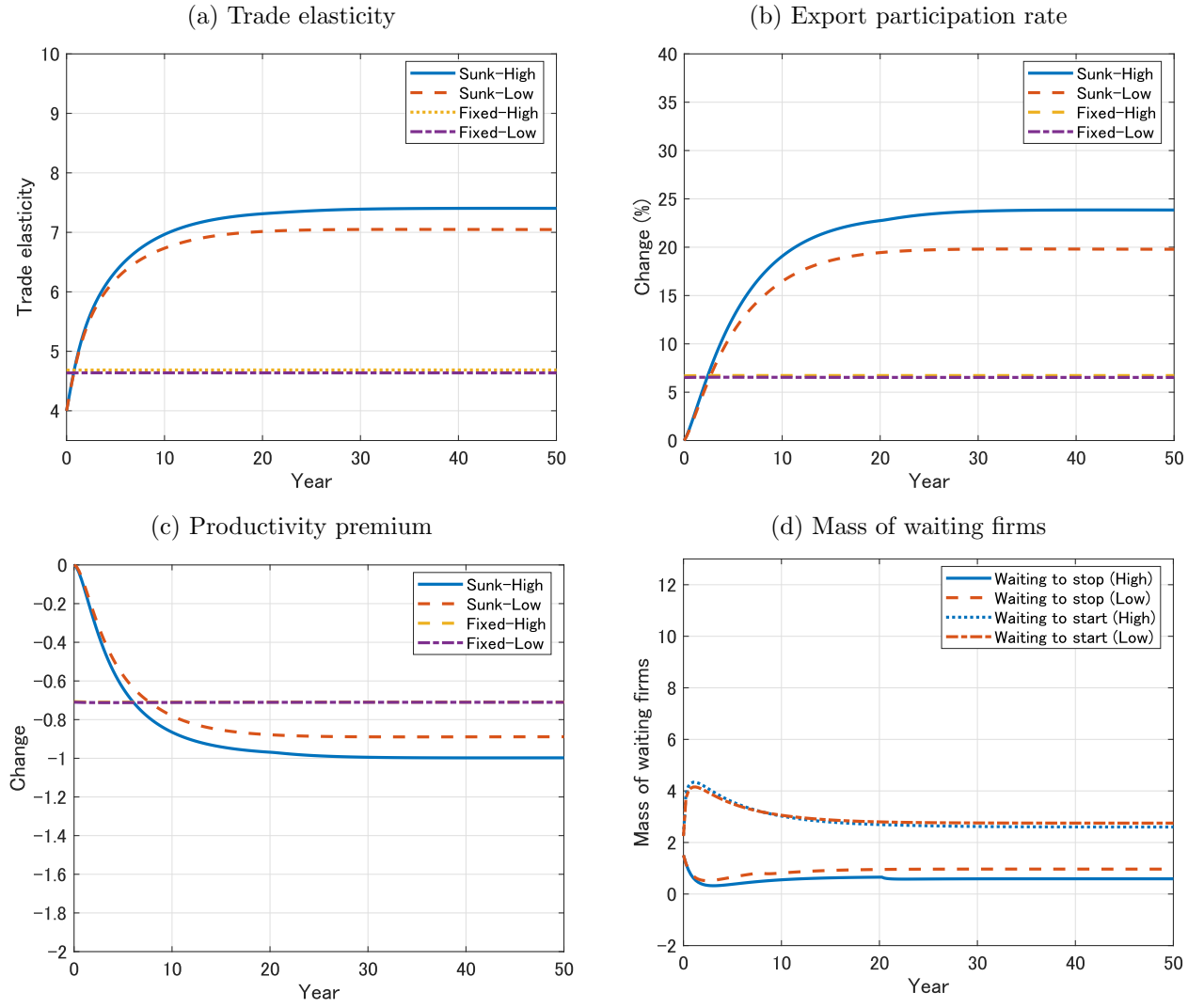




Figure 5: Transition after 8% tariff reduction

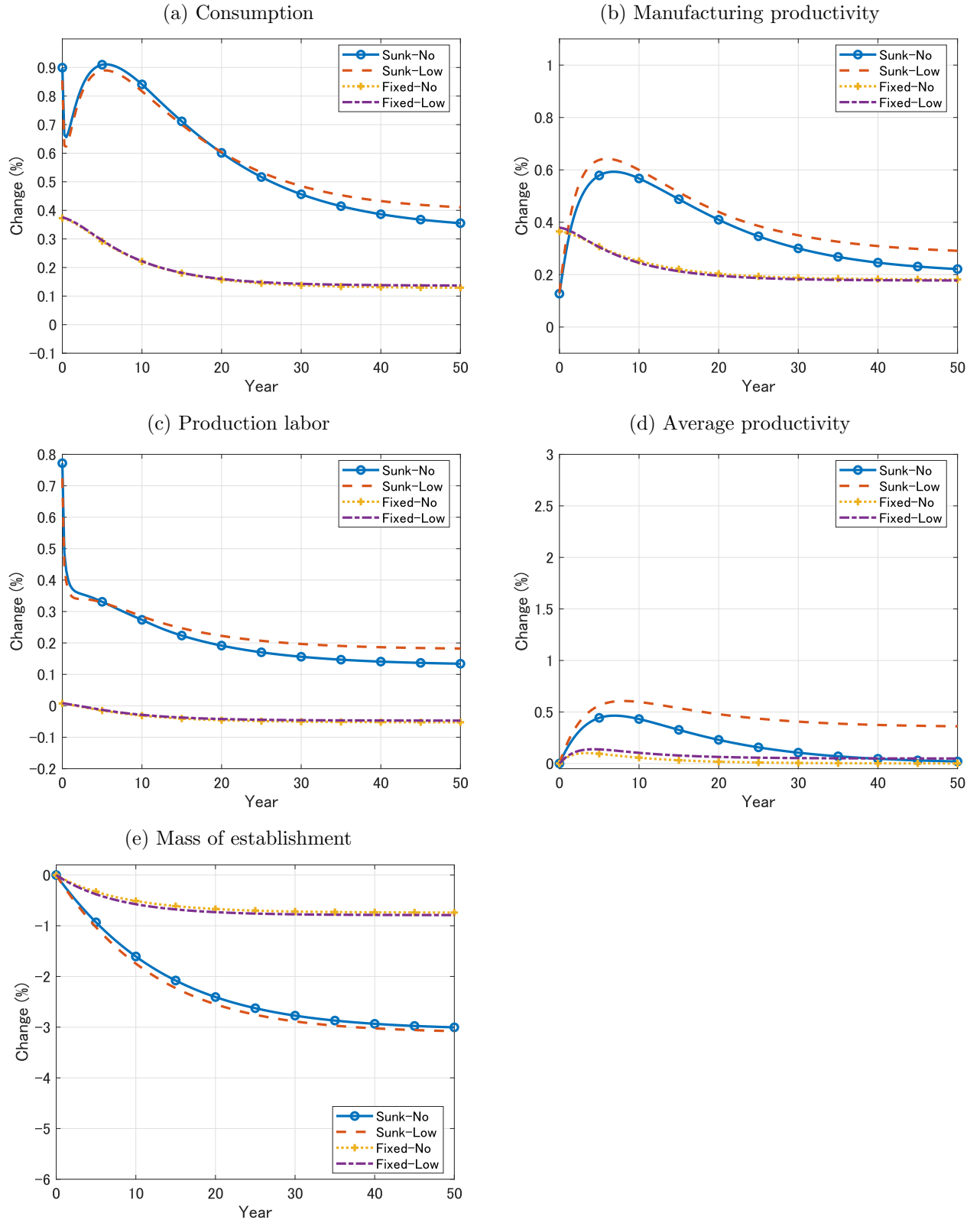


Figure 6: Transition dynamics of trade after 8% tariff reduction

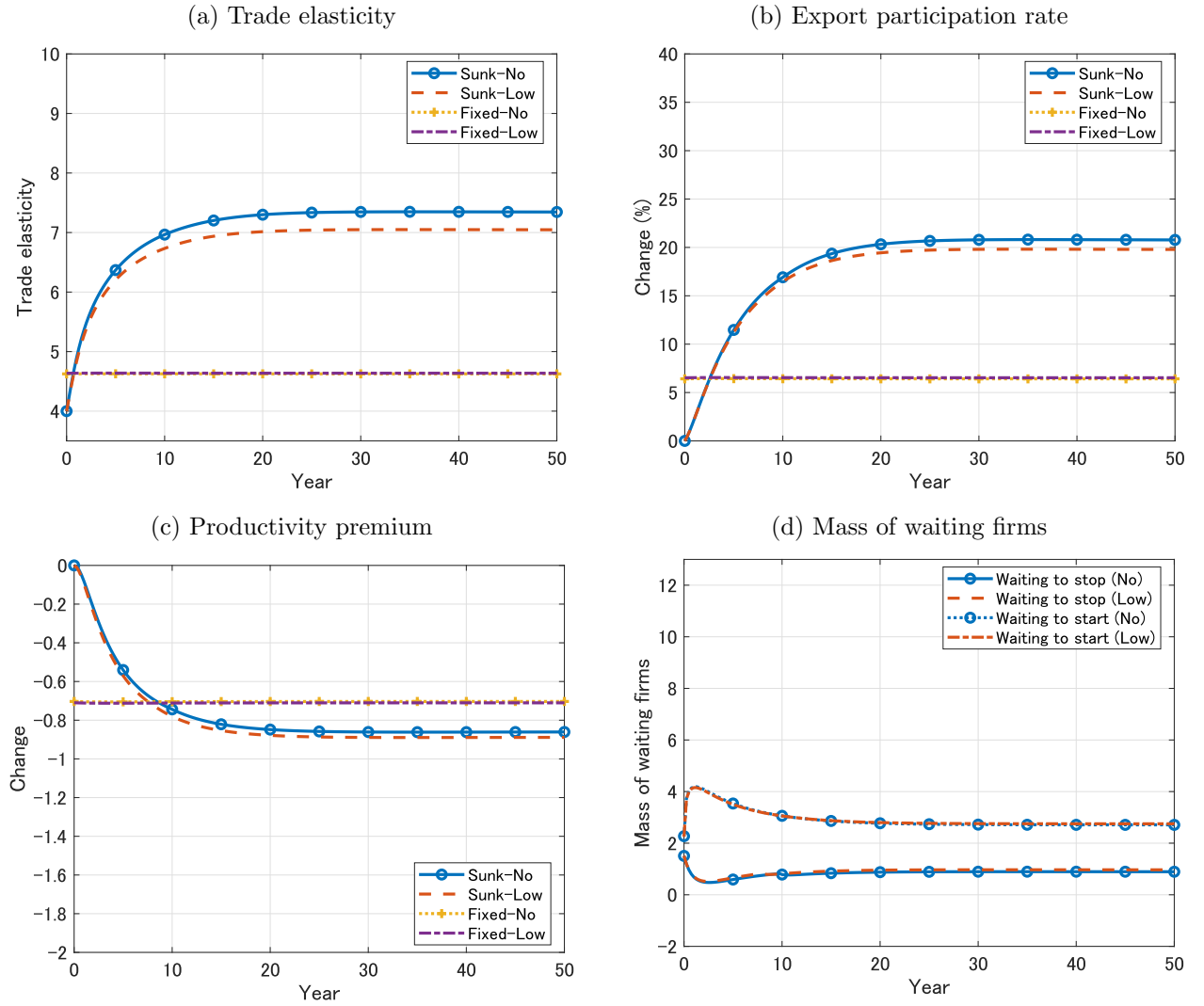


Table 1: Parameter values and Target moments

A. Common Parameters					
		$\rho$	$\theta$	$\gamma$	$\tau$
		0.04	5.0	1	0.08
B. Model Parameters					
		Sunk High	Sunk Low	Fixed High	Fixed Low
Startup cost	$f_0/f_1$	7.16	7.95	-	-
