

**GROWTH PROMOTION POLICIES
WHEN TAXES CANNOT BE RAISED**

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Growth Promotion Policies when Taxes Cannot be Raised*

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Abstract

This paper examines the growth effects of R&D subsidies and public-funded basic research in an R&D-based endogenous growth model when political constraints do not allow the government to increase its revenue. If individuals have enough life-cycle saving motives and R&D productivity is sufficiently high, the growth rate becomes higher than the interest rate in equilibrium, and the government can finance expenses while perpetually rolling over the debt. Given this situation, debt-financed R&D subsidies always enhance short-run growth. However, they increase long-term growth only when R&D productivity exceeds another threshold. Our estimates suggest that this condition holds for most advanced countries, including the United States, but not for low-growth countries such as Japan and Italy. In contrast, enhancing public-funded basic research is effective for economic growth even in low-growth economies. However, such policies reduce the upper bound in the debt-to-GDP ratio, beyond which the economy cannot recover to a steady state.

Keywords: Endogenous growth, R&D, Public debt, Perpetual debt rollover, R minus G, Research subsidy, Basic research

JEL Classification Codes: O38, O41, E62

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1 Introduction

Once one starts to think about [growth], it is hard to think about anything else.

— Robert Lucas Jr. (1988)

As Lucas (1988) mentioned, economic growth is one of the most important objectives in economics. How can we achieve faster economic growth? Modern theories of endogenous growth provide a surprisingly simple answer. In the long run, economic growth is determined by technological change. Technological change is realized via R&D. Therefore, by promoting R&D, e.g., through government subsidies for R&D, economic growth can be accelerated. Nevertheless, many economies face slower economic growth than they desire. Even when they know that R&D promotion policies enhance growth, there are often insufficient funds to implement those policies. Governments typically have a limited ability to raise taxes, which constrains their ability to promote R&D and, therefore, economic growth.

Recent studies on the political economy suggest that it is difficult for the government to raise tax rates for political reasons. For example, Bassetto and Miller (2025) considered an upper bound on tax revenue due to political constraints on austerity. Jiang, Sargent, Wang and Yang (2022) assumed a similar constraint based on the political considerations of Keynes (1923).¹ However, existing studies of long-term growth typically disregard such constraints by implicitly assuming that the government can levy lump-sum taxes to implement policies. (e.g. Romer, 1990; Grossman and Helpman, 1991, Ch. 3). Some studies include distortions caused by factor-income taxation but still assume that the government can set any tax rate.²

In contrast, this paper considers an extreme situation in which political constraints do not allow the government to increase its tax revenue. If the government wants to support R&D, it must be financed entirely by public debt. The government also cannot raise taxes in the future to repay the debt. Therefore, the debt must be rolled over perpetually. In other words, we consider the environment in which growth promotion policies must be financed by a Ponzi scheme. While common sense suggests that such policies would not be sustainable, O’Connell and Zeldes (1988) showed that the government can run a “rational”

¹Through an overlapping generations political economy model, Song, Storesletten and Zilibotti (2012) demonstrated that intergenerational conflict can prevent the government from raising taxes if the political power of the old is stronger than that of the young. See also Alesina and Passalacqua (2016) and Yared (2019) for the literature review.

²Ferraro and Peretto (2020) examined the case where the government can use a distortionary labor tax in the context of fiscal consolidation. See also the references in Chu and Wang (2022).

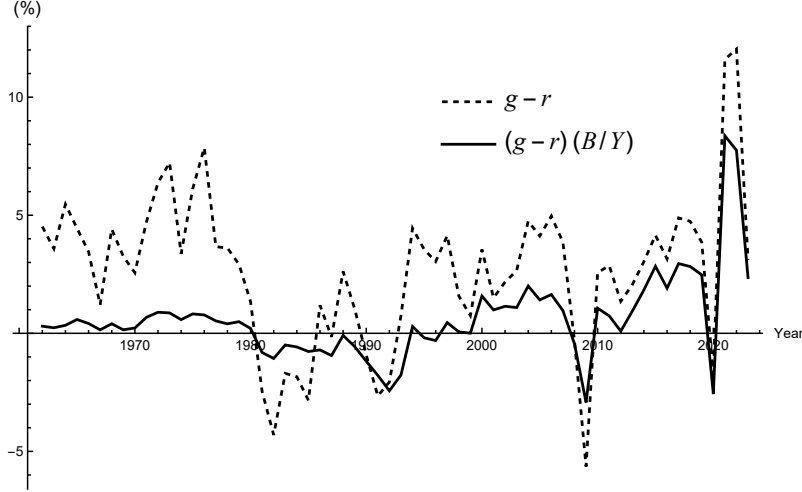


Figure 1: The $g - r$ Gap and Surplus from Government Debt (OECD average)

Ponzi scheme if the rate of economic growth is higher than the interest rate on governmental debt, i.e., $g > r$.

The broken line in Figure 1 shows the time series of the average value of $g - r$ among OECD countries.³ Although $g - r$ was negative in the 1980s and during the Great Recession and the COVID-19 pandemic, it was positive the majority of the time.⁴ When $g - r$ is positive, the debt-to-GDP ratio, B/Y , shrinks by $(g - r)(B/Y)$ if the primary balance (the government revenue minus the government expenditure excluding interest payments on its debts) is zero. In other words, if $g - r$ is positive, the government can spend $(g - r)B$ over its revenue without increasing the debt-to-GDP ratio. The solid line in Figure 1 shows the ratio of this surplus to GDP (i.e., $(g - r)(B/Y)$).⁵ The figure shows that the surplus is significant, particularly after the 2000s (the overall average after 2000 was 1.7% of GDP).

The surplus/GDP ratio is on the rise partly because of the increasing debt-to-GDP ratios among countries. The relationship between surplus and debt can be seen more precisely in Figure 2, which presents a scatter plot of B/Y and $(g - r)(B/Y)$ from OECD countries.⁶ For countries where the debt/GDP ratio is not too high (i.e., below 150%), there is a positive relationship between this ratio and the surplus/GDP ratio, except Turkey

³We construct a $g - r$ dataset by subtracting the short-term nominal interest rate from the nominal GDP growth rate for each OECD country, subject to data availability, and then taking the average. The GDP and interest rate data are obtained from OECD Data Explorer.

⁴Mehrotra and Sergeyev (2021) reported that in the 1946–2006 period, the median value of $g - r$ was 1.0% for the United States and 0.8% for the average of 17 advanced countries. Blanchard (2019) also noted that $g > r$ has been historical norm rather than exception in the U.S. since 1950. Mauro and Zhou (2021) analyzed data on average effective borrowing costs for 55 countries over a span of up to 200 years and reported that $g > r$ prevails for both advanced and emerging economies.

⁵The data on the debt-to-GDP ratio are from the World Economic Outlook (2024).

⁶The figure shows the average value for the 2010–2023 period.

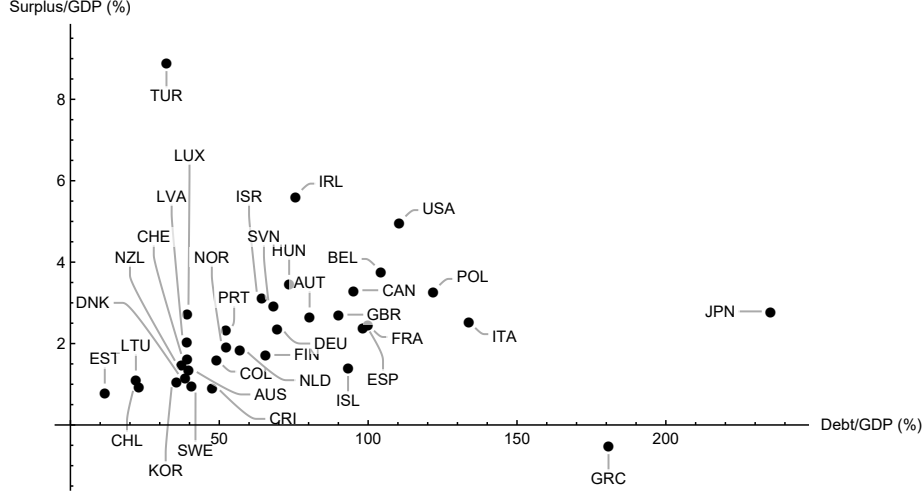


Figure 2: Debt/GDP Ratio and Surplus from Government Debt (Average Since 2010)

(where the inflation rate is extremely high). This finding suggests that a moderate level of government debt can be helpful in financing long-term policies, given the government's constraint on raising taxes. Notably, however, the surplus is not necessarily large (in fact, it is negative for Greece) when the debt-to-GDP ratio is too high. When there is too much government debt, the economy cannot maintain a positive $g - r$ gap, and the surplus vanishes.

Given these findings, our main model examines whether the growth rate can be increased when the government finances R&D subsidies entirely by issuing debt. The result is not obvious because such policies affect both g and r . The direct effect of R&D subsidies is to induce private firms to engage in more R&D, which will speed up technological progress. However, the government's debt increases the equilibrium interest rate in financial markets and may crowd out private R&D investments. More precisely, a higher interest rate implies that the present value of future profits realized by R&D is lower, thereby discouraging R&D.

We find that the overall effects of R&D subsidies crucially depend on the R&D productivity, which is measured by the response of the growth rate of innovations to the fraction of labor devoted to R&D. We show that $g > r$ holds in equilibrium if individuals have enough life-cycle saving motives and if the R&D productivity is high enough. However, this finding does not always mean that the growth rate can be enhanced by debt-financed R&D promotion policies. For these policies to increase the long-term growth rate, the R&D productivity must be even greater. In other words, the $g - r$ gap needs to be not only positive but also greater than a positive threshold value. Our estimates of the R&D productivity show that most advanced countries, including the United States, satisfy this

condition. However, the estimates from Japan and Italy fall short of the threshold. In those countries, our theory suggests that R&D subsidies enhance the growth rate only temporarily, although the increase in economic growth may continue for several hundred years in the transition.

In an extended model, we introduce public-financed basic research that enhances the productivity of private R&D. The government now chooses the pair of the research subsidy and the addition to basic research, while both must be financed by perpetually rolled-over debts. Although R&D subsidies enhance long-term growth only when the initial productivity of private R&D is high enough, our theory suggests that investing more resources into basic research is conducive to long-term growth regardless of the initial productivity, given the existing estimates of the effectiveness of basic research on private R&D. This result suggests that enhancing basic research is preferable to R&D subsidies in low-growth economies.

Still, our theory highlights the cost of debt-financed growth promotion policies even when $g > r$. There are threshold levels for research subsidies and basic research above which no balanced growth exists; i.e., the Ponzi scheme becomes unsustainable. We calculate the maximal debt-to-GDP ratio that can be sustained without raising taxes. While debt-financed R&D subsidies may increase the long-run growth rate, they reduce the fiscal space in the sense that the highest level of the debt-to-GDP ratio beyond which the economy cannot return to a steady state is lower.

A body of literature has examined the sustainability of government debt in an environment where $g > r$ holds in equilibrium (e.g. Blanchard, 2019; Reis, 2021; Ball and Mankiw, 2023; Barro, 2023; Kocherlakota, 2023). Like some of those studies, we use a continuous-time overlapping generations model to explain why the interest rate can be lower than the growth rate in the long run. However, in most studies, the long-term growth rate is exogenous, and the focus is the effect of policies on the safe interest rate. Exceptions include Saint-Paul (1992) and King and Ferguson (1993), who developed AK-type endogenous growth models in which $g > r$ holds in equilibrium. These authors showed that the economy is dynamically efficient even when $g > r$ because, in endogenous growth models, the social return on investment is higher than the interest rate. We also show that the social return to R&D investment in our model, including the benefits from intertemporal knowledge spillovers, is always higher than the growth rate. The most important difference from those studies is that we consider an R&D-based variety-expansion growth model rather than an AK model, where long-term growth is determined solely by capital accumulation. By explicitly modeling the R&D process, we are able to examine the effect of

debt-financed R&D promotion policies on incentives for technological innovations, which unarguably serve as an important source of economic growth.

This paper is also related to Angeletos, Lian and Wolf (2024) in that both show that future tax increases are not necessary after debt-financed policies are implemented. While the abovementioned authors consider a short-term stimulus policy in a new Keynesian setting, we consider a long-term growth promotion policy in an R&D-based endogenous growth model. Apart from the literature on government debt, the relationship between g and r is investigated in studies of rational bubbles because $g > r$ is often a necessary condition for the emergence of rational bubbles. In a classic study by Tirole (1985), individuals invest in pieces of paper, which the author called bubbles,⁷ if $g > r$ holds in the equilibrium without bubbles. The government bonds in our study are related to bubbles in those studies in that $g > r$ is a necessary condition for the government debts to be sustainable, given that the government has no revenue. However, there is a notable difference. In our studies, debts themselves are growth-reducing because they increase the equilibrium interest rates and always reduce the market value of innovation. Debt-financed policies can increase growth only when the revenue from debt is used for growth-promoting policies. In Tirole (1985) and many related studies, bubbles themselves Pareto-improve the welfare of individuals because the equilibrium without bubbles (with $g > r$) is dynamically inefficient. As we show in Section 5, this is not the case in our model.⁸

The remainder of this paper is organized as follows. Section 2 presents the model environment. Section 3 explains the equilibrium dynamics of this economy, including the relationship between g and r . In Section 4, we examine the growth effects of R&D subsidies, assuming that they are financed by perpetually rolled-over debts. Both long-term and transitional results are presented. In Section 5, we investigate the social return of R&D and the dynamic efficiency of the economy. Section 6 examines the extended model where publicly funded basic research is introduced. Section 7 concludes the paper.

2 Model

2.1 Individuals

We consider a continuous-time overlapping generations model. In the economy, \bar{N} new individuals are born per unit of time, and they face a constant Poisson death rate of $\mu > 0$.

⁷In later studies, Olivier (2000) and Hirano and Toda (2025) introduced bubbles for productive assets.

⁸Some recent studies (e.g. Hirano and Yanagawa, 2017; Hori and Im, 2023) have focused on the possibility that bubbles are growth-reducing.

Thus, the number of individuals who are born at time s and are still alive at time t is

$$N_{s,t} = \bar{N}e^{-\mu(t-s)}. \quad (1)$$

The total population is stationary at $N = \bar{N}/\mu$, which is obtained by integrating $N_{s,t}$ over s from $-\infty$ to t . Each individual supplies inelastically $e^{-\delta(t-s)}$ units of effective labor, where $t-s$ is their age.⁹ The effective labor supply decreases with age at the rate of $\delta > 0$ because of the deterioration in productivity and the declining ability to work longer hours as individuals age. Because of this aging effect, individuals have an incentive to make life-cycle savings. In this sense, the parameter δ represents the strength of individuals' saving motives. Then, from (1), the aggregate labor supply is

$$L = \int_{-\infty}^t N_{s,t} e^{-\delta(t-s)} ds = \frac{1}{\mu + \delta} \bar{N}. \quad (2)$$

The expected utility of a generation s individual, accounting for their mortality, is given by

$$U_s = \int_s^{\infty} (\ln c_{s,t}) e^{-(\rho+\mu)(t-s)} dt, \quad (3)$$

where $c_{s,t}$ is the amount of consumption by a generation- s individual at time t and where $\rho > 0$ is the discount rate. Observe that individuals further discount the future by their survival probability, $e^{-\mu(t-s)}$. We assume that the discount rate is not too high. In particular, we assume that $\rho < \delta$ so that individuals have enough incentives to save for their later age. (See, Rachel and Summers, 2019).

Let $k_{s,t}$ be the real asset holding by a generation s and r_t be the real interest rate on bonds. Following Blanchard (1985), we assume that there is a perfect market for annuities. Thus, the rate of return on annuities is $r_t + \mu$ for survivors. Since $r_t + \mu > r_t$, individuals hold all their assets in the form of annuities. We also normalize the price of the final goods to one. Then, the budget constraint is given by

$$\dot{k}_{s,t} = (r_t + \mu) k_{s,t} + e^{-\delta(t-s)} w_t - c_{s,t}, \quad (4)$$

where w_t is the real wage per unit of effective labor. The newborn generation has zero financial assets, which means that $k_{t,t} = 0$. To summarize, each individual maximizes the expedited utility (3) subject to the budget constraint (4), initial condition $k_{t,t} = 0$, and the usual non-Ponzi game condition. In this standard setting, the Euler equation for

⁹More precisely, $t-s$ is the period after each individual starts their economic activity. If the starting age is 20, the actual age of an individual is $20 + (t-s)$.

individuals is $\dot{c}_t = (r_t - \rho)c_t$.

