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INTERNATIONAL KNOWLEDGE DIFFUSION AND PRODUCTIVITY GROWTH IN A CASH-IN-ADVANCE ECONOMY

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Abstract

This paper investigates how the cash-in-advance (CIA) constraints that firms face in production and innovation decisions affect the long-run relationship between monetary policy and innovation-based economic growth. Firms produce differentiated product varieties and invest in process innovation to reduce production costs. With imperfect knowledge diffusion across countries, the country with the greater share of industry has relatively productive firms. We find that when innovation has a stricter CIA requirement than production, an increase in the nominal interest rate in the country with the larger (smaller) share of industry reduces the industrial share of that country, thereby decreasing (increasing) the rate of productivity growth. We also examine the implications of improvements in knowledge diffusion for the optimal nominal interest rate policy of each country.

JEL Classifications: F43; O30; O40
Keywords: Cash-in-advance constraints, International knowledge diffusion, Nominal interest rate policy, Endogenous productivity growth

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1 Introduction

How does monetary policy influence industry location patterns and what are the implications for economic growth? In this paper, we introduce an endogenous growth and endogenous market structure framework (Smulders and van de Klundert, 1995; Peretto, 1996; Etro, 2009) to address this question in an open economy setting. Firms produce differentiated goods for supply to domestic and export markets, while investing in process innovation to reduce future production costs. Both production and innovation activities are tied to monetary policy through cash-in-advance (CIA) constraints that require firms to cover a portion of costs using money procured through short-term loans (Chu and Cozzi, 2014). In addition, the limited nature of international knowledge diffusion links productivity growth with the geographic location of industry (Baldwin et al., 2004). Together the CIA constraints, imperfect knowledge diffusion, and industry location patterns determine the relationship between nominal interest rates, firm-level investment in R&D, and long-run productivity growth.

Recognizing that R&D activity faces strict liquidity requirements, recent work has investigated the effects of monetary policy on economic growth in frameworks that place CIA constraints on the R&D investments of firms (Chu and Cozzi, 2014; Chu et al., 2015; Chu, 2022). A key conclusion of these studies is that the nominal interest rate, and thus the inflation rate, has a negative effect on innovation-based economic growth. Furukawa et al. (2021) examine the relationship between inflation and economic growth in a framework where the costs associated with both market entry and market survival are subject to CIA constraints, and demonstrate that an increase in the inflation rate lowers the rate of market entry. These results are confirmed through an empirically analysis, and suggest that nominal interest rate differentials may affect long-run industry location patterns, as countries with lower inflation rates experience higher rates of market entry.

A fundamental feature of innovation-based economic growth is the essential role of technology diffusion both between firms and across regions and countries. In endogenous growth frameworks, knowledge is created as a byproduct of current innovation efforts, lowering the cost of future R&D investment, given the non-rival nature of knowledge (Romer, 1990). There is broad empirical evidence supporting the existence of knowledge spillovers, but the strength of these spillovers is also recognized to diminish significantly with distance (Jaffe et al., 1993; Mancusi, 2008; and Coe et al., 2009). Thus, the empirical evidence suggests a complex role for industry location patterns in the relationship between monetary policy and economic growth, with nominal interest rates potentially influencing the location of industry and subsequently affecting R&D investment through adjustments in the strength of knowledge diffusion across firms.

In this paper, we study how monetary policy influences aggregate product variety and long-run productivity growth through adjustments in industry location patterns in a two-country model. Imperfect knowledge diffusion links labor productivity in R&D with the geography of industry, ensuring that the country with the larger share of industry has relatively more productive firms. Knowledge spillovers are stronger and labor productivity in R&D is higher when industry is relatively concentrated in one of the countries. In addition, the endogenous market structure of the framework generates a tension between the number of firms in the market and firm-level of employment in process innovation. As a result, long-run equilibria with asymmetric industry location patterns tend to exhibit smaller numbers of firms with larger market shares and faster rates of productivity growth, relative to long-run equilibria with symmetric location patterns.

Firm-level investment in process innovation is connected with the monetary policies of each country through CIA constraints that require firms to obtain short-term loans to cover a portion of the costs incurred by firms in production and innovation, with different money requirements for each activity. Assuming free market entry and exit, the relative CIA constraints determine the ratio of employment in production to innovation at the firm level. Increases in nominal interest rates then raise the cost of employing labor. Accordingly, when production has a stricter CIA constraint, an increase in a country's nominal interest rate raises the country's share of industry and its relative productivity. In contrast, when innovation has a stricter CIA constraint, a rise in the nominal interest rate lowers the country's share of industry and its relative productivity.

Turning to the relationship between monetary policy and economic growth, we find that when firms face stricter CIA constraints in innovation than in production, a decrease in the nominal interest rate in the country with the larger share of industry share of relatively productive firms increases the concentration of industry, pushing the market away from the symmetric equilibrium. Consequently, knowledge spillovers are strengthened, decreasing the number of firms in the market and increasing the rate of productivity growth. Alternatively, a fall in the nominal interest rate of the country with the smaller share of relatively less productive firms decreases the concentration of industry as the market moves towards the symmetric equilibrium. In this case, knowledge spillovers are weakened and the number of firms in the market increases as the rate of productivity growth falls.

Lastly, we introduce a simple numerical analysis to consider the implications of our framework for optimal monetary policy. Assuming zero lower bounds for nominal interest rates, we calculate the optimal nominal interest rates associated with non-cooperative Nash equilibria derived from the countries policy reaction functions. Supposing that innovation has a stricter CIA constraint than production, our numerical example suggests that while it is optimal for the country with the larger share of industry to set a positive nominal interest rate, the country with the smaller share of industry sets its nominal interest rate to zero. The optimal nominal interest rate of the larger country converges to zero, however, with an improvement in international knowledge diffusion.

The remainder of the paper is organized as follows. Section 2 introduces a two-country model of international trade with CIA constraints. In Section 3, we characterize the long-run equilibrium and derive the stability conditions required for convergence to a balanced growth path. In Section 4, we study how nominal interest rates influence product variety and productivity growth through the effects of adjustments in CIA constraints on production and innovation activity. In Section 5 we consider the implications for welfare. The paper concludes in Section 6.

2 Model

In this section, we introduce a two-country model of endogenous productivity growth and endogenous market structure with cash-in-advanced (CIA) constraints. The two countries are labeled home (h) and foreign (f), and in each country monopolistically competitive firms produce differentiated product varieties for consumption by households. In addition, firms invest in process innovation to improve their technologies and lower future production costs. The CIA constraints require firms to secure money to cover a portion of the costs of employing labor in production and innovation. Country j is endowed with a population L_j of households that supply labor inelastically and lend money to firms, where we adopt the subscripts $j, k \in \{h, f\}$ with $j \neq k$ to indicate the variables associated with home and foreign.

2.1 Households

Preferences are symmetric across countries, with dynastic households that maximize lifetime utility over an infinite time horizon. Time flows continuously, and the lifetime utility of a single household in country j is

$$U_j = \int_0^\infty e^{-\rho t} \ln c_j(t) dt, \tag{1}$$

where $c_j(t)$ is consumption at time t and $\rho > 0$ is the subjective intertemporal discount rate. The household's flow budget constraint is expressed in real terms as follows:

$$\dot{a}_j(t) + \dot{m}_j(t) = r_j(t)a_j(t) + i_j(t)b_j(t) - \pi_j(t)m_j(t) + w_j(t) + \tau_j(t) - c_j(t), \tag{2}$$

with a dot over a variable denoting time differentiation. The left-hand side of (2) captures adjustments in the real value of financial assets $a_j(t)$ and money $m_j(t)$ held by the household. On the righthand side, $r_j(t)$ is the real interest rate earned on financial assets and $i_j(t)$ is the nominal interest rate earned on the real value of currency lent to firms $b_j(t)$. The cost of holding real money balances is $\pi_j(t)m_j(t)$, where $\pi_j(t)$ denotes the inflation rate. The household supplies a single unit of labor earning the real wage rate $w_j(t)$. Lastly, $\tau_j(t)$ is a real lump-sum transfer between the government and the household that may be positive or negative.

