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LIFE EXPECTANCY AND BUSINESS CYCLES IN A SMALL OPEN ECONOMY

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Abstract

This paper examines the effects of increased life expectancy on the short-run macroeconomic stability of a typical small open economy. We develop a real-business-cycle (RBC) model of a small open economy that is consistent with the main empirical facts of economic fluctuations in open economies. Different from the previous satisfactory open-economy extensions of the baseline RBC model, given its finite lifetimes feature, our framework also helps rationalize some of the key results from recent empirical literature on the relationship between longevity and business cycles. In our model, changes in life expectancy change the planning horizon of individuals and thus affect their intertemporal choices. Consequently, the cyclical volatilities of aggregate variables are also affected. As a numerical exercise, we quantify how increased life expectancy has impacted Canadian business cycle fluctuations over the past forty years. The results indicate that the fluctuations in physical capital, human capital, and consumption all decrease as life expectancy increases. On the other hand, the fluctuation of hours worked, output, and trade balance ratio are found to increase with life expectancy.

Keywords: Business cycles, Life expectancy, Finite lifetimes, Small open economy

JEL Classification: E32, F41, J11

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1. Introduction

Over the last two centuries life expectancy at birth has doubled in most parts of the world due to advances in medicine, public health, and living standards. While there exists a large theoretical-quantitative literature that focuses on the relationship between life expectancy and long-run economic outcomes, the theoretical link between increased longevity and the short-run macroeconomic performance is ambiguous. In part, this is because the mechanisms behind the relationship are not well understood and explored. The existing theoretical literature includes only few studies, based on overlapping generations models with rich labor market features, that focus on the effect of longevity on the business cycle strictly through changes in the labor market dynamics. However, as pointed out in Dantas Guimarães and Ferreira Tiryaki (2020), intuitively, increased longevity can potentially affect short-run macroeconomic stability through three broad channels: by changing individuals' behavior, by changing the effectiveness of fiscal and monetary policies, and by affecting labor market dynamics. Their empirical results show that the volatility of investment and, to a lesser degree, of consumption decreases as longevity rises. The authors suggest that this outcome is likely to be the result of behavioral changes—including increased aversion to risk, shifts in consumption habits, and changes in savings and financial decisions—observed as individuals grow older. On the other hand, they find that output volatility grows as longevity increases likely due to changes in labor market dynamics and greater net exports' volatility. The latter factor implies that international trade volatility is an additional channel through which longevity appears to influence output volatility.

To help rationalize some of the abovementioned empirical results, the aim of the present paper is to set up a version of the standard small open economy RBC model that formally analyzes how increased longevity affects business cycle fluctuations of an open economy through changes in the individual's intertemporal investment and labor-leisure decisions. Uncovering the potential impacts of life expectancy on economic fluctuations will contribute to our understanding of the determinants of macroeconomic volatility, which is essential for economic policymakers to design stabilization policies and for businesses and households to make informed decisions. In the context of the RBC framework, changes in life expectancy change the planning horizon of individuals and thus affect their intertemporal optimal choices. Consequently, if individuals care more about the future because they live longer, then intuitively, one can expect that the volatility of investment in different types of assets—including physical capital, human capital, and a net foreign financial asset—in the economy will be affected. In addition, when agents expect to live longer, their intertemporal substitution of hours worked and leisure may increase, thereby affecting hours worked volatility.

The model developed in the current paper is an otherwise standard RBC model of a small open economy extended with human capital accumulation and international borrowing constraints à la Barro et al. (1995) and with finite lifetimes à la Blanchard (1985), and builds primarily on the open economy neoclassical growth framework of Tserenkhuu and Kosempel (2023). In addition to allowing for mechanisms for life expectancy to affect economic outcomes, the foregoing features also help deal with the counterfactual predictions inherent in the small open economy macro models, namely, an exaggerated variability of capital stock and investment, and non-stationary equilibrium for consumption and assets. Specifically, the former two features are for slowing down convergence and dampening business cycle movements, while the latter one is for inducing stationarity of the equilibrium dynamics. To study the growth implications of changes in life expectancy from the perspective of an open economy, Tserenkhuu and Kosempel (2023) developed a deterministic continuous-time version of the model we set up in the current paper. However, since their focus was on long run outcomes, unlike the current paper, they abstracted from any shocks affecting the domestic economy in the short run and endogenous labor-leisure decisions by households. Their quantitative results revealed that life expectancy has a substantial impact on the steady state values of the key macroeconomic variables. Given the foregoing results, using the model developed in this paper, we set out to explore the business cycle implications of life expectancy in a small open economy setting. As a numerical exercise, we quantify how changes in life expectancy have impacted Canadian—an economy that has been used frequently in the macroeconomics literature as an example of a small open economy—business cycle fluctuations over the past forty years. To preview our quantitative results, we find that volatility of physical capital, human capital, and consumption all decrease as life expectancy increases. On the other hand, the volatility of hours worked, output, and trade balance ratio are found to increase as life expectancy increases. Followed from the model's feature of diminishing returns to human capital, our results also reveal that changes in life expectancy will have a greater effect—in terms of magnitude—on the volatility of the above mentioned aggregate variables the lower is the initial level of life expectancy.

It is important to study the relationship between life expectancy and macroeconomic volatility at business cycle frequencies in an open economy setting given the substantial borrowing and lending and the trade of goods and services that exist across borders of countries around the world. The widely used closed economy assumption is difficult to justify, especially if the domestic economy's behaviour is likely to be affected by its openness to the world. In fact, there exists a large empirical literature that demonstrates stronger correlations between business cycle fluctuations in real GDP for countries that trade more with one another as in Frankel and Rose (1998), Clark and van Wincoop (2001), Baxter and Kouparitsas (2005), Burstein et al. (2008), and di Giovanni and Levchenko (2010), among others. Moreover, Giovanni and Levchenko's (2009) panel data analysis of 61 countries reveals a positive and economically significant relationship between trade openness and aggregate volatility. Thus, one can expect that the link between life expectancy and volatility can also be affected by the openness of the economy. As already mentioned, Dantas Guimarães and Ferreira Tiryaki's (2020) empirical findings do support this argument.

The paper proceeds as follows. The next section covers the related literature. Section 3 introduces the model. Section 4 analyzes the steady state and the dynamics of the model. Section 5 calibrates the model. Section 6 evaluates the quantitative predictions of the model. Section 7 discusses the results regarding the effects of changes in life expectancy on the steady state levels and cyclical volatilities of the aggregate variables. We also present some quantitative results regarding the small open economy of Canada. Section 8 provides concluding remarks.

2. Related literature

In this section we review two strands of literature that are related to the current paper. As mentioned in the introduction, open-economy RBC models lead to several counterfactual predictions, namely, an exaggerated variability of capital stock and investment, and non-stationary equilibrium for consumption and assets. Thus, we first discuss some of the previous notable modifications to the standard model that are aimed at resolving the foregoing problematic results and explain why we chose to pursue the particular extensions described in the current paper. Second, our paper is also related to the literature—both theoretical and empirical—that aims to quantify the effect of demographics and on business cycle fluctuations.

To moderate the volatility of capital stock and investment, most previous open-economy RBC models have relied on capital-adjustment costs as in Cardia (1991), Mendoza (1991), Correia et al. (1995), Letendre (2004), and Letendre and Luo (2007), among many others. The main assumption here is that the cost of changing capital stock by a fixed amount increases with the speed of the desired adjustment. This gives agents an incentive to undertake investment changes gradually. Nevertheless, there are several reasons to prefer the human capital-borrowing constraint method for our analysis. First, in the growth literature human capital has been identified as an important channel for life expectancy to affect long-run economic outcomes (mostly demonstrated using closed economy models). We cannot, therefore, ignore the possibility that human capital also provides a channel for life expectancy to affect economic conditions in the short term. Second, adjustment cost functions are designed to slow transitional dynamics, but not to affect steady state outcomes. Therefore, an adjustment cost function could be used to replace the borrowing constraint, while maintaining a lower volatility of capital stock and investment, but we would still need human capital otherwise our analysis would overlook the possible impact that changes to steady state values have on short term fluctuations. And once human capital is added to a model with mortality, a borrowing constraint seems very reasonable. The reason for this is that human capital provides poor collateral, because if agents die their human capital is destroyed.

On the other hand, to induce stationarity of the equilibrium dynamics for the framework, the following techniques have been proposed in the existing literature: models with finite lifetimes—following Blanchard's (1985) perpetual youth framework—as in Leiderman and Razin (1989) and Cardia (1991); models with an endogenous discount factor as in Obstfeld (1990), Mendoza (1991), Uribe (1997), and Schmitt-Grohe (1998), among others; models with a debt-elastic interest rate premium as in Senhadji (2003), Mendoza and Uribe (2000), Schmitt-Grohe and Uribe (2001), Letendre (2004), and Letendre and Luo (2007), among others; a model with convex portfolio adjustment costs as in Neumeyer and Perri (2005); and a model with complete asset markets as in Schmitt-Grohe and Uribe (2003). Schmitt-Grohe and Uribe (2003) assess the extent to which most of these stationarity-inducing methods[1] affect the equilibrium dynamics at business-cycle frequencies. According to their results, when calibrated with common parameter values, all models predict virtually identical unconditional second moments and impulse response functions for key macroeconomic variables. The authors conclude that if the reason for modifying the standard non-stationary small open economy model with any of the features mentioned above is simply technical, that is, solely aimed at introducing stationarity, then in choosing a particular modification of the model the researcher should be guided by computational convenience. In this respect, having had equilibrium conditions that contain an additional state variable—human capital—plus finite horizons, our framework might be in disadvantage vis-à-vis the other models. Nonetheless, we adopt the finite horizons feature since it also allows us to study the effects of changes in life expectancy—captured by exogenous changes in the horizon index of our model—on the cyclical volatilities of the aggregate variables for a small open economy.

The two papers that are perhaps the most closely related to ours are Cardia (1991) and Mendoza (1991). Similar to our modelling approach, Cardia (1991) develops a small open economy RBC model with finite lifetimes à la Blanchard (1985) to explore the effects of exogenous shocks to technology, nominal monetary growth, and fiscal spending—while abstracting from shocks affecting the world real interest rate—on the dynamics of a small open economy. The key assumption in the finite horizons framework is that agents face, in each period of their life, a constant probability of death. This drives a wedge between the world real interest rate and the subjective discount rate that becomes a function of financial wealth. Accordingly, adjustments in the subjective discount rate ensure the existence of a stationary equilibrium in which the the two rates are equalized. On the other hand, different from ours, to slow down the movements in capital stock and investment, as in Mendoza (1991), her model has only one type of capital, and relies on capital adjustment costs. Given the focus of the paper, her model also incorporates money via a cash-in-advance constraint as in Cooley and Hansen (1989). In addition, different from us and Mendoza (1991), her model is calibrated to the U.S. economy rather than the Canadian economy. With highly persistent productivity shocks, her model is found to replicate the observed high national saving-investment correlation and the countercyclical behavior of the current account remarkably well. Her results

[1]The exception is the finite lifetimes feature. The authors do not consider this case.

suggest that most of the changes in output, saving, investment, and in the current account are due to technological shocks rather than monetary and fiscal changes.