Let us define aggregate consumption and aggregate asset holding as

$$C_t = \int_{-\infty}^t c_{s,t} N_{s,t} ds, \quad K_t = \int_{-\infty}^t k_{s,t} N_{s,t} ds. \quad (5)$$

In the following, we derive the dynamics for aggregate consumption in terms of C_t and K_t . Using the Leibniz integral rule, we can differentiate C_t in (5) as follows:

$$\dot{C}_t = \int_{-\infty}^t \dot{c}_{s,t} N_{s,t} ds + \int_{-\infty}^t c_{s,t} \dot{N}_{s,t} ds + N_{t,t} c_{t,t}. \quad (6)$$

On the right-hand side (RHS), the first term represents the sum of the changes in consumption by existing individuals. Using the Euler equation of individuals, $\dot{c}_t = (r_t - \rho)c_t$, this term can be written as $(r_t - \rho)C_t$. In the second term, the change in the cohort size $\dot{N}_{s,t}$ can be written $-\mu N_{s,t}$ from (1). Therefore, this term becomes $-\mu C_t$. This value represents the decline in aggregate consumption due to the death of μ portion of individuals per unit time. The last term is the sum of the consumption by newly born individuals. In Appendix A.1, we show that the consumption of a newborn is

$$c_{t,t} = \frac{\mu + \delta}{\mu} (\bar{c}_t - (\rho + \mu)\bar{k}_t), \quad (7)$$

where $\bar{c}_t = C_t/N$ is average consumption and $\bar{k}_t = K_t/N$ is the average asset holding. Equation (7) can be interpreted as follows. Recall that newborns do not have financial assets ($k_{t,t} = 0$). Given the log utility, the consumption propensity out of assets is $\rho + \mu$. Therefore, the lack of financial assets induces a newborn to consume $(\rho + \mu)\bar{k}_t$ less than the average consumption, \bar{c}_t . However, newborns have a greater ability to work than average individuals do. A newborn's working ability (labor supply) is 1, whereas that of an average individual is $L/N = \mu/(\mu + \delta)$. Therefore, the RHS of (7) is multiplied by $(\mu + \delta)/\mu$. By substituting (7) into (6) and using $N_{t,t} = \bar{N} = \mu N$, we obtain the Euler equation for aggregate consumption.

$$\dot{C}_t = (r_t - \rho + \delta) C_t - (\rho + \mu)(\delta + \mu) K_t. \quad (8)$$

2.2 Supply Side

The supply side of this economy is purposefully close to that of the standard variety-expansion model of Grossman and Helpman (1991). It consists of three sectors, namely, the final goods sector, the intermediate goods sector, and the R&D sector. In the final

goods sector, a representative firm competitively produces final goods X_t from a continuum of varieties of intermediate goods $x_t(i)$, where i is the index of intermediate goods. The production function is given by

$$X_t = \left[\int_0^{n_t} x_t(i)^\alpha di \right]^{1/\alpha}, \quad (9)$$

where n_t is the number of intermediate goods available at time t and where $\alpha \in (0, 1)$ is a production parameter. Let $p_t(i)$ be the price of intermediate good i . The representative final goods firm maximizes its profit, which is given by

$$X_t - \int_0^{n_t} p_t(i) x_t(i) di. \quad (10)$$

The first-order condition for profit maximization implies that the demand function of intermediate goods is

$$x_t(i) = p_t(i)^{-\frac{1}{1-\alpha}} X_t. \quad (11)$$

In the intermediate goods sector, there are n_t intermediate goods firms, each of which produces its own variety of goods, $x_t(i)$. The production of one unit of $x_t(i)$ requires one unit of labor; therefore, its profit is given by $\pi_t(i) = (p_t(i) - w_t)x_t(i)$. Given that $x_t(i)$ is determined by the demand function (11), profit-maximizing pricing implies

$$p_t(i) = \frac{w_t}{\alpha}, \quad x_t(i) = \left(\frac{\alpha}{w_t} \right)^{\frac{1}{1-\alpha}} X_t, \quad (12)$$

$$\pi_t(i) = (1 - \alpha) \left(\frac{\alpha}{w_t} \right)^{\frac{\alpha}{1-\alpha}} X_t. \quad (13)$$

The above result shows that all intermediate goods firms produce the same amount of output. Therefore, the output of each intermediate goods firm can be written as $x_t(i) = L_t^P/n_t$ for all i , where L_t^P is the total amount of labor for the production of goods. By substituting this output into the final goods production function (9), we obtain

$$X_t = n_t^{\frac{1-\alpha}{\alpha}} L_t^P. \quad (14)$$

The R&D sector has a representative R&D firm, which competitively creates new goods using labor for R&D, L_t^R . The number of goods increases as follows:

$$\frac{\dot{n}_t}{n_t} = A \frac{L_t^R}{L}, \quad (15)$$

where $A > 0$ is a parameter that measures the efficiency of R&D. When 1% of the labor force is used for R&D, the number of intermediate goods increases $A\%$ per year.¹⁰

Equation (15) implies that the creation of a new intermediate good requires L/An_t units of labor. We assume that the government subsidizes a fraction $\theta \in [0, 1)$ of the R&D cost. Then, the private cost of developing a new intermediate good is $(1 - \theta)w_t L/An_t$. Let v_t be the value of an intermediate goods firm. Then, the free entry condition for R&D is

$$v_t \leq (1 - \theta) \frac{w_t L}{An_t} \text{ with equality if } \dot{n}_t > 0. \quad (16)$$

2.3 Government

As explained above, the government subsidizes a fraction θ of the cost of R&D. Since the presubsidy aggregate cost of R&D is $w_t L_t^R$, the amount of government expenditure is $\theta w_t L_t^R$. We assume that the government cannot collect taxes and that all expenditures are financed by government debt. Then, the amount of government debt, B_t , evolves according to

$$\dot{B}_t = r_t B_t + \theta w_t L_t^R. \quad (17)$$

Note that each bond is repaid at maturity with interest r ; therefore, from the viewpoint of individuals, it is rational to invest in bonds. However, since there is no tax revenue, the repayment and interest must be refinanced entirely by issuing new bonds. Therefore, the government is running a Ponzi scheme.¹¹ We investigate the possibility that the government runs a rational Ponzi game, similar to the one examined by O'Connell and Zeldes (1988), and uses the revenue to promote economic growth.

3 Equilibrium

3.1 The $g - r$ Gap

A necessary condition to run a rational Ponzi scheme is that the growth rate is higher than the interest rate. Given the economy described in the previous section, here, we examine the gap between the growth rate and the interest rate in equilibrium. First, we derive the

¹⁰We can also write equation (15) as $\dot{n}_t = an_t L_t^R$, where $a = A/L$. Although the latter convention is more standard, we use A rather than a because its value is easier to interpret from the data.

¹¹Note that this situation is in contrast to that of individuals, who maximize their lifetime utility subject to the no-Ponzi-game condition. The difference in the ability to borrow between the government and individuals reflects reality. Without collateral, individuals usually cannot borrow large amounts of money for various reasons (e.g., the risk of running away). We can rewrite the model with a borrowing constraint for individuals, which yields the same result as the present setting in the steady state.

real interest rate.¹² The consumers hold all their assets in the form of annuities, and the annuity company invests the assets in government bonds and the shares of intermediate goods firms. Therefore, the equilibrium in the asset market is

$$K_t = B_t + n_t v_t. \quad (18)$$

Since risks are fully diversified, the expected return on holding the shares of an intermediate goods firm should be equal to the interest rate on bonds. This no-arbitrage condition can be written as

$$r_t = \frac{\pi_t + \dot{v}_t}{v_t}, \quad (19)$$

where π_t is given by (13). In the main text, we focus on the case where the amount of R&D is positive, leaving the discussion of the case of $\dot{n}_t = 0$ for the Appendix A.3. Then, v_t is given by (16) with equality. Both π_t and v_t depend on the wage level, which we now derive. Since the final goods sector is competitive, the maximized profit of the final goods firm in Equation (10) should be zero. By substituting $x_t(i)$ and $p_t(i)$ into (10), this condition determines the market wage as

$$w_t = \alpha n_t^{\frac{1-\alpha}{\alpha}}. \quad (20)$$

Substituting the values of π_t and v_t into (19) yields the real interest rate in equilibrium.¹³

$$r_t = \frac{A}{\alpha} \left(\frac{1-\alpha}{1-\theta} \frac{L_t^P}{L} - (2\alpha-1) \frac{L_t^R}{L} \right). \quad (21)$$

Next, we consider the growth rate. The GDP of this economy is defined as the sum of consumption expenditures C_t , private investment expenditures for R&D, $(1-\theta)w_t L_t^R$, and government expenditures, $\theta w_t L_t^R$. Note that the final output is used only for consumption; therefore, the equilibrium in the goods market means that $C_t = X_t$. Then, using (14), and (20), the GDP can be written as

$$\text{GDP}_t = n_t^{\frac{1-\alpha}{\alpha}} (L_t^P + \alpha L_t^R). \quad (22)$$

Since $\alpha < 1$, the GDP is greater when more labor is used for production, given the value of

¹²Note that, in the model, g and r are real variables. However, when we calculate $g - r$ from the data, as shown in Figures 1 and 2, we use the nominal growth rate and the nominal interest rate because the result is the same.

¹³Using (20), the values of π_t and v_t can be obtained as follows: From (14) and (20), Equation (13) gives $\pi_t = (1-\alpha)n_t^{(1-2\alpha)/\alpha} L_t^P$. From (16) with equality and (20), the value of a firm is $v_t = (1-\theta)w_t/an_t = ((1-\theta)\alpha/a)n_t^{(1-2\alpha)/\alpha}$. Using (15), its derivative is $\dot{v}_t = -((2\alpha-1)/\alpha)A(L_t^R/L)v_t$. Substituting these results into (19) yields (21).

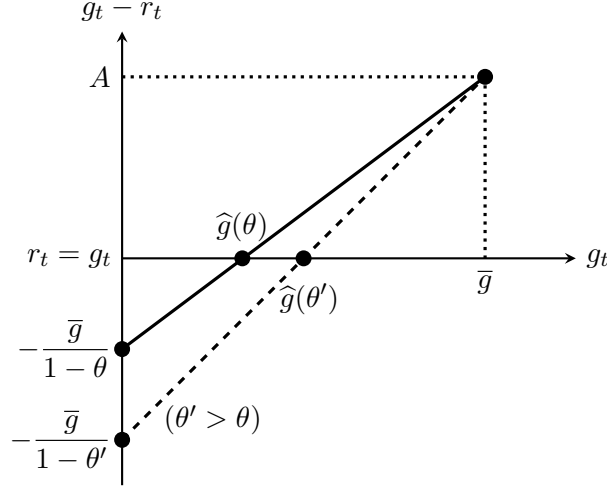


Figure 3: Relationship between the $g_t - r_t$ gap and the growth rate.

n_t . The reason is that there is a positive markup in the intermediate goods sector, whereas there is no markup in the R&D sector. Let us also define potential GDP, Y_t , as the level of GDP when all labor is used for production, given the number of intermediate goods developed by that time, n_t . With $L_t^P = L$ and $L_t^R = 0$, (22) reduces to

$$Y_t = n_t^{\frac{1-\alpha}{\alpha}} L. \quad (23)$$

This value is the upper bound for GDP_t given the state of technology at time t . Y_t can also be viewed as the supply capacity of the economy. From (15), the growth rate of potential GDP in (23) is given by

$$g_t \equiv \frac{\dot{Y}_t}{Y_t} = \bar{g} \frac{L_t^R}{L}, \text{ where} \quad (24)$$

$$\bar{g} \equiv \frac{1-\alpha}{\alpha} A \quad (25)$$

is the maximum growth rate of potential GDP when all labor is used for R&D. Note that on any balanced growth path (BGP), g_t is constant. Then, (24) means that L_t^R is constant; the same holds for $L_t^P = L - L_t^R$. In this case, from (22) and (23), the growth rate of GDP_t coincides with \dot{Y}_t/Y_t . Therefore, g_t also represents the growth rate of GDP in the long run. Below, we simply call g_t the growth rate unless otherwise noted.

Now, we are ready to derive the gap between the growth rate and the interest rate. The labor market equilibrium condition is

$$L_t^P + L_t^R = L. \quad (26)$$

By eliminating L_t^P and L_t^R from (21) via (24) and (26), we can represent the $g - r$ gap in terms of g_t :

$$g_t - r_t = s(\theta)(g_t - \hat{g}(\theta)), \text{ where} \quad (27)$$

$$s(\theta) = \frac{1 - \alpha\theta}{(1 - \alpha)(1 - \theta)} > 1, \quad \hat{g}(\theta) = \frac{1 - \alpha}{1 - \alpha\theta}\bar{g} \in (0, \bar{g}].$$

As shown in Figure 3, given the parameters, $g_t - r_t$ is positively and linearly related to g_t , with a slope of $s(\theta) > 1$. In particular, $g_t > r_t$ holds when g_t is greater than the threshold at $\hat{g}(\theta)$. Therefore, maintaining a high growth rate is crucial for running the government's Ponzi scheme.

When the rate of R&D subsidies, θ , is increased, the thick line in Figure 3 rotates counterclockwise, which lowers $g_t - r_t$ for a given growth rate (as shown by the dashed line). Accordingly, the threshold, $\hat{g}(\theta)$ in (27), increases with θ . While research subsidies may increase the growth rate, g_t needs to be even higher to maintain $g_t > r_t$. Then, how is g_t determined? In the following subsections, we characterize the equilibrium path of g_t by dynamic equations and phase diagrams, first without the research subsidy and then with it.

3.2 Equilibrium Dynamics

The dynamics of this economy can be examined by focusing on two variables, namely, g_t and $D_t \equiv B_t/Y_t$. Here, D_t is the ratio of government debt to potential GDP.¹⁴ We call this the debt-to-GDP ratio. Using (17), (20), (24) and (27), its time derivative is given by

$$\dot{D}_t = -s(\theta)(g_t - \hat{g}(\theta))D_t + \frac{\theta\alpha}{\bar{g}}g_t. \quad (28)$$

The first term of the RHS represents $(r_t - g_t)D_t$. With a balanced budget, the debt-to-GDP ratio would expand or shrink at the rate of $r_t - g_t$. The second term is the ratio of government spending on subsidies to potential GDP.¹⁵ This term accelerates the increase in D_t .

Next, we derive the time evolution of g_t . In this model, final goods are used only for consumption. Therefore, given the equilibrium of the goods market, $X_t = C_t$ holds. Using

¹⁴A benefit of focusing on $D_t \equiv B_t/Y_t$ rather than B_t/GDP_t is that Y_t depends only on the state variable n_t ; therefore, D_t is predetermined. Hence, we can use D_t as an initial condition for the equilibrium dynamics. In contrast, since GDP_t depends on jump variables L_t^P and L_t^R , it is not possible to use B_t/GDP_t as an initial condition. Note also that g_t is a jump variable because $g_t = \bar{g}L_t^R$ from (24).

¹⁵The ratio of government spending on subsidies to potential GDP is $\theta w_t L_t^R/Y_t$. Note that $w_t = \alpha Y_t/L$ from (20) and (23), and that $L_t^R = Lg_t/\bar{g}$ from (24). Accordingly, $\theta w_t L_t^R/Y_t = \theta\alpha g_t/\bar{g}$.

(8), (14), (16), (18), (20), (23), (24) and (27), we obtain¹⁶

$$\dot{g}_t = (\bar{g} - g_t)(s(\theta)(g_t - \hat{g}(\theta)) - \delta + \rho) + \bar{g}(\rho + \mu)(\delta + \mu) \left(D_t + \frac{\alpha(1 - \theta)}{A} \right). \quad (29)$$

In the first term, $s(\theta)(g_t - \hat{g}(\theta)) - \delta + \rho$ represents $g_t - (r_t - \rho + \delta)$. There is a positive effect of g_t on \dot{g}_t because growth in Y_t means that fewer production workers are required to produce a given amount of C_t ; therefore, more labor can be used for R&D, increasing \dot{g}_t . Additionally, as explained in Section 2.1, the Euler equation for aggregate consumption (Equation 8) implies that an increase in $r_t - \rho + \delta$ raises \dot{C}_t , which reduces the labor used for R&D, thereby decreasing \dot{g}_t . In the second term, $D_t + \alpha(1 - \theta)/A$ represents the sum of government debt and the value of all firms, divided by potential GDP (i.e., K_t/Y_t).¹⁷ A larger amount of the aggregate asset negatively affects \dot{C}_t since those who pass away, on average, have assets of $\bar{k}_t = K_t/N$, and are replaced by newborns who do not have financial assets. Then, \dot{g}_t increases since more labor will be allocated to R&D.

3.3 Dynamics without Research Subsidies

We start the analysis of the phase diagram with the case of no research subsidy ($\theta = 0$) because it clearly provides the condition under which existing debts can be rolled over perpetually and how much debts can be sustained. We first examine the $\dot{D}_t = 0$ locus. With $\theta = 0$, \dot{D}_t in (28) becomes zero when either $g_t = \hat{g}(0)$ or $D_t = 0$ holds. On the $g_t = \hat{g}(0)$ line, $r_t = g_t$ holds, which means that government debt is growing at the same rate as potential GDP; therefore, the debt-to-GDP ratio is stationary. D_t is also stationary on the $D_t = 0$ line because there is no income or expenditure by the government. These lines are drawn in red in Figure 4(a)-(c).