Households set consumption, real money holding, and money lending to maximize lifetime utility (1) subject to the flow budget constraint (2) and a CIA lending constraint $b_j(t) \leq m_j(t)$. The CIA lending constraint binds in equilibrium, and the optimal consumption path then satisfies the following Euler condition:

$$\frac{\dot{c}_j(t)}{c_j(t)} = r_j(t) - \rho. \tag{3}$$

Although we assume that national money markets are segmented, the international financial market features perfect mobility for financial capital, ensuring a common evolution for home and foreign consumption, $\dot{c}_h(t)/c_h(t) = \dot{c}_f(t)/c_f(t) = r(t) - \rho$, with the real interest rate equalized across countries: $r_h(t) = r_f(t) = r(t)$. Naturally, household lending behavior satisfies the Fisher identity: $i_j(t) \equiv r(t) + \pi_j(t)$.

At each moment in time, households allocate real expenditure across the mass of product varieties $N(t)$ available in the economy. The consumption composite and the corresponding price index have a constant elasticity of substitution formulation (Dixit and Stiglitz, 1977):

$$c_j(t) = \left(\int_0^{n_j(t)} q_{jj}(\omega,t)^{(\sigma-1)/\sigma} d\omega + \int_0^{n_k(t)} q_{kj}(\omega,t)^{(\sigma-1)/\sigma} d\omega \right)^{\sigma/(\sigma-1)}, \tag{4}$$

$$\bar{P}_j(t) = \left(\int_0^{n_j(t)} P_j(\omega,t)^{1-\sigma} d\omega + \int_0^{n_k(t)} \left(E_{jk} P_k(\omega,t)\right)^{1-\sigma} d\omega \right)^{1/(1-\sigma)}. \tag{5}$$

The masses of product varieties produced in home and foreign are measured by $n_h(t)$ and $n_f(t)$, with $N(t) \equiv n_h(t) + n_f(t)$. The quantity of product variety ω produced in country k and consumed in country j is denoted by $q_{kj}(\omega,t)$, with a nominal price of $P_k(\omega,t)$. We suppose there are no impediments to trade, indicating that the "law of one price" holds for each good and that the purchasing power parity nominal exchange rate is $E_{jk}(t) \equiv \bar{P}_j(t)/\bar{P}_k(t)$. The elasticity of substitution across product varieties is $\sigma > 1$.

We denote the real price of variety ω produced in country j by $p_j(\omega,t) \equiv P_j(\omega,t)/\bar{P}_j(t) = E_{kj}P_j(\omega,t)/\bar{P}_k(t)$. The household demands in country j for the representative product varieties produced in each country are then derived as

$$q_{jj}(\omega,t) = p_j(\omega,t)^{-\sigma}c_j(t), \qquad q_{kj}(\omega,t) = p_k(\omega,t)^{-\sigma}c_j(t). \tag{6}$$

2.2 Production

Firms produce differentiated product varieties for supply to home and foreign households, and compete monopolistically following Dixit and Stiglitz (1977). As each firm produces a single product variety, there is an exact correspondence between the number of firms and the number of product varieties produced in each country. There are no costs associated with market entry, but firms incur innovation costs with both fixed and variable components, at each moment in time.

The production technology of a representative firm ω in country j is

$$x_j(\omega,t) = \theta_j(\omega,t)^{\varepsilon} l_{Xj}(\omega,t), \tag{7}$$

where $x_j(\omega,t)$ is firm-level output, $\theta_j(\omega,t)$ is a firm-specific productivity coefficient, $l_{Xj}(\omega,t)$ is employment in production, and $\varepsilon \in (0,1)$ is the productivity elasticity of output. While each firm employs a specific production technology for its unique product variety, we assume that productivity is initially symmetric across firms within a given country: $\theta_j(\omega,0) = \theta_j(0)$.

The per-period real profit of a representative firm ω in country j is

$$\Pi_j(\omega,t) = p_j(\omega,t)x_j(\omega,t) - (1+\alpha_j i_j(t))w_j l_{Xj}(\omega,t) - (1+\beta_j i_j(t))w_j(l_{Rj}(\omega,t)+\zeta), \tag{8}$$

where $l_{Rj}(\omega,t)$ and $\zeta > 0$ are respectively the variable and fixed components of firm-level labor employed in process innovation. A key feature of our framework is that firms borrow money from households at the nominal interest rate (i_j) to cover a portion of the wage payments to labor employment in production and innovation. The fractions of wage payments requiring money in production and innovation are respectively $\alpha_j \in [0,1]$ and $\beta_j \in [0,1]$.

Given the current state of technology, each firm sets supply to meet the combined demands from home and foreign households. Thus, firm-level output is $x_j(\omega,t) = q_{jj}(\omega,t)L_j + q_{jk}(\omega,t)L_k$ in country j. Under monopolistic competition, the large mass of firms in the market eliminates strategic interaction between firms. Referencing the demand functions (6) and the production technology (7), firms set employment in production ($l_{Xj}(\omega,t)$) to maximize per-period profit (8), generating an optimal price that is equal to a constant markup over unit cost; that is, for a firm in country j, we have

$$p_j(\omega,t) = \frac{\sigma(1+\alpha_j i_j(t))w_j(t)}{(\sigma-1)\theta_j(\omega,t)^{\varepsilon}}. \tag{9}$$

In addition, substituting the pricing rule (9), the demand conditions (6), and the production function (7) back into firm-level output, $x_j(\omega,t) = q_{jj}(\omega,t)L_j + q_{jk}(\omega,t)L_k$, yields the following expression for optimal employment in production in country j:

$$l_{Xj}(\omega,t) = \frac{(\sigma-1)p_j(\omega,t)^{1-\sigma}(c_h(t)L_h + c_f(t)L_f)}{\sigma(1+\alpha_j i_j(t))w_j(t)}. \tag{10}$$

2.3 Process Innovation

At each moment in time, firms invest in process innovation with the aim of improving labor productivity in production ($\theta_j(\omega,t)$). Specifically, firms employ a variable quantity ($l_{Rj}(\omega,t)$) and a fixed quantity (ζ) of labor in process innovation each period. We assume that the fixed cost of innovation is symmetric across countries. Firm-level productivity evolves over time according to the following differential equation:

$$\dot{\theta}_j(\omega,t) = K_j(t)l_{Rj}(\omega,t)^{\gamma}, \tag{11}$$

where $K_j(t)$ is firm-level labor productivity in process innovation, and $\gamma \in (0,1)$ generates diminishing marginal products of labor in R&D investment. We suppose that firm-level productivity $\theta_j(\omega,t)$ measures the current stock of technical knowledge associated with the production process of firm ω in country j. Accordingly, process innovation both improves the productivity of labor in production and increases the stock of technical knowledge in the firm (Smulders and van de Klundert, 1995; Peretto, 1996).

We model labor productivity in process innovation as a weighted average of the technical knowledge current observable by a firm in country j:

$$K_j(t) = \frac{1}{N(t)} \left(\int_0^{n_j(t)} \theta_j(\omega,t) d\omega + \delta \int_0^{n_k(t)} \theta_k(\omega,t) d\omega \right), \tag{12}$$

where the technical knowledge of domestic firms has a stronger weighting given the greater difficulty associated with observing the production processes of firms operating in the foreign country (Baldwin and Forslid, 2000). Specifically, the parameter $\delta \in (0,1)$ regulates the degree of knowledge diffusion, with $\delta = 0$ implying that knowledge spillovers are completely local in scope and $\delta = 1$ indicating perfect knowledge diffusion. Under this specification, current R&D efforts increase the stock of technical knowledge, raising the productivity of labor in future innovation efforts and potentially generating endogenous productivity growth (Romer, 1990).