Mendoza (1991) also extends the standard RBC model to the case of a small open economy. Different from Cardia (1991), whose objective was to study the impact of government policy shocks in a small open economy; Mendoza, like us, studies shocks to technology and the world real interest rate. The author shows that, due to the separation of saving and investment that characterizes a small open economy, the basic RBC model without capital adjustment costs performs poorly in terms of mimicking the observed business-cycle statistics of investment. In the open economy framework, when the productivity shock hits the economy, the capital stock is rapidly and freely adjusted to maintain equality of the expected marginal returns of domestic capital and foreign assets. To moderate the variability of capital accumulation and thus investment, rather than relying on imperfections in international capital mobility as in the current paper, Mendoza (1991) embeds capital-adjustment costs into the model and assumes perfect capital mobility. In addition, in the spirit of Uzawa (1968) and Obstfeld (1981), an endogenous rate of time preference—captured in the proposed stationary cardinal utility function—that increases with past consumption levels is introduced into his model to ensure a well-defined stationary equilibrium for the holdings of foreign assets. Given this preference formulation, as mentioned in the paper, the dynamics of the model works as follows: as long as the time-preference rate is smaller (greater) than the interest rate, households choose to accumulate (deplete) foreign assets in order to fund an increasing (decreasing) consumption. Once the rate of time preference adjusts to the world's real interest rate, the accumulation of external assets reaches its steady state level. Arguably, there are at least two attractions in our modelling approach compared to Mendoza (1991). First, in our framework, the effective discount rate varies in response to levels of consumption and assets and is truly endogenous in the sense that this feature materializes out of the finite horizon structure of the model, rather than simply being imposed on the utility function. Second, in our model, the horizon index can be chosen arbitrarily anywhere between zero and infinity and this allows us to study the changes in life expectancy on the behavior of the economy. In Mendoza's (1991) analysis, in contrast, the individuals are assumed to be infinitely lived.

The literature that focuses on the relationship between demographics and the business cycle is relatively thin. By developing a closed economy overlapping generations labor search model with matching frictions and productivity shocks, Lugauer (2012a) shows how the age distribution in the population affects aggregate fluctuations and demonstrates that this effect is quantitatively large. On the empirical side, using panel data methods, Jaimovich and Siu (2009) find that changes in the age composition of the workforce account for a significant fraction of the variation in business cycle volatility observed in G7 economies. Likewise, based on standard panel data methods with lagged birth rates as an instrument, Lugauer (2012b) finds a strong statistical relationship between the relative supply of young workers and GDP volatility across the United States.

The abovementioned studies consider the effects of longevity on the business cycle strictly through changes in the labor market dynamics. However, in their recent work, Dantas Guimarães and Ferreira Tiryaki (2020) provide empirical evidence that increased longevity may also affect the volatility of macroeconomic aggregates through changes in individuals' behavior as well as through international trade volatility. Our work contributes to the ongoing research by means of a theoretical framework which explains how increased life expectancy affects the aggregate volatility of a small open economy through changes in individuals' intertemporal investment and labor-leisure decisions.

3. The Model

3.1. Households

Following Blanchard (1985), we introduce finite lifetimes into the model by assuming that agents face a constant probability of death, λ, in each period. Consequently, the survival probability from one period to the next is $1-\lambda$. Without loss of generality, the size at birth of a cohort born at any date s is normalized to λ. Thus, the size at date t of a cohort born at date s is $\lambda(1-\lambda)^{t-s}$, and the population size at any date t, L_t, is $\sum_{s=0}^{\infty}\lambda(1-\lambda)^s = \frac{\lambda}{1-(1-\lambda)} = 1$. Consequently, the model will be used to study economywide per capita average values. The life expectancy of a member of cohort born at date s is $\sum_{t-s=1}^{\infty}(t-s)\lambda(1-\lambda)^{t-s-1} = \frac{1}{\lambda}$.

In period t, a representative agent of cohort s maximizes his expected lifetime utility of the form:

$$E_t\left\{\sum_{v=t}^{\infty}(\beta(1-\lambda))^{v-t}\left[\pi ln(c_{s,v})+(1-\pi)ln(1-n_{s,v})\right]\right\} \tag{1}$$

where $\beta \in (0,1)$ is the pure rate of time preference, $\beta(1-\lambda)$ captures the effective rate of time preference, π is the weight on consumption in utility, and $c_{s,v}$ and $n_{s,v}$ are, respectively, time-v consumption and labor by an agent born at time s. Agents are born with no wealth and have no bequest motives. As in Yaari (1965) and Blanchard (1985), uncertainty about death and the absence of a bequest motive create the need for a life insurance market whereby agents sell contingent claims on their assets to insurance companies. Assets, except for human capital which is destroyed when the agent dies, are collected in each period from λ agents who died and transferred to the surviving agents who are of the size $1-\lambda$. With a perfectly competitive insurance market, each surviving agent receives a premium payment of $\frac{\lambda}{1-\lambda}$ and the gross rate of return on the insurance contract is given by $1+\frac{\lambda}{1-\lambda}=\frac{1}{1-\lambda}$. Given this gross return and assuming that agents are endowed with one unit of time that can be allocated between labor and leisure, the budget constraint in period t for an agent born at date s is given as follows:

$$c_{s,t}+k_{s,t+1}+f_{s,t+1}+h_{s,t+1}=\left(\frac{1+r_t}{1-\lambda}\right)(k_{s,t}+f_{s,t})+(1+q_t-\delta_h)h_{s,t}+w_t n_{s,t} \tag{2}$$

where $k_{s,t}$, $h_{s,t}$, and $f_{s,t}$ denote, respectively, the stock of physical capital, the stock of human capital, and net foreign assets at period t; and δ_h, r_t, q_t, and w_t denote, respectively, the depreciation rate for an agent's human capital, and the real interest rate, the real rental rate of human capital, and the wage rate in period t.

As in Barro et al. (1995), an international borrowing constraint is introduced into the model by assuming that only physical capital is an acceptable collateral on foreign loans, while human capital and raw labor are not[2]. Consequently, the foreign debt can be positive but cannot exceed

[2]The authors motivate the borrowing constraint as follows. "Physical capital is more easily repossessed and more readily monitored than human capital and is therefore more easily financed with debt. Moreover, physical capital is also more amenable to direct foreign investment: one can own a factory but not someone else's stream of labor income." The finite horizons feature of our model provides additional motivation. In the event of death, an agent's human capital is destroyed, whereas for physical capital both debt and equity can be transferred to surviving members of the economy.

the amount of physical capital:

$$-f_{s,t} \leq k_{s,t}. \tag{3}$$

Note that if the borrowing constraint (3) is slack, then the economy has sufficient collateral to borrow and will adjust both types of capital rapidly. This situation would lead to counterfactually high transitional growth and investment volatility. Therefore, to avoid the exaggerated variability of capital and having instantaneous convergence to the balanced growth path, we restrict our attention to the case where the borrowing constraint is binding:

$$-f_{s,t} = k_{s,t} \quad \forall t. \tag{4}$$

Given (4), the flow budget in (2) can now be written as follows:

$$c_{s,t} + h_{s,t+1} = (1 + q_t - \delta_h)h_{s,t} + w_t n_{s,t}. \tag{5}$$

In the spirit of Frenkel and Razin (1986), for the purpose of deriving the present value at date t of agent's lifetime budget constraint, we define the following present-value factor, τ_t, which is composed of one-period real rental rates of human capital compounded from period 1 to period t:

$$\tau_t = \{(1 + q_1 - \delta_h)(1 + q_2 - \delta_h)...(1 + q_t - \delta_h)\}^{-1}. \tag{6}$$

Therefore, τ_t/τ_{t+1} equals $(1 + q_{t+1} - \delta_h)$, which is the gross rate of return to individual human capital stock in period $t + 1$. Given τ_t and the survival probability from one period to the next, $1 - \lambda$, the present-value lifetime budget constraint can be derived by consolidating the periodic budget constraints for the current period t and the future periods (i.e., periods t+1, t+2, ...):

$$E_t \sum_{v=t}^{\infty} (1-\lambda)^{v-t} \frac{\tau_v}{\tau_t} c_{s,v} = E_t \left(\sum_{v=t}^{\infty} (1-\lambda)^{v-t} \frac{\tau_v}{\tau_t} w_v n_{s,v} + (1 + q_t - \delta_h)h_{s,t} \right). \tag{7}$$

Note that, in deriving equation (7), we have imposed the transversality condition given by:

$$lim_{v\to\infty}(1-\lambda)^v \tau_v h_{s,v} = 0. \tag{8}$$

The agent's problem is to maximize the expected lifetime utility in equation (1) subject to the lifetime budget constraint in (7), and the Lagrangian associated with the optimization problem can be written as follows:

$$\mathcal{L} = E_t \sum_{v=t}^{\infty} (\beta(1-\lambda))^{v-t}[\pi ln(c_{s,v}) + (1-\pi)ln(1-n_{s,v})] + \\ + \gamma E_t \left[\sum_{v=t}^{\infty} (1-\lambda)^{v-t} \frac{\tau_v}{\tau_t} w_v n_{s,v} + (1+q_t-\delta_h)h_{s,t} - \sum_{v=t}^{\infty} (1-\lambda)^{v-t} \frac{\tau_v}{\tau_t} c_{s,v} \right] \tag{9}$$

where γ is the Lagrange multiplier associated with the budget constraint. The first order conditions with respect to $c_{s,v}$ and $n_{s,v}$, respectively, imply:

$$c_{s,v} = E_t \left[\beta^{v-t} \pi \left(\gamma \frac{\tau_v}{\tau_t} \right)^{-1} \right] \quad \text{for} \quad v = t, t+1, t+2, ... \tag{10}$$

$$n_{s,v} = E_t \left[1 - \beta^{v-t}(1-\pi) \left(\gamma \frac{\tau_v}{\tau_t} w_v \right)^{-1} \right] \quad \text{for} \quad v = t, t+1, t+2, ... \tag{11}$$

Using equations (7) and (10), we can express consumption at time t as a function of wealth as follows:

$$c_{s,t} = \Delta^{-1} E_t \left[\sum_{v=t}^{\infty} (1-\lambda)^{v-t} \frac{\tau_v}{\tau_t} w_v n_{s,v} + (1+q_t-\delta_h)h_{s,t} \right] \tag{12}$$

where $\Delta \equiv \sum_{v=t}^{\infty} ((1-\lambda)\beta)^{v-t} = \frac{1}{1-(1-\lambda)\beta}$; and Δ^{-1} and $\widetilde{w}_t \equiv \sum_{v=t}^{\infty} (1-\lambda)^{v-t} \frac{\tau_v}{\tau_t} w_v n_{s,v}$ denote, respectively, the propensity to consume out of wealth and the present value at date t of the agent's lifetime wage income. From (10) and (11), we derive:

$$n_{s,t} = 1 - \frac{(1-\pi)c_{s,t}}{\pi w_t}. \tag{13}$$

3.2. Firms

A representative firm in the perfectly competitive market produces output, Y_t,[3] according to the following Cobb-Douglas technology with physical capital, K_t, human capital, H_t, and labor, N_t, as factor inputs :

$$Y_t = z_t K_t^{\alpha} H_t^{\eta} (A_t N_t)^{1-\alpha-\eta} \tag{14}$$

where A_t and z_t are the deterministic level of technology and the stochastic total factor productivity shock, respectively, and they evolve exogenously as follows:

$$A_t = A_0 e^{xt} \tag{15}$$

$$lnz_t = \rho_z lnz_{t-1} + \varepsilon_{z,t}, \quad \varepsilon_{z,t} \sim i.i.d.\, N(0,\, \sigma_z^2). \tag{16}$$

We perform a balanced growth transformation and define variables in effective worker units using lowercase letters with hats ($\wedge$)[4]. Consequently, equation (14) can be expressed in its intensive form as follows:

$$\hat{y}_t = z_t \hat{k}_t^{\alpha} \hat{h}_t^{\eta} N_t^{1-\alpha-\eta}. \tag{17}$$

The firm's profit maximization implies that factors of production are paid their marginal products and that no arbitrage opportunities exist between physical capital and foreign assets:

$$w_t = (1-\alpha-\eta) A_t \frac{\hat{y}_t}{N_t} \tag{18}$$

$$q_t = \eta \frac{\hat{y}_t}{\hat{h}_t} \tag{19}$$

$$r_t = \alpha \frac{\hat{y}_t}{\hat{k}_t} - \delta_K \tag{20}$$

where δ_K denotes the rate of depreciation for physical capital.