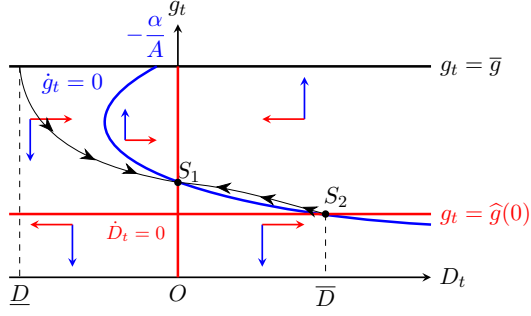
Next, we consider the $\dot{g}_t = 0$ locus. With $\theta = 0$, \dot{g}_t in (29) becomes zero when

$$D_t = \frac{\bar{g} - g_t}{\bar{g}(\rho + \mu)(\delta + \mu)} (\delta - \rho - s(0)(g_t - \hat{g}(0))) - \frac{\alpha}{A}. \quad (30)$$

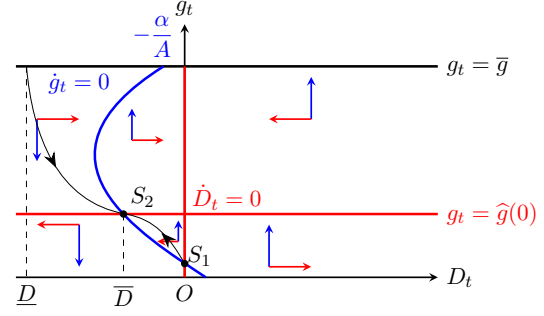
As depicted by the blue curves in Figure 4(a)-(c), the $\dot{g} = 0$ locus is a parabola that opens toward the right. Recall that g_t can only take values between $[0, \bar{g}]$, where $\bar{g} \equiv (1 - \alpha)A/\alpha$ is the growth rate of potential GDP when all the labor is used for R&D. Therefore, we limit

¹⁶From (24) and (26), $\dot{g}_t = -\bar{g}(\dot{L}_t^P/L)$. Additionally, from (14), (23) and $X_t = C_t$, we have $L_t^P = LX_t/Y_t = LC_t/Y_t$. Since L is constant, the rate of change of this equation is $\dot{L}_t^P/L_t^P = \dot{C}_t/C_t - \dot{Y}_t/Y_t$. Therefore, $\dot{g}_t = -\bar{g}(L_t^P/L)(\dot{L}_t^P/L_t^P) = -\bar{g}(L_t^P/L)(\dot{C}_t/C_t - \dot{Y}_t/Y_t)$. In the RHS, \dot{C}_t/C_t is given by the Euler equation (8), and $\dot{Y}_t/Y_t = g_t$. The Euler equation depends on aggregate assets K_t . From (16), (20), and (23), $n_t v_t = \alpha(1 - \theta)Y_t/A$. Substituting this result into (18) yields $K_t/Y_t = D_t + \alpha(1 - \theta)/A$. Finally, we substitute these results and (27) into $\dot{g}_t = -\bar{g}(L_t^P/L)((\dot{C}_t/C_t - g_t))$ and obtain (29).

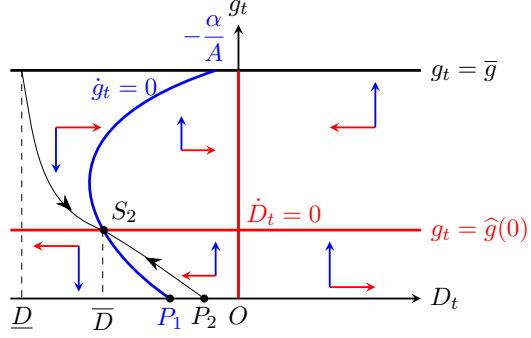
¹⁷See footnote 16.



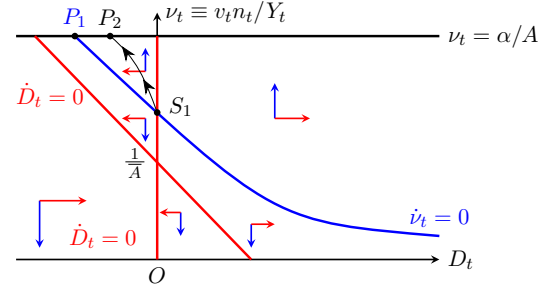
(a) When $A > \bar{A}$, S_1 is a saddle point with $g_1^* > r_1^*$, and S_2 is a source with $g_2^* = r_2^*$.



(b) When $A \in (\tilde{A}, \bar{A})$, S_2 is a saddle point with $r_2^* = g_2^*$; S_1 is a source with $g_1^* < r_1^*$.



(c) When $A \in (0, \tilde{A})$, S_2 is a saddle point with $r_2^* = g_2^*$, which is the only steady state in the $g_t > 0$ region.



(d) When $A \in (0, \tilde{A})$, there is another unstable steady state in the $g_t = 0$ region.

Figure 4: Phase diagram when there is no government expenditure ($\theta = 0$).

our attention to the area of $g_t \in [0, \bar{g}]$. At the upper end of $g_t = \bar{g}$, the $\dot{g} = 0$ parabola starts from $D_t = -\alpha/A$. Since this value is negative, the parabola intersects with the $D_t = 0$ line at most once. When the intersection exists (as in Figure 4(a)-(b)), it is a steady state with $\dot{D}_t = \dot{g}_t = 0$, which we label S_1 and denote its coordinates as $D_1^* = 0$ and $g_1^* \in [0, \bar{g}]$. Additionally, the parabola crosses the $g_t = \hat{g}(0)$ line exactly once before reaching the lower end of 0. We call this crossing point S_2 . The coordinates of S_2 are $D_2^* = \bar{D}$ and $g_2^* = \hat{g}(0)$, where

$$\bar{D} \equiv \frac{\alpha(\delta - \rho)}{(\rho + \mu)(\delta + \mu)} - \frac{\alpha}{A}. \quad (31)$$

The pattern of the dynamics changes depending on whether \bar{D} is positive or negative and whether S_1 exists. We explain three cases in turn.

Case 1: Saddle-stable steady state with $g_t > r_t$ ($A > \bar{A}$)

Since we assume that $\delta > \rho$, $D_2^* = \bar{D}$ is positive if and only if the research productivity parameter A is higher than

$$\bar{A} \equiv \frac{(\rho + \mu)(\delta + \mu)}{\delta - \rho} > 0. \quad (32)$$

When $A > \bar{A}$, as shown in Panel (a) of Figure 4, the steady state S_1 is above the $g_t = \hat{g}(0)$

line (i.e., $g_1^* \in (\hat{g}(0), \bar{g})$). From (27), this observation implies that the growth rate in the steady state is higher than the interest rate. In Appendix A.2, we show that S_1 is saddle stable, whereas S_2 is totally unstable (a source). Therefore, there is a stable arm that originates from S_2 and converges to S_1 . Hence, whenever the initial value of the debt-to-GDP ratio D_0 is less than $D_2^* = \bar{D} > 0$, there is an equilibrium path that converges to S_1 , where the debt-to-GDP ratio is zero in the long run.¹⁸ Even when the government has no revenue, its debt-to-GDP ratio can be stabilized as long as the initial ratio is not too large, given that $A > \bar{A}$.

Case 2: Saddle-stable steady state with $g_t = r_t$ ($A < \bar{A}$)

When $A < \bar{A}$, the steady state S_2 is located in the $D_t < 0$ region.¹⁹ In addition, if the $\dot{g}_t = 0$ parabola crosses the $D_t = 0$ line, another steady state S_1 exists, with $g_1^* \in (0, \hat{g}(0))$, as depicted in Figure 4(b). This case happens when $\tilde{A} < A < \bar{A}$, where the threshold is given by

$$\tilde{A} \equiv \frac{-(\delta - \rho) + \sqrt{(\delta - \rho)^2 + 4(1 - \alpha)(\rho + \mu)(\delta + \mu)}}{2(1 - \alpha)/\alpha} > 0. \quad (33)$$

In this case, as formally discussed in the Appendix A.2, S_2 is saddle stable, and S_1 is totally unstable (a source). Therefore, the stability property is the opposite of that described in Case 1. There is a stable arm originating from S_1 and converging to S_2 . Given that the economy starts from a positive government net asset ($D_t < 0$), there is an equilibrium path that stabilizes the net asset-to-GDP ratio in the long run.²⁰ In this steady state, government assets grow at the same rate as potential GDP does ($r_t = g_t$), hence stabilizing the ratio.

When $A < \tilde{A}$, there is only one steady state in the phase diagram shown in Figure 4(c). Similar to Case 2, S_2 is saddle stable, and a stable arm converges to it. However, the stable arm starts from point P_2 , which is located left of the origin. Hence, this diagram shows that the economy can converge to the steady state only when the asset-to-GDP ratio is larger than $|P_2|$. What happens if the economy starts from an initial asset smaller than that? Note that the phase diagram in Figure 4 is drawn under the assumption that the free entry condition (16) holds with equality. In fact, this model economy has another phase

¹⁸ Strictly speaking, the economy can converge to the steady state only when $D_0 \in (\underline{D}, \bar{D})$, where $\underline{D} < 0$ is the point where the downward-sloping stable arm crosses the $g_t = \bar{g}$ line. Numerically, we find that the absolute value of \underline{D} is very large under various parameter values; thus, it is not realistic to consider the case where the initial asset-to-GDP ratio is larger than $|\underline{D}|$. Therefore, in the main text, we disregard this possibility.

¹⁹ We ignore the border case of $A = \bar{A}$ because this case has a zero probability.

²⁰ Strictly speaking, the economy converges to the stable steady state when $D_0 \in (\underline{D}, 0)$. See the discussion in footnote 18.

diagram in the D_t - ν_t space, where $\nu_t = n_t v_t / Y_t$ is the ratio of the value of all firms to the potential GDP. This phase diagram describes the movement of the economy when the amount of R&D is zero (i.e., $g_t = 0$), which implies that the free entry condition (16) does not need to hold with equality. In the Appendix A.3, we show that there is an unstable steady state in this region if $A < \tilde{A}$. There is a stable arm that originates from this steady state and connects to point P_2 , as shown in Figure 4(d). Therefore, if the economy starts from a slightly negative D_t (i.e., a positive asset), it will experience a period of zero growth before arriving at point P_2 ; then, g_t gradually increases until the economy reaches the saddle-stable steady state S_2 . Therefore, the economy converges to the stable steady state if $D_0 \in (\underline{D}, 0)$, which is similar to the case of $\tilde{A} < A < \bar{A}$.

The following proposition summarizes the results.

Proposition 1 *Suppose that $\theta = 0$. The growth rate in the saddle-stable steady state is higher than the interest rate if and only if $A > \bar{A}$. In this case, there is an equilibrium path converging to this steady state if the initial debt is less than $\bar{D} > 0$. If $A < \bar{A}$, then the growth rate in the saddle-stable steady state is the same as the interest rate, and there is an equilibrium path converging to this steady state only when the initial debt is less than zero.*

Let us briefly mention what happens when the initial debt is too large to satisfy the condition of Proposition 1. Suppose that $A > \bar{A}$ and $D_0 > \bar{D}$ or that $A < \bar{A}$ and $D_0 > 0$. In these cases, the dynamics in the phase diagram eventually reach either the point of $g > \bar{g}$ or $\nu_t = v_t n_t / Y_t < 0$, regardless of the initial values of the jump variables (g_t and ν_t). Since $g > \bar{g}$ means that $L^P < 0$ and $\nu_t < 0$ means that $v_t < 0$, this equilibrium path is not feasible. Therefore, the economy has no equilibrium when $A > \bar{A}$ and $D_0 > \bar{D}$ or when $A < \bar{A}$ and $D_0 > 0$. We can consider this situation as the bankruptcy of the government because individuals cannot hold the rational belief that the aggregate value of government bonds is D_0 and they do not want to hold bonds at face value.

Proposition 1 implies that the government is able to roll over the existing debt perpetually if the R&D productivity is higher than \bar{A} and the initial debt-to-GDP ratio is less than \bar{D} . This result suggests the possibility that the government can spend some money without collecting taxes. If so, the money could be used to promote growth. The following subsection considers this possibility.

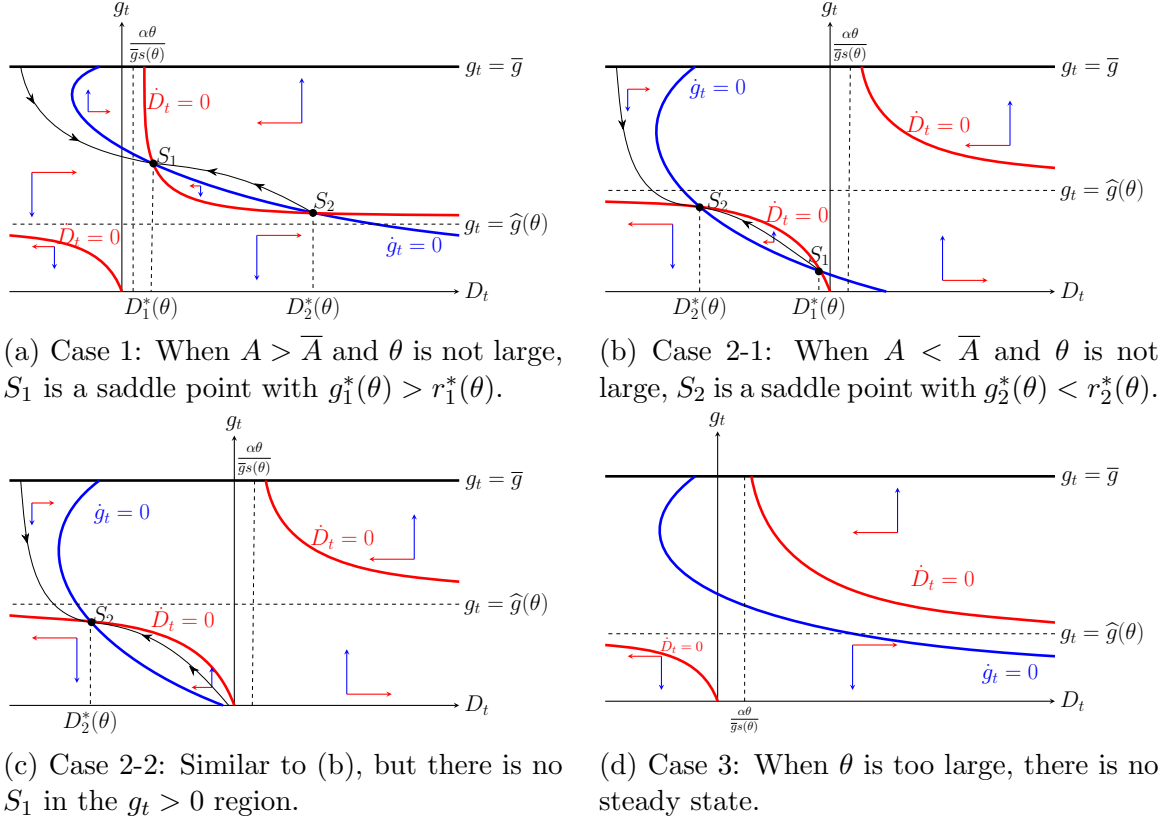


Figure 5: Phase diagram for the case when the rate of debt-financed research subsidy θ is positive

3.4 Dynamics with Research Subsidies

Now, we consider the effect of research subsidies on the dynamics of the economy while maintaining the assumption that the government does not have any revenue. With a research subsidy rate of $\theta \in [0, 1)$, Equation (29) implies that the $\dot{g} = 0$ locus is as follows:

$$D_t = \frac{\bar{g} - g_t}{\bar{g}(\rho + \mu)(\delta + \mu)} (\delta - \rho - s(\theta)(g_t - \hat{g}(\theta))) - \frac{\alpha(1 - \theta)}{A}. \quad (34)$$

From (28), the $\dot{D} = 0$ locus is

$$D_t = \frac{\alpha\theta}{\bar{g}s(\theta)} \frac{g_t}{g_t - \hat{g}(\theta)}. \quad (35)$$

The $\dot{g} = 0$ locus is a parabola, and $\dot{D} = 0$ is a rectangular hyperbola, with asymptotes of $D_t = \alpha\theta/\bar{g}s(\theta)$ and $g_t = \hat{g}(\theta)$. Whether they intersect with each other depends on the parameter values, particularly θ . Figure 5 shows three possible cases.

Case 1: Saddle-stable steady state with $g_t > r_t$ ($A > \bar{A}$ and θ is not too large)

Recall that if $A > \bar{A}$, the economy has a saddle-stable steady state with $g^* > r^*$ when $\theta = 0$. (See Figure 4(a).) Because the phase diagram moves continuously with θ , the economy can still have a saddle-stable steady state with $g_t > r_t$ if $A > \bar{A}$ and θ is not too

large, as shown by S_1 in Figure 5(a). We denote the coordinate of S_1 in the D_t - g_t space by $(D_1^*(\theta), g_1^*(\theta))$ since it changes with θ . As long as the $\dot{g}_t = 0$ locus intersects with the upper right portion of the $\dot{D}_t = 0$ locus, the value of g_t in the steady state is always higher than the horizontal asymptote at $g_t = \widehat{g}(\theta)$. Then, (27) implies that g_t is larger than r_t .

Additionally, the long-term level of the debt-to-GDP ratio, $D_1^*(\theta)$, is positive since the stable steady state S_1 is to the right of the asymptote line at $D_t = \alpha\theta/\bar{g}s(\theta) > 0$. Intuitively, the government issues a new bond each year to finance the research subsidy and pays interest on it forever. Nevertheless, the debt-to-GDP ratio converges to a positive constant because the economy is growing faster than the interest rate.

Another steady state S_2 at $(D_2^*(\theta), g_2^*(\theta))$ is a source (totally unstable). There is a saddle path originating from S_2 and converging to S_1 . Therefore, the debt-to-GDP ratio can be stabilized in the long run if the initial debt-to-GDP ratio is less than $D_2^*(\theta)$.

Case 2: Saddle-stable steady state with $g_t < r_t$ ($A < \bar{A}$ and θ is not too large)

In contrast to Case 1, if $A < \bar{A}$ and θ is not too large, then the $\dot{g}_t = 0$ locus intersects with the lower left portion of the $\dot{D}_t = 0$ locus, as shown in Figure 4(b)-(c). In this case, S_2 at $(D_2^*(\theta), g_2^*(\theta))$ is a saddle-stable steady state, and there may or may not exist an unstable steady state (S_1) in the $g_t > 0$ region depending on the parameters, as discussed in Case 2 of Section 3.4.

The value of $g_2^*(\theta)$ is always lower than the horizontal asymptote at $\widehat{g}(\theta)$. Therefore, from (27), the interest rate in this steady state, denoted by $r_2^*(\theta)$, is higher than the growth rate $g_2^*(\theta)$. Additionally, $D_2^*(\theta)$ is negative because the hyperbola slopes downward and moves through the origin. Thus, in the steady state S_2 , the government holds a positive net asset that grows at the rate of economic growth so that the asset-to-GDP ratio is constant at $|D_2^*(\theta)|$. Given that $r_2^*(\theta) > g_2^*(\theta)$, the government can use the $(r_2^*(\theta) - g_2^*(\theta)) |D_2^*(\theta)| Y_t$ portion of interest revenue while keeping the asset-to-GDP ratio constant. In the steady state, this surplus is just enough to finance the expenditure for the research subsidy.