Continuation cost	$f_1$	0.02	0.02	0.02	0.02
Iceberg cost	$\xi$	0.45	0.45	0.45	0.45
Death rate	$\lambda_D$	5.52	5.79	6.08	5.71
	$n_{d0}$	0.02	0.02	0.02	0.02
Entry disadvantage	$\mu_E$	-0.28	-0.25	-0.41	-0.24
Entry cost	$f_E$	1.63	2.45	1.12	1.42
Productivity persistence	$\alpha$	0.17	0.24	0.61	0.44
Productivity SD	$\sigma_z$	0.31	0.32	0.51	0.43
C. Target moments		Data			
5-year exit rate	37.0	36.8	37.0	37.0	37.0
Startups' labor	1.5	1.4	1.8	1.6	1.6
Exiters' labor	2.3	2.1	2.4	2.4	2.3
Stopper rate	17.0	17.0	16.9	65.2*	56.4*
Exporter ratio	22.3	22.4	22.3	22.3	22.3
Trade share	13.3	13.3	13.3	13.3	13.3
Distribution					
Establishments		1.42	0.64	0.77	0.97
Employment		1.01	0.67	0.67	0.55

Note: Sunk and Fixed denote the Sunk-cost model and the Fixed-cost model, respectively. The value with \* denotes the non-target moments. High represents the model with  $\phi = 20$  and Low represents the model with  $\phi = 100$ .