To capture transmission of technology shocks across countries, we assume that the real rental rate of physical capital faced by the agents in the small open economy, $r_t + \delta_K$, consists of three

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[3] Aggregate variables are denoted by capital letters.

[4] Note that, since the population size at any date t, L_t, is equal to 1, the aggregate and per worker/average variables are the same in the model. Thus, $\hat{k}_t \equiv \frac{K_t}{A_t L_t} = \frac{K_t}{A_t}$.

components, a constant corresponding to the average value, R, a random disturbance affecting the world's real interest rate, g_t, and the stochastic total factor productivity shock, z_t. Note that in closed economy models the technology variable and the real interest are strongly correlated, and this is because a positive (negative) shock will raise (lower) the rate of return to an investment in physical capital, and consequently the optimal reallocation of domestic savings requires a rise (fall) in the interest rate to clear the market. In comparison, a key assumption about small open economies is that the interest rate is invariant to domestic savings and investment activities. Therefore, in order for interest rates to be procyclical, as observed in the data, business cycle activity will need to be synchronized internationally. In fact, there is a large literature that demonstrates stronger correlations between business cycle fluctuations in real GDP for countries that trade more with one another as in Frankel and Rose (1998), Clark and van Wincoop (2001), Baxter and Kouparitsas (2005), Burstein et al. (2008), and di Giovanni and Levchenko (2010), among others. Thus, we assume the following functional form $F(\cdot)$ for $r_t + \delta_K$:

$$r_t + \delta_K = F(r, z_t, g_t) = R z_t^{\theta} g_t^{1-\theta} \tag{21}$$

where $0 \leq \theta \leq 1$ captures the degree of transmission of technology shocks across countries. Accordingly, higher θ implies stronger synchronization of business cycle movements across countries. Moreover, g_t is assumed to evolve exogenously as follows:

$$ln g_t = \rho_g ln g_{t-1} + \varepsilon_{g,t}, \quad \varepsilon_{g,t} \sim i.i.d.\, N(0, \sigma_g^2). \tag{22}$$

Combining (20) and (21), we derive:

$$\hat{k}_t = \frac{\alpha \hat{y}_t}{R z_t^{\theta} g_t^{1-\theta}}. \tag{23}$$

From (17) and (23):

$$\hat{y}_t = z_t^{\frac{1}{1-\alpha}} \left[\frac{\alpha}{R z_t^{\theta} g_t^{1-\theta}} \right]^{\frac{\alpha}{1-\alpha}} \hat{h}_t^{\frac{\eta}{1-\alpha}} N_t^{\frac{1-\alpha-\eta}{1-\alpha}}. \tag{24}$$

Note that $0 < \alpha + \eta < 1$ implies that $0 < \frac{\eta}{1-\alpha} < \alpha + \eta < 1$. Consequently, from (24), we can see that our model features diminishing returns with respect to human capital.

3.3. Competitive equilibrium

We now combine the behavior of households and firms to characterize the structure of a competitive market equilibrium. For each time period t, the competitive equilibrium consists of an allocation $\left\{\hat{c}_{s,t},\ \hat{h}_{s,t},\ \hat{k}_{s,t},\ \hat{f}_{s,t},\ \hat{a}_{s,t}\right\}$ for all living households of each cohort $s \leq t$, a production plan $\{Y_t,\ H_t,\ N_t,\ K_t\}$ for each firm, and a set of prices $\{r_t,\ w_t,\ q_t\}$, such that:

(i) given prices, the allocations for the households solve their optimization problem given by (1)-(8);

(ii) given prices, the allocations for the firms solve their profit maximizing conditions given by (18)-(20);

(iii) all markets clear; below $-F(t)$ stands for the aggregate foreign debt of the domestic economy:

$$N_t = \sum_{s=0}^{\infty} \lambda(1-\lambda)^s n_{s,t} \tag{25}$$

$$H_t = \sum_{s=0}^{\infty} \lambda(1-\lambda)^s \hat{h}_{s,t} \tag{26}$$

$$K(t) = -F(t) \tag{27}$$

$$C_t + H_{t+1} = (1-\alpha)Y_t + (1-\delta_h-\lambda)H_t + w_t N_t; \tag{28}$$

(iv) individual decisions are consistent with aggregate outcomes. As in Blanchard (1985), aggregate variables can be found as the summation across cohorts, weighted by their respective sizes. For example, for aggregate consumption, $C_t = \sum_{s=0}^{\infty} \lambda(1-\lambda)^s c_{s,t}$, and so on. Using the foregoing equations that connect aggregate to individual outcomes, the first order conditions to the individuals' and firms' optimization problems, and the market clearing conditions, we can derive expressions for the aggregate resource constraint, the Euler equation for the accumulation of human capital, and the efficiency condition for aggregate hours worked, and these are given, respectively by:

$$(1-\alpha)\hat{y}_t = \hat{c}_t + e^x \hat{h}_{t+1} - (1-\delta_h-\lambda)\hat{h}_t \tag{29}$$

$$\hat{c}_t = \frac{\Delta^{-1}\lambda}{\beta(1-\lambda)}\left(\hat{h}_t + e^x \hat{h}_{t+1}\right) + \frac{e^x}{\beta} E_t \left\{ \frac{\hat{c}_{t+1}}{1 + \eta \frac{\hat{y}_{t+1}}{\hat{h}_{t+1}} - \delta_h} \right\} \tag{30}$$

$$N_t = 1 - \frac{(1-\pi)\hat{c}_t}{\pi(1-\alpha-\eta)\frac{\hat{y}_t}{N_t}}. \tag{31}$$

4. Steady state and dynamics

In this section, we characterize the steady state and dynamics of the model. In order for a stable steady state to exist for the model, the variables in equations (29)-(31) must be stationary in the long run. To study the nonstochastic steady steady, we set $\hat{c}_t = \hat{c}^*$ and $\hat{h}_t = \hat{h}^*$ for all t. Then, from the Euler equation in (30), the required condition for having consumption per effective unit of labor to be constant in the long run is given by:

$$1 + \eta \frac{\hat{y}^*}{\hat{h}^*} - \delta_h = \left[\frac{\beta}{e^x} - \frac{\Delta^{-1}\lambda}{1-\lambda} \frac{(1+e^x)}{e^x} \frac{\hat{h}^*}{\hat{c}^*} \right]^{-1}. \tag{32}$$

Notice that the term on the left hand side of equation (32) is the rate of return to human capital and it is decreasing in $\hat{h}$, whereas the term on the right hand side of the equation is the steady state effective discount rate of the domestic economy and, for $\lambda \neq 0$, it can be shown to be increasing in $\hat{h}$. Due to the model's finite lifetimes feature, adjustments in the ratio $\frac{\hat{h}}{\hat{c}}$ in the effective discount rate help to ensure that the long run value for the rate of return to human capital will be above the steady state world interest rate, r[5]. Recall that the model equations (29)-(31) above were derived under the assumption of a binding debt constraint—given by equation (4)—and this assumption is only valid if the rate of return to human capital remains above r. Therefore, when analyzing the dynamics of the model, there are two cases to consider: (i) $1+\eta\frac{\hat{y}^*}{\hat{h}^*}-\delta_h > r$ and (ii) $1+\eta\frac{\hat{y}^*}{\hat{h}^*}-\delta_h = r$. We note that the case (ii) implies a situation in which the borrowing constraint no longer holds with equality. If during the transition to the steady state the accumulation of human capital causes its return to fall and equal the world real interest rate before equation (32) was satisfied—so the case (ii)—then agents would stop investing in human capital and would start accumulating foreign assets or build up equity from physical capital. Our companion paper, Tserenkhuu and Kosempel

[5] Note that, from equation (21), $r \equiv R - \delta_k$.

(2023), discusses how to deal with this situation. Specifically, once the rate of return to human capital equals r, then the rate of return to human capital and human capital are replaced in the Euler equation with r and a new variable—say $\hat{b}$, where $\hat{b}$ denotes the sum of all assets: $\hat{b} = \hat{k}+\hat{h}+\hat{f}$. As the economy accumulates more assets, the effective discount rate rises until it equals r, and (32) is satisfied. In this case, having the effective discount rate rise with the assets to consumption ratio, which materializes out of the finite-horizons structure, is critical to guarantee a stable outcome for consumption and assets. Note, however, that our calibration exercise in the next section produces parameter values for which the borrowing constraint in the model is permanently binding, and thus we consider only the case (i) in the rest of this paper. Readers will find a detailed analysis of the case (ii) in our companion study which focuses on the transitional dynamics and long term outcomes in a deterministic version of this model.

The steady state consumption to human capital and output to human capital ratios—$\frac{\hat{c}^*}{\hat{h}^*}$ and $\frac{\hat{y}^*}{\hat{h}^*}$, respectively—can be found by solving equations (29) and (30) when evaluated at the steady state. Given N^* and a positive unique root for $\frac{\hat{y}^*}{\hat{h}^*}$, equation (24) at the steady state implies:

$$\hat{h}^* = \left[\left(\frac{\alpha}{R}\right)^{\alpha}\left(\frac{\hat{y}^*}{\hat{h}^*}\right)^{\alpha-1}\right]^{\frac{1}{1-\alpha-\eta}} N^*. \tag{33}$$

Next, given $\frac{\hat{c}^*}{\hat{h}^*}$, $\hat{h}^*$, and N^*, we can derive the steady state levels of consumption and output per effective labor, and the domestic saving rate for human capital, s^*, as follows:

$$\hat{c}^* = \left(\frac{\hat{c}^*}{\hat{h}^*}\right)\hat{h}^* \tag{34}$$

$$\hat{y}^* = \left[\frac{\alpha}{R}\right]^{\frac{\alpha}{1-\alpha}} \hat{h}^{*\frac{\eta}{1-\alpha}} N^{*\frac{1-\alpha-\eta}{1-\alpha}} \tag{35}$$

$$s^* = 1 - \frac{\hat{c}^*}{(1-\alpha)\hat{y}^*}. \tag{36}$$

To analyze the dynamics of the model, we log-linearize the model equations (29)-(31) around its deterministic steady state and solve the resulting system for the recursive equilibrium laws of motion following Uhlig's (2001) method of undetermined coefficients. Once the law of motions for human capital, consumption, and labor are known and given the initial values $\left\{\hat{h}_0,\ z_0,\ g_0\right\}$ and a sequence of disturbances $\{\varepsilon_{z,t},\ \varepsilon_{q,t}\}_{t=0}^{\infty}$, we can then derive the dynamic time paths for all model variables.