While the situation might seem desirable, note that this steady state can be reached only when the government has sufficient initial assets. Suppose that the unstable steady state S_1 exists in the $g_t > 0$ region, as depicted in Figure 5(b). Then, the stable steady state S_2 can be reached only when the initial value of D_t is less than $D_1^*(\theta)$. This means that the government needs to start with an asset-to-GDP ratio that is greater than $|D_1^*(\theta)|$. Otherwise, it is not possible to maintain a positive θ in this case.

Case 3: No steady state (θ is larger than a certain threshold)

If θ is too large, then there is no intersection between the $\dot{g}_t = 0$ locus and the $\dot{D}_t = 0$

locus in the phase diagram, as shown in Figure 5(d). Therefore, regardless of the initial value of D_t , the economy cannot reach a steady state. Starting from any value of g_t , we find that the economy will eventually violate the non-negativity condition for some variables (e.g., L_t^P or v_t eventually become negative). Thus, there is no equilibrium path in this economy. Such a large debt-financed subsidy is not sustainable regardless of the initial amount of government debt or assets.

4 The Growth Effects of Debt-Financed Research Subsidies

As discussed in the Introduction, the economic growth rate tends to be higher than the interest rate on government bonds in the U.S. and other developed countries (see Figures 1 and 2). Given this fact, the analysis outlined in the previous section suggests that the R&D productivity, A , is greater than \bar{A} in these economies. In this case, the government can provide research subsidies for firms while rolling over their debts, given that the initial debt-to-GDP ratio is less than $D_2^*(\theta)$ (see Case 1 of Section 3.4). Here, we examine whether such a policy can actually enhance economic growth. In the first subsection, we examine the effects on the long-term growth rate by focusing on the steady state. In the second subsection, we investigate the transitional effects.

4.1 Long-Term Effects

A research subsidy has two opposing effects on the growth rate. First, it promotes research activity by reducing the R&D cost incurred by private firms. Second, the government expense for the subsidy increases the long-term debt-to-GDP ratio $D_1^*(\theta)$, which in turn increases the equilibrium interest rate. A higher interest rate reduces the value of firms and hinders the incentives for R&D. The following proposition corresponds to the case when the first effect dominates.

Proposition 2 *A marginal increase in θ from $\theta = 0$ increases the growth rate in the saddle-stable steady state if and only if*

$$A > 2\bar{A} + \frac{\delta - \rho}{2} \equiv \hat{A}. \quad (36)$$

Proof: See the Appendix A.4.

The proposition shows that a high value of A is necessary for the R&D subsidies to have a positive effect on long-term growth. In Proposition 1, we have shown that $A > \bar{A}$ is the

Parameter	Description	Value	Source
ρ	Discount rate	0.01	Standard
μ	Mortality rate	$1/(80.3-20)$	OECD (2023b)
δ	Labor deterioration rate	$1/(64-20)$	OECD (2023a)
α	Inverse of the markup rate	$1/1.2$	Ball and Mankiw (2023)
A	R&D Productivity ($A > \hat{A}$)	0.536	Estimated from U.S. data
	R&D Productivity ($A < \hat{A}$)	[0.152]	Estimated from Japanese data

Table 1: Parameters for Numerical Simulations

condition for the economy to have a saddle-stable steady state with $g_t > r_t$. Since $\hat{A} > \bar{A}$, Proposition 2 implies that $g_t > r_t$ is not sufficient for such policies to have positive long-run effects. The following corollary shows that research subsidies enhance long-term growth if and only if the $g_t - r_t$ gap exceeds a certain threshold.

Corollary 1 *Suppose that in the absence of research subsidies ($\theta = 0$), the economy has a saddle-stable steady state with $g_1^*(0) > r_1^*(0)$. A marginal increase in θ from $\theta = 0$ increases the growth rate in the saddle-stable steady state if and only if the following holds:*

$$g_1^*(0) - r_1^*(0) > \frac{\delta - \rho}{2}. \quad (37)$$

Proof: See the Appendix A.4.

Note that the RHS of (37) is strictly positive because we assume that $\delta > \rho$. In our model environment, $\delta > \rho$ is necessary for the economy to have a saddle-stable steady state with $g_1^*(0) > r_1^*(0)$. When δ is greater, the decline in the individual effective labor supply with age is steeper, and the individual has more incentives to save for their old age. These saving incentives keep the interest rate lower than the growth rate. However, Corollary 1 shows that a larger δ requires a wider gap between g_t and r_t for debt-financed research subsidies to have a positive effect on long-term growth.

In the following, we examine the effect of research subsidies via numerical simulations. Table 1 summarizes the parameters used in all the simulations in Section 4. We set a standard value for the discount rate $\rho = 0.01$. The mortality rate is $\mu = 1/(80.3 - 20)$, where 80.3 is the average life expectancy in the OECD in 2021 (OECD, 2023b), and 20 is the age from which we assume that the agents start economic activities. The deterioration rate of labor productivity δ is set to $1/(64-20)$, where 64 is the average normal retirement age in the OECD as of 2022 (OECD, 2023a).²¹ We set $\alpha = 1/1.2$ so that the markup

²¹Using similar data from the OECD, Rachel and Summers (2019) developed a probabilistic retirement model, whereas we consider the continuous depreciation of the working ability to simplify the analysis. The evolution of the average productivity is the same in these two specifications.

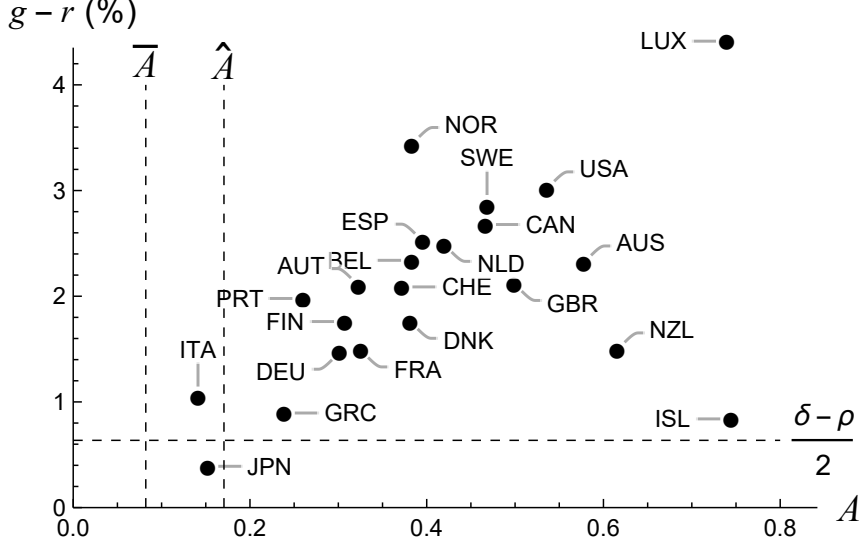


Figure 6: Estimates of research productivity A and the $g - r$ gap

rate of intermediate goods is $1/\alpha = 1.2$, following Ball and Mankiw (2023). Given these parameters, we find that the threshold values of A are $\hat{A} = 0.171$, $\bar{A} = 0.0821$, and $\tilde{A} = 0.0414$.

To account for the difference in research productivity across countries, we separately estimate the value of A for each of the countries that were included in the OECD by 1973. We focus on those developed countries because they can be seen as already on the BGP in the year 2000. Given that all investment in our model takes the form of R&D and that R&D uses only labor, the value of A in country i can be estimated as²²

$$A_i = \alpha \left(1 + \frac{\alpha}{1 - \alpha} \frac{1}{(wL^R/\text{GDP})_i} \right) g_i, \quad (38)$$

where we use the average real GDP growth rate and the average investment-to-GDP ratio of country i for the 2000–2023 period to specify g_i and $(wL^R/\text{GDP})_i$.²³ Figure 6 plots the estimates of A along with the average value of the $g - r$ gap for the same 2000–2023 period. We find that all the estimates of A_i exceed the threshold value of $\bar{A} = 0.0821$, which implies that those economies have a steady state with $g - r > 0$ from Proposition 1. Indeed, Figure 6 confirms that the average values of $g - r$ in the 2000–2023 period are positive for the counties shown. Except for Japan and Italy, all other countries exceed the

²²From (15) and (24), $A = (L_t^R/L)/(\dot{n}_t/n_t) = (\alpha/(1 - \alpha))\sigma_t^L/(w_t L_t^R/\text{GDP}_t)$, where $\sigma_t^L = w_t L/\text{GDP}_t$ is the labor share. Using (20), (22) and (24), $\sigma_t^L = \alpha L/(L_t^P + \alpha L_t^R) = \alpha/(1 - \alpha g_t/A)$. By substituting this σ_t^L into the first equation and then solving for A , we obtain (38).

²³Both the real GDP growth rate and the investment-to-GDP ratio (gross fixed capital formation as a percentage of GDP) are obtained from the World Development Indicators (World Bank). The $g - r$ gap is from the same data used in Figures 1 and 2.

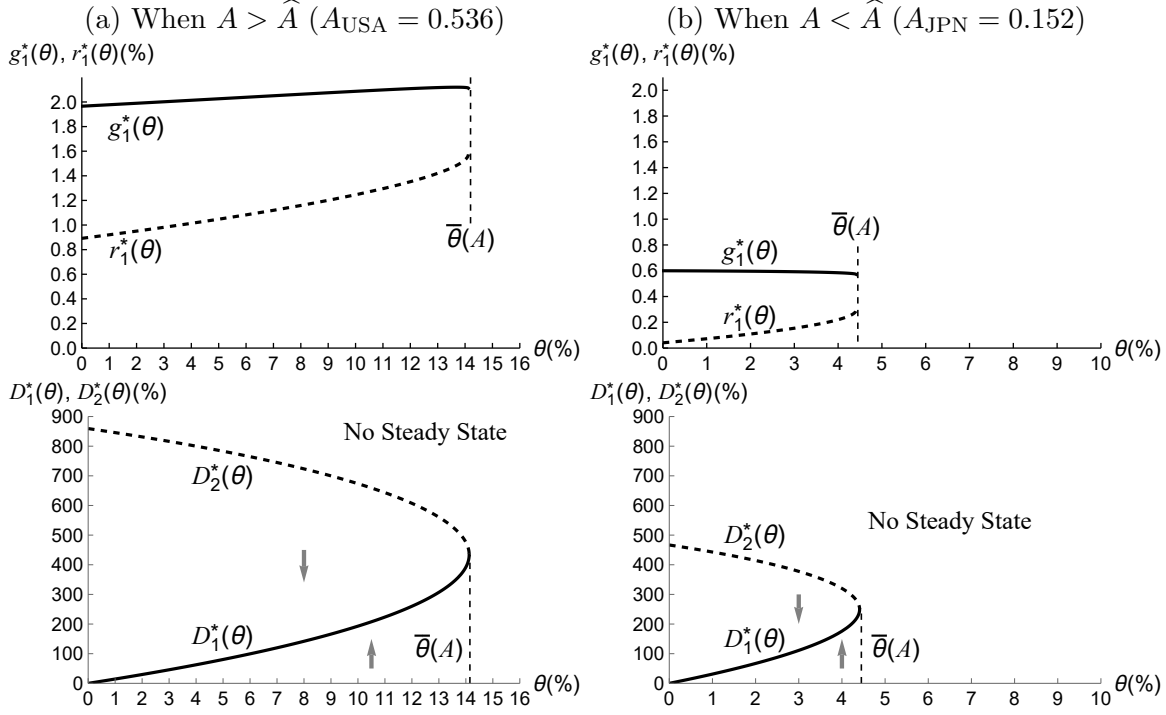


Figure 7: The long-term effect of the research subsidy rate θ on the growth rate $D_1^*(\theta)$, the interest rate $r_1^*(\theta)$, the debt-to-GDP ratio $D_1^*(\theta)$, and the maximum limit of the debt-to-GDP ratio $D_2^*(\theta)$.

threshold at $\hat{A} = 0.171$. These countries also have $g - r$ gaps that are greater than $(\delta - \rho)/2$. Proposition 2 and Corollary 1 state that it is possible to increase growth via debt-financed research subsidies in those countries. We take the United States ($A_{\text{USA}} = 0.536$) as a representative of these countries and examine the effect of the policy in detail below. The estimates of A_i for Japan and Italy fall short of \hat{A} .²⁴ We take Japan ($A_{\text{JPN}} = 0.152$) as a representative of the countries with $A_i < \hat{A}$.

Figure 7 numerically plots the long-term growth rate, interest rate and debt-to-GDP ratio against the rate of research subsidies θ . We can confirm the result of Proposition 2 in the upper panels of Figure 7. Column (a) shows the baseline case of $A_{\text{USA}} = 0.536 > \hat{A}$. In this case, the value of $g_1^*(\theta)$ is increasing in θ unless θ is close to the upper limit, where the BGP disappears. We denote this upper limit of θ by $\bar{\theta}(A)$ because it depends on A . When θ exceeds this point, the phase diagram of the dynamics changes from Figure 5(a) to (d), and there is no steady-state level of the debt-to-GDP ratio. Therefore, it is not possible to increase the rate of research subsidies beyond $\bar{\theta}(A)$ because the government debt becomes unsustainable. Column (b) shows that $g_1^*(\theta)$ is always decreasing with θ when $A < \hat{A}$, which is consistent with Proposition 2. The upper panels also show the response of the

²⁴Italy does not perfectly fit the theory in that the $g - r$ gap is larger than $(\delta - \rho)/2$ despite $A < \hat{A}$.

interest rate to changes in θ .²⁵ The interest rate is increasing in θ , but it is always lower than the growth rate as long as the BGP exists. This result confirms the analysis in Case 1 of Section 3.4. As shown in Figure 5(a), $g_1^*(\theta)$ is greater than $\hat{g}(\theta)$; therefore, from (27), $g_t > r_t$ holds. In other words, the government can roll over its debt only when its policy allows for the existence of a steady state with $g_t > r_t$.

The lower panels show that the steady-state debt-to-GDP ratio, $D_1^*(\theta)$, against the subsidy rate, θ . $D_1^*(\theta)$ increases with θ , especially when θ is near $\bar{\theta}(A)$. Note that $r_1^*(\theta)$ and $D_1^*(\theta)$ follow similar patterns. This similarity demonstrates that the accumulation of government debt is causing the equilibrium interest rate to rise, which in turn crowds out private R&D. In fact, even when $A > \hat{A}$ (i.e., in Column (a)), $g_1^*(\theta)$ is decreasing in θ when θ is close to the upper limit. This phenomenon occurs because the response of the interest rate to θ is sharp when θ is close to $\bar{\theta}(A)$.

The $D_2^*(\theta)$ curves in the lower panels represent the maximum value of the debt-to-GDP ratio beyond which the economy cannot converge to the saddle steady state (see the phase diagram in Figure 5(a)). The arrows in the panels represent the direction of movement of D_t on the saddle path. If D_t exceeds $D_2^*(\theta)$ or if $D_2^*(\theta)$ does not exist (i.e., $\theta > \bar{\theta}(A)$), the economy has no equilibrium path. Observe that $D_2^*(\theta)$ is decreasing in θ , and it eventually connects to $D_1^*(\theta)$ just before the steady state disappears. Therefore, even when a higher θ increases the growth rate, it entails two kinds of costs, namely, it increases the steady-state level of the debt GDP ratio, $D_1^*(\theta)$, and it reduces the highest maintainable debt-to-GDP ratio, $D_2^*(\theta)$.²⁶ The curves of $D_1^*(\theta)$ and $D_2^*(\theta)$ eventually coincide, which means that the steady state cannot exist beyond this point. This point defines the highest sustainable subsidy rate, $\bar{\theta}(A)$, and the highest sustainable debt-to-GDP ratio in the steady state, $D_1^*(\bar{\theta}(A)) = D_2^*(\bar{\theta}(A))$.²⁷

Figure 8 shows the contour plot of the long-term growth rate, $g_1^*(\theta)$, against A and θ . The growth rate is greater when the color is lighter. The thick black curve shows the relationship between the upper limit of the subsidy rate, $\bar{\theta}(A)$, and the R&D productivity, A . Since it slopes upward, a higher rate of research subsidies is sustainable when the research productivity is also higher. The area above the $\bar{\theta}(A)$ curve, which is shown in

²⁵From (27), the interest rate in the steady state S_1 is obtained by $r_1(\theta) = s(\theta)\hat{g}(\theta) - (s(\theta) - 1)g_1^*(\theta)$.

²⁶The debt-to-GDP ratio could be (temporarily) affected by various kinds of shocks, although we do not explicitly consider these shocks. If the current debt-to-GDP ratio is close to the highest maintainable debt-to-GDP ratio, $D_2^*(\theta)$, these shocks may push D_t above $D_2^*(\theta)$, which would make the government go bankrupt. Therefore, the decline in $D_2^*(\theta)$ can be viewed as a cost since it negatively affects the sustainability of government debt.

²⁷In these numerical examples, the highest sustainable debt-to-GDP ratio is 433% when $A = 0.536$ (A_{USA}) and 249% when $A = 0.152$ (A_{JPN}).