Firms set the the optimal level of investment in process innovation (l_{Rj}) with the objective of maximizing real firm value,

$$v_j(\omega,0) = \int_0^{\infty} \Pi_j(\omega,t) e^{-\int_0^t r(t')dt'} dt, \tag{13}$$

subject to the technology constraint (11). We solve this intertemporal optimization problem using a current-value Hamiltonian function: $H_j(\omega,t) = \Pi_j(\omega,t) + \mu_j(\omega,t) K_j(t) l_{Rj}(\omega,t)^{\gamma}$, where $\mu_j(\omega,t)$ is the current shadow value associated with an improvement in the production technology of firm ω in country j. Referencing the profit function (8), optimal firm-level employment in process innovation is obtained from the static efficiency condition, $\partial H_j(\omega,t)/\partial l_{Rj}(\omega,t) = 0$, and the dynamic efficiency condition, $\partial H_j(\omega,t)/\partial\theta_j(t) = r(\omega,t)\mu_j(\omega,t) - \dot{\mu}_j(\omega,t)$. Together these conditions yield a no-arbitrage condition for investment in process innovation in country j:

$$r(t) = \frac{\varepsilon\gamma K_j(t) l_{Xj}(\omega,t)}{\Gamma_j \theta_j(\omega,t) l_{Rj}(\omega,t)^{1-\gamma}} + \frac{(1-\gamma)\dot{l}_{Rj}(\omega,t)}{l_{Rj}(\omega,t)} + \frac{\dot{w}_j(t)}{w_j(t)} - \frac{\dot{K}_j(t)}{K_j(t)} + \frac{\beta_j \dot{i}_j(t)}{1+\beta_j i_j(t)}, \tag{14}$$

where the relative CIA constraint $\Gamma_j(t) \equiv (1+\beta_j i_j(t))/(1+\alpha_j i_j(t))$ captures the ratio of the CIA constraints for innovation and production. Under monopolistic competition, each firm has a small market share and therefore takes real expenditures (4), the price indices (5), and knowledge spillovers (12) as constant when considering the impact of changes in its production technology on firm value.

2.4 Monetary Authority

In each country, the monetary authority sets a policy target for the nominal interest rate. With perfectly segmented currency markets, the nominal money supply in country j is $M_j(t)$. Thus, the real money balance of a single household becomes $m_j(t) = M_j(t)/\bar{P}_j(t)L_j$, and growth in real money holdings is $\dot{m}_j(t)/m_j(t) = \dot{M}_j(t)/M_j(t) - \pi_j(t)$. With the real interest rate determined in the international market for financial assets, the Fisher identity $i_j(t) \equiv r(t) + \pi_j(t)$ implies that the monetary authority adopts the rate of money supply growth ($\dot{M}_j/M_j$) as a nominal anchor to achieve its policy target for the nominal interest rate. The seigniorage revenue generated by money supply growth is then returned to households in the form of a real lump-sum transfer (Chu and Cozzi, 2014):

$$\tau_j(t) = \dot{m}_j(t) + \pi_j(t) m_j(t). \tag{15}$$

With the monetary authority setting the rate of money supply growth to meet its exogenous policy target for the nominal interest rate, we have $\dot{i}_j(t) = 0$ at all moments in time. Moreover, combining real growth in money holdings and the Fisher identity with the Euler condition (3), we find that growth in real household consumption equals the rate of growth in real money holdings in country j; that is, $\dot{c}_j(t)/c_j(t) = \dot{m}_j(t)/m_j(t)$. Hence, we confirm that the monetary authority sets the nominal interest rate through its control over the rate of growth in the nominal money supply (i.e., $i_j = \rho + \dot{M}_j/M_j$).

2.5 Market Equilibrium

In this section, we examine the equilibrium conditions for the labor, financial asset and lending markets to derive real household expenditure as a function of the real wage rate in each country. In addition, we show that free market entry and exit in the product market determines national shares of industry. First, full employment in the labor market implies

$$L_j = n_j(t)(l_{Xj}(t) + l_{Rj}(t) + \zeta). \tag{16}$$

Next, free market entry and exit reduces firm value to zero. The time derivative of firm value (13) yields a no-arbitrage condition for market entry: $r(t)v_j(\omega, t) = \Pi(\omega, t) + \dot{v}_j(\omega, t)$. Then, with no costs incurred in market entry, firm value determines incentives for market entry and exit at each moment in time (Novshek and Sonnenchein, 1987). New firms enter when firm value is positive ($v_j(\omega, t) > 0$), reducing firm value. Alternatively, firms exit when firm value is negative ($v_j(\omega, t) < 0$), raising firm value. The entry and exit process immediately drives firm value to zero ($v_i(t) = 0$). Consequently, referencing the production function (7), per-period profit (8), and the pricing rule (9), we derive the free market entry condition in country j:

$$l_{Xj}(t) = (\sigma - 1)\Gamma_j(l_{Rj}(t) + \zeta). \tag{17}$$

This condition shows that the ratio of firm-level employment in production to process innovation depends on the relative CIA constraint $\Gamma_j \equiv (1 + \beta_j i_j)/(1 + \alpha_j i_j)$, with $\dot{\Gamma}_j = 0$ given that the nominal interest rate is an exogenous policy variable. For example, a rise in the money requirement for innovation increases Γ_j, thereby raising the production to innovation employment ratio ($l_{Xj}/(l_{Rj} + \zeta)$). In addition, as the real value of financial assets equals the real value of market capitalization $a_h(t)L_h + a_f(t)L_f = n_h(t)v_h(t) + n_f(t)v_f(t)$, we conclude that free market entry and exit drives the real value of financial assets to zero: $a_h(t) = a_f(t) = 0$.

Equilibrium in the labor and product markets leads to constant national shares of industry at each moment in time. Combining the labor market clearing condition (16) and the free market entry condition (17), we obtain country j's share of firms as

$$s_j \equiv \frac{n_j}{N} = \frac{L_j/L_k}{L_j/L_k + (1+(\sigma-1)\Gamma_j)/(1+(\sigma-1)\Gamma_k)}. \tag{18}$$

From this expression, we observe that country j's share of industry expands with an increase in market size (i.e., $ds_j/dL_j > 0$) or a decrease in the relative CIA constraint (i.e., $ds_j/d\Gamma_j < 0$).

Lastly, the total real money balances held by households matches the real cash-in-advance demand from production and innovation yielding the following market clearing condition:

$$b_j(t)L_j = n_j(t)w_j(t)(\alpha_j l_{Xj}(t) + \beta_j(l_{Rj}(t) + \zeta)).$$

We substitute this expression with the income transfer (15), the labor market clearing condition (16), and the free market entry condition (17) into the household flow budget constraint (2) to obtain real expenditure in country j as

$$c_j(t) = \frac{\sigma(1+\beta_j i_j)w_j(t)}{1+(\sigma-1)\Gamma_j}. \tag{19}$$

As the monetary authority sets the nominal interest rate as an exogenous policy variable, the time derivative of (19) indicates that real wages also grow at the same rate as real money holdings: $\dot{w}_j(t)/w_j(t) = \dot{c}_j(t)/c_j(t) = \dot{m}_j(t)/m_j(t)$ at all moments in time.