5. Calibration

To derive the numerical solutions to the model and evaluate the quantitative predictions of the model, it is necessary to assign values for the model parameters. We calibrate the model to Canadian quarterly data and values for most parameters are available from the existing literature. The share of physical capital in output is set at $\alpha = 1/3$ and the rate of depreciation for physical capital is set at $\delta_K = 0.025$. These values are commonly used in models calibrated to small open economies as in Mendoza (1991) Schmitt-Grohe and Uribe (2003), Letendre (2004), Nason and Rogers (2006), and Letendre and Luo (2007), among others. The pure rate of time preference is set at $\beta = 0.989$ to target the average quarterly rate of return to human capital of 2% as in the data[6]. The value of $\beta = 0.989$ is also very close to 0.99—a value estimated by Prescott (1986).

In our benchmark calibration, life expectancy is set at 74 years or 296 quarters (i.e. $\lambda = 1/296$) to match life expectancy at birth in Canada at the start of our sample. Then, in Section 7, we study how using the end of period value (82 years or 328 quarters) impacts the results.

Based on Mendoza (1991) and Schmitt-Grohe and Uribe (2003), who use the value suggested by Kydland and Prescott (1982) and Prescott (1986) for the real interest rate in the U.S. economy, we set the steady state world's real interest rate at $r = R - \delta_K = 0.01$, which implies an annual real interest rate of four percent. The value of four percent is also consistent with the estimates suggested by Hansen (1985) and McGrattan and Prescott (2001).

As mentioned in Alessandrini et al. (2015), regarding the depreciation rate for human capital, there is no agreement in the literature and estimates vary in the range 0.5% – 5% per annum. As a compromise, we choose the midpoint of the range and set δ_h at its quarterly equivalence of 0.6875%. Similarly, regarding the human capital share parameter in the production function (η), estimates used are typically in the range 1/3 – 2/3. For example, Mankiw et al. (1992) suggest

[6] Psacharopoulos and Patrinos (2018) estimate that the private return to schooling is 8% annually for high income countries like Canada.

that a production function that is consistent with their empirical work has equal factor shares of 1/3 across the three inputs: K, H, N. While their estimate for α is consistent with physical capital's share of income as measured in the National Accounts, their value of η seems low. For example, in the endogenous growth literature, η is typically set to 2/3, implying an income share of unimproved labor (N) of zero[7]. Empirical support for using a near zero share of unimproved labor is provided by Gundlach (1995), who re-examines the work of Mankiw et al., but using a more broadly defined measure of human capital. In the current paper, as a compromise, we choose the midpoint of $\eta = 1/2$.

As it is a common practice in the RBC literature, the weights on consumption and leisure, π and $1-\pi$ are set to target the hours worked in the steady state to 1/3, a value that is routinely used in the RBC literature. The resulting values are $\pi = 0.5449$ and $1-\pi = 0.4551$.

Given parameter values for α and η and using Canadian quarterly data for the period 1976Q1 to 2019Q4—from CANSIM by Statistics Canada—on GDP, business gross fixed investment in nonresidential structures, machinery and equipment[8] (a proxy for investment in physical capital), total expenditures on education[9] (a proxy for investment in human capital), and labor hours, we construct a series of Solow residuals, SR, as follows:

$$SR_t = lnY_t - \alpha lnK_t - \eta lnH_{t-1} - (1-\alpha-\eta)lnN_t. \tag{37}$$

According to the production function in equation (14), the above Solow residuals are related to the total factor productivity shock, z_t, and the growth rate in deterministic level of technology, x, as follows:

$$SR_t = (1-\alpha-\eta)lnA_0 + (1-\alpha-\eta)x \cdot t + lnz_t. \tag{38}$$

Based on equation (38), we regress the SR_t on a time trend, t, to find estimate of x. The estimate

[7]This includes RBC models with endogenous growth as in Gomme (1993).

[8]Given the series, we can derive the series for stock of physical capital as follows. First, we assume that the economy is on a balanced growth path and thus flow and stock of physical capital are growing on average at the same rate. Then, given this assumption and given parameter value for δ_K, the series for stock of physical capital can be computed from the following law of motion equation: $K_{t+1} = I_{K,t+1} + (1-\delta_K)K_t$, where $I_{K,t+1}$ is the investment in physical capital in period $t+1$.

[9]The series for the stock of human capital is derived following the same procedure used to derive the stock of physical capital. This time, the law of motion for human capital, which is derived from equation (39), is given by: $H_{t+1} = I_{h,t} + (1-\delta_h-\lambda)H_t$, where $I_{h,t}$ is the investment in human capital in period t.

is found to be $x = 0.0041$, which corresponds to an annual growth rate of 1.64%. Note that the constant, A_0, is simply a scaling factor and can be normalized to 1.

Next, given the series on GDP and physical capital, to estimate the parameter for the degree of transmission of technology shocks across countries, θ, based on equation (20), we first construct a series for natural logarithm of the real rental rate of physical capital as follows:

$$ln(r_t + \delta_K) = ln\left(\frac{\alpha\hat{y}_t}{\hat{k}_t}\right). \tag{39}$$

According to equation (21), the above series for the real rental rate of physical capital is related to the total factor productivity shock, z_t, and the random disturbance affecting the world's real interest rate, g_t, as follows:

$$ln(r_t + \delta_K) = lnR + \theta lnz_t + (1-\theta)lng_t. \tag{40}$$

Based on equation (40), we regress the series on the left hand on a constant and the residuals from equation (38) to find an estimate of θ, which is found to be $\theta = 0.9598$. The value of 0.9598 corresponds to a high degree of transmission of technology shocks, thereby capturing the strong correlations between business cycle fluctuations in real GDP for the small open economy and its trading partners as supported by the literature. While we consider the aforementioned high degree of transmission of technology shocks in our baseline calibration, when we present our numerical results in Section 6, we will test the sensitivity by using low and mid values for θ as well.

Next, while the residuals from the regression in (38) can be regressed on their first lag to derive estimates for the value of the autocorrelation coefficient in the stochastic total factor productivity process, ρ_z, and the standard deviation of the innovation to the process, σ_z, there are at least two reasons why this approach might not be appropriate for our analysis[10]. First, as already highlighted in Mendoza (1991), Solow residuals obtained from Canadian data exhibit too much variability and serial autocorrelation for the model to be consistent with the stylized facts. Second,

[10]According to our regression results, we find $\rho_z = 0.9887$ and $\sigma_z = 0.748\%$. While the value of 0.748% is in the range 0.7% – 1% that is suggested in the literature (See Hansen 1985, for example), the value of 0.9887 is higher than 0.95, a value that is often used in the RBC literature.

in constructing the series for investment in human capital in (37), we converted our raw annual data on total expenditures on education to quarterly frequency by interpolating linearly between points and this procedure will add additional persistence to the stock of human capital and also to the Solow residuals. Therefore, to avoid these issues and to help facilitate a comparison with existing RBC studies, we choose to adopt the following standard values used in the literature[11]: $\rho_z = 0.95$ and $\sigma_z = 0.763\%$.

Regarding the value of the autocorrelation coefficient in the process for stochastic world interest rate, ρ_g, and the standard deviation of the innovation to the process, σ_g, there is no agreement in the literature. For example, to facilitate his numerical analysis by restricting the set of free parameters to be specified, for simplicity, Mendoza (1991) sets $\rho_g = \rho_z$. For σ_g, he considers various different values—in the range $0.678\% - 3.39\%$[12]—to explore his model's sensitivity to international disturbances. In our baseline calibarion, we set $\rho_g = \rho_z (= 0.95)$ following Mendoza, and the value for the parameter σ_g is set at 1.567% to mimic the volatility of investment in physical capital in the data. While $\sigma_z = 0.763\%$ and $\sigma_g = 1.567\%$ are our preferred values, in Section 6.5, we will demonstrate the sensitivity of the model to the shocks by analyzing how the results change in the absence of one shock, that is, when the volatility of one shock is set to zero. Finally, since the parameter θ was already introduced to account for the potential international transmission of technology shocks, we consider mutually uncorrelated disturbances, that is, we set $\rho_{z,g} = 0$.

6. Numerical results: The model properties

6.1. Transitional dynamics in the absence of shocks

Figure 1 shows the time paths of the model variables in the absence of any shocks (i.e. $z_t = z^* = 1$ and $g_t = g^* = 1$ for all t) and when the hypothetical economy's initial level of human capital per effective labor is set at $\hat{h}_0 = 0.25 \cdot \hat{h}^*$. Setting $\hat{h}_0 < \hat{h}^*$ allows us to study the transitional dynamics of the model; however, the actual level of $\hat{h}_0$ is set arbitrarily. From panels A and B, we can see that output per capita increases during the transition while the growth of output per

[11] See Prescott (1986), for example.

[12] These correspond to the quarterly equivalents of the annual values used by Mendoza (1991).

capita decreases. These results are consistent with the evidence that the growth rate declines as income increases.

As can be seen from panels C and G, in our model, both the domestic saving rate and aggregate hours worked decrease during the transition until they reach their respective steady state values. Higher values of hours worked and savings early on in the model imply that agents in the economy are taking advantage of high returns to human capital that exist during the transition by working longer and saving to accumulate more human capital for later on when returns are lower.

Panels D, E, F, and H show, respectively, that consumption, human capital, physical capital, and foreign debt per effective labor all increase gradually during the transition until they reach their respective positive steady state values. The gradual increase in human capital during the transition reflects the feature of diminishing returns to human capital in our model, which can be seen from the production function in its intensive form in equation (24). Moreover, due to the constraint of domestic saving on the accumulation of human capital and the complementarity between the two types of capital in the Cobb-Douglas production function, the physical capital stock and level of foreign debt also rise gradually towards their steady-state values. We also see that transition paths for the variables measured in effective labor units all have concave shapes. Over time growth slows and values approach positive constants. This stability is achieved due to the model's finite-lifetimes feature. The observed slow transition to the steady state is an implication of the borrowing constraint, which is critical to prevent capital stocks from adjusting instantaneously to their long run values.