Table 2: Consumption decomposition

R&D elasticity	Sunk High	Sunk Low	Fixed High	Fixed Low
A. Discounted				
$\hat{C}$	0.89	0.63	0.20	0.20
$\hat{L}_p$	0.45	0.25	-0.04	-0.03
$\frac{1}{\theta-1}\hat{\Psi}$	0.50	0.11	0.04	0.02
$\frac{1}{\theta-1}\hat{N}$	-0.91	-0.51	-0.19	-0.15
$\frac{1}{\theta-1}\hat{\lambda}$	-1.52	-1.45	-1.27	-1.27
$\hat{S}$	-0.67	-0.67	-0.89	-0.90
$\hat{Z}_p$	0.44	0.38	0.23	0.24
$-\varepsilon_T$	6.77	6.52	4.62	4.58
B. Long-run				
$\hat{C}$	0.70	0.40	0.13	0.14
$\hat{L}_p$	0.42	0.18	-0.05	-0.05
$\frac{1}{\theta-1}\hat{\Psi}$	0.65	0.09	0.05	0.01
$\frac{1}{\theta-1}\hat{N}$	-1.38	-0.78	-0.25	-0.20
$\frac{1}{\theta-1}\hat{\lambda}$	-1.68	-1.58	-1.29	-1.28
$\hat{S}$	-0.67	-0.68	-0.91	-0.91
$\hat{Z}_p$	0.28	0.22	0.18	0.18
$-\varepsilon_T$	7.40	7.05	4.69	4.64

Note:  $Z_p$  represents the manufacturing productivity, where  $Z_p = C/L_p$ . The welfare gain is given by the value of  $x$  that satisfies  $\int_0^\infty e^{-\rho t} U(C_{-1}e^x)dt = \int_0^\infty e^{-\rho t} U(C_t)dt$ , where  $C_{-1}$  is the consumption level in the initial steady state. The values are shown in percent. Time subscript is omitted.

Table 3: Change in the labor allocation

		Sunk High	Sunk Low	Fixed High	Fixed Low
A. Discounted					
$\hat{L}_p$	production	0.34	0.20	-0.03	-0.03
$\hat{L}_r$	research	-0.043	-0.008	-0.014	-0.004
$\hat{L}_x$	firm creation	-0.72	-0.60	-0.11	-0.11
$\hat{L}_e$	export	0.42	0.41	0.15	0.14
$\hat{L}_0$	startup	0.10	0.12	-	-
$\hat{L}_1$	continuation	0.33	0.29	-	-
B. Long-run					
$\hat{L}_p$	production	0.32	0.14	-0.04	-0.04
$\hat{L}_r$	research	-0.093	-0.018	-0.016	-0.004
$\hat{L}_x$	firm creation	-0.69	-0.56	-0.09	-0.09
$\hat{L}_e$	export	0.46	0.43	0.15	0.14
$\hat{L}_0$	startup	0.05	0.08	-	-
$\hat{L}_1$	continuation	0.41	0.36	-	-

Note: In the equilibrium,  $L = L_p + L_r + L_x + L_c$ . The export labor is the sum of startup and continuation labor,  $L_e = L_0 + L_1$ . The values are shown in level. Time subscript is omitted.

Table 4: Parameter values and Target moments

A. Model Parameters			
		Sunk	Fixed
Startup cost	$f_0/f_1$	7.48	-
Continuation cost	$f_1$	0.02	0.02
Iceberg cost	$\xi$	0.45	0.45
Death rate	$\lambda_D$	6.65	6.37
	$n_{d0}$	0.02	0.02
Entry disadvantage	$\mu_E$	-0.37	-0.42
Entry cost	$f_E$	2.82	1.64
Productivity persistence	$\alpha$	0.29	0.72
Productivity SD	$\sigma_z$	0.33	0.53
B. Target moments		Data	
5-year exit rate	37.0	37.0	37.0
Startups' labor	1.5	1.5	1.4
Exiters' labor	2.3	2.4	2.4
Stopper rate	17.0	17.1	65.1*
Exporter ratio	22.3	22.4	22.3
Trade share	13.3	13.3	13.3
Distribution			
Establishments		0.63	1.35
Employment		0.61	0.59