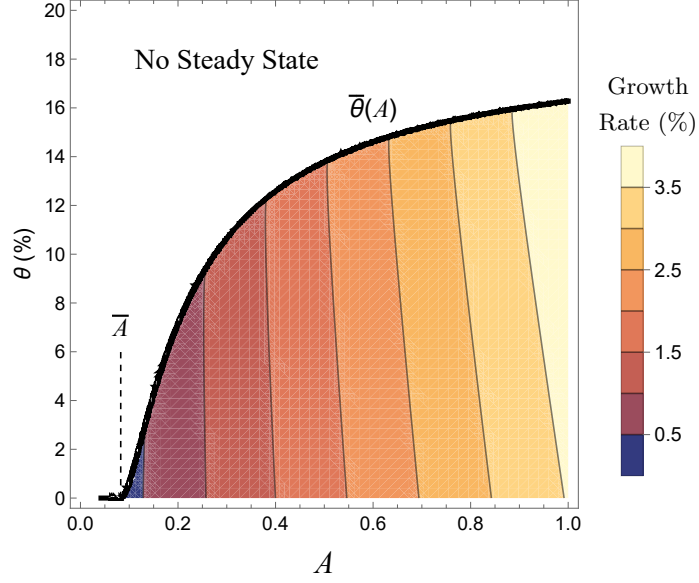


Figure 8: Contour plot of the growth rate in the steady state $g_1^*(\theta)$ against the R&D productivity, A , and the R&D subsidy rate, θ

white, indicates that there is no steady state with $g_t > r_t$ for the given combination of A and θ ; i.e., such a policy is not sustainable. Recall that, according to Proposition 1, the government can roll over the debt only when $A > \bar{A}$. Therefore, the $\bar{\theta}(A)$ curve intersects with the horizontal axis to the right of the origin at $A = \bar{A}$.

4.2 Transitional Effects

Thus far, we have shown that debt-financed R&D subsidies enhance long-term growth in economies with high A (which means high $g_1^*(0)$), but they are detrimental to long-term growth when A is low (i.e., $g_1^*(0)$ is low). From this finding, it can be tempting to conclude that the policy will be beneficial for economies only when the growth rate before the policy is already sufficiently high. However, this conclusion may not hold for two reasons. First, the growth rate may change during the transition from the old steady state to the new steady state. Second, the transition may take a very long time. If the growth rate is higher than the steady-state value for a long period, the eventual difference in the GDP level can be significant.

Figure 9 shows the transitional response of the economy when the subsidy rate θ is permanently raised from zero to the neighborhood of the maximal sustainable rate, $\bar{\theta}(A)$.²⁸ The horizontal axis shows the time after the policy change in years. Column (a) presents

²⁸The transitional paths of g_t and D_t are obtained by numerically solving the differential equations (28) and (29) backward in time. This method is called the reverse shooting algorithm. We start the calculation from (the neighborhood of) the new steady state with an increased θ and terminate the calculation when the state variable D_t reaches the point $D_t = D_1^*(0) = 0$.

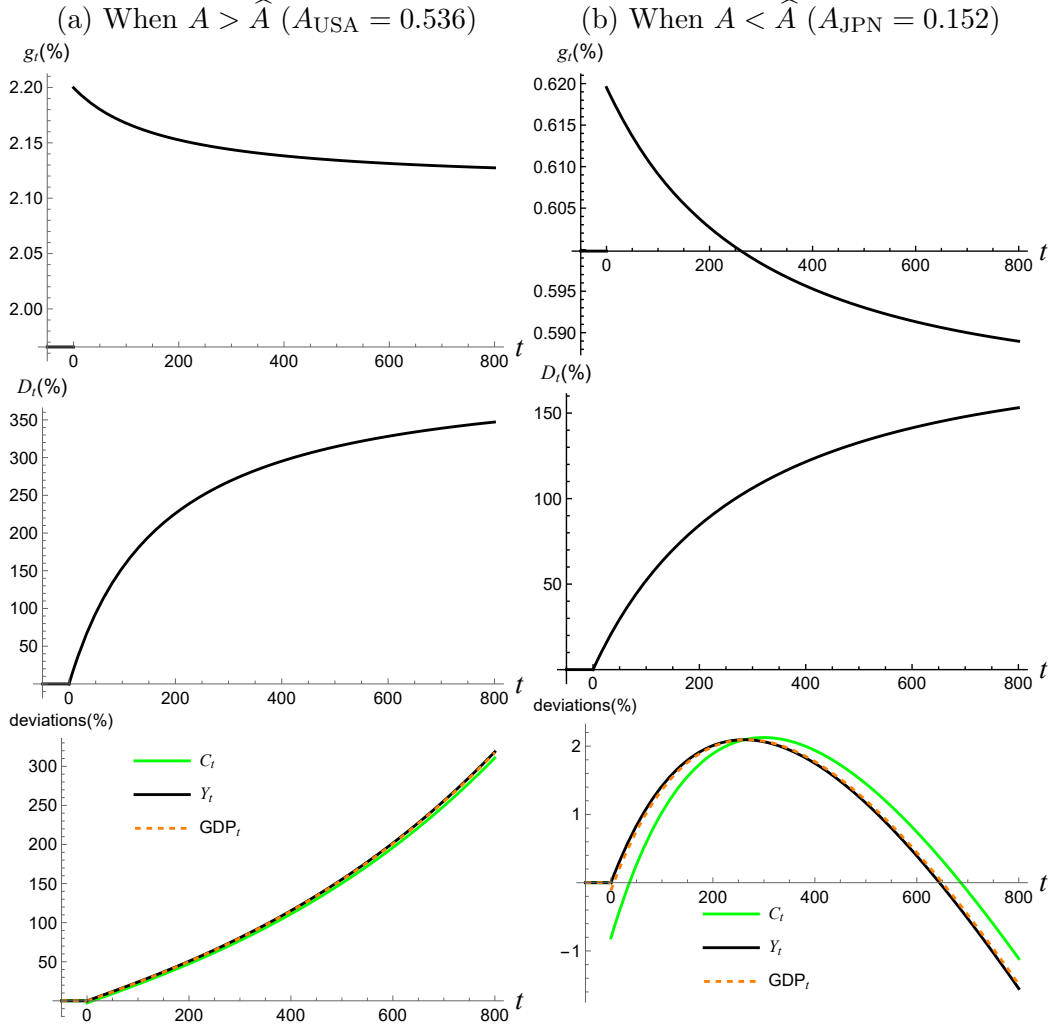


Figure 9: Transitional dynamics after research subsidies are introduced.

the results for the benchmark case of $A = 0.536$, and Column (b) presents the results for a low-growth economy with $A = 0.152 < \hat{A}$. The economy is assumed to be on the BGP with $\theta = 0$ before time $t = 0$. At time $t = 0$, the subsidy rate is raised to 14% ($A = 0.536$) and 4% ($A = 0.152$).²⁹ The other parameters are the same as those in the previous subsection.

The upper panels show the response of g_t , i.e., the growth rate of potential GDP. g_t increases immediately after the R&D subsidy is introduced, even when $A < \hat{A}$. The growth rate gradually decreases, and it takes hundreds of years to converge to the long-term value. The reason for the initial overshoot of g_t and the slow convergence can be observed from the middle panels, which depict the response of D_t , the debt-to-GDP ratio. Specifically, the panels show that it takes a long time for the debt-to-GDP ratio to increase, given that $g_t > r_t$. As explained in the previous section, debt-financed R&D subsidies have both positive and negative effects on growth. Among these effects, the positive and direct

²⁹These values are slightly below $\bar{\theta}(A)$ (see Figures 7 and 8).

effects of subsidies on research take effect immediately. In contrast, the negative crowding-out effect comes from the stock of government debt, which only gradually accumulates. Therefore, in the short run, the positive effect always dominates.

The bottom panels show the percentage deviation of paths of potential GDP Y_t , aggregate consumption C_t , and actual GDP, given by (22), from their respective paths without research subsidies.³⁰ The response of the deviation of Y_t is monotonically increasing when $A > \hat{A}$ and is hump shaped when $A < \hat{A}$. Since the growth rate of Y_t is g_t , this pattern directly reflects the results shown in the upper panels.³¹ The aggregate consumption amount decreases when the research subsidy is introduced.³² The reason for this result is that more resources are devoted to R&D than to the production of consumption goods. After approximately 10 to 20 years, the production of consumption goods overtakes the case of no subsidy, owing to increased productivity (i.e., a higher level of potential GDP Y_t). The response of the deviation in GDP is between those of Y_t and C_t . The GDP is the sum of the values of consumption and R&D. In our model, consumption goods are produced with a positive markup, where R&D is subject to free entry without markups. Therefore, when more resources are devoted to R&D, the GDP slightly decreases because the total value of markups decreases.

The results in column (b) of Figure 9 suggest that debt-financed R&D subsidies have sizable positive effects on the levels of GDP and consumption for several centuries even when $A < \hat{A}$. The reason is that the growth rate is higher in the transition, and the transition takes a very long time. However, the level of GDP will eventually become lower than in the case of no policy change. Since it takes hundreds of years before the negative effect dominates the initial positive response, the desirability of such a policy depends on how we evaluate the utility of current and future generations.

³⁰Let $Y_t^{\theta=0}$ be the value of potential GDP on the BGP without research subsidies. Then, the deviation of Y_t from $Y_t^{\theta=0}$ is defined by $(Y_t - Y_t^{\theta=0})/Y_t^{\theta=0}$. The deviations of C_t and GDP_t are defined similarly.

³¹Note that $Y_t = Y_0 \exp\left(\int_0^t g_\tau d\tau\right)$ and $Y_t^{\theta=0} = Y_0 e^{g_1^*(0)t}$, where $g_1^*(0)$ is the growth rate on the BGP with $\theta = 0$. Based on the above, the deviation of Y_t can be calculated as $(Y_t - Y_t^{\theta=0})/Y_t^{\theta=0} = \exp\left(\int_0^t (g_\tau - g_1^*(0)) d\tau\right) - 1$. Therefore, the slope of the deviation of Y_t is positive when $g_t - g_1^*(0) > 0$, and vice versa.

³² Using Equations (14), (23), (24) and (26), $C_t = X_t = n_t^{(1-\alpha)/\alpha} L_t^P = (L_t^P/L) Y_t = (1 - g_t/\bar{g}) Y_t$. From this result, the deviation of C_t from the BGP with $\theta = 0$ is $(C_t - C_t^{\theta=0})/C_t^{\theta=0} = \exp(\log C_t - \log C_t^{\theta=0}) - 1 = \exp(\log(\bar{g} - g_t) - \log(\bar{g} - g_1^*(0)) + \int_0^t (g_\tau - g_1^*(0)) d\tau) - 1$. Similarly, we can write $GDP_t = n_t^{(1-\alpha)/\alpha} (L_t^P + \alpha L_t^R) = (1 - (1 - \alpha)g_t/\bar{g}) Y_t$ from (22). Its deviation from the BGP with $\theta = 0$ is $(GDP_t - GDP_t^{\theta=0})/GDP_t^{\theta=0} = \exp(\log(\bar{g} - (1 - \alpha)g_t) - \log(\bar{g} - (1 - \alpha)g_1^*(0)) + \int_0^t (g_\tau - g_1^*(0)) d\tau) - 1$.

5 Social Rate of Return of R&D and Dynamic Efficiency

We have shown the possibility that R&D subsidies financed by perpetually rolled-over debt may stimulate R&D investments and increase the rate of economic growth, depending on the parameters. Here, we examine whether such increased R&D investments are beneficial to the economy. Since we explicitly consider overlapping generations of consumers, how we should measure the welfare of the overall economy is not obvious. As shown in Column (b) of Figure 9, a policy can be beneficial for the current generation but detrimental to distant future generations. Rather than weighing the utility of different generations, here, we consider the most conservative criterion: we examine whether the economy is dynamically efficient.

Previous studies have shown that the economy is dynamically inefficient if the rate of return on investment is lower than the rate of economic growth (e.g., Tirole, 1985; Abel, Mankiw, Summers and Zeckhauser, 1989). In this case, the aggregate consumption for all periods can be increased by reducing investment; therefore, stimulating investments is actually welfare-reducing.³³ This possibility is a particular concern in the literature on rolling over debts because most papers focus on the case of $r < g$. Recent studies (e.g., Blanchard, 2019; Ball and Mankiw, 2023; Barro, 2023) have shown that the economy can be dynamically efficient despite $r < g$ when the marginal product of capital is higher than the interest rate for various reasons, including risk premiums and markups on capital production.

In our model, R&D investments have positive externalities on future R&D. In Equation (15), the output of R&D, \dot{n}_t , depends positively on today's stock of knowledge capital n_t , i.e., the cumulative number of ideas produced in the past. Similarly, today's R&D creates new knowledge, which will be used in the future. Therefore, the social rate of return on investment r^s , including this externality, is higher than the interest rate r . Thus, we cannot say that $r^s < g$ holds even when the interest rate on the government bond is lower than the growth rate. In fact, in the following, we show that the social rate of return of R&D investments in our model is higher than the rate of economic growth on any BGP.

³³To understand this point, let us consider a standard neoclassical growth model in discrete time. Suppose that on the BGP, the economy is growing at a rate of g , and the social rate of return on investment is $r^s < g$. Now, reduce the investment by 1% from period t_0 onward. Then, the consumption at period t_0 is increased by this amount. In period $t_0 + 1$, the output, including the undepreciated capital, is reduced by $(1 + r^s)\%$ of the investment of period t . Note that on the BGP before manipulation, the amount of investment also grows at a rate of g . Therefore, this reduction is $((1 + r^s)/(1 + g))\% < 1\%$ of the investment in period $t_0 + 1$. Recall that in period $t_0 + 1$, the investment is reduced by 1%, which can be used for consumption. In total, the consumption in period $t_0 + 1$ is increased by $((g - r^s)/(1 + g))\% > 0$ of the investment in that period. Similar arguments hold for all $t \geq t_0 + 1$. Therefore, reducing investments is Pareto improving if $r^s < g$.

Following Jones and Williams (1998), we define the social rate of return as

Definition 1 *On the BGP, the social rate of return of R&D (r^s) is defined as follows:*

$$r^s \equiv \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \left(\frac{dX_{t+\Delta t}}{-dX_t} - 1 \right), \quad (39)$$

where $dX_t < 0$ and $dX_{t+\Delta t} > 0$ are the changes in X_t and $X_{t+\Delta t}$, respectively, from the respective BGP values when the following operations are conducted in the model, where time is discretized with an infinitesimally small Δt .

1. At time t , L_t^R marginally increases by a small value $dL_t^R > 0$.
2. At time $t + \Delta t$, $L_{t+\Delta t}^R$ decreases by $dL_{t+\Delta t}^R < 0$. The magnitude of $dL_{t+\Delta t}^R$ is determined so that all variables at and after time $t + 2\Delta t$ are on the original BGP.

Note that in this definition, the final goods production X_t at time t decreases by $dX_t < 0$ because the economy allocates more resources to R&D. This result is considered an increase in the R&D investment measured in terms of final goods. At time $t + \Delta t$, the return on this R&D investment is harvested by increasing final goods production by $dX_{t+\Delta t} > 0$ so that the reduction in R&D investment at $t + \Delta t$ exactly offsets the effect of the initial increase in R&D. The social rate of return of R&D is measured by how large the magnitude of the harvest ($dX_{t+\Delta t}$) is relative to the initial R&D cost ($-dX_t$). Since this value is the social rate of return for the duration of Δt , the rate of return for unit time is given by dividing it by Δt , as in (39). We determine the value of r^s as follows.

Proposition 3 *In any BGP of this model, the social rate of return of R&D, defined by Definition 1, is $r^s = \bar{g}$; therefore, $r^s \geq g$ holds.*

Proof: See the Appendix A.5.

Proposition 3 holds regardless of the value of θ , as long as the BGP exists. Note that \bar{g} is given by (25), which represents the maximum rate of economic growth when all labor is used for R&D. Therefore, the social rate of return $r^s = \bar{g}$ is always higher than the rate of economic growth g . This result implies that the economy satisfies a necessary condition for dynamic efficiency ($r^s \geq g$), even though $r < g$. The following proposition establishes the essence of dynamic efficiency in a more direct way.

Proposition 4 *For any BGP in this model, it is not possible to increase the aggregate consumption for a short period while keeping the aggregate consumption outside this interval unchanged.*

Proof: See the Appendix A.6.

Given the state of technology (i.e., n_t), the only way to increase aggregate consumption is by reducing R&D and allocating more resources to production. If the amount of R&D investments is reduced today, then the proof of the proposition shows that it is inevitable to reduce aggregate consumption sometime in the future. This property holds in any BGP, including the BGP with positive R&D subsidies. Therefore, a reduction in the research subsidy today (which reduces R&D investments) will lead to a reduction in consumption in the future. In other words, enhancing R&D by subsidies is never a waste of resources.

6 Extension: Basic Research

Thus far, we have examined the effect of debt-financed research subsidies on economic growth. The results critically depend on the research productivity of private firms, A . R&D subsidies always enhance growth in the short run, but they hinder long-term growth if A is lower than \hat{A} .

Although we treat A as a fixed parameter in our main model, recent studies suggest that the government can influence this parameter by funding basic research, which is typically conducted in government agencies and universities. For example, Akcigit, Hanley and Serrano-Velarde (2020) considered spillovers across different types of research activities and concluded that the policies geared toward basic public research improve welfare. Gersbach, Sorger and Amon (2018b) reported that basic research extends the knowledge base for applied research and considers a variety-expansion model where private applied R&D and publicly funded basic research are complementary in creating new goods.³⁴ Given these studies, this section considers a simple extension of our model, where the government can improve the research productivity of private firms by investing more resources into publicly funded basic research.

Now, we assume that a part of the aggregate labor supply, L , is used for publicly funded basic research. We label this amount L^B .³⁵ We assume that individuals are employed by the government at the market wage w_t . The research productivity of private R&D depends

³⁴In a related study, Gersbach, Schetter and Schneider (2018a) introduced public-funded basic research into a quality-ladder endogenous growth model, where public funding increases the probability of success in quality-upgrading innovation. Leon-Ledesma and Shibayama (2023) extended Romer (1990) by introducing the basic research sector subsidized by the government.