6.2. Total factor productivity shocks

Figure 2 shows the impulse response functions for the aggregate variables to a positive one percent TFP shock. The graphs represent the percent deviation of each variable from steady state after the shock. We can see that all the reported variables are procyclical. The results are consistent with the standard results in the RBC literature. A positive TFP shock has a direct positive impact on output via the production function. Consequently, marginal products of all factors of production increase. Given these increases, the agents are encouraged to accumulate more human capital and increase their labor supply. The procyclical transitional dynamics for physical capital stock and

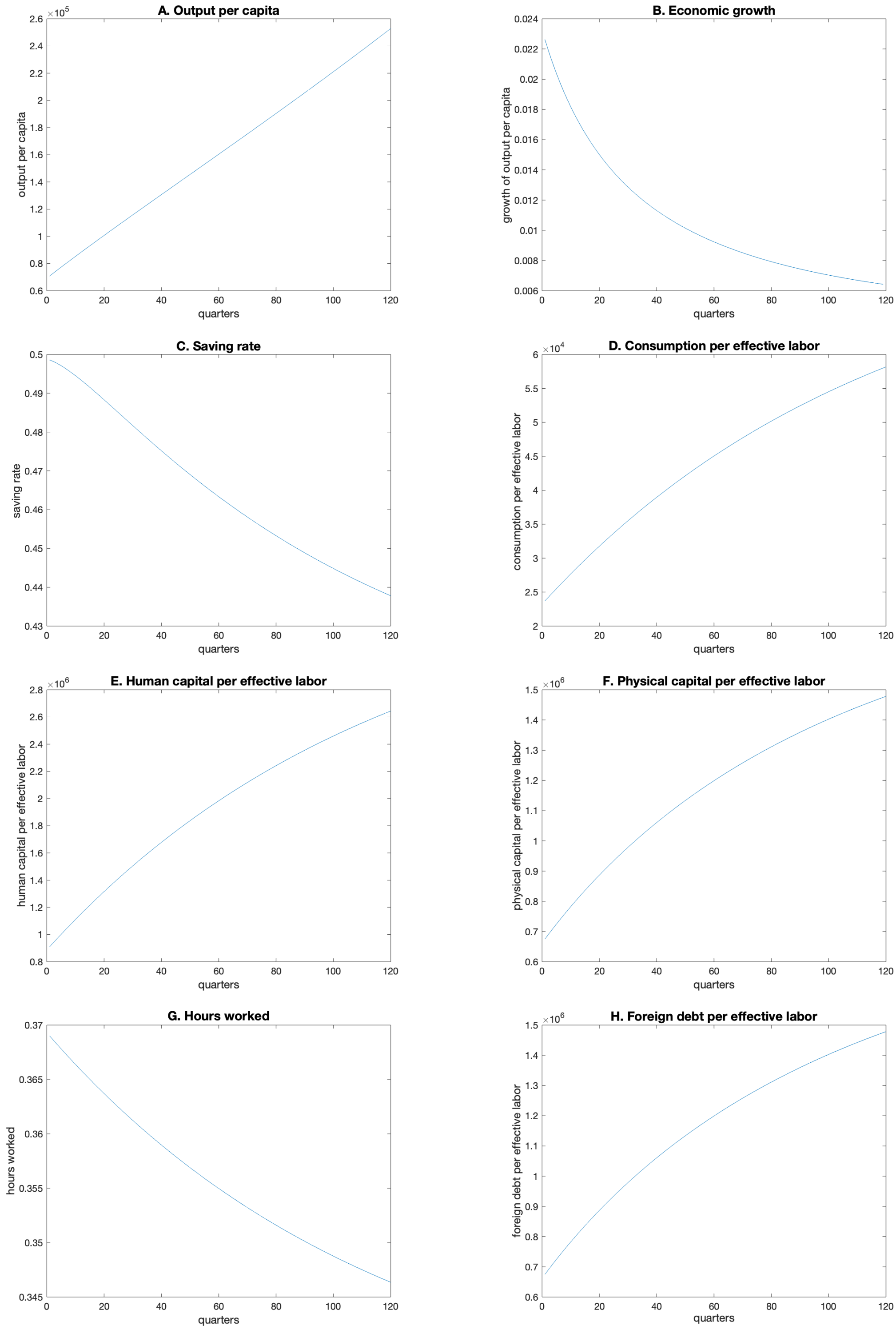

Figure 1: Time paths of the model variables in the absence of shocks

the interest rate following the TFP shock are determined by equations (20) and (21). Recall that the world interest rate will respond positively to a productivity shock, and the strength of the response will depend on the parameter θ in equation (21), which restricts/permits the international transmission of productivity shocks. Having $\theta > 0$ is necessary to reproduce procyclical movements in the interest rate. We will revisit this result in Section 6.5 when we test the sensitivity our our results to low, medium, and high values for theta. Next, given that consumption is a normal good, it also increases following the positive shock. Finally, we can see that all the variables eventually—around 40-140 quarters after the shock—converge back to their steady state values.

6.3. Interest rate shocks

Figure 3 shows the impulse response functions for the aggregate variables to a positive one percent shock to the exogenous component of the world interest rate, g. We can see that all the variables are affected negatively by the shock. A positive interest rate shock makes borrowing from abroad more expensive and thus lowers the amount of foreign loans for the domestic economy. Since foreign loans require physical capital as a collateral, the domestic economy's physical capital stock also decreases following the positive interest rate shock. Moreover, since physical capital is the only acceptable form of collateral, it responds directly to the shock and is the most sensitive. Other variables will respond to the shock, but only indirectly via its impact on physical capital. For example, a fall in physical capital will in turn lead to decreased marginal products of human capital and labor. As a result, the agents accumulate less human capital and decrease their labor supply. Consequently, the output level also decreases. Since consumption is a normal good, it is also affected negatively by the positive interest rate shock. Lastly, owing to the model's finite-lifetimes feature, we can see again that all the key variables eventually—around 40-140 quarters after the shock—converge back to their steady state values.

6.4. Business cycle statistics

Table 1 reports the quarterly business cycle statistics for Canadian economy and three different versions of our model economy: (i) Model with TFP shock only; (ii) Model with interest rate shock only; and (iii) Model with both types of shocks. For the model economies, the average business cycle statistics are computed from 1,000 simulations and each simulation consists of 175 periods