3 Long-run Equilibrium

In this section, we characterize a long-run equilibrium with constant intersectoral allocations of labor in each country, constant national shares of production ($\dot{s}_j = 0$), and a constant mass of firms ($\dot{N} = 0$). Defining the rate of productivity growth in country j as $g_j \equiv \dot{\theta}_j/\theta_j$, growth in real expenditure, growth in real wages, and growth in real money holdings all converge to a rate that is proportionate with the common rate of productivity growth across countries ($g = g_h = g_f$): $\dot{c}_j/c_j = \dot{w}_j/w_j = \dot{m}_j/m_j = \varepsilon g$. Hereafter, we suppress time notation (t) to simplify notation.

As home and foreign are linked through international knowledge spillovers and the terms of trade, the steady-state characterization of the economy depends closely on relative labor productivity, which we now formally define as $\tilde{\theta}_j \equiv \theta_j/\theta_k$. The evolution of the relative productivity of country j is regulated by the following differential equation:

$$\frac{\dot{\tilde{\theta}}_j}{\tilde{\theta}_j} = g_j - g_k = \frac{K_j l_{Rj}^{\gamma}}{\theta_j} - \frac{K_k l_{Rk}^{\gamma}}{\theta_k}, \tag{20}$$

where we have referenced the technology constraints (11). As discussed above, the rate of productivity growth equalizes across countries in the long run. Setting (20) equal to zero yields a steady-state condition for the determination of relative labor productivity:

$$\frac{K_h l_{Rh}^{\gamma}}{\theta_h} = \frac{K_f l_{Rf}^{\gamma}}{\theta_f}. \tag{21}$$

Referencing (18), there are constant masses of firms in each country ($\dot{s}_j = 0$), and taking the derivative of (12) with respect to time therefore demonstrates that growth in knowledge spillovers converges to the rate of productivity growth (i.e., $g = \dot{K}_h/K_h = \dot{K}_f/K_f$).

We now turn to the steady-state conditions for firm-level employment in process innovation. Rewriting the no-arbitrage condition for R&D investment (14), we have

$$\rho = R_j \equiv \left[\varepsilon\gamma(\sigma - 1)\left(1 + \frac{\zeta}{l_{Rj}}\right) - 1\right] \frac{K_j l_{Rj}^{\gamma}}{\theta_j}, \tag{22}$$

where we have referenced the Euler condition (3), the technology constraint (11), and the free market entry condition (17), and made use of the fact that $\dot{c}_j(t)/c_j(t) = \dot{w}_j(t)/w_j(t)$ from (19). The internal rate of return on investment in process innovation is denoted by R_j. Thus, invoking

(21), we find that home and foreign firms employ the same quantity of labor in process innovation: $l_R + \zeta$. It is then clear that $l_{Xh}/\Gamma_h = l_{Xf}/\Gamma_f$. As such, the relative firm-level scale of production across countries depends solely on the relative CIA constraints in the long run.

In Appendix A, we evaluate a linear expansion of the dynamic system around the long-run values for $\tilde{\theta}_h$, l_{Rh} and l_{Rf}, as determined by (21) and (22), in combination with the free market entry conditions (17). The following lemma summarizes the conditions required for a stable balanced growth path.

Lemma 1 *The long-run equilibrium is saddle-path stable for*

$$\frac{\partial R_j(l_{Rj})}{\partial l_{Rj}} = -\left[1 - \varepsilon\gamma(\sigma - 1) + \frac{\varepsilon(1-\gamma)(\sigma-1)\zeta}{l_{Rj}}\right]\frac{K_j\gamma l_{Rj}^{\gamma-1}}{\theta_j} < 0.$$

Proof: See Appendix A.

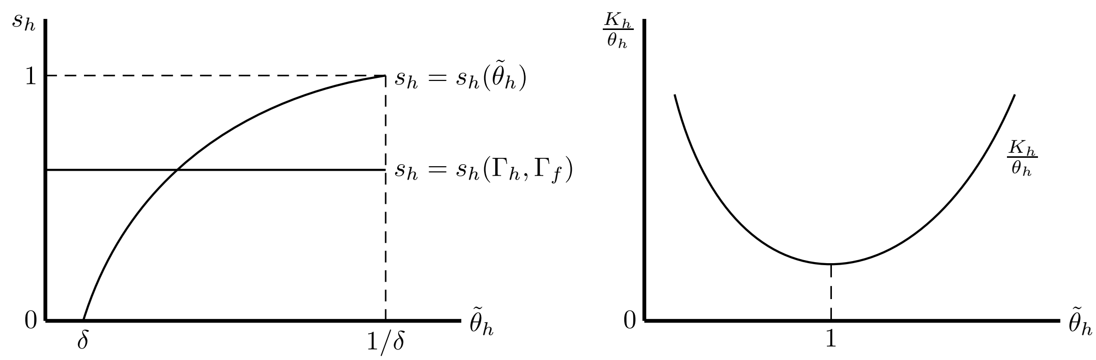
At each moment in time, firms set the optimal level of variable employment in R&D to maximize firm value, raising investment in process innovation when the internal rate of return ($R_j(l_{Rj})$) is greater than the discount rate (ρ), and reducing investment in process innovation when $R_j(l_{Rj}) < \rho$. This investment behavior requires diminishing marginal internal returns to employment in process innovation ($\partial R_j(l_{Rj})/\partial l_{Rj} < 0$) to ensure that the economy converges to a balanced growth path with positive and finite levels of variable employment in R&D. Henceforth, we assume that the necessary conditions for saddle-path stability hold as we investigate how nominal interest rate policy affects long-run product variety and productivity growth.

We solve for steady-state relative productivity ($\tilde{\theta}_j$) as a function of relative market size and the relative CIA constraints of each country using two conditions.

$$s_h = \frac{1 - \delta\tilde{\theta}_h^{-1}}{2 - \delta\tilde{\theta}_h - \delta\tilde{\theta}_h^{-1}} = \frac{L_h/L_f}{L_h/L_f + (1+(\sigma-1)\Gamma_h)/(1+(\sigma-1)\Gamma_f)}, \tag{23}$$

where $\Gamma_j \equiv (1+\beta_j i_j)/(1+\alpha_j i_j)$. First, the common scale of firm-level employment in innovation ($l_{Rh} = l_{Rf}$) indicates that productivity-adjusted knowledge spillovers equalize across

Figure 1: Productivity Differential and Knowledge Spillovers



(a) Home's Share of Firms

(b) Knowledge Spillovers

countries. Referencing (12), we solve $K_h/\theta_h = K_f/\theta_f$ for the home country's share of firms as a function of relative labor productivity: $s_h = s_h(\tilde{\theta}_h)$. As shown in Figure 1(a), the $s_h(\tilde{\theta}_h)$ curve has a strictly positive slope, with the feasible range for relative productivity determined by the degree of international knowledge diffusion: $\tilde{\theta}_h \in (\delta, 1/\delta)$. In addition, the country with the greater share of firms employs relatively advanced technologies: for $s_h \geq 1/2$ we have $\tilde{\theta}_h \geq 1$. A rise in the degree of knowledge diffusion dampens this result, with home and foreign productivity converging as δ approaches one: $d\tilde{\theta}_h/d\delta = (1 - \tilde{\theta}_h^2)/((1 - \delta\tilde{\theta}_h)\delta\tilde{\theta}_h^{-1} + (1 - \delta\tilde{\theta}_h^{-1})\delta\tilde{\theta}_h)$.

Second, the labor market clearing (16) and the free market entry (17) conditions determine the home country's share of firms as a function of the market sizes (L_h and L_f) and the relative CIA constraints (Γ_h and Γ_f): $s_h = s_h(\Gamma_h, \Gamma_f)$. This expression is depicted by the horizontal line in Figure 1(a). Thus, the intersection of the $s_h(\tilde{\theta}_h)$ curve and the $s_h(\Gamma_h, \Gamma_f)$ line implicitly determine the relative productivity of home firms, as depicted in Figure 1(a). Referencing (23), we find that an increase in country j's market size raises its share of firms ($ds_j/L_j > 0$) and its relative labor productivity ($d\tilde{\theta}_j/L_j > 0$), following the results of Davis and Hashimoto (2015). Similarly, the share of firms and relative productivity increase with a decrease in the relative CIA constraint ($ds_j/\Gamma_j < 0$ and $d\tilde{\theta}_j/\Gamma_j < 0$).