³⁵We omit the time subscript in L^B because it is a policy variable.

Parameter	Description	Value	Source
A_0	Ex-ante R&D productivity	0.536	from the U.S. data
L_0^B/L	Fraction of ex-ante basic research labor	[0.152] 0.00549	from Japanese data from the U.S. data
ϕ	Elasticity of A to L^B	[0.00448] 0.28,0.1,0.04	from Japanese data Jaffe (1989), Acs et al.(1992)

Table 2: Parameters when the R&D productivity is determined by basic research

on the amount of basic research in the following way:³⁶

$$A = A_0 \left(\frac{L^B}{L_0^B} \right)^\phi, \quad L^B \geq L_0^B, \quad (40)$$

where A_0 is R&D productivity before the policy change, L_0^B is the initial amount of basic research labor, and ϕ is the elasticity of private R&D productivity with respect to the amount of basic research. Public basic research is financed entirely by the government. We assume that there is an existing labor tax that just finances wages for L_0^B .³⁷ For political reasons, as we discussed in the Introduction, we assume that the government cannot increase the tax rate. In the analysis of this section, we consider the case where the government cannot raise the existing tax rate at all. Therefore, if it wants to enhance basic research by increasing L^B above L_0^B , the added cost, $w_t(L^B - L_0^B)$, must be financed by issuing bonds. The other parts of the model are the same as those in Section 2. In this setting, we examine how government policies, characterized by the pair of $\theta \geq 0$ and $L^B \geq L_0^B$, affect long-term economic growth. The details of the analysis of this extended model are presented in the Appendix A.7.

Table 2 summarizes the parameters used to specify Equation (40). Similar to the numerical analysis in Section 4, we focus on two cases, one with $A > \hat{A}$, represented by the U.S., and the case with $A < \hat{A}$, represented by Japan.³⁸ We treat L_0^B/L as the average value of the fraction of basic research workers in the 2000–2023 period. Then, A_0 should approximately equal the average value of A in the same period, which can be estimated by (38).³⁹

³⁶For simplicity, we assume that the government cannot reduce government-financed basic research labor from L_0^B .

³⁷Specifically, we assume that labor income is taxed at the rate of L_0^B/L . The revenue from this tax is then wL_0^B .

³⁸Strictly speaking, the threshold values \bar{A} and \hat{A} slightly change when we explicitly include basic research in the model. We present the equations for the modified threshold values for A_0 in the Appendix A.7. With the data from the United States ($L_0^B = 0.00549$), we obtain $\bar{A}_0 = 0.0826$ and $\hat{A}_0 = 0.172$. With the data from Japan ($L_0^B = 0.00448$), $\bar{A}_0 = 0.0825$ and $\hat{A}_0 = 0.171$. These values are nearly the same as $\bar{A} = 0.0821$ and $\hat{A} = 0.171$, respectively, in the main model without basic research because L_0^B is small relative to L .

³⁹Strictly speaking, the average of A can be slightly lower than A_0 since (40) is a concave function of L^B .

To calibrate L_0^B/L , we use the data from the OECD Main Science and Technology Indicators. According to them, basic research expenditures as a percentage of GDP in the U.S. is 0.47% and that in Japan is 0.39% for the average of the 2000–2023 period.⁴⁰ In the Appendix A.7.1, we explain the relationship between the ratio of basic expenditure to GDP and the fraction of basic research workers in the labor force. Using this relationship, we calibrate $L_0^B = 0.00549$ for the United States and $L_0^B = 0.00448$ for Japan.

For parameter ϕ , we rely on previous studies that have estimated this elasticity by focusing on the magnitude of local spillovers from basic research activities to private R&D. As mentioned by Bloom, Van Reenen and Williams (2019), there appears to be a correlation between areas with strong science-based universities and private sector innovation (for example, Silicon Valley in California, Route 128 in Massachusetts, and the Research Triangle in North Carolina). This correlation suggests that there are significant spillovers from universities to corporate R&D. More specifically, Jaffe (1989) estimated the elasticity of corporate patent numbers with respect to the amount of university research in the same U.S. state and reported that the elasticity ranges between 0.04 and 0.28 across industries. Acs, Audretsch and Feldman (1992) estimated the same elasticity via an innovation index that includes data on inventions that were not patented but were ultimately introduced into the market. Their result shows that the elasticity is 0.241 when data from all areas are used. Given the variations in the estimates among these studies, we consider three values for ϕ : the upper bound, $\phi = 0.28$; an intermediate value, $\phi = 0.1$; and the lower bound, $\phi = 0.04$. The remaining parameters are the same as those in Section 4 (shown in Table 1).

Figure 10 presents the results from the simulations. We calculate the long-term growth rate when L^B/L is changed from L_0^B/L to the value on the horizontal axis and θ is changed to the value on the vertical axis. We perform the calculation for six combinations of A_0 and ϕ : $A_0 = 0.536$ ($A_{0,\text{USA}}$) and 0.152 ($A_{0,\text{JPN}}$) and $\phi = 0.28$ (the upper bound), 0.1 (an intermediate value), and 0.04 (the lower bound). The black square in each panel indicates the pair of L^B and θ that maximizes the long-term growth rate. The white area means that there is no steady state because the government debt is not sustainable given the pair of L^B and θ . The border between the colored area and the white area is downward-sloping. The reason is that the research subsidy (θ) and added basic research ($L^B - L_0^B$) are both financed by perpetually rolled-over government debts. When the government devotes more

However, given that the movement of L^B is moderate, we ignore this difference.

⁴⁰In our model, the basic research expenditure as a percentage of GDP is given by $\sigma_t^B = w_t L^B / \text{GDP}_t$, where GDP_t is slightly modified from (22) due to the introduction of L^B . See Appendix A.7.

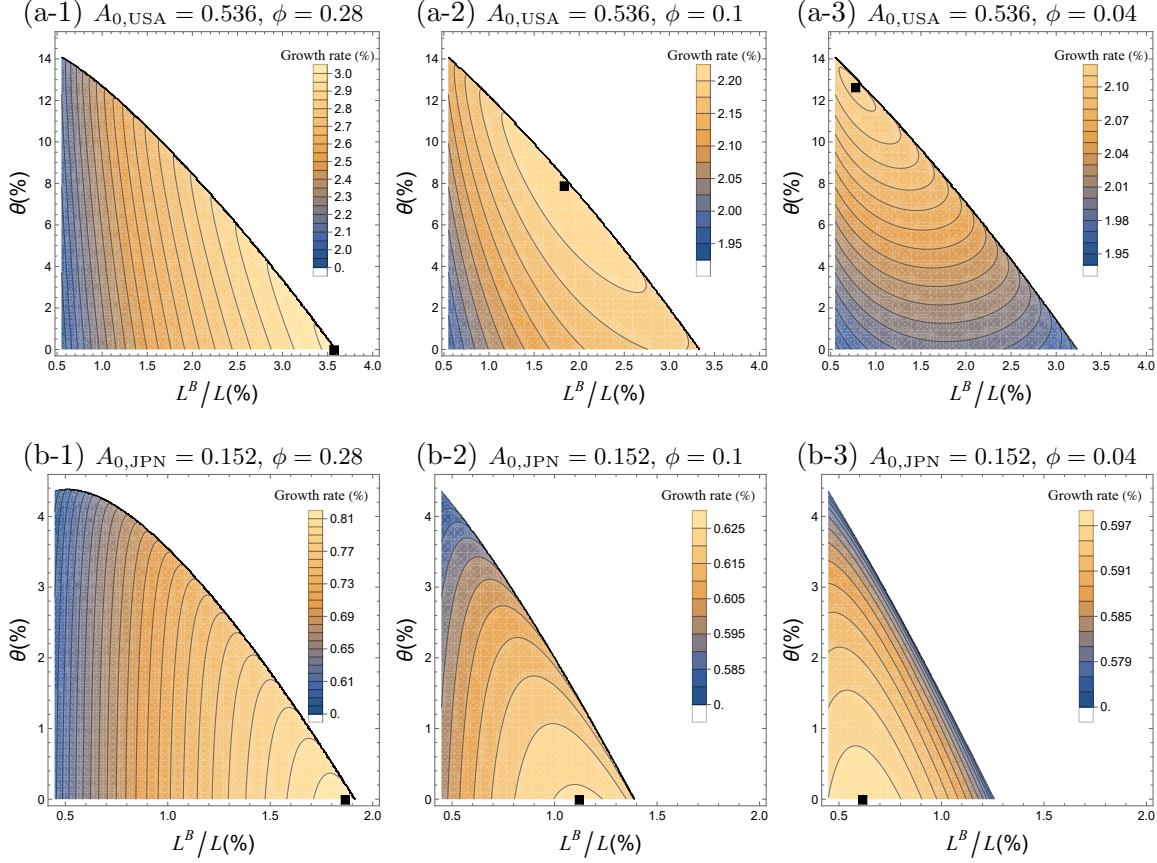


Figure 10: Long-Term Growth Effects of Basic Research, L^B , and the Research Subsidy, θ

resources to basic research, the maximum sustainable rate of research subsidies decreases.

The three panels in the upper row show the graphs for the baseline case of $A_0 = 0.536$ ($A_{0,USA}$). Unsurprisingly, when ϕ is larger, allocating more funds to basic research is desirable. Indeed, if $\phi = 0.28$ (the upper bound of its estimate), the growth-maximizing policy is to use all available funds to promote basic research rather than to subsidize private R&D. When ϕ is 0.1 (an intermediate value) and 0.04 (the lower bound), the growth-maximizing policy combines the research subsidy and the added basic research. Notably, added basic research is beneficial for long-term economic growth even when the elasticity ϕ is rather small. Note that $\phi = 0.04$ means that a doubling of basic research labor results in only a 2.8% increase in private R&D productivity, A .⁴¹ Even when basic research has only a modest effect on private R&D productivity, our numerical simulations show that expanding public-financed basic research should be a part of a growth-maximizing policy.

The three panels in the lower row in Figure 10 show the graphs for the low-growth economy with $A_{0,JPN} = 0.15 < \hat{A}$. Consistent with Proposition 2, research subsidies do

⁴¹This value is obtained by $A = A_0(2L_0^B/L_0^B)^\phi = 2^\phi A_0$. When $\phi = 0.04$, this equation results in $A = 1.028A_0$.

not enhance long-term growth in this case.⁴² Still, panels (b-1)-(b-3) show that adding basic research financed by perpetually rolled-over debts enhances growth even in these economies, and it is desirable to use funds more aggressively for basic research when ϕ is larger. This result highlights the importance of investing more resources into basic research, especially for economies where the initial rate of economic growth is low.

7 Conclusion

This paper has examined the effect of growth promotion policies in an R&D-based endogenous growth model, assuming that political constraints do not allow the government to increase its revenue and that all expenses must be financed by perpetually rolled-over debt. Such policies are sustainable only when the interest rate is lower than the growth rate. We find that $g > r$ is realized in the steady state if the R&D productivity, A , is high enough. In accordance with recent findings in the literature that $g > r$ is more historical than the exception, we focused on the case where the abovementioned condition is satisfied.

We find that subsidies of private R&D always enhance innovation activities and thus the growth in potential GDP in the short run. In addition, when A is higher than another higher threshold (i.e., when the $g - r$ gap is large enough), the policy can also increase the long-term growth rate. While our estimates of A suggest that this condition holds for most advanced countries, including the United States, we find that it does not hold for Japan and Italy. For those countries, our theory implies that the R&D subsidy initially increases the growth rate but lowers it in the long run because the effect of the increased government debt on the interest rate dominates the positive direct effect of the subsidy. Nevertheless, the initial positive response can dominate the negative effect for a long time; therefore, the desirability of such a policy depends on how we evaluate the utility of current and future generations. We confirm that the social return of R&D in equilibrium is always higher than the growth rate; therefore, enhancing R&D by subsidies is never a waste of resources.

In an extended model, where the productivity of private R&D depends on the amount of publicly funded basic research, we find that the government should allocate a portion of the funds obtained from issuing debts to enhance basic research. Notably, our theory suggests that expanding basic research will increase long-term growth even in low-growth economies such as Japan, where research subsidies for private R&D are not conducive to

⁴²When L^B is increased from L_0^B , A becomes higher than A_0 and may exceed \hat{A} even when $A_0 < \hat{A}$. Nevertheless, in numerical simulations, we find that research subsidies do not increase growth when $A_0 < \hat{A}$. We infer that this result occurs because L^B and θ are financed by the same means. When L^B is large, the debt-to-GDP ratio is already high even when $\theta = 0$, which makes increasing θ less attractive.

growth in the long run.

Nevertheless, it is worth remembering that growth-enhancing policies are not without costs. They reduce the upper bound in the debt-to-GDP ratio (or fiscal space), beyond which the economy cannot recover to a steady state. If the government tries to increase the subsidy rate or expense for basic research over certain values, the economy has no equilibrium path. Thus, such a policy cannot be implemented in the rational expectation equilibrium. The government will go bankrupt unless the tax rates are eventually raised.

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A Appendix

A.1 Derivation of the Consumption of Newborns

In this section, we derive the consumption of a newly born individual (7). As explained in the main text, each individual maximizes the expedited utility (3) subject to the budget constraint (4), initial condition $k_{t,t} = 0$, and the non-Ponzi game condition:

$$\lim_{T \rightarrow \infty} e^{-\int_t^T (r_\tau + \mu) d\tau} k_{s,T} \leq 0. \quad (\text{A.1})$$

From (4), (A.1), and the Euler equation for individuals, $\dot{c}_{s,t} = (r_t - \rho)c_{s,t}$, we obtain

$$c_{s,t} = (\rho + \mu)(k_{s,t} + h_{s,t}), \quad (\text{A.2})$$

where $h_{s,t}$ is the present value of future labor income, or “human wealth,” of an individual of generation s evaluated at time t . It is defined by

$$h_{s,t} \equiv \int_t^\infty e^{-\delta(t'-s)} w_{t'} e^{-\int_t^{t'} (r_\tau + \mu) d\tau} dt'. \quad (\text{A.3})$$

By averaging (A.2) for all individuals, we obtain the average consumption as follows:

$$\bar{c}_t = (\rho + \mu) (\bar{k}_t + \bar{h}_t), \quad (\text{A.4})$$

where \bar{h}_t is the average human wealth, defined by

$$\bar{h}_t \equiv \frac{1}{N} \int_{-\infty}^t N_{s,t} h_{s,t} ds. \quad (\text{A.5})$$

Note that, from (A.3), (A.5) and (2), \bar{h}_t and the human wealth of a newborn $h_{t,t}$ are related as follows:

$$\begin{aligned} \bar{h}_t &= \frac{1}{N} \int_{-\infty}^t N_{s,t} \int_t^\infty e^{-\delta(t'-s)} w_{t'} e^{-\int_t^{t'} (r_\tau + \mu) d\tau} dt' ds \\ &= \frac{1}{N} \int_{-\infty}^t N_{s,t} e^{-\delta(t-s)} \int_t^\infty e^{-\delta(t'-t)} w_{t'} e^{-\int_t^{t'} (r_\tau + \mu) d\tau} dt' ds \\ &= \frac{1}{N} L h_{t,t} = \frac{\mu}{\mu + \delta} h_{t,t}. \end{aligned} \quad (\text{A.6})$$

This relationship means that $h_{t,t} = ((\mu + \delta)/\mu) \bar{h}_t$; i.e., a newborn has $(\mu + \delta)/\mu > 1$ times the human wealth of an average individual. Equation (A.2) with $s = t$ and $k_{t,t} = 0$ implies

that the consumption of a newborn is

$$c_{t,t} = (\rho + \mu)h_{t,t} = \frac{(\rho + \mu)(\mu + \delta)\bar{h}_t}{\mu}. \quad (\text{A.7})$$

Eliminating \bar{h}_t from (A.4) by using (A.7) results in (7).

A.2 Stability of the Steady State When There is no Subsidy

Substituting $\theta = 0$ into (28) and (29) and linearizing the system around a steady state (S_1 or S_2) yields

$$\begin{bmatrix} \dot{D}_t \\ \dot{g}_t \end{bmatrix} = \mathbf{J} \begin{bmatrix} D_t - D^* \\ g_t - g^* \end{bmatrix},$$

where D^* and g^* are the values of D_t and g_t in the steady state, respectively, and where \mathbf{J} is the Jacobian matrix. \mathbf{J} is given by

$$\mathbf{J} \equiv \begin{bmatrix} -s(0)(g^* - \widehat{g}(0)) & -s(0)D^* \\ \bar{g}(\rho + \mu)(\delta + \mu) & s(0)(\widehat{g}(0) + \bar{g} - 2g^*) + \delta - \rho \end{bmatrix}, \quad (\text{A.8})$$

where $s(0) = 1/(1 - \alpha)$, $\widehat{g}(0) = (1 - \alpha)\bar{g}$, and $\bar{g} = ((1 - \alpha)/\alpha)A$. Since $D_1^* = 0$ in the steady state S_1 , the determinant and the trace of \mathbf{J} in S_1 are

$$\det \mathbf{J}|_{S_1} = -s(0)(g_1^* - \widehat{g}(0)) \{s(0)[(2 - \alpha)\bar{g} - 2g_1^*] + \delta - \rho\}, \quad (\text{A.9})$$

$$\text{tr } \mathbf{J}|_{S_1} = \delta - \rho + s(0)(2\widehat{g}(0) + \bar{g} - 3g_1^*). \quad (\text{A.10})$$

In the steady state S_2 , $D_2^* = \bar{D}$, $g_2^* = \widehat{g}(0)$; therefore,

$$\det \mathbf{J}|_{S_2} = \bar{g}(\rho + \mu)(\delta + \mu)\bar{D}, \quad (\text{A.11})$$

$$\text{tr } \mathbf{J}|_{S_2} = A + \delta - \rho > 0. \quad (\text{A.12})$$

In the following, we examine the stability of the steady states (S_1 and S_2) in the three cases discussed in the main text.