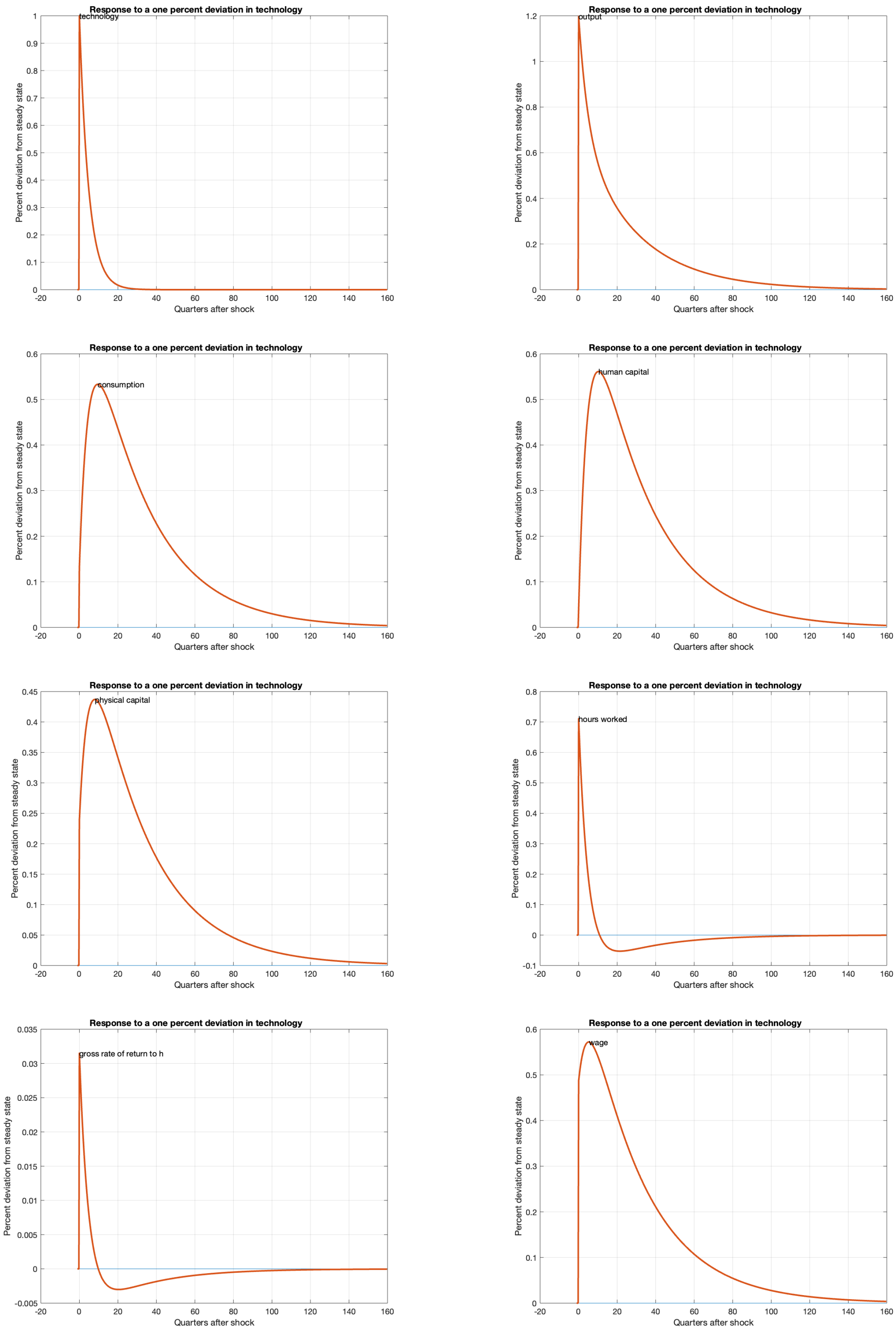

Figure 2: Impulse response functions for the aggregate economy, a TFP shock

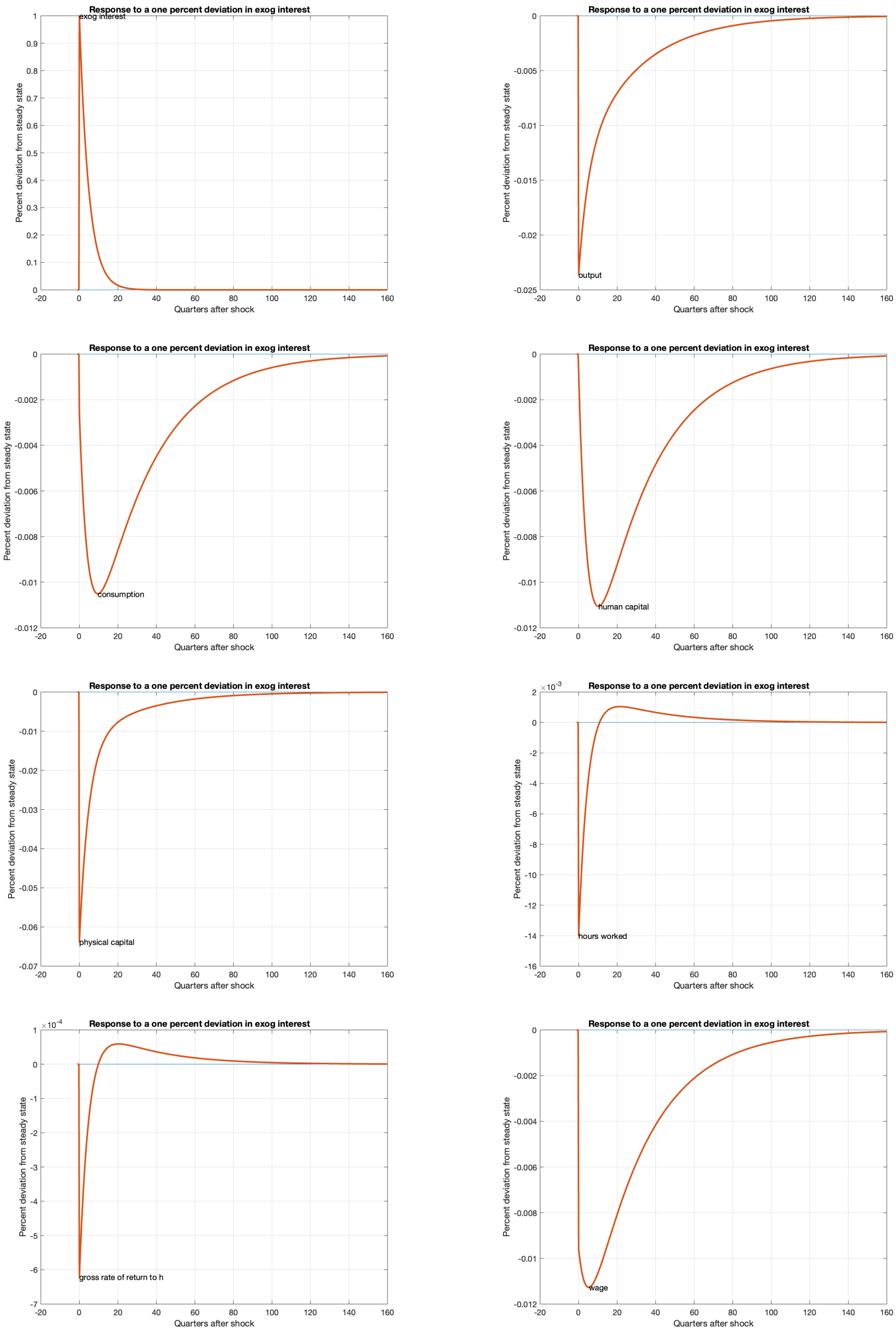

Figure 3: Impulse response functions for the aggregate economy, an interest rate shock

as in the data. As it is standard, before computing the statistics, each simulated series was logged and detrended using the Hodrick-Prescott filter with the smoothing parameter of 1,600, which is an appropriate value for quarterly frequency data. We can see that the fluctuations in all the reported variables except for savings and trade balance ratio are larger for the actual economy than for the model economies. However, a care must be taken in making comparisons. As pointed out in Hansen (1985), there are likely to be measurement errors and sampling errors that may lead to the much higher fluctuations of the actual economy. The much lower fluctuation of consumption series in the model economies than the actual economy is the typical result for the RBC models that do not distinguish between consumer durables and non-durables. In addition, the much lower variability of hours worked in the model economies is also a well-known issue for the RBC literature, especially in models where labor supply is perfectly divisible. Despite these shortcomings, overall, the model economies (i) and (iii) are both consistent with the low volatility of consumption and large volatility of investment in physical capital relative to that of output. Further, they are in line with the high positive correlation of consumption, savings, hours worked, and investment in physical capital with output. The model economies (i)-(iii) are also able to capture the counter-cyclicality of trade balance ratio satisfactorily.

Comparisons of the model economies (i)-(iii) also reveal that fluctuations in our framework are mainly driven by TFP shocks rather than interest rate shocks. In our model, due to the borrowing constraint, the effects of interest rate disturbances on the domestic economy operate only indirectly through their effect on physical capital—which can be seen from equation (23). The relative neutrality of interest rate shocks is supported by the work of Mendoza (1991). His model also predicts that interest rate shocks have little effect on the economy. Mendoza points out that in models calibrated to the small open economy of Canada where the average interest rate is low and foreign interest payments are a small fraction of GDP, the effects generated by these shocks tend to be small. However, the author further points out that, in economies with a higher debt-service ratio, such as the heavily indebted developing countries, interest rate shocks are likely to play a more significant role.

In the model, a rise in the world interest rate has a negative effect on output, and this can be seen from the negative correlation coefficient reported for capital productivity in Model (ii)—when only interest rate shocks are present, and from the impulse response functions reported earlier. However, in the Canadian data capital productivity is strongly procyclical. Since small open economies are assumed to be price takers in international financial markets, in order to match this feature of the data, it is necessary that in the model we account for the international synchronization of business cycle activity, or more specifically, the positive correlation observed between productivity and the rate of return to physical capital. In this way, the dominate shock—productivity (z)—in the model moves both output and the world real interest rate in the same direction. Therefore, these results provide additional support for setting the degree of international synchronization (θ) to be positive. In the next subsection, we provide additional support for setting θ to be a high value.

Finally, as reported in the last row of the table, the model economies capture well the observed positive correlation between domestic savings and investment in physical capital. Having two types of capital helps to achieve this result. For example, following a positive technology shock, the rate of return for both types of capital increases. As a result, agents borrow from abroad to finance physical capital investment, and domestic savings rises to support human capital investment.

6.5. Sensitivity analysis

In this subsection, we test the sensitivity of some of our results using different values for parameters capturing the degree of transmission of technology shocks across countries, θ, the standard deviation of the innovation to the stochastic total factor productivity process, σ_z, and the standard deviation of the innovation to the process for stochastic world interest rate, σ_g. Based on equation (40), we estimated θ to be 0.9598, corresponding to a high degree of international transmission of technology shocks as supported by the literature. In Table 2, we report how the volatility of the two main macro aggregates—output and investment—changes when we use low ($\theta = 0$) and mid ($\theta = 0.5$) values for θ while keeping σ_z and σ_g at their benchmark values of 0.763% and 1.567%, respectively. The table illustrates the role of a higher degree of international transmission of technology shocks in lowering investment volatility in the model. We can see that, while the volatility of output (and other key macro aggregates not reported in the table) increases moderately with low and mid values for θ, the volatility of investment jumps to a very high level of 47.51% with $\theta = 0.5$, and further to 93.01% with $\theta = 0$. Thus in our model, although the international bor-

Table 1: Statistical Moments: Canadian data and the model economies

Series	Canadian data* (1976Q1-2019Q4)		(i) Model with TFP shock		(ii) Model with interest rate shock		(iii) Model with both types of shocks	
	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
Consumption	0.83	0.77	0.17 (0.03)	0.74 (0.04)	0.006 (0.001)	0.74 (0.04)	0.17 (0.02)	0.73 (0.04)
Savings	2.14	0.97	2.71 (0.31)	0.99 (0.00)	0.11 (0.01)	0.99 (0.00)	2.68 (0.27)	0.99 (0.00)
Hours worked	1.4	0.86	0.7 (0.08)	0.99 (0.00)	0.03 (0.003)	0.99 (0.00)	0.69 (0.07)	0.99 (0.00)
Output	1.42	1	1.16 (0.13)	1 (0.00)	0.047 (0.005)	1 (0.00)	1.1474 (0.11)	1 (0.00)
Investment in physical capital	6.63	0.62	5.67 (0.3)	0.53 (0.02)	3.26 (0.21)	0.4 (0.04)	6.63 (0.42)	0.47 (0.03)
Trade balance ratio	0.64	-0.12	1.35 (0.08)	-0.35 (0.05)	0.85 (0.05)	-0.38 (0.04)	1.62 (0.1)	-0.32 (0.04)
Capital productivity	1.84	0.87	0.94 (0.1)	0.99 (0.00)	0.08 (0.009)	-0.99 (0.00)	0.93 (0.09)	0.99 (0.00)
Labor productivity	0.75	0.28	0.48 (0.05)	0.99 (0.00)	0.02 (0.002)	0.99 (0.00)	0.47 (0.05)	0.99 (0.00)
$S-I$ correlation:	0.59		0.55 (0.02)		0.42 (0.04)		0.49 (0.03)	

Notes: Standard deviations in percent (a) and correlations with output (b).
Small sample standard errors are reported in brackets.
*Output refers to gross domestic product (GDP); consumption to personal expenditures on consumer non-durables, semi-durables, and services; investment in physical capital to business gross fixed investment in nonresidential structures, machinery, and equipment; and hours worked to actual hours at work. Savings equals output minus consumption. Trade balance ratio equals one plus the ratio of net exports to GDP. $S-I$ correlation stands for the contemporaneous correlation between savings and investment in physical capital. All data is from CANSIM by Statistics Canada. Before statistics were reported all series were converted to chained 2012 dollars, seasonally adjusted, deflated by the population, logged, and then detrended via the Hodrick-Prescott (HP) filter.

rowing constraint works to dampen the fluctuations in most of the key macro variables, the high degree of transmission of technology shocks across countries is needed to moderate the variability of investment in physical capital. In Mendoza (1991), on the other hand, the feature of capital adjustment costs works to dampen the variability of most variables, including investment.

Next, we test whether we can lower the volatility of investment in the model enough to better match the data by lowering either σ_g or σ_z[13]. In Table 3, we report the results when we set $\sigma_g = 0\%$ for three different levels of θ: high, mid, and low. We can see that, while the volatility of the reported variables are dampened by lowering σ_g, for the mid and low values of θ, the variability of investment is still at a counterfactually high level: 24.26% for $\theta = 0.5$ and 44.4% for $\theta = 0$.

Lastly, in Table 4, we present the results from setting $\sigma_z = 0\%$. Overall, the volatility of the reported variables are dampened by lowering σ_z. However, similar to the previous case, for the mid and low values of θ, the variability of investment is still at a counterfactually high level: 41.06% for $\theta = 0.5$ and 82.12% for $\theta = 0$. We can also see that, for a higher values of θ, the volatility of output becomes much smaller compared to the data counterpart.

The main conclusion from the sensitivity analysis in Tables 2, 3, and 4 is as follows. In order for the model to produce a moderate level of volatility of investment in physical capital, we require a high degree of transmission of technology shocks to the world interest rate, and also human capital with borrowing restrictions to mitigate the indirect effects that technology has on investment volatility through the other inputs in the production function. We would not be able to match investment volatility without accounting for both the borrowing restriction and the international transmission of technology shocks.