The steady-state value of productivity-adjusted knowledge spillovers plays a central role in the transmission of nominal interest rate policy to long-run product variety and productivity growth. We substitute $s_h = s_h(\tilde{\theta}_h)$ into observable knowledge (12) to obtain an expression for productivity-adjusted knowledge spillovers:

$$\frac{K_h}{\theta_h} = \frac{K_f}{\theta_f} = \frac{1-\delta^2}{2-\delta\tilde{\theta}_h - \delta\tilde{\theta}_h^{-1}}. \tag{24}$$

Long-run knowledge spillovers are directly linked with industry location patterns (s_j) as a consequence of imperfect international knowledge diffusion. As a result, productivity-adjusted knowledge spillovers strengthen as industry becomes more concentrated in a given country. For example, an increase in the home share of firms (s_h) raises the relative productivity of home firms ($\tilde{\theta}_h$), weakening knowledge spillovers for $s_h < 1/2$ and $\tilde{\theta}_h < 1$, and strengthening knowledge spillovers for $s_h > 1/2$ and $\tilde{\theta}_h > 1$. Therefore, as illustrated in Figure 1(b), the steady-state value of productivity-adjusted knowledge spillovers is convex in relative productivity with a minimum at $\tilde{\theta}_h = 1$. Productivity-adjusted knowledge spillovers are maximized when firms concentrate fully in either home ($s_h = 1$) or foreign ($s_h = 0$). This result is a common feature of new economic geography models that assume imperfect knowledge diffusion between countries (Baldwin and Martin, 2004).

The following lemma summarizes the relationship between nominal interest rates (i_h and i_f) and relative labor productivity ($\tilde{\theta}_j$):

Lemma 2 *In country j, an increase in the nominal interest rate (i_j) raises relative productivity ($\tilde{\theta}_j$) when production has a larger money requirement ($\alpha_j > \beta_j$), but lowers relative productivity when innovation has a larger money requirement ($\alpha_j < \beta_j$).*

Proof: Total differentiation of (23) gives

$$\frac{d\tilde{\theta}_j}{di_j} = -\frac{(\sigma-1)(1-s_j)(1-\delta\tilde{\theta}_j^{-1})}{(1+(\sigma-1)\Gamma_j)\delta(s_j + s_k\tilde{\theta}_j^{-2})}\frac{d\Gamma_j}{di_j},$$

with $1-\delta\tilde{\theta}_j^{-1} > 0$, $d\Gamma_j/di_j = (\beta_j - \alpha_j)/(1+\alpha_j i_j)^2$, and $d\tilde{\theta}_k/di_j = -\tilde{\theta}_k^2(d\tilde{\theta}_j/di_j)$.

After an increase in the nominal interest rate (i_j) of country j, the subsequent adjustment in the relative CIA constraint (Γ_j) depends on the balance of the money requirements for production (α_j) and process innovation (β_j). Consider an increase in the nominal interest rate of the home country (i_h). On the one hand, when $\alpha_h > \beta_h$, the relative CIA constraint ($d\Gamma_h/di_h < 0$) falls, lowering per-period innovation costs. The increase in per-period profit raises country j's share of firms as the $s_h = s_h(\Gamma_h, \Gamma_f)$ line shifts upwards in Figure 1(a). In response, the relative productivity of home firms rises until productivity-adjusted knowledge spillovers are once again equalized across countries ($d\tilde{\theta}_h/di_j > 0$). On the other hand, when $\alpha_h < \beta_h$, the relative CIA constraint increases ($d\Gamma_h/di_h > 0$), raising per-period innovation costs, lowering per-period profit, and decreasing the home country's share of firms. In this case, the $s_h = s_h(\Gamma_h, \Gamma_f)$ line shifts downwards, and the relative productivity of home firms falls ($d\tilde{\theta}_h/di_j < 0$).

With relative productivity determined as a function of market sizes and the relative CIA constraints, and productivity-adjusted knowledge spillovers determined as a function of relative productivity, it is now possible to examine how changes in nominal interest rates influence firm-level employment in process innovation ($l_R + \zeta$).

Lemma 3 *When production has a larger money requirement ($\alpha_j > \beta_j$), an increase in country j's nominal interest rate (i_j) lowers firm-level employment in process innovation (l_{Rj}) for $\tilde{\theta}_j < 1$ and raises it for $\tilde{\theta}_j > 1$. Alternatively, when innovation has a larger money requirement ($\alpha_j < \beta_j$), an increase in the nominal interest rate raises firm-level employment in process innovation for $\tilde{\theta}_j < 1$ and lowers it for $\tilde{\theta}_j > 1$.*

Proof: Substituting (24) into (22) and taking the total derivative with respect to l_{Rj} and $\tilde{\theta}_j$ yields

$$\frac{dl_{Rj}}{di_j} = -\frac{\rho\delta(1-\tilde{\theta}_j^{-2})(K_j/\theta_j)}{(1-\delta^2)(\partial R_j/\partial l_{Rj})}\frac{d\tilde{\theta}_j}{di_j}.$$

This derivative is signed using the results of Lemmas 1 and 2.

Nominal interest rates influence firm-level employment in innovation (l_{Rj}) indirectly through the link between relative productivity ($\tilde{\theta}_j$) and productivity-adjusted knowledge spillovers (K_j/θ_j).

Consider, for instance, the case where home has a larger share ($s_h > 1/2$) of more productive firms ($\tilde{\theta}_h > 1$). The effect of a rise in the nominal interest rate of home depends on the relative money requirements of production (α_h) and innovation (β_h). Following the results of Lemma 2, when $\alpha_h > \beta_h$ relative productivity increases ($d\tilde{\theta}_h/di_h > 0$), raising productivity-adjusted knowledge spillovers ($d(K_h/\theta_h)/di_h > 0$). As a result, the internal rate of return on process innovation increases ($dR_h/di_h > 0$) inducing firms to expand employment in process innovation ($dl_{Rj}/di_h > 0$). In contrast, when $\alpha_h < \beta_h$, a rise in i_h lowers both $\tilde{\theta}_h$ and K_h/θ_h, causing a decrease in the internal rate of return that reduces firm-level investment in process innovation ($dl_{Rj}/di_h < 0$). These results are reversed if home has a smaller share ($s_h < 1/2$) of less productive firms ($\theta_h < 1$), highlighting the role that national shares of industry play in the effects of nominal interest rate policy in our framework.