Case 1: $A > \bar{A}$. First, we show that S_1 is saddle stable. As explained in the main text, The $\dot{g}_t = 0$ parabola intersects the $g_t = \bar{g}$ line at $D_t = -\alpha/A < 0$. This result implies that the vertex of the parabola is in the $D_t < 0$ region, and given that S_1 exists, g_1^* is necessarily lower than the g -coordinate of the vertex of the parabola, $g_{\text{vertex}} = [(\delta - \rho)(1 - \alpha) + (2 - \alpha)\bar{g}]/2$. On the RHS of (A.9), the term $s(0)[(2 - \alpha)\bar{g} - 2g_1^*] + \delta - \rho$ is a decreasing function of g_1^* and becomes 0 when $g_1^* = g_{\text{vertex}}$. Since $g_1^* < g_{\text{vertex}}$, as shown

above, this term is positive. Additionally, the term $g_1^* - \widehat{g}(0)$ is positive since $g_1^* > \widehat{g}(0)$ in Case 1. Based on the above, $\det \mathbf{J}|_{S_1}$ in (A.9) is negative. Therefore, only one eigenvalue is negative; hence, S_1 is saddle stable.

Since $\overline{D} > 0$ in Case 1, (A.11) implies that $\det \mathbf{J}|_{S_2} > 0$. Additionally, since $\delta > \rho$, (A.12) implies that $\text{tr} \mathbf{J}|_{S_2} > 0$. Based on the above, S_2 has two positive eigenvalues and is hence totally unstable.

Case 2-1: $\tilde{A} < A < \overline{A}$. Similar to Case 1, $s(0)[(2-\alpha)\bar{g}-2g_1^*]+\delta-\rho > 0$ holds because $g_1^* < g_{\text{vertex}}$. However, since $g_1^* < \widehat{g}(0)$, $\det \mathbf{J}|_{S_1} > 0$ from (A.9). Additionally, substituting $g_1^* < \widehat{g}(0)$ into (A.10), which is a decreasing function of g_1^* , yields $\text{tr} \mathbf{J}|_{S_1} \geq \delta - \rho + \alpha s(0)\bar{g} > 0$. From $\det \mathbf{J}|_{S_1} > 0$ and $\text{tr} \mathbf{J}|_{S_1} > 0$, S_1 has two positive eigenvalues and is therefore totally unstable.

Since $\overline{D} < 0$ in Case 2, (A.11) implies that $\det \mathbf{J}|_{S_2} < 0$ in S_2 . Therefore, S_2 has only one negative eigenvalue and is hence saddle stable.

Case 2-2: $A < \tilde{A}$. Similar to Case 2-1, S_2 is saddle stable. There is no S_1 steady state in the $g_t > 0$ region. However, we show in the Appendix A.3 that there is an unstable steady state (S_1) when we consider the dynamics in the $g_t = 0$ region.

A.3 Dynamics in the $g_t = 0$ region

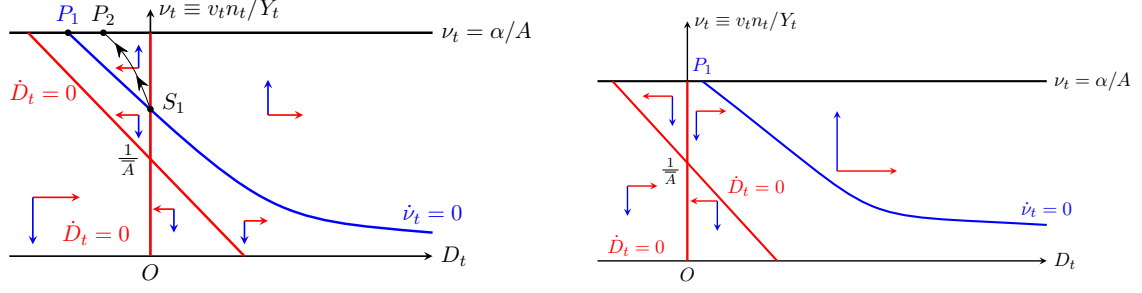
This section explains the dynamics of the economy when g_t becomes 0, which means that $\dot{n}_t = 0$, $L_t^R = 0$ and $L_t^P = L$ from (24) and (26). For simplicity, we focus on the case of $\theta = 0$. In the last part of this section, we explain that the analysis can be extended to the case of $\theta > 0$.

When $g_t = 0$, Y_t is constant from (23). Additionally, $L_t^P = L$ means that $X_t = Y_t$ is constant from (14) and (23). Since the final goods are used only for consumption, C_t is also constant. Let $\nu_t \equiv v_t n_t / Y_t$ represent the ratio of the total market value of firms to potential GDP. When $\dot{n}_t = 0$, the free entry condition (16) holds whenever $\nu_t \leq \alpha/A$. Substituting $\dot{C}_t = 0$ and $K_t = B_t + v_t n_t$ from the equilibrium of the asset market into the Euler equation (8) determines the interest rate:

$$r_t = (\delta - \rho)(\overline{A}(D_t + \nu_t) - 1). \quad (\text{A.13})$$

The dynamics of the economy in the $g_t = 0$ region can be examined by ν_t and D_t .⁴³ Since the government has no revenue or expense in this region, its debt B_t grows at the

⁴³Depending on whether the free entry condition (16) holds with equality or not, either g_t or ν_t can move, while the other is fixed.



(a) When $A < \tilde{A}$, S_1 exists in the $g_t = 0$ region. (b) When $A > \tilde{A}$, no steady state exists in the $g_t = 0$ region.

Figure A.1: Phase diagram in the $g_t = 0$ region

rate of r_t . Since Y_t is constant, $\dot{D}_t/D_t = \dot{B}_t/B_t = r_t$. Therefore,

$$\dot{D}_t = (\delta - \rho)(\bar{A}(D_t + \nu_t) - 1)D_t. \quad (\text{A.14})$$

Since Y_t and n_t are constant, the definition of $\nu_t \equiv n_t v_t / Y_t$ implies that $\dot{\nu}_t / \nu_t = \dot{v}_t / v_t$. From $L_t^P = L$, $\pi_t = (1 - \alpha)Y_t / n_t$.⁴⁴ Using these equations, the no-arbitrage condition (19) in the $g_t = 0$ region can be written as

$$\dot{\nu}_t = (\delta - \rho)(\bar{A}(D_t + \nu_t) - 1)\nu_t - (1 - \alpha). \quad (\text{A.15})$$

From (A.14) and (A.15), the phase diagram of the economy when $g_t = 0$ can be represented as

$$\text{The } \dot{D}_t = 0 \text{ locus: } D_t + \nu_t = \frac{1}{\bar{A}}, \text{ and} \quad (\text{A.16})$$

$$D_t = 0.$$

$$\text{The } \dot{\nu}_t = 0 \text{ locus: } D_t + \nu_t = \frac{1}{\bar{A}} + \frac{1 - \alpha}{(\rho + \mu)(\delta + \mu)} \frac{L}{\nu_t}. \quad (\text{A.17})$$

Recall that the free entry condition (16) can be written as $\nu_t \leq \alpha/A$. Therefore, the phase diagram in the D_t - ν_t plane is defined only for $\nu \in [0, \alpha/A]$. The phase diagram is shown in Figure A.1 for the case when the $\dot{D}_t = 0$ locus intersects with the ν_t axis ($A < \tilde{A}$) and when it does not ($A > \tilde{A}$). From (A.17), the D_t coordinate of the $\dot{\nu}_t = 0$ locus at $\nu_t = \alpha/A$ (denoted as point P_1 in Figure A.1) is

$$\frac{1 - \alpha}{(\rho + \mu)(\delta + \mu)} \frac{A}{\alpha} - \frac{\alpha}{A} + \frac{1}{\bar{A}}, \quad (\text{A.18})$$

⁴⁴See footnote 13.

which is positive if and only if $A > \tilde{A}$. We can confirm that (A.18) coincides with the D_t -intercept of the $\dot{g}_t = 0$ locus in the phase diagram of the $g_t > 0$ region, as shown in Figure 4(b) and 4(c).

We now consider the steady state. According to (A.17), the $\dot{\nu}_t = 0$ locus slopes downward, and it is always above the sloping part of the $\dot{D}_t = 0$ locus (A.16). Therefore, the $\dot{\nu}_t$ locus intersects with the vertical portion of the $\dot{D}_t = 0$ locus (at $D_t = 0$) given that $A < \tilde{A}$, i.e., when point P_1 is to the left of the vertical axis, as shown in Figure A.1(a). In this case, we denote the steady state by S_1 and let $\nu^* > 1/\bar{A}$ represent its ν_t value.⁴⁵ If $A > \tilde{A}$, then there is no steady state, as shown in Figure A.1(b).

Next, we show that S_1 in the $g_t = 0$ region, if it exists, is unstable. Linearizing (A.14) and (A.15) around $(0, \nu^*)$ yields

$$\begin{bmatrix} \dot{D}_t \\ \dot{\nu}_t \end{bmatrix} = \mathbf{J}_0 \begin{bmatrix} D_t \\ \nu_t - \nu^* \end{bmatrix}, \text{ where} \\ \mathbf{J}_0 \equiv \begin{bmatrix} L(\rho + \mu)(\delta + \mu)\nu^* - (\delta - \rho) & 0 \\ L(\rho + \mu)(\delta + \mu)\nu^* & 2L(\rho + \mu)(\delta + \mu)\nu^* - (\delta - \rho) \end{bmatrix}. \quad (\text{A.19})$$

Its determinant is

$$\det \mathbf{J}_0 = 2(\delta - \rho)^2 \bar{A}^2 \left(\nu^* - \frac{1}{\bar{A}} \right) \left(\nu^* - \frac{1}{2\bar{A}} \right), \quad (\text{A.20})$$

which is positive since $\nu^* > 1/\bar{A}$. The trace of \mathbf{J}_0 is positive:

$$\text{tr } \mathbf{J}_0 = 3L(\rho + \mu)(\delta + \mu)\nu^* - 2(\delta - \rho) > \delta - \rho > 0, \quad (\text{A.21})$$

where the first inequality comes from $\nu^* > 1/\bar{A}$ and (32). From (A.20) and (A.21), we can conclude that both eigenvalues are positive.

Finally, we explain that there is a saddle path that originates from S_1 and connects to the saddle-stable steady state S_2 in the $g_t \geq 0$ region, as depicted in Figure 4(c). As shown in Figure 4(c), there is a saddle path that originates from a point on the horizontal axis (i.e., $g_t = 0$). This point is located between the origin and the intercept of the $\dot{g}_t = 0$ locus (point P_1); we call it point P_2 . Note that point P_2 also belongs to Figure A.1(a) because it satisfies both $g_t = 0$ and $\nu_t = \alpha/A$.⁴⁶ Since the steady state S_1 is completely unstable,

⁴⁵We can confirm $\nu^* > 1/\bar{A}$ by substituting $D_t = 0$ in (A.17).

⁴⁶When $g_t > 0$, the free entry condition for R&D (16) needs to hold with equality. This condition is equivalent to $\nu_t \equiv v_t n_t / Y_t = \alpha/A$.

there exists a point in the neighborhood of S_1 from which the path leads to P_2 . This path is the saddle path of the economy. Once the economy reaches P_2 in Figure A.1(a), it experiences a phase transition to Figure 4(c). If the economy with $A < \tilde{A}$ starts from a small negative value of D_t , g_t remains at 0 for some time and then starts to increase until it converges to $g_t = \hat{g}(0)$.

The case of $\theta > 0$: Thus far, we have focused on the case of $\theta = 0$. Even when θ is positive, the phase diagram within the $g_t = 0$ region is nearly the same as that in the case of $\theta = 0$ because $g_t = 0$ means that no firms undertake R&D; therefore, the government expenditure is zero. The only difference is the border between the phase diagrams. With $\theta > 0$, the $\nu_t = (1 - \theta)\alpha/A$ line in the diagram of the $g_t = 0$ region will be connected to the $g_t = 0$ line in the phase diagram of the $g_t \geq 0$ region. Intuitively, with $\theta > 0$, firms are more eager to invest in R&D even when the index of firm values, ν_t , is not as high as in the case of $\theta = 0$. Therefore, the economy transitions from the $g_t = 0$ region to the $g_t \geq 0$ region with a smaller ν_t .

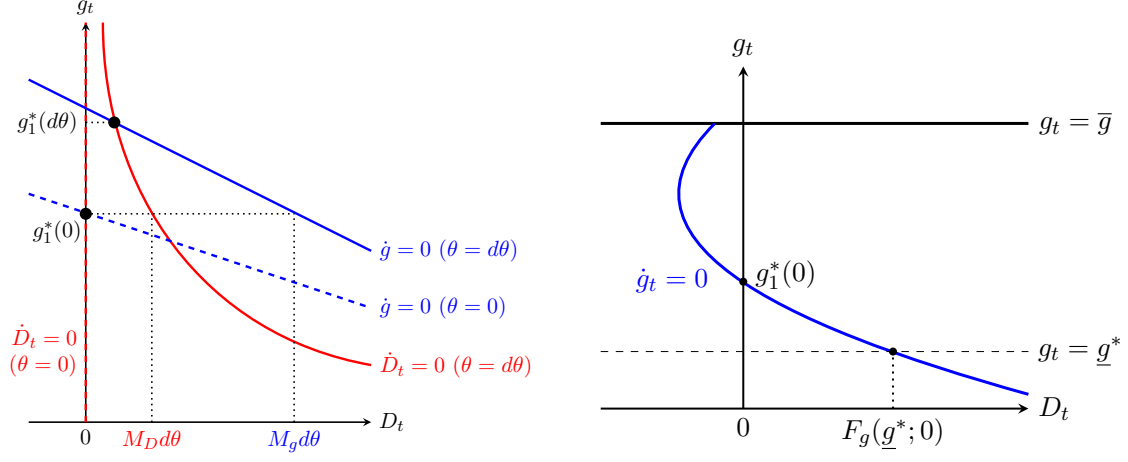
A.4 Proof of Proposition 2 and Corollary 1

A.4.1 Proposition 2

We assume that the economy is in a saddle-stable steady state S_1 before the increase in the subsidy. As illustrated as S_1 in Figure 4(a), this steady state is given by an intersection between the $\dot{g}_t = 0$ locus and the $\dot{D}_t = 0$ locus.

We consider the situation where the rate of the R&D subsidy, θ , is increased marginally by $d\theta$ from 0. Then, both the $\dot{g}_t = 0$ locus and the $\dot{D}_t = 0$ locus shift to the right. The location of the steady state always moves to the right, which means that D_t in the new steady state is higher than before. Whether g_t in the new steady state is higher or lower depends on the relative magnitude of the size of the shifts of the two loci. The slope of the \dot{D}_0 locus at S_1 is steeper than the slope of the \dot{g}_0 locus. Then, as illustrated in Panel (a) of Figure A.2, g_t in the new steady state should be higher than before if the size of the shift of the $\dot{g} = 0$ locus (denoted by $M_g d\theta$), measured at steady state S_1 (located at $D_t = 0, g_t = g_1^*(0)$), is larger than the size of the shift of the $\dot{D} = 0$ locus (denoted by $M_D d\theta$).

Let $F_g(g_t; \theta)$ be the RHS of (34), and let $F_D(g_t; \theta)$ be the RHS of (35). $D_t = F_g(g_t; \theta)$ and $D_t = F_D(g_t; \theta)$ represent the $\dot{g} = 0$ locus and the $\dot{D} = 0$ locus, respectively. Then, M_g



(a) The shifts in the $\dot{g}_t = 0$ and $\dot{D}_t = 0$ loci and the movement of the steady state. (b) Condition $g_1^* > \underline{g}$ holds if and only if $F_g(\underline{g}^*; 0) > 0$.

Figure A.2: Proof of Proposition 2

and M_D are calculated as

$$M_g \equiv \frac{\partial F_g(g_1^*(0); 0)}{\partial \theta} = \frac{\alpha}{A} + \frac{(g_1^*(0) - \bar{g})^2}{\bar{g}(\rho + \mu)(\delta + \mu)}, \quad (\text{A.22})$$

$$M_D \equiv \frac{\partial F_D(g_1^*(0); 0)}{\partial \theta} = \frac{\alpha^2 g_1^*(0)}{A} \frac{1}{g_1^*(0) - \widehat{g}(0)}. \quad (\text{A.23})$$

Equations (A.22) and (A.23), together with $\widehat{g}(0) < g_1^* < \bar{g}$, indicate that $M_g - M_D > 0$ holds if and only if

$$(\bar{g} - g_1^*)(g_1^* - \widehat{g}(0)) > (1 - \alpha)^2(\rho + \mu)(\delta + \mu). \quad (\text{A.24})$$

The fact that the steady state before the shift ($D_t = 0, g_t = g_1^*(0)$) is on the $\dot{g}_t = 0$ locus means that $F_g(g_1^*(0); 0) = 0$. This equation can be rearranged to

$$(1 - \alpha)^2(\rho + \mu)(\delta + \mu) = (\bar{g} - g_1^*(0))((1 - \alpha)(\delta - \rho) + \widehat{g}(0) - g_1^*(0)). \quad (\text{A.25})$$

Substituting (A.25) into the RHS of (A.24) and rearranging yield

$$g_1^*(0) > \frac{(1 - \alpha)(\delta - \rho)}{2} + \widehat{g}(0) \equiv \underline{g}^*. \quad (\text{A.26})$$

This inequality shows that when θ is slightly increased from 0, the growth rate in the saddle-stable steady state increases if and only if $g_1^*(0) > \underline{g}^*$.