Our small open economy model is perhaps most similar to Mendoza (1991) in that it has been calibrated to Canada and we consider both technology and interest rate shocks. However, different from us, Mendoza finds that, even for high values for the correlation coefficient between technology and interest rate shocks, he is not able to lower investment volatility enough unless capital adjustment costs are introduced. In order to understand differences between these models, we need to think about what causes fluctuations in physical capital investment, and why they

[13]Note that we cannot set both parameters simultaneously at a very low value because then there will be almost no business cycle fluctuations in the model.

may be different between these two models. These fluctuations are the result of shocks that form temporary gaps (or arbitrage opportunities) between the rate of return to physical capital and the world real interest rate. When a gap forms, investment in physical capital must adjusts to restore equality. For example, a technology shock affects the marginal product of physical capital directly, and also the interest rate via international transmission of technology shocks as discussed earlier. Gaps between rates of return will be larger and investment in physical capital more volatile the smaller the degree of international synchronization of business cycle activity. But this feature applies to both to our setup and Mendoza's setup. However, a technology shock will also affect the rate of return to physical capital indirectly through the other inputs in the production function, since changes in these other inputs affect the marginal product of capital in a complementary way. In our model, these indirect effects will be smaller than in Mendoza's model, leading to additional stability for investment in physical capital. There are two reasons for this. First, compared to Mendoza's model, we have incorporated human capital as an additional input. And by including human capital, the share parameter on labor reduces from $2/3$ $(= 1 - \alpha)$ to $1/6$ $(= 1 - \alpha - \eta)$. This makes the marginal product of physical capital, and therefore physical capital investment, less sensitive to fluctuation in hours worked. Second, the borrowing constraint has the impact of mitigating business cycle fluctuations for most variables, but particularly for human capital investment which is assumed to provide poor collateral for international investors. Although the share parameter on human capital is relatively large, the volatility of the human capital stock is relatively low.

7. Implications of changes in life expectancy on the small open economy

In the next two subsections, we analyze how changes in life expectancy affect, respectively, the steady state levels and business cycle fluctuations of the model variables when the parameters of the model are set at their benchmark values from Section 5. As a numerical exercise, we quantify the contribution of life expectancy to the long-run economic outcome of Canada over the past four decades. As well, we quantify the effect of life expectancy on Canadian business cycle statistics over the same period. It is worth noting that, while our model is calibrated to Canada, the parameter values are also appropriate for other developed countries, such as those in the OECD. We can also

Table 2: Sensitivity analysis: The degree of transmission of technology shocks, θ, with $\sigma_z = 0.763\%$ and $\sigma_g = 1.567\%$

Series	Canadian data* (1976Q1–2019Q4)	$\theta = 0.9598$	$\theta = 0.5$	$\theta = 0$
Output	1.42	1.1474	1.509	2.02
		(0.11)	(0.15)	(0.21)
Investment in physical capital	6.63	6.63	47.51	93.01
		(0.42)	(2.69)	(5.22)

Notes: Standard deviations in percent. Small sample standard errors are reported in brackets. *Output refers to gross domestic product and investment in physical capital to business gross fixed investment in nonresidential structures, machinery, and equipment. All data is from CANSIM by Statistics Canada. Before statistics were reported all series were converted to chained 2012 dollars, seasonally adjusted, deflated by the population, logged, and then detrended via the Hodrick-Prescott (HP) filter.

Table 3: Sensitivity analysis: The degree of transmission of technology shocks, θ, with $\sigma_z = 0.763\%$ and $\sigma_g = 0\%$

Series	Canadian data* (1976Q1–2019Q4)	$\theta = 0.9598$	$\theta = 0.5$	$\theta = 0$
Output	1.42	1.1472	1.4058	1.687
		(0.11)	(0.14)	(0.17)
Investment in physical capital	6.63	5.78	24.26	44.4
		(0.35)	(1.47)	(2.68)

Notes: Standard deviations in percent. Small sample standard errors are reported in brackets. *Output refers to gross domestic product and investment in physical capital to business gross fixed investment in nonresidential structures, machinery, and equipment. All data is from CANSIM by Statistics Canada. Before statistics were reported all series were converted to chained 2012 dollars, seasonally adjusted, deflated by the population, logged, and then detrended via the Hodrick-Prescott (HP) filter.

see in Figure 4 that changes in the life expectancy over the time period being investigated are similar between Canada and other OECD countries.

7.1. The effect of life expectancy on the steady state

As panel A of Figure 5 shows, increased life expectancy is found have a positive and notable impact on the steady state level of output per effective labor. For example, in Canada, between 1976 and 2018, life expectancy at birth increased from around 74 years to 82 years, which translates to an estimated 5.5% increase in the long-run output per effective labor.

Our model suggests that long-run output per effective labor is more sensitive to changes in life expectancy at low levels of life expectancy than it is at high levels. This can be seen more clearly

Table 4: Sensitivity analysis: The degree of transmission of technology shocks, θ, with $\sigma_z = 0\%$ and $\sigma_g = 1.567\%$

Series	Canadian data* (1976Q1–2019Q4)	$\theta = 0.9598$	$\theta = 0.5$	$\theta = 0$
Output	1.42	0.046	0.573	1.1456
		(0.004)	(0.05)	(0.11)
Investment physical capital	6.63	3.301	41.06	82.12
		(0.17)	(2.2)	(4.39)

Notes: Standard deviations in percent. Small sample standard errors are reported in brackets. *Output refers to gross domestic product and investment in physical capital to business gross fixed investment in nonresidential structures, machinery, and equipment. All data is from CANSIM by Statistics Canada. Before statistics were reported all series were converted to chained 2012 dollars, seasonally adjusted, deflated by the population, logged, and then detrended via the Hodrick-Prescott (HP) filter.

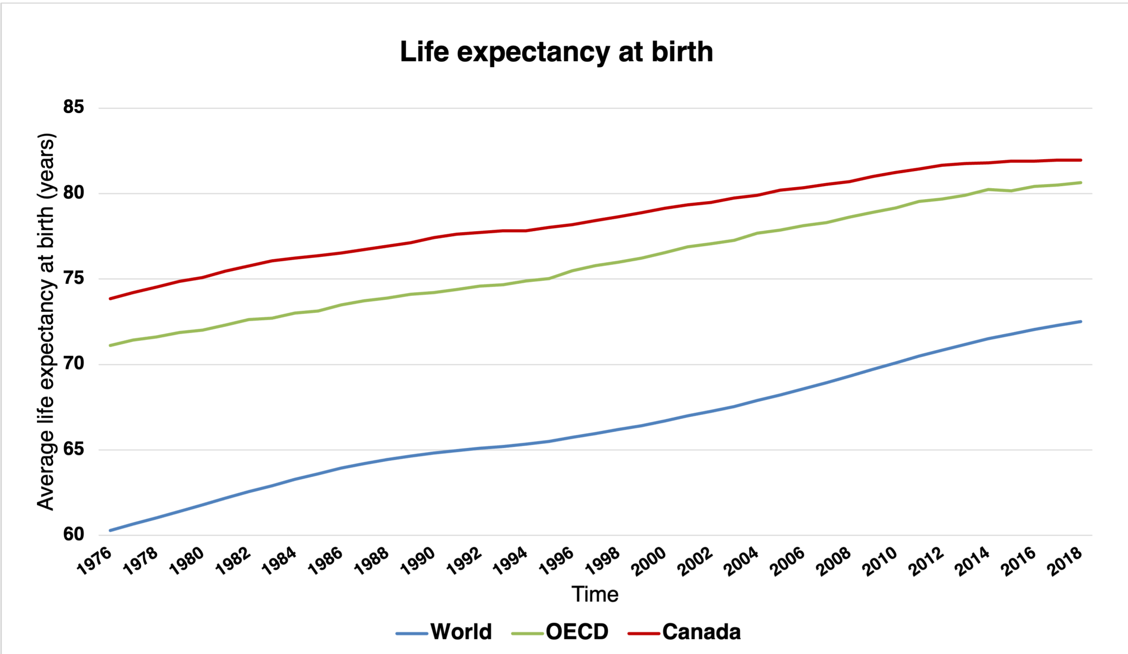

Figure 4: Average life expectancy at birth. Life expectancy at birth is defined as the average number of years a newborn is expected to live if mortality patterns at the time of its birth remain constant throughout its life. Data are from the World Development Indicators by the World Bank.

from panel B of Figure 5, which illustrates the percentage changes in the steady state output per effective labor as life expectancy increases. We can see that increased life expectancy has positive but diminishing marginal effect on long-run output per effective labor. This result follows from the model's feature of diminishing returns to human capital.

Finally, panels C through F of Figure 5 show how the steady state levels of consumption, aggregate hours worked, human capital, and physical capital change following changes in life expectancy. We note that, in our model, consistent with standard theory of human capital, increased life expectancy makes investment in human capital more profitable and thus encourages its accumulation. Given the complementarity between the two types of capital in the production function, the increase in the stock of human capital then leads to an increase in the stock of physical capital. As can be seen from panel D, average hours worked per period decreases, albeit rather moderately, as life expectancy increases. This suggests that increased life expectancy allows the agents to enjoy more leisure per period. The rise in both leisure and consumption are due to having greater lifetime wealth. Given the foregoing changes in human capital, physical capital, and hours worked, the output level of the domestic economy rises as life expectancy increases.

7.2. The effect of life expectancy on business cycle fluctuations

Figure 6 below shows the effect of changes in life expectancy on the standard deviations of the model variables when the parameters are set at their benchmark values from Section 5. We can see that volatility of consumption, human capital, and physical capital all decrease as life expectancy increases. In the model, an increase in life expectancy raises the effective time preference of the domestic economy and the agents become more patient. As a result, the volatility of human capital decreases as life expectancy increases, shown in panels C. Given the complementarity between the two types of capital, the volatility of physical capital also decreases as life expectancy increases as shown in panel D. Shown in panel F, the volatility of trade balance ratio is found to increase as life expectancy increases.

Next, as shown in panel E, the volatility of average hours worked per period increases as life expectancy increases. When life expectancy is low, the agents have no luxury to increase or adjust their leisure hours and hence there is less volatility in the hours worked. On the other hand, when life expectancy is high, the agents become more flexible to increase their leisure hours and thus there is more volatility in the hours worked. As can be seen from panel B, following the decreased volatilities of physical and human capital and the increased volatility of hours worked, output ends up becoming more volatile as life expectancy increases. Panel A shows that consumption becomes less volatile as life expectancy increases, suggesting an improvement in consumption smoothing.