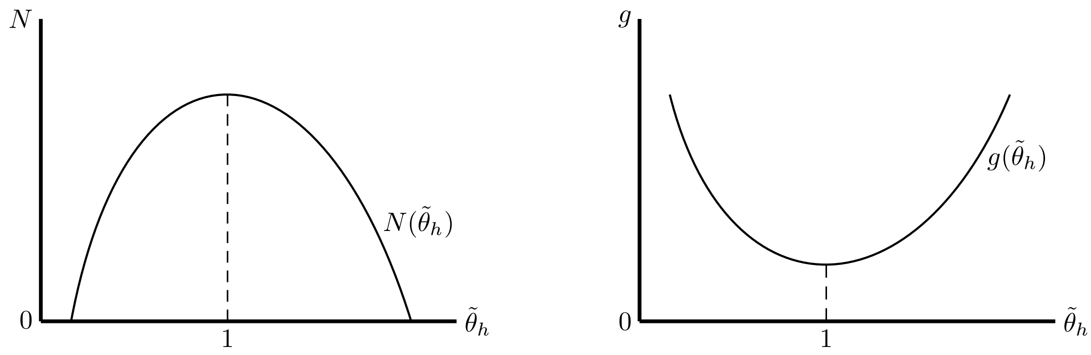
4 Product Variety and Productivity Growth

This section considers how nominal interest rates influence long-run product variety and productivity growth. Beginning with steady-state product variety, we combine the labor market clearing (16) and the free market entry (17) conditions with $l_R = l_{Rh} = l_{Rf}$ to obtain

$$N(\tilde{\theta}_h) = \frac{1}{(l_R(\tilde{\theta}_h) + \zeta)} \left(\frac{L_h}{1 + (\sigma - 1)\Gamma_h} + \frac{L_f}{1 + (\sigma - 1)\Gamma_f} \right), \tag{25}$$

where $\Gamma_j \equiv (1 + \beta_j i_j)/(1 + \alpha_j i_j)$. Figure 2(a) plots product variety (N) as a concave function of the relative productivity of home firms ($\tilde{\theta}_h$), with a maximum at $\tilde{\theta}_h = 1$. When, $\tilde{\theta}_h < 1$, an increase in $\tilde{\theta}_h$ lowers productivity-adjusted knowledge spillovers (K_j/θ_j), as shown in Figure 1(b), causing firms to reduce employment in innovation (l_R). The decrease in R&D costs raises per-period profit, inducing market entry. On the other hand, when $\tilde{\theta}_h > 1$, Figure 1(b) indicates that an increase in $\tilde{\theta}_h$ raises productivity-adjusted knowledge spillovers (K_j/θ_j), and firms therefore increase employment in innovation. In this case, the rise in per-period costs lowers per-period profit and firms exit the market.

Figure 2: Relative Productivity, Product Variety, and Productivity Growth



(a) Product Variety　　(b) Productivity Growth Rate

The concave relationship between product variety and relative productivity has important implications for the effects of adjustments in nominal interest rates, as outlined in the following proposition.

Proposition 1 *When production has a larger money requirement ($\alpha_j > \beta_j$), an increase in country j's nominal interest rate (i_j) raises product variety (N) for $\tilde{\theta}_j > 1$, but has an ambiguous effect on product variety for $\tilde{\theta}_j < 1$. Alternatively, when innovation has a larger money requirement ($\alpha_j < \beta_j$), an increase in the nominal interest rate lowers product variety for $\tilde{\theta}_j < 1$, but has an ambiguous effect on product variety for $\tilde{\theta}_j > 1$.*

Proof: Taking the total derivative of (25), we have

$$\frac{dN}{di_j} = -\frac{(\sigma-1)L_j}{(l_R+\zeta)(1+(\sigma-1)\Gamma_j)^2}\frac{d\Gamma_j}{di_j} - \frac{N}{(l_R+\zeta)}\frac{dl_R}{di_j},$$

where $d\Gamma_j/di_j = (\beta_j - \alpha_j)/(1+\alpha_j i_j)^2$. This derivative is signed using Lemma 3.

Changes in nominal interest rates affect product variety both directly through the relative CIA constraints and indirectly through adjustments in firm-level employment in process innovation. Consider, for example, an increase in the nominal interest rate of the home country (i_h). On the one hand, when production has a greater money requirement ($\alpha_h > \beta_h$), the relative CIA constraint decreases ($d\Gamma_j/di_j < 0$), reducing firm-level employment in production through the free market entry condition (17). This positive direct effect shifts the $N(\tilde{\theta}_h)$ curve upwards in Figure 2(a). The indirect effect exhibits as a rightward movement along the $N(\tilde{\theta}_h)$ curve, with firms adjusting optimal employment in process innovation. Thus, the direct and indirect effects align for $\tilde{\theta}_h < 1$, and product variety increases. But for $\tilde{\theta}_h > 1$, the opposing direct and indirect effects generate an ambiguous relationship between i_h and N. On the other hand, when innovation has a larger money requirement ($\alpha_h < \beta_h$), firm-level employment in production increases, with the $N(\tilde{\theta}_h)$ curve shifting downwards, and adjustments in optimal investment in innovation moving the economy leftward along the $N(\tilde{\theta}_h)$ curve. Therefore, the negative direct effect and indirect effect align for $\tilde{\theta}_h < 1$, with product variety decreasing, but are in opposition for $\tilde{\theta}_h > 1$, resulting in an ambiguous adjustment in $N(\tilde{\theta}_h)$.

Next, we derive steady-state productivity growth as an implicit function of relative productivity. Combining the innovation technology (11) and the no-arbitrage conditions (22) gives

$$g(\tilde{\theta}_h) = \frac{\rho}{\varepsilon\gamma(\sigma-1)\left(1+\zeta/l_R(\tilde{\theta}_h)\right)-1}. \tag{26}$$

This expression indicates that long-run productivity growth is scale neutral: proportionate changes in the market sizes of home and foreign (L_h and L_f) are fully absorbed by adjustments in product variety, leaving productivity growth unaffected. Figure 2(b) plots productivity growth as a convex function of the relative productivity of home firms ($\tilde{\theta}_h$). Following the results of Lemma 3, when $\tilde{\theta}_h < 1$, an increase in $\tilde{\theta}_h$ lowers productivity-adjusted knowledge spillovers (K_h/θ_h), reducing firm-level investment in process innovation ($dl_R/d\tilde{\theta}_h < 0$) and slowing growth ($dg/d\tilde{\theta}_h < 0$). But, when $\tilde{\theta}_h > 1$, productivity-adjusted knowledge spillovers rise, increasing firm-level employment in innovation ($dl_R/d\tilde{\theta}_h > 0$) and raising the rate of productivity growth ($dg/d\tilde{\theta}_h > 0$). Long-run productivity growth is minimized at $\tilde{\theta}_h = 1$ where home and foreign have equal shares of industry ($s_h = 1/2$). Thus, productivity growth is faster when industry concentrates in one of the two countries ($s_j > 1/2$).

We outline the relationship between nominal interest rates and long-run productivity growth in the following proposition:

Proposition 2 *When production has a larger money requirement ($\alpha_j > \beta_j$), an increase in country j's nominal interest rate (i_j) slows productivity growth (g) for $\tilde{\theta}_j < 1$ and raises productivity growth for $\tilde{\theta}_j > 1$. Alternatively, when innovation has a larger money requirement $\alpha_j < \beta_j$, an increase in the nominal interest rate raises productivity growth for $\tilde{\theta}_j < 1$ and slows productivity growth for $\tilde{\theta}_j > 1$.*

Proof: Taking the total derivative of (26) and applying the results of Lemma 3, we have

$$\frac{dg}{di_j} = \frac{\varepsilon\gamma(\sigma-1)\zeta g}{(\varepsilon\gamma(\sigma-1)(1+\zeta/l_R)-1)l_R^2}\frac{dl_R}{di_j}.$$

This derivative is signed using Lemma 3.