Recall that for a given value of g_t , $F_g(g_t; 0)$ gives the value of D_t on the $\dot{g}_t = 0$ locus with $\theta = 0$. Therefore, $F_g(\underline{g}^*; 0)$ gives the value of D_t where the $\dot{g}_t = 0$ locus reaches $g_t = \underline{g}^*$, as

shown in Figure A.2. Note that the condition $g_1^*(0) > \underline{g}^*$ indicates the intersection of the $\dot{g}_t = 0$ locus with $\theta = 0$ and the vertical axis ($D_t = 0$) is above the $g_t = \underline{g}^*$ line. Because the $\dot{g}_t = 0$ locus slopes downward in the $D_t > 0$ region, this condition holds if and only if $F_g(\underline{g}^*; 0) > 0$ (see Figure A.2). When we solve this condition for A , we can confirm that it is equivalent to (36).

A.4.2 Corollary 1

In this proof, we show that condition (A.26) holds if and only if (37) is satisfied. Note that the relationship between $g_t - r_t$ and g_t in (27) always holds in equilibrium; therefore, it also holds in the saddle-stable steady state with $\theta = 0$:

$$g_1^*(0) - r_1^*(0) = s(0)(g_1^*(0) - \widehat{g}(0)). \quad (\text{A.27})$$

Since $s(0) = 1/(1 - \alpha) > 0$,

$$g_1^*(0) > \underline{g}^* \Leftrightarrow s(0)(g_1^*(0) - \widehat{g}(0)) > s(0)(\underline{g}^* - \widehat{g}(0)) = \frac{\delta - \rho}{2}. \quad (\text{A.28})$$

Combining (A.27) and (A.28), we have

$$g_1^*(0) > \underline{g}^* \Leftrightarrow g_1^*(0) - r_1^*(0) > \frac{\delta - \rho}{2}.$$

A.5 Proof of Proposition 3

Here, we derive the social rate of return of R&D. As explained in Definition 1, we discretize time in the model with a step size of Δt . We later take the limit of $\Delta t \rightarrow 0$. We assume that the economy is on the BGP before time t . The state variable n_t at time t is also on the BGP because it is predetermined by time t . Then, we marginally increase L_t^R by $dL_t^R > 0$, which reduces the production of consumption goods at time t (denoted by $dX_t < 0$) but increases the number of goods at $t + \Delta t$ (denoted by $dn_{t+\Delta t} > 0$). Then, at $t + \Delta t$, we marginally decrease $L_{t+\Delta t}^R$ by $dL_{t+\Delta t}^R < 0$ so that $n_{t+2\Delta t}$ returns to the original BGP. The production of consumption goods at $t + \Delta t$ will be higher than that on the BGP. We denote the difference by $dX_{t+\Delta t} > 0$.

In the following, we derive the relationships among $dn_{t+\Delta t}$, dL_t^R , and $dL_{t+\Delta t}^R$. The discrete version of (15) at the time between t and $t + \Delta t$ is

$$n_{t+\Delta t} = n_t + aL_t^R n_t \Delta t, \quad (\text{A.29})$$

where $a = A/L$. Taking total derivative of Equation (A.29) while keeping n_t constant, yields

$$dn_{t+\Delta t} = an_t \Delta t dL_t^R. \quad (\text{A.30})$$

Similar to (A.29), the discrete version of (15) at from time $t + \Delta t$ to time $t + 2\Delta t$ is

$$n_{t+2\Delta t} = n_{t+\Delta t} + aL_{t+\Delta t}^R n_{t+\Delta t} \Delta t. \quad (\text{A.31})$$

By taking total derivative of (A.31), while keeping $n_{t+2\Delta t}$ unchanged from the BGP, we obtain

$$0 = (1 + aL_{t+\Delta t}^R \Delta t) dn_{t+\Delta t} + an_{t+\Delta t} \Delta t dL_{t+\Delta t}^R. \quad (\text{A.32})$$

We substitute $n_{t+\Delta t}$ in (A.29) and $dn_{t+\Delta t}$ in (A.30) into (A.32), which yields

$$0 = (1 + aL_{t+\Delta t}^R \Delta t) dL_t^R + (1 + aL_t^R \Delta t) dL_{t+\Delta t}^R. \quad (\text{A.33})$$

Recall that the economy is on the BGP before the manipulation of L_t^R , where L_t^R is constant for all t . Let this constant value be L^{R*} . Then, $L_t^R = L^{R*} + dL_t^R$ and $L_{t+\Delta t}^R = L^{R*} + dL_{t+\Delta t}^R$. Substituting these values into (A.33) gives

$$0 = (1 + aL^{R*} \Delta t)(dL_t^R + dL_{t+\Delta t}^R) + 2a\Delta t dL_t^R dL_{t+\Delta t}^R. \quad (\text{A.34})$$

This equation implies that the ratio of the changes in L_t^R and $L_{t+\Delta t}^R$ is

$$-\frac{dL_{t+\Delta t}^R}{dL_t^R} = 1 + \frac{2a\Delta t dL_{t+\Delta t}^R}{1 + aL^{R*} \Delta t}. \quad (\text{A.35})$$

Now, we consider the changes in the final output X_t in periods t and $t + \Delta t$. Note that in period t , n_t is still on the BGP. Therefore, from (14), the changes in X_t and L_t^R are related by

$$dX_t = \frac{\partial X_t}{\partial L_t^R} dL_t^R = n_t^{\frac{1-\alpha}{\alpha}} (-dL_t^R). \quad (\text{A.36})$$

In period $t + \Delta t$, $n_{t+\Delta t}$ deviates from the BGP by $dn_{t+\Delta t}$. Therefore, the change in $X_{t+\Delta t}$

can be expressed as

$$\begin{aligned}
dX_{t+\Delta t} &= \frac{\partial X_{t+\Delta t}}{\partial n_{t+\Delta t}} dn_{t+\Delta t} + \frac{\partial X_{t+\Delta t}}{\partial L_{t+\Delta t}^R} dL_{t+\Delta t}^R \\
&= \frac{1-\alpha}{\alpha} n_{t+\Delta t}^{\frac{1-\alpha}{\alpha}-1} (L - L_{t+\Delta t}^R) dn_{t+\Delta t} - n_{t+\Delta t}^{\frac{1-\alpha}{\alpha}} dL_{t+\Delta t}^R \\
&= n_{t+\Delta t}^{\frac{1-\alpha}{\alpha}} \left[\frac{1-\alpha}{\alpha} (L - L_{t+\Delta t}^R) \frac{a\Delta t dL_t^R}{1 + aL_t^R \Delta t} - dL_{t+\Delta t}^R \right].
\end{aligned} \tag{A.37}$$

where we use (A.29) and (A.30) in the third line.

We obtain the social return of R&D investment by substituting (A.36) and (A.37) into Definition 1:⁴⁷

$$\begin{aligned}
r^s \Delta t &= \frac{dX_{t+\Delta t}}{-dX_t} - 1 \\
&= \left(\frac{n_{t+\Delta t}}{n_t} \right)^{\frac{1-\alpha}{\alpha}} \left(\frac{1-\alpha}{\alpha} (L - L_{t+\Delta t}^R) \frac{a\Delta t}{1 + aL_t^R \Delta t} - \frac{dL_{t+\Delta t}^R}{dL_t^R} \right) - 1.
\end{aligned} \tag{A.38}$$

In the RHS of the above equation, we can use the Taylor expansion as follows:

$$\left(\frac{n_{t+\Delta t}}{n_t} \right)^{\frac{1-\alpha}{\alpha}} = (1 + aL_t^R \Delta t)^{\frac{1-\alpha}{\alpha}} = 1 + \frac{1-\alpha}{\alpha} aL_t^R \Delta t + o((\Delta t)^2),$$

where $o((\Delta t)^2)$ is the collection of terms that are of order $(\Delta t)^2$ and higher. We can also eliminate $dL_{t+\Delta t}^R/dL_t^R$ via (A.35). Then, (A.38) becomes

$$r^s = \frac{1-\alpha}{\alpha} aL_t^R + \frac{\frac{1-\alpha}{\alpha} a(L - L_{t+\Delta t}^R) + 2a dL_{t+\Delta t}^R}{1 + aL_t^R \Delta t} + o(\Delta t). \tag{A.39}$$

Recall that Δt is infinitesimally small. In the continuous-time limit, $\Delta t \rightarrow 0$. Additionally, we consider marginal perturbations of dL_t^R and $dL_{t+\Delta t}^R$ from the BGP. Therefore, $dL_{t+\Delta t}^R \rightarrow 0$. Applying these to (A.39) and $aL = A$, we obtain

$$r^s = \frac{1-\alpha}{\alpha} A = \bar{g}. \tag{A.40}$$

Since $g_t \leq \bar{g}$ from (24) and $L_t^R \leq L$, we have $r^s = \bar{g} \geq g_t$.

A.6 Proof of Proposition 4

Let g^* be the growth rate in aggregate consumption on the original BGP, and let L^{R*} and L^{P*} be the constant amounts of R&D labor and production labor, respectively. Suppose

⁴⁷In Definition 1, r^s is defined as the limit of the first line of (A.38) as $\Delta t \rightarrow 0$. We calculate the limit after we rewrite (A.38) as (A.39).

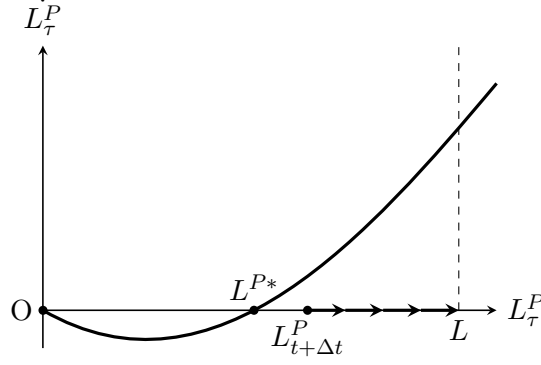


Figure A.3: Movement of L_t^P after $\tau = t + \Delta t$

that from this BGP, the path of aggregate consumption, $C_t = X_t$, is increased by dX_t from time t to time $t + \Delta t$, where Δt is a small time interval. In the following, we show that the aggregate consumption from $t + \Delta t$ cannot be maintained on the original BGP.

As explained in the Proof of Proposition 3 in the Appendix A.5, the only way to increase aggregate consumption at time t is to reduce R&D labor and increase production labor. Then, at $t + \Delta t$, $n_{t+\Delta t}$ should be lower than the BGP value, which means that the production technology of the final goods is inferior to that of the original BGP case. To keep $X_{t+\Delta t}$ at the original BGP value despite a smaller $n_{t+\Delta t}$, we need to allocate more workers to production, which means that $L_{t+\Delta t}^P > L^{P*}$.

Below, we examine the dynamics of the economy from $t + \Delta t$ in continuous time. For $\tau \geq t + \Delta t$, the aggregate consumption should be the same as that of the original BGP, which is growing at the rate of g^* . By taking the log of (14) and then differentiating with respect to τ , we obtain

$$\frac{\dot{X}_\tau}{X_\tau} = \frac{1 - \alpha}{\alpha} \frac{\dot{n}_\tau}{n_\tau} + \frac{\dot{L}_\tau^P}{L_\tau^P} = g^* \text{ for all } \tau \geq t + \Delta t.$$

Using (15), (24) and (26), the above equation can be rewritten as

$$\begin{aligned} \frac{\dot{L}_\tau^P}{L_\tau^P} &= g^* - \frac{1 - \alpha}{\alpha} A \frac{L_\tau^R}{L} \\ &= \frac{1 - \alpha}{\alpha} A \frac{L^{R*} - L_\tau^R}{L} \\ &= \frac{1 - \alpha}{\alpha} A \frac{L_\tau^P - L^{P*}}{L}. \end{aligned}$$

Therefore, we have the following autonomous differential equation for L_τ^P for $\tau \geq t + \Delta t$:

$$\dot{L}_\tau^P = \frac{1-\alpha}{\alpha} A \frac{L_\tau^P - L^{P*}}{L} L_\tau^P. \quad (\text{A.41})$$

As shown in Figure A.3, the RHS of (A.41) is positive when $L_\tau^P > L^{P*}$. Recall that $L_{t+\Delta t}^P > L^{P*}$. Therefore, $\dot{L}_\tau^P > 0$ for all $\tau \geq t + \Delta t$, and L_τ^P exceeds the total labor supply L in a finite period. Therefore, such a path is not feasible.

A.7 Analysis of the Extended Model

In this section, we analyze the extended model presented in Section 6. Note that the labor supply is fixed at L ; therefore, it is not affected by the labor tax. Additionally, the labor tax does not affect the individual's Euler equation. Therefore, the equation for the dynamics of aggregate consumption (8) is the same as that in the main model. As explained in the main text, the government pays wages to basic research workers, the sum of which is $w_t L^B$. Additionally, there is an existing labor tax at the rate of L_0^B/L , the revenue of which is $w_t L_0^B$. Therefore, the dynamics of the government debt (17) change to

$$\dot{B}_t = r_t B_t + \theta w_t L_t^R + w_t (L^B - L_0^B). \quad (\text{A.42})$$

The labor demand now consists of that for production, L_t^P ; for private R&D, L_t^R ; and for basic research, L^B . Therefore, the equilibrium condition of the labor market changes from (26) to

$$L_t^P + L_t^R + L^B = L. \quad (\text{A.43})$$

In this setting, the $g - r$ gap is written as

$$g_t - r_t = s(\theta)(g_t - \hat{g}(\theta, L^B)), \text{ where} \quad (\text{A.44})$$

$$s(\theta) = \frac{1-\alpha\theta}{(1-\alpha)(1-\theta)} > 1, \quad \hat{g}(\theta, L^B) = \frac{1-\alpha}{1-\alpha\theta} \bar{g} \frac{L - L^B}{L} \in (0, \bar{g}].$$

Note that the threshold level of growth, $\hat{g}(\theta, L^B)$, now depends on the pair of policy parameters θ and L^B . From (A.42) and (A.44), the dynamics for the debt-to-GDP ratio change from (28) to

$$\dot{D}_t = -s(\theta) (g_t - \hat{g}(\theta, L^B)) D_t + \frac{\theta\alpha}{\bar{g}} g_t + \alpha \frac{L^B - L_0^B}{L}. \quad (\text{A.45})$$

The last term of (A.45) comes from the expenses used for the added basic research. The

time evolution of g_t is

$$\dot{g}_t = \left(\bar{g} \frac{L - L^B}{L} - g_t \right) (s(\theta) (g_t - \hat{g}(\theta, L^B)) - \delta + \rho) + \bar{g}(\rho + \mu)(\delta + \mu) \left(D_t + \frac{\alpha(1 - \theta)}{A} \right). \quad (\text{A.46})$$

Note that, from (A.43), the inclusion of basic research means that less labor can be allocated to production and R&D. Therefore, (A.46) differs from (29) in that the first term contains the $(L - L^B)/L$ term. We performed numerical simulations via (A.45) and (A.46). The results are presented in the main text.

Given that the equations for the dynamics are changed as above, the threshold values for A in Propositions 1 and 2 are also affected. Since A is given by function (40), the thresholds are now defined in terms of the parameter A_0 . Specifically, the economy has a stable steady state with $g^* > r^*$ when $\theta = 0$ and $L^B = L_0^B$ if and only if A_0 is larger than

$$\bar{A}_0 = \frac{(\rho + \mu)(\delta + \mu)}{(\delta - \rho)(1 - L_0^B/L)}. \quad (\text{A.47})$$

Additionally, a marginal increase in θ from 0 increases the long-term growth rate if and only if A_0 is higher than

$$\hat{A}_0 = 2\bar{A}_0 + \frac{\delta - \rho}{2(1 - L_0^B/L)}. \quad (\text{A.48})$$

Given that L_0^B is small relative to L , as in our calibration presented in Table 2, \bar{A}_0 and \hat{A}_0 are close to \bar{A} and \hat{A} in the main model.

A.7.1 Calibration of L_0^B/L in the Extended Model

The GDP of this economy is defined as the sum of consumption expenditures $C_t = X_t$, private investment expenditures for R&D, $(1 - \theta)w_t L_t^R$, and government expenditures, $\theta w_t L_t^R + w_t L^B$. Since (14) and (20) are unchanged, GDP becomes

$$\text{GDP}_t = n_t^{\frac{1-\alpha}{\alpha}} (L_t^P + \alpha L_t^R + \alpha L^B).$$

Therefore, the ratio of basic research expenditure to GDP is given by

$$\frac{w_t L^B}{\text{GDP}_t} = \frac{\alpha L^B}{L_t^P + \alpha L_t^R + \alpha L^B} \equiv \sigma_t^B. \quad (\text{A.49})$$

Eliminating L_t^P from (A.49) via (A.43) and then solving for L^B/L yields

$$\frac{L^B}{L} = \frac{\sigma_t^B}{\alpha + (1 - \alpha)\sigma_t^B} \left(1 - (1 - \alpha)\frac{L_t^R}{L} \right).$$

Using (24), the above equation becomes

$$\frac{L^B}{L} = \frac{\sigma_t^B}{\alpha + (1 - \alpha)\sigma_t^B} \left(1 - \frac{\alpha}{A_0} g_t \right). \quad (\text{A.50})$$

For σ_t^B , we use the average value of the basic research expenditures as a percentage of GDP in the 2000–2023 period from the OECD Main Science and Technology Indicators. Accordingly, for g_t , we use the average value of the real GDP growth rate in the same period from the World Bank World Development Indicators. Then, we obtain the estimate of the average value of L^B/L in the 2000–2023 period, which we treat as L_0^B/L .