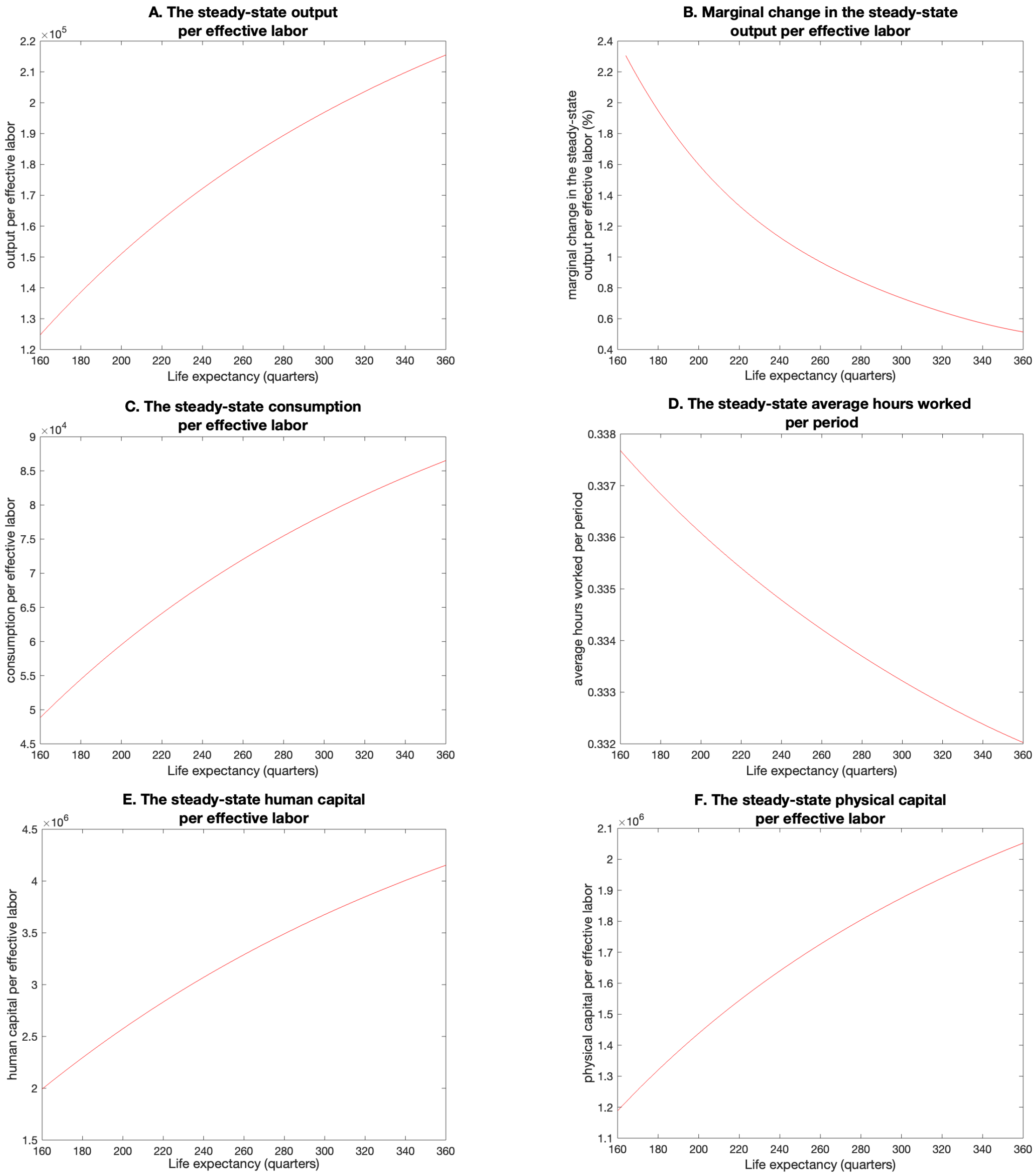

Figure 5: The steady state and life expectancy

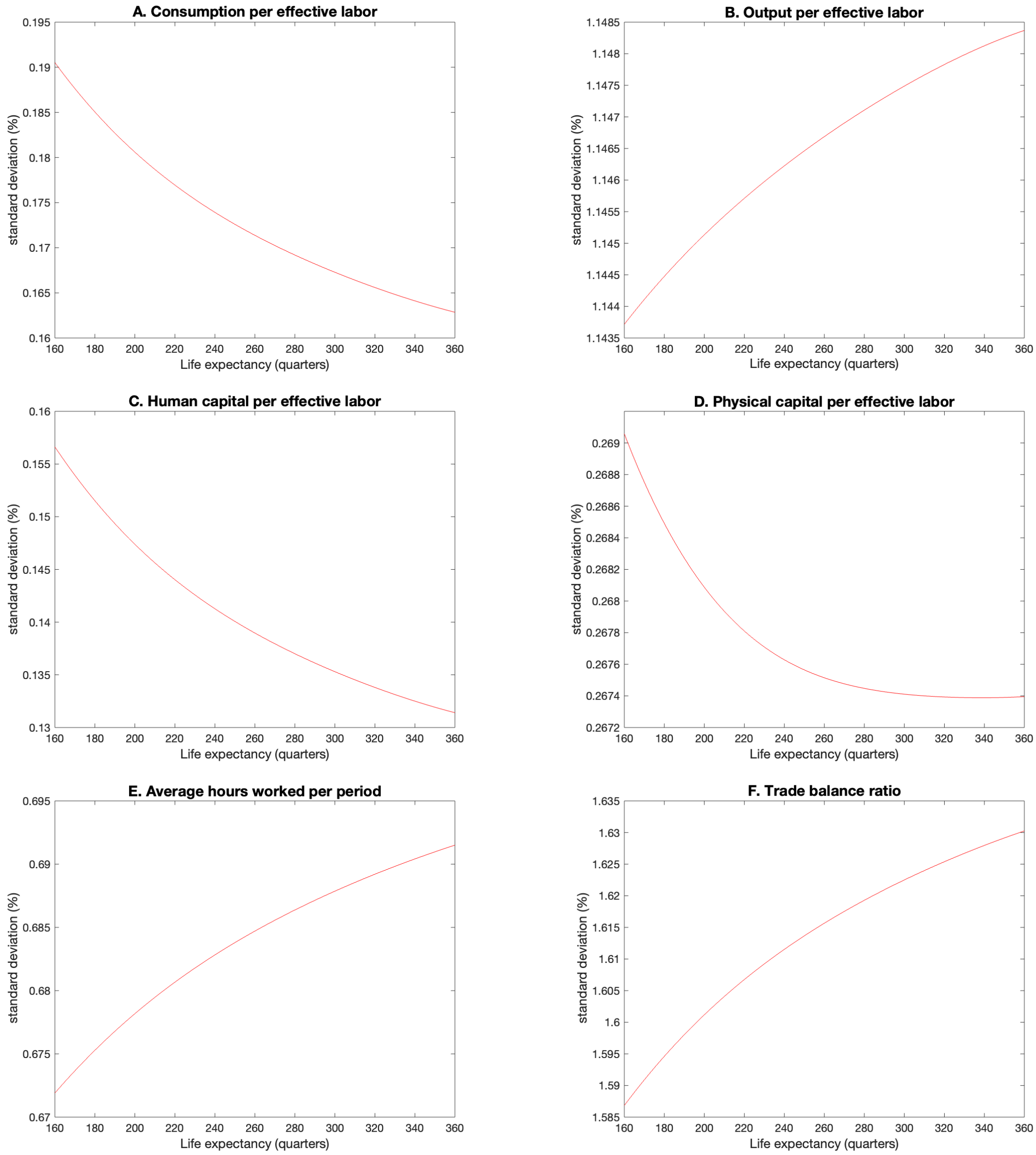

Figure 6: Life expectancy and standard deviation of the model variables

Followed from the model's feature of diminishing returns to human capital, we find that changes in life expectancy will have a greater effect—in terms of magnitude—on the volatility of the above mentioned aggregate variables the lower is the initial level of life expectancy. This can be seen Figure 7, which illustrates the percentage changes in the standard deviations of consumption and average hours worked per period due to changes in life expectancy. Although we report the results for the given two variables only, we get the similar pattern for the other variables as well: As life expectancy increases, the marginal effects it has on the standard deviations of the model variables decrease in magnitude.

Lastly, our numerical exercise reveals that, in Canada, due to the change in life expectancy between 1976 and 2018, the volatility of consumption has decreased by an estimated 1.55%. On the other hand, the volatility of hours worked, output, and trade balance ratio have increased by an estimated 0.3%, 0.043%, and 0.277%, respectively.

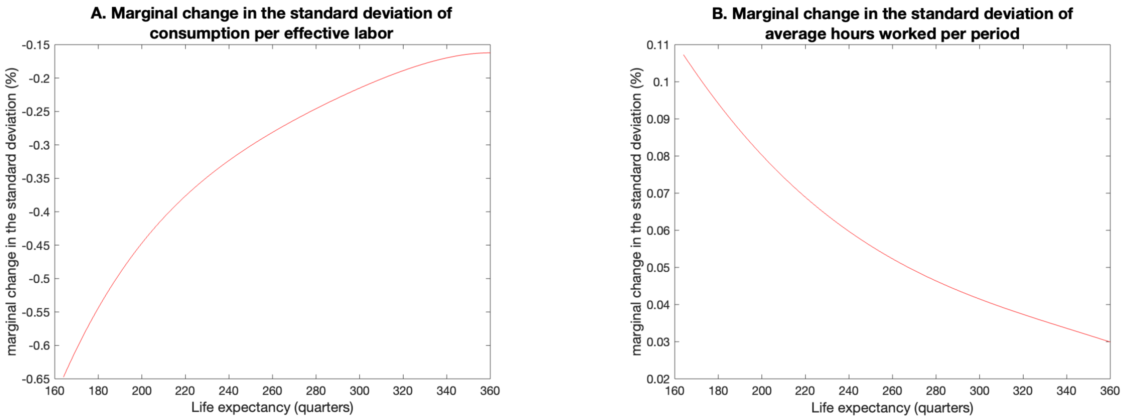

Figure 7: Life expectancy and marginal change in standard deviation of the model variables

8. Conclusion

We develop a standard small open economy RBC model driven by neutral productivity shocks and interest-rate shocks that is consistent with the main empirical facts of economic fluctuations in an actual open economy. The model is calibrated to the Canadian economy. As in the data, the model is consistent with the low volatility of consumption and large volatility of investment relative to that of output. The model is also able to capture the high positive correlations of consumption, savings, hours worked, and investment in physical capital with output; the sufficiently countercyclical trade balance ratio; and the procyclical interest rate. The observed positive correlation between domestic savings and investment in physical capital is also well captured by the model. Further, the impulse responses of aggregate variables to a TFP shock as well as an interest-rate shock are found to be consistent with the standard results in the literature.

Given its finite lifetimes feature, our framework allows us to study the effects of life expectancy on the cyclical fluctuations of the aggregate variables within the context of a small open economy. Our results reveal that the fluctuations in physical capital, human capital, and consumption all decrease as life expectancy increases. On the other hand, the fluctuation of hours worked, trade balance ratio, and output are found to increase with life expectancy. We also find that changes in life expectancy will have a greater effect—in terms of magnitude—on the volatility of the above mentioned variables the lower is the initial level of life expectancy. Using our model, we have quantified how changes in life expectancy have impacted the long-run economic outcome and business cycle fluctuations of Canada over the past forty years.

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Appendix: Statistics Canada (CANSIM) Data Series Label Retrieval Codes

Series title	Series label
1. Gross domestic product at market prices	V62305752
2. Household consumption of semi-durable goods	V62305727
3. Household consumption of non-durable goods	V62305728
4. Household consumption of services	V62305729
5. Business investment in non-residential structures, machinery and equipment	V62305735
6. Exports of goods and services	V62305745
7. Imports of goods and services	V62305748
8. Total actual hours worked, all industries	V4391505
9. Population	V1
10. Total expenditures on education (1976–1999)	V1996800
11. Public and private elementary and secondary education expenditures (2000–2019)	V1200358
12. Expenditures of universities and degree granting colleges (2000–2019)	V80711164
13. GDP Implicit price index (2012=100)	V62471023

Notes: Most data is quarterly (1976Q1–2019Q4), in chained 2012 dollars, and seasonally adjusted at annual rates. The exception is data used to construct a series for human capital investment, which is availably only annually and in current dollars. These values were converted to constant 2012 dollars using the GDP deflator and then to quarterly frequency by interpolating linearly between points.