Adjustments in nominal interest rates influence the long-run rate of productivity growth indirectly through changes in firm-level employment in process innovation. Continuing with the example of an increase in the nominal interest rate of the home country (i_h). When production has a greater money requirement ($\alpha_h > \beta_h$), the $s_h = s_h(\Gamma_h, \Gamma_f)$ line shifts upwards in Figure 1(a), expanding the home share of firms and raising the relative productivity of home firms. As $\tilde{\theta}_h$ rises, the economy moves rightwards along the $g(\tilde{\theta}_h)$ curve in Figure 2(b). For $\tilde{\theta}_h < 1$, firm-level employment in process innovation decreases ($dl_R/di_j < 0$), slowing growth ($dg/di_j < 0$). For $\tilde{\theta}_h > 1$, however, firm-level employment in innovation increases ($dl_R/di_j > 0$), thereby raising productivity growth ($dg/di_j > 0$). In contrast, when $\alpha_h < \beta_h$, the $s_h = s_h(\Gamma_h, \Gamma_f)$ line shifts downwards in Figure 1(a). The home share of firms decreases, lowering the relative productivity of home firms. The economy now moves leftward along the $g(\tilde{\theta}_h)$ curve, with firm-level employment in innovation and productivity growth decreasing for $\tilde{\theta}_h > 1$ and increasing for $\tilde{\theta}_h < 1$. In either case, productivity growth is minimized when home and foreign have equal shares of firms and productivity is equalized across countries ($\tilde{\theta}_h = 1$).

5 Welfare Analysis

In this section, we derive the welfare of home and foreign households along the balanced growth path, and identify the channels through which adjustments in nominal interest rates influence steady-state welfare. As the complex nature of our framework makes a theoretical analysis of welfare intractable, we employ a simple numerical example to examine the optimal interest rates that maximize welfare in a non-cooperative Nash equilibrium.

Our derivation of steady-state welfare begins with the calculation of the terms of trade. Combining the pricing rules (9), optimal employment in production (10), and the labor market clearing conditions (17), we obtain the terms of trade for country j as

$$\frac{p_j}{p_k} = \left(\frac{\Gamma_k}{\tilde{\theta}_j^{\varepsilon} \Gamma_j} \right)^{1/\sigma}, \tag{27}$$

where we have used $l_R = l_{Rh} = l_{Rf}$. Next, we turn to the derivation of the real wage rate for each country. Substituting the pricing rules (9) and the terms of trade (27) into the price index (5) yields the real wage for a given value of labor productivity in country j:

$$w_j = \frac{(\sigma - 1)\theta_j^{\epsilon} N^{1/(\sigma-1)}}{\sigma(1 + \alpha_j i_j)} \left(s_j + s_k \left(\frac{p_j}{p_k} \right)^{\sigma-1} \right)^{1/(\sigma-1)}, \tag{28}$$

where we have used $p_j = P_j/\bar{P}_j$. Combining this expression with real consumption (19), we confirm that real wages and real consumption grow proportionately with productivity growth along the balanced growth path: $\dot{c}_j/c_j = \dot{w}_j/w_j = \varepsilon g$. Supposing that the economy converges to a balanced growth path at time $t = 0$, real consumption at time t becomes $c_j(t) = e^{\varepsilon g t} c_j(0)$, and the present value of utility flows to households in country j is $\rho U_j(0) = \ln c_j(0) + \varepsilon g/\rho$. Thus, we use (19), (27) and (28) to obtain the steady-state welfare of a household in country j as

$$\rho U_j(0) = \ln \left(\frac{(\sigma - 1)\theta_j(0)^{\varepsilon} \Gamma_j}{1 + (\sigma - 1)\Gamma_j} \right) + \ln \left(s_j + s_k \left(\frac{p_j}{p_k} \right)^{\sigma-1} \right)^{1/(\sigma-1)} + \ln N^{1/(\sigma-1)} + \frac{\varepsilon g}{\rho}, \tag{29}$$

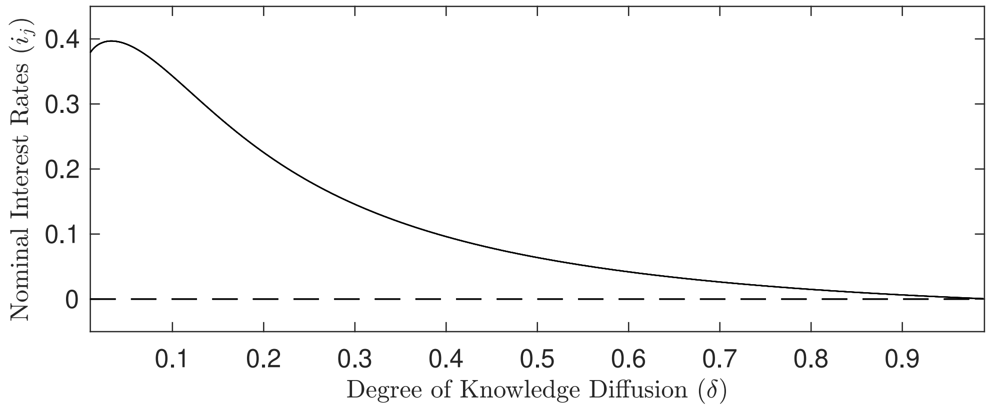
where we normalize the initial productivity of home firms to one ($\theta_h(0) = 1$) and the initial productivity of foreign firms becomes $\theta_f(0) = 1/\tilde{\theta}_h(0)$.

Adjustments in the nominal interest rates influence household welfare through five channels. The first term on the righthand side of (29) captures the effects of nominal interest rates on real income. The second term describes the effect of changes in national shares of industry and the terms of trade. The third term describes the love-of-variety effect. The fourth term shows the productivity growth effect. The opposing directions of the described channels renders a general analysis of the relationship between welfare and nominal interest rates intractable. As an alternative, we present a numerical example to examine how increases in the degree of knowledge diffusion (δ) affect the optimal nominal interest rate for each country. Specifically, we examine the non-cooperative Nash equilibria associated with the policy reaction functions of each country, while assuming a zero lower bound for the nominal interest rates.

We adopt the following parameter values. A value of ρ = 0.05 is assumed for the discount rate, referencing Acemoglu and Akcigit (2012). The elasticity of substitution is set to σ = 5, generating a cost price markup of $\sigma/(\sigma - 1) = 1.25$, matching evidence presented by Gali et al. (2007). The CIA parameters for production and innovation are set to α_h = α_f = 0.16 and β_h = β_f = 0.33, following Chu et al. (2015). We assume that the population of home is sufficiently large ($L_h = 14$ and $L_f = 10$) to ensure that home always has a greater share of firms ($s_h > 1/2$) that are relatively productive ($\tilde{\theta}_h > 1$). The productivity elasticity of output and the labor elasticity of productivity growth are ε = 0.2 and γ = 0.4. The per-period fixed cost is set to ζ = 0.01.

Figure 3 plots the optimal nominal interest rates associated with non-cooperative Nash equilibria over the range $\delta \in (0.01, 0.99)$ for the degree of international knowledge diffusion. Under the assumed parameter set, the smaller foreign country always sets its nominal interest rate at the zero lower bound. In contrast, the larger home country sets a strictly positive rate. For example, fixing the degree of knowledge diffusion to $\delta = 0.35$, the optimal nominal interest rates of home and foreign are $i_h = 0.118$ and $i_f = 0$. With these nominal interest rates, the home share of industry is $s_h = 0.541$ and relative productivity is $\tilde{\theta}_h = 1.161$. The long-run product variety

Figure 3: Optimal Nominal Interest Rates



These figures adopt the following parameters: α_h = α_f = 0.16, β_h = β_f = 0.33, γ = 0.4, ε = 5, σ = 1.25, ζ = 0.01, ρ = 0.05, $L_h = 14$, and $L_f = 10$. Plots for the foreign country are indicated with dashed lines.

and productivity growth are $N = 1088.121$ and $g = 0.025$. The welfare of home and foreign households are $U_h = 134.898$ and $U_f = 134.610$. Figure 3 shows that as international knowledge spillovers improve the optimal nominal interest rate of the home country also converges to the lower zero bound.

6 Conclusion

In this paper, we investigate how monetary policy affects aggregate product variety and long-run productivity growth in a two-country model of international trade. Monopolistically competitive firms supply differentiated products to the households of each country, and invest in process innovation to lower future production costs. The endogenous market structure of the framework creates a tension between market entry and productivity growth in the long run. Imperfect international knowledge diffusion links labor productivity in R&D with industry location patterns, ensuring that the country with the larger share of industry has relatively productive firms. As a result, knowledge spillovers between firms are greater when industry is relatively concentrated in one of the countries, and industry location patterns that feature asymmetric shares of industry across countries generate higher rates of productivity growth, but lower levels of product variety. In contrast, symmetric equilibria are characterized by a larger number of firms and slower productivity growth.

Monetary policy influences aggregate product variety and the rate of productivity growth through cash-in-advance (CIA) constraints that require firms to obtain short-term loans to secure money to cover a portion of the labor costs associated with production and process innovation. Thus, increases in nominal interest rates raise the cost employing labor. Free market entry then ties the firm-level ratio of production and innovation employment with the relative CIA constraints. On the one hand, when production has a larger money requirement, an increase in the nominal interest rate raises firm-level employment in innovation relative to production, expanding the country's share of firms and raising its relative productivity. On the other hand, when innovation has a larger money requirement, firm-level employment in innovation falls relative to production, reducing the country's share of firms and lowering its relative productivity.

Connecting the impact of nominal interest rates on CIA constraints with the influence of industry location patterns on the strength of knowledge spillovers yields interesting results for aggregate product variety and productivity growth. For example, focusing on the case where innovation has a larger money requirement, a decrease in the nominal interest rate of the country with the larger share of industry raises the concentration of industry, pushing the market away from the symmetric equilibrium. Knowledge spillovers are strengthened, leading to a decrease in the number of firms in the market and an increase in the rate of productivity growth. Alternatively, a decrease in the nominal interest rate of the country with the smaller share of industry lowers the concentration of industry as the market moves towards the symmetric equilibrium. In this case, knowledge spillovers are weakened, and the number of firms in the market increases as the rate of productivity growth falls.

Appendix: Stability of Symmetric Equilibrium

In this appendix, we show that $\partial R_j/\partial l_{Rj} < 0$ is a sufficient condition for the saddle-path stability of long-run equilibrium. First, we use the labor market clearing conditions (16) with the free market conditions (17) to obtain

$$s_j = \frac{(1 + (\sigma - 1)\Gamma_k(l_{Rk} + \zeta))L_j/L_k}{(1 + (\sigma - 1)\Gamma_j(l_{Rj} + \zeta)) + (1 + (\sigma - 1)\Gamma_k(l_{Rk} + \zeta))L_j/L_k}.$$

Next, referencing (12), we define productivity-adjusted knowledge spillovers as follows:

$$z_j(\tilde{\theta}_j, l_{Rj}, l_{Rk}) \equiv \frac{K_j}{\theta_j} = s_j(l_{Rj}, l_{Rk}) + \delta s_k(l_{Rj}, l_{Rk})\tilde{\theta}_k.$$

The dynamics of relative productivity (20) are then defined in terms of $\tilde{\theta}_h$, l_{Rh}, and l_{Rf}. Second, we rewrite the no-arbitrage conditions for investment in process innovation (14) as

$$\rho = R_j + \left(1 - \gamma + \frac{s_j s_k(1 - \delta\tilde{\theta}_k)l_{Rj}}{(l_{Rj} + \zeta)z_j}\right)\frac{\dot{l}_{Rj}}{l_{Rj}} - \frac{s_j s_k(1 - \delta\tilde{\theta}_k)l_{Rk}}{(l_{Rk} + \zeta)z_j}\frac{\dot{l}_{Rk}}{l_{Rk}} + \frac{\delta s_K\tilde{\theta}_k}{z_j}\frac{\dot{\tilde{\theta}}_j}{\tilde{\theta}_j}. \qquad \text{(A1)}$$

Using (A1), we solve the no-arbitrage conditions for the dynamics of employment in process innovation as follows:

$$\frac{\dot{l}_{Rj}}{l_{Rj}} = \left(1 - \gamma + \frac{s_j s_k(1 - \delta\tilde{\theta}_j)}{(1 + \zeta/l_{Rk})z_k}\right)\left(\rho - R_j - \frac{\delta s_k\tilde{\theta}_k}{z_j}\frac{\dot{\tilde{\theta}}_j}{\tilde{\theta}_j}\right)\frac{1}{(1 - \gamma)\Omega}$$
$$+ \frac{s_j s_k(1 - \delta\tilde{\theta}_k)}{(1 + \zeta/l_{Rk})z_j}\left(\rho - R_k + \frac{\delta s_j\tilde{\theta}_j}{z_k}\frac{\dot{\tilde{\theta}}_j}{\tilde{\theta}_j}\right)\frac{1}{(1 - \gamma)\Omega}, \qquad \text{(A2)}$$

where $\Omega = 1 - \gamma + s_h s_f(1 - \delta\tilde{\theta}_f)/((1 + \zeta/l_{Rh})z_h) + s_h s_f(1 - \delta\tilde{\theta}_h)/((1 + \zeta/l_{Rf})z_f)$. Together (20) and (A2) provide a system of three differential equations to describe the dynamics of $\tilde{\theta}_j$, l_{Rj} and l_{Rk}. Setting country j as the country with the larger share of more productivity firms ($s_j > 1/2$ and $\tilde{\theta}_j > 0$), we evaluate a linear expansion of the dynamic system around the steady state to obtain the following values for the leading principal minors of the Jacobian matrix J:

$$\frac{\partial \dot{\tilde{\theta}}_j}{\partial \tilde{\theta}_j} = -\delta(s_j\tilde{\theta}_j + s_k\tilde{\theta}_k)l_{Rj}^{\gamma} < 0,$$

$$\frac{\partial \dot{\tilde{\theta}}_j}{\partial \tilde{\theta}_j}\frac{\partial \dot{l}_{Rj}}{\partial l_{Rj}} - \frac{\partial \dot{\tilde{\theta}}_j}{\partial l_{Rj}}\frac{\partial \dot{l}_{Rj}}{\partial \tilde{\theta}_j} = \frac{((1-\gamma)s_j + \Omega s_k)(s_j\tilde{\theta}_j + s_k\tilde{\theta}_k)l_{Rj}^{1+\gamma}}{(1-\gamma)\Omega}\frac{\partial R_j}{\partial l_{Rj}} - \frac{\rho\delta\gamma(1-\gamma)s_k\tilde{\theta}_k l_{Rj}^{\gamma}}{(1-\gamma)\Omega} - \frac{(s_j^2\tilde{\theta}_j^2 - s_k^2)(\Omega^2 - (1-\gamma)^2)\rho l_{Rj}^{\gamma}}{(1-\gamma)\Omega\tilde{\theta}_j} < 0,$$

$$|J| = \frac{\delta\gamma(\sigma-1)\zeta(s_j\tilde{\theta}_j + s_k\tilde{\theta}_k)z_j l_{Rj}^{2\gamma}}{(1-\gamma)^2\Omega}\frac{\partial R_j}{\partial l_{Rj}} < 0.$$

Setting $\tilde{\theta}_j$ as a state variable, and l_{Rh} and l_{Rf} as control variables, saddle-path stability requires that the system have one negative characteristic root and two positive characteristic roots; that is, we require $|J| < 0$. This is the case when $\partial R_j/\partial l_{Rj} < 0$. Then, because we set country j as the country with a larger share of more productive firms ($s_j > 1/2$ and $\tilde{\theta}_j > 1$). The second leading principal minor is also negative if $\partial R_j/\partial l_{Rj} < 0$. If all three leading principal minors are negative, J is an indefinite matrix and does not have three character roots with the same sign (Chiang, 1984, 323-330). Thus, the system is saddle-path stable with one negative and two positive characteristic roots when $\partial R_j/\partial l_{Rj} < 0$.

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